Multiple Solutions and Bubbles in Stochastic Models of the Exchange Rate.

Alan Sutherland
University of Warwick

No. 348

Research support from the ESRC is gratefully acknowledged (grant no. R000231417) as are valuable comments from Marcus Miller.

May 1990

Introduction

A number of recent papers (Krugman (1988), Miller and Weller (1988), Froot and Obstfelt (1989a) and Flood and Garber (1989)) make use of continuous time stochastic rational expectations models to analyse the operation of currency bands. Although this type of model yields a unique solution for the exchange rate when a currency band is in operation, there appear to be many possible solutions when the exchange rate is allowed to float freely. In deterministic rational expectations models there are also many possible solutions to a free float regime - but only the stable manifold has the property that the asset price is the integrated discounted value of fundamentals. For this reason the stable manifold is usually regarded as the "fundamental" solution in deterministic models (see Blanchard and Fischer (1989)). By contrast, the other solutions are referred to as "bubbles" solutions because at any point in time the current exchange rate is sustained by future capital gains or losses rather than any consideration of fundamentals.

The linear stable manifold is also one of the possible solutions to a stochastic model and this paper shows that, as in deterministic models, it is the only solution which yields a value for the asset price that is the integrated discounted value of (expected) fundamentals. It is argued, therefore, that the stable manifold is the fundamental solution to stochastic models in exactly the same sense as it is the fundamental solution to deterministic models. By implication the other stochastic solutions can be regarded as involving bubbles[1]. Although these bubbles appear very different from deterministic bubbles they nevertheless possess the property that the

---

[1] Froot and Obstfeld (1989b) called these bubbles "intrinsic bubbles" and tested for their presence in US stock prices.

current exchange rate is sustained by expected capital gains or losses rather than by fundamentals.

In a deterministic context it is normal to assume that a rational market will rule out bubbles solutions on the ground that they explode (see Blanchard and Fischer (1989) and Blanchard and Watson (1982) for a discussion of the plausibility of this assumption). This paper seeks to establish whether the same condition leads to the elimination of bubbles in stochastic models of the exchange rate.

The paper is divided into three sections. In Section 1 the argument outlined above is applied to the simple Krugman (1988) "monetary" model of the exchange rate where the level of velocity follows Brownian motion. Section 2 then extends the results to the case where velocity is mean reverting but independent of the exchange rate. In Section 3 the sticky price model of Miller and Weller (1988) is considered where there is feedback from the asset price to the dynamics of the fundamental. In all three cases there are strong results on the explosiveness of bubbles which provide support for ruling them out.

1. Brownian Motion Fundamentals.

The Krugman model consists of two basic equations. The first is an exchange rate equation

$$s = m + v + \lambda \frac{E(ds)}{dt} \tag{1}$$

where s is the log of the exchange rate, m is the log of the money stock and v is the log of velocity. The second equation determines velocity - which is assumed to be a Brownian motion process as follows

$$dv=\sigma dz \tag{2}$$

The model is solved by postulating a deterministic mapping between the exchange rate (the asset price) and velocity (the fundamental) of the following form

$$s=f(v) \tag{3}$$

Applying Ito's lemma and substituting from (1) yields the following differential equation

$$\frac{\sigma^2}{2}f''(v)=\frac{1}{\lambda}f(v)-\frac{1}{\lambda}(m+v). \tag{4}$$

Krugman shows that the general solution to this equation is

$$s=m+v+Ae^{\rho v}+Be^{-\rho v} \tag{5}$$

where $\rho=(2/\lambda\sigma^2)^{1/2}$ and A and B are constants to be determined by boundary conditions.

Krugman constructed this model as a means of analysing the effects of currency bands. A band provides the necessary boundary conditions to determine A and B, but when the exchange rate is allowed to float freely there are no boundary conditions available and this apparently leaves an infinity of possible solutions.

