MONETARY CONTRACTING BETWEEN CENTRAL BANKS AND THE DESIGN OF SUSTAINABLE EXCHANGE-RATE ZONES***

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Abstract

An exchange-rate system is a set of contracts which commits Central banks to intervene in the foreign-exchange market. The design features of the system include: the rules of intervention, the limits placed on exchange rates and the "crisis scenario" which describes possible transitions to new regimes in case one Central bank runs out of reserves or borrowing capacity. This paper considers the various trade-offs one faces in designing an exchange-rate system. Svensson (1989) has already analyzed the degree of variability in the exchange rate, the interest rate and the fundamentals. But the tradeoff also pertains to the amount of reserves which the Central banks must have on hand in order to forestall a speculative attack and make the system sustainable. The amount of reserves needed depends crucially on the assumed crisis scenario.

An exchange-rate system is a set of contracts between Central banks which commits them to intervene in the foreign-exchange market. The design of these contracts can be viewed as a problem in financial engineering, analogous to the design of covenants attached to a bond issue.[1] As the terms of the contract are modified, so does the way in which financial variables (exchange rates, interest rates and reserve levels) behave while the agreement is in force, and the way in which the system may ultimately come to an end.

The terms of the contracts, which are the design features of the system, are meant to cope with the various contingencies which may arise in a stochastic world. The design features can be classified into three categories: (i) The *rules of intervention* spell out the circumstances and the manner in which the Central banks sell reserves against their own currency in the foreign-exchange market. What event triggers compulsory intervention? If intervention takes place, who intervenes and is the volume of intervention massive or incremental? What reserve assets are used in the intervention?[2] (ii) *Exchange rate limits* are a special way of specifying the event which triggers intervention: It is triggered when the rate has reached some upper or lower bound.[3] If there is only one upper or lower bound, a one-sided zone is created. If there is both an upper and a lower bound, a target zone is in effect. The center and the width of the target zone are the essential parameters of the contract. When the two bounds are equal to each other, the exchange rate is fixed. When there are no bounds whatsoever, the system is one of free float;

(iii) Most important are *crisis scenarios*. The contract specifies, explicitly or implicitly, what will happen if and when a Central bank runs out of

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[1] See Merton (1974) and Black and Cox (1976).

[2] More details are provided in section I below.

[3] Intervention may also take place at all times according to some special formula. We do not consider here such "intramarginal intervention" but ultimately it should be considered.

reserves or out of borrowing capacity. It is then unable to carry out the interventions it was committed to. In fact, a speculative attack may have anticipated that event and may have wiped out the bank's remaining reserves. The current regime is no longer sustainable, so that a transition to a new regime must be declared. *What this new regime will be -- free float, one-sided zone or realigned target zone -- is part of the design of the original system.*

The purpose of the present paper is to investigate the effects of these design features on the operation of the system and the conditions of its ultimate collapse. Svensson (1989) has studied the changes in exchange-rate, interest-rate and fundamentals volatility brought about by target zones of different widths, under the assumption that the underlying stochastic disturbance affecting the economic system follows a Brownian motion, all but ignoring the impact of the drift on that motion. We consider a Brownian motion with a drift. This is an important extension because the drift introduces an asymmetry: one currency in the long run becomes more inflation prone than the other and the defense of that currency will require more reserves.

In order for the exchange-rate system to be sustainable over a strictly positive period of time, the Central banks party to the contract must each have on hand a minimum amount of reserves. This has been shown by Salant and Henderson (1978) and by Krugman (1979, 1989): If these levels are not on hand, a speculative attack is triggered. We follow Krugman's lead and determine the needed levels of reserves as functions of the design features of the system. We emphasize the relation between these needed levels of reserves and the assumed crisis scenario.[4]

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[4]It may be noted that this study is in the same spirit as the analysis of indenture provisions attached to financial contracts. This sort of analysis is routinely carried out by financial economists, using a technique known as Contingent Claims Valuation, an extension of option pricing. The analogy between the two problems is almost complete: the differential equation for the exchange-rate function [Equation (5) *infra*] takes the place of the Black-and-Scholes (1973) partial differential equation. The rules of intervention are analogous to the covenants on a bond or financial security which specify what the issuing firm may or may not do, and the crisis scenario is akin to the

The paper is organized as follows. Section I spells out the assumptions of the analysis concerning the rules of foreign-exchange intervention. Section II describes, interprets and solves the simple one-equation model which is used throughout. Section III defines and illustrates the crucial concept of "the amount of reserves needed by a Central bank" and examines the reserves needed to forestall a transition to free float, starting from a fixed-rate system, a one-sided zone or a two-sided zone. We determine in this context the amount of reserves needed by the Central bank of the currency which is strong in the long run, and solve a paradox which arises in this connection (Section IV). Section V examines the reserves needed by both Central banks to forestall a transition from a fixed-rate system or from a target zone to a one-sided band.

I INTERVENTION RULES

We construct a model of an exchange-rate system involving two currencies only. In this section we list the main assumptions made when developing the model and discuss their degree of realism, comparing them in particular to the theoretical and practically observed workings of the European Monetary System.[5] These pertain to the rules of foreign-exchange intervention which

default scenario on a bond which stipulates to what assets the various categories of bondholders have title in case of default. It is impossible to list the vast literature on Contingent Claims Valuation; but see Cox and Rubinstein (1985), section 7-3 or Mason and Merton (1985).

[5] A concise reference to the rules of the EMS and the way they were effectively applied or violated is Micossi (1985). It is sometimes hard to justify the imposition of exchange-rate target zones, but it is possible to find some parameter values for a central planner trying to minimize a welfare loss function such that the target zone dominates [See for example de Kock and Grilli (1989) and the references given there]. In the present article, we avoid entirely the issue of the proper choice of the Central banks' objective functions; instead, we provide a number of assumptions which specify their behavior

keep the system operative. We state the rules of intervention in terms which are applicable to a generic system of the target-zone type; their meaning when applied to other regimes is straightforward. They are four in number.

. *Assumption 1*: Interventions in the foreign-exchange market occur only when the exchange rate reaches the bounds of the target zone.

The original rules of the EMS did not literally forbid intramarginal interventions; they only made them subject to prior approval. The subsequent practice has been that more than 85% of the interventions have been intramarginal;[6,7]

. *Assumption 2*: When a bound of the target zone is reached, the burden of intervention falls entirely on the Central bank of the currency which is currently at its minimum value, the currency which is currently weak. Equivalently, it is stipulated that the intervention of a central bank in the market never leads to an increase in its reserves and its money supply. A bank, when intervening, only reduces its money supply, buying back its currency with reserves.[8] *This rule, as all others, is applied to fixed-rate regimes as well.*

The rules of the EMS call for both Central banks to intervene in tandem but do not make clear exactly how the burden of intervention is to be shared. Our model could easily be adjusted to accomodate a rule according to which the weak currency central bank performs x% of the intervention and the strong one (1 - x)%, x being given;[9]

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[6]See Micossi (1985), page 332, for instance.

