The Role of Exogenous and Endogenous Learning and Economic Factors in the Diffusion of New Technology: An Epidemic Based Study of the Spread of Colour Television Ownership in the UK

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Abstract

In this paper existing epidemic models of diffusion in the literature are modified to give freer interaction between exogenous and endogenous factors in the diffusion process and to also reflect the role of economic factors in that process. A new encompassing model is proposed and its properties explored. The model is applied to the diffusion of colour television ownership in the UK and outperforms the existing models in the literature. The exogenous factors play a dominant role in the diffusion of colour television in the UK.

Keywords: Technology Diffusion, Colour Television.

JEL Classification No.620

1. Introduction[1]

Recent theoretical analysis of the process of technological diffusion has generated two main classes of models in the literature. The first class, discrete choice models, incorporates probit and game theoretic models (see for example David, 1969 and Reinganum, 1981), and the second class includes information based (epidemic or logistic) models (see, for example, Mansfield, 1968). The literature has been surveyed by Stoneman (1983 and 1987). Empirical work on diffusion has been largely concentrated on the epidemic models, and there has only been a limited amount of empirical work on probit models (see, for example, Davies, 1979). To the best of our knowledge the game theoretic approach has not been used empirically, although elements of that approach are reflected in the work of Hannan and McDowell(1987). The work reported in this paper is part of a larger research project[2] exploring the empirical validity of different diffusion models.

Deaton and Muellbauer (1980) state, 'it is likely to be difficult in practice to disentangle diffusion processes associated with discrete choice problems from those associated with the spread of information'. In line with this statement (which is particularly relevant, when as here, panel data on individual adopters is not available) our work has proceeded in two directions. In that reported here we begin with information based models which are largely devoid of any active role for economic forces and modify them to provide such a role. In other work proceeding, we begin with discrete choice models which stress economic factors but ignore information problems, and adapt them to incorporate such information factors. Thus the work reported in here is firmly based in the epidemic tradition.

To be precise, in this paper we make two modifications to the standard logistic model. First we allow for the speed of diffusion to be a function of economic variables. Secondly, we

[1] We are grateful for comments from Hashem Pesaran, Anne Gibbons and participants in industrial organization and technology conferences at the universities of Bristol and Warwick. Of course, all errors that remain are the responsibility of the authors.

[2] The project, funded by the ESRC, has the title 'The theory, empirics, and policy of technological diffusion', and is part of the ESRC/DTI research initiative on new technologies and the firm.

modify the standard assumptions made with regard to the learning process in such models. In particular we argue that the standard epidemic model is unsatisfactory in that it always gives dominance to endogenous elements in the learning process. We then set up a more general epidemic model (which nests the standard logistic models in the literature) that does not assume such dominance a priori, and can thus be used to explore the relative strengths of the exogenous and endogenous forces.

The paper proceeds with a discussion of the standard epidemic models followed by the specification of a new model. We explore the mathematical properties of this model and also discuss its stochastic specification. Finally, an application of the model to the growth of colour television ownership in the UK is made, and the results are compared with those of other models suggested in the literature.

2. The Logistic Type Growth Curves

In recent years different logistic type[3] growth curves have found a wide range of applications in characterizing the diffusion of new products and technologies (for some recent reviews of the literature see, Stoneman 1983, Mead 1984, and Mahajan and Wind 1986). The empirical results of fitting the logistic type models to the data on product diffusion, however, have not been entirely satisfactory. The following problems have been observed in the empirical case studies. First, the fit of the model varies from one case to another. While a particular curve fits the data for certain new products adequately, it performs poorly with respect to others. Further, it is often noted that different growth curves with different properties and divergent forecasts fit a particular set of data equally well. Second, the parameter estimates often show wrong sign or magnitude compared to what theory would suggest. Third, forecasting performance varies widely across different products, and sometimes the calculation of the forecast error is too complicated and not reported. As has been pointed out in a series of

[3] By logistic type curves we refer to the family of exponential growth curves with saturation, such as the Logistic, Gompertz, Log-Logistic, etc., which are alternatively referred to by Gregg et.al. (1964) as the modified exponentials.

recent papers some of these problems arise from the stochastic specification of the models and the estimation methods used (see e.g., Mead 1985, 1988, Oliver and Yang 1988). These issues will be taken up in later sections. In this section we are instead mainly concerned with the functional form of the logistic type models and their theoretical underpinnings.

In part of the literature the logistic curve is introduced as a summary device without any attempt at a theoretical justification. Griliches (1957) and Mansfield (1968), for example, use the logistic curve as a device for parsimonious representation of the data, where the coefficients of the fitted curve are later used in cross-section analysis for testing the effect of economic variables. This 'curve fitting' approach has, paradoxically, become popular in the marketing literature where the main interest is in forecasting. Different 'modified' or 'generalized' logistic curves have been devised in order to achieve better fits to empirical data through the adoption of more flexible functional forms, and in particular to derive a skewed logistic function more in conformity with empirical growth curves (see, e.g., Chow, 1967, Hutchesson, 1967, Tanner 1978, Easingwood etal 1981, McGowan, 1986).

In much of the literature, however, the rationale for the use of the logistic is sought on behavioural grounds. The behavioural justifications for the use of the logistic in characterizing the diffusion process is often made by analogy to the spread of epidemics as discussed in biological sciences. It is thus appropriate to begin with a brief discussion of a simple epidemic model before moving to the economic applications of the model. This would help to underline the assumptions which are necessary to justify the use of the logistic for characterizing the diffusion process (see also Bain, 1964, Davies, 1979).

Assume a community with a number of persons susceptible to a new infection, N, a number of already infected people S, and a constant rate of infection β (i.e., β = probability of contracting the infection after a contact is made). Under the assumption of a homogeneously mixing population, it is plausible to assume that the probability for a susceptible to meet an infected person and contract the disease in a small time interval dt is $\beta(S/N)$. In a population of (N-S) susceptibles the average number of infections in a small time interval dt would therefore be:

$$dS = \beta \frac{S}{N} (N-S)\, dt \qquad (1)$$

Integrating this equation gives the simple deterministic logistic curve for the spread of the epidemic as a single valued function of time. Bartlett (1955) and Bailey (1950) provide stochastic versions of the model with properties which for medium sized populations are particularly at variance with the deterministic case. However, since in economic applications we mainly deal with large populations, it would be more appropriate here to work with the deterministic version of the model. This, in addition to avoiding some of the intractable complexities of the stochastic case, also helps to illustrate the relation of the new model proposed to the logistic type models which in the economics literature are largely formulated in deterministic terms.

The following assumptions are made in the above derivation of the simple logistic growth curve (see, Bailey 1957); i)- infection spreads through contact between the members of the community, and it is not sufficiently serious for cases to be withdrawn from circulation by isolation or death, ii)- each infected individual has the same chance of coming into contact with a susceptible member of the community independently of the age of his infection or his location, and iii)- no case becomes clear of infection during the course of the epidemic. These assumptions are additional to the assumption made above that β is the same for different individuals, i.e., individuals are equally susceptible to the infection once a contact is made.