Consider, however, setting A and B to zero to yield the solution

$$s=m+v \tag{6}$$

This is the solution that would be chosen if the model were deterministic (i.e. if $\sigma=0$) - it is the so called (semi) stable manifold. It is also the solution that emerges by integrating the discounted value of expected fundamentals as follows

$$s(t)=E_t\frac{1}{\lambda}\int_t^\infty e^{-\frac{1}{\lambda}(\tau-t)}(m(\tau)+v(\tau))d\tau \tag{7}$$

where E_t is the expectations operator conditioned on information at time t. Assuming no anticipated changes in monetary policy (as is the case in a free float regime) and noticing that $E_t v(\tau)=v(t)$, solving integral (7) yields equation (6). The integral of the discounted value of expected fundamentals has been called the "fundamental" solution by Blanchard and Fischer (1989) (in the context of a deterministic model) and it is a very natural choice for the value of an asset.

To show that none of the other stochastic solutions yields a value for the asset price that is the discounted value of fundamentals is simply a matter of noting that the dynamics of the fundamental are independent of the asset price. There is therefore a unique expected path for velocity from any given starting point and a unique fundamental solution for the asset price.

Blanchard and Fischer show that the other solutions to a deterministic rational expectations model are given by the fundamental solution plus a bubble term, where the bubble term satisfies arbitrage. The solution to the Krugman model consists of the fundamental solution (m+v) plus the term $(Ae^{\rho v}+Be^{-\rho v})$. It therefore seems logical to regard the second term as a "bubble".

In deterministic models, when considering rational markets, bubbles are usually ruled out because they explode. The assumption is that a rational market will not choose a solution which eventually leads to an "unreasonable" level for the exchange rate. It is a simple matter to observe that the bubble term in (6) explodes for extreme values of v whenever A or B are non-zero. Of course it may be argued that, since v is stochastic, it will not necessarily reach an extreme value. But it is a basic result in stochastic processes that the probability of a Brownian motion variable, such as v, passing through any point on the real line tends to unity as time tends to infinity. The probability of v taking on an extreme value must therefore tend to unity and any bubble is certain to explode sooner or later. These stochastic bubbles can thus be ruled out in a rational market in exactly the same way as deterministic bubbles are ruled out.

Before proceeding to the mean reversion case it is important to note that the description of the general solution as the fundamental solution plus a bubble is only appropriate in the free float context. If there is a currency band in operation (or any other state contingent regime switch is in prospect) then the solution will involve non-zero A and B. In this case the bubble should, more correctly, be regarded as the price of the "option" that is implicit in holding the asset in the presence of a potential regime switch. Weller (1989) discusses this implicit option with reference to currency bands while Dixit (1989) uses a similar interpretation in the context of the valuation of a firm.

2. Mean-Reverting Fundamentals

In Krugman (1989) the model of the previous section is extended by making velocity follow a mean reverting process such as

$$dv=-\theta v dt+\sigma dz \tag{8}$$

In this case the differential equation takes the following form

$$\frac{\sigma^2}{2}f''(v)-\theta v f'(v)-\frac{1}{\lambda}f(v)=-\frac{1}{\lambda}(v+m) \tag{9}$$

This is a linear equation with varying coefficients and its general solution can be expressed in terms of the Confluent Hypergeometric (CHG) function (see Slater (1960)). The solution is

$$f(v)=m+\frac{1}{1+\lambda\theta}v+AvM\left(\frac{1}{2}+\frac{1}{2\lambda\theta},\frac{3}{2},\frac{\theta v^2}{\sigma^2}\right)+BM\left(\frac{1}{2\lambda\theta},\frac{1}{2},\frac{\theta v^2}{\sigma^2}\right) \tag{10}$$

where A and B are constants to be determined by boundary conditions and $M(\alpha, \beta, z)$ is the CHG function which has a series representation as follows

$$M(\alpha,\beta,z) = 1 + \frac{\alpha}{\beta}z + \frac{\alpha(\alpha+1)}{\beta(\beta+1)}z^2 + \frac{\alpha(\alpha+1)(\alpha+2)}{\beta(\beta+1)(\beta+2)}z^3 \ldots$$