[7]It would therefore be of great interest to study these interventions and we plan to do so in a sequel to this paper: Delgado and Dumas (1990).

[8]Without this restriction, *any* speculative attack against a country's currency could always be accommodated by a sufficiently large increase in the reserves and the money supply of the other country's Central bank.

[9]A joint modification of Assumptions 1 and 2 would lead to a richer model in which a Central bank would loose reserves in defending its currency when it is weak but would also replenish them when its currency is strong, at the same time easing the intervention burden of the other Central bank.

. *Assumption 3*: The manner in which intervention is performed is such as to produce an instantaneous change in the stocks of money and reserves.

These stocks receive a "push" over an infinitely small period of time. This is "Blitz intervention". Intervention, each time it occurs, remains infinitesimal in size but it takes place at an infinite speed.[10,11] It is repeated as many times as the exchange rate hits a boundary, within the limitations of Assumption 4 below.

. *Assumption 4*: The assets of a Central bank fall into two categories. The first category called "Reserve assets", in an amount noted R, contains assets whose stock can be reduced *ad libitum* for the purpose of contracting the money supply and supporting the exchange value of the currency. The second category, labelled "Domestic Credit", in an amount noted D, contains assets whose stock, for present purposes, cannot be regulated, presumably because their regulation is assigned to serve other policy goals, such as internal ones. D is a positive constant.

. *Assumption 5*: Reserve assets of both Central banks are entirely in the form of some outside assets available in infinitely elastic net supply.

Reserve assets are made up of some stores of value which bring an equilibrium rate of return, so that anyone -- Central banks and private investors -- are willing to hold them. The fact that private investors are willing to hold them is what creates the infinite supply (or demand). For instance, reserves may be *interest-earning* foreign-currency denominated

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[10] Under Assumption 8 below, the shocks to the money supplies or demands are themselves going to be infinitesimal.

[11] This type of intervention is called "instantaneous control". Flood and Garber (1989) present a model in which the policy is not specified by infinitesimal interventions but by discrete ones. This implies the same exchange rate function for the same bounds placed on the exchange rate. The type of regulation implied by discrete interventions is called "impulse control".

claims; but they are not made up of foreign currency *per se*.[12] Neither are reserve assets made up of monetized gold, the fixed total stock of which would shuttle back and forth between the two Central banks, as in models of the Gold Standard.[13]

. *Assumption 6*: It is assumed that a Central bank will be forced to stop intervening forever, if and when its level of reserves reaches the zero mark.

Zero is an arbitrary choice for the level of reserves which forces suspension of intervention. In real life, lines of credit and swap agreements buttress actual reserves. For all analytical purposes, such arrangements would simply allow a Central bank to continue intervention even after its reserve level has reached the zero mark. Nothing much is lost by setting the minimum level of reserves equal to zero rather than equal to some finite borrowing capacity.[14] Something is lost, however, in assuming that a Central bank's borrowing capacity is exogenous.

II THE MODEL AND ITS VARIOUS SOLUTIONS

II.1 The model

This paper deals entirely with various solutions to *one* differential equation governing exchange rate behavior. Under a set of three *Assumptions numbered 7 to 9*, -- (7) Commodities prices are flexible and Purchasing-power parity prevails,[15] (8) The demand for real balances in both countries is log-

[12]If they were, we would have to introduce a reason for private investors to hold foreign currency, over and beyond their transactions needs, and the money-market equilibrium conditions would have to reflect the balances held by central banks.

[13]See Krugman (1989) and Buiter and Grilli (1989).

[14]See Grilli (1989) for a study of optimal Central bank borrowing to prevent the drop in reserves when maintaining a fixed exchange rate.

[15]See Miller and Weller (1990) for a model incorporating price inertia.

linear with the same interest semi-elasticity in both countries, (9) Capital flows are free between the two currencies,[16] -- several authors[17] have shown that the exchange rate is governed by:

$$S = m - m^{*} + v + \gamma E\{dS \mid \Phi(t)\}/dt \tag{1}$$

where:

S is the logarithm of the exchange rate prevailing between two currencies (the home currency value of foreign exchange),

$m = \ell n(R + D)$ and $m^{*} = \ell n(R^{*} + D^{*})$ are home and foreign measures of money supply,

v is an exogenous monetary shock,

γ is a strictly positive coefficient interpreted as an interest semi-elasticity of money demand,

and $E\{dS \mid \Phi(t)\}/dt$ is the conditionally expected instantaneous change in the exchange rate,

$\Phi(t)$ is the information set of economic agents acting in the foreign exchange market.

In this study R and R^{*}, and therefore m and m^{*}, are deemed controllable

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[16] Note a major implication of Assumption 9: No sterilized intervention is possible and there is no distinction between modifications in the two currencies' money supplies and intervention in the foreign-exchange market. To put it another way, an intervention on the foreign-exchange market which would not modify money supplies -- and would, for instance, involve money-market instruments only -- would have no effect on the exchange rate. International economists typically prefer to distinguish "fundamental" aspects of policy, such as monetary and fiscal policy, from purely "palliative" measures such as foreign-exchange intervention. No such distinction is possible here. In a moment, a variable X will be introduced, lumping together all the determinants of the exchange rate which include the two currencies' money supplies as well as an exogenous disturbance to the two money supplies and money demands. No distinction will be made between money supply changes brought about by monetary policy and changes brought about by foreign-exchange intervention. This variable X will be labelled "the fundamentals", without reference to the above distinction.

[17] Frenkel (1976), Mussa (1976), Bilson (1978, 1979).

because foreign-exchange intervention by Central banks modifies reserves; D and D^* stand for home and foreign domestic credit, assumed constant. Over intervals of time during which no foreign-exchange intervention takes place, m - m^* is a constant.[18]

We consider the case (*Assumption 10*) where v follows a Brownian motion with a non-zero constant trend. The formulation is:

$$dv = \mu dt + \sigma dW; \qquad \mu, \sigma \text{ constant and } > 0 \qquad (2)$$
$$v_o > 0 \text{ given.}$$

In (2), μ and σ are constants and dW is the increment of a standard Wiener process. It is conceivable to interpret v either as a supply or a demand shock.[19]

For $\mu > 0$, which is assumed for the sake of definiteness,[20] the home currency is inherently *weaker in the long run* because the trend in fundamentals works against it. Without intervention the home currency would be expected to depreciate. In order to convey that notion, we call the home currency the "Franc" and the foreign currency the "Deutschemark". S is then the log-Franc value of the Deutschemark (FRF/DEM).

Define the "fundamentals" X as: $X \equiv m - m^* + v$. Equation (1) is rewritten as:

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[18]Strictly speaking, fluctuations in the exchange rate could conceivably have valuation effects on R and R^*. But there is no mechanism to monetize these capital gains and losses. They would truly accrue to the stockholders of the two Central banks, whoever they may be. Since these capital gains and losses have no effect on m and m^*, we have simply chosen to ignore them.