The analogy often made between the spread of epidemics and the diffusion of a new technology or product is either based on the learning processes involved in the use of new technology and its transmission through human contact, with the 'infection' being information, or based on pressures of social emulation and competition. Bain (1962) derives a simple logistic curve for the growth of monochrome television ownership in the UK on the basis of such behavioural reasoning. In doing so he in effect retains all of the assumptions of the simple epidemic model enumerated above. Bass (1969), however, partially modifies the assumption that the diffusion process takes place as an endogenous learning process within a homogeneous population. Following Rogers (1962), Bass distinguishes between two homogeneous groups; the innovators, who are not subject to social emulation or endogenous learning, and the imitators, for whom the diffusion process mainly takes the epidemic form discussed above. This gives rise to the positively skewed logistic curve which has become influential in the marketing literature. Different interpretations of the Bass model have appeared in this literature. For example Dodson and Muller (1978) attribute the exogenous part of the Bass model to promotional advertising, and the endogenous part to 'word of mouth' (see also Lekvall and Wahlbin, 1973). Tanny and Derzko (1988) rightly argue that the model originally formulated by Bass does not abandon the assumption of a homogeneous population as is commonly maintained in the literature that followed it, but rather, it distinguishes between the endogenous and exogenous influences at work in the process of adoption in a homogeneous population. They in turn set up a logistic model which explicitly distinguishes between the innovators and imitators in the population. It is interesting to note that in empirical studies, either based on the Bass model or on its new reformulation by Tanny and Derzko, the exogenous factor turns out to play an insignificant role in the diffusion process. As we shall argue below, this may result from the a priori assumption of a dominant endogenous growth factor in these models. In other words, given the underlying mathematical structure of these models, in the interaction between the endogenous and exogenous factors it is the endogenous element which would have the dominant influence. None of the above variants of the Bass model, therefore, essentially alter the underlying assumptions of the simple epidemic model and its dynamic behaviour.

3. A New Model

To illustrate the basis of the model that we construct, consider the standard logistic model as separating the acquisition decision into two parts. The first is that information spreading or the pressures of social emulation generate a 'desire to acquire', and second is that some proportion of this is converted to actual acquisition decisions through β, the probability of acquisition (contracting the infection) once the 'desire to acquire' (contact) has been established. In the new model proposed here two basic changes to the standard model are made.

First we allow for β to be a function of economic variables rather than being taken as a fixed parameter. This in itself is not a major revision. Mansfield (1968) provides a theoretical model justifying such a variation, and a whole body of empirical work (starting for example with Griliches, 1957) proceeds on the assumption that β is dependent on economic variables. In Mansfield's work, where new process technologies are being considered, β is taken to be a function of for example, profitability, firm size and liquidity. As colour televisions are household durables we consider β to be a function of disposable income (YD), the price of colour televisions (P) and credit conditions (CRED). Where our model differs from Mansfield's and much of the other empirical work on diffusion is that we allow β to vary *over time* as a result of variations in income, the price of colour televisions and credit conditions. In Mansfield's work, which is based on cross section data, β is constant over time but differs across industries or technologies. As our data covers a single aggregate time series, only intertemporal variations can be accommodated.

The second modification to the standard epidemic model that we introduce is to respecify the nature of the learning or emulation process. The respecification allows for the standard logistic as a special case. The reason for doing this is that as we noted above, a major weakness of the standard epidemic, logistic or Bass type models is that they, a priori, assume that endogenous factors throughout the diffusion process remain dominant in the learning or emulation process. It is more plausible, however, to set up a more general model which could test this proposition (even though it may after all turn out to be correct under certain circumstances). This objective is met by the new model through dropping some of the restrictive assumptions of the simple epidemic model.

One assumption which has an important bearing on the behaviour of the simple epidemic model, and does not seem to be plausible in the case of the diffusion of new products, is that of a homogeneously mixing population. It is more reasonable to assume that each individual has contact with only a limited number of individuals in the society, and therefore his direct influence in terms of social emulation and/or learning gradually wears off as his immediate contacts adopt the new product. In addition, with the existence of exogenous sources of information and learning, one would also expect the effect of endogenous learning through personal contact to gradually wear off as the news about the existence and the qualities of the new product become common knowledge.

To pursue this idea we distinguish at each point of time between three subsets of the total population (or total maximum adopters) N in the following way: the number of owners of the new product at time t who continue to be instrumental in social emulation or learning (X); the number of owners at time t who no longer contribute to the diffusion of the new product (Y); and the number of non-adopters who are 'susceptible' at time t (Z), such that N=X+Y+Z. As before we define S=X+Y as the total number of owners at time t. The non-homogeneous mixing is catered for by assuming that only X and not S influences learning and emulation. In addition we allow for exogenous factors to influence learning and emulation by incorporating a factor q in the learning or emulation process. Specifically we have that:

$$\frac{dS}{dt} = \beta(q+X/N)Z = \beta(Nq+S-Y)(N-S)/N \qquad (2)$$

To complete the model we specify that the number of 'influential owners' X will 'depreciate' over time according to (3):

$$\frac{dY}{dt} = \alpha X = \alpha(S-Y) \qquad (3)$$

where α is the coefficient of decay of the 'active' adopters. Given the initial values (S, Y)=(S_0, 0) at time t=0, we can solve the above equations in the following way. Substituting for (S-Y) from equation 3 into equation 2 we get:

$$dS/dt=\beta[Nq+(1/\alpha)(dY/dt)](N-S)/N. \qquad (4)$$

Solving this equation for Y we get:

$$Y = N(\alpha/\beta)(\text{Log}(\frac{N-So}{N-S}) - \beta qt)$$

Substituting for Y in equation 2 gives the following solution:

$$\frac{dS}{dt} = \beta[q + S/N - (\alpha/\beta)(\text{Log}(\frac{N-So}{N-S}) - \beta qt)](N-S) \qquad (5)$$

For the values of q=0 and α=0 this equation reduces to the simple logistic, and for q>0 and α=0 it gives the Bass model. The new model thus nests these two other models. This model meets the aim of allowing for a more free interaction between endogenous and exogenous growth factors, without imposing any a priori assumptions about the dominance of one factor over another. The coefficient α/β in this model could be interpreted as an index of the strength of the endogenous growth factors. It could be said that with the value of $(\alpha/\beta)\geq1$, the endogenous growth factors are weak and exogenous factors play a dominant role in diffusion, and the reverse with $(\alpha/\beta)<1$. The value of this index, however, has to be estimated empirically. The existing epidemic models in the literature, by assuming $(\alpha/\beta)=0$, give an a priori dominance to the endogenous growth element.