As before, consider the solution obtained by setting A and B to zero

$$f(v) = m + \frac{1}{1+\lambda\theta}v \qquad (11)$$

This is again the solution that would be chosen if the model were deterministic (that is, it is the stable manifold) and again it is the fundamental solution or the solution that gives the asset price as the integral of the discounted value of expected fundamentals. This can be checked by evaluating integral (7) and noting that, given (8), $E_t v(\tau) = v(t)e^{-\theta(\tau-t)}$ (see Jazwinski (1972)) - the result is equation (11). Since the dynamics of v are still independent of the exchange rate, there is a unique expected path for v for any given v(t) so, as before, the fundamental solution is unique. The general solution (10) can therefore be interpreted as the fundamental solution plus a bubble term.

Having demonstrated that the set of free float solutions consists of a unique fundamental solution plus an infinity of bubbles solutions, it is necessary to investigate the asymptotic properties of the bubbles to establish whether or not they explode. It is apparent that, even though the bubbles term explodes for extreme value of v, the argument of the previous section no longer applies. As v follows a mean reverting process and has a well defined asymptotic distribution, it is not possible to argue that the probability of an extreme value approaches unity as time tends to infinity.

There is, however, an alternative definition of "explosiveness" which can be used in this situation and which does suggest that a rational market would rule out these bubbles. First notice that, because of mean reversion, the model now possesses a well defined equilibrium point where E(dv)=E(ds)=0. It seems reasonable to restrict attention to solution paths which pass through this point. This implies that B should be set to zero and the bubbles term can be written as

$$b(v) = AvM\left[\frac{1}{2}+\frac{1}{2\lambda\theta},\frac{3}{2},\frac{\theta v^2}{\sigma^2}\right]$$

The following proposition can now be proved.

<u>Proposition 1</u>. If A is not equal to zero the asymptotic variance of the bubble is infinite.

<u>Proof</u>: The asymptotic variance of the bubble is defined as

$$Var(b) = \int_{-\infty}^{\infty}[b(v)]^2 g(v)dv$$

where g(v) is the steady state or asymptotic probability density of v. Notice that, given (8), v is distributed asymptotically Normal as follows

$$g(v) = Ce^{-\frac{\theta v^2}{\sigma^2}}$$

where C is a constant. Also notice that, for any given values of A and v, b(v) is strictly increasing in $1/\lambda$. Thus, to prove the proposition it is sufficient to prove that Var(b) is infinite for $1/\lambda=0$. Setting $1/\lambda=0$ reduces b(v) to

$$b(v) = AvM\left(\frac{1}{2},\frac{3}{2},\frac{\theta v^2}{\sigma^2}\right)$$

Define H(v) to be $[b(v)]^2 g(v)$. H(v) is symmetric about v=0 and strictly positive for finite v. It will be shown that H'(v) is positive for all positive v and, therefore, the integral of H(v) must diverge. The derivative of H(v) is as follows

$$H'(v) = 2Ah(z)g(v)b(v)$$

where $z = \frac{\theta v^2}{\sigma^2}$

$$h(z) = M\left(\frac{1}{2},\frac{3}{2},z\right) + z\frac{2}{3}M\left(\frac{3}{2},\frac{5}{2},z\right) - zM\left(\frac{1}{2},\frac{3}{2},z\right)$$

From the series definition of the CHG function the following series representation of h(z) can be constructed

$$h(z) = 1 + \frac{1}{6}z^2 + \frac{1}{15}z^3 \ldots \quad \ldots + \left[\frac{n-1}{2n-1}\right]\frac{1}{n!}z^n \ldots$$

from which it can be seen that h(z) is positive for all positive z and H'(v) is positive for all positive v. This completes the proof.

Since the asymptotic variance of the bubble is the expectation of the long run value of b^2 it can be thought of as a measure of the expected absolute magnitude of the bubble far into the future. The fact that it is infinite suggests that a bubble, if it exists, is expected to explode to either a large positive or large negative value as time tends to infinity. This result shows that the bubbles in this model are at least as explosive as the bubbles in the model of Section 1 and, in a relevant stochastic sense, are at least as explosive as the bubbles in an equivalent deterministic model. If the tendency for bubbles to explode is a sufficient reason to rule out bubbles in deterministic models then it seems equally powerful as a means of ruling out bubbles in the stochastic models discussed above, when markets are rational.

