A NORMATIVE ANALYSIS OF CAPITAL INCOME TAXES IN THE PRESENCE OF AGGREGATE RISK

By

Vidar Christiansen
University of Warwick and University of Oslo

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Abstract

A simple portfolio model is used to examine the efficiency effects of capital income taxes when the economy faces aggregate risk. To achieve a first best optimum the use of state contingent lump-sum taxes is required. Through the tax policy the riskiness of total consumption is partly assigned to the private consumption and partly to the public consumption. State independent income taxes may generate a misallocation of risk and distort the allocation of resources between assets. The second best optimum, representing a trade-off between these inefficiencies, is characterized. Uniform taxation is shown to be optimal only in very special cases. Finally, the second best optimality rule for public consumption is extended to the case of uncertainty.

1. Introduction

Most of the literature on capital income taxation and risk-taking has focused on the effects of taxes on portfolio and savings decisions.[1] Early modern articles are Mossin (1968), Stiglitz (1969) and Sandmo (1969). This means that most contributions have been positive theories of behaviour under uncertainty. There has been little concern with welfare considerations. However, there are discussions in the literature that reflect an implicit concern with welfare. In particular, it has been pointed out that there are discrepancies between private and social risk-taking, and there are suggestions that private incentives in the presence of taxes may not be compatible with social concerns. See for instance Atkinson and Stiglitz (1980, lecture 4). An explicit analysis of the choice of tax rates under uncertainty, when taxes are distortionary, is provided by Richter (1988). Recently Ahsan has analysed the choice of tax base under uncertainty (Ahsan (1989). An analysis of how to correct for incomplete markets by means of taxes when the imperfections are related to uncertainty (lack of insurance markets) is presented in Varian (1980). Even though the latter question has not received much attention in the public finance or public economics literature, there is an important body of literature on the performance of markets under uncertainty, which is of relevance, and which occasionally even derives some tax implications.

This literature raises a number of important and complex questions that may be worth mentioning even though an elaboration will be beyond the purpose of the present paper. Some of the main achievements have been to establish efficiency criteria (such as the constrained Pareto optimality criterion of Diamond (1967)) and to identify respective circumstances under which markets perform efficiently or fail to be efficient. The latter case may seem to provide a justification for a tax or subsidy intervention. However, some of the authors have forcefully argued that one should take great care not to jump to this conclusion unless

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1 Other dimensions of risk-taking have been dealt with in other parts of the tax literature, but will not be discussed here. For references, see Sandmo (1985).

it can be demonstrated that the government is less affected by or more able to cope with the causes of market failure. (In particular, see Stiglitz (1982) and Dixit (1986)). For instance if there would be a moral hazard problem in private insurance markets, problems of the same nature would be faced by a government considering a tax-transfer device to implement risk-sharing.

Even though there is in some respects a rather sophisticated literature that is of relevance, it seems that some of the basic questions in normative tax theory have not received proper attention in the presence of uncertainty. In particular, there is a need to clarify the meaning of such concepts as tax distortions and tax neutrality under uncertainty. And it is desirable to examine the trade-offs that have to be observed in the optimization of distortionary taxes under uncertainty. The emphasis will then be on the second best aspects of taxation rather than on the potential for improving on (incomplete) market allocations that would somehow be inefficient in the absence of taxes.

The possible role of taxes depends heavily on the nature of the uncertainty. If the risk is purely individual, some people have good luck and some have bad luck, while the average outcome is fixed, the possible insurance motive for tax-transfer devices becomes essential. If there is pure aggregate risk, everybody sharing the same luck or misfortune, the role of taxes as a distortionary source of government finance will be at the heart of the analysis.

The primary purpose of the present paper is to extend the analysis of tax induced distortions to the case of uncertainty. In order to isolate the public finance aspects from the concern with market failure, only aggregate risk will be considered. Only a single consumer will be considered. The consumer is conceived of as a representative agent of a population of homogeneous individuals.[2] Since there is only aggregate risk, the homogeneity

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2 At least in cases where public goods are considered the consumer is assumed to represent the total population (rather than an average individual) in the sense that the total benefit is perceived of as accruing to this consumer.

assumption is given the strict interpretation that individuals face the same risk ex ante as well as the same outcome ex post. Another implication of the approach is that distributional concerns are neglected. Only efficiency aspects are considered.

The basic model is presented in section 2 of the paper. Section 3 characterizes the first best optimum of the perfectly controlled economy, while a decentralized implementation by means of state contingent lump-sum taxes is discussed in section 4. A general discussion of the distinction between tax level and tax structure is provided in section 5. An important point is how to formulate the government budget constraint in the presence of uncertainty. The subsequent sections 6 through 9 examine the nature of the welfare effects and the second best optimality of state independent income taxes. Section 10 concludes the paper.

2. The Model

We shall establish a model which allows us to analyse some aspects of taxation and risk-taking. In addition to the arguments presented above two concerns govern our choice of model. Firstly, a simple model is wanted in order to isolate and bring out the uncertainty considerations. Secondly, it seems useful to stick to the modelling tradition of the positive analysis of taxation and risk-taking. Only one individual, or, equivalently, a representative individual of a population of homogeneous persons, is considered. The time horizon is limited to two periods. Desicions are taken in the first period. The uncertainty is associated with the state of the economy that will come into being in the second period.