[19]See, for example, Froot and Obstfeld (1989a, b), Svensson (1989), Klein (1989).

[20]Svensson (1989) has examined limiting properties of some solutions to (1) under the assumption that the shock v follows a zero-drift arithmetic Brownian motion: $\mu = 0$.

$$S = X + \gamma E\{dS \mid \Phi(t)\}/dt. \tag{3}$$

Under Assumption 1, *we define a target zone by specifying the bounds on the exchange rate itself*. An infinitesimal intervention policy, which is triggered when some bounds $\underline{S}$ and $\bar{S}$ are reached, can be characterized as an instantaneously regulated Brownian motion.[21] Given the behavior of v [Equation (2)] and the type of intervention, the stochastic differential equation for X is:

$$dX = \mu dt + \sigma dW - dU + dL, \tag{4}$$

U and L being two continuous, non negative, non decreasing processes, U increasing only when $S = \bar{S}$ and L increasing only when $S = \underline{S}$. U and L stand for the cumulative amounts of intervention done by the two countries. The same fixed trend μ drives v and the fundamentals X when no intervention takes place.

Assume (*Assumption 11*) that the value of the exchange rate is a twice continuously differentiable function of X and apply Itô's lemma to calculate $E\{dS \mid \Phi(t)\}/dt$ explicitly. Then Equation (3) becomes:

$$S = X + \gamma[\mu S'(X) + 0.5\sigma^2 S''(X)]. \tag{5}$$

This equation applies over the domain of X where no intervention takes place.

<u>II.2 The solutions</u>

The two roots of the characteristic equation associated with (5) are of

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[21] Harrison (1985) is the standard reference on regulated Brownian motion, but economic application to optimal decisions have been used previously. See Dumas (forthcoming) for a presentation of instantaneous and impulse regulated Brownian motion.

opposite signs since their product is: $- 2/\gamma\sigma^2 < 0$. We denote the positive root α and the negative one $-\beta$. The following property of the roots will prove useful later:

$$1/\alpha - 1/\beta = \gamma\mu. \tag{6}$$

The *general solution* of (5) is given as Equation (7) in Table 1 where A and B are constants of integration which must be solved for, using the boundary conditions implied by the exchange rate policy.

INSERT TABLE 1 AND FIGURE 1 HERE

The *free-float* particular solution to (5) is easily identified. Resorting to a no-bubble assumption (*Assumption 10*), one imposes: A = B = 0. The free-float solution, given in Table 1 as Equation (8) and shown in Figure 1 and other figures, is a 45^o line with intercept at $S = \gamma\mu$.

In Figure 1, *point* FX on the first diagonal represents a strict *fixed-exchange regime* solution. Indeed, if the exchange rate is not expected to change $[E\{dS \mid \Phi(t)\}/dt = 0]$, Equation (3) implies: $S = X$. If the authorities wish to peg the exchange rate at some level S_o, they must strictly maintain the fundamentals at a level $X = S_o$. Strictly speaking, the fixed rate regime cannot be regarded as a solution to differential equation (3) since it implies that the domain contains only one point. As we will verify later, it can, however, be regarded as the limit of a sequence of target-zone solutions.

Target-zone solutions may correspond to a one-sided or a two-sided zone. In the case of a *one-sided zone with an upper bound* $\bar{S}$, but no lower bound, the value of the Deutschemark is limited from above or, equivalently, the value of the Franc is limited from below. At $S = \bar{S}$ intervention is undertaken by the Banque de France. In order to eliminate explosive solutions when $X \to -\infty$, let B

= 0 in (7). The boundary conditions for this one-sided control are shown as Equations (11) and (12) in Table 1. In (11)-(12) $\bar{X}$ is the implied limit for the fundamentals. Equation (11) is just a restatement of the fact that, at the boundaries of the zone, $\bar{S} = S(\bar{X})$, which is the definition of $\bar{X}$. Equation (12) is a statement of the "Value matching" principle that the path of the exchange rate cannot include a jump when intervention takes place: $\bar{S} = S(\bar{X} - dU)$ implies $S'(\bar{X}) = 0$.[22]

The solution of (11)-(12) for the two unknowns A and $\bar{X}$ is explicit. The values of $\bar{X}$ and A are given by equations (13) and (14) of Table 1. Equation (13) demonstrates that there exists a *one-to-one relationship* between the bound on fundamentals and the bound on the exchange rate. Hence, one can be imposed or the other equivalently. In fact, the locus of intervention points, such as C_1, in the (S, X) plane is a straight line represented in Figure 1 by the line UU' in the case of an upper bound. In the case of a lower bound, the line LL' which will appear in Figure 2 would represent the analogous relationship.

In the case of a *two-sided target zone*, the determination of the constants of integration A and B and the bounds on fundamentals implied by the bounds on exchange rates is done by solving simultaneously a system of four equations with four unknowns A, B, $\underline{X}$, and $\bar{X}$. This system is made up of Equations (15)-(18) of Table 1. In (15)-(18), $\bar{S}$ and $\underline{S}$ are the upper and lower limits which the authorities will allow the exchange rate to reach before intervention. $\bar{X}$ and $\underline{X}$ are the implied limits for the fundamentals. Equations (15) and (16) are analogous to Equation (11) above. Equations (17) and (18) are analogous to

[22] The no-arbitrage requirement explains this condition: As X is pushed back instantaneously into the $(-\infty, \bar{X})$ domain (Assumption 3), S is not allowed to move, as otherwise market participants would be provided with an arbitrage opportunity. Alternatively, the result follows from the assumptions that X is an instantaneously controlled Brownian motion without costs of regulation and that S is given by: $S_t = E\{\int_t^\infty (1/\gamma)\exp\{-(1/\gamma)(\tau - t)\}X_\tau d\tau \mid \Phi(t)\}$. It is also a direct implication of the fact that the boundaries are reflecting. See Dumas (forthcoming).

Equation (12).

The system made up of Equations (15) to (18) is to be solved for the general case of *non-zero drift* in fundamentals, in which *a symmetric band placed on the exchange rate translates into an asymmetric band placed on the fundamentals*. One such solution is shown as curve B_2C_2 in Figure 1.

III TRANSITIONS TO THE FREE FLOAT AND THE CONCEPT OF "NEEDED AMOUNT OF RESERVES"

III.1 Collapse of a fixed-rate regime

Even though it is seldom an explicit policy statement, governments are willing, or able, to intervene in the foreign-exchange market only until their reserves have been exhausted. At that point they let the pegging collapse. How much reserves must Central banks have on hand in order to forestall speculative attacks occurring prior to the exhaustion of reserves? This question, which Triffin (1960) raised regarding the Bretton-Woods system, can be given a specific answer.