As with the previous version of the model the description of the general solution as <u>the fundamental solution plus a bubble</u> is only appropriate in the free float context. Again, if there is some regime switch in prospect, the "bubble" is actually an option price.

A crucial aspect of the models considered in these first two sections is that the fundamental, v, follows an autonomous process. This yields explicit and strong results concerning the "explosiveness" of bubbles which allows them to be ruled out. The next section considers the same question in the context of the Miller and Weller

(1988) model of the exchange rate, where the dynamics of the fundamental are not independent of the asset price.

3. Non-Autonomous Fundamentals.

Miller and Weller (1988) use the stochastic version of the Dornbusch (1976) model given in equations (12) to (15)

$$m - p = \kappa y - \lambda i \tag{12}$$

$$y = -\gamma i + \eta(s - p) \tag{13}$$

$$dp = \phi y dt + \sigma dz \tag{14}$$

$$E(ds) = (i - i^{*})dt \tag{15}$$

where m=log of the money supply

p=log of the price level

y=log of real output

i=nominal interest rate

z=Brownian motion

s=log of the exchange rate

Equation (1) is the demand for money function and equation (2) is the demand for real output function. The Phillips curve in equation (3) relates inflation to excess real demand and a stochastic shock while equation (4) is the arbitrage condition between foreign and domestic assets. An asterisk indicates a foreign variable.

$$\begin{bmatrix} dp \\ E(ds) \end{bmatrix} = A \begin{bmatrix} p \\ s \end{bmatrix} dt + \begin{bmatrix} \sigma \\ 0 \end{bmatrix} dz$$

where $A = \frac{1}{\Delta} \begin{bmatrix} -\phi(\gamma + \eta\lambda) & \phi\lambda\eta \\ 1 - \kappa\eta & \kappa\eta \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $\Delta = \kappa\gamma + \lambda$

and where it is assumed that m=i*=0. The matrix A is assumed to have one stable eigenvalue and one unstable eigenvalue. Notice that the term a_{12} ensures that the dynamics of the price level depend on the level of the exchange rate.

As in the previous sections, a mapping from the asset price to the fundamental (here the goods price level) is postulated and the following differential equation is obtained

$$a_{21}p + a_{22}f(p) = a_{11}pf'(p) + a_{12}f(p)f'(p) + f''(p)\frac{\sigma^2}{2}$$

This equation is non-linear and does not have closed form solutions. However, it is simple to check that the solutions can be written in the following form

$$f(p)=\theta p+b(p) \tag{16}$$

where θ is the slope of the stable eigenvector of matrix A and b(p) satisfies

$$a_{22}b(p) = a_{12}pb'(p) + a_{12}b(p)b'(p) + \frac{\sigma^2}{2}b''(p)$$

It again seems reasonable to restrict consideration to solutions which pass through the point of equilibrium and these are illustrated in Figure 1a (see Miller and Weller (1988) for further discussion of this picture). As in the previous sections there are an infinite number of free float solutions. But consider the solution given by setting b(p)=0 for all p,

$$f(p)=\theta p. \tag{17}$$

This is the stable manifold and, as in the simpler models, it is the fundamental solution or the solution that gives the asset price as the integral of the discounted value of fundamentals. That is, the solution for which the following is true,

$$s(t) = -E_t\int_{-t}^{\infty} e^{-a_{22}(\tau-t)}a_{21}p(\tau)d\tau. \tag{18}$$

This is easily checked by substituting s=θp into the equation for the dynamics of p and solving for the expected future path of prices, which is

$$E_t p(\tau) = p(t)e^{(a_{11}+a_{12}\theta)(\tau-t)}.$$

Substituting this into integral (18) and evaluating yields (17).

