# Imperfect Competition with Intermediate Goods:

## A Simulation Analysis of a Two-Sector Model.*

**Gareth D. Myles**

**University of Warwick**

**May 1990.**

No. 349

**Abstract:** A two-sector model of imperfect competition with intermediate goods is developed and analysed by numerical simulation. It is shown how an objective notion of demand can be derived and employed in three concepts of equilibrium that differ in the possibilities for price-discrimination and collusion. The results indicate that there may be excessive use of labour relative to produced input in production, that price discrimination reduces both welfare and profits and that collusion between firms is beneficial to both the firms and the consumer. In addition, collusion may result in produced inputs being sold at a price less than marginal cost.

**Acknowledgements**: Thanks are due to Norman Ireland, Umberto Galmirini and numerous seminar participants.

***Address**: Department of Economics, University of Warwick, Coventry, CV4 7AL, U.K.

## 1. Introduction

The analysis of this paper focuses upon a two sector general equilibrium model in which the outputs of imperfectly competitive industries are used both as intermediate inputs and for final consumption. It is shown how the objective demand function facing each industry can be constructed thus overcoming one of the difficulties raised by Hart (1975). The model is analysed by numerical simulation since the interdependences do not make it tractable for analytical investigation. As this work is primarily an exploratory one, the implications of a number of equilibrium concepts that differ in the possibilities for price discrimination and collusion are investigated. In addition, the simulations also consider the effect of the form of production technology and the demand relationship of the two goods upon the equilibrium outcomes.

The analysis of imperfectly competitive economies in which intermediate goods are employed has received little attention in the economics literature, a result, perhaps, of the pessimistic comments of Hart (1985) and Roberts and Sonnenschein (1977). A partial exception has been the work on vertical integration, for instance Panzar and Sibley (1989), but that work has been based firmly within a partial equilibrium framework. The intention here is to present a general equilibrium model with explicit profit and utility maximisation and a complete "circular flow" of income. The model has two produced goods with each good produced by a single firm. Each firm produces using as inputs labour and the output of the other firm. Although specialised, this model captures all the essential features of the situation.

One particular area of focus is the consequence of price discrimination between final and intermediate consumers. It is commonly observed that many goods are marketed with two distinct prices: one for final consumers of the good and a lower "trade" price for producers wishing to use the good as an input. The existing literature is at a loss to provide a convincing model of this phenomenon but it is clear that it will be the natural outcome in the presence of imperfect competition if producers are able to distinguish between the two classes of customer. Given that this practice occurs, it is natural to investigate its welfare consequences and to consider whether the model provides a justification for why trade prices are observed to be lower than final prices.

The results demonstrate that the model generates expected conclusions when no price discrimination occurs, although the extent to which the substitution of labour for produced input takes place is slightly surprising. In contrast, the results that emerge when price discrimination is allowed, and the relation of these to the no-discrimination case, are in some cases rather unexpected. That both firms can lose and the consumer can gain through price discrimination is contrary to expectations. The same is also true when collusion is permitted; two results of note being that both firms and the consumer can gain by collusion in the setting of trade prices and that trade prices may be below marginal cost.

The paper is organised as follows. Section 2 provides a formal description of the model, derives the objective demand functions and characterises the three equilibria. Section 3 describes the specification employed in the numerical simulations. Results of the simulations are given in section 4 and section 5 contains the conclusions.

## 2. Description of model

This section describes the structure of the model and the derivation of the aggregate demand functions. The three equilibrium concepts that are employed are also introduced.

The model has two firms, labelled 1 and 2, each producing a single good using labour and the product of the other firm. Two different prices are distinguished: intermediate, or "trade", prices denoted $p_1$, $p_2$ and final consumer prices $q_1$ and $q_2$. If there is no price discrimination then clearly $p_i = q_i$, i = 1, 2. Writing w for the wage rate, the cost function of firm i is given by

$$C^i = C^i(p_j, w, X^i), i, j = 1, 2, i \neq j, \tag{1}$$

where $X^i$ is total production of firm i. Using Shephard's lemma intermediate good demand facing firm j, $X^{jF}$, is given by

$$X^{jF} = \frac{\partial C^i(p_j, w, X^i)}{\partial p_j} \equiv C^i_1(p_j, w, X^i), i, j = 1, 2, i \neq j. \tag{2}$$

The difficulty in providing a notion of objective intermediate demand is clear from (2): $X^{jF}$ is not determined uniquely by the parameters $p_j$, w but is also conditional on $X^i$, which can only be determined by the equilibrium of the system. Hence $X^{jF}$ given above cannot be employed directly in the description of firm i's objective function, a reflection of the circularity in the model. It will be shown below how a derived notion of intermediate demand can be developed that can be used in place of (2).

The demand for final consumption goods and the supply of labour are derived from the actions of a single, aggregate, utility-maximising consumer. Utility is represented by the function

$$U = U(X^{1C}, X^{2C}, L), \tag{3}$$

and the consumer's budget constraint is

$$\pi + wL = q_1X^{1C} + q_2X^{2C}, \tag{4}$$

where $\pi$ is profit income, $X^{iC}$ consumption of good i and L total labour supply. The budget constraint indicates that the consumer is the recipient of the firms' profits, so that

$$\pi = \pi^1 + \pi^2. \tag{5}$$

From (3) and (4), utility maximisation results in demands of the general form

$$X^{jC} = X^{jC}(q_1, q_2, w, \pi) \geq 0, j = 1, 2, \tag{6}$$

where $q_1$ and $q_2$ are the consumer prices. Note that given w and $\pi$ this is uniquely defined by the choice variables of the firms.

To construct the objective aggregate demand functions it is first observed that the system must be consistent with demands equal to production. This implies

$$X^1 = X^{1C}(q_1, q_2, w, \pi) + C_1^2(p_1, w, X^2), \tag{7}$$

and

$$X^2 = X^{2C}(q_1, q_2, w, \pi) + C_1^1(p_2, w, X^1). \tag{8}$$

Substitution of (8) into (7) then yields

$$X^1 = X^{1C}(q_1, q_2, w, \pi) + C_1^2\left(p_1, w, X^{2C}(q_1, q_2, w, \pi) + C_1^1(p_2, w, X^1)\right) \tag{9}$$

If this equation can be solved for $X^1$, the solution will be of the form

$$X^1 = X^1(q_1, q_2, w, p_1, p_2, \pi). \tag{10}$$

Equation (10) represents the derived objective aggregate demand facing firm 1 incorporating the effects of input demand from firm 2. Repeating the construction, the demand facing firm 2 can be written

$$X^2 = X^2(q_1, q_2, w, p_1, p_2, \pi). \tag{11}$$

It follows from the above that objective intermediate demand can now be defined as

$$X^{jF} = X^j(q_1, q_2, w, p_1, p_2, \pi) - X^{jC}(q_1, q_2, w, \pi),\ j = 1, 2. \tag{12}$$

Intermediate demand expressed in this form is dependent upon the choice variables of the firms and can be used to replace (2).

To investigate the existence of a solution to (7) and (8), first assume

A1. For all $q_1$, $q_2$, w > 0, $X^{jC}(q_1, q_2, w, \pi) \geq 0$, j = 1,2.

A2. $C^i(p_j, w, 0) \geq 0$, i, j = 1,2.

Employing A1 and A2, the following lemma provides a sufficient condition for the existence of a solution.

*Lemma.*

There exists a non-negative solution to (7) and (8) if

A3. $1 - C^1_{10}C^2_{10} > 0, \ C^j_{10} \equiv \dfrac{\partial C^j_1}{\partial X^j} \geq 0$, j= 1,2.

*Proof.*

Write (9) in the form

$$g^1(X^1) = X^{1C}(q_1,q_2,w, \pi) + C^2_1\left(p_1,w, X^{2C}(q_1,q_2,w, \pi) + C^1_1(p_2,w,X^1)\right).$$

It is clear that the solution occurs when $X^1 - g^1(X^1) = 0$. Now let $X^1 = 0$. From A1 and A2, for all $p_1$, $p_2$, $q_1$, $q_2$, w, $\pi$, $X^1 - g^1(0) \leq 0$; if it is zero for a particular set of parameter values then a non-negative solution has been found. Now take the case that $X^1 - g^1(0) < 0$. Using A3 $g^1(X^1)$ is a contraction mapping and thus has a unique fixed point which is clearly positive. Furthermore, it follows that this solution is continuously dependent on the parameters. The same construction can be applied to (8). •

The content of the restriction in A3 can be understood by noting that $C^i_{10}$ represents marginal intermediate input use. The condition is therefore satisfied if the production of an extra unit of good j requires less than one unit of good i in the chosen units of measurement. Condition A3 is thus connected to the notion that the system is productive in the sense of being able to produce positive net outputs of both goods. In general, the restriction may hold for some price levels but not at others. In particular, if substitution can take place of intermediate good for labour, at low price levels input demand may be such that it is optimal to employ more than one unit of intermediate good per unit of output.