A certain wealth (resource endowment) in the initial period is allocated between two assets. The assets may be interpreted as the financial assets of an external capital market or as inputs into two different linear production activities which yield an amount of output in the future period. One asset yields an uncertain return, while there is a safe return to the other asset. We shall now treat the initial wealth as exogenous. This assumption could be relaxed by introducing a trade-off between consumption and savings in the initial period. With exogenous initial wealth we are only concerned with consumption in the future period. The resources available in the future period after taking the return to the assets into account, the terminal wealth, can be used for private and publicly provided consumption[3]. The rate of transformation between the two is set equal to unity.

Let A denote the initial wealth of which an amount m is allocated to an asset that yields a sure rate of return, r, and an amount a is allocated to an asset yielding a random rate of return, x. The feasible allocations are then defined by the constraint

(1) $$A = a + m.$$

Using the notation c to denote private consumption and g to denote the amount of goods provided by the public sector, the following resource constraint defines the consumption opportunities:

(2) $$c = A + xa + rm - g$$

The consumption opportunities are stochastic as a reflection of the terminal wealth being stochastic. But the probability distribution of each consumption variable is not defined until an allocation rule for consumption between the private and public part has been specified. In a specific state, say state i, the consumption opportunities are expressed by

(3) $$c_i = A + x_i a + rm - g_i,$$

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3 Publicly provided consumption may be consumption of genuine public goods or public provision of private goods free of charge. In the present paper the term public consumption is for simplicity frequently used to cover either case.

where a quantity with subscript i denotes the value of a variable in state i.

Preferences are assumed to be compatible with the axioms of the Expected Utility Theorem. The von Neumann-Morgenstern utility function of the consumer is written as

(4) $u\,(c,g)$.

Letting single and double subscripts denote first and second derivatives, respectively, the properties attributed to the utility function can be expressed as

$$u_c > 0,\ u_g > 0,\ u_{cc} < 0,\ u_{gg} \leq 0.$$

Hence risk aversion is implied with the exception that $u_{gg} = 0$ will be considered as a special case.

The expected utility is

(5) $E = E(u) = \sum_i \pi_i\, u\,(c_i,g_i),$

where π_i is the probability of state i.

The basic features of the economy as described by the available resources, technology, uncertainty and preferences have then been presented. So far no tax system and institutional framework have been imposed. But the model we have established provides the tools required for characterizing the first best optimum defined as the optimum achieved in a perfectly controlled economy.

3. The first best optimum

The first best optimum has a long tradition as a useful benchmark in tax analysis in spite of its hypothetical nature, and we shall derive its characteristics within the present model. Also in the perfectly controlled economy the allocation of initial wealth between assets has to be determined in the initial period when there is uncertainty about the future state. The allocation between private and publicly provided consumption can however be chosen as the best possible allocation of the terminal wealth that actually emerges. Thus the consumption decision can be described as a state contingent decision.

The optimization problem can then be formulated as that of maximizing the expected utility with respect to a (resources allocated to the uncertain asset) and the public expenditure levels in all states. The amount allocated to the safe asset and the private consumption levels then follow from the equations (1) and (3). Let E denote the expectations operator. We derive the first order conditions

$$(6)\qquad \frac{\partial E(u(A+rA+(x-r)a-g,g))}{\partial a} = E(u_c(x-r)) = 0$$

and

$$(7)\qquad \frac{\partial E}{\partial g_i} = \pi_i(\partial u/\partial g_i-\partial u/\partial c_i) = 0 \qquad \text{for all i.}$$

The latter set of conditions simply implies that

$$(7')\qquad \partial u/\partial g_i = \partial u/\partial c_i.$$

The marginal utility of private consumption is equal to the marginal utility of public consumption in each state. Or, in other words, the marginal rate of substitution is equal to the marginal rate of transformation in each state. This is the classical optimal allocation rule.

The optimality condition (6) can be rewritten as

$$(6')\qquad E(u_c x) = \bar{u}_c r,$$

saying that the risk-adjusted expected marginal utility of the return to a is equal to the expected marginal utility of the return to m, where the bar indicates an expected value. This condition also implies that

$$(6'')\qquad \bar{x} = r + (-(\text{cov}(u_c,x))/\bar{u}_c).$$

The expected rate of return to a should be equal to the safe rate of return plus a risk premium measured by the absolute value of the relative covariance measure on the right hand side.

A special case may help to shed light on the marginal trade-offs at the optimum. Let us consider the class of utility functions

(8) $u = v(c) + kg,$

where v is an increasing, strictly concave function and k is a constant.

The marginal utility of publicly provided consumption is constant. Then it follows immediately from (7') that the private consumption must be adjusted in such a way that the marginal utility of private consumption also becomes the same in every state. This implies that the amount of private consumption must be the same in all states. Only publicly provided consumption varies across states. This result is easily explained. The constant marginal utility implies that the consumer is risk neutral with respect to public consumption. In other words the consumer is not bothered by variability in the consumption of publicly provided goods, while he has an aversion to variability in private consumption. The optimal response is obviously to shift all the risk, i.e. all the variability in consumption, to the publicly provided part of it.

Having established the first best conditions we are in a position to define different kinds of distortions as deviations from the first best allocation. But rather than discussing distortions at the general level it seems advantageous to carry out that discussion within the framework of a specified second best model in a later section of the paper.

