British Economic Fluctuations, 1851-1913 : A Perspective Based on Growth Theory

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No. 362

September 1990

An earlier version of this paper was presented to the Warwick Summer Workshop in 1989 ; a revised version will appear in S. N. Broadberry and N. F. R. Crafts (eds.), Britain in the World Economy 1870-1939 : Essays in Honour of Alec Ford (Cambridge University Press, forthcoming).

1. Introduction

Alec Ford's analysis of economic fluctuations, synthesised in his contributions to Aldcroft & Fearon (1972) and Floud & McCloskey (1981), remains the focal point of discussions of the trade cycle before 1914. Perhaps the best known element of this work concentrates on the statistical investigation of nominal demand variables, notably in terms of deviations from nine-year moving averages. The most widely cited recent paper on pre-1914 business cycles (Eichengreen, 1983a) also concentrates its quantitative analysis on short-term correlations between changes in money, exports, output, prices *etc.* by using a vector autoregression approach.

Although Ford's exposition takes changes in effective demand as central to the analysis, he does not favour an endogenous cycle of a multiplier-accelerator kind as capturing the essential nature of the Victorian economy. His view of the underlying sources of fluctuations in this period places its emphasis on long run real factors associated with an international process of capital accumulation and long run economic development such that he would

> "support the use of a weak multiplier-accelerator model with erratic shocks (autonomous investment abroad could be one strong source) with emphasis on 'real' forces, although the monetary or interest rate factor must not be neglected. The 'trade cycle' in this period for Britain is seen as inextricably linked with the growth and development process not only of Britain but of the primary producers and borrowers" (1972, p. 159)

Many other authors have argued for the importance of this perspective: for example, Matthews argued that the apparent phenomenom of a 7 to 10 year cycle inherent in the working of the economy was illusory, being a reflection of unsynchronised long waves of capital accumulation at home and abroad (1959, pp. 220-226). This argument has since been significantly developed by Edelstein (1982) and Solomou (1987).

Edelstein established that British savings were sensitive to interest rates and that shocks to foreign investment would impact on home investment and vice versa. Solomou produced some statistical evidence in favour of long swings, which he interpreted as episodic events (based on investment shocks associated with structural changes) leading to transitions between steady-state growth paths. Nevertheless, such approaches have not been embedded in the basic models of economic growth which have been analysed in a hitherto separate new economic history literature concerned with the alleged failure of the late Victorian economy: see, for example, Crafts (1979), Kennedy (1974, 1987) and McCloskey (1970).

In this paper we analyse a formal model looking, in particular, at links between growth and cycles in order to provide some statistical evidence relevant to Ford's underlying view, given that these recent contributions have emphasised its general plausibility. We would argue that it is important to consider economic fluctuations and growth explicitly together, since it seems probable that there were strong interactions between the two.

The strong likelihood that this is the case has dominated recent developments in macroeconomics which follow up insights from the real business cycle literature and the econometric testing for unit roots in output to develop the notion that unforecasted changes in long run economic prospects, working in the context of a neoclassical growth model, can generate fluctuations much like the conventional conception of the business cycle (Stock & Watson, 1988). A major concern of this approach is whether trend growth should be seen as deterministic or stochastic. This in its turn is reflected (much less formally, of course) in the controversy over late nineteenth century British growth, with McCloskey firmly of the deterministic position while Kennedy and earlier writers like Phelps-Brown & Handfield Jones (1952), in their famous account of the climacteric, seem much more inclined to the stochastic view.

Time series analysis of British growth has already thrown up some important results in this context. Mills (1991), whose work is summarised below, has demonstrated by using a variety of tests that World War I appears to mark a boundary between an earlier period in which the growth process was trend stationary and a later period when the appropriate characterisation was of a random walk with drift, a result which seems to be rather general for twentieth century Western economies (see Campbell & Mankiw, 1989, and Kormendi & Meguire, 1990, for example). Such results are essentially statistical, however, and leave unanswered the question of what model may have generated the observed behaviour. Resolution of this issue is central to any serious attempt to view economic fluctuations as arising from shocks to a (neoclassical) growth process. Given Mills' results, we might expect the basic neoclassical growth model, which embodies trend stationarity, to be a good starting point for the analysis of pre-World War I fluctuations, although not for later periods.

Our main concern is then with the question of whether an essentially neoclassical growth model of the refined types presented in the recent literature, notably King, Plosser & Rebelo (1988a, b), is capable of replicating the British experience of fluctuations, particularly before 1914. A further issue needs also to be addressed, however. In some circumstances it should be very difficult (indeed impossible with realistic sample sizes) to reject correctly the hypothesis of a unit root in output. For example, West (1988) shows that there are configurations of aggregate demand and supply relations which, when combined with particular policy rules followed by the authorities, will give near unit root behaviour of output. A full understanding of growth and fluctuations in the pre-1914 world needs to be able to explain the absence of this near unit root outcome.

2. Models of Persistence in Output

Before proceeding to an analysis of a formal real business cycle model, it is important to clarify some basic statistical ideas concerning the effects of shocks on output growth and future levels of output.

We begin by assuming that the logarithm of output, denoted y_t, follows a first-difference stationary linear process: in other words, that the growth rate of output is stationary. If this is the case, y_t has a moving average representation of the form

$$\nabla y_t = (1-B)y_t = \mu + A(B)\varepsilon_t = \mu + \sum_{j=0}^{\infty} a_j \varepsilon_{t-j}, \tag{1}$$

where B is the lag operator, defined such that $B^k y_t = y_{t-k}$, the first equality defines the equivalent notations ∇y_t and $(1-B)y_t$ for first differences of y_t, and the last equality defines the lag polynomial notation $A(B)$. The ε_t are independent and identically distributed errors with common variance σ_ε^2: *i.e.* ε_t is white noise. The constant μ is the "drift", representing the long-run growth of y_t.

From (1), the impact of a shock in period t on the growth rate of output in period $t+k$, ∇y_{t+k}, is a_k. The impact of the shock on the *level* of output in period $t+k$, y_{t+k}, is therefore $1+a_1+\ldots+a_k$. The ultimate impact of the shock on the level of output is the infinite sum of these moving average coefficients, defined as

$$A(1) = 1 + a_1 + a_2 + \ldots = \sum_{j=0}^{\infty} a_j.$$

The value $A(1)=\sum a_j$ can then be taken as a measure of how persistent shocks to output are. For example, $A(1)=0$ for any trend stationary series, since $A(B)$ must contain a factor $(1-B)$, whereas $A(1)=1$ for a random walk, since $a_j=0$ for $j>0$. Other positive values of $A(1)$ are, of course, possible, depending upon the size and signs of the a_j.

Difficulties arise in estimating $A(1)$ because it is an infinite sum, thus requiring the estimation of an infinite number of coefficients. Various measures have thus been proposed in the literature to circumvent this problem. Mills (1991) compares the results of tests based on the approaches proposed by Campbell & Mankiw (1987), Cochrane (1988) and Clark (1987): these being, respectively, an ARMA model approach, a nonparametric model of persistence, and a structural time series model.

These alternative estimates of the persistence of output innovations for the U.K. are remarkably consistent, in contrast to the analagous findings for the U.S. summarised, for example, in Stock & Watson (1988). For the post-World War II quarterly data and for annual data post-1921, innovations to output have been largely persistent: a 1 per cent unforecasted increase in output will change the forecast of the long-run level of output by around 1 per cent. For the pre-1919 data, however, innovations were largely temporary: an forecasted increase in output would tend to have no impact on the long-run forecast. This last finding is consistent with the results of unit root tests reported in Crafts, Leybourne & Mills (1989) and Mills & Taylor (1989), and the evidence in favour of trend stationarity in the pre-World War I period thus appears quite strong.

There are several implications of this discussion of persistence in output which are of interest for this paper. First, it suggests that it may well be that a different model of growth and cycles is required for the Victorian economy than would be appropriate under modern conditions, as Ford's work itself strongly suggests. Second, also in line with Ford's expectations, there would be a reversion to long run trend following shocks such as changes in home or foreign investment opportunities in the pre-1914 economy but not after World War I. This could be consistent with Ford's emphasis on fluctuations in planned spending around a steady growth in productive potential (1981, p. 32). It also corresponds with the constant natural rate of growth idea at the heart of McCloskey's discussion of growth (1970), though it is not so easy to square with Solomou's (1987) emphasis on growth traverses. Third, the finding of no persistence prior to 1914 could be compatible with a neoclassical model of growth and cycles such as that put forward by King, Plosser & Rebelo (1988a), which is considered in detail in Section 3 below.

This model is in the Real Business Cycle tradition, however, which, while stressing real shocks as an explanation of fluctuations, as does Ford, has a different emphasis from the Keynesian tradition, in that it seeks to work out predicted implications in a choice theoretic framework in markets that clear. Nevertheless, the Real Business Cycle viewpoint is similar to Ford's in stressing real shocks rather than monetary disturbances as providing a very sizeable fraction of output fluctuations, and it seems to offer an interesting way of formalising the underlying Ford view of pre-1914 fluctuations. We proceed next to investigate how well the basic neoclassical model performs in attempts to replicate British cyclical experience.

3. A Formal Modelling Approach to Growth and Cycles

(a) The Basic Neoclassical Model

We begin by setting out briefly the key features of the basic one-sector, neoclassical model of capital accumulation that forms the basis of King, Plosser & Rebelo's (1988a) analysis of real business cycles.

The preferences, technology and endowments of the environment are defined in the following manner.

Preferences

The economy is assumed to be populated by many identical infinitely-lived individuals, of sufficient number that each perceives his influence on aggregate quantities to be insignificant, and whose preferences over goods and leisure are represented by the utility function

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \quad , \quad \beta<1 \quad , \tag{2}$$

where C_t and L_t are commodity consumption and leisure in period t, respectively. Momentary utility, $u(\cdot)$, is assumed to be strictly increasing, concave, twice continuously differentiable and to satisfy Inada-type conditions that ensure that the optimal solution for C_t and L_t is always (if feasible) interior. Restrictions can also be imposed on β to guarantee that life-time utility U is finite.