To observe the role of the coefficient (α/β) in the model more clearly, it would be helpful at this point to explore the mathematical properties of the new model. For this purpose, we treat β as a constant parameter. Like the simple logistic and the Bass models, the new model has a stable equilibrium at S=N. This can be easily checked if we solve for S from equation 4 to get:

$$S = N - [\exp(-(\beta/\alpha)Y/N)] [\exp(-\beta qt)]$$

Clearly, as $t \rightarrow \infty$, S tends towards its stable equilibrium value of N, the saturation level. The new model, nevertheless has totally different underlying properties both from the simple epidemic and the Bass models. To see this more clearly we shall set q=0 and rewrite equations 2 and 3 as:

$$\frac{dS}{dt} = \beta XZ/N = \beta(S-Y)(N-S)/N \tag{6}$$

$$\frac{dY}{dt} = \alpha X = \alpha(S-Y) \tag{7}$$

This set of equations, which characterize the endogenous side of the more general model, has entirely different dynamic properties from the simple logistic model. In addition to the solution S=N, this system has an infinite number of other solutions in the interval {0, N} when S=Y. None of these solutions, however, constitutes a stable node. A comparison between the phase diagrams of the system of differential equations 6 and 7 with those of the more general model (i.e., equations 2 and 3) would help to clarify some important aspects of the dynamic behaviour of the model. For this purpose we divide equations 6 by 7, and 2 by 3 respectively to derive the following set of equations in terms of S and Y:

$$\frac{dS}{dY} = (\beta/\alpha)(N-S)/N \tag{8}$$

for the restricted model, and:

$$\frac{dS}{dY} = (\beta/\alpha)\left(\frac{Nq+S-Y}{S-Y}\right)(N-S)/N \tag{8'}$$

for the general model. In both cases $(dS/dY) \geq 0$ for values of $S \leq N$. The phase diagrams of the restricted model (with q=0) and the general model are shown in figures 1 and 2 respectively. As we are interested in values of S and Y which obey the restriction $0 \leq Y \leq S$

≤ N, the diagrams are depicted only for values in the upper right hand quadrant above the 45° line.

Figure 1

Adjustment Paths with q=0

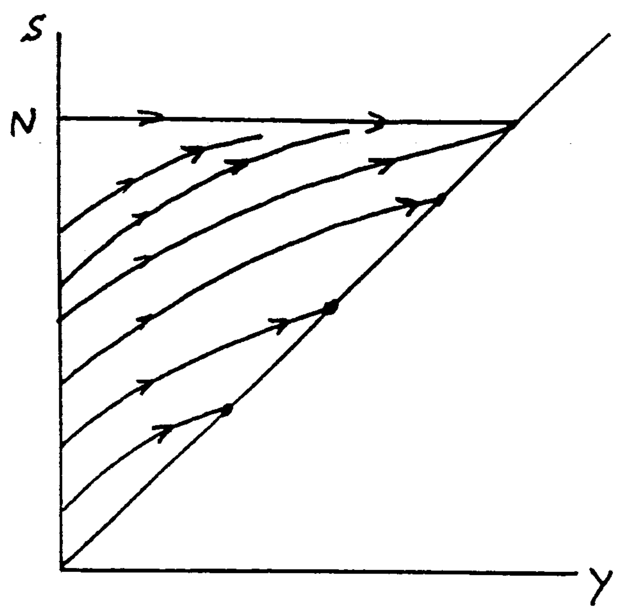
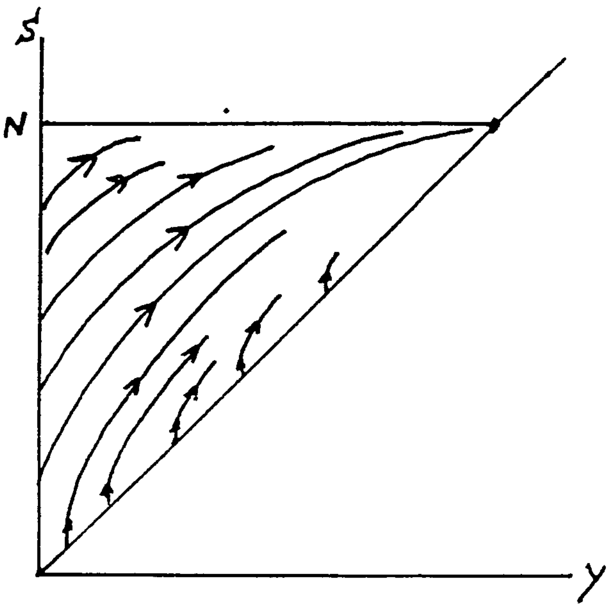


Figure 2

Adjustment Paths with q>0



As was pointed out, the trajectories in both phase diagrams have positive slopes. Equations 8 and 8' also indicate that in both cases the overall slope depends on the relative values of β and α, but declines as S approaches the saturation level N. In the purely endogenous diffusion case (figure 1), the terminal point of the trajectories - which could be anywhere in the {0, N} interval on the 45° line - depends on the initial values as well as the coefficient (α/β). None of these points, however, constitute a stable solution. The model depicted in figure 1, therefore, characterizes a degenerate case, where the diffusion process in the absence of exogenous factors may come to an end well below the potential saturation level N. This could well characterize many instances of brand competition where endogenous diffusion factors are weak (i.e., β/α is small) and exogenous factors - e.g., effective advertising campaign, price competitiveness etc. - are absent.

In the case of the general model, depicted by the phase diagram 2, the system starting from any initial conditions would end up at the stable solution Y=S=N. As can be seen from equation 8', at values of Y=S the slope of the trajectories is ∞, and as S tends towards N the slope approaches 0. The general model proposed here, therefore, characterizes the process of diffusion as one of interaction between the endogenous and exogenous factors without setting any a priori restrictions on which one would be the dominant factor. This is the major distinction between this model and the logistic type models so far proposed in the literature. The assignment of a priori dominance to the endogenous factors in those models totally obliterates the significance of exogenous factors. This could be an important reason for past econometric studies based on such models generating insignificant coefficient estimates or the wrong signs for exogenous economic variables.

A Discrete Analogue of the Model

For the purposes of estimation it is more convenient to work with a discrete time version of the above model. To derive the discrete time version we write equations 2 and 3 in the following way:

$$\Delta S_t = \beta((X_{t-1}/N)+q)Z_{t-1} = \beta(Nq+S_{t-1} - Y_{t-1})(N-S_{t-1})/N \qquad (9)$$

$$\Delta Y_t = \alpha X_{t-1} = \alpha(S_{t-1} - Y_{t-1}) \qquad (10)$$

Equation 10 can be expanded in the following form:

$$Y_t = Y_{t-1}(1-\alpha) + \alpha S_{t-1} = \alpha \sum_{i=0}^{t-1} (1-\alpha)^i S_{t-i-1}$$

substituting in 9 yields (11):

$$\Delta S_t = \beta[Nq + S_{t-1} - \alpha \sum_{i=0}^{t-1} (1-\alpha)^i \; S_{t-i-1})](N-S_{t-1})/N \qquad (11)$$

This is a general distributed lag model which reproduces the simple logistic model with α=0 and q=0, the Bass model with α=0 and q>0, and the degenerate diffusion case discussed above with q=0 and $\alpha \geq \beta$. It is thus an encompassing model which allows for the testing of those restrictions which are in the existing literature usually imposed on diffusion models on a priori grounds.