4. Decentralization under lump sum taxation

It is easy to verify that the first best optimum is achieved by private, decentralized optimization if state contingent lump-sum taxes are imposed to finance the optimum levels of g. The expected utility to be maximized with respect to the choice of portfolio is

$$(9) \qquad E(u) = \Sigma \, \pi_i \, u(A+rA+(x_i-r)a-T_i, \, g_i),$$

where the lump-sum tax in state i, T_i, is set equal to the first best optimum value of public expenditure in that state, g_i. Maximizing expected utility with respect to a yields the first best condition (6). Thus the result that lump-sum taxes can be used to achieve a social first best optimum carries over to the case of uncertainty with the qualification that the taxes must also in general be state contingent.

In the special case where the marginal utility of publicly provided consumption is constant, first best private consumption is constant and only the government expenditure varies. The implication is that if the allocation is implemented by state contingent lump-sum taxes, a rise in income due to the emergence of a more favourable state is fully confiscated by the tax authorities. In this sense the marginal tax rate is 100 percent. But these are income changes beyond the control of the consumer. The marginal rate of taxation on income changes that the consumer can obtain through decisions of his own is of course zero by the definition of lump-sum taxes.

If only a state independent lump-sum tax is feasible, only private consumption becomes state contingent. All variability in consumption is assigned to the private part of it. The marginal utility of private consumption equals the marginal utility of public consumption only in expected terms at the tax optimum.

From the theory of taxation under certainty we are used to associate the effects of changes in lump-sum taxes with pure income effects. In the presence of uncertainty we have to notice that the income effects of changing lump-sum taxes depend on the states in which the taxes are changed. An exogenous loss of income in a specific state makes it desirable to change the portfolio so as to make up for some of the income loss if that state comes into being.

To establish a reference case it is useful to consider the allocation effects of arbitrary lump-sum taxes. Let us consider a tax reform whereby marginal changes in an arbitrary vector of lump-sum taxes are introduced. Let $dT_i = dg_i$ denote the marginal change in the lump-sum tax (= government expenditure) in state i. Ex ante we can define $dT = dg$ as the stochastic change in tax and expenditure. Let dE denote the corresponding change in expected utility. We find that

$$
\begin{aligned}
(10) \qquad dE &= E(u_g\, dg) - E(u_c\, dT) \\
&= E(u_g - u_c)\, dg \\
&= (\bar{u}_g - \bar{u}_c)\, d\bar{g} + \mathrm{cov}\,(u_g - u_c, dg) \\
&= (\bar{u}_g - \bar{u}_c)\, d\bar{g} + [\mathrm{cov}\,(u_g, dg) - \mathrm{cov}\,(u_c, dg)],
\end{aligned}
$$

where a bar denotes expected values.

By the definition of an optimum the effects on expected utility is obviously zero if the initial lump-sum taxes are optimal. If not a tax reform will have welfare effects. In formula (10) the effect on expected utility is decomposed into the effect of the reallocation between expected private and public consumption (the first term on the right hand side) and the effects of changes in the variability of public and private consumption, respectively (the term in square brackets). The former effect is similar to the effects of changing the lump-sum tax under certainty, while the latter effect is due to the state specific taxes of the uncertainty case. Further details will be discussed when comparing with non-lump-sum taxes in a later section.

5. Tax level and tax structure

Before we embark on the analysis of distortionary taxes it is useful to discuss a general problem.

The optimum tax policy is the choice of tax parameters that maximizes the expected utility. In the optimum tax literature under certainty it is common to distinguish between the choice of tax level and the design of tax structure. Optimum tax studies are usually concerned with the latter and characterize the optimum tax structure for a fixed total tax revenue requirement. Under uncertainty the tax revenue is random, and a fixed tax revenue has no meaning without a further interpretation.

One possibility would be to derive the optimal tax structure for predetermined levels of public expenditure in all states. But since there are several states, the predetermination of expenditure levels imposes a number of constraints as opposed to the single budget constraint in the absence of uncertainty. Unless the number of tax parameters is large enough, there may be no degress of freedom left for the optimization, or one may not even be able to choose independent expenditure levels for all states. And even if there are enough degrees of freedom, one may only be able to characterize relatively few properties of the optimal tax system when the analysis is constrained by predetermined revenue requirements in all states.

A natural and less restrictive way to obtain a distinction between the tax level and the tax structure is to define the tax level as the expected tax revenue. The optimal tax structure can then be analysed for a predetermined expected tax revenue. The case for this procedure is particularly strong if the marginal utility of publicly provided goods is constant (= k). Then the utility function belongs to the class introduced by formula (8) above, $u(c,g) = v(c) + kg$, and the expected utility is

$$E(u) = E(v) + kE(g). \quad (11)$$

This functional form implies that the consumer is risk neutral with respect to government expenditure. It is only the expected government expenditure that matters, and the policy optimization is naturally separated into the design of the tax structure for a fixed expected revenue (= government expenditure) and the choice of expected tax revenue (the tax level). This is the case studied by Richter (1988).

In general the government expenditure does not enter the utility function only through its expected value. Nevertheless it is possible to derive the optimum tax policy corresponding to a fixed tax level defined as a predetermined <u>expected</u> tax revenue. But the separation between the concern with the tax structure and the concern with government expenditure is not complete since varying tax rates subject to expected proceeds being fixed, will in general affect the variability of government expenditure in a way <u>which has welfare implications</u>. This was not the case with the utility function (8) which reflects indifference to the variability of g as long as the mean value is preserved.

Since in general the tax structure is not easily separated from the tax level(s) under uncertainty the more convenient approach may be to carry out a simultaneous optimization of tax rates and expenditure levels. But the final choice of approach should depend on the actual specification of the model (the tax system, preferences, etc.) in each case.

