Entering an Preannounced Currency Band*

Masaki Ichikawa, Marcus Miller and Alan Sutherland
University of Warwick

No. 347

Abstract

Krugman (1988) provides a stationary characterisation of an exchange rate inside a currency band. Here we derive the non-stationary solution for a floating exchange rate which applies when currency dealers expect such a band to be imposed at a known future date.

* This paper has been produced as part of an ESRC project on The International Monetary System, and resources supplied (under grant number R000231417) are gratefully acknowledged. Masaki Ichikawa is indebted to the Economic Planning Agency, Tokyo for leave to work on an MPhil at Warwick. We have benefited from discussions with Andrew Carverhill of the Financial Options Research Centre at the University of Warwick; but responsibility for any errors remains with us.

March 1990

1. Introduction

What are the dynamics of a freely floating exchange rate in circumstances where entry into a currency band is widely anticipated? The answer will surely depend *inter alia* upon whether the transition is expected only "when conditions are right" or if it is to take place at a given date, irrespective of such conditions. The former case (that of a state dependent switch from a floating to a managed rate) has been examined for a stochastic environment first by Flood and Garber (1983) and more recently by Smith and Smith (1988) and Froot and Obstfeld (1989). (The specific episode they focus on is, in fact, sterling's return to the Gold Standard in 1925.) As far as we are aware the latter case, that of a purely time dependent switch, has not been analysed; and it is the subject of this paper.

In the context of a monetary model with flexible prices, continuous Purchasing Power Parity and a "random walk" in the velocity of money, it is found that the exchange rate satisfies a partial differential equation (PDE) before the date of entry. The relevant boundary condition applying at that time is the stationary solution obtained by Paul Krugman (1988) for such a model - see Section 2 below. By transforming the model into the "heat equation of physics", we are able to obtain an explicit general solution; and we provide an intuitive explanation of the integrals involved. An illustration for the special case of an anticipated return to a fixed rate (i.e. a band with zero width) is given, for which the formula takes a much simpler exponential form.

2. The Monetary Model and the Stationary Solution Within the Band

The monetary model and its stationary solution inside a currency band in Krugman (1988) may briefly be summarised. There are three equations

$$m(t)+v(t)=p(t)-\lambda\frac{E[ds(t)]}{dt} \tag{1}$$

$$p(t)=s(t) \tag{2}$$

$$dv(t)=\sigma dz(t) \tag{3}$$

where s(t) in the log of the exchange rate, p(t) is the log of the price level, m(t) is the log of the domestic money supply and v(t) is the disturbance affecting the velocity of money. (For notational convenience the time "subscripts" are typically dropped in what follows, except where they are essential to the argument.) Inside the band the money supply is constant, but the random evolution of the velocity shock [equation (3) where z is a Brownian motion process with unit variance] has an impact on both the exchange rate and the price level [equation (2)] as they evolve so as to satisfy money market equilibrium [equation (1)].

The required solution is obtained as a stationary function relating the exchange rate to the "velocity adjusted money stock" (m+v) of the form

$$s=f(m+v)=m+v+Ae^{\rho(m+v)}+Be^{-\rho(m+v)} \tag{4}$$

where $\rho=(2/\lambda\sigma^2)^{1/2}$. The coefficients A and B are determined by boundary conditions which, for a fully credible band, are

$$f(\overline{m+v}) = \bar{s}$$

$$f(\underline{m+v}) = \underline{s} \qquad (5)$$

$$f'(\overline{m+v}) = f'(\underline{m+v}) = 0$$

where $\overline{m+v}$ and $\underline{m+v}$ are the values of the fundamental at which the rate is driven onto the edges of the band, $\bar{s}$ and $\underline{s}$ respectively. This solution, for the symmetric case A=-B, is shown as the familiar S shaped curve labelled KK in Figure 1 where the boundary conditions ensure that the exchange rate "smooth pastes" onto the edges of the band. At the edges of the band, the money supply is varied (by "marginal intervention") as necessary to offset any velocity shocks that would take the velocity adjusted money stock outside the range shown, see Krugman (1988).

Note that, if there is no band and the currency floats freely with the money supply held constant, then the exchange rate moves *pari passu* with the velocity shock, as shown by the 45^0 line in Figure 1, marked FF.

3. Anticipations of Managed Exchange Rates

Consider now how the exchange rate will depart from the freely floating solution FF under the impact of an announcement at time 0 that the monetary authorities are to defend the band ($\bar{s}$,$\underline{s}$) after time T. Assume for convenience that the announcement is fully credible, and that the currency band can be successfully defended, so that the solution at time T is the curve KK itself; and no further regime shifts are anticipated.