4. Stochastic Specification

In fitting the logistic type growth curves to the data, a variety of stochastic specifications have been put forward in the literature. The most common approach has been to superimpose an additive disturbance term on a growth curve which is a deterministic function of time (see e.g., Mar Molinero, 1980, Oliver, 1981, and Mead, 1984 for further references). The general stochastic model takes the following form:

$$S_t = F(\omega ;t) + u_t$$

where ω is typically a three or four parameter vector, depending on the specific form of the growth curve assumed, and u_t is assumed to be a white noise disturbance term. Different empirical studies have shown that alternative parameterizations of this type of global trend model often fit the data equally well, but produce widely diverging forecasts. This may be evidence of spurious regression, which is not uncommon in such type of models (see e.g., Granger and Newbold, 1977). Unfortunately very few of the empirical studies report the value of the Durbin Watson test statistic, or other test statistics which could substantiate the adequacy of the stochastic assumptions of their models. In one case, Mar Molinero (1980), where the DW test statistic is reported, there is strong evidence of first order autocorrelation in the error terms which supports the hypothesis of spurious regression. Such type of global trend models have also been criticized on various other grounds in the literature (see, e.g., Harvey, 1984, Mead, 1985, and Oliver and Yang, 1988).

The second common approach is to introduce the random element in the new adoptions in each period (ΔS_t), rather than in the stock (S_t) as in the above models. The change in stock is characterized as a deterministic function of the previous period level of stocks, upon which an additive disturbance term is superimposed (see, e.g., Bain, 1962, Williams, 1972, and Bass, 1969). The general form of the stochastic model which is estimated in this type of approach could be written as:

$$\Delta S_t = F(\omega ; S_{t-1}) + u_t$$

where ω is a three to four parameter vector and u_t is a whitenoise error term. One problem with this type of specification is that it fails on account of the possibility that the variance in the error term near the saturation level of the diffusion process will be very different from the variance during the high growth period in the early and middle parts. To deal with this problem Mead (1988) introduces a heteroskedastic error term, $u_t \sim N(0,V_t)$, where $V_t = S^{\pi}(N-S)^{\tau}\sigma^2$. He then conducts a numerical search for the values of π and τ which best fit the variance structure of the model.

A second criticism levelled against the above type of models relates to the assumed constancy of the parameters of the growth curve over the entire diffusion process. Mead (1985, 1988) estimates variable coefficient models, where parameters such as the saturation level and the growth coefficient are assumed to follow a random walk. He uses Kalman filters adapted for non-linear equations to estimate the model. It should be noted, however, that varying parameters may be due to mis-specification of the model or omission of important explanatory variables.

In the model we use here for estimation the random element is attributed to the flow or change in stocks rather than the level of stocks. The criticisms of this raised in the literature, however, have to be explicitly addressed. In particular, we must test for the structural stability of the model. This should of course be done after including in the model the economic variables which are likely to have important effects on the diffusion process. A common shortcoming of the recent literature is that such relevant economic variables, e.g., prices, incomes, credit conditions, etc., are often excluded. Once included, the coefficient estimates of such variables often show signs which are contrary to what theory would suggest (see, e.g., Bain 1964, Bonus, 1968). This suggests that theory consistency should be another important criterion of model selection. Further, the error specification of the model should conform with the mathematical structure of the model. This is particularly important in non-linear models which are subject to saturation. Thus we provide the relevant asymptotic test statistics to check the consistency of the error assumptions of the model. Finally, we compare the performance of the model with that of some of the existing models in the literature, in terms of data consistency, structural stability and predictive performance.

The following versions of equations 9 and 10 represent the basis for our estimation:

$$\Delta S_t = \beta_t[q + S_{t-1} - Y_{t-1}(\alpha)](1 - S_{t-1}) \tag{12}$$

$$Y_{t-1}(\alpha) \equiv \alpha \sum_{i=0}^{t-1} (1-\alpha)^i S_{t-i-1}$$

where the ownership level S is shown as a ratio of saturation level N which is taken as fixed in this formulation. For simplicity only the β parameter is considered to be a variable function of economic variables, while both q and the saturation level parameters are fixed. One could further extend the influence of the economic factors by allowing for example that q, characterizing the exogenous component of the growth curve, to be a function of economic variables. In the marketing literature it is sometimes made a function of advertising expenditure. Similarly N (the saturation level) is often characterized as a function of relevant economic variables such as prices, incomes, etc. (see, e.g., Bain, 1962, and Williams, 1972). We have not allowed for such influences, partly because of the limitations of the data but primarily because we believed that a more parsimonious model would allow for a better appraisal of the comparative properties of the model. We, however, test the sensitivity of the model to the variations in the saturation level.

For television ownership, the case study below, following Bain (1964), we have assumed the saturation level to be equal to the total number of households in each period, which seems plausible given that in the early 1980s more than 80 per cent of households owned colour televisions. We shall nevertheless consider the sensitivity of the model to the assumed saturation level by allowing it to equal a θ fraction of the number of households and then varying the value of θ in following version of equation 12, by increments between 0.8 and 1.0:

$$\Delta S_t = \beta_t[\theta q + S_{t-1} - Y_{t-1}(\alpha)](\theta - S_{t-1})/\theta \qquad (13)$$

where the ownership level S_t is shown as a proportion of total number of households and θ is the ratio of saturation level to the number of households.

To take care of the possibility that the variance of ΔS_t changes as the saturation level is approached, we propose to estimate the following transformation of the model:

$$(\theta \Delta S_t/(\theta - S_{t-1})) = \beta_t(\theta q + S_{t-1} - Y_{t-1}(\alpha)) + u_t \qquad (14)$$

where u_t is assumed to be an iid$(0,\sigma^2)$ error term. In the type of models we have been discussing in previous section, we would have $\Delta S>0$ as long as S remains below the saturation level. In this case we may force the error term to be positive by assuming a multiplicative exponential error term of the form $\exp(u_t)$, where u_t is an iid$(0,\sigma^2)$ error term as before. In this case the above equation may be estimated in log terms. The choice between the two functional forms has to be decided by the data.

5. Estimation and Inference

The model was applied to the data for colour television ownership in the UK over the 1968-86 period. Quarterly figures for colour television ownership were collected on the basis of monthly license[4] figures published in the Monthly Digest of Statistics. The charts in Figures 3 and 4 show the one Qtr. and four Qtr. lag difference in ownership for the 1968-86 period. As can be seen the one Qtr. differences are extremely volatile. This is partly due to seasonal variations, but more significantly due to the leads and lags arising from delayed response in license fee payments, postal delays etc. Considering that the flow figures have been

[4] We would like to thank Anne Gibbons for supplying us with her data on the relevant variables.