<u>6. State independent income taxes</u>

We shall now analyse distortionary taxes in the presence of uncertainty. The feasible tax policy is assumed to be such that differential taxation of capital income from the two asssets is possible, while state contingent taxation is not. Let t_x be the tax rate applied to ax, while t_r is the tax rate on rm. Perfect loss offset provision is assumed, implying that a grant is received if x is negative. The tax rates are fixed preannounced parameters. Let g be the tax revenue which equals some kind of government expenditure.

$$(12) \qquad g = t_r\ rm + t_x\ xa$$
$$= t_r\ rA + (t_x x - t_r r)a.$$

The private consumption (net terminal wealth) is then

$$(13) \qquad c = (1-t_r)rm + (1-t_x)xa + A$$
$$= (1+(1-t_r)r)A + ((1-t_x)x - (1-t_r)r)a.$$

The variables c and g are both stochastic variables. The representative consumer (investor) is assumed to maximize the expected utility

$$(14) \qquad E = E(u(c,g)),$$

taking the tax policy and the government expenditure as given. The maximizing behaviour is characterized by the first order condition

$$(15) \qquad dE/da = E(u_c \cdot (x(1-t_x)-r(1-t_r)) = 0,$$

where u_c is the marginal utility of private consumption. We can rewrite the condition as

$$(16) \qquad E(u_c(x-r)) - E(u_c(t_x x-t_r r)) = 0.$$

The second term can be interpreted as the risk-adjusted expected marginal tax on a. As usual a marginal tax rate causes a distortion. Comparing with (6) we see that the first best optimum choice of portfolio is only satisfied if the marginal tax on a is zero. If not there is a <u>portfolio distortion</u>. The risk-adjusted marginal tax can be written as

$$(17) \qquad E(u_c(t_x x-t_r r)) = E(u_c s(x-r))$$

where $$s = \frac{t_x x-t_r r}{x-r}.$$

We can interpret s as the marginal net tax on the return to the a-asset. It is the tax on the excess return of this asset over the other. In general s is a stochastic tax rate. It depends on the state of the economy[4]. (We could make this explicit by writing $s = s(x)$.) Making use of s we can write the first order condition of the individual choice of portfolio as

$$(18) \qquad E(u_c(1-s)(x-r)) = 0$$

We easily see that the first best condition is normally only satisfied if s is constant which in turn requires that

$$(19) \qquad t_x = t_r = t.$$

Uniform capital income taxation is equivalent to constant net taxation which in turn is equivalent to a zero marginal tax on changes in the portfolio composition[5]. Hence this taxation is non-distortive.

If uniform proportional income taxation yields the desired public provision of goods in all states, there are no distortions. But this is an unlikely event. In general one cannot expect the first best conditions for optimal public consumption (7) to be satisfied even if a portfolio distortion is permitted, unless a sufficiently large number of tax instruments is available. In general one must expect a portfolio distortion as well as a misallocation between private and public consumption. Both these effects of the tax policy must be allowed for in policy considerations.

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4 This net tax concept is slightly more general than the net tax concept usually adopted in the literature on taxation and risk-taking. The latter is defined as a constant rate of tax on the excess return. See for instance Sandmo (1985).

5 The taxes are only equivalent as far as marginal effects are concerned. In the case of income taxation the tax revenue can be expressed as $t_r rA + s(x-r)a$. Hence the income tax can be interpreted as a mixture of a lump sum tax (on A) and a net tax, of which only the latter has a marginal effect.

Before discussing the allocation problems in more detail it is useful to take a closer look at the allocation effects of taxes. Let us consider a marginal tax reform whereby the tax rates are slightly altered. Let dE, dt_r, dt_x, da and dg denote the marginal changes in the respective variables. The first order change in expected utility is then

$$
\begin{aligned}
(20)\qquad dE &= E(u_c(-m\ r\ dt_r - a\ x\ dt_x) \\
&\quad + E(u_g(m\ r\ dt_r + a\ x\ dt_x + (t_x x - t_r r)da)) \\
&\quad + E(u_c(t_x x - t_r r)da) - E(u_c(t_x x - t_r r)da) \\
&= E((u_g - u_c)dg + E(u_c(t_x x - t_r r)da) \\
&= (\bar{u}_g - \bar{u}_c)\ d\bar{g}\ + \mathrm{cov}\ (u_g - u_c,\ dg) + E(u_c(t_x x - t_r r)da \\
&= (\bar{u}_g - \bar{u}_c)\ d\bar{g} + [\mathrm{cov}\ (u_g, dg) - \mathrm{cov}\ (u_c, dg)] \\
&\quad + E(u_c(t_x x - t_r r))da,
\end{aligned}
$$

where a bar is used to indicate expected values.

The effect on expected utility has been decomposed into three terms, counting the expression in square brackets as one. In general a change in tax policy may change the expected tax revenue and government expenditure. The net marginal social benefit is equal to the marginal valuation of public expenditure ($\bar{u}_g$) minus the marginal valuation of the private consumption foregone ($\bar{u}_c$). This net effect is expressed by the first term. The same kind of effect would occur in the absence of uncertainty.