A collapse of a fixed-rate system directly to a free-float regime would imply a discrete jump from point FX to point C_o of Figure 1, at an unchanged level of the exchange rate.[23] This speculative attack has a constant log-size $\gamma\mu$; i.e., it implies a drop in fundamentals, X, equal to that amount. Recall that X is defined as $m - m^* + v$ or $\ell n(R + D) - \ell n(R^* + D^*) + v$. Since shock v follows a continuous, exogenously specified process (2), the $\gamma\mu$ drop in X is necessarily a drop in $\ell n(R + D) - \ell n(R^* + D^*)$. At each point in time, speculators in the market ask the question: Can fundamentals drop by this amount and still leave stocks of reserves above their exhaustion levels? If the answer is yes, the attack is forestalled. If the answer is no, the attack

[23] See Krugman (1979), Flood and Garber (1984) or Obstfeld (1986).

and the collapse are triggered.

Define the log-amount of reserves of the Banque de France as:

$$r \equiv \ell n(R + D) - \ell n(D). \tag{19}$$

If the Banque starts with any amount of r greater than $\gamma\mu$ and if it is forced to intervene, it gradually depletes its reserves down to the level $\gamma\mu$. At that point, speculators attack and eliminate all remaining reserves.[24] Krugman (1979) developed this basic result in a perfect-foresight model.

Define then any exchange-rate system as being *"sustainable"* if the amount of time before collapse is strictly positive, when starting the system at $S = \underline{S}$ or at $S = \bar{S}$.[25] This notion is one of short-term sustainability: The system is deemed unsustainable if, as soon as intervention is called for, a collapse occurs.[26]

Define the level $\underline{r}$ of reserves as being *"needed by the Banque de France"* if it is true that the system is sustainable only if $r > \underline{r}$ (strictly). In that sense, $\gamma\mu$ is the level of reserves needed by the Banque to make the fixed-rate system sustainable when the crisis scenario is one of transition to free float.

Meanwhile, the Bundesbank must only have on hand an amount of reserves

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[24] One might question why the Central bank which is the target of an attack continues intervention even though there is no more hope of saving the fixed-rate regime. Observe, however, that a loss of reserves is not a loss of value since it is accompanied by a drop in the liabilities of the bank. The bank does not mind losing reserves especially if the next regime is going to be one of free float. Furthermore the bank's commitment to all out intervention is what made the fixed-rate regime sustainable in the first place.

[25] The definition is extended to the fixed-rate system by letting the two exchange rate bounds approach each other.

[26] It would be interesting to extend this investigation and determine the conditions for a system to be sustainable over an expected period of time before collapse equal to a given finite number T. By contrast with the notion we have just defined, this would be a notion of "Medium-term sustainability". The difference between the two notions is that the latter incoporates the frequency with which $S = \underline{S}$ or $S = \bar{S}$ while the former one does not.

strictly greater than 0.

III.2. One-sided upper-bound zone collapsing to free float

Consider now a one-sided upper-bound zone. When S reaches $\bar{S}$ and the Banque de France (which intervenes at that point) has on hand a log-amount of reserves less than or equal to $\gamma\mu + 1/\beta - 1/\alpha$ (which is the distance C_oC_1 in Figure 1), a speculative attack against the Franc is triggered. The reserves are immediately depleted, which means that the fundamentals jump from point C_1 of Figure 1 to point C_o. The *one-sided upper-bound zone collapses to free float*. When a one-sided zone with an upper bound is in force, the amount of reserves needed by the Banque de France is $1/\alpha$.[27]

III.3 Target zones of various widths

INSERT FIGURE 2 HERE

Part of our exposition of what happens as the target zone is widened or narrowed, is based on Figure 2. We proceed to explain it at this point. The figure is constructed by changing the width of the band around S_o. This means solving the system (15)-(18) for different values of $\bar{S}$ and $\underline{S}$ *positioned symmetrically around* S_o (= 4.5 in Figure 2). The two slanted straight lines in the middle of the figure are the 45^o diagonal line which contains the fixed-exchange point, and the free-float line.[28] The thickly drawn line in Figure 2

[27] In the case of a one-sided lower-bound target zone, in which the Bundesbank intervenes occasionally to support the Deutschemark, the required log-amount of reserves $[\ell n(R^* + D^*) - \ell nD^*]$ would only be $1/\beta$.

[28] As we know, the free-float solution is also a 45^o line but translated up a distance $\gamma\mu$. Furthermore, in Figure 2 -- as is clear from the basic Equation (3) which indicates that the interest-rate differential is equal to $(S - X)/\gamma$,-- iso-interest-rate differential lines would be equally-spaced 45^o lines, the diagonal line corresponding to a zero value of the interest-rate differential, while the free-float line corresponds to the level μ of the differential.

is the locus of intervention points B_2 and C_2. Points along the locus to the left of S_o represent pairs $(\underline{X},\underline{S})$; points to the right of S_o represent pairs $(\bar{X},\bar{S})$.

It is visible on Figure 2 that the following statement holds:

The locus of tangency points establishes a monotonic (non decreasing) relationship between the positioning of the bounds on the exchange rate and the positioning of the bounds on the fundamentals.

Proof: By total differentiation of system (15)-(18) [available on request].

This result authorizes us, under the current assumption of infinitesimal intervention, to define a target zone in terms of exchange rate bounds. The assumption of declared bounds imposed on the exchange rate, rather than on the fundamentals, is preferable because, in practice, exchange rates are directly observable by the financial markets, while fundamentals are less easily observable.

In the case of a two-sided target zone with an agreement to switch to free float in case of crisis, *the amounts of reserves needed by the two partners depend on the width of the band*. These amounts can be read off Figure 2 as the horizontal distance between the locus of tangency points (thick line) and the free-float straight line, above and below S_o.

III.4 Wide target zones

We verify now that, for wide enough bands, the distance of $\bar{X}$ from the diagonal line is the same, asymptotically, as the distance of $\underline{X}$ from the free-float line and that both tend to a constant value. Let this distance be δ and the width of the band 2ε, $\bar{S} = S_o + \varepsilon$ and $\underline{S} = S_o - \varepsilon$. The following relationships hold by definition:

$\bar{X} = S_o + \varepsilon + \delta$; $\underline{X} = S_o - \varepsilon - \gamma\mu - \delta$; and: $\underline{X} = \bar{X} - 2(\varepsilon + \delta) - \gamma\mu$.

