THE INTERTEMPORAL DEMAND FOR CONSUMER TECHNOLOGIES REQUIRING JOINT HARDWARE AND SOFTWARE INPUTS

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ABSTRACT

In this paper we explore the demand for consumer technologies that involve joint inputs in the generation of the service flow. An intertemporal model is used in the analysis. The impact of hardware and software prices on hardware and software demand as well as the impact of changes in the size of the software catalogue are analysed. The model allows copying and it is shown that the ease of copying has a significant impact on hardware and software demands and that the presence of copying has a significant impact on the reactions of hardware and software purchases to changes in the size of the software catalogue and its intertemporal availability. The role of copying in the competition between technologies is also addressed, and it is shown that the prospect of a copying facility on a forthcoming technology will tend to reduce the demand for a current technology without such a facility and moreover will reduce the demand for software for that technology.

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1. INTRODUCTION

In this paper the demand for an increasingly common type of consumer technology is explored. The peculiar characteristic of the technologies with which we are concerned is that joint inputs are involved in generating the service flow, the two inputs being loosely labelled hardware and software. Examples of such technologies are record players or hi-fi systems, tape recorders, CD players, video recorders and personal computers. There are also producer technologies that exhibit similar joint input characteristics, e.g. business computers, and in principle the analysis could be applied to these, but that is not pursued here.

The modelling framework is one that places demand in an intertemporal context. There are three main reasons for this. The first is that it would appear to be common for the catalogue of software products associated with any one of these technologies to be continually changing over time (for example the catalogue of music available on disc has grown and changed extensively over time) and it is important to take account of this. Secondly, the technologies that are being discussed are in many cases quite new and are still on their diffusion paths, e.g. CD players. To take account of this requires an intertemporal analysis. Thirdly, the origin of this work is in a continuing study of the diffusion process which requires an intertemporal framework. In many ways this can be considered to be a paper on the diffusion of new technologies, however it should be noted that throughout the paper the supply side is ignored, and as we have argued elsewhere (e.g. Ireland and Stoneman (1987)) a full diffusion model does require consideration of both demand and supply. The intertemporal character of the analysis is actually introduced by the use of a simple two period model. Although in principle a multi-period framework might be preferred on grounds of realism, it was felt that very few extra insights were likely to be generated that might justify the increased degree of complexity involved.

Throughout the paper it is considered that hardware is an homogeneous product but it is unrealistic to make any such assumption with respect to software. Software is thus treated as a differentiated product. Also, because in many cases the technologies that we are discussing enable software to be copied, the model allows software to be acquired by both purchasing and copying.

The main purpose of the paper is to explore, in general, the determinants of the demand for both hardware and software associated with such joint input technologies. Certain issues are, however, concentrated upon. The first is the impact of the size of the software catalogue and its intertemporal availability on the demands for hardware and software. The second is the impact of a copying capability on the pattern of demand and the third is the impact of the ease of copying on the pattern of demand. To facilitate this the paper is structured as follows. First a general model of software demand is constructed and a no-copying scenario is analysed as a special case and compared to a scenario with copying. Secondly hardware demand is modelled and again the no-copying scenario is considered as a special case and compared to a scenario with copying.

A third section is added that has its origins in recent debates on whether Digital Audio Tape (DAT) technology should or should not have a capacity to copy from CD discs. We address a situation in which one new technology is faced with competition from another newer technology and explore the demands for the hardware and software related to the two technologies and the impact on these of the newest technology having a copying facility.

To illustrate the importance of the time dimension of the model, initial thoughts suggested that if a greater variety of software products were available then this would increase the demand for hardware and software. It is shown in the paper that (a) not only is the extent of variety important but also its temporal availability and (b) that in the presence of copying a greater variety of software in an early period, contrary to initial thoughts, can reduce the demand for hardware in that period.

In Johnson (1985) a similar type of model is analysed. That paper, however, does not have a time dimension. Recently there has also been a burgeoning literature on standards and compatibility (e.g. Katz and Shapiro (1986), Farrell and Saloner (1988), Matutes and Regibeau (1988)) which obviously addresses issues closely related to the demand for technologies with joint inputs. That literature however is mostly concerned with the incentives for product compatibility on the supply side and does not appear to directly address the issues addressed in this paper.

Prior to moving to the detailed modelling we can make a few observations on aspects of the problem from which we abstract in this analysis. First we abstract from uncertainty about the future availability of hardware and software and assume that consumers know future prices (and quality) of hardware and software. Secondly, except as prices can be interpreted as quality adjusted, we do not allow hardware or software to improve in quality over time. Thirdly we do not allow hardware products to be differentiated. Fourthly we do not discuss the supply of hardware or software. Of course these aspects of the real world are relevant to the issues under discussion, but the paper is already sufficiently long without these extra complications. The model could be extended to incorporate such issues, but, in our view, some useful insights into the issues addressed in this paper can be generated without these further additions.

2. THE DEMAND FOR SOFTWARE

2.1 The General Model

In this paper software products are treated as horizontally differentiated and use is made of the standard model of horizontal product differentiation introduced by Salop (1979). The assumption of horizontal differentiation does not seem to need justification (for further detail on modelling product differentiation see Ireland (1987)).

To derive the software utility function the product space for software is assumed to be a unit circle. Assume there to be n software products available (the catalogue is of size n) equidistantly placed around the circle. Assume a population size N .

For consumer $j(j = 1 .. N)$ one may specify a point in product space l^*_j that represents the characteristic of his/her ideal software product. These l^*_j are assumed to be evenly distributed around the unit circle. A consumer with the appropriate hardware is then assumed to derive, in each period of ownership of a software product with characteristic l_i, utility given by equation (1).

$$U_j = v_j - \theta | l_i - l^*_j | \tag{1}$$

We assume that v_j is distributed across consumers independent of l^*_j, such that the density function equals k on [a, b] where $a > 0$, $a < b$, and zero otherwise and assumed constant over time (the total population N thus equals $k(b - a)$). θ is assumed to be the same for all j.