Given that the solution for the rate at time T (and after) is known, the solution between 0 and T may be obtained by introducing a time varying element into the the relationship between the exchange rate and economic fundamentals, so (4) becomes

$$s=g(m+v,t) \qquad (4')$$

and the application of Ito's lemma produces a partial differential equation of the form

$$\frac{\sigma^2}{2}g_{11}(m+v,t)+g_2(m+v,t)=\frac{1}{\lambda}(s-m-v)\cdot \qquad (6)$$

The evolution of the exchange rate after the announcement involves finding the solution for this equation subject to the boundary condition that will hold when the band is actually implemented. In general numerical techniques are required, but for the special case where the currency band is in fact a fixed exchange rate ($\bar{s}=\underline{s}=s^*$ and the curve KK coincides with the horizontal axis), an explicit solution is easily available.

Consider, therefore, a dated return to a fixed rate, where the solution to equation (6) subject to the boundary condition $g(m+v,T)=s^*$ can be obtained by the separation of variables and takes the convenient form

$$g(m+v,t) = (m+v)(1-e^{-\frac{1}{\lambda}(T-t)}) + s^* e^{-\frac{1}{\lambda}(T-t)}. \qquad (7)$$

Note that the exchange rate in the interval [0,T] is a weighted average of its value under a free float (when it is equal to the velocity adjusted money stock) and of the target value s* announced by the monetary authority, with weights that vary with time (measured backwards from T). At time T the weight on the second term is unity, ensuring no jump in the exchange rate at that time.

The implication of this solution is that the credible announcement of future stabilisation has immediate effects as the rate jumps from FF to g(m+v,0). (Graphically equation (7) implies that at the time of announcement the rate will jump onto a linear solution - not shown - with a slope of less than 45^0 passing through s*; subsequently this line swivels through the point s* becoming horizontal at t=T). Clearly the further into the future is the anticipated "return to a fixed rate", the smaller the stabilising jump at the time of announcement, *ceteris paribus*.

Turning to the more general case where the announcement is one of entry into a currency band, the need to ensure no anticipated jump in the exchange rate requires the solution to (6) to coincide with Krugman's stationary S-shaped solution at the time of implementation. Formally the relevant boundary (or terminal) condition is

$$
\begin{aligned}
g(m+v,T) &= \overline{s} && m+v > \overline{m+v} \\
g(m+v,T) &= f(m+v) && \underline{m+v} \le m+v \le \overline{m+v} \qquad (8) \\
g(m+v,T) &= \underline{s} && m+v < \underline{m+v}
\end{aligned}
$$

where, as before, $\overline{m+v}$ and $\underline{m+v}$ are the values of the fundamental (the velocity adjusted money stock) at which the stationary solution is tangent to the edges of the currency band. Note that the money stock will be constant before T, but may require discrete adjustment at that time, if m+v lies outside the range consistent with the currency band. So, if $m+v>\overline{m+v}$ at T, for example, the money stock will have to be reduced by just enough to keep the exchange rate at the upper edge of the band. Once the band is established its support will from time to time require the marginal adjustment to the money stock already mentioned.

There is a close parallel between the analysis of a preannounced band and the analysis of option prices in Black and Scholes (1973). In the case of options, Black and Scholes used Ito's lemma to obtain a PDE in the option pricing function which (if allowance is made for their use of levels rather than logs) is identical in form to (6). Similarly, there is a terminal condition at the date the option expires (provided by the need for the option price to coincide with the underlying stock price less the strike price or zero whichever is the greater).

Black and Scholes solve their PDE by a change of variables which transforms the equation into the heat equation of physics, for which solutions can be obtained by Fourier transform methods [see Guenther and Lee (1988, pp166-178)]. The same approach is applied to the solution of (6) subject to condition (8) and the solution that emerges is

$$g(m+v,t)=(m+v)-\frac{e^{-\frac{1}{\lambda}(T-t)}}{\sqrt{2\pi\sigma^2(T-t)}}\int_{-\infty}^{\infty}\exp\left[-\frac{(x-m-v)^2}{2\sigma^2(T-t)}\right][(m+v)-\phi(x)]dx \tag{9}$$

where $\phi(x)=\bar{s}$ $\quad x>\overline{v+m}$

$\phi(x)=f(x)$ $\quad \underline{m+v}\le x\le\overline{m+v}$

$\phi(x)=\underline{s}$ $\quad x<\underline{m+v}$.