Fig 3. UK Colour TV Ownership, 1 Qtr Lag Difference (% Households)

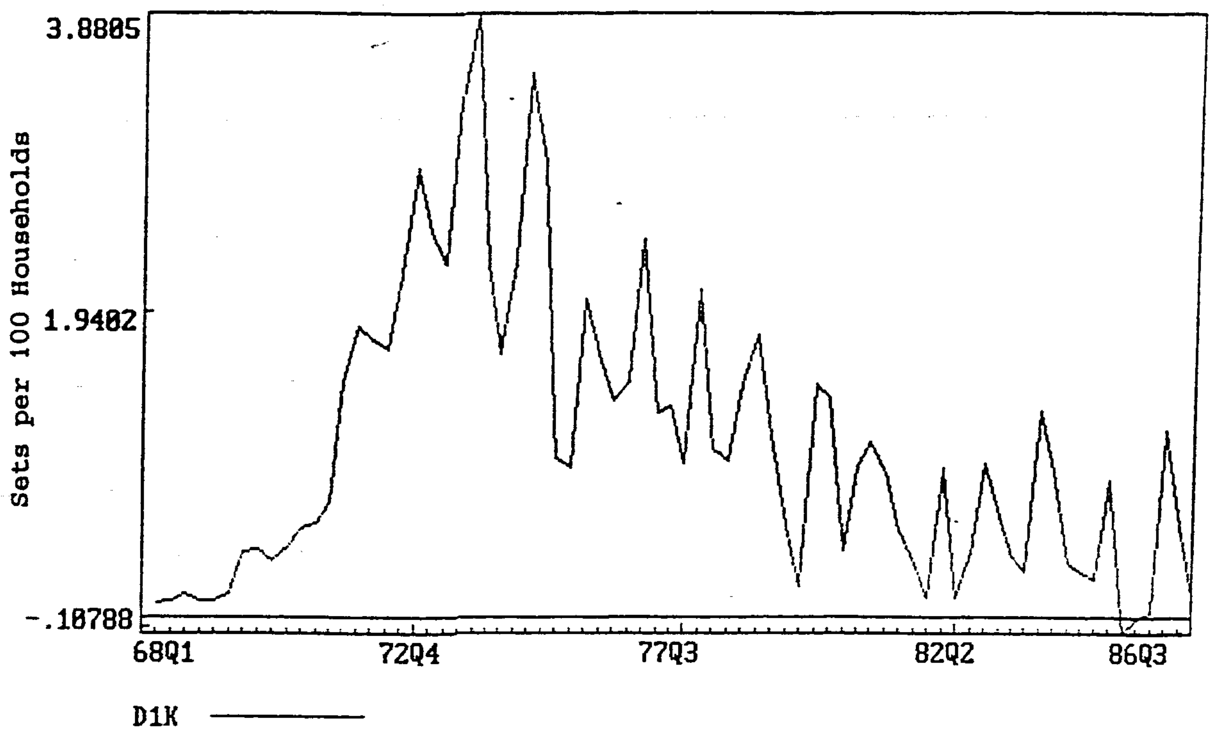
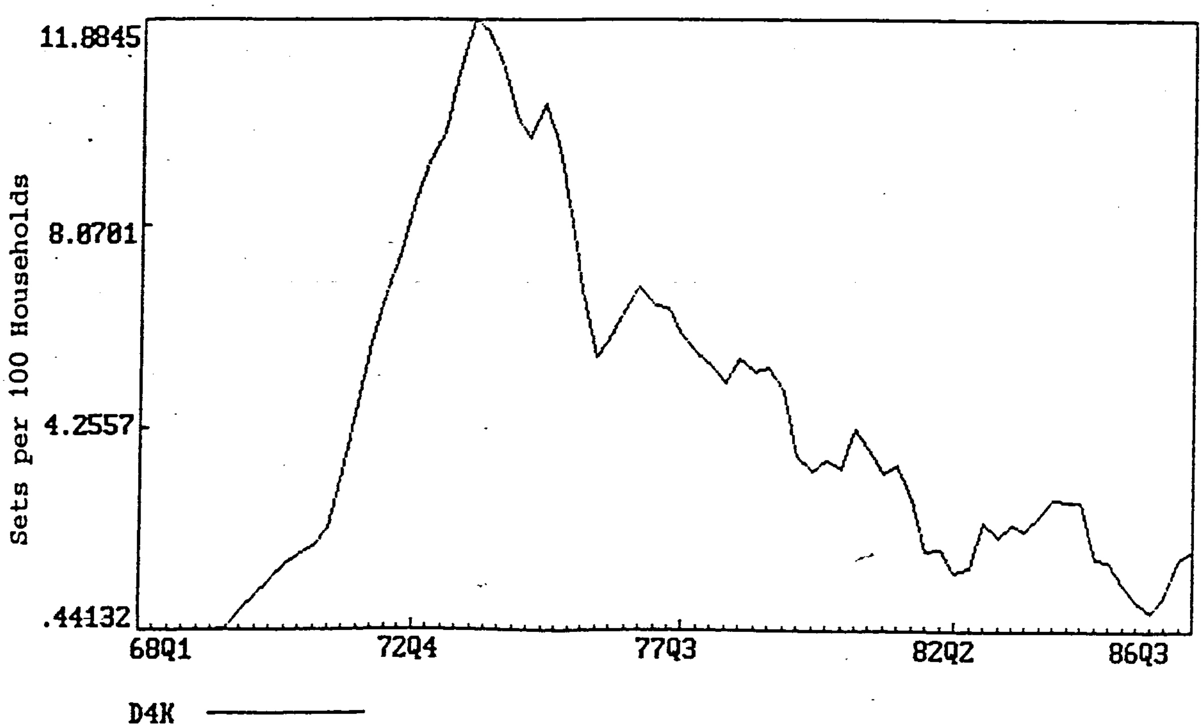


Fig 4. UK Colour TV Ownership, 4 Qtr Lag Difference (% Households)



calculated as quarterly differences in stocks, such 'measurement errors' could easily overshadow the additions to the stock, particularly near the saturation level. To overcome this, we have estimated the model in four Qtr. lag difference, using a four Qtr. moving average transformation of the quarterly stock data. Denoting the original data series by K_t, we applied the following linear filter, $S_t=(1/4)(1+L+L^2+L^3)K_t$ which gives $\Delta S_t=\Delta^4 K_t$. The model was estimated in terms of ΔS_t and S_t. The economic variables included in the model as determinants of β were, the retail price of colour television (P) (deflated by the general retail price index), real personal disposable income (YD), and a proxy for credit conditions(CRED). The quarterly price variable was lagged four periods to safeguard against possible simultaneity bias arising from the interaction between demand and price. The income variable was also lagged one period. The hire purchase deposit was taken as a proxy for the credit conditions variable (the other relevant variable, i.e., the maximum repayment period, is highly correlated with the hire purchase deposit and hence is not included)[5]. The credit variable is meant to reflect the effects of liquidity constraint on consumer demand.