Under uncertainty a change in the tax policy will change the allocation between private and government consumption in all states. This may happen to a different extent in different states. Even the direction of the reallocation may differ between states. The effect is to shift risk betweeen the private and the public consumption. The net effect on the variability of u and hence on the risk burden of the consumer is measured by the difference between the covariances expressed within the square brackets. If this effect is positive (negative) we can say that the risk has been shifted to a more (less) efficient risk bearer, and the social risk-taking is reduced (increased). This effect is termed <u>the risk-shifting effect</u>.

Finally, the tax changes may induce a change in the portfolio which has welfare implications in the presence of a tax distortion. It is an advantage to have a reallocation of the portfolio in favour of the more highly taxed asset in which there is an underinvestment initially. The opposite reallocation would be socially harmful. We shall call this effect in either direction the <u>portfolio effect</u>. It is captured by the last term of (20).

From the first order condition (15) it follows that

$$(21) \qquad E(u_c x) = \bar{u}_c\, r(1-t_r)/(1-t_x),$$

where the bar is used to indicate an expected value. Making use of this result the expected risk-adjusted marginal tax can be expressed as

$$(22) \qquad E(u_c(t_x x - t_r r)) = (t_x - t_r) r\, \bar{u}_c/(1-t_x),$$

and we see that the sign of the marginal tax is equal to the sign of t_x-t_r. If the marginal tax rate is positive there is a distortion in disfavour of the risky asset and a tax reform which increases a implies that the social allocation of wealth is being improved.

Let us now make a comparison between the effects of the income tax reform under survey and the lump-sum tax reform considered in section 4. Comparing formula (10) and formula (20) we see that the effect on expected government expenditure and the risk-shifting effect are similar to the effects of reforming non-optimal lump-sum taxes. These effects are due to the failure to implement the (first best) desirable tax levels in all states. If a uniform income tax is imposed, these are the only effects since there is no portfolio distortion. <u>The uniform income tax can be characterized as equivalent to a constrained lump sum-tax</u>. It is equivalent to a lump-sum tax because the effective marginal tax rate on changes in portfolio composition is zero. But because of its special (linear) form, it can only be used to mimic a subset of lump-sum taxes. There is a constraint on the extent to which the levels can be tailored to the states of the economy.

A tax which closely resembles the uniform income tax, and which is very well known from the literature, is the net tax mentioned earlier in the paper. A couple of references to the literature are Sandmo (1985) and (1988). Formally, we can define a net tax in our model as a tax imposed at a rate t on the excess return to the risky asset, (x-r)a. Hence the tax paid is t(x-r)a. In the literature the tax is analysed without allowing for effects on behaviour of the changes in public expenditure that will accompany changes in the tax rate. To be able to adopt the well-known results from the literature let us now make the explicit assumption that there is no effect from public consumption on behaviour, presumably because of separability in the utility function.

With a constant net tax the first order condition for individual optimization is

(23) $$E(u_C(x-r)(1-t)) = 0.$$

Since t is a constant, the first best condition for optimal choice of portfolio (6) is obviously satisfied. There is no portfolio distortion.

If the tax rate is increased, it is well known that more is invested in the risky asset, while private consumption remains the same as before in all states. It follows that there is no change in private risk-taking, while public and social risk-taking increase. Since the expected value of x is greater than r, there is also an increase in expected tax revenue.

We should now be in a position to understand the welfare aspects of the effects of the net tax. We may note that the effect on the choice of portfolio is an income effect equivalent to the effect of corresponding changes in lump-sum taxes in the respective states. This change in portfolio may have a welfare effect. This is not because the choice of portfolio as such is distorted. It is not. The welfare effect emerges because the tax levels are imperfectly adjusted to the states of the economy. More or less desirable tax levels may be obtained. In general we cannot tell whether the total effect of changing the net tax rate is positive or negative since the presumably harmful effect of having more risk-taking may be (more than) offset by the possibly beneficial effect of a higher expected tax revenue.

The emphasis in the discussion above has been on the possible failure of the tax system to generate the desirable tax proceeds. This is very different from the focus of tax analysis under certainty. The difference in emphasis is clearly due to the different nature of government expenditure in the two cases. As was discussed in section 4, the analysis of the tax structure can be kept apart from the concern with the tax and expenditure level under certainty. Even if the tax level is not assumed to be optimal it makes little or no difference to the qualitative characterization of the tax structure. In the presence of uncertainty this is different because the tax revenues in all states matter. Therefore a more integrated analysis of tax structure and expenditure levels is normally required.

7. Optimum income taxation and evaluation of uniformity

To assess the total effect of a tax reform all the marginal effects expressed by (20) have to be taken into account. At the optimum any arbitrary marginal tax reform must yield effects that just cancel out. In particular, if we consider a tax reform that keeps the expected tax revenue and government expenditure unchanged the two last terms of (20) must cancel out. The risk-shifting effect and the portfolio effect must be offsetting.

If we also have the special case where the marginal utility of publicly provided consumption is constant, only the latter covariance in the square brackets matters. Then the risk-shifting effect is always positive if the tax is increased in a high consumption state and lowered in a low consumption state. At the optimum the corresponding portfolio effect has to be negative.