The form of intermediate demand in (12) is clearly an objective notion. It would have possible to define intermediate demand in a subjective manner following the practice adopted for final demand in Negishi (1961). There are two reasons why this approach was not adopted. Firstly, the choice of functional restrictions for subjective demand and the manner in which the subjective demands are formed by the firms are clearly open to discussion and a specific choice would be essentially arbitrary. Secondly if the firms' beliefs are not to be proved wrong it is natural to assume subjective and objective coincide at the equilibrium, the actual equilibrium is then invariant to the choice.

Using the derived demands, the description of the model is completed by specifying the profit functions

$$\pi^1 = q_1.X^{1C} + p_1.[X^1 - X^{1C}] - C^1, \qquad (13)$$

and

$$\pi^2 = q_2.X^{2C} + p_2.[X^2 - X^{2C}] - C^2. \qquad (14)$$

At this point it is possible to define the three equilibrium concepts that are analysed. The first assumes that there is no price discrimination between intermediate and final consumers. In this case, it is natural to define equilibrium as the Nash equilibrium in choice of prices with payoffs given by (13) and (14). Equilibrium therefore occurs when

$$q_1^* = \text{argmax} \{\pi^1 \mid q_2 = q_2^*\},$$

and

$$q_2^* = \text{argmax} \{\pi^2 \mid q_1 = q_1^*\},$$

with $q_i = p_i$, $i = 1, 2$. Computation of this equilibrium simply requires the simultaneous solution of the first-order conditions.

It is worth commenting on the treatment of income effects, via the entry of profit levels in the demand functions, in this equilibrium and the two that follow. One possibility would be to assume that the firms do not take account of the income effect when maximising, an assumption that is probably appropriate for a large economy. More attractive for the present small economy are the following two possibilities. Either the firms are completely objective and take account of how their actions affect both their profit level and the profit level of the other firm or they take account of the effect of their profit level but treat the other firm's level as fixed (thus forming a "Nash" conjecture on the price and profit of the other firm). However, the linear specification of the model used in the simulation obviates the need to settle on one of these options. If the model were to be approached analytically, a choice would need to be made.

The second equilibrium concept permits price discrimination. For this purpose the model is interpreted as a two - stage game in which the firms first set producer prices and then consumer prices are determined given the known producer prices. Equilibrium is then defined as the perfect equilibrium of this two stage game. The institutional arrangement behind this choice of form is that the firms enter longer-term contracts with their fellow producers and these are settled before consumer prices are determined.

Analytically, the equilibrium is found by obtaining the first-order conditions for optimal choice of $q_1$ and $q_2$ from (13) and (14) conditional on $p_1$ and $p_2$. Solving these will provide solutions

$$q_1 = q_1(p_1, p_2, w), \tag{15}$$

and

$$q_2 = q_2(p_1, p_2, w). \tag{16}$$

Substituting (15) and (16) into the definitions of profit then determine profits in terms of trade prices alone. Solving for the resulting Nash equilibrium gives the choice of trade prices and (15), (16) the consumer prices.

The final equilibrium concept is a variant of the second in that it permits price discrimination but also has an element of collusion. As before, it is assumed that the firms compete in their choice of final prices conditional upon previously selected trade prices. However, trade prices are selected collusively to maximise joint profits given the behaviour of final prices at the second stage. Optimal trade prices therefore solve

$$\max_{\{p_1,p_2\}} \pi^1 + \pi^2 \text{ subject to } q_i = q_i(p_1, p_2, w),\ i = 1, 2.$$

The motivation behind this form of equilibrium is that collusion over final prices is generally prevented by legislation but is rather less well regulated with respect to trade prices. In addition, collusion over final prices is subject to regulation as it is widely viewed to be harmful to the interests of the purchaser but, in direct contrast, collusion over trade prices will usually be welcomed by the "trade" purchaser.

To close this section some comments on the choice of model are offered. The model can be seen as an extension to an imperfectly competitive environment of the standard two-sector competitive model, discussed in detail in Atkinson and Stiglitz (1980). It improves upon the model of Harberger (1962) by including explicit profit maximising behaviour rather than imposing mark-up pricing. Although the assumption of profit maximisation has been subject to some criticism in this form of model, as it is not in the interests of the consumers who are assumed to be the final owners of the firms, it would still appear to be the most natural assumption and a defence of this approach has been offered by Gabszewicz and Vial (1972). It still seems worth noting that if the aggregate consumer is composed of many "small" consumers, each with little direct control over the firms, then profit maximisation seems natural. Even if the single consumer is literally interpreted as the sole owner of the firm (an interpretation I feel should be avoided), it can still be argued that they may have a dichotomous personality and when acting as the owner of the firm myopically seeks to maximise the firm's profits regardless of how this eventually affects their welfare.

It is a natural consequence of having only two sectors that the interactions and linkages between the two firms will be exaggerated compared to any larger model. In defence of this, it does have the benefit of highlighting the causes and effects involved in the model. Finally, with respect to the choice of equilibrium concepts, the first two reflect the obvious possibility, or otherwise, of price discrimination. The third is rather a hybrid but its consequences will be shown to be rather interesting and may go some way to explaining the observed relation between trade and producer prices. Other equilibria could also be defined but the intention here is to illustrate possibilities rather than to be encyclopaedic.

## 3. Simulation specification

Although conceptually simple, the above model is only analytically tractable in a limited number of special cases. It therefore appears more valuable to investigate the model via numerical simulation. In constructing the simulation two general factors are captured: the elasticity of substitution between labour and produced goods as inputs and the complementarity/substitutability on the final goods markets.

To allow the degree of substitutability to vary in production, the cost function for firm i is chosen to be of the C.E.S. form

$$C^i\left(p_j,\ w,\ X^i\right) = K_i\left[(m_i)^{1/1-\rho_i} p_j^{\rho_i/\rho_i-1} + (1-m_i)^{1/1-\rho_i} w^{\rho_i/\rho_i-1}\right]^{\rho_i-1/\rho_i} X^i \equiv C^i(p_j,\ w).X^i. \quad (17)$$

$\rho_i$ and $m_i$ are the parameters that define the underlying production function. By varying $\rho_i$ it is possible to vary the elasticity of substitution between the intermediate input and labour. When $\rho_i$ is 1, the technology is linear with an infinite elasticity of substitution. It is Cobb-Douglas, with unit elasticity, at $\rho_i = 0$ and cost function

$$C^i\left(p_j, w, X^i\right) = K_i\left[p_j{}^{m_i} w^{(1-m_i)}\right] X^i.$$

At the other extreme, costs are Leontief as $\rho_i \to -\infty$, so the elasticity of substitution is zero, with cost function

$$C^i\left(p_j, w, X^i\right) = K_i\left[p_j + w\right] X^i.$$

To satisfy the restriction on productivity of the system, $K_i$, the input of intermediate good per unit of output in the Leontief case, must be less than 1. It follows from (17) that the intermediate demand facing j is

$$X^{jF} = K_i\left[(m_i)^{1/_{1-\rho_i}} p_j{}^{\rho_i/_{\rho_i-1}} + (1-m_i)^{1/_{1-\rho_i}} w^{\rho_i/_{\rho_i-1}}\right]^{-1/_{\rho_i}} (m_i)^{1/_{1-\rho_i}} (p_j)^{1/_{\rho_i-1}} X^i \equiv c^i(p_j, w).X^i. \quad (18)$$

The assumed form of utility function is

$$U = \alpha_1 X^{1C} - \frac{\beta_1 X^{1C2}}{2} + \alpha_2 X^{2C} - \frac{\beta_2 X^{2C2}}{2} + \delta X^{1C} X^{2C} - L, \quad (19)$$

where $\alpha_i$, $\beta_i$, i =1, 2 are positive constants. By varying the value of $\delta$, it is possible to investigate the consequences of alternative substitutability and complementarity relations between the two goods. It should also be noted that additivity in labour supply eliminates income effects.