Define x_j as the number of units of software acquired from the catalogue by consumer j. We may assume that the consumer will acquire first those products closest to his or her ideal (on both sides of the ideal) and thus with some minor approximation we may write the utility per period derived from the ownership of x_j units as (2).

$$U_j(x_j) = v_j x_j - \theta x_j^2/4n \tag{2}$$

Assuming that no consumer reaches satiation, marginal utility is positive and $U''(x)$ is negative for all x for all j .

To develop the intertemporal character of the model consider two time periods, 1 and 2, today and tomorrow, and assume that the technology under consideration is available in period 1 for the first time but will still be available in period 2.

Assume that each consumer has income in period 1 of Y_1 and in period 2 of Y_2 . In diffusion models based on demand patterns of the kind modelled here it is often assumed that consumers' incomes differ (see, for example, Stoneman 1989) and it is these differences that enable a diffusion path to be generated. In the first instance this was the path to be pursued in this paper. It soon became apparent, however, that to introduce this into the model required a much more involved specification of the marginal utility of income than that which we assume below, and thus in preference we have taken a route along which consumers' inherent valuations of the technology, v_j , differ, as assumed above, and it is these differences that enable the generation of an intertemporal demand path for the technology. There is a discount rate, r , and consumers may borrow and lend at this rate. The present value of income remaining after the purchase of hardware and software is assumed to have a constant marginal utility u , which is invariant across consumers.

The population may be split into three groups, those who acquire the hardware in period 1, those who acquire it in period 2 and those who do not acquire it at all. It will be assumed that hardware and software do not depreciate and there is no second hand market. Thus a buyer in period 1 is an owner in period 2. Obviously those who do not buy the hardware do not acquire software. We will assume that buyers of hardware in period 2 do not acquire software in advance of their hardware purchase. The conditions to guarantee this can be spelt out (it will depend inter alia on software prices in period 1 and 2) but exclusion by assumption saves space.

Assume that in period 1, αn different software products are available (the first period catalogue) equidistantly placed round a unit circle. In period 2, $(1 - \alpha) n$ new software products are launched, (the second period catalogue) equidistantly placed round a unit circle, and the first period catalogue is also still available. Thus in total there are n different software products available in period 2. Let

x_{11} be the number of units of software <u>purchased</u> by an by an individual consumer in period 1 from the first period catalogue.

x_{21} be the number of units of software <u>purchased</u> by an individual consumer in period 2 from the first period catalogue, and

x_{22} be the number of units of software <u>purchased</u> by an individual consumer in period 2 from the second period catalogue.

An important dimension of many of the joint input technologies is that they allow copying, thus providing an alternative to purchase as a source of software. There are basically two types of copying, from broadcasts and from the software of other hardware owners. In this piece I am primarily concerned with the latter although in principle the model could be adapted to apply to the former. In Johnson (1985) copying is modelled in such a way that the cost of copying is equal to the value of time spent copying and the value of time is assumed to differ across consumers. The result in that model is that some consumers only copy and do not buy software whereas other consumers only buy and do not copy software. This seems to be an unrealistic characterisation of behaviour in that we would expect all consumers to buy some software and to copy some software, not to exclusively buy or copy. Two potential reasons why we might expect to see this non exclusive behaviour are that (i) new software releases will not generally be available in the existing software stock to be copied and (ii) there may well be search costs involved in tracing desired pieces of software among software owners.

In the model here we take account of these and other factors by assuming that copying takes place under the following conditions.

(a) copying can only occur in the second period on the grounds that only then is there a stock of software to be copied.

(b) In period 2 only software from the first period catalogue can be copied, on the grounds that the "new" software is not yet in household stocks and is thus not available for copying. We define x_{31} as the number of units of software <u>copied</u> by a consumer in period 2.

(c) The recording medium onto which software is to be copied has a cost δ, and this is the only cost attached to copying that explicitly enters the utility maximisation.

(d) Within a period only a proportion of the software that is available to copy can be copied by a consumer because of problems locating owners of software. We assume that this proportion, Ω, is linearly related to the average proportion of the first period catalogue owned by a member of the population, i.e.

$$\Omega = \beta \cdot \frac{M_1}{N} \cdot \frac{X_{11}}{M_1} \cdot \frac{1}{\alpha n} = \beta \frac{X_{11}}{N\alpha n} \tag{3}$$

where M_1 is the total number of owners of hardware in period 1, X_{11} is the total stock of software purchased in period 1, and β is a parameter reflecting the "ease" of obtaining software to copy. The greater is β the easier it is to copy. We assume that copying is evenly distributed over the software characteristics interval.

(e) We assume that the part of the first period catalogue that has neither been purchased in period 1 nor copied in period 2 but it is utility maximising to purchase at its second period price will be purchased in the second period.

Let the cost of buying a unit of software in period 1 be p_1 and in period 2 be p_2. Assume $\delta < p_2$. Write the cost of hardware purchase as F_1 in period 1 and F_2 in period 2. Throughout the paper it is assumed that all these prices are known at the beginning of period 1. The alternative, allowing second period prices to only be revealed at the beginning of period 2 would necessitate the introduction of price expectations into the model and although this would increase the degree of realism it would generate a further degree of complication and only lead to a reiteration of issues already addressed in Ireland and Stoneman (1986).

With this modelling framework we may now proceed to define the demand for software, first by an acquirer of hardware in period 1 and then by an acquirer of hardware in period 2. In this we take the j subscripts as read. A first period acquirer of hardware will obtain in the second period from his ownership of software from the first period catalogue (either purchased or copied) utility U_2 as defined in (4).

$$U_2 = \int_0^{x_{11}} \left(v - \frac{\theta x}{2\alpha n}\right) dx + \int_{x_{11}}^{f_1} \left(v - \frac{\theta x}{2\alpha n}\right) dx + \Omega \int_{f_1}^{f_2} \left(v - \frac{\theta x}{2\alpha n}\right) dx \qquad (4)$$

where $f_1 = x_{21}\left(\frac{1}{1 - \Omega}\right) + x_{11}$

$f_2 = x_{31} \cdot \frac{1}{\Omega} + x_{11}$.