The model was estimated both in the form shown in equation 14, with $\theta=1$, and in log transform form. As the latter showed slightly better results we report here only the results from the log-transformed model. Incorporating the relevant economic variables in the β coefficient (assuming constant elasticities):

$$\beta t = a_0 P_{t-4}^{a_1} \; YD_{t-1}^{a_2} \; CRED_t^{a_3}$$

and taking logs from both sides of equation 14 gives equation 15 which is final form used for estimation:

[5] The data are based on the following sources. Quarterly data on real personal disposable income and the general retail price index are from the Monthly Digest of Statistics, 1967-86, CSO. Wholesale price index of colour televisions is based on, Price Index Numbers for Current Cost Accounting, 1968-85, CSO. Purchase tax or VAT plus retailers margins were added to this to derive the retail price index for colour televisions (sources; Retailing, CSO, and British Radio and Electronics Manufacturers Association). The data for hire purchase deposits and minimum repayment period are provided by the British Radio and Electronics Manufacturers Association. The data on the number of households are based on the Census of Population, 1961, 1966, 1971, 1981 (at 1st April). Quarterly data are estimated by linear extrapolation. The original data is available on request from the authors.

$$LS_t = a_0 + a_1 P_{t-4} + a_2 YD_{t-1} + a_3 CRED_t + Log(q + W_{t-1}(\alpha)) + u_t \qquad (15)$$

where for brevity $Log(\Delta S_t/(1-S_{t-1}))$ is denoted by LS_t and $(S_{t-1}-Y_{t-1}(\alpha))$ by $W_{t-1}(\alpha)$. On theoretical grounds we expect the following coefficient signs: $a_1 \leq 0$, $a_2 \geq 0$, $a_3 \leq 0$, and $q \geq 0$.

The above equation was estimated by Nonlinear Least Squares (NLS), which, adding the assumption of normality of the error term replicates the results from Maximum Likelihood estimation (see, e.g., Gallant, 1987). Though the data series start from 1968Q1, the equation was estimated for a sample of 67 quarterly observations, from 1970Q1 to 1986Q3. The starting point of 1970Q1 was chosen because colour transmission up to the fourth quarter of 1969 were confined to BBC 2, and in any case the license figures for those early years may not accurately reflect colour tv ownership. A grid search was conducted for the value of α which minimized the standard error of regression (or assuming normality of errors maximized the likelihood function), and $\alpha=1.2$ was chosen as the optimum value. The estimated equation for the value of $\alpha=1.2$ was:

$$LS_t = \underset{(5.10)}{4.17} - \underset{(-8.16)}{0.801} P_{t-4} + \underset{(1.86)}{0.981} YD_{t-1} - \underset{(-2.18)}{0.359} CRED_t$$

$$+Log(\underset{(4.18)}{0.0011} + W_{t-1}(1.2)) \qquad (16)$$

$T=67$, $R^2=0.937$, $\sigma'=0.1540$, $LL=32.8$,

$F,AR1=1.58$, [4.0] $F,AR4=1.45$, [2.35] $F,Hetr=.089$, [4.0] $Z1=0.933$ [5.99]

The asymptotic t ratios of the coefficient estimates are given in brackets. As can be seen, all of the coefficients have the correct sign and they are all statistically significant at the 5% significance level. In addition, a number of other test statistics are provided for testing the overall goodness of fit and the adequacy of the underlying error assumptions of the model. R^2 is the multiple correlation coefficient adjusted for the degrees of freedom, σ' is the asymptotic standard error of regression, and LL is the maximized value of log likelihood function. F,AR1 and F,AR4 are the asymptotic F test statistics for 1st order and joint 1st-4th order serial correlation of residuals. These test statistics, together with all other tests which are based on some kind of parameter restriction in linear models (i.e., the Lagrange Multiplier, Likelihood Ratio, and Wald type tests) are all asymptotically valid for the non-linear models as well (see, e.g., Judge et.al. 1980, ch.17, Gallant 1987). F,Hetr is the asymptotic F test for heteroskedasticity based on the hypothesis of correlation between the second power of fitted values and the second power of the residuals. Z1 is the Chi-Sq test for normality of the disturbances based on skewness and kurtosis of the NLS residuals. The 5% significance critical values of these test statistics are given in square brackets. As can be seen none of the prior assumptions regarding the stochastic behaviour of the model are rejected by these tests.

One of the interesting results of the above estimate is the relatively high value of α, 1.2. This implies that the epidemic or endogenous growth factor was not very important in the diffusion process. This is a perfectly plausible result in the case of colour television ownership, for by 1968 the majority of the households already owned monochrome tv sets and were familiar with the qualities of the new product in terms of tv programs etc. Furthermore, colour television transmission in the UK had a relatively late arrival - by 1968 colour tv ownership was already widespread in the US. It seems natural therefore that the endogenous learning processes through the epidemic mechanisms would play only a small role in the diffusion of colour television.

We next proceeded to test the appropriateness of the assumption that the ultimate saturation level was the total number of households. For this purpose we replaced the dependent variable by the new variable $Log(\theta \Delta S_t/(\theta - S_{t-1}))$ and after transferring $Log(\theta/(\theta - S_{t-1}))$ to the right hand side of the equation, estimated the parameter θ together with other parameters by NLS. The estimated value of θ was 0.9 which was significantly different from 1.0. To test the sensitivity of this result to the assumption of α=1.2 we conducted a new grid search for the value of α. The outcome was that the values of α=1.2 and θ=0.9 were optimum. The maximized log-likelihood measures for selected values of α and θ are shown in Table 1. Since the maximum LL values were relatively close, and as we are here mainly concerned with the general properties of the model rather than exact elasticity estimates, it was decided to keep the value of θ=1.0 for the rest of the analysis. A more thorough estimation procedure, of course, may require modelling of the saturation level in terms of economic variables.

Table 1
Maximized Log-Likelihood Values

θ \ α	1.1	1.2	1.3
1.00	32.73	32.85	32.35
0.90	33.44	33.66	33.39
0.85	30.92	31.29	31.30

Structural Stability and Predictive Performance

Equation 15 was re-estimated over the 1970-80, 1970-82, 1970-83, and 1970-84 sub-periods by NLS in order to test the structural stability and predictive power of the model. The coefficient estimates together with their asymptotic standard errors are shown in Table 2. As the Table shows the coefficient estimates remain fairly stable over the different sample periods. The Chow test for structural stability (CHOW1 in Table 2) does not reject the hypothesis of structural stability for any of the sample periods (the test was not possible for periods after 1983, as the matrix of regressors in the linearized 'pseudomodel' used in estimation became singular for the period 1983-86 and its successive sub-periods. Had it been possible, however, we consider that there is little doubt that the above result would be confirmed for these subperiods as well).