From (19), consumer demand is determined by the linear demand functions

$$X^{1C} = a_1 - b_1 q_1 + d_1 q_2, \quad (20)$$

and

$$X^{2C} = a_2 + b_2 q_1 - d_2 q_2, \quad (21)$$

where

$$a_1 = \frac{\alpha_2\delta+\beta_2\alpha_1}{\beta_1\beta_2-\delta\delta}, \quad b_1 = \left(\frac{1}{w}\right)\frac{\beta_2}{\beta_1\beta_2-\delta\delta}, \quad d_1 = \left(\frac{1}{w}\right)\frac{-\delta}{\beta_1\beta_2-\delta\delta},$$

$$a_2 = \frac{\alpha_1\delta+\beta_1\alpha_2}{\beta_1\beta_2-\delta\delta}, \quad b_2 = \left(\frac{1}{w}\right)\frac{-\delta}{\beta_1\beta_2-\delta\delta}, \quad d_2 = \left(\frac{1}{w}\right)\frac{\beta_1}{\beta_1\beta_2-\delta\delta}.$$

It is clear from these that the two goods are gross substitutes if $\delta < 0$ and gross complements if $\delta > 0$. In addition, the parameters must be restricted so that the inequality $\beta_1\beta_2-\delta\delta > 0$ is satisfied.

While the utility function above is fairly flexible, the linear structure of demand is more restrictive than would be ideal. However, although the linearity is not required for the analysis of the no-discrimination model, it is one of the few forms that permits explicit solution of the second stage of the two-stage models.

Combining the specifications of intermediate demand and solving as in (9) and (10), the objective aggregate demands are found to be

$$X^1 = \left(\frac{a_1+c^2a_2}{1-c^1c^2}\right) - \left(\frac{b_1-c^2b_2}{1-c^1c^2}\right).q_1 + \left(\frac{d_1-c^2d_2}{1-c^1c^2}\right).q_2, \tag{22}$$

and

$$X^2 = \left(\frac{a_2+c^1a_1}{1-c^1c^2}\right) + \left(\frac{b_2-c^1b_1}{1-c^1c^2}\right).q_1 - \left(\frac{d_2-c^1d_1}{1-c^1c^2}\right).q_2. \tag{23}$$

Note that these demands are dependent upon trade prices via the terms $c^1$ and $c^2$ from the first derivatives of the cost functions.

The characterisation of equilibrium for the no-discrimination model can be calculated by substituting from (19) - (23) into (12) and (13), differentiating with respect to the choice variables and solving the resulting simultaneous equations.

For the two-stage, price discrimination model, with trade prices taken as given the optimal values of $q_1$ and $q_2$ can be found to take the form

$$q_j = h_j + j_j(p_1) + k_j(p_2),\ j = 1, 2. \quad (24)$$

Substitution of these forms into the profit functions then allows the optimal trade prices to be calculated by an iterative procedure. In the case of Leontief costs, equations (24) are affine functions of the producer prices and a closed form solution can be derived.

The collusive equilibrium can be calculated by substituting the functions (24) into the definition of profits and then choosing producer prices to maximise the sum of profits.

## 4. Simulation results

The simulations reported below impose one further restriction on the model: only symmetric equilibria are considered. This restriction greatly simplifies the computation. Nine tables of results are given. The first three relate to the no-discrimination case and the remainder to the model with discrimination, with the no-collusion results being presented first. In each case the tables are distinguished by the value of $\delta$, which captures the substitutability/complementarity relation. For each value of $\delta$ equilibrium prices and quantities are given for a range of values of $\rho$ from 0.99, representing an elasticity of substitution close to infinity, to -10000 which is almost a zero elasticity of substitution.

In all tables $X^C$, $X^F$ and X represent respectively final, intermediate and total consumption of each good. As the equilibria are symmetric, the prices and quantities are the same for both goods. L is total labour use and profit is that of a single firm.

The remaining parameter values, which are constant throughout, are as follows:

$$a_1 = a_2 = 2000,\ b_1 = b_2 = 0.8,\ m_1 = m_2 = 0.5,\ K_1 = K_2 = 0.5,\ w = 1.$$

It should be noted that as the model possesses standard homogeneity properties, the choice of value for w represents a normalisation procedure for the nominal variables.

Table 1. $\delta$ = -.4 (Gross Substitutes), no discrimination.

| $\rho$ | p | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|
| .99 | 667.3 | 1110.5 | 0 | 1110.5 | 2237 | 740000 | 2959967 |
| .9 | 667.4 | 1110.5 | 0 | 1110.5 | 2398 | 739948 | 2959948 |
| .8 | 667.5 | 1110.4 | 0 | 1110.4 | 2641 | 739882 | 2959395 |
| .7 | 667.6 | 1110.3 | 5.8E-7 | 1110.3 | 2989 | 739764 | 2958936 |
| .6 | 667.7 | 1110.2 | 1.5E-4 | 1110.2 | 3524 | 739579 | 2958234 |
| .5 | 668.0 | 1110.0 | 4.9E-3 | 1110.0 | 4427 | 739266 | 2957053 |
| .4 | 668.6 | 1109.5 | 0.05 | 1109.5 | 6076 | 738773 | 2954746 |
| .3 | 670.3 | 1108.0 | 0.42 | 1108.4 | 9158 | 738196 | 2949701 |
| .2 | 677.5 | 1102.0 | 2.08 | 1104.1 | 14438 | 739469 | 2936333 |
| .1 | 700.9 | 1082.6 | 7.48 | 1090.1 | 21715 | 747925 | 2902234 |
| .08 | 708.6 | 1076.2 | 9.28 | 1085.4 | 23262 | 750941 | 2891643 |
| .05 | 722.0 | 1065.0 | 12.51 | 1077.5 | 25542 | 756159 | 2873387 |
| .01 | 743.4 | 1047.2 | 17.86 | 1065.0 | 28397 | 764265 | 2844400 |
| -.01 | 755.4 | 1037.2 | 20.99 | 1058.2 | 29700 | 768626 | 2828109 |
| -.05 | 781.2 | 1015.7 | 28.11 | 1043.8 | 31981 | 777448 | 2792791 |
| -.1 | 815.6 | 987.0 | 38.47 | 1025.5 | 34112 | 787941 | 2744885 |
| -.2 | 883.7 | 930.2 | 62.85 | 993.1 | 35858 | 804132 | 2646703 |
| -.5 | 1025.8 | 811.8 | 144.4 | 956.2 | 29376 | 818090 | 2427069 |
| -.8 | 1092.6 | 756.2 | 215.8 | 971.9 | 21042 | 815666 | 2317479 |
| -1 | 1118.6 | 734.5 | 254.7 | 989.2 | 17037 | 813093 | 2273575 |
| -2 | 1184.1 | 679.9 | 374.4 | 1054.3 | 7922 | 801128 | 2157000 |
| -5 | 1252.8 | 622.7 | 480.4 | 1103.0 | 3154 | 778499 | 2022255 |
| -10 | 1288.3 | 593.1 | 518.8 | 1111.9 | 1989 | 763074 | 1948246 |
| -100 | 1328.7 | 559.4 | 551.7 | 1111.2 | 1185 | 742704 | 1860945 |
| -1000 | 1333.2 | 555.7 | 554.9 | 1110.6 | 1118 | 740255 | 1851030 |
| -10000 | 1333.7 | 555.2 | 555.2 | 1110.4 | 1111 | 739981 | 1849925 |

Table 2. $\delta = 0$, no discrimination.