(4) may be simplified to (5).

$$U_2 = vx_{11} + vx_{21} + vx_{31}$$

$$- \frac{\theta}{4\alpha n} [(x_{21} \frac{1}{1 - \Omega} + x_{11})^2 \cdot (1 - \Omega) + \Omega(x_{31} \cdot \frac{1}{\Omega} + x_{11})^2] \qquad (5)$$

Such a first period acquirer of hardware will then determine his purchases and copying of software so as to maximise W as given by (6).

$$W = vx_{11} - \frac{\theta x_{11}^2}{4\alpha n} + z(vx_{22} - \frac{\theta x_{22}^2}{4(1 - \alpha) n}) + zU_2$$

$$+ u(Y_1 + zY_2 - F_1 - p_1 x_{11} - zp_2 x_{21} - z\delta x_{31} - zp_2 x_{22}) \qquad (6)$$

where $z = \frac{1}{(1 + r)}$.

Assuming that each buyer considers X_{11} to be exogenous this yields, from the first order conditions, optimal software acquisition as in (7) - (10).

$$x_{11}^* = \frac{2\alpha n}{\theta} (v - up_1 + uz(\Omega\delta + (1 - \Omega) p_2)) \qquad (7)$$

$$x_{22}^* = \frac{2(1 - \alpha) n}{\theta} (v - up_2) \qquad (8)$$

$$x^{*}_{21} = \frac{2\alpha n}{\theta} \cdot (1 - \Omega)\ u(p_1 - p_2 - z(\Omega\delta + (1 - \Omega)\ p_2)) \tag{9}$$

$$x^{*}_{31} = \frac{2\alpha n}{\theta} \cdot \Omega u(p_1 - \delta - z(\Omega\delta + (1 - \Omega)\ p_2)) \tag{10}$$

A purchaser of hardware in period 2 will obtain in period 2 from his purchases and copies from the first period catalogue utility as given by (11)

$$\bar{U}_2 = vx_{21} + vx_{31} - \frac{\theta}{4\alpha n} [(x_{21} \cdot \frac{1}{1 - \Omega})^2 (1 - \Omega) + \Omega(x_{31} \cdot \frac{1}{\Omega})^2] \tag{11}$$

Such a purchaser will determine his software demands by the maximisation of (12)

$$W = z\bar{U}_2 + z(vx_{22} - (\frac{\theta x^2_{22}}{4(1 - \alpha)\ n}) + u[Y_1 + zY_2 - zF_2 - zp_2x_{21} - z\delta x_{31} - zp_2x_{22}] \tag{12}$$

On the assumption that X_{11} is considered exogenous by the consumer the first order conditions yield software acquisitions as given by (13) - (15)

$$x^{**}_{22} = \frac{2(1 - \alpha)\ n}{\theta} (v - up_2) \tag{13}$$

$$x_{21}^{**} = \frac{2\alpha n}{\theta} (1 - \Omega) \quad (v - up_2) \tag{14}$$

$$x_{31}^{**} = \frac{2\alpha n}{\theta} \cdot \quad \Omega \cdot \quad (v - u\delta) \tag{15}$$

We may note that certain conditions must be satisfied for software demands of both types of hardware acquirers to be positive in both periods. First $x_{21}^{*} > 0$ iff $p_1 - p_2 - z(\Omega\delta + (1 - \Omega)\, p_2) > 0$. If this condition does not hold there is no benefit to a first period owner of hardware waiting until period 2 in order to purchase from the first period catalogue. We will assume that this condition holds. (The model has been analysed with this condition violated but it is much less interesting and yields no new insights). If the condition holds, then given $\delta < p_2$, x_{31}^{*} is also positive. Secondly purchases from the second period catalogue are only positive for first and second period acquirers of hardware if $v - up_2 > 0$. This can be shown to hold for all purchasers of hardware in periods 1 and 2. Given $\delta < p_2$, if $v - up_2 > 0$ then x_{31}^{**} is also positive. Finally x_{11}^{*} will only be positive if $v - up_1 + uz\ (\Omega\delta + (1 - \Omega)\, p_2) > 0$. This can be shown to hold for all acquirers of hardware in period 1.

2.2 The No-Copying Case

A useful case to analyse as a base model is that where the technology does not permit copying. In this situation $\beta = \Omega = 0$. Substituting this into (7) - (10) and (13) - (15) yields expressions for the software demands. Consider first the software demands of a first period acquirer of hardware. In Table 1 below the signs of the derivatives of such an acquirer's software demands in the two periods with respect to the exogenous parameters are summarised. These signs, especially with respect to prices are as would be expected taking account of the possibilities in this model for intertemporal arbitrage. It is however worth briefly discussing the impact of changes in the size and intertemporal availability of the software catalogue and the relationships between software demands and v . Consider the latter first. It is clear that x^*_{11} and x^*_{22} are positively related to v as are total purchases from the first period catalogue $(x^*_{11} + x^*_{21})$. Thus those consumers for whom software yields the greatest utility buy the most.

This is not surprising. However, x^*_{21} is independent of v , but dependent on the relative costs of purchasing software in the two periods. The rationale for this is that given the total purchases from the first period catalogue, x^*_{21} will just reflect intertemporal arbitrage, and the relative benefits of first and second period purchase to an individual are independent of v.

Of more interest is the effect of changes in α and n . A larger n , i.e. a larger catalogue, is associated with greater software purchases in both periods. This is not surprising for the higher is n the greater is the amount of software located close to a consumers' ideal specification. For a given n the impact of an increase in α , the proportion of the catalogue available in the first period catalogue, is to increase software purchases from the first period catalogue in both periods and to reduce purchases from the second period catalogue. This is as one would expect, but we highlight it here for when copying is re-introduced into the model below it is with respect to α and n that the most interesting results appear.