Proving that the stable manifold is the unique fundamental solution is slightly more complicated than before because, in this model, the expected path of the price level depends on the stochastic solution being considered. The proof of uniqueness can be illustrated with reference to Figure 2 where SS is the stable manifold. Suppose the current price is $p(t_0)$ and that the exchange rate is on a stochastic solution below the stable manifold - such as AA. Notice from the sign of a_{12} that a lower exchange rate at a given level of p implies that the price level is expected to return more quickly to equilibrium and thus the expected path of prices is lower than if the exchange rate were given by the stable manifold. Integral (18) must, therefore, be higher (less negative) than on the stable manifold. But the current exchange rate is lower on AA so this cannot be a fundamental solution. A similar argument works for solutions above the stable manifold. The stable manifold is therefore the unique fundamental solution.

Having established this result it is again possible to argue that the set of free float solutions represented by equation (16) is made up of the unique fundamental solution plus an infinity of bubbles solutions, where b(p) is the bubbles term. It is now necessary to consider the explosive properties of these bubbles solutions. As in Section 2 the asymptotic variance of the bubbles term is investigated. It is given by

$$\mathrm{Var}(b) = \int_{-\infty}^{\infty} [b(p)]^2 g(p)\,dp$$

where g(p) is the asymptotic density function of p.

Because the dynamics of the fundamental are no longer autonomous, the asymptotic distribution of the fundamental is more complicated than in the previous models. The density function, g(p), can be obtained by solving Kolmogorov's forward equation at its steady state (see Jazwinsky (1970)). The latter equation takes the following form in the present context

$$-\frac{\sigma^2}{2}g''(p)+u(p)g'(p)+u'(p)g(p)=0 \tag{19}$$

where

$$u(p)=E(dp)=a_{11}p+a_{12}f(p)$$

$$=(a_{11}+a_{12}\theta)p+a_{12}b(p)$$

and the following boundary conditions are required

$$g'(0)=0 \tag{20}$$

$$\text{and } \int_{-\infty}^{\infty} g(p)dp=1. \tag{21}$$

The solution to (19) for positive values of p is simply

$$g(p)=Ce^{\frac{2}{\sigma^2}\int_0^p u(s)ds} \tag{22}$$

where C is a constant chosen so as to satisfy condition (21). (Notice that this expression collapses to the Normal density function when a_{12} is zero.)

Figure 1b shows the bubbles that correspond to the solutions in Figure 1a. It is convenient to split the various bubbles into two basic categories and deal with each separately. Bubbles in the first category are referred to as "Type 1" bubbles and have a slope at the origin greater than zero in Figure 1b. Bubbles in the second category are referred to as "Type 2" bubbles and have a slope at the origin which is less than zero.

Type 1 Bubbles: The following proposition can be stated and proved.

Proposition 2. The asymptotic variance of Type 1 bubbles is infinite.

Proof: The proposition is proved with reference to Figure 1b which shows the $E(dp)=u(p)=0$ schedule. Points to the left of this schedule imply $u(p)>0$ while points to the right imply $u(p)<0$. Since $g(p)$ is symmetric about the origin it is sufficient to consider positive p. Starting from the origin and moving to the right it is a fact (which follows from the results of Miller and Weller (1988)) that a Type 1 bubble either begins in the $u(p)>0$ region or eventually enters it. Once in that region Type 1 bubbles do not leave it. But from the definition of $g(p)$ given in equation (22) it is clear that $g(p)$ must therefore tend to positive infinity as p tends to infinity. This implies that Var(b) is infinite and the proposition is proved.

Type 2 Bubbles: Type 2 bubbles are considerably more complicated to deal with than Type 1 bubbles and require the application of numerical analysis. Accurate and revealing representations of $b(p)$ can be obtained by power series methods while a Taylor's Series expansion of (22) can be used to generate a corresponding power series for $g(p)$.

An examination of the power series solution for b(p) reveals that Type 2 bubbles have a finite radius of convergence and are therefore asymptotic to vertical lines placed symmetrically about the origin. An example is illustrated in Figure 3 where the solution tends to positive infinity at $-p_0$ and negative infinity at $+p_0$.