To study the behavior of wide bands, we substitute these relationships into (15)-(18), simplify, and neglect terms that approach zero as δ and ε go to infinity. A simple linear relationship is obtained:[29]

$$\delta = \varepsilon - \gamma\mu + 1/\alpha = \varepsilon + 1/\beta \tag{21}$$

One implication of this linear-asymptote result is that, irrespective of how much one widens the exchange rate band, some characteristics of the system are not affected: The maximum attack is $\gamma\mu + 1/\beta = 1/\alpha$ on the Franc, and $1/\beta$ on the Deutschemark.[30] Since $\gamma\mu > 0$, the maximum attack on the franc is greater than the maximum attack on the Deutschemark. In order to prevent attacks, the Central banks of both countries *must have on hand the corresponding amounts of reserves* and the Banque de France needs more reserves than does the Bundesbank. Even though the behavior of the exchange rate would resemble the free-float behavior over most of the band, an extremely wide target zone remains qualitatively different, in this respect, from a free-float regime. In fact, when attempting to sustain a wide target zone, it is necessary to have on hand amounts of reserves which are larger than would be required to sustain a fixed-exchange rate system.

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[29] Svensson (1989) obtains a similar relationship for the special case of zero drift ($\mu = 0$) in which the two roots are of equal magnitude: $\alpha = \beta$. The distance from the free-float (identical in his case with the 45° line) was $1/\alpha = 1/\beta$.

[30] For wide bands, this statement is true whether we consider a "coordinated collapse" (transition to free float) or a "unilateral collapse" (transition to a one-sided zone). These terms are defined in Section IV.1 below.

III.5 Narrow target zones

A fixed-rate regime is, in theory, an exchange rate system in which the authorities are committed to doing whatever is necessary to maintain the exchange rate fixed. In practice, there may never have existed such a system. Under most historical fixed rate regimes, there has been a difference between the price at which governments bought either foreign currency or gold and the price at which they sold them. This difference was a bid/ask spread. The U.S. buying and selling prices of gold were $34.9125 and $35.0875 in the Gold-Exchange system. Before that, the British price for gold in the Gold standard system was L-3.17s. 10.5d for sale and L-3.17s. 9d. for purchases.[31] The price of gold could oscillate within these bands without triggering intervention. The presence of transactions costs would further widen these bands, up to what is normally referred to as the gold points. One may wonder, as does Krugman (1989), whether it may be more appropriate to model a so-called fixed-rate regime as a narrow target zone.[32] But, if one is the limit of the other, a strict fixed-rate model may be an acceptable rendition of what may be, in fact, a narrow band.

Figure 2 can again help in visualizing the process of narrowing the band. As one tightens the band around a given exchange rate value S_o, the system converges to the fixed-rate solution ($X = S = S_o$).

The interesting aspect, however, is the rate at which this convergence takes place:

As the width ε of the band in terms of the exchange rate approaches zero, the width of the band δ in terms of fundamentals is of order $\varepsilon^{1/3}$.

[31] See Yeager (1976) page 306.

[32] In a somewhat different setting, involving mean-reverting fundamentals, Krugman (1989) conjectured that the limit of a very narrow band may not be identical to the fixed rate regime.

Proof: By expansion around zero of system (15)-(18) [Available on request].

This generalizes a similar result obtained by Svensson (1989) in the symmetric ($\mu = 0$) case. It has an important policy implication. *Even a very tight target zone provides some room for the fundamentals to move about*: the bounds on the fundamentals are two orders of magnitude wider apart than the bounds on the exchange rate! As compared to a strict fixed-rate system, in which fundamentals would be absolutely immutable, a narrow target zone buys a lot of temporary flexibility. Foreign-exchange traders do not move the exchange rate in response to a deviation in the fundamentals because, under the target-zone intervention policy, they know that this deviation is temporary. The anticipated reversion in the fundamentals, which is bound to be triggered in the near future, is what keeps the current exchange rate from reacting to the current value of the fundamentals.

There is a flip side to this argument. In order for this future intervention to be credible, it is necessary that the narrow target zone be buttressed by reserves which are markedly greater than those needed under a fixed-rate regime. In fact, another way of reading the above proposition is to say that the excess reserves needed, over an above those needed in a fixed-rate regime, shrink in the limit as the cubic root of the width of the band.

IV A PARADOX CLARIFIED

IV.1 The paradox under fixed rates

If a regime switch from fixed rates to free float is declared whenever the Bundesbank runs out of reserves, the resulting shift from point FX to point C_0 in Figure 1 clearly corresponds to an attack of size $\gamma\mu$ against the *Franc*.

Whether the transition to free float is triggered by the Banque de France reaching a reserve level equal to $\gamma\mu$ (as explained in section III.1) or by the Bundesbank reaching a reserve level of zero -- whichever occurs first,-- the end result is the same: Fundamentals X jump *down* by $\gamma\mu$, which implies that the *Banque de France* loses reserves,[33] while the exchange rate, of course, remains unchanged.

It is counter-intuitive that the Central bank which has a reserve problem may not be the one which is the target of the attack. This paradox was first described by Krugman (1989).[34]

The paradox will be cleared away if one considers the economic situation which prevails while the fixed-rate regime is in effect, and the conditions under which the transition takes place. Because $\mu > 0$, a fixed exchange rate (point FX in Figure 1) implies a value for the Deutschemark lower than its shadow free-float value. Evidently, the intervention of the Banque de France in support of the Franc, or its commitment to intervene, is responsible for this "undervaluation" of the Deutschemark.

As for the conditions under which the transition takes place, it is being assumed that, in case the fixed-rate regime is abandoned, *both countries* simultaneously stop intervening forever. We can call this a *coordinated*

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[33] *Under our burden-sharing assumption (Assumption 2), intervention can never be such as to increase a currency's supply. Hence, no one ever gains reserves as a result of intervention: The Central Bank whose currency is being supported loses reserves.*

[34] *Krugman considered a Gold Standard system and mean-reverting fundamentals. As we just found out, the paradox is not restricted to Krugman's setting but the Gold Standard system makes the observation "even more" paradoxical. In this system, reserve assets are in fixed supply so that any reserves lost by the Banque de France are automatically received by the Bundesbank. The attack against the Franc replenishes the coffers of the Bundesbank and makes one wonder why in the first place the Bundesbank's lack of reserves has triggered an attack. On that, see Buiter and Grilli (1989). In fact, as has been pointed out to us by Daniel Cohen, the replenishment of the Bundesbank's coffers on the occasion of the transition to free float is totally irrelevant. When these reserves become available, considering the agreement on the crisis scenario, the Bundesbank is committed not to use them to intervene.*

collapse. *Two* announcements simultaneously impact the foreign-exchange market:

(i) The Bundesbank has run out of reserves and will in the future no longer support the Deutschemark;

(ii) Neither will the Banque de France henceforth support the Franc. It means that the Banque de France has decided no longer to maintain the "overvaluation" of the Franc or, equivalently, the "undervaluation" of the Deutschemark. This second event is what causes the attack against the Franc.

The alternative to a coordinated collapse is a *one-sided collapse* in which the country which happens to run out of reserves stops intervening unilaterally, while the other one maintains its commitment to intervene; this is event (i) taken in isolation. In that case, the switch is not to a free float regime but *to a one-sided zone*. The one-sided zone may itself just be an intermediate stage and a further collapse may later take place to free float; this would be event (ii).