Consider now the demands of a second period acquirer of hardware. From the first order conditions it is clear that, as would be expected, higher software purchases are positively related to v and n, but negatively related to p_2. The relative size of purchases from the first and second period catalogues is given by $\alpha/(1 - \alpha)$, i.e. the relative sizes of the two catalogues. The signs of these and other derivatives are summarised in Table 1.

2.3 The General Case

At this stage, when there is copying in the model $(\Omega > 0$, $\beta > 0)$, we can only discuss signs of derivatives dependent on X_{11}. X_{11} is in fact endogenous, but until we model hardware demand we are unable to treat it as such. However, given this proviso, we may use (7) - (10) and (13) - (15) above to derive the derivatives detailed in Table 2. The most interesting aspects of this Table refer to the effect of the copying capability on software demand.

One might first note from Table 2, that the sign of the effect of changes in α and n are the same as in Table 1, where there was no copying, but from Table 2 we see that, in addition, with copying, changes in α and n negatively affect the amount of software copied. (We will however reconsider this finding below when X_{11} is made endogenous.)

In addition we may gain further insight by exploring the impact of changes in β, the ease of copying. Considering first the software demand of a first period purchaser we may see that easier copying (an increase in β) reduces x^*_{11} and increases x^*_{31}, leaving x^*_{22} unaffected. This is as one would expect. However the effect on x^*_{21}, purchases from the first period catalogue in period 2, can only be definitively signed (as negative) if $\Omega < 0.5$. The rationale for this is that as β changes there is a shift of purchases from period 1 to period 2 which may offset the switch from purchasing to copying in period 2. For a second period purchaser of hardware, one may observe that an increase in β will lead to an increase in second period copying of software and a decrease in second period purchases of software from the first period catalogue, but will leave purchases of software from the second period catalogue unaffected.

3. THE DEMAND FOR HARDWARE

3.1 The General Framework

We assume that a buyer will only acquire one unit of hardware at the most, in which case he/she may choose to acquire hardware in periods 1 or 2 or not at all. In the case of no acquisition the consumer will generate utility $u(Y_1 + zY_2)$. In the case of first period acquisition the consumer will obtain utility as given by W^* which results from optimal software purchase decisions summarised in (7) - (10) being substituted into (6). In the case of second period hardware acquisition the consumer will obtain utility as given by W^{**} which results from optimal software purchase decisions summarised in (13) - (15) being substituted into (12). It should be noted that both W^{*} and W^{**} are increasing in v.

A buyer will acquire in period 1 if

$$W^{*} > W^{**}$$

and $$W^{*} > u(Y_1 + zY_2)$$

will acquire in period 2 if

$$W^{**} > W^{*}$$

and $$W^{**} > u(Y_1 + zY_2)$$

and will not acquire at all if

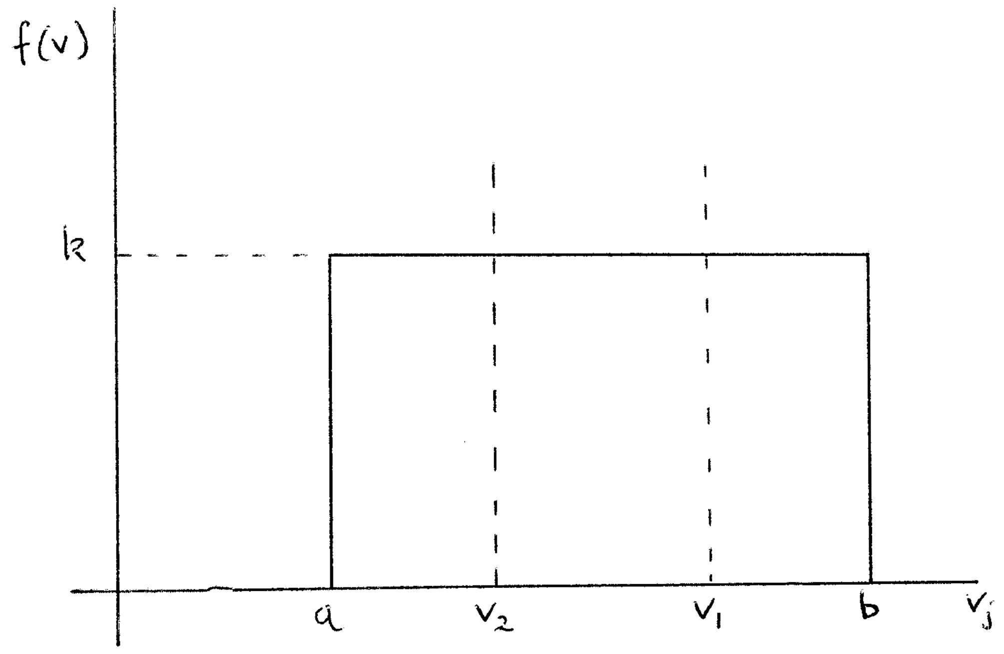
$$W^{*} < u(Y_1 + zY_2)$$

$$W^{**} < u(Y_1 + zY_2)$$

Define two critical values of v_j, v_1 and v_2, such that for $v_j > v_1$, $W^{*} > W^{**}$ and for $v_j > v_2$, $W^{**} > u(Y_1 + zY_2)$. On the condition that $v_1 > v_2$, all consumers for whom $v_j > v_1$ satisfy both condition for purchase in period 1, all consumers for whom $v_1 > v_j > v_2$, satisfy both conditions for purchase in period 2, and all consumers for whome $v_j < v_2$ satisfy both conditions for not purchasing hardware at all. On the further assumption that the density function of v is such that $v_1 < b$, $v_2 > a$, there will be consumers in each of the three categories.

Given that the parameters are such that $v_1 > v_2$ we may diagramatically represent the demand for hardware across the population as in Figure 1.

FIGURE 1 : The Demand for Hardware



With reference to Figure 1, consumers in the right hand block have $v_j > v_1$ and acquire in period 1. Consumers in the middle block have $v_1 > v_j > v_2$ and acquire in period 2. Consumers in the left hand block have $v_j < v_2$ and do not acquire the hardware at all.