| $\rho$ | p | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|
| .99 | 1000.6 | 1249.2 | 0 | 1249.2 | 2516 | 1248742 | 3745983 |
| .9 | 1000.6 | 1249.2 | 0 | 1249.2 | 2698 | 1248650 | 3745801 |
| .8 | 1000.6 | 1249.2 | 0 | 1249.2 | 2971 | 1248514 | 3745528 |
| .7 | 1000.7 | 1249.1 | 1.7E-7 | 1249.1 | 3362 | 1248318 | 3744887 |
| .6 | 1000.8 | 1249.0 | 6.3E-5 | 1249.0 | 3965 | 1248017 | 3744034 |
| .5 | 1001.0 | 1248.7 | 2.5E-3 | 1248.7 | 4985 | 1247506 | 3742514 |
| .4 | 1001.5 | 1248.1 | 0.03 | 1248.2 | 6887 | 1246553 | 3739360 |
| .3 | 1002.6 | 1246.7 | 0.27 | 1247.0 | 10624 | 1244680 | 3732868 |
| .2 | 1007.8 | 1240.2 | 1.55 | 1241.8 | 17556 | 1241146 | 3712868 |
| .1 | 1027.1 | 1216.1 | 6.28 | 1222.4 | 27908 | 1235128 | 3653424 |
| .08 | 1033.9 | 1207.6 | 7.98 | 1215.6 | 30168 | 1233479 | 3633645 |
| .05 | 1046.3 | 1192.1 | 11.10 | 1203.2 | 33500 | 1230570 | 3598070 |
| .01 | 1066.6 | 1166.7 | 16.44 | 1183.2 | 37621 | 1225645 | 3540335 |
| -.01 | 1078.2 | 1152.2 | 19.61 | 1171.9 | 39460 | 1222626 | 3507396 |
| -.05 | 1103.4 | 1120.7 | 26.92 | 1147.7 | 42563 | 1215354 | 3435572 |
| -.1 | 1136.8 | 1079.0 | 37.70 | 1116.7 | 45217 | 1203999 | 3339390 |
| -.2 | 1201.0 | 998.7 | 63.22 | 1062.0 | 46576 | 1176211 | 3150423 |
| -.5 | 1318.2 | 852.2 | 149.0 | 1001.3 | 35834 | 1105519 | 2792102 |
| -.8 | 1361.9 | 797.6 | 226.2 | 1023.8 | 24926 | 1073823 | 2656610 |
| -1 | 1376.4 | 779.5 | 269.3 | 1048.8 | 19979 | 1062914 | 2611925 |
| -2 | 1409.2 | 738.5 | 406.4 | 1144.9 | 9112 | 1036138 | 2508582 |
| -5 | 1447.9 | 690.1 | 532.4 | 1222.5 | 3581 | 997441 | 2375901 |
| -10 | 1470.3 | 662.1 | 579.1 | 1241.3 | 2248 | 972398 | 2295524 |
| -100 | 1496.9 | 628.9 | 620.2 | 1249.1 | 1334 | 940696 | 2197779 |
| -1000 | 1500.0 | 625.0 | 624.1 | 1249.1 | 1257 | 936871 | 2186243 |
| -10000 | 1500.3 | 624.6 | 624.5 | 1249.2 | 1249 | 936499 | 2185125 |

Table 3. $\delta = .4$ (Gross Complements), no discrimination.

| ρ | p | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|
| .99 | 1200.5 | 1998.7 | 0 | 1998.7 | 4025 | 2397487 | 6392974 |
| .9 | 1200.5 | 1998.7 | 0 | 1998.7 | 4317 | 2397340 | 6392682 |
| .8 | 1200.5 | 1998.7 | 0 | 1998.7 | 4754 | 2397122 | 6392245 |
| .7 | 1200.6 | 1998.5 | 1.5E-7 | 1998.5 | 5380 | 2396709 | 6391019 |
| .6 | 1200.7 | 1998.2 | 6.3E-5 | 1998.2 | 6344 | 2396127 | 6389455 |
| .5 | 1200.8 | 1998.0 | 2.8E-3 | 1998.0 | 7979 | 2395209 | 6387220 |
| .4 | 1201.2 | 1997.0 | 0.04 | 1997.0 | 11051 | 2393271 | 6381745 |
| .3 | 1202.1 | 1994.7 | 0.34 | 1995.1 | 17207 | 2389285 | 6370182 |
| .2 | 1205.9 | 1985.2 | 2.04 | 1987.3 | 29043 | 2379491 | 6335470 |
| .1 | 1221.5 | 1946.2 | 8.81 | 1955.1 | 47419 | 2353635 | 6222425 |
| .08 | 1227.3 | 1931.7 | 11.30 | 1943.0 | 51490 | 2345092 | 6182847 |
| .05 | 1237.8 | 1905.5 | 15.97 | 1921.5 | 57506 | 2329875 | 6112122 |
| .01 | 1255.3 | 1861.7 | 24.06 | 1885.8 | 64913 | 2304598 | 5995642 |
| -.01 | 1265.3 | 1836.7 | 28.92 | 1865.7 | 68190 | 2289945 | 5929350 |
| -.05 | 1287.1 | 1782.2 | 40.22 | 1822.5 | 73628 | 2257120 | 5784806 |
| -.1 | 1315.9 | 1710.2 | 57.00 | 1767.2 | 78085 | 2211476 | 5592933 |
| -.2 | 1369.8 | 1575.5 | 97.06 | 1672.6 | 79790 | 2118225 | 5229330 |
| -.5 | 1458.5 | 1353.7 | 235.2 | 1588.9 | 60495 | 1944197 | 4621449 |
| -.8 | 1484.6 | 1288.5 | 364.5 | 1653.0 | 42143 | 1891835 | 4447764 |
| -1 | 1491.4 | 1271.5 | 438.6 | 1710.1 | 33876 | 1879377 | 4405439 |
| -2 | 1504.7 | 1238.2 | 681.2 | 1919.5 | 15612 | 1855389 | 4324083 |
| -5 | 1527.2 | 1182.0 | 911.8 | 2093.8 | 6188 | 1802057 | 4162962 |
| -10 | 1542.9 | 1142.7 | 999.5 | 2142.3 | 3896 | 1761201 | 4044752 |
| -100 | 1562.9 | 1092.7 | 1077.8 | 2318.5 | 2318 | 1706700 | 3891041 |
| -1000 | 1565.2 | 1087.0 | 1085.5 | 2172.5 | 2187 | 1700279 | 3873185 |
| -10000 | 1565.5 | 1086.2 | 1086.1 | 2172.3 | 2173 | 1699437 | 3870851 |

Reviewing tables 1 - 3, it can be seen that these display generally the same pattern of results. The equilibrium price rises as the elasticity of substitution falls, explained by aggregate demand becoming less elastic, and this is accompanied by a fall in final demand. At high elasticities of substitution no intermediate input is used and production is entirely by labour alone. Labour use rises at first and then falls, it is highest around the Cobb-Douglas case of $\rho = 0$. At these values around zero, production is taking place at extreme points on the isoquants: labour is being substituted for intermediate input despite the low degree of substitutability.

For $\delta$ equal to 0 and 0.4, profits fall monotonically as $\rho$ falls. The firms therefore prefer the high elasticity of substitution, because it prevents them being exploited by their rival, despite the reduction it causes in their own market power. As $\rho$ falls the firms are able to capture each other's surplus, a process that ultimately reduces the welfare of both. In contrast, for $\delta = -0.4$ profit first falls then rises, with a peak at $\rho = -0.5$, and then falls again. The period of rising profit coincides with a rapidly rising price and substitution of intermediate input for labour.

Utility is highest in all cases when the elasticity of substitution is high, the consumer benefits from the firms being unable to exploit each other's demand for intermediate input. The consumer thus prefers a high elasticity of substitution. It is interesting to compare the reasoning behind this conclusion with the motivation for a similar result in Ireland (1989).

Finally, contrasting the tables, the prices are lowest when the goods are substitutes on the final market and highest when they are complements - the expected conclusion.

Table 4. $\delta = -.4$ (Gross Substitutes), price discrimination.

| $\rho$ | p | q | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|---|
| .99 | 16.4 | 667.3 | 1110.5 | 0 | 1110.5 | 2237 | 739994 | 2959976 |
| .9 | 21.8 | 667.4 | 1110.5 | 0 | 1110.5 | 2399 | 739940 | 2959763 |
| .8 | 34.5 | 667.5 | 1110.4 | 2.7E-5 | 1110.4 | 2641 | 739860 | 2959440 |
| .7 | 64.7 | 667.6 | 1110.3 | 1.3E-3 | 1110.3 | 2989 | 739744 | 2958978 |
| .6 | 163.0 | 667.7 | 1110.2 | 5.1E-3 | 1110.2 | 3522 | 739566 | 2958271 |
| .5 | 692.5 | 668.0 | 1110.0 | 4.6E-3 | 1110.0 | 4427 | 739265 | 2957060 |
| .4 | 8418.1 | 668.5 | 1109.5 | 8.9E-4 | 1109.5 | 6239 | 738663 | 2954632 |
| .3 | 1448986 | 670.0 | 1108.3 | 8.7E-6 | 1108.3 | 11086 | 737046 | 2948149 |
| .2* | 8.7E9 | 677.2 | 1102.3 | 1.0E-8 | 1102.3 | 34703 | 729110 | 2916469 |
| .1* | 4.6E12 | 908.2 | 908.2 | 0 | 908.2 | 634408 | 474691 | 2011571 |
| .08* | 1.5E13 | 1539.4 | 383.8 | 0 | 383.8 | 937895 | 11894 | 420629 |
| .05* | 3.5E7 | 1898.7 | 84.4 | 1.0E-4 | 84.4 | 203539 | 58450 | 125445 |
| .01 | 3.9E6 | 1878.4 | 101.3 | 0.01 | 101.3 | 161091 | 109747 | 231875 |
| -.01 | 1229608 | 1860.2 | 116.0 | 0.07 | 116.1 | 152737 | 139452 | 295311 |
| -.05 | 283351 | 1846.6 | 127.8 | 0.39 | 128.2 | 122082 | 174982 | 371015 |
| -.1 | 95464 | 1836.2 | 136.5 | 1.38 | 137.9 | 92919 | 204218 | 435868 |
| -.2 | 30147 | 1830.2 | 141.5 | 5.22 | 146.7 | 56507 | 230719 | 504571 |
| -.5 | 9308.7 | 1827.9 | 143.4 | 22.79 | 166.2 | 20177 | 252049 | 611500 |
| -.8 | 6289.3 | 1799.6 | 167.0 | 45.91 | 212.9 | 11838 | 294629 | 785441 |
| -1 | 5429.7 | 1776.0 | 186.7 | 63.35 | 250.0 | 9337 | 326875 | 915801 |
| -2 | 3921.8 | 1696.1 | 253.3 | 138.94 | 392.2 | 4382 | 427369 | 1379874 |
| -5 | 3237.6 | 1649.8 | 291.9 | 225.05 | 516.9 | 1731 | 480631 | 1745264 |
| -10 | 3075.5 | 1645.7 | 295.2 | 258.22 | 553.4 | 1072 | 485359 | 1845224 |
| -100 | 2966.0 | 1651.1 | 290.7 | 286.73 | 577.4 | 621 | 479714 | 1909076 |
| -1000 | 2957.1 | 1652.2 | 289.8 | 289.39 | 579.2 | 583 | 478519 | 1913615 |
| -10000 | 2956.2 | 1652.3 | 289.7 | 289.66 | 579.4 | 580 | 478404 | 1914093 |