IV.2 The paradox in the case of a target zone

Refer again to Figure 2 and imagine tightening the band. A conundrum appears. On the Franc side there is no problem: The needed reserves drop, at first very slowly, and then precipitously (at a cubic rate, as mentioned) from $1/\alpha$ to $\gamma\mu$. But on the Deutschemark side, it is not immediately apparent how the needed reserves drop from $1/\beta$ to zero. Calling A the point of intersection, in Figure 2, of the thickly drawn tangency locus with the free-float line, it would seem that, in order to sustain bands narrow enough to have their lower bound above point A, the Bundesbank needs a negative level of reserves!

The conumdrum can also be described in another way. Refer to Figure 1 and imagine a band narrow enough to have its lower bound above point A. Then one must also imagine point B_2 being positioned differently than it is in Figure

1: It would be located *to the right* of the free float locus. The difficulty arises in figuring out how there can be a transition from point B_2 so positioned to the free float line. The transition cannot involve an upward jump in the exchange rate since the event that the Bundesbank has run out of reserves is not a surprise. Nor can such a transition involve a jump downward in the fundamentals $X = \ell n(R + D) - \ell n(R^* + D^*)$. Indeed, at $S = \underline{S}$ the Banque de France cannot be selling reserves R since it is not intervening and the Bundesbank is certainly not buying reserves R^* (see Assumption 2). How then can there be a transition to free float?

We have just encountered another version of the paradox already identified in the case of fixed rates; we can attempt to resolve it the same way. The assumption made here is that, in case of foreign-exchange crisis, both Central banks stop intervening simultaneously. For narrow enough bands, *the Deutschemark is undervalued throughout the allowed domain of fluctuation of the exchange rate, relative to its shadow free-float value*. While the target-zone agreement is in effect, the Bundesbank needs reserves to intervene in support of the D-mark, not because it is overvalued -- as it would have been when $S = \underline{S}$, if the band had been wider, -- but only because the Banque de France is committed to intervene in the opposite direction. If the Bundesbank runs out of reserves and a switch to free float is declared, the commitment of the Banque de France is ended. Coordinated suspension of intervention shifts market demand in favor of the Deutschemark. It would then seem logical that the franc should be attacked. Unfortunately, this cannot be the solution of the paradox: Since the Banque de France is not intervening when $S = \underline{S}$, the franc cannot be the target of an attack at that time. Any attack on the franc must have been preemptive and must have occurred earlier on, at a time when S was equal to $\bar{S}$. It is not clear as yet what critical level of German reserves would have triggered this preemptive attack.

V TRANSITIONS FROM A FIXED-RATE SYSTEM OR FROM A TWO-SIDED TARGET ZONE TO A ONE-SIDED ZONE

V.1 The collapse of a fixed-rate regime to a one-sided zone

Suppose that the Bundesbank having run out of reserves is not sufficient reason for the Banque de France to stop supporting its currency. I.e., suspension of intervention is not coordinated. If the Banque has on hand reserves large enough to sustain the new regime (i.e., $r > 1/\alpha$), the economy *de facto* shifts from a fixed-rate regime *to a one-sided upper-bound zone*. Or, in Figure 1, the economy shifts from point FX to point C_1 and thereafter starts moving along the curve stemming from that point.

A shift from FX to C_1, is an attack against the Deutschemark, as would have been expected. Generally, if the Banque de France has on hand an amount of reserves equal to $r > 1/\alpha$, the Bundesbank must to have on hand, in order to forestall this attack, an amount of reserves strictly greater than $1/\beta$. If $r < 1/\alpha$ a one-sided zone would not be sustainable and the only possible switch in regime is a direct transition from fixed rate to free float (section III.1).

V.2 The collapse of a two-sided target zone to a one-sided zone

Consider a target-zone system in which no coordinated transition agreement exists. If one Central bank stops its intervention activities, the economy switches to a one-sided zone rather than to a free-float system.

Recall that, at a point like B_2 of Figure 1, the Bundesbank intervenes to support its currency. At that point the Deutschemark is overvalued compared to the value which would prevail if the Banque de France alone were to pursue its intervention policy. The *amount of reserves needed by the Bundesbank, to forestall a transition to a one-sided regime*, is given by the distance between point B_2 and point R_1, at which the horizontal line drawn at level $\underline{S}$

intersects the one-sided exchange-rate curve. Under the one-sided zone crisis scenario, the amount of reserves needed by the Bundesbank never reaches zero, no matter how narrow the band may be.

INSERT FIGURE 3 HERE

Figure 3 plots the amount of reserves needed as the lower bound of the band is progressively pushed up,[35] it being understood that the upper bound not shown is symmetrically pushed down. The asymptote for wide bands on the left of the figure is at a level $1/\beta$. As the band is tightened, the needed amount of reserves increases progressively and then precipitously drops back to $1/\beta$ as one takes the limit to an infinitely narrow band. Hence, the limiting zero-width band (or the fixed-rate regime) as well as the limiting large-width band require the same level of reserves. Every band in-between requires larger reserves, with some very narrow band of finite width requiring a maximum amount. A similar construction on the Franc side would indicate that an increased level is also needed by the Banque de France when the band reaches a small critical width.

CONCLUSION

This paper has illustrated the tradeoffs which policy makers face when designing an exchange-rate system. We have worked out the specifics of the intuitive idea that the amount of reserves needed by Central banks is a function of what scenario is envisaged for the case in which one of them runs out of reserves. For instance, we have been careful to distinguish coordinated

[35] We assume $r > 1/\alpha$; i.e., the Banque de France has enough reserves to sustain the one-sided zone one switshes to.

(or synchronized) collapses, in which both Central banks stop intervening at the same time, from unilateral collapses, which transform a two-sided target-zone regime or a fixed-rate regime into a one-sided zone. We have found that the amount of reserves needed in the case of unilateral collapses is much larger than in the case of coordinated collapses, and especially so for target zones of comparatively small but finite width.

While it was preferable, for analytical clarity, to consider the pure cases of transition from one regime to another, it would certainly be worthwhile, in future research, to consider hybrid, or two-stage, scenarios: first realignment, then transition to free float etc... Hybrid scenarios would help determine in greater detail the total amount of reserves which Central banks, in practice, need to have on hand. These scenarios would have to be chosen in such a way as to represent as closely as possible real-life arrangements, such as those of the Very Short Term Financing Facility of the EMS.[36] We hope to have laid the foundations of such a calculus.

One very important aspect has been neglected in this analysis. While they commit themselves to a policy of intervention, Central banks and governments, in the real world, retain many degrees of freedom. Their policies are not confined to foreign-exchange dealings; their compacts with other Central banks, therefore, do not tie their hands nearly as tightly as has been assumed. One should not design an exchange-rate system without having in mind that they create a set of incentives which influence Central bank behavior in these other respects.