Let us now consider in more detail the optimum tax structure for a fixed expected tax revenue. Then the following constraint is imposed:

$$(24) \qquad E(t_r rm + t_x \, xa) = \bar{g},$$

where $\bar{g}$ is fixed. Let $\bar{x}$ denote the expected value of x. Let t_x be slightly increased and let dt_r/dt_x and da/dt_x be the corresponding changes in t_r and a which are consistent with private optimization and the constraint on expected tax revenue. It follows from (24) that

$$rm \; dt_r/dt_x + \bar{x}a + (t_x\bar{x} - t_r r) \; da/dt_x = 0$$

Solving for dt_r/dt_x, yields

$$(25) \qquad \frac{dt_r}{dt_x} = - \frac{a\bar{x} + (t_x\bar{x} - t_r r) \; da/dt_x}{mr}.$$

Moreover, it follows that under the expected revenue constraint the change in revenue as a function of x is

$$(26) \qquad dg/dt_x = (a + t_x \; da/dt_x) \; (x-\bar{x}).$$

If the first term on the right hand side is positive, the effect is to increase the variability of tax revenue (and government expenditure).

An interesting question is whether uniform taxation ($t_x = t_r$) can be optimal. To explore this question let us consider the relationship between c and g. We know that

$$(27) \qquad c = A + (1-t_r) \; rm + (1-t_x) \; ax$$

and

$$g = t_r \; rm + t_x \; ax.$$

Using the latter equation to substitute for x in the former, yields

$$(28) \qquad c = A + rm\,(t_x(1-t_r) - t_r(1-t_x))/t_x + (1-t_x)\,g/t_x$$

It follows that when taxation is uniform ($t_r = t_x = t$),

$$(29) \qquad c = A + (1-t)g/t$$

The first order condition for optimum taxation is that the first order change in expected utility for an arbitrary marginal tax reform is zero. We then see from (20) that uniform taxation will be optimal only if no marginal tax reform which preserves the expected tax revenue, can be devised in such a way that a positive risk-shifting effect is obtained.

A special case is the one in which the marginal utilities of private and public consumption are equal for every combination of c and g that may arise. This requires that the utility function is of a special quasi-homothetic kind which implies that marginal utilities are equal along a curve

$$(30) \qquad c = A + bg,$$

where A is the initial resource endowment, and b is a constant. When the tax rate is adjusted in such a way that $(1-t)/t = b$, (30) and (31) will coincide, and marginal utilities are equal for all c, g - combinations that can materialize.

To sum up, we have the following <u>result</u>:

If the utility function belongs to the quasi-homothetic class which implies that the marginal utilities of private and public consumption are equal along the ray $c = A + bg$, and there is a uniform tax rate t, such that $(1-t)/t = b$, no welfare - improving marginal tax reform can be devised.

This is obviously a very special case. An assumption which may seem more reasonable is the

Monotonicity assumption: For a uniform tax rate $u_g - u_c$ is monotonically increasing or decreasing as a function of x.

As the income increases and a constant marginal share is allocated to private and public consumption, respectively, the net marginal valuation of public consumption increases or decreases. This is obviously true in the special case where the marginal utility of public consumption is constant.

Let us also assume that for a tax reform preserving expected revenue (as defined by (24)-(26))

(31) $$a + t_x \, da/dt_x \neq 0.$$

From (26) we have that

$$dg = (a + t_x \, da/dt_x)\ (x-\bar{x})\ dt_x,$$

and $d\bar{g} = 0$. Then we can always find a small change in t_x such that dg is made an increasing or decreasing function of x. It follows that dg can be made positively correlated to $u_g - u_c$, and expected utility can be increased by implementing this tax reform. Hence uniform taxation will not be optimal. Strictly speaking it is sufficient to get this result that the monotonicity assumption and assumption (31) hold for the uniform tax rate which generates the optimum level of expected revenue. Thus we have demonstrated the result:

Under the monotonicity assumption and assumption (31) a uniform tax rate is second best inefficient.

If the monotonicity assumption is not satisfied uniform taxation may happen to be optimal. We may also note that if (31) is not satisfied, it follows from (26) that dg = 0, and (20) and (22) imply that the optimum tax rates are uniform. But uniform taxation being optimal is clearly a special case that has no particular claim for policy attention.

I shall now consider the case of constant marginal utility of public consumption. The focus is on the optimal tax structure for a predetermined expected tax revenue. This is the case analysed in Richter (1988) except for the difference that Richter considered a risky investment project that might yield a decreasing return to scale while in the current analysis a linear technology is assumed. For the purpose of studying the role of uncertainty this is a minor difference.

The following first order optimality condition is easily derived.

(32) $$dE/dt_x = E(u_c(-rm\ dt_r/dt_x - ax)) = 0,$$

where dt_r/dt_x is the change in t_r which preserves the expected tax revenue when t_x is increased. The first order condition of the individual optimization according to formula (21) implies that

$$E(u_c x) = \bar{u}_c(1-t_r)r/(1-t_x).$$

Making use of this result and (25) in (32) we get

(33) $$dE/dt_x = \bar{u}_c(a\bar{x}+(t_x\bar{x}-t_r r)da/dt_x-a(1-t_r)r/(1-t_x)) = 0,$$

which implies that at the optimum

(34) $$\hat{a} = -(1-t_x)(da/dt_x)/a = ((1-t_x)\bar{x}-(1-t_r)r)/(t_x\bar{x}-t_r r),$$

where $\hat{a}$ is defined by the first equation. It is the elasticity of a with respect to $(1-t_x)$ when t_x and t_r are changed simultaneously.