Production Technology

The economy has only one final good, Y_t, and it is produced according to a constant returns to scale production technology given by

$$Y_t = A_t F_t(K_t, N_t X_t) \quad , \tag{3}$$

where K_t is the predetermined capital stock (chosen at t-1) and N_t is labour input. By allowing the scale variable A_t to be time varying, temporary changes in total factor productivity are permitted, although, as we discuss below, permanent technology variations are restricted to be in labour productivity, X_t. We assume that $F(\cdot)$ has standard neoclassical properties, i.e. that it is concave, twice continuously differentiable, satisfies the Inada conditions, and implies that both factors of production are essential.

Capital Accumulation

In this simple neoclassical framework the single commodity can either be consumed or invested, i.e. stored for use in production next period. The evolution of the capital stock is thus

$$K_{t+1} = (1-\delta_K)K_t + I_t \quad , \tag{4}$$

where I_t is gross investment and δ_K is the rate of depreciation of capital.

Resource Constraints

In each period, an individual faces two resource constraints: (i) total time allocated to work and leisure cannot exceed the endowment, which is normalised to unity, and (ii) total uses of the commodity must not exceed output. These conditions are

$$L_t + N_t \leq 1 \quad , \tag{5}$$

and

$$C_t + I_t \leq Y_t \quad . \tag{6}$$

There are also the non-negativity constraints $L_t \geq 0$, $N_t \geq 0$, $C_t \geq 0$ and $K_t \geq 0$.

Steady State Growth

A characteristic of most industrialised economies is that variables such as output per capita and consumption per capita exhibit sustained growth over long periods of time: this long-run growth occuring at rates that are roughly constant over time within economies but which differ across economies. King, Plosser & Rebelo interpret this pattern as evidence of steady state growth: that levels of certain key variables grow at constant, but possibly different, rates. For the economic system described by equations (2)-(6) to exhibit steady state growth, additional restrictions on preferences are required.

Restrictions on Production

For steady state growth to be feasible, permanent technical change must be expressible in a labour augmenting form. While there are various functional forms for $F(\cdot)$ that will ensure this, the most tractable is the Cobb-Douglas:

$$Y_t = A_t K_t^{1-\alpha} (N_t X_t)^{\alpha} \quad . \tag{7}$$

Since variation in A_t is assumed to be temporary, it can be ignored in terms of steady growth, so that we can work with the assumption that A_t is constant for all time, i.e. $A_t = A$. As the amount of time devoted to work (N) has to be between zero and one, the only feasible per capita constant growth rate for N is zero, i.e. on denoting $\gamma_N = N_{t+1}/N_t$, we must have $\gamma_N=1$. The production function (7) (indeed, *any* constant returns to scale production function) and the capital accumulation equation (4) then imply that the steady state rates of growth of output, consumption, capital and investment per capita are all equal to the growth rate of labour augmenting technical progress, i.e.

$$\gamma_Y = \gamma_C = \gamma_K = \gamma_I = \gamma_X \tag{8}$$

Restrictions on Preferences

The feasible steady state given by the growth rates above will be compatible with an (optimal) competitive equilibrium if two restrictions on preferences are imposed: (i) the intertemporal elasticity of substitution in consumption must be invariant to the scale of consumption, and (ii) the income and substitution effects associated with sustained growth in labour productivity must not alter labour supply per person. These conditions imply the following class of admissable utility functions:

$$u(C,L) = \frac{1}{(1-\sigma)} C^{1-\alpha} v(1-N) \qquad (9a)$$

for $0<\sigma<1$ and $\sigma>1$, while for $\sigma=1$,

$$u(C,L) = \log(C) + v(1-N) \quad . \qquad (9b)$$

For this class of utility functions, the constant intertemporal elasticity of substitution in consumption is $1/\sigma$.

Stationary Economies and Steady States

The standard method of analysing models with steady state growth is to transform the economy into a stationary one, which can be done here by dividing all variables by the growth component X, so that $c=C/X$, $k=K/X$, etc. This alters the capital accumulation equation (4) to

$$\gamma_X k_{t+1} = (1-\delta_K)k_t + i_t \qquad (10)$$

and transforms the utility function (2) to

$$U = \sum_{t=0}^{\infty} (\beta^*)^t u(C_t, L_t) \quad , \tag{11}$$

where $\beta^* = \beta(\gamma_X)^{1-\sigma} < 1$ to guarantee finiteness of lifetime utility. Substituting (5) into (10) and combining (3), (6) and (9) into a general resource constraint, we can form the Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} (\beta^*)^t u(c_t, 1-N_t) + \sum_{t=0}^{\infty} \Lambda_t \left[A_t F(k_t, N_t) - c_t - \gamma_X k_{t+1} + (1-\delta_K) k_t \right] \tag{12}$$

The first order (efficiency) conditions for this transformed economy are given below as equations (13)-(16), in which $\mathcal{D}_i$ is the first partial derivative operator with respect to the ith argument and, for convenience, we discount the Lagrange multipliers to current values, i.e. $\lambda_t = \Lambda_t / (\beta^*)^t$.

$$\mathcal{D}_1 u(c_t, 1-N_t) - \lambda_t = 0 \tag{13}$$

$$\mathcal{D}_2 u(c_t, 1-N_t) - \lambda_t A_t \mathcal{D}_2 F(k_t, N_t) = 0 \tag{14}$$

$$\beta^* \lambda_{t+1} \left[A_{t+1} \mathcal{D}_1 F(k_{t+1}, N_{t+1}) + (1-\delta_K) \right] - \lambda_t \gamma_X = 0 \tag{15}$$

$$A_t F(k_t, N_t) + (1-\delta_K) k_t - \gamma_X k_{t+1} - c_t = 0 \tag{16}$$

for all $t=1,2,\ldots$ There is also a 'transversality condition',

$$\lim_{t \to \infty} (\beta^*)^t \lambda_t k_{t+1} = 0 \tag{17}$$

which ensures that the non-negativity constraint on k_t is imposed as $t \to \infty$ (the economy's initial capital stock, k_0, is assumed to be given).

For a given sequence, $\{A_t\}_{t=0}^{\infty}$, of technology shifts, optimal per capita quantities for this economy are sequences of consumption, $\{c_t\}_{t=0}^{\infty}$, work effort, $\{N_t\}_{t=0}^{\infty}$, capital stock, $\{k_t\}_{t=0}^{\infty}$, and shadow prices, $\{\lambda_t\}_{t=0}^{\infty}$, that satisfy the efficiency conditions (13)-(17), which are both necessary and sufficient for an optimum to be achieved. Thus, as real shocks impact on A_t, the economy will be characterised by intertemporal substitutions which will temporarily change investment, consumption, work effort, etc. and these transitory dynamics lie at the heart of fluctuations. It should further be noted that, although it is common in expositions of this type of model to think of A_t as the outcome of technology shocks, other factors could have similar effects. In general, any shock to the value of Tobin's *q* (Tobin, 1961) which triggers off a change in the desired capital stock will have similar effects, and in an open economy such as late Victorian Britain the source of such changes might well be developmemts abroad rather than at home, as Edelstein (1982) and Solomou (1987) have suggested.

(b) Near Steady State Dynamics

The basic one-sector neoclassical model with stationary technology has the property that the optimal capital stock converges monotonically to a stationary point. Our focus of attention will be on the approximate *linear* dynamics of the model in the neighbourhood of the steady state denoted by (A, k, N, c and y).

The initial step in obtaining a system of linear difference equations is to approximate (13)-(16) near the stationary point. This is done by expressing each condition in terms of the percentage deviation from the stationary value, which we indicate using a circumflex [e.g. $\hat{c}_t=\log(c_t/c)$, $\hat{k}_t=\log(k_t/k)$, etc.], and then linearising each condition in terms of these deviations. Equations (13) and (14) imply that

$$\xi_{cc}\hat{c}_t - \xi_{cl}\frac{N}{1-N}\hat{N}_t - \hat{\lambda}_t = 0 \quad , \tag{18}$$

$$\xi_{lc}\hat{c}_t - \frac{N}{1-N}\xi_{ll}\hat{N}_t - \hat{\lambda}_t - \hat{A}_t - (1-\alpha)\hat{k}_t + (1-\alpha)\hat{N}_t = 0 \quad , \tag{19}$$

where ξ_{ab} is the elasticity of the marginal utility of a with respect to b and where we have used the Cobb-Douglas production function (7). The ξ's depend on the utility function employed. We shall use the additively separable function (9b), from which it follows that $\xi_{cc}=-1$, $\xi_{cl}=\xi_{lc}=0$ and $\xi_{ll}=L\mathcal{D}^2v(L)/v(L)$.

Approximation of the intertemporal efficiency condition (15) implies that

$$\hat{\lambda}_{t+1} + \eta_A\hat{A}_{t+1} + \eta_K\hat{k}_{t+1} + \eta_N\hat{N}_{t+1} = \hat{\lambda}_t \quad , \tag{20}$$

where η_A is the elasticity of the gross marginal product of capital with respect to A evaluated at the steady state, etc. With the Cobb-Douglas assumption, it follows that $\eta_A=[\gamma_X-\beta^*(1-\delta_K)]$, $\eta_K=-\alpha\eta_A$ and $\eta_N=\alpha\eta_A$. Approximation of the resource constraint (16) implies

$$\hat{y}_t = \hat{A}_t + \alpha\hat{N}_t + (1-\alpha)\hat{k}_t \tag{21}$$
$$= s_c\hat{c}_t + s_i\phi\hat{k}_{t+1} - s_i(\phi-1)\hat{k}_t \quad ,$$

where s_c and s_i are consumption and investment shares in output and $\phi=K_{t+1}/I_t=\gamma_X/[\gamma_X-(1-\delta_K)]>1$.