Note that while v is a stochastic process evolving through time, the money supply is kept constant until T.

Equation (9) has a simple interpretation. To see this, observe that the distribution of (m+v(T)) conditional on (m+v(t)) is N(m+v(t),σ^2(T-t)), bearing in mind that m is constant before T. This implies that the expected value of the exchange rate at time T, conditional on the level of the fundamental at time t, is obtained as

$$E(s(T)|m+v(t),t)=\frac{1}{\sqrt{2\pi\sigma^2(T-t)}}\int_{-\infty}^{\infty}\exp\left[-\frac{(x-m-v(t))^2}{2\sigma^2(T-t)}\right]\phi(x)dx. \tag{10}$$

It is therefore possible to rewrite (9) as follows

$$g(m+v,t)=(m+v)(1-e^{-\frac{1}{\lambda}(T-t)})+E(s(T)|m+v,t)e^{-\frac{1}{\lambda}(T-t)} \tag{11}$$

(Where once again v(t) is written simply as v.) The similarity between this solution and that obtained for the case of a move to a fixed rate is clear. The only difference is the appearance of the conditional expectation of s(T) in (11) in the place of s* in (7).

The explanation for this is obvious - in the case of a move to a fixed rate the value of the exchange rate at time T is known with certainty, while in the case of a preannounced band the exchange rate at time T is uncertain, so its expected value is used instead.

We have evaluated equation (11) numerically, but rather than report these numerical results, we show that the qualitative nature of the solution can be deduced directly from the formula itself, as follows. Suppose the level of the velocity adjusted money stock at time 0 is given by m+v(0) as shown in Figure 1. Immediately before the announcement the exchange rate lies on the free float line FF and is thus $s(0^-)$. When the announcement is made agents use their knowledge of the distribution of m+v(T) to form their expectation of the exchange rate that will hold at time T. Since velocity follows Brownian motion without drift, their expectation of m+v(T) is simply its current level, m+v(0). But the curvature of the S-shaped stationary solution (marked KK) at this level of m+v implies that the expectation of s(T) is slightly below that given by KK. Equation (11) tells us that the level of the current exchange rate is a weighted average of the expected exchange rate at time T and the current free float solution. This is shown by the point marked $s(0^+)$ in Figure 1. If this procedure is carried out for each possible level of m+v(0) the curved line marked AA - the isochronous solution to the PDE at time 0 - is traced out. It is apparent that a credible announcement of a band has an immediate stabilising effect.

The passage of time has two effects on the solution. Firstly the variance of m+v(T) decreases as time T approaches, thus the expectation of s(T) moves closer to the KK. The second and much larger effect is the increase in the weight attached to the expectation of s(T) in equation (11). The isochronous solution therefore moves closer to KK, and the stabilising effect of the anticipated band increases.

4. Conclusion

Where an anticipated transition from floating to managed exchange rates is treated as state contingent, others have shown that the solution for the exchange rate as a function of fundamentals is stationary. This is not the case when the commitment to stabilise rates is to take effect from a fixed future time. The partial differential equation describing the rate can, however, be solved using the appropriate stationary solution as a boundary or terminal condition. An explicit solution is available for the case of a credible return to a fixed rate. Although numerical methods are necessary for evaluating the exchange rate when entry to a currency band is anticipated, a qualitative interpretation of the path of the rate is available.

Two possible extensions of the techniques developed here are worth mentioning: first is the idea of combining state and time contingent elements; second is that of allowing for lack of credibility in the commitment to stabilise the rate.

References

Black, Fischer and Myron Scholes, 1972, The Valuation of Options and Corporate Liabilities, Journal of Political Economy 81, 637-654.

Flood, Robert P. and Peter M. Garber, 1983, A Model of Stochastic Process Switching, Econometrica 51, 537-551.

Froot, Kenneth A. and Maurice Obstfeld, 1989, Exchange-Rate Dynamics under Stochastic Regime Shifts: A Unified Approach, (mimeo) MIT and University of Pennsylvania.

Guenther, Ronald B. and John W. Lee, 1988, Partial Differential Equations of Mathematical Physics and Integral Equations (Prentice Hall, Englewood Cliffs, New Jersey)

Krugman, Paul, 1988, Target Zones and Exchange Rate Dynamics, NBER Working Paper No. 2481, forthcoming in the Quarterly Journal of Economics.

Smith, G. W. and T. Smith, 1988, Stochastic Process Switching and the Return to Gold, 1925, (mimeo) Queen's University.

Figure 1