Then the following inverse elasticity rule, equivalent to that of Richter, is obtained:

(35) $$\frac{t_x\bar{x} - t_r r}{(1-t_x)\bar{x} - (1-t_r)r} = \frac{1}{\hat{a}}.$$

The right hand side is the inverse of the elasticity of a with respect to $(1-t_x)$. We then turn to the left hand side. The numerator is the expected marginal net tax on a. In other words it is the expected net increase in tax liability due to a marginal increase in the risky investment. The denominator has a similar interpretation as the expected marginal net return to the risky asset. Hence the left hand side may be interpreted as a relative marginal tax rate which is inversely related to the elasticity of the risky investment at the optimum. This interpretation is formulated in somewhat different terms from that of Richter. The formula is similar to other inverse elasticity formulae in the optimum tax literature. But I would like to argue that the similarity is of a rather formal nature. The effects that are involved are of a very different kind from those of a Ramsey type model of commodity taxation and similar models. In that model the distortion of one price has to be set against the distortion of some other price. But in the present model the distortion between two assets has to be set against the shifting of risk between private and publicly provided consumption.

Let us then consider the general case. We have that

$$(36) \qquad dE/dt_x = E(u_c(-mr\, dt_r/dt_x - ax)) + E(u_g\, dg/dt_x))$$

The first term on the right hand side is the same as before. Hence we can make use of (32) and (33) to get the first order condition

$$(37) \qquad dE/dt_x = \bar{u}_c(a\bar{x}+(t_x\bar{x}-t_r r)da/dt_x - a(1-t_r)r/(1-t_x)) + E(u_g(\bar{x}-x)(a+t_x da/dt_x)) = 0,$$

where (26) has been inserted. Some simple manipulations yield

$$\bar{x}(1-t_x) - r(1-t_r) + (t_x\bar{x}-t_r r)(1-t_x)(da/dt_x)/a + \text{cov}\ (u_g,x)(1-t_x)/\bar{u}_c + \text{cov}\ (u_g,x)t_x(1-t_x)(da/dt_x)/a\bar{u}_c = 0$$

Moreover we get,

$$\bar{x}(1-t_x)(1+\frac{\text{cov}\ (u_g,x)}{\bar{x}\ \bar{u}_c}) - r(1-t_r)$$

$$- \hat{a}[t_x\bar{x}\ (1+\frac{\text{cov}\ (u_g,x)}{\bar{x}\ \bar{u}_c}) - t_r r] = 0,$$

which implies that

$$\hat{a} = \frac{\bar{x}(1-t_x)(1+\frac{\text{cov}\ (u_g,x)}{\bar{x}\ \bar{u}_c}) - r(1-t_r)}{t_x\bar{x}\ (1+\frac{\text{cov}\ (u_g,x)}{\bar{x}\ \bar{u}_c}) - t_r r}$$

or

$$\text{(38)} \qquad \frac{t_x\bar{x}(1+\frac{\text{cov}\ (u_g,x)}{\bar{x}\ \bar{u}_c}) - t_r r)}{\bar{x}(1-t_x)(1+\frac{\text{cov}\ (u_g,x)}{\bar{x}\ \bar{u}_c} - r(1-t_r)} = \frac{1}{\hat{a}}\ .$$

The interpretation of this formula is analogous to that of (35) except that the expected tax on x in the numerator and the expected x in the dominator have been replaced by risk-adjusted expectations. Normally the covariances are assumed to be negative and the expected values of x are given less weight due to the risk adjustment. The left hand side may be interpreted as the ratio of the risk-adjusted expected marginal tax rate to the risk-adjusted marginal expected return.

9. The trade-off between private and public consumption

Optimality rules for public provision of goods have been derived in detail in models of deterministic economies. It is well known that if non-distortive sources of finance are available the public sector should push its provision of goods to the point where the marginal rate of substitution between publicly provided goods and private goods is equated to the marginal rate of tranformation[6]. Or, in other words, the marginal valuation is equal to the marginal cost. When distortionary taxation is the source of public funds, the optimality rule is of the following form:

(38) MRS = MRT + marginal cost of taxation.

(See for instance Atkinson and Stiglitz (1980, lecture 16)). When taxes are distortionary, one must take into account that the marginal cost of increasing the public sector supply of goods is not only the cost of diverting resources from the private sector but also the cost (deadweight loss, excess burden) of further distorting the economy.

We can now establish a similar optimality rule within the framework of our stochastic economic environment. Setting the first order change in expected utility equal to zero in formula (20) the following condition is derived

$$(39) \qquad \frac{E(u_q dg)}{E(u_c dg)} = 1 - E(u_c(t_x x - t_r r))da/E(u_c dg)$$

Both the numerator and the denominator on the left hand side are assumed to be positive, implying that we consider a use of resources which has a positive impact on welfare whether it is allocated to the public or private sector. Since the marginal rate of tranformation has been set equal to unity, this is an optimality condition of the same form as (38). The only difference is that due to the uncertainty the various terms are expected values. The last term can be interpreted as the expected marginal cost of taxation that goes along with an increase in public provision of goods. This effect is positive if a further distortion of the portfolio allocation is induced. The left hand side is the marginal rate of substitution between public and private consumption in expectation terms, or, we may say the

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6 If the good provided by the public sector is actually a public good the marginal rate of substitution for the population as a whole is of course the sum of individual rates of substitution.

risk-adjusted marginal rate of substitution. Applying the decomposition of formula (20), and setting $d\bar{g} = 1$, this term can be expressed as

$$\frac{E(u_g dg)}{E(u_c dg)} = \frac{\bar{u}_g(1+ \text{cov } (u_g,dg)/\bar{u}_g)}{\bar{u}_c(1+ \text{cov } (u_c,dg)/\bar{u}_c)},$$

where the relative covariance terms serve as risk-adjustment operators. If, for instance the adjustment term of the numerator is larger than that of the denominator it implies that the marginal valuation of (willingness to pay for) public consumption in terms of private consumption is greater than in the absence of these terms. The reason is that the reallocation from private to public consumption in this case reduces the risk (the variability of utility across states).