Table 2
Coefficient Estimates for Different Sample Periods

Coefficient Estimates	Sample Periods				
	70Q1-86Q3	70Q1-84Q1	70Q1-83Q1	70Q1-82Q1	70Q1-80Q1
a_0	4.17 (5.1)	4.17 (5.5)	4.22 (5.3)	4.79 (6.6)	5.51 (8.4)
a_1	-0.801 (-8.1)	-0.819 (-9.2)	-0.827 (-8.7)	-0.918 (-10.2)	-1.03 (-11.8)
a_2	0.981 (1.9)	1.12 (2.2)	1.09 (2.1)	0.941 (2.1)	0.782 (1.9)
a_3	-0.359 (-2.2)	-0.463 (-3.2)	-0.474 (-3.0)	-0.574 (-3.7)	-0.647 (-4.7)
q	0.0011 (4.2)	0.0014 (4.3)	0.0014 (4.2)	0.0015 (4.7)	0.0018 (4.9)
CHOW1	---	n.a.	n.a.	1.27 [2.37]	2.22 [2.37]
CHOW2	---	0.724 [1.53]	0.858 [1.53]	1.84 [1.59]	4.91 [1.59]

The Table also shows the results of Chow's predictive failure tests (CHOW2) for different sample periods. The results are fairly satisfactory for sub-periods up to 1970-82 when the test just misses the 5% critical value, but for the 1970-80 subperiod the test clearly fails. It could be said that Chow's predictive failure test, or any test of predictive failure for that matter, is a joint test of variability of parameters and the variance of the disturbance terms. It is quite possible therefore that the assumption of variance stability does not hold, and that the F test reported in equation 15 has not been adequate to detect this. This seems plausible, given that there are various a priori reasons to support it. First, the assumption of a constant q, which was made here for simplicity may not be correct. The parameter q is meant to capture the effect of exogenous factors, such as advertising expenditure, which are likely to have systematic variation over time, and therefore their omission could lead to non-stationary error variances. The second, and perhaps more important, source of variance instability is the possible measurement errors to which we have already referred in the previous section. As we have pointed out such measurement errors are likely to become increasingly significant as we approach the saturation level and thus lead to non-stationary error variances. It can be shown that if this phenomenon exists, the fourth order moving average filter which we used in the previous section is not adequate to remove the effects on error variances. To further test the assumption of variance stationarity we thus made a second test of heteroskedasticity, a fourth order ARCH test, which with an asymptotic CH-SQ(4) value of 14.88 clearly rejected the hypothesis of homoskedastic errors. A more elaborate version of the model with varying q parameter, or a more accurate data series, may go a long way to alleviate this problem. However, since we are here less interested in measuring accurate elasticities than to demonstrate relative strength of the present model compared to alternative models, we shall leave this issue as it is with the risk of a possible loss of efficiency arising from heteroskedastic errors. A more crucial issue is the comparison of the model with other models used in the literature, which will be attempted in the next section.

6. Comparison with Alternative Models

This section compares the results of the present model with those of other models put forward in the literature, all of which have a dominant endogenous growth component. The logistic and the Bass models are two obvious candidates for comparison because they arise from imposing restrictions on the present model. We have also included the Gompertz curve in the comparison, as it is a left side skewed growth curve and is expected to have a better fit to the television ownership data than the logistic. A fourth model considered is the model suggested by Harvey (1984), which is a more general model incorporating the logistic and Gompertz as special cases.

The NLS estimate of Bass model, which is derived from our proposed model when α is set equal to 0, is the following:

$$LS_t = -12.6 + 1.965\ P_{t-1} + 0.438\ YD_{t-1} - 0.832\ CRED_t$$
$$\quad (-9.8) \quad (13.11) \qquad (0.46) \qquad (-2.98)$$

$$+Log(0.0052 + W_{t-1}(0.0)) \qquad (17)$$
$$(1.58)$$

T=67, R^2=0.817, σ'=0.2625, LL=-2.88,

F,AR1=50.46 [4.0], F,AR4=36.74 [2.35], F,Hetr=1.67 [4.0], Z1=1.67 [5.99]

The estimation period was the same as in equation 16, and similar test statistics as in the former equation are presented above. As can be seen the only coefficient which both has the correct sign and is statistically significant is that of the credit variable. The price coefficient, though significant has the wrong sign, and the coefficient on income is not significant. As expected, the coefficient of q, the exogenous growth factor, is not significantly different from zero. The estimated equation also showed strong signs of 1st and joint 1st-4th order autocorrelation.

By setting q=0 in equation 17 we get the simple logistic function. Though this was not expected to dramatically change the estimates of the Bass model, we preferred to re-estimate the above equation with q=0, as it would lead to a linear equation (in logs) which allows for correction for autoregressive errors in a more straightforward manner than in the non-linear case. The simple logistic function in that case takes the following form:

$$L_t = a_0 + a_1\ P_{t-4} + a_2\ YD_{t-1} + a_3\ CRED_t + u_t$$

where $L_t = Log(\Delta S_t/(S_{t-1}(1-S_{t-1})))$, and the economic variables are in log terms as before. The OLS estimate of the equation was:

$$L_t = -11.9 + 1.948\ P_{t-4} - 0.458\ YD_{t-1} - 0.772\ CRED_t$$
$$(-9.4) \quad (12.75) \quad (-0.537) \quad (-2.71)$$

$$\{\ \hat{u}_t = 0.98\ \hat{u}_{t-1} - 0.42\ \hat{u}_{t-2} + e_t\ \}$$
$$(8.36) \quad (-3.52)$$

T=67, R^2=0.911, σ'= 0.2679, LL=-4.77, DW=0.611

F,AR1=54.8, F,AR2=48.7, F,Hetr=8.78, Z1=0.408
[4.0] [3.15] [4.0] [5.99]

To deal with the second order autocorrelation in residuals the equation was also estimated by Cochrane-Orcutt AR(2) methods with the following results:

$$L_t = -11.1 + 1.77\ P_{t-4} - 0.406\ YD_{t-1} - 0.309\ CRED_t$$
$$(-10.1) \quad (10.99) \quad (-0.59) \quad (-0.94)$$

T=67, R^2=0.961, σ'= 0.1770, LL=23.5, DW=1.94

As expected the OLS estimates are very similar to equation 17. Though the C-O estimation reduces the standard error of regression to a large extent, there is no sensible change in parameter signs, and the credit coefficient is no longer significant. We further checked whether the relaxation of the assumption that the saturation level was equal to the maximum level of household numbers, i.e., $\theta=1$, helped to improve the performance of the simple logistic equation. This was done by changing the dependent variable to $Log(\theta \Delta S_t/(\theta - S_{t-1}))$ and a free estimation of θ and other parameters by NLS, after transferring $Log(\theta/(\theta - S_{t-1}))$ to the right hand side of the equation. The results were not significantly different from the above case and the hypothesis of $\theta=1.0$ was not rejected at the 5% significance level.