Table 5. $\delta = 0$, price discrimination.

| ρ | p | q | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|---|
| .99 | 16.5 | 1000.5 | 1249.4 | 0 | 1249.4 | 2516 | 1248741 | 3746225 |
| .9 | 21.6 | 1000.5 | 1249.3 | 0 | 1249.3 | 2699 | 1248650 | 3745951 |
| .8 | 34.0 | 1000.6 | 1249.3 | 3.3E-5 | 1249.3 | 2971 | 1248514 | 3745542 |
| .7 | 61.5 | 1000.7 | 1249.2 | 1.8E-3 | 1249.2 | 3362 | 1248318 | 3744959 |
| .6 | 143.5 | 1000.8 | 1249.0 | 8.0E-3 | 1249.0 | 3961 | 1248018 | 3744070 |
| .5 | 413.1 | 1001.0 | 1248.7 | 9.4E-3 | 1248.8 | 4976 | 1247511 | 3742547 |
| .4 | 4030.2 | 1001.4 | 1248.2 | 3.4E-3 | 1248.2 | 6992 | 1246502 | 3739498 |
| .3 | 169037 | 1002.5 | 1246.9 | 2.1E-4 | 1246.9 | 12330 | 1243827 | 3731447 |
| .2* | 5.1E8 | 1007.8 | 1240.3 | 2.4E-7 | 1240.3 | 38394 | 1230725 | 3692054 |
| .1* | 6.6E12 | 1183.6 | 1020.5 | 0 | 1020.5 | 722221 | 813370 | 2526691 |
| .08* | 4.5E13 | 1700.1 | 374.8 | 0 | 374.8 | 985412 | -76813 | 401442 |
| .05* | 6.3E8 | 1949.3 | 63.4 | 4.9E-5 | 63.4 | 179008 | 34026 | 71262 |
| .01 | 6.6E6 | 1943.2 | 71.0 | 9.2E-3 | 71.0 | 144493 | 65650 | 135364 |
| -.01 | 2.1E6 | 1939.4 | 75.8 | 0.04 | 75.8 | 132265 | 80874 | 166487 |
| -.05 | 471866 | 1932.0 | 84.9 | 0.22 | 85.2 | 111082 | 108562 | 223744 |
| -.1 | 155969 | 1924.7 | 94.1 | 0.85 | 94.9 | 89143 | 136525 | 283394 |
| -.2 | 48036 | 1916.2 | 104.7 | 3.65 | 108.4 | 58111 | 171601 | 365941 |
| -.5 | 14209 | 1888.5 | 139.4 | 21.86 | 161.3 | 25644 | 250441 | 598598 |
| -.8 | 9241.2 | 1834.9 | 206.4 | 56.45 | 262.8 | 18024 | 369660 | 977989 |
| -1 | 7790.1 | 1797.9 | 252.5 | 85.45 | 338.0 | 15084 | 446503 | 1245470 |
| -2 | 5195.3 | 1682.9 | 396.4 | 217.4 | 613.8 | 7530 | 663367 | 2146274 |
| -5 | 4034.6 | 1612.9 | 483.8 | 373.0 | 856.9 | 2977 | 778890 | 2837127 |
| -10 | 3775.4 | 1601.1 | 498.6 | 436.0 | 934.6 | 1844 | 797346 | 3037723 |
| -100 | 3612.8 | 1599.7 | 500.4 | 493.5 | 993.9 | 1070 | 799896 | 3184154 |
| -1000 | 3600.5 | 1600.1 | 499.8 | 499.1 | 998.9 | 1006 | 799284 | 3186481 |
| -10000 | 3599.3 | 1600.2 | 499.8 | 499.7 | 1000.2 | 1000 | 799216 | 3197698 |

Table 6. $\delta = .4$ (Gross Complements), price discrimination.

| ρ | p | q | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|---|
| .99 | 16.4 | 1200.4 | 1999.0 | 0 | 1999.0 | 4026 | 2397584 | 6393557 |
| .9 | 20.8 | 1200.4 | 1999.0 | 0 | 1999.0 | 4318 | 2397408 | 6393089 |
| .8 | 33.4 | 1200.5 | 1998.8 | 5.7E-5 | 1998.8 | 4754 | 2397147 | 6392391 |
| .7 | 58.5 | 1200.5 | 1998.6 | 3.5E-3 | 1998.7 | 5379 | 2396771 | 6391398 |
| .6 | 128.0 | 1200.6 | 1998.4 | 0.02 | 1998.4 | 6337 | 2396196 | 6389896 |
| .5 | 401.5 | 1200.8 | 1998.0 | 0.02 | 1998.0 | 7952 | 2395225 | 6387323 |
| .4 | 2391.1 | 1201.1 | 1997.2 | 0.01 | 1997.2 | 11141 | 2393307 | 6382174 |
| .3 | 52567 | 1202.0 | 1995.1 | 1.7E-3 | 1995.1 | 19487 | 2388284 | 6368732 |
| .2* | 3.2E7 | 1206.0 | 1984.9 | 1.2E-5 | 1984.9 | 594589 | 2364149 | 6304266 |
| .1* | 1.3E13 | 1350.4 | 1624.0 | 0 | 1624.0 | 1180006 | 1546067 | 4261101 |
| .08* | 9.7E12 | 1685.1 | 787.3 | 8.0E-9 | 787.3 | 1862820 | 401726 | 1038377 |
| .05* | 1.0E9 | 1946.5 | 133.6 | 7.2E-5 | 133.6 | 427059 | 46575 | 100312 |
| .01 | 1.0E7 | 1902.1 | 244.6 | 0.02 | 244.7 | 602833 | 163933 | 351904 |
| -.01 | 3.3E6 | 1909.0 | 227.7 | 0.09 | 227.8 | 510936 | 179165 | 379404 |
| -.05 | 763731 | 1887.2 | 282.1 | 0.62 | 282.7 | 497614 | 283558 | 601292 |
| -.1 | 253674 | 1865.5 | 336.1 | 2.71 | 338.8 | 443647 | 405225 | 865751 |
| -.2 | 78193 | 1837.2 | 406.9 | 13.4 | 420.3 | 319946 | 587679 | 1290686 |
| -.5 | 22739 | 1784.1 | 539.9 | 83.5 | 623.4 | 134070 | 896094 | 2203982 |
| -.8 | 14300 | 1724.3 | 689.3 | 187.6 | 876.9 | 76356 | 1150307 | 3123614 |
| -1 | 11745 | 1686.4 | 784.0 | 264.5 | 1048.5 | 57340 | 1293412 | 3696933 |
| -2 | 7044.6 | 1572.0 | 1070.0 | 586.4 | 1656.3 | 22482 | 1670690 | 5505351 |
| -5 | 4957.4 | 1501.8 | 1245.5 | 960.3 | 2205.8 | 7931 | 1866524 | 6869000 |
| -10 | 4521.7 | 1488.3 | 1279.2 | 1118.7 | 2398.0 | 4809 | 1901447 | 7286843 |
| -100 | 4275.2 | 1484.1 | 1289.9 | 1272.2 | 2562.0 | 2764 | 1912843 | 7619758 |
| -1000 | 4258.6 | 1484.1 | 1289.7 | 1287.9 | 2577.7 | 2597 | 1912792 | 7650307 |
| -10000 | 4257.1 | 1484.1 | 1289.7 | 1289.5 | 2579.2 | 2581 | 1912762 | 7653282 |

The results presented in tables 4 - 6 also have a similar general pattern but there are a number of interesting differences when they are viewed closely. In all cases trade prices first rise, almost become unbounded and then fall, the asterisks indicate difficulties with achieving convergence. The reason for this behaviour is that these prices are determined by the resolution of two factors: the intermediate good demand and the effect on the final market price. Without the latter of these effects, trade prices would be unbounded for values of $\rho$ less than or equal to zero. The fact they are bounded in the results above is a consequence of the second effect but the strength of this effect is also dependent on $\rho$; for positive values of $\rho$ it is clearly very weak. The ranking of trade prices across the values of $\delta$ is of some interest. For high values of $\rho$ they are lowest when the goods are complements but for high values the ranking is reversed and they are lowest for the substitutes case.