[36] See, e.g. Micossi (1985).

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TABLE 1

Regime	Boundary conditions		Solutions	
General	None		$S(X) = X + \gamma\mu + Ae^{\alpha X} + Be^{-\beta X}$ A, B arbitrary	(7)
Free float	No bubbles		$A = B = 0$ $S = X + \gamma\mu$	(8)
Fixed rate	$E(dS\|\Phi(t))/dt = 0$		$S = X$	(9)
One-sided zone	No bubbles $\bar{S} = \bar{X} + \gamma\mu + Ae^{\alpha\bar{X}}$ $0 = 1 + \alpha Ae^{\alpha\bar{X}}$	 (11) (12)	$B = 0$ $\bar{X} = \bar{S} + 1/\beta$ $A = -e^{-\alpha\bar{X}}/\alpha.$	 (13) (14)
Two-sided zone	$\bar{S} = \bar{X} + \gamma\mu + Ae^{\alpha\bar{X}} + Be^{-\beta\bar{X}}$ $\underline{S} = \underline{X} + \gamma\mu + Ae^{\alpha\underline{X}} + Be^{-\beta\underline{X}}$ $0 = 1 + \alpha Ae^{\alpha\bar{X}} - \beta Be^{-\beta\bar{X}}$ $0 = 1 + \alpha Ae^{\alpha\underline{X}} - \beta Be^{-\beta\underline{X}}$	(15) (16) (17) (18)	A, B, $\underline{X}$, $\bar{X}$ solve (15) - (18).	