The general form of the Gompertz growth curve in discrete terms could be written as:

$$\Delta S_t = \beta S_{t-1}\ (Log(N/S_{t-1}))$$

where N is the saturation level, β is the speed of adjustment, and S is in level terms. To get a stochastic form similar to the simple logistic case, we took logs from both sides of the equation, and included the economic variables in the β coefficient to get:

$$G_t = a_0 + a_1\, P_{t-4} + a_2\, YD_{t-1} + a_3\, CRED_t + u_t$$

where $G_t = Log(\Delta S_t/(S_{t-1}(Log(1/S_{t-1}))))$, with S measured as a proportion of saturation level, and the right hand side variables being the economic variables in log terms as before. The OLS estimates of this equation were:

$$G_t = -11.2 + 1.388\, P_{t-4} + 1.834\, YD_{t-1} - 0.333\, CRED_t$$
$$(-9.6) \quad (9.79) \qquad (2.32) \qquad (-1.26)$$

$$\{\ \hat{u}_t = 0.94\, \hat{u}_{t-1} - 0.35\, \hat{u}_{t-2} + e_t\ \}$$
$$(7.85) \qquad (-2.87)$$

T=67, R^2=0.791, σ'=0.2487, LL=0.232, DW=0.592

F,AR1=59.2, [4.0] F,AR2=37.2, [3.15] F,Hetr=9.25, [4.0] Z1=2.29, [5.99]

The overall results are very similar to the simple logistic case, with the difference that the income coefficient is now significant. The residuals also showed strong second order autocorrelation. The Cochrane-Orcutt AR(2) estimation method results were:

$$G_t = -8.2 + 0.986\, P_{t-4} + 0.284\, YD_{t-1} - 0.070\, CRED_t$$
$$(-8.1) \quad (6.40) \qquad (0.468) \qquad (-0.235)$$

T=67, R^2=0.916, σ'=0.1580, LL=31.1, DW=2.00

This gives a much more improved fit to the model, but parameter estimates still remain problematic.

The general model suggested by Harvey (1984) is the following:

$$Log(\Delta S_t) = b_0 + b_1\, Log(S_{t-1}) + b_2\, T + u_t$$

Where T is a time trend variable. This model reduces to the simple logistic with b_1=2, and to Gompertz with b_1=0 and S measured in log terms. Since there is no straightforward manner in which the economic variables could be incorporated into this equation (e.g., b the speed of adjustment enters into b_2 and b_1 in a complicated manner), we decided to estimate this equation in the original form proposed by Harvey. The OLS estimate of the model, again using S as a ratio of the number of households, were:

$$\text{Log}(DS_t) = \underset{(-5.7)}{-1.12} \underset{(15.4)}{+0.819} \text{Log}(S_{t-1}) \underset{(-18.5)}{-0.062} T$$

$$\{ \quad \hat{u}_t = \underset{(13.1)}{0.848} \hat{u}_{t-1} + e_t \quad \}$$

T=67, R^2=0.838, σ'= 0.2849, LL=-9.4, DW=0.345

As the residuals showed strong sign of first order autocorrelation we re-estimated the equation by C-O AR(1) method which resulted in:

$$\text{Log}(DS_t) = \underset{(-2.2)}{-1.24} \underset{(6.32)}{+0.837} \text{Log}(S_{t-1}) \underset{(-6.11)}{-0.058} T$$

T=67, R^2=0.947, σ'=0.1631, LL=27.8, DW=1.383

This gives a much better fit to the data than the OLS estimate, but the overall fit as measured for example by the maximized value of the log-likelihood function is still not better than the Gompertz curve.

As to the comparison between these models and our proposed model, the latter clearly outperforms as far as theory consistency is concerned. In none of the above models could we get parameter signs consistent with theory, particularly with regard to the price variable. Though we could not corroborate this with regard to Harvey's model, it would not be surprising to find the same phenomenon if economic variables are somehow included in that model as well (given that it belongs to the same family of exponential growth curves with a dominant endogenous growth component).

In terms of the general fit also it appears that the proposed model outperforms the existing models. This could be seen through a comparison of the values of maximized log-likelihood functions of the different models reported in Table 3. Since the dependent variables in the different models are not the same, it is necessary to rescale the maximized LL values for such a comparison to be possible. However, as it happens the Jacobean of the transformation in each case turns out to be exactly the same. This is due to the fact that the dependent variables are all in log forms and the denominators are all composed of lagged dependent variables. As a result, the Jacobean only depends on the numerators which turn out to be the same in all models. As the Table shows, the proposed model outperforms the alternative models no matter by what method the latter are estimated.

Table 3
Maximized Log-Likelihood Values of Alternative Models

Model	Estimation Method	Maximized LL Value
Proposed Model	NLS	32.85
Logistic	OLS	-4.77
	C-O AR(2)	23.52
Gompertz	OLS	0.232
	C-O AR(2)	31.11
Harvey	OLS	-9.42
	C-O AR(1)	27.84

7. Concluding Remarks

In this paper the diffusion of a new consumer durable has been explored empirically in the epidemic tradition. A new model was set up with the aim of allowing for a more free interaction between exogenous and endogenous forces in the learning process, and incorporating economic factors as determinants of the speed of diffusion.

The coefficient α/β in this model could be interpreted as an index of the strength of the endogenous growth factors. It could be said that with the value of $(\alpha/\beta)\geq 1$, the endogenous growth factors are weak and exogenous factors play a dominant role in diffusion, and the reverse with $(\alpha/\beta)<1$. The value of this index, however, has to be estimated empirically. The existing epidemic models in the literature, by assuming $(\alpha/\beta)=0$, give an a priori dominance to the endogenous growth element.

The model was tested with the empirical data on the diffusion of colour tv ownership in the UK. Despite the simplifying assumptions made about the constancy of saturation level, etc. and the data problems arising from possible measurement errors in ownership figures, the results were encouraging, and certainly superior to other types of epidemic models. A relatively high value of 1.2 for α in the case of colour tv ownership indicated weak endogenous forces, contrary to the a priori assumptions of other models.

There are various ways in which the present model can be improved, particularly in its empirical applications. For example, our assumption of a simple linear decay factor, could, subject to mathematical tractability, be improved. One should also note that the work reported upon here takes no account of supply factors in the diffusion process (see, Ireland and Stoneman, 1986). Perhaps of greatest importance, however, is to explore more fully the role of economic factors in the diffusion process by taking into account the heterogeneity of adopters. These have been considered in this paper to the extent that the aggregate time series data on colour tv ownership allowed, but it is in other work proceeding on discrete choice models that adopter heterogeneity will figure more prominently.

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