Final consumer prices at the highest elasticities of substitution are very similar to the no-discrimination case. This indicates that for these values of $\rho$, the trade prices are chosen only to achieve the optimal consumer prices and have no other purpose since no intermediate inputs are actually employed. The ranking of final consumer prices bears the same relation to $\delta$ as in the no-discrimination case for high elasticities but is reversed as the elasticity falls; for the Leontief case final prices are lower when the goods are complements and highest when they are substitutes. This is the converse of the no-discrimination result and contrary to the natural expectation.

Contrasting trade and final prices it can be seen that trade prices are not always lower than final prices. The strategic importance of the trade price eliminates the incentive to charge a low price in order to receive a lower-priced input in return. Trade prices are only lower at very high elasticities of substitution when little or no intermediate input is used.

Final consumption mirrors that for no-discrimination: falling and then rising. In contrast intermediate demand starts at zero, becomes slightly positive, falls to zero again and then rises, reaching a maximum either at or just before the Leontief case. Aggregate consumption mirrors final consumption. Labour use begins low, rises to a maximum and then falls to reach a minimum at the zero elasticity point. Around the Cobb-Douglas point, labour use is high and production is taking place on the tail of the relevant isoquant.

Profits fall, rise and then fall slightly again. When trade prices reach their maximum, profits are markedly reduced: even to a negative value in one case (this has been left in the table despite the question mark that hangs over its interpretation). The strategies of the firms are therefore having a "beggar thy neighbour" effect with the firms inflicting considerable harm on each other.

For all values of $\delta$ utility falls and then rises. As $\delta$ increases, the value of utility at $\rho = 0.99$ and $\rho = -10000$ becomes closer. For $\delta = -0.4$, utility is at a maximum when the elasticity of substitution is high, as for the no discrimination case, but for $\delta = 0.4$ it is highest in the zero elasticity case, in contrast to previous findings. The consumer is obviously better-off with extremes of substitutability since this reduces the firms' market power but precisely which extreme depends on the form of demand. For the complements case, the high elasticity eliminates market power on the intermediate market but for the substitutes case the dominating effect seems to be via the effect upon the final goods market so that a low elasticity is preferred.

Comparing across the no-discrimination and discrimination cases for given values of $\delta$, it can be seen that the final market price is lower for the no-discrimination case for $\delta$ equal to -0.4 and 0. In contrast, the final market price is higher with no-discrimination when $\delta = 0.4$. Final consumption has the same pattern: it is higher for no-discrimination for the two lower values of $\delta$ but lower when the goods are substitutes. Consequently, the final market pricing and consumption depend on both the structure of demand and the institutional arrangement of the market.

Labour use at the extreme elasticities of substitution corresponds to relative rankings of final consumption but around $\rho = 0$ is always substantially higher in the price-discrimination case. This reflects the high levels of producer prices forcing production around the isoquants, with labour substituted for intermediate input despite technologies which do not encourage substitution.

With a minor number of exceptions, most noticeably close to the Leontief technology with the goods as substitutes, profits are greater when there is no price discrimination. Price discrimination is therefore harmful to the firms since it results in high levels of trade prices and the use of techniques that replace intermediate input with labour. In general the firms would actually welcome the elimination of price discrimination.

For all three values of $\delta$, utility is greater for the no-discrimination case for high elasticities of substitution than for price discrimination. The ranking is reversed for low elasticities of substitution. Consequently, whether a utilitarian government should encourage or eliminate price discrimination is highly dependent upon the elasticity of substitution.

Table 7. $\delta = -.4$ (Gross Substitutes), collusion.

| $\rho$ | p | q | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|---|
| .99 | 1.01 | 667.3 | 1110.5 | 1110.5 | 2221.1 | 2221 | 740000 | 2962222 |
| .9 | 1.03 | 667.3 | 1110.5 | 827.9 | 1938.4 | 2225 | 740003 | 3242462 |
| .8 | 1.08 | 667.4 | 1110.5 | 760.5 | 1871.0 | 2235 | 740007 | 3280997 |
| .7 | 1.14 | 667.4 | 1110.5 | 725.9 | 1836.4 | 2247 | 740012 | 3296505 |
| .6 | 1.20 | 667.4 | 1110.5 | 715.9 | 1826.4 | 2258 | 740016 | 3300465 |
| .5 | 1.29 | 667.4 | 1110.5 | 683.6 | 1794.1 | 2279 | 740021 | 3311558 |
| .4 | 1.40 | 667.5 | 1110.4 | 657.4 | 1767.8 | 2304 | 740027 | 3318744 |
| .3 | 1.52 | 667.5 | 1110.4 | 640.5 | 1750.9 | 2330 | 740034 | 3322479 |
| .2 | 1.69 | 667.5 | 1110.4 | 614.7 | 1725.1 | 2369 | 740042 | 3326863 |
| .1 | 1.93 | 667.6 | 1110.3 | 584.0 | 1694.3 | 2425 | 740051 | 3329988 |
| .08 | 2.00 | 667.6 | 1110.3 | 574.8 | 1685.1 | 2442 | 740053 | 3330482 |
| .05 | 2.12 | 667.7 | 1110.3 | 560.2 | 1670.5 | 2471 | 740057 | 3330844 |
| .01 | 2.31 | 667.7 | 1110.2 | 538.4 | 1648.6 | 2520 | 740062 | 3330424 |
| -.01 | 2.45 | 667.7 | 1110.2 | 525.3 | 1635.5 | 2551 | 740065 | 3329615 |
| -.05 | 2.82 | 667.8 | 1110.1 | 491.9 | 1602.0 | 2641 | 740072 | 3325681 |
| -.05043 | 2.82 | 667.8 | 1110.1 | 492.4 | 1602.6 | 2640 | 740073 | 3325764 |
| | 2680.5 | 730.7 | 1057.8 | 17.9 | 1075.6 | 65641 | | 2848525 |
| -.1 | 7570.3 | 852.3 | 956.4 | 18.8 | 975.2 | 126134 | 752087 | 2633416 |
| -.2 | 5210.1 | 932.5 | 889.6 | 42.7 | 932.3 | 106957 | 776053 | 2579318 |
| -.5 | 2383.1 | 983.7 | 846.9 | 143.0 | 989.9 | 51011 | 807605 | 2732702 |
| -.8 | 1767.9 | 993.9 | 838.4 | 236.1 | 1074.5 | 30081 | 818261 | 2882527 |
| -1 | 1586.1 | 996.6 | 836.2 | 288.1 | 1124.3 | 22951 | 821848 | 2957422 |
| -2 | 1263.1 | 1000.0 | 833.3 | 458.8 | 1292.1 | 9918 | 828374 | 3155022 |
| -5 | 1097.7 | 1000.6 | 832.8 | 642.6 | 1475.4 | 4128 | 831269 | 3285316 |
| -10 | 1047.7 | 1000.6 | 832.9 | 728.6 | 1561.4 | 2742 | 831962 | 3317299 |
| -100 | 1005.0 | 1000.5 | 832.9 | 821.5 | 1655.5 | 1759 | 832453 | 3331395 |
| -1000 | 1000.9 | 1000.5 | 832.9 | 831.8 | 1664.7 | 1675 | 832496 | 3331654 |
| -10000 | 1000.4 | 1000.4 | 833.0 | 832.8 | 1665.8 | 1667 | 832500 | 3331666 |

Table 8. $\delta = 0$, collusion.

| $\rho$ | p | q | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|---|
| .99 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .9 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2489.8 | 2498.8 | 1248750 | 4997500 |
| .8 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .7 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .6 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .5 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .4 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .3 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .2 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .1 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .08 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .05 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| .01 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -.01 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -.05 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -.1 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -.2 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -.5 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -.8 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -1 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -2 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -5 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -10 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -100 | 1.000 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -1000 | 1.001 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |
| -10000 | 1.005 | 1000.5 | 1249.4 | 1249.4 | 2498.8 | 2498.8 | 1248750 | 4997500 |