In this case, the convergence of Var(b) can be tested by generating a power series for H(p) (where $H(p)=[b(p)]^2g(p)$), integrating it term by term and applying a standard convergence test (such as the comparison test - see Hirschman (1962)) at $p=p_0$. Table 1 lists a number of parameter sets to which this procedure has been applied. In all cases the convergence test (when applied to the first 500 terms of the power series) indicated that Var(b) is finite.

Table 1

Parameters						
η	0.5	0.5	0.5	4.0	4.0	4.0
λ	0.4	0.8	2.0	0.4	0.8	2.0
ϕ	0.5	0.5	0.5	0.5	0.5	0.5
γ	0.5	0.5	0.5	0.5	0.5	0.5
κ	1.0	1.0	1.0	1.0	1.0	1.0
σ	1.0	1.0	1.0	1.0	1.0	1.0

Given the great similarity in shape between Type 2 solutions to this model and all the solutions to the model in Section 2, it seems surprising that there is such a great difference in their implications for the variance of the bubbles. However, the explanation is quite intuitive. In the model in this section the asset price acts on the dynamics of the fundamental through the a_{12} term. On a Type 2 bubble solution the asset price adds to the mean reverting tendency of the price level and therefore reduces the amount of price variability. That this is a powerful effect can be seen if (22) is rewritten as follows

$$g(p) = \left(Ce^{\frac{(a_{11}+a_{12}\theta)p^2}{\sigma^2}} \right)\left(e^{\frac{2a_{12}}{\sigma^2}B(p)} \right) \qquad \text{where } B(p) = \int_0^p b(s)ds.$$

The first bracketed term is the Normal density function that would arise if there were no bubble. The expression shows that the density function is compressed by a factor (the second bracketed term) which depends on the integral of the bubble. This powerful tendency for Type 2 bubbles to stabilise the price level is obviously sufficient to make the variance of the bubble finite.

The fact that the variance of Type 2 bubbles is finite implies that they do not explode in the same sense as the bubbles in the model of Section 2. It is therefore necessary to turn to some alternative notion of explosiveness in order to justify ruling out Type 2 bubbles. One possible route is to argue that the fact that Type 2 bubbles explode with at least a non-zero probability is sufficient to rule them out in a rational market. This argument is made considerably stronger by noting that, as illustrated in Figure 3, Type 2 bubbles explode at a finite level of the fundamental. Indeed, in this respect they are more explosive than the bubbles in the other models which only explode as the fundamental tends to infinity. The consequences of an exploding bubble are also much more far reaching in this model than in the previous models. For instance a bubble which drives the exchange rate to an extreme value will also drive output to an extreme value. The fact that this happens at a finite level of the fundamental is surely so unreasonable that a rational market will rule it out by avoiding Type 2 bubbles solutions.

Conclusion

This paper has examined three stochastic rational expectations models of the exchange rate. It was found that the multiplicity of free float solutions which these models possess is made up of a unique fundamental solution plus an infinity of bubbles solutions. The fact that a fundamental solution exists and that it is unique shows that there is a clear theoretically preferred free float solution.

The paper also considered the asymptotic properties of the bubbles solutions in the various models, to determine whether there was a case for assuming that a rational market would rule them out. In the monetary model of Section 1 (where the fundamental follows Brownian motion) bubbles were seen to explode with certainty as time tends to infinity. With mean-reverting fundamentals, as shown in Section 2, the same model produces bubbles with an absolute value that explodes in expectation as time tends to infinity. In the Miller and Weller model, where the fundamental does not follow an autonomous process, it was found that some bubbles do not explode in expectation. Nevertheless, it was argued that a strong case still exists for ruling out these bubbles in a rational market because they explode with some non-zero probability at a finite level of the fundamental and with devastating consequences for the real economy.

References

Blanchard, Olivier and Mark Watson (1982) "Bubbles, Rational Expectations and Financial Markets" in P. Wachtel (ed.) *Crises in the Economic and Financial Structure,* Lexington MA, Lexington Books.