Define M_1 and M_2 as the total number of units of hardware acquired in periods 1 and 2 respectively, and define $M = M_1 + M_2$. Then assuming $v_2 > a$, $v_1 < b$,

$$M_1 = k(b - v_1) \tag{16}$$

$$M_2 = k(v_1 - v_2) \tag{17}$$

$$M = k(b - v_2) \tag{18}$$

3.2 The No Copying Case

In this case $\Omega = \beta = 0$. By use of (5) - (15) we may derive v_1 , that value of v that makes $W^* = W^{**}$, as in (19).

$$v_1 = \left[\frac{u\theta(F_1 - zF_2)}{\alpha n} \right]^{\frac{1}{2}} + u(p_1 - zp_2) \tag{19}$$

Also v_2 , that value of v that makes $W^{**} = u(Y_1 + zY_2)$ is given by (20).

$$v_2 = \left[\frac{u\theta F_2}{n} \right]^{\frac{1}{2}} + up_2 \tag{20}$$

Considering only the positive values of the square roots, we may then note that $v_1 > v_2$ if $F_1 - zF_2 > \theta\alpha F_2$ which we assume to hold. Given this assumption we may also then note that for first period acquirers of hardware $v_1 > u(p_1 - zp_2)$ and for second period acquirers of hardware $v_2 > up_2$, and thus $v_1 > up_2$. These conditions then ensure that x^{*}_{11}, x^{*}_{22}, x^{*}_{21}, x^{**}_{21}, x^{**}_{22} are all positive.

Write (19) in its implicit form as

$$v_1 = v_1(F_1\ ,\ F_2\ ,\ \alpha\ ,\ n\ ,\ p_1\ ,\ p_2) \tag{21}$$

and the partial derivatives may be signed as follows

$$v_{11} > 0\ ,\ v_{12} < 0,\ v_{13} < 0,\ v_{14} < 0,\ v_{15} > 0,\ v_{16} < 0\ .$$

The signs of these derivatives do not really need further discussion, it suffices to point out for later reference that as the size of the total software catalogue increases and the proportion of that software available in the first period increases so v_1 will be lowered and a greater proportion of the population will prefer first period acquisition. A similar effect results from a fall in the first period prices of hardware and software and a rise in the second period prices of hardware and software.

Writing v_2 as

$$v_2 = v_2(F_2 , n , p_2) \tag{22}$$

then

$$v_{21} > 0 , \quad v_{22} < 0 , \quad \text{and} \quad v_{23} > 0 .$$

All the partial derivatives of v_1 and v_2 with respect to the exogenous parameters are summarised in Table 1. Given these, then by use of (16) - (18) one can determine the impact on M_1 , M_2 and M, (i.e. the ownership of hardware) of changes in the exogenous parameters, which impacts are detailed in Table 1.

Define X_{11} as the total demand for software in period 1, X_{21} as the total demand for software from the first period catalogue in period 2, X_{22} as the total demand for software from the second period catalogue, X_2 as the total demand for software in period 2 and X as the total software sales across both periods. Then

$$X_2 = X_{21} + X_{22} \tag{23}$$

$$X = X_2 + X_{11} \tag{24}$$

$$X_{11} = k2\alpha n \int_{v_1}^{b} ((v - u(p_1 - zp_2))/\theta)\, dv \tag{25}$$

$$X_{21} = k2\alpha nu \int_{v_1}^{b} ((p_1 - (1 + z)\, p_2))/\theta)\, dv = 2k\alpha n \int_{v_2}^{v_1} ((v - up_2)/\theta)\, dv \tag{26}$$

$$X_{22} = (k/\theta)\; 2(1 - \alpha)\; n \int_{v_2}^{b} (v - up_2)\, dv \tag{27}$$

By the use of equations (23) - (27) we can determine the impact on the ownership of hardware and software of changes in the exogenous parameters. The signs of derivatives, to the extent that it has been possible to calculate them are detailed in Table 1.

In many ways the signs of the derivatives as detailed in Table 1 are not very surprising, and thus it is not necessary to discuss them in great detail. As would be expected for any joint input technology the demand for hardware in total will be affected by software as well as hardware prices and the demand for software in total will be affected by hardware prices as well as software prices. Moreover given the intertemporal nature of the model and the possibility of intertemporal arbitrage it is no surprise that first period demands are affected by second period prices. The role of the first period prices is to shift demand between periods one and two rather than to affect total purchases over the two periods, which are determined, on the price side, by second period prices. This is again as one would expect.

Throughout this discussion we are particularly interested in the impact of changes in the size and intertemporal distribution of the software catalogue. Let us return then to that issue. As can be seen from Table 1, the distribution of the catalogue over time (α) has no impact on M, total hardware sales, and this is as one would expect (we have not been able to definitively sign the effect on overall software sales, although we would expect it to be zero). However increases in α do affect the time distribution of sales, encouraging first period hardware and software sales and not surprisingly encouraging sales from the first period catalogue. The effect of an increase in n, the overall size of the software catalogue is to increase overall sales of hardware and software and to increase first period sales of hardware and software. We have been unable to definitively sign the impact on second period sales of software and hardware, although one might expect both to be positive. At this point we will draw this section to a close and move to consideration of the model with copying, where the signs of some of these derivatives may be different.

3.3 The General Case

Just as in the model without copying, specify values of v_j, v_1 and v_2, above which first and second period acquisition respectively occur. Using similar procedures to above we determine

TABLE 1 : Impacts of Parameter Changes [The No Copying Case]

	Changes in						
	v	p_1	p_2	F_1	F_2	n	α
Effect upon							
x_{11}^*	+	−	+	0	0	+	+
x_{21}^*	0	+	−	0	0	+	+
x_{22}^*	+	0	−	0	0	+	−
$x_{11} + x_{21}^*$	+	0	−	0	0	+	+
x_{21}^{**}	+	0	−	0	0	+	+
x_{22}^{**}	+	0	−	0	0	+	−
v_1		+	−	+	−	−	−
v_2		0	+	0	+	−	0
M_1		−	+	−	+	+	+
M_2		+	−	+	−	?	−
M		0	−	0	−	+	0
X_{11}		−	+	−	+	+	+
X_{21}		?	?	?	?	?	+
X_{22}		?	?	?	?	?	?
X_2		+	−	+	−	?	?
X		0	−	0	−	+	?