EXCHANGE RATE FUNCTIONS

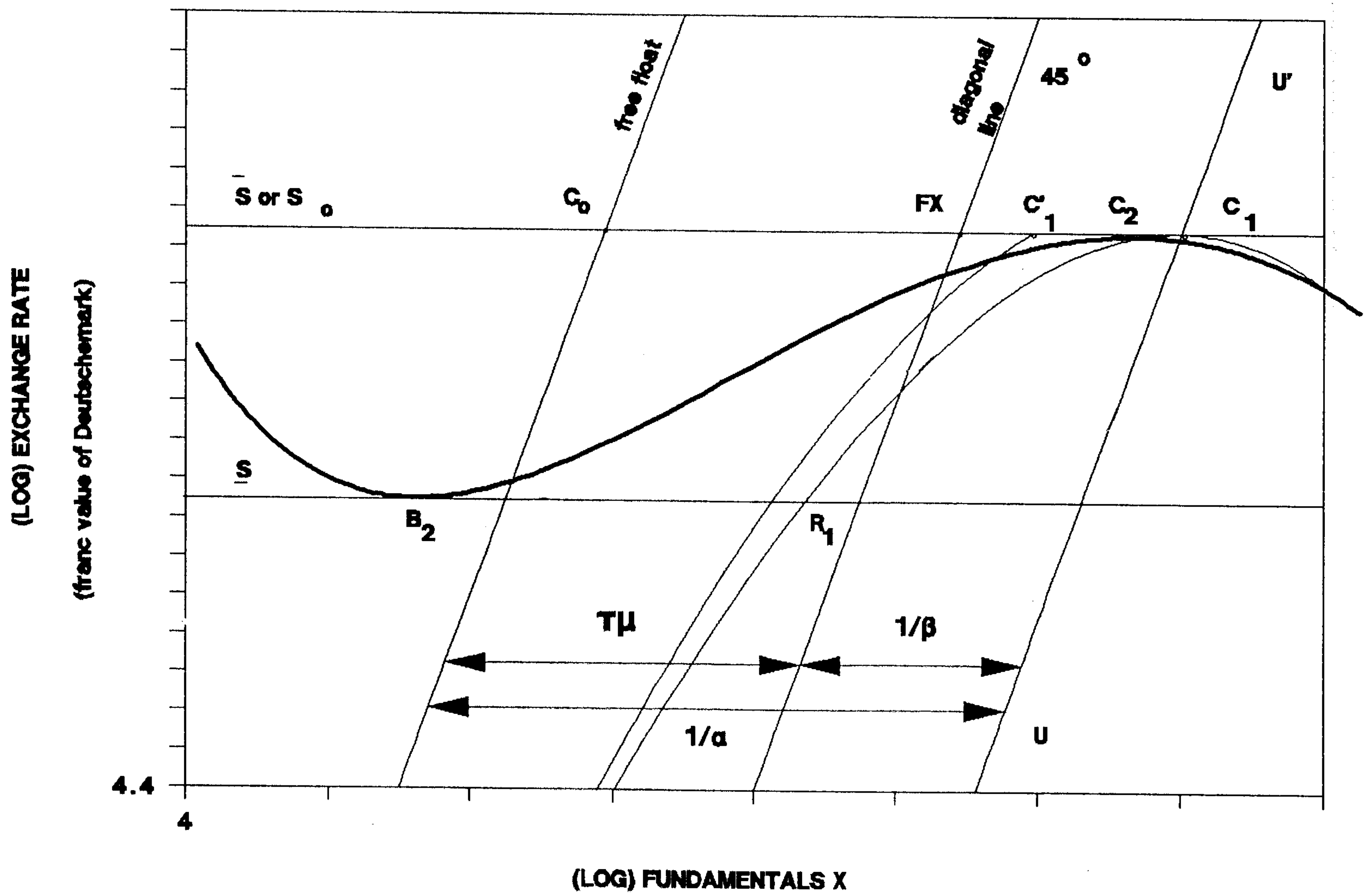

FIGURE 1

TARGET ZONES OF DIFFERENT WIDTHS

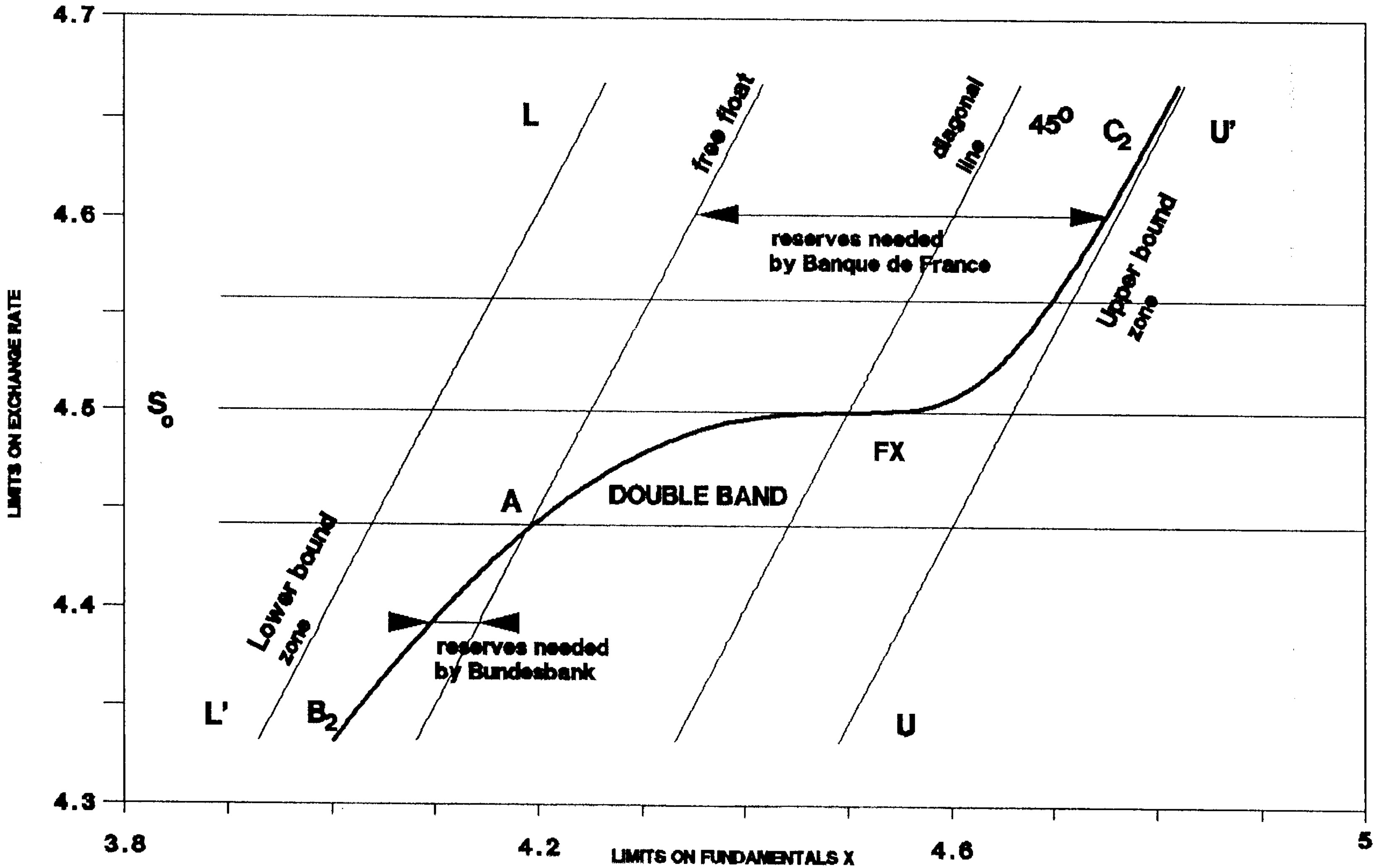

FIGURE 2

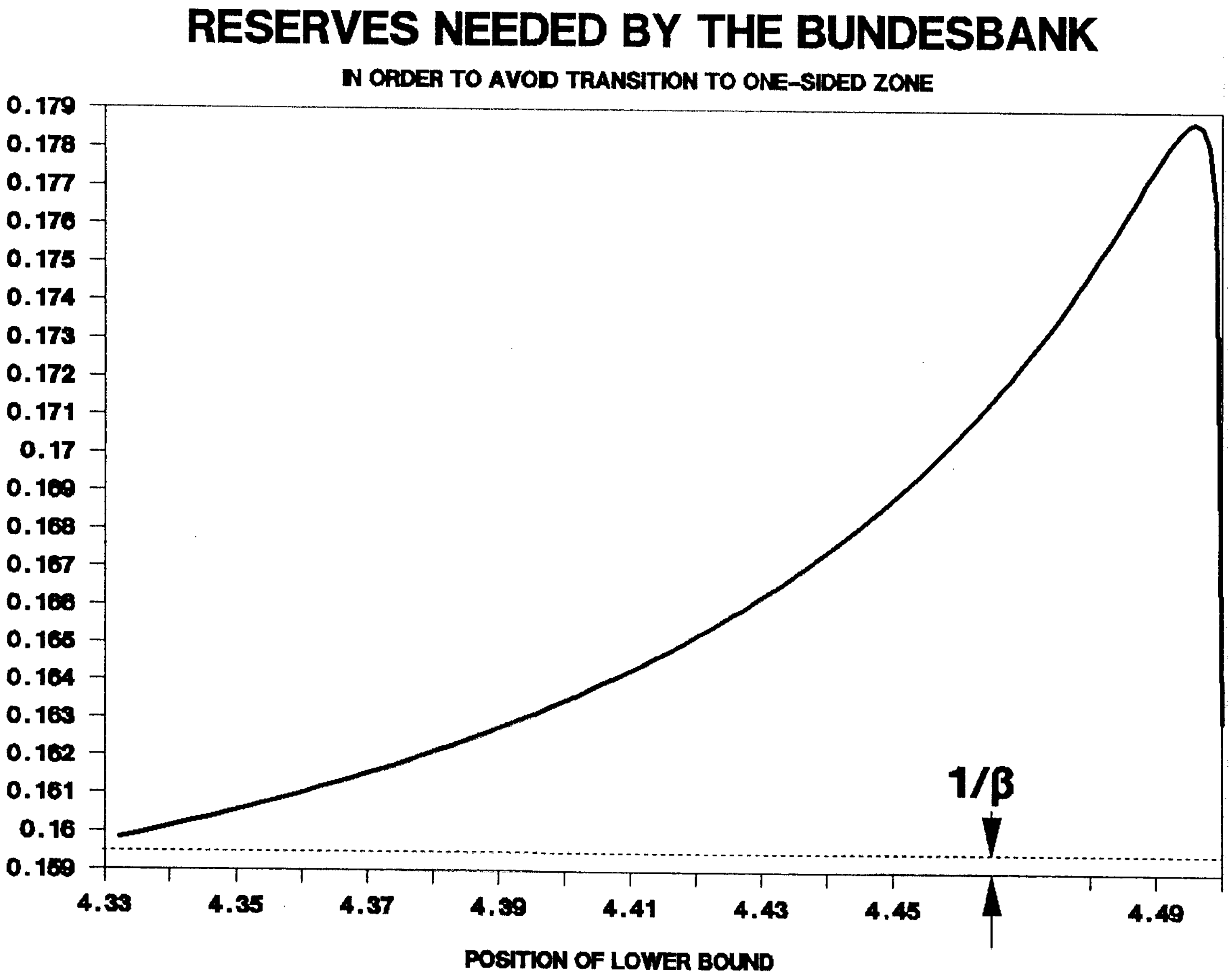
RESERVES NEEDED BY THE BUNDESBANK
IN ORDER TO AVOID TRANSITION TO ONE-SIDED ZONE
0.179
0.178
0.177
0.176
0.175
0.174
0.173
0.172
0.171
0.17
0.169
0.168
0.167
0.166
0.165
0.164
0.163
0.162
0.161
0.16
0.159
1/β
4.33
4.35
4.37
4.39
4.41
4.43
4.45
4.49
POSITION OF LOWER BOUND

FIGURE 3