Equations (18)-(20) can be combined to eliminate $\hat{c}_t$, $\hat{N}_t$ and $\hat{y}_t$, yielding a difference equation system in $\hat{k}$ and $\hat{\lambda}$, which can then be solved, subject to the transversality condition, to produce unique solution sequences for capital accumulation $\{\hat{k}_t\}_{t=0}^{\infty}$ and shadow prices $\{\hat{\lambda}_t\}_{t=0}^{\infty}$, given a specification for the exogenous sequence $\{\hat{A}_t\}_{t=0}^{\infty}$. The time path of capital accumulation can, in fact, be written in the form

$$\hat{k}_{t+1} = \mu_1\hat{k}_t + \psi_1\hat{A}_t + \psi_2\sum_{j=0}^{\infty} \mu_2^{-j}\hat{A}_{t+j+1} \quad , \tag{22}$$

where μ_1 and μ_2 are the roots of the quadratic

$$\mu^2 - \left[1/\beta^* - s_c\eta_k/\sigma s_i\phi + 1\right]\mu + 1/\beta^* = 0 \quad ,$$

and which satisfy the inequalities $\mu_1<1<\beta^{*-1}<\mu_2$. The parameters ψ_1 and ψ_2 are given by

$$\psi_1 = \frac{1}{\mu_2\phi s_i}$$

and

$$\psi_2 = \psi_1\left[\left(\frac{s_c}{\sigma}\eta_A - 1\right) + \mu_2^{-1}\right]$$

and are thus complicated functions of the underlying parameters of preferences and technology.

(c) Real Business Cycles

We now incorporate uncertainty, in the form of temporary productivity shocks, into the basic neoclassical model discussed above. The time path of efficient capital production, given by equation (22), contains the future time path of productivity shocks 'discounted' by μ_2. If we posit a particular stochastic process for $\hat{A}$, we may replace the sequence $\{\hat{A}_{t+j}\}_{j=1}^{\infty}$ with its conditional expectation given information available at t. In particular, if $\hat{A}_t$ follows a first-order autoregressive process with parameter ρ, then $\hat{A}_{t+j}$ can be replaced by $\rho^j\hat{A}_t$. This then allows the 'state dynamics' of the model to be given by the linear system

$$s_{t+1} \equiv \begin{bmatrix} \hat{k}_{t+1} \\ \hat{A}_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_1 & \pi_{kA} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{A}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_{A,t+1} \end{bmatrix} = Ms_t + \varepsilon_{t+1} \tag{23}$$

where $\pi_{kA}=\psi_1+\psi_2\rho/(1-\rho\mu_2^{-1})$ and $s_t'\equiv(\hat{k}_t,\hat{A}_t)$ is the state vector.

Given (23), the efficiency conditions (18)-(21) and the further equations

$$\hat{w}_t = \hat{y}_t - \hat{N}_t \tag{24}$$

and

$$\hat{i}_t = \frac{1}{s_i}\hat{y}_t - \frac{s_c}{s_i}\hat{c}_t \quad , \tag{25}$$

then the vector $z_t'=(\hat{c}_t,\hat{N}_t,\hat{y}_t,\hat{i}_t,\hat{w}_t)$ is related to the state variables through the system of linear equations

$$z_t = \begin{bmatrix} \hat{c}_t \\ \hat{N}_t \\ \hat{y}_t \\ \hat{i}_t \\ \hat{w}_t \end{bmatrix} = \begin{bmatrix} \pi_{ck} & \pi_{kA} \\ \pi_{Nk} & \pi_{NA} \\ \pi_{yk} & \pi_{yA} \\ \pi_{ik} & \pi_{iA} \\ \pi_{wk} & \pi_{wA} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{A}_t \end{bmatrix} = \Pi s_t \tag{26}$$

where the π coefficients, which are elasticities with respect to deviations of the capital stock from its stationary value, are complicated functions of the parameters of the model, i.e. α, σ, δ_K, β and γ_X. This formulation allows computation of impulse response functions for the system and population moments of the joint (z_t, s_t) process.

Impulse Responses

Impulse response functions provide information on the system's average *conditional* response to a technology shock at date t, given the posited stochastic process for $\hat{A}_t$. The response of the system in period $t+m$ to a technology shock at $t+1$ is

$$s_{t+m} - E(s_{t+m} | s_t) = M^{m-1} \varepsilon_{t+1}$$

and

$$z_{t+m} - E(z_{t+m} | s_t) = \Pi M^{m-1} \varepsilon_{t+1} .$$

Population Moments

Population moments provide additional, *unconditional*, properties of the time series generated by this model economy.

The linearity of the system implies that it is relatively straightforward to calculate population moments. The following procedure may be employed. The system matrix M is first decomposed as

$$M = PM \cdot DM \cdot PM^{-1} ,$$

where PM is the matrix of eigenvectors of M and DM contains the eigenvalues on its diagonal (and zeros elsewhere). Transformed states and innovations are then defined as

$$s_t^* = PM^{-1} s_t$$

and

$$\varepsilon_t^* = PM^{-1} \varepsilon_t$$

respectively. The covariance between any two elements of s_t^*, s_{jt}^* and s_{it}^*, say, is given by

$$E\left[s_{jt}^* s_{it}^*\right] = \left[1 - dm_j dm_i\right]^{-1} E\left[\varepsilon_{jt}^* \varepsilon_{it}^*\right] ,$$

where dm_i is the *i*th diagonal element of DM. Calculation of the variance-covariance matrix of the original (untransformed) state variables is then given by reversing the transformation:

$$\Sigma_{ss} = E\left[s_t s_t'\right] = PM \cdot E\left[s_t^* s_t^{*\prime}\right] \cdot PM^{-1} .$$

The autocovariance of the states at any desired lead or lag $m \geq 0$ is then given by lags: $E\left[s_t s'_{t-m}\right] = M^m \cdot \Sigma_{ss}$

leads: $E\left[s_t s'_{t+m}\right] = \Sigma_{ss}(M')^m$,

while the autocovariance of z is given similarly by

lags: $E\left[z_t z'_{t-j}\right] = \Pi M^m \Sigma_{ss} \Pi'$

leads: $E\left[z_t z'_{t+m}\right] = \Pi \Sigma_{ss}(M')^m \Pi'$.

(d) Alternative Parameterisations of the Model

To compute the M and Π matrices, and hence obtain, for example, the population moments, values are required for the taste and technology parameters σ, α, s_c, γ_X, β^*, ξ_{cc}, ξ_{cl}, ξ_{ll}, ξ_{lc}, N, ρ and δ_K. Labour's share of output was set at α=0.52, the mean share over the period 1855-1913. Consumption's share of output was similarly set at its mean value over the period of s_c=0.87, so that s_i was thus 0.13 (Feinstein, 1972). The growth parameter was set at γ_X=1.02, using the common growth rate estimated from the deterministic trend models reported later in the text. As noted above, we assume that the momentary utility function is of the additively seperable form (9b): this specification implies zero cross-elasticities ($\xi_{lc}=\xi_{cl}=0$) and unitary elasticity of consumption ($\sigma=-\xi_{cc}=1$). The steady state value of work effort was set at N=0.35, reflecting the fact that hours worked per person has been estimated as 65 hours per week until 1870 and 56 hours thereafter (Matthews et al., 1982, p. 566). Given this value, and an estimate of the elasticity of labour supply of 0.4, taken from

Beenstock & Warburton's (1986, p. 164) research on interwar Britain, the elasticity of the marginal utility of leisure with respect to leisure (ξ_{ll}) is then estimated to be ξ_{ll}=-4.5, *cf.* King, Plosser & Rebelo (1988a, footnote 28). It should be recognised, however, that econometric evidence on labour supply elasticities is negligible and quite possibly unreliable.

Since the price level was stationary over this period (see Mills, 1990), the real interest rate and the nominal interest rate coincide. This averaged approximately 3% per annum during the sample period, thus yielding a value of β^*=0.99. The remaining pair of parameters, the rate of depreciation of capital (δ_K) and the persistence of technology shocks (ρ), were allowed to vary, taking the values 0.017 and 0.10 (the latter as in King, Plosser & Rebelo, 1988a) and 0, 0.5 and 0.9, respectively. There is no way of estimating the value of ρ, but Feinstein (1988, p. 427) tentatively suggests a lifetime of about 60 years for capital (i.e. δ_K=0.017).

The coefficients of the M and Π matrices obtained under these various combinations are shown in Table 1, while the implied population moments, i.e. relative standard deviations, correlations and auto- and cross-correlations, are shown in Tables 2 and 3.

The Feinstein depreciation coefficient is much lower than King, Plosser & Rebelo's 'realistic depreciation'. As Table 1 shows, this has a number of implications. The adjustment parameter μ_1 increases as δ_K falls, indicating that the capital stock adjusts more slowly at lower depreciation rates. The elasticity π_{kA}, on the other hand, declines as δ_K falls but, unlike μ_1, is sensitive to the serial correlation properties of $\hat{A}$, declining as ρ increases. These responses can be explained in terms of the basic economics of lowering the depreciation rate. First, when there is a lower depreciation rate, it follows that there is a higher steady state capital stock and a lower output-capital ratio: using the result that $(y/k) = (\gamma_X - \beta^*(1-\delta_K))/\beta^*(1-\alpha)$, then as δ_K goes from 0.10 to 0.017, y/k falls from 0.19 to 0.015. This suggests a substantial decline in the elasticity π_{kA}. Second, the change in μ_1 and the sensitivity of π_{kA} to ρ reflect the implications that δ_K has for the relative importance of wealth and intertemporal substitution effects. With lower depreciation, the intertemporal technology that links consumption today with consumption tomorrow becomes more linear near the stationary point. This means that the representative agent faces less sharply diminishing returns in intertemporal production possibilities and will choose a temporally smooth consumption profile that requires more gradual elimination of deviations of the capital stock from its stationary level. The depreciation rate also impinges on the relative importance of substitution and wealth effects associated with future shifts in technology.