Legend: + positive effect
− negative effect
0 no effect
? effect unsigned
blank, not applicable

that v_1 and v_2 are given by (28) and (29)

$$[v_1 - up_1 + uz(\Omega\delta + (1 - \Omega)\ p_2)]^2 = \frac{\theta}{\alpha n} \cdot u(F_1 - zF_2) \qquad (28)$$

$$(v_2 - up_2)^2\ (1 - \alpha\Omega) + \alpha\Omega(v_2 - u\delta)^2 = \frac{uF_2\theta}{n} \qquad (29)$$

It is not necessary to again repeat the derivation of the conditions ensuring that $v_1 > v_2$ and to show when software demands are positive. Similarly, to be brief the partial derivatives of v_1 and v_2 with respect to their arguments, for a given X_{11}, are detailed in Table 2.

To proceed further we must move away from the consideration of partial derivatives based on the assumption that X_{11} is given, for it is in fact endogenous. For example, to examine the impact of a change in β on x^*_{11}, we know that the direct impact, for a given X_{11}, is negative, but there will also be an indirect impact, as the ownership pattern of hardware and software changes, that will feed back via Ω.

In Appendix 1 we detail the derivation of the signs of the derivatives of X_{11} and v_1 with respect to the exogenous parameters, and these are summarised in Table 3. By the use of these we may determine derivatives relating to M_1. By the use of these derivatives plus the partial derivatives in Table 2 we may proceed to calculate the other derivatives in Table 3. Where ? appears, the direct effect of a change in the parameter is counteracted by the impact brought about by changes in X_{11}. As can be seen by a comparison of Tables 1 and 3, the introduction of copying reduces the

TABLE 2 : Impacts of Parameter Changes (Given X_{11}), with copying

Effect upon	Changes in x_{11}	p_1	p_2	F_1	F_2	δ	β	n	α
x_{11}	–	–	+	0	0	+	–	+	+
x^{*}_{22}	0	0	–	0	0	0	0	+	–
x^{*}_{21}	?	+	–	0	0	–	?	+	+
x^{*}_{31}	+	+	–	0	0	–	+	–	–
x^{**}_{22}	0	0	–	0	0	0	0	+	–
x^{**}_{21}	–	0	–	0	0	0	–	+	+
x^{**}_{31}	+	0	0	0	0	–	+	+	+
v_1	+	+	–	+	–	–	+	–	–
v_2	–	0	+	0	+	+	–	–	0

Legend: + positive effect, - negative effect,
0, no effect, ? unsigned

degree of certainty concerning the impact of changes in parameter values. Although we could find the conditions for signing some of the ? entries in Table 3, such signs will depend on parameter values and the process will be very tedious and not very informative. Of more interest is to consider the relationships that we have actually been able to sign starting with the impact of an increase in β, the ease of copying. As can be seen this will reduce first period sales of hardware. This is not particularly surprising, for in the model copying can only take place in period 2 and thus an increase in the ease of copying is equivalent to a reduction in the cost of software in period 2 which encourages second period as opposed to first period acquisition. However if there are fewer first period acquirers and as a result fewer purchases of software in period 1, this reduces the stock of software available from which copying can occur in period 2. It is this reduction that counteracts the direct effect of an increase in β on second period hardware and software acquisitions.

TABLE 3 : Impacts of Parameter Changes (X_{11} endogenous)

	Changes in							
Effect upon	p_1	p_2	F_1	F_2	δ	α	n	β
X_{11}	–	+	–	+	+	+	+	–
v_1	+	–	+	–	–	+	+	+
M_1	–	+	–	+	+	–	–	–
v_2	+	?	+	?	?	–	–	?
M	–	?	–	?	?	+	+	?
M_2	?	?	?	?	?	+	+	?
x_{11}^{*}	–	?	+	–	?	?	?	?
x_{22}^{*}	0	–	0	0	0	–	–	0
x_{21}^{*}	?	?	?	?	?	?	?	?
x_{31}^{*}	+	?	–	+	?	?	?	?
x_{22}^{**}	0	–	0	0	0	–	+	0
x_{21}^{**}	+	–	+	–	–	?	?	?

Legend: + positive effect, – negative effect,
0, no effect, ? counteracting effect.

Turn then to the effect of changes in the size of the software catalogue and its distribution over time, which issue we have addressed in the model without copying. We see by comparison with Table 1 that the possibility of copying reverses the sign of the impact of changes in n on first period hardware sales and makes the sign of the impact on second period hardware sales definitely positive. Why should an increase in n lead to a fall in first period hardware sales? The rationale appears to be that as an increase in n leads to an increase in first period software sales this leads to an increase in Ω in the second period, making software acquisition cheaper in period 2, thus encouraging second period hardware acquisition at the expense of first period hardware acquisition.

The effect of the presence of copying on the impact of changes in α is to reverse the signs on derivatives relating to M_1 (to negative) and M_2 (to positive) and to change the derivative relating to M from zero to positive. Basically what is happening here is that the increase in α leads to greater software purchases in the first period which makes Ω greater in period 2 thus encouraging second period acquisition. The "cheaper" software in period 2 also encourages acquisition in period 2 thus extending total ownership.

As can be seen the presence of copying has a significant effect on the impact of changes in the size of the software catalogue and its distribution over time. Moreover the ease of copying itself has significant impacts on hardware and software demands.