Blanchard, Olivier and Stanley Fischer (1989) *Lectures on Macroeconomics,* Cambridge MA, MIT.

Dixit, Avinash (1989) "Entry and Exit Decisions under Uncertainty", *Journal of Political Economy,* vol 97, 620-838.

Dornbusch, Rudiger (1976) "Expectations and Exchange Rate Dynamics", *Journal of Political Economy,* vol. 84, 1161-1176.

Flood, Robert P. and Peter M. Garber (1989) "The Linkage Between Speculative Attack and Target Zone Models of Exchange Rates", NBER Working Paper No. 2918.

Froot, Kenneth A. and Maurice Obstfelt (1989) "Exchange-Rate Dynamics under Stochastic Regime Shifts: A Unified Approach", mimeo MIT and University of Pennsylvania.

Froot, Kenneth A. and Maurice Obstfeld (1989) "Intrinsic Bubbles: The Case of Stock Prices", mimeo MIT and University of Pennsylvania.

Hirschman, I.I, (1962) *Infinite Series,* New York, Holt, Rinehart and Winston.

Jazwinski, Andrew H. (1970) *Stochastic Processes and Filtering Theory,* London, Academic Press.

Krugman, Paul (1988) "Target Zones and Exchange Rate Dynamics", NBER Working Paper No. 2841.

Krugman, Paul (1989) "Target Zones with Limited Reserves", mimeo MIT.

Miller, Marcus H. and Paul Weller (1988) "Solving Stochastic Saddlepoint Systems: A Qualitative Treatment with Economic Applications", Warwick Economic Research Paper No. 309.

Slater, L. J. (1960) *Confluent Hypergeometric Functions*, Cambridge, CUP.

Weller, Paul (1989) "Currency Options and Exchange Rate Bands" mimeo, University of Warwick.