Capital accumulation is less responsive to technological conditions when shocks are more persistent (i.e. π_{kA} falls as ρ rises). For the same reason, more persistent technology shocks imply that consumption is more responsive (π_{CA} rises as ρ rises) and investment is less responsive (π_{iA} falls). Altering the character of intertemporal tradeoffs also has implications for labour supply via intertemporal substitution channels, with more persistent shifts in technology producing smaller, but still positive, changes in work effort ($\pi_{NA}>0$, but falls as ρ rises). Alternative parameterisations of the model imply then that the nature of the intertemporal substitutions will change and thus, as we now discuss, different patterns of variability in the time series properties of the key variables.

Table 1

Parameter Values for the Linear System (23) and (26)

	Persistence					
	None (ρ=0)		Moderate (ρ=0.5)		Strong (ρ=0.9)	
δ_K	0.017	0.100	0.017	0.100	0.017	0.100
μ_1	0.931	0.801	0.931	0.801	0.931	0.801
π_{kA}	0.257	0.718	0.248	0.661	0.206	0.500
π_{ck}	0.649	0.625	0.649	0.625	0.649	0.625
π_{cA}	0.246	0.369	0.277	0.429	0.422	0.598
π_{Nk}	-0.059	-0.050	-0.059	-0.050	-0.059	-0.050
π_{NA}	0.262	0.219	0.251	0.198	0.201	0.140
π_{yk}	0.449	0.454	0.449	0.454	0.449	0.454
π_{yA}	1.136	1.114	1.131	1.103	1.104	1.073
π_{ik}	-0.889	-0.689	-0.889	-0.689	-0.889	-0.689
π_{iA}	7.096	6.104	6.846	5.616	5.672	4.247
π_{wk}	0.508	0.504	0.508	0.504	0.508	0.504
π_{wA}	0.874	0.895	0.879	0.905	0.904	0.933

Table 2

Population Moments for the Linear System (23) and (26): Relative Variabilility

		Standard Deviation relative to $\hat{A}$					Standard Deviation relative to $\hat{y}$			
δ_K	ρ	$\hat{y}$	$\hat{c}$	$\hat{i}$	$\hat{N}$	$\hat{w}$	$\hat{c}$	$\hat{i}$	$\hat{N}$	$\hat{w}$
0.017	0	1.18	0.52	7.12	0.27	0.95	0.44	6.04	0.22	0.80
0.017	0.5	1.54	0.97	7.75	0.28	1.32	0.63	5.04	0.18	0.86
0.017	0.9	4.03	3.51	11.1	0.37	3.84	0.87	2.76	0.09	0.95
0.100	0	1.24	0.84	6.16	0.23	1.08	0.67	4.97	0.18	0.87
0.100	0.5	1.77	1.46	6.17	0.22	1.65	0.82	3.49	0.12	0.93
0.100	0.9	4.36	4.13	7.49	0.20	4.28	0.95	1.72	0.05	0.98

Table 3

Population Moments for the Linear System (23) and (26): Auto- and Cross-Correlations

$\rho = 0;\ \delta_K = 0.017$

	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
Variable	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\hat{y}$	.16	.09	.06	.05	.05	.06	.09	.16	1	.16	.09	.06	.05
$\hat{c}$	.88	.71	.64	.59	.18	.21	.29	.53	.69	.26	.22	.20	.18
$\hat{i}$	-.02	-.00	.00	.01	-.02	-.02	-.03	-.05	.94	.08	.00	-.01	-.02
$\hat{N}$	-.03	.01	.02	.02	-.03	-.04	-.05	-.09	.91	.06	-.01	-.03	-.03
$\hat{w}$	.26	.16	.13	.11	.08	.09	.12	.23	.99	.19	.11	.09	.08

$\rho = 0;\ \delta_K = 0.10$

	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
Variable	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\hat{y}$	.39	.29	.22	.17	.17	.22	.29	.39	1	.39	.29	.22	.17
$\hat{c}$	.88	.69	.54	.42	.34	.45	.60	.80	.79	.43	.34	.26	.21
$\hat{i}$	-.07	-.05	-.03	-.02	-.05	-.07	-.09	-.12	.83	.21	.15	.10	.07
$\hat{N}$	-.10	-.06	-.04	-.03	-.10	-.13	-.18	-.24	.75	.16	.11	.07	.05
$\hat{w}$	.53	.40	.30	.23	.21	.28	.37	.50	.99	.41	.31	.24	.18

$\rho = 0.5;\ \delta_K = 0.017$

	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
Variable	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\hat{y}$	.67	.40	.27	.21	.21	.27	.40	.67	1	.67	.40	.27	.21
$\hat{c}$	.96	.79	.70	.64	.39	.44	.54	.79	.80	.60	.43	.35	.30
$\hat{i}$	.46	.24	.12	.06	-.00	.05	.16	.36	.86	.52	.25	.13	.07
$\hat{N}$	.45	.25	.13	.07	-.05	-.01	.09	.28	.80	.47	.20	.09	.04
$\hat{w}$	.74	.48	.36	.29	.26	.32	.45	.72	.99	.68	.42	.30	.24

$\rho = 0.5$; $\delta_K = 0.10$

Variable	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\hat{y}$	.78	.51	.34	.24	.24	.34	.51	.78	1	.78	.51	.34	.24
$\hat{c}$	.94	.69	.52	.40	.35	.48	.66	.93	.91	.72	.49	.35	.26
$\hat{i}$	.41	.18	.08	.03	-.03	.00	.08	.26	.77	.59	.34	.20	.12
$\hat{N}$	.36	.16	.07	.03	-.12	-.12	-.08	.04	.60	.46	.25	.13	.07
$\hat{w}$	.84	.56	.39	.28	.27	.38	.56	.83	.99	.78	.51	.35	.25

$\rho = 0.9$; $\delta_K = 0.017$

Variable	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\hat{y}$	.96	.84	.76	.70	.70	.76	.84	.96	1	.96	.84	.76	.70
$\hat{c}$	.99	.87	.79	.73	.69	.75	.83	.96	.95	.91	.81	.73	.67
$\hat{i}$	.86	.80	72	.64	.48	.54	.60	.65	.78	.75	.65	.58	.53
$\hat{N}$	.84	.80	.72	.65	.30	.35	.39	.40	.55	.54	.45	.40	.36
$\hat{w}$	.97	.85	.77	.71	.70	.77	.85	.97	.99	.96	.84	.76	.70

$\rho = 0.9$; $\delta_K = 0.10$

Variable	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\hat{y}$	.97	.79	.66	.56	.56	.66	.79	.97	1	.97	.79	.66	.56
$\hat{c}$	.99	.77	.62	.53	.51	.62	.77	.99	.99	.95	.77	.64	.54
$\hat{i}$	.82	.80	.76	.70	.63	.67	.69	.68	.83	.83	.68	.57	.49
$\hat{N}$	.72	.72	.69	.65	.50	.46	.37	.20	.40	.44	.36	.31	.27
$\hat{w}$	.97	.78	.65	.54	.54	.65	.79	.98	1	.96	.79	.66	.55

(e) Time Series Implications

A major feature of economic fluctuations is the differential variability in the use of inputs (labour and capital) and in the components of output (consumption and investment). With purely temporary technology shocks (ρ=0), consumption is much less variable than output (about a half as variable), while investment is far more variable (about five times as variable). Labour input $\hat{N}$ is somewhat less variable than consumption while the real wage rate $\hat{w}$ is rather more variable.

When shifts in technology become more persistent, there are important changes in the relative variabilities. Consumption increases in variability relative to output, although it is still less volatile in absolute terms, and this accords with the permanent income perspective. Real wages become somewhat more variable relative to output while labour input becomes much less variable, this fundamentally reflecting the diminished desirability of intertemporal substitution of effort in the face of more persistent technology shocks. Investment declines in variability relative to output, but still remains considerable more volatile. These relative variabilities appear to be only marginally affected by changes in the rate of depreciation.

A notable feature of the time series implications of the model is that $\hat{y}$, $\hat{i}$ and $\hat{N}$ exhibit almost no serial correlation in the absence of serially correlated technology shocks, irrespective of the rate of depreciation. This is not true of real wages or consumption, however, the latter being considerably smoother and also cross-correlated with output, unlike investment and labour.

Serial and cross-correlation patterns become stronger as ρ increases, although again the value of δ_K has little influence.

We have considered deterministic labour augmenting technological change that grows at a constant proportionate rate as the source of sustained growth. The neoclassical model then predicts that all quantity variables (with the exception of N) grow at the same rate γ_X. The non-deterministic components of consumption, output and investment are then

$$\hat{y}_t = \log(Y_t) - \log(X_t) - \log(y)$$

$$\hat{c}_t = \log(C_t) - \log(X_t) - \log(c)$$

$$\hat{i}_t = \log(I_t) - \log(X_t) - \log(i)$$

Assuming that X grows at a constant proportionate rate

$$X_t = X_0 \gamma_X^t$$

i.e.

$$\log(X_t) = \log(X_0) + t \cdot \log(\gamma_X)$$

then

$$\hat{y}_t = \log(Y_t) - \beta_0 - \beta_1 t$$

where

$$\beta_0 = \log(y/X_0) \qquad \beta_1 = \log(\gamma_X)$$

and similarly for $\hat{c}_t$ and $\hat{i}_t$. Hence $\log(Y_t)$, $\log(C_t)$ and $\log(I_t)$ are *trend stationary* and possess a *common* deterministic trend. Therefore we should consider deviations of the log levels of output, consumption and investment from a common linear trend as empirical counterparts to $\hat{y}_t$, $\hat{c}_t$ and $\hat{i}_t$. Work effort, on the other hand, possesses no trend and thus $\hat{N}$ is simply the deviation of the log of hours from its mean. The real wage rate will also be trend stationary, although it will not necessarily have the common growth rate of γ_X.