4. COMPETITION BETWEEN TECHNOLOGIES

In this section we explore a general question that is amenable to analysis using the models above, but which has been prompted by an ongoing dispute in the media and elsewhere regarding the impact of allowing Digital Audio Tape technology (DAT) to have a copying facility. Basically we wish to enquire what the impact of a copying facility would be on the outcome of a competition between one existing new technology and a newer technology on the horizon. The general scenario is that there is a new technology (CD) that is in the process of diffusion, but has no copying facility, and there is another newer technology (DAT) known to be soon available. The newest technology may be capable of copying the software of the other (may be compatible with the other). The two major questions to be addressed here are (a) what impact will the ability of the new technology to copy have on the overall sales of software and (b) what impact will the ability of the new technology to copy have on total hardware purchases of the two technologies, and their time distribution?

We characterise the problem as follows. In the first period only CD technology is available and a software catalogue of size αn^c is also available. However the prospect of DAT in period 2 is known. In period 1 a buyer must choose whether to buy CD and associated software or to wait. In period 2 both CD and DAT are available. For CD an additional amount of software is available, $(1 - \alpha) n^c$, however the technology does not allow copying. For DAT technology an amount of software n^d is available. We have a parameter ϕ, such that if $\phi = 1$, then the technology allows copying of CD software and if $\phi = 0$, CD software cannot be copied for use on DAT. We assume that the benefit obtained from a piece of software is the same whether it is played on CD or DAT. Copying is subject to the same conditions as in the previous model. Hardware and software prices are represented as before by F and p with c and d superscripts to indicate CD and DAT respectively. The price of the DAT recording medium is δ .

We may ask in general terms what will influence a buyer to choose DAT in preference to CD. There will be three basic reasons; DAT may allow copying and thus the cheaper acquisition of software; the software for DAT may be more numerous or different from CD software, and DAT hardware and software may be cheaper. In preliminary investigations it soon became apparent that without some simplification the whole issue becomes very involved. To simplify matters we are going to assume that the benefits that DAT provides over and above those provided by CD are never sufficient to encourage any buyer to own both. Thus an acquirer of CD in period 1 will not acquire DAT in period 2, nor will a buyer in period 2 acquire both CD and DAT. The choices open to the consumer are thus to acquire CD in period 1, acquire CD in period 2, acquire DAT in period 2, or to not acquire either. One further simplifying assumptions is introduced. We assume that the second period DAT software catalogue contains only items that differ from the first period CD catalogue, although the DAT catalogue is once again distributed around a unit circle.

Consider first an acquirer of a CD player in period 2. As this technology does not permit copying we may immediately derive from (11) - (15) with $\Omega = \beta = 0$ both the utility of such a buyer, W^{**c} , and his software demands. An acquirer of DAT in period 2 will, if the copying facility exists be able to copy from the existing CD software catalogue. We may then modify equations (11) - (15) to generate his software demands and his utility W^{**d}. The software demands and utility (W^{*}) of a purchaser of a CD player in period 1 can be derived from (5) - (10) with $\Omega = \beta = 0$.

After the relevant substitutions we may express the utility of the four potential strategies as

(1) Buy CD in period 1

$$W^{*} = u(Y_1 + zY_2 - F_1^c) + \frac{\alpha n^c}{\theta}(v - u(p_1^c - zp_2^c)) + \frac{zn^c}{\theta}(v - up_2^c)^2 \tag{30}$$

(2) Buy CD in period 2

$$W^{**c} = \frac{zn^c}{\theta}(v - up_2^c) + u(Y_1 + zY_2 - zF_2^c) \tag{31}$$

(3) Buy DAT in period 2

$$W^{**d} = \frac{zn^d}{\theta}(v - up_2^d)^2 + \phi\,\frac{z\alpha n^c\Omega}{\theta}(v - u\delta)^2 + u(Y_1 + zY_2 - zF_2^d) \tag{32}$$

(4) Not acquire hardware

$$\bar{U} = u(Y_1 + zY_2) \tag{33}$$

It is clear that the consumer's choices are a function of a number of parameters. As we particularly wish to address the copying issue we will simplify matters further by assuming that without copying households are indifferent between DAT and CD in period 2, i.e. $n^c = n^d$, $p_2^d = p_2^c$ and $F_2^d = F_2^c$. This is somewhat unrealistic but it will maximise the impact of the copying facility. (Of course if, even with $\phi = 1$, households prefer CD, then the copying facility would have no impact.) We may then specify that the increase in utility derived from acquiring hardware in period 2 (DAT if $\phi = 1$, either CD or DAT if $\phi = 0$) relative to not acquiring hardware is given by (34).

$$W^{**} - \bar{U} = \frac{zn^d}{\theta}(v - up_2^d)^2 + \frac{\phi z\alpha n^c\Omega}{\theta}(v - u\delta)^2 - uzF_2^d \qquad (34)$$

which enables us to define a critical level of v_j , v_2 below which no acquisition takes place. It is immediately obvious that v_2 is negatively related to ϕ yielding our first result, that a copying facility on the later technology can extend hardware ownership at the end of period 2.

Comparing first period acquisition of CD to second period acquisition,

$$W^* - W^{**} = uzF_2^d - uF_1^c + \frac{\alpha n^c}{\theta}(v - u(p_1^c - zp_2^c))^2 - \frac{\phi z\alpha n^c\Omega}{\theta}(v - u\delta)^2 \qquad (35)$$

From which it is clear that v_1 , the critical value of v above which first period acquisition occurs is higher if $\phi = 1$ than if $\phi = 0$ and thus the copying facility will discourage first period CD acquisitions. Moreover given that x_{11}^* is independent of ϕ , a copying facility will also reduce first period software sales.

We may therefore argue that the copying facility would have the impact of reducing first period acquisitions of hardware and software and extending second period acquisitions of hardware. We have not been able to show whether the second period extensions of ownership in the face of copying will lead to greater or less purchases of software, and there is no presumption either way. What we can say however, is that in the model, second period software cannot be copied and thus the copying facility would increase demands on the second period's catalogue.