10. Conclusion

The purpose of this paper has been to display the welfare effects of taxes under uncertainty. The focus of the analysis has been on the fiscal role of taxes. The concern has been with taxes as a source of revenue for the public sector and the allocative distortions induced by these taxes. In this respect the present analysis is in contrast to analyses of tax-transfer devices as means of curing market imperfections, in particular due to incomplete markets. (In the case of uncertainty lack of insurance markets may be of particular relevance.) To isolate the aspects of concern only aggregate risk has been considered. A simple portfolio model has been judged as sufficient to show how tax considerations in principle are affected by uncertainty. A larger model would have increased the number of decisions and distortions under survey, but would not necessarily have improved our understanding of the special nature of tax considerations under uncertainty, "the grammar of arguments" in this case, to use the words of Hahn (1973). (This is not to dismiss the argument that a richer model may be required for other purposes).

The analysis that has been presented, captures three main aspects: the role of taxes in reallocating resources between (expected) private and (expected) publicly provided consumption, the effect of taxes in shifting risk between private and public consumption, and taxation as a source of allocative distortion. The trade-off between the expected use of resources in the two sectors is hardly any different from the corresponding trade-off under certainty. Tax wedges as distortionary elements are not much different either. One only has to allow for the fact that it is decisions under uncertainty that are being distorted. Thus the important difference is the role of taxes in assigning risk to the private or public part of consumption in the presence of uncertainty.

This aspect of taxation under uncertainty makes it difficult to separate sharply between the tax structure and the level of tax revenue as is usually done in the deterministic optimum tax models. Apart from special cases a more intergrated analysis is required.

It has been an important contribution of the conventional optimum tax theory to distinguish between income effects and substitution effects. The former are necessary consequences of the appropriation of private resources that would also be induced by lump-sum taxes. The latter represent the distortionary effects. Under uncertainty the appropriation of private resources may be inefficient across states. Hence income effects equivalent to the effects of raising lump-sum taxes in specific states, may have welfare implication as they change the appropriation of private income in different states towards or away from the efficient pattern. Thus the distinction between neutral income effects and distortionary substitution effects is not easily maintained when uncertainty is allowed for.

Under certainty the emphasis of efficiency considerations is on tax induced distortions, that for instance lead to too little investment in highly taxed assets. A misallocation between the private and public sector is usually seen as a consequence of taxes being distortionary and therefore unfit for furnishing the public sector with the amount of resources that would be desirable by first best standards. In the presence of uncertainty the available tax instruments may fail to bring about an efficient allocation of risk. Whether this problem should be defined as a distortion is a matter of taste or semantics. The important reality is that taxes may be distorting the allocation in the traditional sense as well as failing to assign the inescapable variability in consumption efficiently to the private and public parts of it. Even if taxes might be neutral (non-distortive) in one of these respects it would usually be desirable to sacrifice such a partial neutrality in order to alleviate the misallocation in the other respect. This is in harmony with the general insight from the second best theory. As shown in the analysis it will normally be optimal to accept some inefficiency in the allocation between assets in order to obtain a better allocation between public and private consumption across states.

To obtain the first best allocation state contingent lump-sum taxes are required. In the present analysis state independent income taxes have been considered. It would also be interesting to analyse income taxes that could be made state contingent. That problem is left for a separate paper. Another extension would be to include a broader range of allocative decisions. Yet another would be to integrate the pure public finance aspect of taxation with its possible role as a remedy for market imperfections under uncertainty.

REFERENCES

Ahsan, S.M., 1989, Choice of tax base under uncertainty: Consumption or income?, Journal of Public Economics 40, 99-134.

Atkinson, A.B. and J.E. Stiglitz, 1980, Lectures on public economics (McGraw-Hill, London).

Dixit, A.K., 1986, Trade and insurance with moral hazard, Journal of International Economics 23, 201-220.

Diamond, P.A., 1967, The role of a stock market in a general equilibrium with technological uncertainty, American Economic Review 67, 759-776.

Hahn, F.H., 1973, On optimum taxation, Journal of Economic Theory 6, 96-106.

Mossin, J., 1968, Taxation and risk-taking: An expected utility approach, Economica 35, 74-82.

Richter, W.F., 1988, The optimal taxation of risky capital income: An elasticity rule, Discussion Paper, University of Dortmund.

Sandmo, A., 1969, Capital risk, consumption and portfolio choice, Econometrica 37, 586-599.

Sandmo, A., 1985, The effects of taxation on savings and risk-taking, in: A.J. Auerbach and M. Feldstein, eds., Handbook of Public Economics, vol. I (North-Holland).

Sandmo, A., 1988, Differential taxation and the encouragement of risk-taking, Discussion Paper no. 15, The Norwegian School of Economics and Business Administration.

Stiglitz, J.E., 1969, The effects of income, wealth and capital gains taxation on risk-taking, Quarterly Journal of Economics 83, 262-283.

Stiglitz, J.E., 1982, The inefficiency of the stock market equilibrium, Review of Economic Studies 49, 241-261.

Varian, H.R., 1980, Redistributive taxation as social insurance, Journal of Public Economics 14, 49-68.