Data for the period 1851 to 1913 for output, consumption and investment were taken from Mitchell (1988, pp. 837-9). The real wage series is for own-product real wages and was obtained by deflating a money wage series from Mitchell (1988, pp. 149-50) spliced to Feinstein (1990) by the GDP deflator from the national accounts tables in Mitchell. The labour input series N was generated by scaling the percentage employed (Feinstein, 1972, p. T125) by hours worked per week (65/168 up to 1870 and 56/168 from 1871 onwards). The following trend stationary models were then estimated.

$$\log(Y_t) = 6.12 + .0193t, \qquad s = .0355,$$

$$\log(C_t) = 6.02 + .0193t, \qquad s = .0364,$$

$$\log(I_t) = 3.67 + .0206t, \qquad s = .1546,$$

$$\log(W_t) = 4.03 + .0081t, \qquad s = .0353,$$

where s is the residual standard error of the regression. Note that, as predicted by the model, the slope coefficients (i.e. trend growth rates) are very similar for output, consumption and investment, but not for real wages. Imposing a common trend yields the following models, which show very little deteriation of fit when compared to the unrestricted trend models:

$$\log(Y_t) = 6.10 + .0197t, \qquad s = .0363,$$

$$\log(C_t) = 6.00 + .0197t, \qquad s = .0375,$$

$$\log(I_t) = 3.72 + .0197t, \qquad s = .1555.$$

Given the above models, the empirical counterparts to $\hat{y}_t$, $\hat{c}_t$ $\hat{i}_t$ and $\hat{w}_t$ were generated as

$$\hat{y}_t = \log(Y_t) - 6.10 - 0.02t$$
$$\hat{c}_t = \log(C_t) - 6.00 - 0.02t$$
$$\hat{i}_t = \log(I_t) - 3.72 - 0.02t$$

and

$$\hat{w}_t = \log(W_t) - 4.03 - 0.008t$$

Figure I plots $\log(N_t)$ and $\hat{N}_t$, the latter defined as the deviation of $\log(\hat{N}_t)$ from its mean value, which is allowed to shift in 1870 when hours worked per week was reduced. As predicted by the model, work effort shows no trend whatsoever.

Figures II-V show the empirical counterparts to $\hat{c}$, $\hat{i}$, $\hat{w}$ and $\hat{N}$ plotted against the reference variable $\hat{y}$, while Table 4 presents the sample moments of the series. Consumption is highly cross-correlated with output, investment, work effort and real wages somewhat less so. Consumption, real wages and investment are also more highly serially correlated than output.

Consumption, real wages and output have almost identical variability, while investment is some four times more volatile, but work effort is rather less variable, than output. Comparisons of Table 4 with Tables 2 and 3 suggests approximate correspondence with a neoclassical growth model having moderate technology shocks and a depreciation rate more in the range of 10% per annum rather than that assumed by Feinstein. Nevertheless, there are some noticeable deviations from the predictions of the model, most particularly in investment, which seems too volatile and serially correlated, and in real wages, which is almost uncorrelated with output. Moreover, the standard deviation of $\hat{N}$ is much larger than in Table 2. These features are particularly noticeable in the Figures, the close correspondence in serial correlation and variability between $\hat{c}$ and $\hat{y}$ being clearly seen, as is the markedly more volatile and persistent behaviour of $\hat{i}$.

Further evidence concerning the performance of the model is obtained by examining the processes generating the empirical counterparts to $\hat{y}$, $\hat{c}$ and $\hat{i}$. It is easy to show that these series should be generated by ARMA(2,1) processes with identical autoregressive polynomials, but different moving average parts. Fitting such processes obtains the following models:

Table 4

Sample Moments

Variable	Std. Dev	Std. Dev Relative to $\hat{y}$
$\hat{y}$	.036	1.00
$\hat{c}$	.037	1.03
$\hat{i}$	.154	4.29
$\hat{N}$	.024	0.67
$\hat{w}$	.035	0.97

	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
Variable	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\hat{y}$	.57	.54	.44	.35	.35	.44	.54	.57	1	.57	.54	.44	.35
$\hat{c}$	.88	.79	.71	.58	.43	.54	.64	.71	.84	.65	.59	.56	.43
$\hat{i}$	.91	.73	.51	.29	.33	.43	.50	.57	.58	.46	.35	.24	.15
$\hat{N}$	.58	.05	-.32	-.40	-.01	.06	.13	.19	.39	.19	.05	-.12	-.19
$\hat{w}$	.82	.72	.58	.46	.23	.15	.03	-.04	-.15	-.15	-.21	-.19	-.18

$$\hat{y}_t = .61\hat{y}_{t-1} + .23\hat{y}_{t-2} + \varepsilon_t - .27\varepsilon_{t-1}, \quad \sigma_{\varepsilon_A} = .028$$

$$\hat{c}_t = .70\hat{c}_{t-1} + .24\hat{c}_{t-2} + \varepsilon_t + .08\varepsilon_{t-1}, \quad \sigma_{\varepsilon_A} = .016$$

$$\hat{i}_t = 1.52\hat{i}_{t-1} - .65\hat{i}_{t-2} + \varepsilon_t - .65\varepsilon_{t-1}, \quad \sigma_{\varepsilon_A} = .052$$

The implied ARMA structure is apparent for output and consumption but is not found for investment, the autoregressive parameters of which reflect the cyclical behaviour of $\hat{i}_t$ shown in Figure III.

The low and slightly negative correlation between $\hat{w}$ and $\hat{y}$ represents a serious lack of correspondence with the basic neoclassical model. It also represents an interesting difference from the results obtained by King, Plosser & Rebelo (1988a, Table 6) in applying the basic neoclassical model to the post World War II United States. By contrast they found a high correlation between $\hat{w}$ and $\hat{y}$ but a very low correlation between $\hat{N}$ and $\hat{y}$.

(f) Stochastic Growth

An alternative growth model has been proposed by King, Plosser & Rebelo (1988b) which might be thought capable of replicating the behaviour of investment more satisfactorily. Rather than assume that labour productivity grows at a constant proportionate rate as in the basic neoclassical growth model, so that consumption, investment and output are trend stationary with a common trend growth rate, here we assume that, in general, labour productivity follows a *stochastic trend*, specifically a random walk:

$$\nabla\log(X_t) = \log(X_t) - \log(X_{t-1}) = \log(\gamma_X) + \varepsilon_t$$

or (27)

$$\log(X_t) = \log(X_0) + t\cdot\log(\gamma_X) + \sum_{i=0}^{\infty} \varepsilon_{t-i} \quad .$$

Hence shocks to the stochastic trend at time t, ε_t, result in a *permanent* shift in the level of X_t. The (transformed) capital stock is then driven by the permanent technology shock ε_t:

$$\hat{k}_t = \mu_1 \hat{k}_{t-1} - \varepsilon_t \quad . \tag{28}$$

Since all other stationary variables of the system respond only to the position of the transformed capital stock (there being no transitory components of technology under the random walk assumption), we have

$$\hat{c}_t = \pi_{ck}\hat{k}_t, \qquad \hat{N}_t = \pi_{Nk}\hat{k}_t, \qquad \hat{y}_t = \pi_{yk}\hat{k}_t,$$
$$\hat{i}_t = \pi_{ik}\hat{k}_t, \qquad \hat{w}_t = \pi_{wk}\hat{k}_t \tag{29}$$

Concentrating attention on the behaviour of output, consumption and investment, then substituting the appropriate relationships into

$$\log(Y_t) = \log(X_t) + \log(y) + \hat{y}_t$$

$$\log(C_t) = \log(X_t) + \log(c) + \hat{c}_t$$

$$\log(I_t) = \log(X_t) + \log(i) + \hat{i}_t$$

yields

$$\nabla\log(Y_t) = \log(\gamma_X) + (1-\pi_{yk})\varepsilon_t + \pi_{yk}(\mu_1-1)\hat{k}_{t-1}$$

or

$$\nabla\log(Y_t) = (1-\mu_1)\log(\gamma_X) + \mu_1\nabla\log(Y_{t-1}) + (1-\pi_{yk})\varepsilon_t - (\mu_1-\pi_{yk})\varepsilon_{t-1}$$

with similar expressions for $\log(C_t)$ and $\log(I_t)$, *i.e.* the *growth rates* of Y, C and I should follow ARMA(1,1) processes with identical AR parts.

Utilising the parameter values $\mu_1=0.8$, $\pi_{yk}=0.4$, $\pi_{ck}=0.6$ and $\pi_{ik}=-0.7$, taken as representatives of the values given in Table 1, the implied processes are

$$\left(1 - 0.8B\right)\nabla\log(Y_t) = 0.2 + \left(1 - 0.7B\right)\varepsilon^*_{yt}$$

$$\left(1 - 0.8B\right)\nabla\log(C_t) = 0.2 + \left(1 - 0.5B\right)\varepsilon^*_{ct}$$

$$\left(1 - 0.8B\right)\nabla\log(I_t) = 0.2 + \left(1 - 0.9B\right)\varepsilon^*_{it}$$

where $\varepsilon^*_{yt}=0.6\varepsilon_t$, $\varepsilon^*_{ct}=0.4\varepsilon_t$ and $\varepsilon^*_{it}=1.7\varepsilon_t$.

Since the autoregressive and moving average parameters are fairly similar, the autocorrelations and cross-correlations of the series will be small, although the contemporaneous correlation between them will be high. Given the above models and taking $\sigma^2_\varepsilon=1$, the implied standard deviations of the growth rates of the series are $\sigma_y=0.61$, $\sigma_c=0.45$ and $\sigma_i=1.72$, respectively.

The corresponding sample moments are given in Table 5. The variability of consumption and investment relative to output are both somewhat too low to that implied by the model, while the presence of large autocorrelations and cross-correlations at low lags and leads is also in contrast to that predicted.