5. CONCLUSIONS

In this paper we have explored the demand for consumer technologies that involve joint inputs in the generation of the service flow. Such technologies are now reasonably common, but in many cases being modern are still on their diffusion path. For this reason an intertemporal model is used in the analysis. A special case where there was no copying of software was presented and the impact of hardware and software prices on hardware and software demand was analysed. In addition the impact of changes in the size of the software catalogue and its distribution over time was investigated. The general model involving copying was also investigated and it was shown that the ease of copying had a significant impact on hardware and software demands and that the presence of copying had a significant impact on reactions of hardware and software acquisitions to changes in the size of the software catalogue and its intertemporal availability. Finally we addressed the role of copying in the competition between technologies, having the CD/DAT issue in the background. It was shown that the prospect of a copying facility on a forthcoming technology would reduce the demand for a current technology without such a facility and moreover would reduce the demand for software in the early period. We were unable to calculate the impact of the copying facility on second period software sales although the facility would encourage hardware sales.

APPENDIX 1

By definition

$$X_{11} = \int_{v_1}^{b} kx_{11}^{*} \, dv \tag{A1}$$

Equations (28) and (A1) are a pair of simultaneous equations in X_{11} and v_1 that we may in principle solve.

From (28) and (27) we may derive (A2)

$$X_{11} = \frac{N\alpha n}{\beta} \cdot \frac{1}{uz(p_2 - \delta)} \cdot [v_1 - up_1 + uzp_2 - (\frac{\theta}{\alpha n} \cdot u \cdot (F_1 - zF))^{\frac{1}{2}}] \tag{A2}$$

and from (A2)

$$\frac{\partial X_{11}}{\partial v_1} = \frac{N\alpha n}{\beta} \cdot \frac{1}{uz(p_2 - \delta)} > 0 \, . \tag{A3}$$

From (A1) we may derive that

$$\frac{\partial X_{11}}{\partial v_1} \cdot \frac{1}{k} = \int_{v_1}^{b} \frac{\partial x_{11}^{*}}{\partial v_1} \cdot dv - x_{11}^{*} (v_1) \tag{A4}$$

where $x_{11}^{*}(v_1)$ is x_{11}^{*} evaluated at v_1. Substituting from (7) into (A4) yields (A5).

$$\frac{\partial X_{11}}{\partial v_1} = \frac{-\frac{2\alpha n}{\theta}\,(v_1 - up_1 + ux\,[\frac{\beta x_{11}\delta}{N\alpha n} + (1 - \frac{\beta X_{11}}{N\alpha n})\,p_2]}{\frac{1}{k} - \frac{2\alpha n}{\theta}\,.\,u\,.\,z\,.\,(\delta - p_2)\,.\,\frac{\beta}{N\alpha n}\,.\,(b - v_1)} \qquad (A5)$$

Given that $\delta < p_2$, from (A5) $\frac{\partial X_{11}}{\partial v_1} < 0$ along (A1).

Thus along (28) X_{11} is an increasing function of v_1, and along (A1) X_{11} is a decreasing function of v_1, and therefore in principle (28) and (A1) can be solved for X_{11} and v_1.

By exploring the partial derivatives of X_{11} with respect to the exogenous parameters (p_1, P_2, F_1, F_2, δ, α, n and β) for a given v_1 along (28) and (A1) we may determine the impact on v_1 and X_{11} of changes in these parameters. Writing (A1), for a given v_1, as (A6).

$$X_{11} = H(p_1, p_2, F_1, F_2, \delta, \alpha, n, \beta) \qquad (A6)$$

the partial derivatives are signed as

$$H_1 < 0,\ H_2 > 0,\ H_3 = 0,\ H_4 = 0,\ H_5 > 0,\ H_6 > 0,\ H_7 > 0,\ H_8 < 0\ .$$

Writing (28) for a given v_1 as (A7)

$$X_{11} = G(p_1,\ p_2,\ F_1,\ F_2,\ \delta,\ \alpha,\ n,\ \beta) \tag{A7}$$

the partial derivatives are signed as

$$G_1 < 0,\ G_2 > 0,\ G_3 < 0,\ G_4 > 0,\ G_5 > 0,\ G_6 > 0,\ G_7 > 0,\ G_8 < 0\ .$$

Given the derivatives we may then determine that the relationship of X_{11} to the exogenous parameters is given by (A8) with derivatives as indicated

$$X_{11} = J(p_1,\ p_2,\ F_1,\ F_2,\ \delta,\ \alpha,\ n,\ \beta) \tag{A8}$$

$$J_1 < 0,\ J_2 > 0,\ J_3 < 0,\ J_4 > 0,\ J_5 > 0,\ J_6 > 0,\ J_7 > 0,\ J_8 < 0.$$

This method also enables us to sign $\frac{\partial v_1}{\partial F_1} > 0$ and $\frac{\partial v_1}{\partial F_2} < 0$. To sign the other derivatives on v_1 we use (A1) and (28) to yield (A9).

$$[v_1 - up_1 + uzp_2 - (\frac{\theta}{\alpha n}\ .\ u\ .\ (F_1 - zF_2)^{\frac{1}{2}})]\ [\frac{N}{\beta uz(p_2 - \delta)}]$$

$$= \frac{2k}{\theta}\ [v_1(v_1 - b) - [\frac{\theta}{\alpha n}\ u(F_1 - zF_2)]^{\frac{1}{2}}\ (b - v_1) + \frac{b^2}{2} - \frac{v_1^2}{2}] \tag{A9}$$

from which we may derive that

$$\frac{\partial v_1}{\partial p_1} > 0 \ , \ \frac{\partial v_1}{\partial p_2} < 0, \ \frac{\partial v_1}{\partial \delta} < 0, \ \frac{\partial v_1}{\partial \alpha} > 0, \ \frac{\partial v_1}{\partial n} > 0, \ \frac{\partial v_1}{\partial \beta} > 0 \ .$$

In Table 3 we summarise the derivatives we have found relating X_{11} and v_1 to the parameters.

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