Table 9. $\delta = .4$ (Gross Complements), collusion.

| $\rho$ | p | q | $X^C$ | $X^F$ | X | L | Profit | Utility |
|---|---|---|---|---|---|---|---|---|
| .99 | 0.9995 | 1200.40 | 1999.0 | 2101.5 | 4100.5 | 3998.0 | 2397601 | 9672384 |
| .9 | 0.9943 | 1200.40 | 1999.0 | 2116.8 | 4115.8 | 3998.3 | 2397601 | 9683262 |
| .8 | 0.9887 | 1200.40 | 1999.0 | 2116.2 | 4115.2 | 3998.7 | 2397601 | 9682875 |
| .7 | 0.9832 | 1200.40 | 1999.0 | 2115.7 | 4114.7 | 3999.0 | 2397601 | 9682477 |
| .6 | 0.9779 | 1200.40 | 1999.0 | 2114.5 | 4113.5 | 3999.3 | 2397601 | 9681681 |
| .5 | 0.9727 | 1200.40 | 1999.0 | 2113.6 | 4112.6 | 3999.6 | 2397601 | 9681026 |
| .4 | 0.9674 | 1200.40 | 1999.0 | 2113.5 | 4112.5 | 3999.9 | 2397602 | 9680961 |
| .3 | 0.9623 | 1200.40 | 1999.0 | 2113.0 | 4112.0 | 4000.2 | 2397602 | 9680560 |
| .2 | 0.9573 | 1200.40 | 1999.0 | 2112.4 | 4111.4 | 4000.5 | 2397602 | 9680139 |
| .1 | 0.9523 | 1200.40 | 1999.0 | 2112.0 | 4111.0 | 4000.7 | 2397602 | 9679881 |
| .08 | 0.9513 | 1200.40 | 1999.0 | 2111.9 | 4111.0 | 4000.8 | 2397602 | 9679845 |
| .05 | 0.9499 | 1200.40 | 1999.0 | 2111.6 | 4110.7 | 4000.9 | 2397602 | 9679627 |
| .01 | 0.9479 | 1200.40 | 1999.0 | 2111.6 | 4110.6 | 4001.0 | 2397602 | 9679584 |
| -.01 | 0.9470 | 1200.40 | 1999.0 | 2111.3 | 4110.3 | 4001.0 | 2397602 | 9679407 |
| -.05 | 0.9449 | 1200.40 | 1999.0 | 2111.5 | 4110.5 | 4001.2 | 2397602 | 9679545 |
| -.1 | 0.9427 | 1200.40 | 1999.0 | 2110.9 | 4109.9 | 4001.3 | 2397603 | 9679084 |
| -.2 | 0.9379 | 1200.40 | 1999.0 | 2110.6 | 4109.6 | 4001.5 | 2397603 | 9678860 |
| -.5 | 0.9241 | 1200.39 | 1999.0 | 2109.3 | 4108.3 | 4002.3 | 2397603 | 9677935 |
| -.8 | 0.9108 | 1200.39 | 1999.0 | 2108.1 | 4107.1 | 4003.0 | 2397604 | 9677127 |
| -1 | 0.9024 | 1200.39 | 1999.0 | 2107.2 | 4106.2 | 4003.4 | 2397604 | 9676451 |
| -2 | 0.8627 | 1200.39 | 1999.0 | 2103.8 | 4102.8 | 4005.5 | 2397605 | 9674032 |
| -5 | 0.7687 | 1200.38 | 1999.0 | 2094.8 | 4093.8 | 4009.9 | 2397609 | 9667530 |
| -10 | 0.6585 | 1200.38 | 1999.0 | 2084.8 | 4083.9 | 4014.2 | 2397614 | 9660269 |
| -100 | 0.2207 | 1200.35 | 1999.1 | 2039.9 | 4039.0 | 4019.3 | 2397641 | 9626610 |
| -1000 | 0.0322 | 1200.34 | 1999.2 | 2010.3 | 4009.4 | 4006.8 | 2397660 | 9603514 |
| -10000 | 0.0033 | 1200.33 | 1999.2 | 2001.2 | 4000.3 | 4000.1 | 2397666 | 9596273 |

The effect of introducing collusion can be seen from tables 7 - 9 to have quite a dramatic effect on the form of the results. In table 7, with the goods as gross substitutes, trade prices start low and then "leap" to a higher level of price. This does not reflect a discontinuity in the model but instead is explained by there being two local maxima at values of ρ around -.05, one at a low price and the other with a far higher price. Below -.05043 the low price maximum is the global maximum, the two give the same level of profit at -.05043 and above this the high price becomes the global maximum. This behaviour would appear to arise from the two conflicting aims of preferring a low priced input but at the same time competing on the final goods market. When substitution of labour for produced input is easy, the first option will raise more profit. The converse is true when substitution becomes more difficult.

Final prices rise slowly and then jump and continue to rise. They are lower than the price discrimination case for all values of ρ and lower than the no-discrimination level except at the jump. Corresponding to this pattern, final consumption falls, drops at the jump and then falls again. However, it is higher than for the other two equilibria except around the jump. Intermediate input use is also higher than in previous cases, due to its lower price. This is mirrored by the use of labour which has a far lower range than it had without discrimination. Collusion thus results in less extreme combinations of labour and produced input in production.

Profits are far higher with collusion than for the other two cases; the expected conclusion. However, what is most surprising is that utility is also higher, with the exception that the no-discrimination case has slightly higher utility around the price jump, so that the collusion is in the interest of both the firms and the consumers. This can mostly be explained by the more effective use of intermediate input in production and the move away from labour intensive techniques.

Table 8 is especially interesting for its almost complete lack of variation. The explanation for this must lie in the fact that the firms are not interacting at all on the final goods market and the collusive setting of trade prices is then aimed solely at aiding their independent profit maximisation at the final stage. A trade price of 1 indicates that the collusion results in marginal cost pricing of intermediate inputs hence the only inefficiency in this case is arising through the monopoly pricing on the final market. The equality of $2X^F$ and L indicates that production takes with equal quantities of labour and intermediate input so that this case has none of the extreme substitution of labour for produced input seen in previous tables.

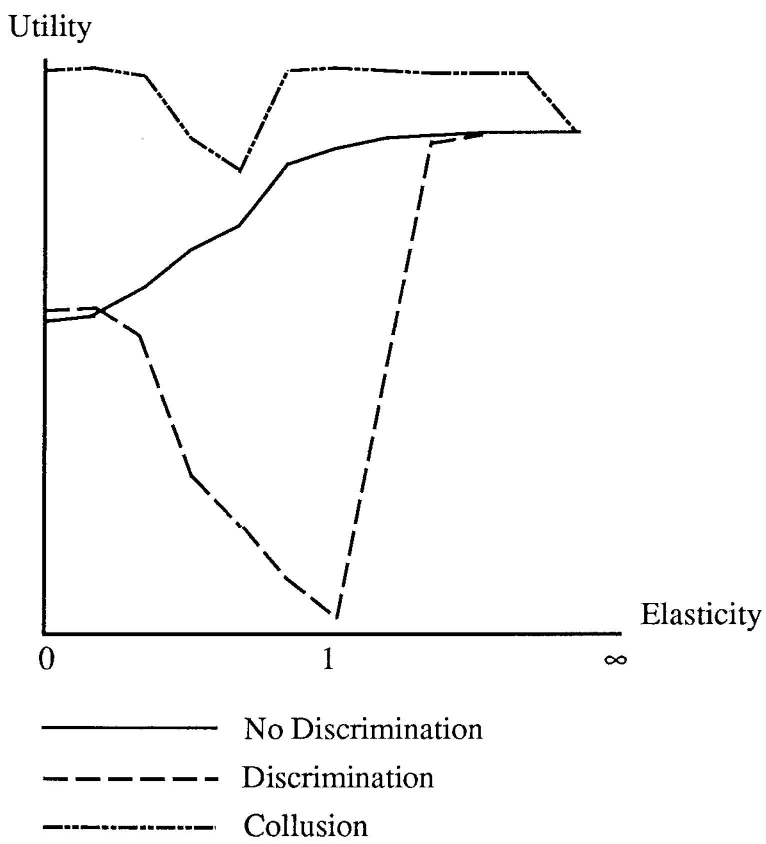
Final consumption, profit and utility are all higher than for the corresponding tables for the two other equilibrium concepts. As in table 7, the collusion is benefiting both the firms and the consumer by removing the excessive use of labour, the lack of interaction on the final market having the consequence of reducing trade prices to marginal cost.