Table 5

Sample Moments

Variable	Std. Dev	Std. Dev Relative to $\hat{y}$
$\nabla\log(Y_t)$	.033	1.00
$\nabla\log(C_t)$	.016	0.48
$\nabla\log(I_t)$	.064	1.94

	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
Variable	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\nabla\log(Y_t)$	-.48	.09	-.02	.05	.05	-.02	.09	-.48	1	-.48	.09	-.02	.05
$\nabla\log(C_t)$	-.22	.01	.19	-.02	-.05	-.01	.14	-.25	.75	-.35	-.05	.21	-.01
$\nabla\log(I_t)$	.50	.22	-.04	-.09	.07	.05	.01	.14	.31	-.04	.01	-.07	.04

The fitted ARMA(1,1) models confirm these findings:

$$\nabla\log(Y_t) = .019 - .21\nabla\log(Y_{t-1}) + \varepsilon^*_{yt} - .40\varepsilon^*_{yt-1}, \quad \sigma_\varepsilon = .029$$

$$\nabla\log(C_t) = .019 - .19\nabla\log(C_{t-1}) + \varepsilon^*_{ct} - .19\varepsilon^*_{ct-1}, \quad \sigma_\varepsilon = .016$$

$$\nabla\log(I_t) = .018 + .45\nabla\log(I_{t-1}) + \varepsilon^*_{it} + .14\varepsilon^*_{it-1}, \quad \sigma_\varepsilon = .056$$

The models for output and consumption are close to those predicted by the model: the problem is again that of investment, whose parameters are considerably different. This would therefore appear to confirm that assuming stochastic growth is not, in itself, capable of correcting the failure of the basic neoclassical growth model adequately to model the behaviour of investment.

Indeed, we would argue that, of the two variants, the constant growth model is to be favoured, giving overall a better modelling of the growth and fluctuations of the pre-world War I British economy. In particular, the evidence of the various tests discussed earlier seems strong enough to reject the notion of a random walk model of the trend in GDP with some confidence. For the post World War I period, however, we would expect the stochastic growth model to be the better of the two, recollecting that in the Introduction we reported the findings of Mills (1991) to the effect that the behaviour of output was very different before and after World War I: while output was trend stationary in the earlier period, it was difference stationary from 1922 onwards. This suggests that the constant growth model should provide a poor fit to the post-World War I data, being dominated by the stochastic growth variant.

Evidence that this is indeed the case is now provided. Concentrating again on the behaviour of just output, consumption and investment, trend stationary models for the period 1922-1986 were estimated to be

$$\log(Y_t) = 5.66 + .0206t, \qquad s = .0736,$$

$$\log(C_t) = 5.60 + .0197t, \qquad s = .0753,$$

$$\log(I_t) = 2.14 + .0356t, \qquad s = .3147,$$

Imposing a common trend considerably worsens the fit of the models, which is not surprising given that the slope coefficient of the investment equation is substantially larger than those of the other two:

$$\log(Y_t) = 5.08 + .0253t, \qquad s = .1159,$$

$$\log(C_t) = 4.91 + .0253t, \qquad s = .1303,$$

$$\log(I_t) = 3.42 + .0253t, \qquad s = .3706,$$

Given the above models, the empirical counterparts to $\hat{y}_t$, $\hat{c}_t$ and $\hat{i}_t$ were generated as

$$\hat{y}_t = \log(Y_t) - 5.08 - 0.025t$$
$$\hat{c}_t = \log(C_t) - 4.91 - 0.025t$$
$$\hat{i}_t = \log(I_t) - 3.42 - 0.025t$$

and the sample moments associated with these series are shown in Table 6. From them we see that it would be difficult to find a constant growth model that would adequately fit the observed data. Indeed, the autocorrelations of the series show that each contains a root that is close to, if not equal to, unity.

The sample moments of the logarithmic first differences for the period 1922-1986 are shown in Table 7. The relatively low autocorrelations and cross-correlations are reasonably consistent with a stochastic growth model in which technology shocks are persistent and thus confirm that such a model presents a better explanation of the post-World war I data than the constant growth model.

Table 6

Sample Moments

Variable	Std. Dev	Std. Dev Relative to $\hat{y}$
$\hat{y}$	.111	1.00
$\hat{c}$	.125	1.13
$\hat{i}$	.371	3.34

	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
Variable	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\hat{y}$	.93	.82	.70	.56	.56	.70	.82	.93	1	.93	.82	.70	.56
$\hat{c}$	.94	.86	.77	.69	.39	.42	.46	.50	.57	.60	.63	.66	.68
$\hat{i}$	.91	.77	.64	.53	-.62	-.66	-.67	-.64	-.56	-.45	-.34	-.21	-.11

Table 7

Sample Moments

Variable	Std. Dev	Std. Dev Relative to $\hat{y}$
$\nabla\log(Y_t)$	.034	1.00
$\nabla\log(C_t)$	.027	0.79
$\nabla\log(I_t)$	.149	4.38

	Autocorrelations				Crosscorrelations with $\hat{y}_{-j}$								
Variable	1	2	3	4	4	3	2	1	0	-1	-2	-3	-4
$\nabla\log(Y_t)$	.28	.14	.12	-.14	-.14	.12	.14	.28	1	.28	.14	.12	-.14
$\nabla\log(C_t)$	.44	.07	-.08	-.22	-.01	-.13	-.24	-.36	-.19	-.12	-.15	.04	.29
$\nabla\log(I_t)$	.35	-.03	-.14	-.08	-.22	-.33	-.36	-.39	-.02	-.05	-.12	.23	.34

4. Further Consideration of the Results

The results presented in section 3 are basically consistent with our expectations based on the observed pattern of persistence in output. The basic neoclassical growth model, with serial correlation in technology shocks and trend stationary growth, has some promise as a foundation for the analysis of pre-1914 fluctuations but is not well-suited for the later, post-World War I, period. The first topic to be addressed in this section is the conditions which permitted the absence of the unit root (or random walk) in output in the Victorian economy. We then go on to explore the apparent failings of the real business cycle model revealed in section 3, namely its inability to replicate the behaviour of investment and the absence of a procyclical pattern of real wage fluctuations.

(a) Near Random Walk Behaviour in Output

In a recent paper, West (1988) demonstrated that, in some circumstances, simple aggregate demand and supply models subjected to nominal shocks can generate a highly persistent (near random walk) process for output. Since we can reject the hypothesis of a unit root in pre-1914 output, it is useful to clarify that this outcome would be expected on the basis of West's approach.

In fact, West offers two models: one for a case where the government pursues an interest rate target, and a second for the case of a money supply target. This second model is the one we investigate here, as it is appropriate to the operations of the

Bank of England in the pre-1914 Gold Standard period. It is widely agreed that the overwhelming priority of the Bank of England's actions in the money markets was to maintain convertibility and to sustain a target level of reserves (Collins, 1988, p. 181; Dutton, 1984, p. 192; Goodhart, 1984, p. 223). Given the stable behaviour of the money to high powered money and high powered money to gold ratios (Bordo & Schwartz, 1981, pp. 114-5), this effectively translates into a money supply rule followed over anything other than the very short term, with the rule chosen so as to be consistent with the fixed exchange rate.

The model is based on overlapping wage contracts as an endogenous source of persistence and where the money supply rule allows a choice as to the degree of accommodation of inflation and of past control errors. The model can be set out as follows.

$$x_t = .5x_{t-1} + .5E(x_{t+1|t-1}) + .5\gamma E(\hat{y}_{t|t-1}+\hat{y}_{t+1|t-1}) \tag{30}$$

$$p_t = .5(x_t+x_{t-1}) \tag{31}$$

$$\hat{y}_t + p_t = m_t \tag{32}$$

$$m_t = \varphi p_t + \lambda(m_{t-1}-\varphi p_{t-1}) + u_t \tag{33}$$

In these equations, x_t is the logarithm of the nominal wage rate, $\hat{y}_t$ is the logarithm of output, i_t is the nominal interest rate, p_t is the logarithm of the price level, m_t is the logarithm of the money supply and u_t is a serially uncorrelated shock. All variables are zero mean deviations from trend.

In each period (year), one half of the labour force fixes its nominal wage for the next two periods. Equation (30) then says that the nominal wage depends on actual and expected wages, as well as expected demand pressure, measured by expected deviations of output from trend. Equation (31) is a price markup equation, while (32) is a simple quantity equation. Equation (33) is a money supply target with $0 \le \lambda \le 1$.

Values of the unknown parameters γ, φ and λ were then obtained by estimating regression counterparts of equations (30) and (33). In the former equation, expectations were replaced by functions of observed values using the results that both x_t and $\hat{y}_t$ can be modelled adequately by AR(2) processes. The estimates thus obtained were $\gamma=0.2$, $\varphi=0.4$ and $\lambda=0.9$. West shows that the model has a simple solution when the monetary accomodation parameter λ equals unity, this being

$$\hat{y}_t = a\hat{y}_{t-1} + u_t + .5(1-a)u_{t-1} ,$$

where

$$a = \begin{cases} c - (c^2-1)^{1/2}, & \text{if } c>1 \\ c + (c^2-1)^{1/2}, & \text{if } c<-1 \end{cases}$$

with

$$c = \left(1 + .5(1-\varphi)\gamma\right)\left(1 - .5(1-\varphi)\gamma\right)^{-1}$$

Since our estimate of λ is reasonably close to unity, making this approximation and using the estimates of γ and φ found above yields

$$c = 1.13 \qquad \text{and} \qquad a = 0.60 ,$$

so that the implied process for $\hat{y}_t$ is

$$\hat{y}_t = 0.6\hat{y}_{t-1} + u_t + 0.2u_{t-1}$$

or, approximately

$$\hat{y}_t = 0.40\hat{y}_{t-1} + 0.16\hat{y}_{t-2} + u_t \quad .$$

Since the AR(2) model fitted to $\hat{y}_t$ has estimated parameters of 0.38 and 0.36, we see that a reasonably close correspondence is obtained, and that a near unit root result, as would have been implied by a being close to 1, is not found.