Figure 1a

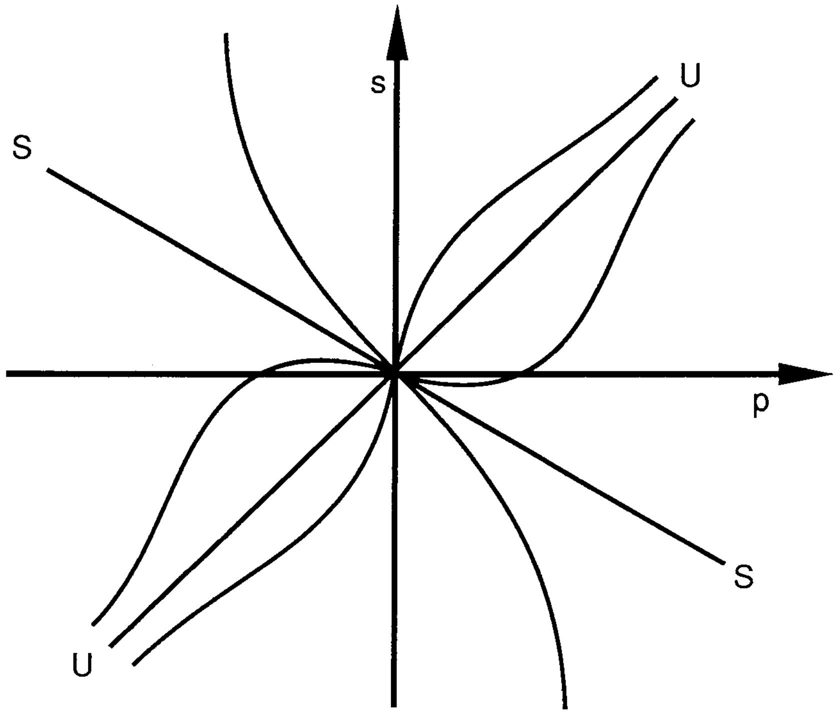

Figure 1b

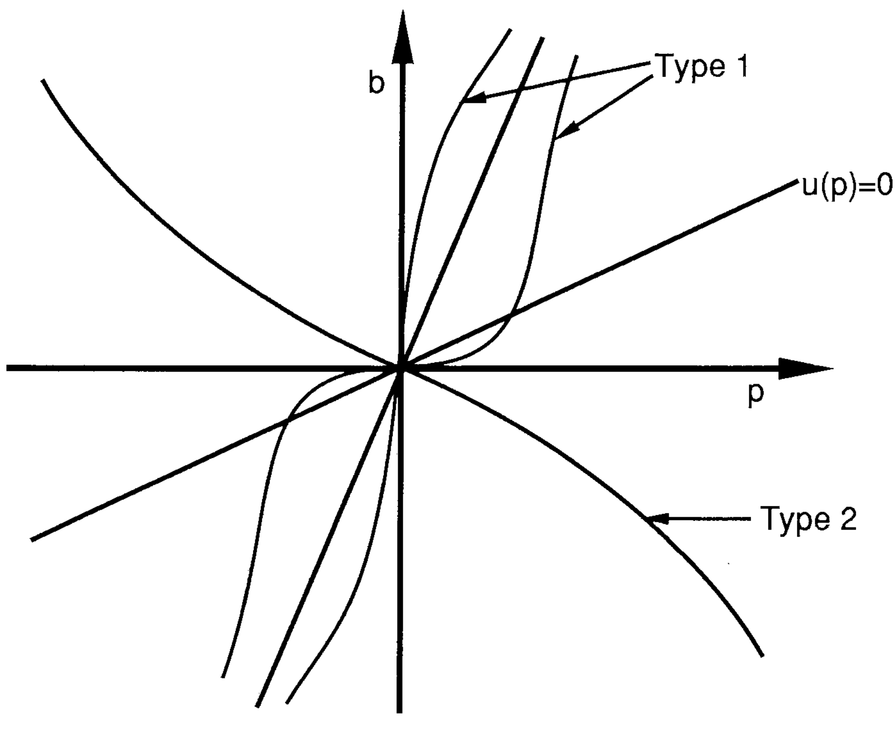

Figure 2

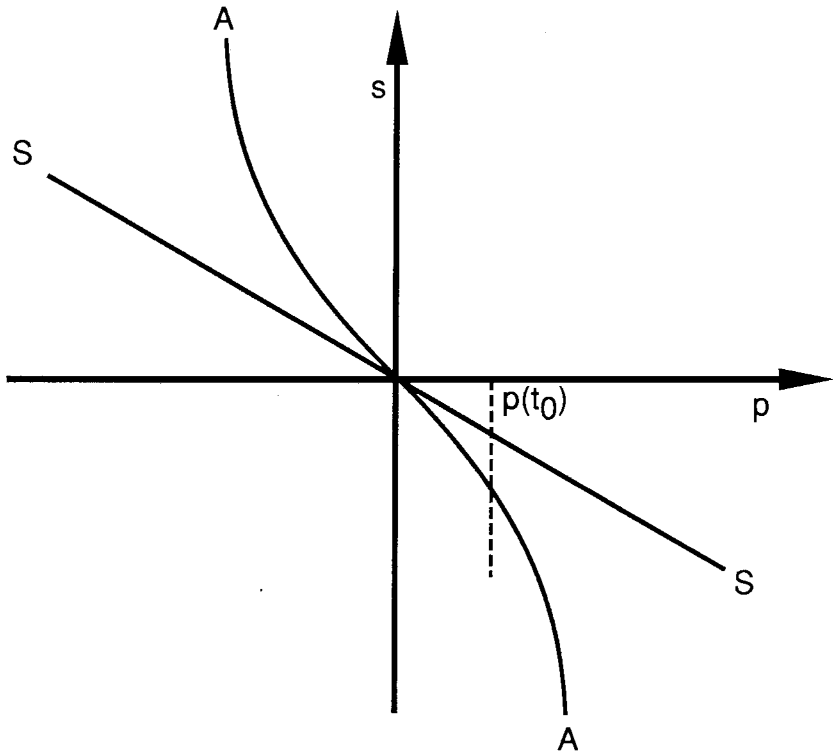

Figure 3