The final table, involving gross complements and collusion, continues the natural progression apparent in tables 7 and 8. The effect of the complementarity on the final goods market is to reduce the trade price below marginal cost. For high elasticities of substitution it is only a little below marginal cost, so that labour and produced input are used fairly equally in production, but falls substantially below cost as the Leontief case is approached. Final prices remain almost constant at a level well below that reported in the corresponding tables for the equilibria with no collusion. The low trade and final prices results in high final consumption, intermediate use and total production. Labour use is also correspondingly high.

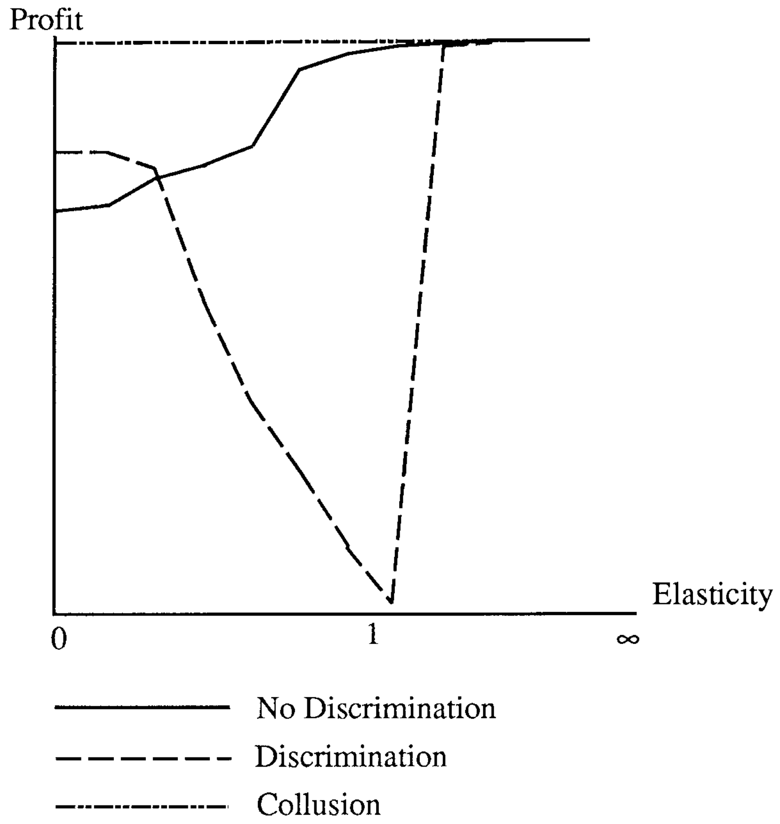
Despite the loss taken on the sales of intermediate input, profits are high and rise as the elasticity of substitution falls. Utility follows a undulating path but its range of variation is small and it remains at a level well in excess of that attained in tables 3 and 6. As in the previous two tables, collusion is beneficial for both the firms and the consumer.

Returning to the question raised in the introduction of the observed relation between final and producer prices, only in tables 8 and 9 are trade prices everywhere less than consumer prices. Interpreted literally, this result suggests that the explanation for trade prices being lower is that firms are colluding and consider their products to be either complements or entirely independent in respect of final demand. This may be too exact an application of the model, but these elements are likely to form the basis of any explanation.

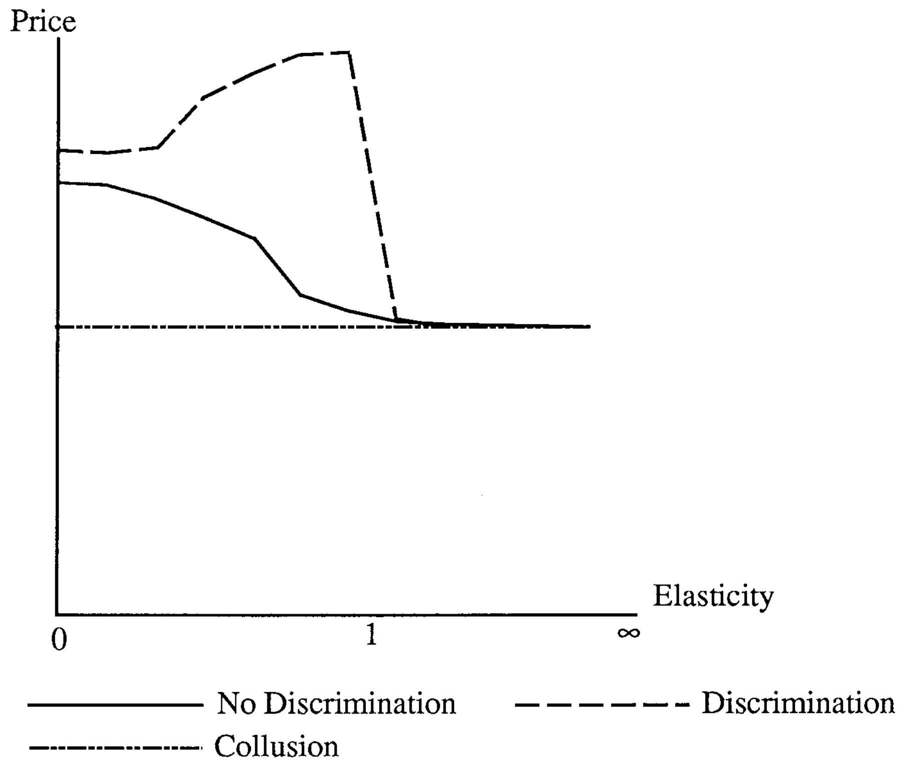
Finally the graphs below provide a qualitative illustration of the typical equilibrium values of utility, profit and final consumer prices.




Graph 1. Utility for Gross Substitutes




Graph 2. Profit for Gross Complements




Graph 3. Consumer Price for no interaction

## 5. Conclusions

As the numerical results have been discussed at length in the previous section, the conclusions will constitute some broad reflections on the outcome of the analysis. The first point is it has proved possible to construct a consistent model of general equilibrium with imperfect competition and intermediate goods and, furthermore, to expose some special forms of this model to numerical analysis. The model has been the simplest conceivable but a number of extensions could be made, although I would hesitate to claim they are easy. The symmetry could be dropped but at the cost of introducing computing difficulties. Retaining symmetry it would not be too difficult to increase the number of firms in each industry provided a switch was made to Cournot quantity-setting. Also possible would be an increase in the number of sectors.

Extensions aside, the present analysis has highlighted several features. It is clear that the form of market organisation is of critical importance for the equilibrium that emerges. A thorough knowledge of institutional features is therefore required to predict the behaviour of such markets. The application of objectivity throughout and the small scale of the model has in most cases made the equilibrium values highly sensitive to the value of the elasticity of substitution, rather more so than would have been initially suspected. This is particularly true of the price-discrimination model, although I would suggest that some of the very high values of trade prices should be treated as a reflection of the particular special case analysed. However, the important lesson to be drawn from this case is that price discrimination may be mutually harmful, a result that may not be new in the trade literature but may be surprising in the context of imperfect competition. In addition, collusion has been seen to be beneficial both to the firms and to the consumer and to lead to prices that fall below marginal cost.

Finally, the model has also shown that there may also be excessive substitution of labour for produced input. This has the effect of reducing the total output of society to the detriment of all concerned and represents a welfare loss additional to that typically identified in the analysis of monopoly.

## References

Atkinson, A. B. and J. E. Stiglitz (1980) "Lectures on Public Economics", London: McGraw - Hill.

Gabszewicz, J. J. and J.-P. Vial (1972) "Oligopoly 'a la Cournot' in a general equilibrium analysis", Journal of Economic Theory, 4, 381 - 400.

Harberger, A. C. (1962) "The incidence of the corporation income tax", Journal of Political Economy, 70, 215 - 240.

Hart, O.D. (1985) "Imperfect competition in general equilibrium: an overview of recent work", in Frontiers of Economics, K.J. Arrow and S. Honkapohja (eds.), Oxford: Basil Blackwell.

Ireland, N. J. (1989) "Profit sharing to counter restrictive practices", European Journal of Political Economy, 5, 347 - 357.

Negishi, T. (1961) "Monopolistic competition and general equilibrium", Review of Economic Studies, 28, 196 - 201.

Panzar, J. C. and D. S. Sibley (1989) " Optimal two-part tariffs for inputs", Journal of Public Economics, 40, 237 - 249.

Roberts, J. and H. Sonnenschein (1977) "On the foundations of the theory of monopolistic competition", Econometrica, 45, 101 - 113.