Comparison with West's results for the postwar United States (1988, p. 206) indicates that the absence of the near unit root in output derives both from a rather steeper short-run aggregate supply curve and, especially, from a lower degreee of monetary accomodation by the pre-1914 Bank of England than the post-World War II Federal Reserve System. This is not particularly surprising given the evidence for a Phillips Curve in this period (Hatton, 1991) and recent econometric estimates of Bank behaviour (Pippenger, 1984, p. 208).

(b) The Behaviour of Home Investment

Certain peculiarities of British investment expenditure in the late nineteenth century are well known. In particular, Edelstein has stressed the existence of an inverse pattern of long swings in home and foreign investment and their relative profitability (1982, p. 30, 153). The tendency to quite long periods where home investment grew consistently well above or below trend is indeed reflected in Figure III. However, econometric investigation has shown that, in practice, foreign investment conditions had little effect in crowding out home investment, although the volume of foreign investment responded both to surges of opportunity abroad and to the home marginal efficiency of capital (Edelstein, 1982, p. 224). Within the real business cycle framework, this suggests that our attention can be confined to the domestic determinants of home investment.

The best known recent discussion of home investment in the Victorian economy is that of Eichengreen (1983b). In particular, Eichengreen stressed the positive, though lagged, impact of rising share prices based on optimistic re-assessments of home profitability in raising *q* (the ratio of asset market valuation of capital relative to its replacement cost) in the home investment boom of the 1890s, followed by a subsequent reversal in both *q* and investment. Share prices exhibited substantial volatility in this period, presumably as investors generally (not merely in the London equity markets) re-assessed future prospects. It should be recognised, however, that there is a considerable body of evidence which suggests that even in modern economies there appears to be substantial excess volatility of share prices and thus, presumably, investor confidence about profits, compared with what would have turned out to be ex post rational (Bulkley & Tonks, 1989). If this can be shown also to apply to our period, this might account for the divergence of investment from the behaviour predicted by the basic neoclassical model, which embodies ex post rational forecasting.

The presence of excess volatility in share prices can be established by utilising the methodology proposed by Shiller (1981) and since used and debated extensively: Bulkley & Tonks (1989) provide a British application. This is based upon a simple efficient markets model, where the detrended real equity price P_t at period t is given by

$$P_t = \sum_{k=0}^{\infty} \delta^{k+1} \mathrm{E}(D_{t+k|t}) \quad , \tag{34}$$

where D_t is the detrended real dividend paid at t and δ is the constant detrended real discount factor. This model can also be written in terms of the ex post rational price, or 'perfect foresight', series P_t^*, i.e. P_t^* is the present value of actual subsequent dividends:

$$P_t = \mathrm{E}(P_t^*) \quad , \tag{35}$$

where

$$P_t^* = \sum_{k=0}^{\infty} \delta^{k+1} D_{t+k} \quad .$$

P_t^* can be approximated by working backwards from a terminal date P_T^* using the recursion

$$P_t^* = \delta(P_{t+1}^* + D_t) \quad . \tag{36}$$

From (35) we thus have

$$P_t^* = E(P_t^*) + e_t = P_t + e_t \quad , \tag{37}$$

where $e_t = P_t^* - P_t$ is a 'forecast error' uncorrelated with the 'forecast' P_t of P_t^*: it therefore follows that, since $var(P_t^*) = var(P_t) + var(e_t)$, the inequality

$$var(P_t^*) \geq var(P_t) \tag{38}$$

must hold or else P_t is 'too volatile' to be explained by the simple efficient markets model.

To investigate the volatility of equity prices in this period, data on detrended real equity prices and dividends are required. The nominal share price index is taken from Mitchell (1988, pp. 687-8) and is deflated by Feinstein's (1972) GDP deflator. The series is then detrended by removing an exponential trend with growth rate of 0.8 per cent per annum, the average rate over the sample period. Real dividends are proxied by Mitchell's (1988, pp. 828-9) profits series, deflated by the GDP deflator and detrended by removing an exponential trend with growth rate 1.8 per cent per annum. The detrended real discount factor δ is given by the ratio of one plus the dividend growth rate to one plus the real interest rate, i.e. δ=0.99. Using the terminal condition $P_{1914}^* = P_{1914}$, the recursion (36) was then computed, yielding the series shown in Figure VI. It is quite clear that P_t^* is considerably smoother than P_t: indeed, the standard deviation of P_t, at 15, is almost twice as large as that of P_t^*, calculated to be 7.6. Given P_t^*, a 'perfect foresight' q can be computed: this series, q_t^*, plotted along with q_t itself, is shown in Figure VII.

The perfect foresight series is seen to be much smoother than q, the large increase in the value of this latter series around the turn of the century being entirely absent in q_t^*. (A similar exercise was carried out using a proxy for real dividends given by capital's share of output from the Cobb-Douglas production function of section 3, i.e. $Y_t^{1-\alpha}$, where $\alpha=0.52$, detrended and scaled to the equity index. Even greater evidence of excess volatility was found, the ratio of standard deviations in this case being approximately five).

Figures VIII and IX show q and q^* plotted against investment. It is clear that the correspondence between investment and q is much closer than that with q^*. We thus conclude that a prima facie case exists for the argument that the source of the real business cycle approaches's failure to model investment successfully may well lie in a tendency to waves of undue optimism and pessimism among investment decision makers. Arguably, expectations may have been formed in a more Keynesian mode in a world of relatively poor information and a quite imperfect capital market (Kennedy, 1987). Certainly Ford would not be surprised by this, as he pointed to the importance of 'animal spirits', 'exaggerated gloom and revulsion' and 'bloated expectations of gain' during this period (1981, p. 37).

(c) <u>The Absence of Procyclical Real Wages</u>

The real business cycle model proponents in the American literature have wanted to argue that technology shocks are quantitatively more important than monetary disturbances as factors initiating business cycle fluctuations, or even that monetary disturbances are unimportant in that context (McCallum, 1989, p. 17). In the market clearing tradition, the implication of the recessionary tendency for individuals to increase the amount of leisure they demand simultaneously with reducing their demand for goods must be that the real wage (the price of leisure) has fallen as a result of an adverse technology shock. By contrast, the Phillips Curve or "Lucas surprise supply function" approaches to short term macroeconomic fluctuations see output rising as real wages fall through wage-bargainers' failure fully to anticipate changes in monetary policy and inflation. As Mankiw (1989) points out, in a pure real business cycle framework there would be no room for a Phillips Curve.

Hatton (1991) is the latest in a long line of investigators to confirm the existence of a Phillips Curve in pre-war 1914 Britain. Moreover, he finds that wages were relatively insensitive to price changes, as if there were money illusion in the economy. In this context we would expect that any demand shocks or, indeed, monetary disturbances, would tend to generate a tendency for countercyclical movement in real wages. In fact, we observe virtually no correlation between output and real wages, a result also found by some researchers for the recent United States experience (McCallum, 1989, p.23). This suggests the possibility that <u>both</u> real (technology) shocks <u>and</u> demand shocks were present, with neither strongly dominant over the other.

Thus we regard the notion that all short run movements in output and employment result from real shocks as implausible, but do not believe that this need imply rejection of the view that fluctuations were substantially affected by the long run growth and development process. In this we arrive at a position which, in important respects, is probably similar to that of Phelps-Brown. He stressed the strong impact on the British economy of a drying-up of technological and investment opportunities at the end of the nineteenth century (Phelps-Brown & Handfield Jones, 1952), while also emphasising the presence of a Phillips Curve in a world where nominal wages were insensitive to prices (Phelps-Brown & Browne, 1968). We would, however, not accept the notion of a climacteric in trend growth which looms large in Phelps-Brown's account of this period, as we have shown elsewhere that the time series evidence appears to reject this hypothesis (Crafts, Leybourne & Mills, 1989).

5. Summary and Conclusions

1) We have shown that a neoclassical model of growth disturbed by real shocks has some success in accounting for fluctuations in the Victorian and Edwardian economies. Such a model seems to require both serial correlation of shocks and depreciation over realistic time periods to replicate key features of the historical experience.

2) There seems to be strong evidence that shocks to output were not persistent and that growth tended to revert to a constant trend rate of around 2% per annum. This matches the early vintage (Solow-type) neoclassical growth model, as used in McCloskey (1970), rather than new vintage examples like Romer (1986), which do not have diminishing returns to capital accumulation. It would

follow that the very slow growth of 1899-1907 or the very strong growth prior to 1873 should be seen as 'blips'.

3) The major failings of this real busines cycle model are its inability to replicate fluctuations in investment and the absence of pro-cyclical real wages in late Victorian Britain. These are serious difficulties and seem likely to result from features of the economy stressed by writers in a more Keynesian tradition, namely, 'animal spirits' and a short-run Phillips Curve.

4) In general, the results of this paper are consistent with Alec Ford's own views of the trade cycle in the late nineteenth century. Real shocks have an important part to play, but so also do 'Keynesian factors' that are assumed away in a perfect-foresight, choice theoretic framework.

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LABOUR INPUT (WORK EFFORT)

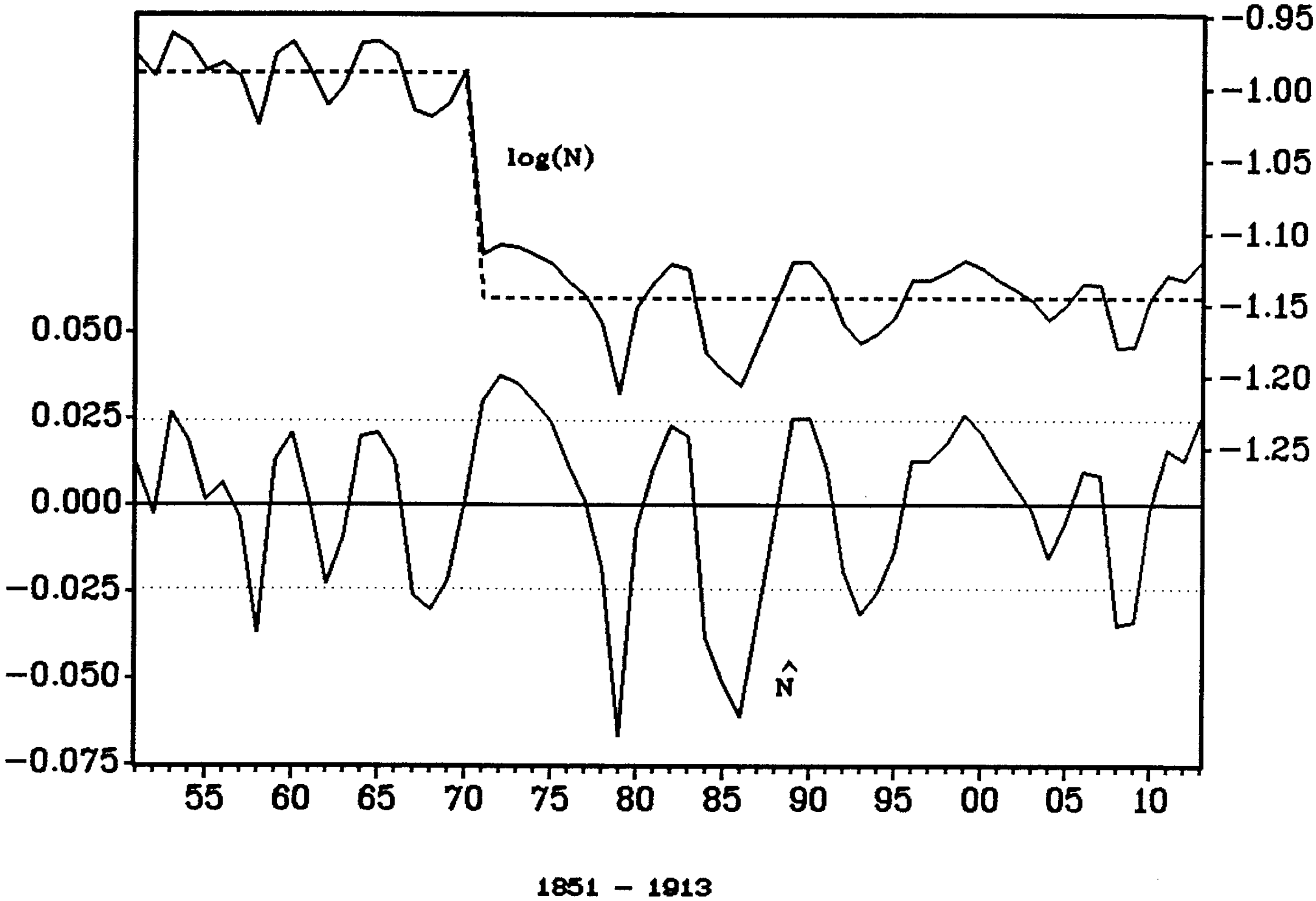
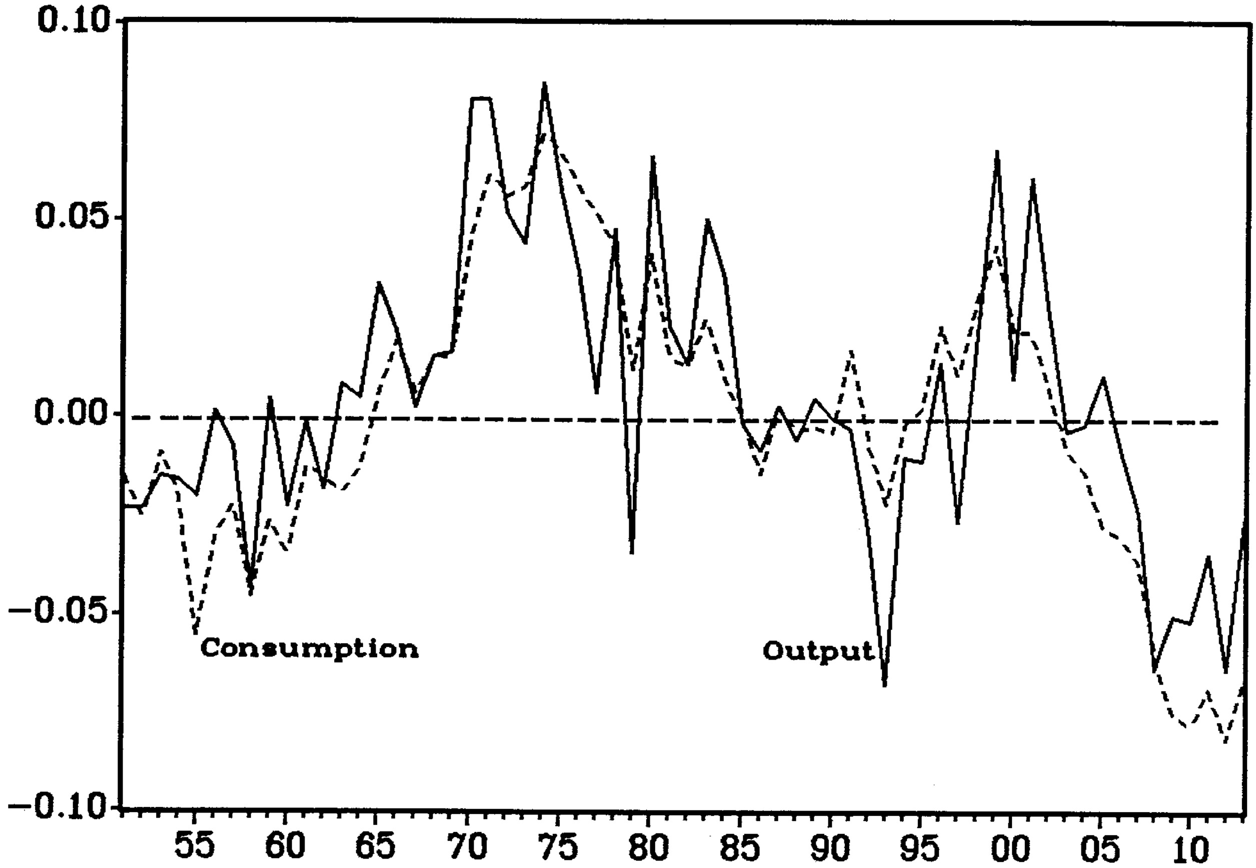


Figure I

ESTIMATED DEVIATIONS FROM COMMON TREND

Output and Consumption

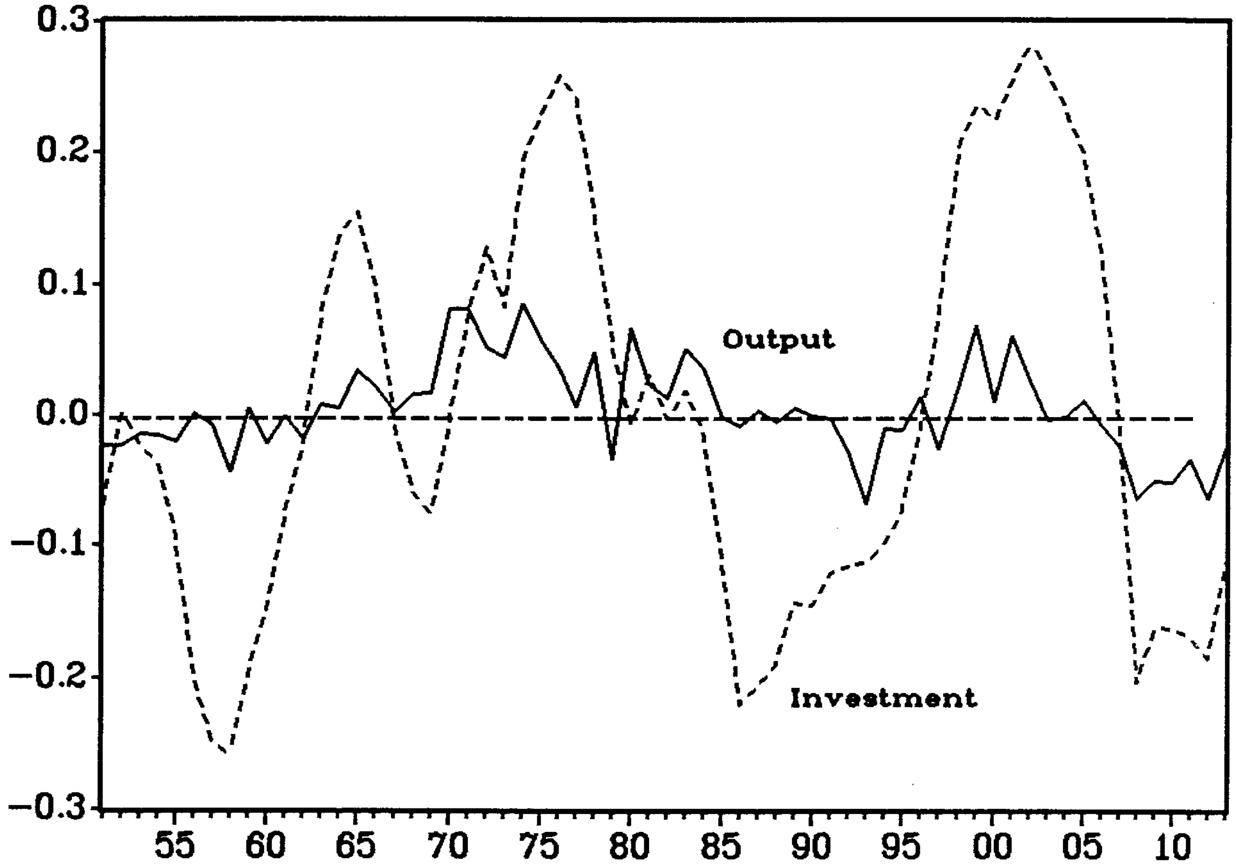


1851 – 1913

Figure II

ESTIMATED DEVIATIONS FROM COMMON TREND

Output and Investment



1851 – 1913

Figure III

ESTIMATED DEVIATIONS FROM TREND

Output and Real Wages

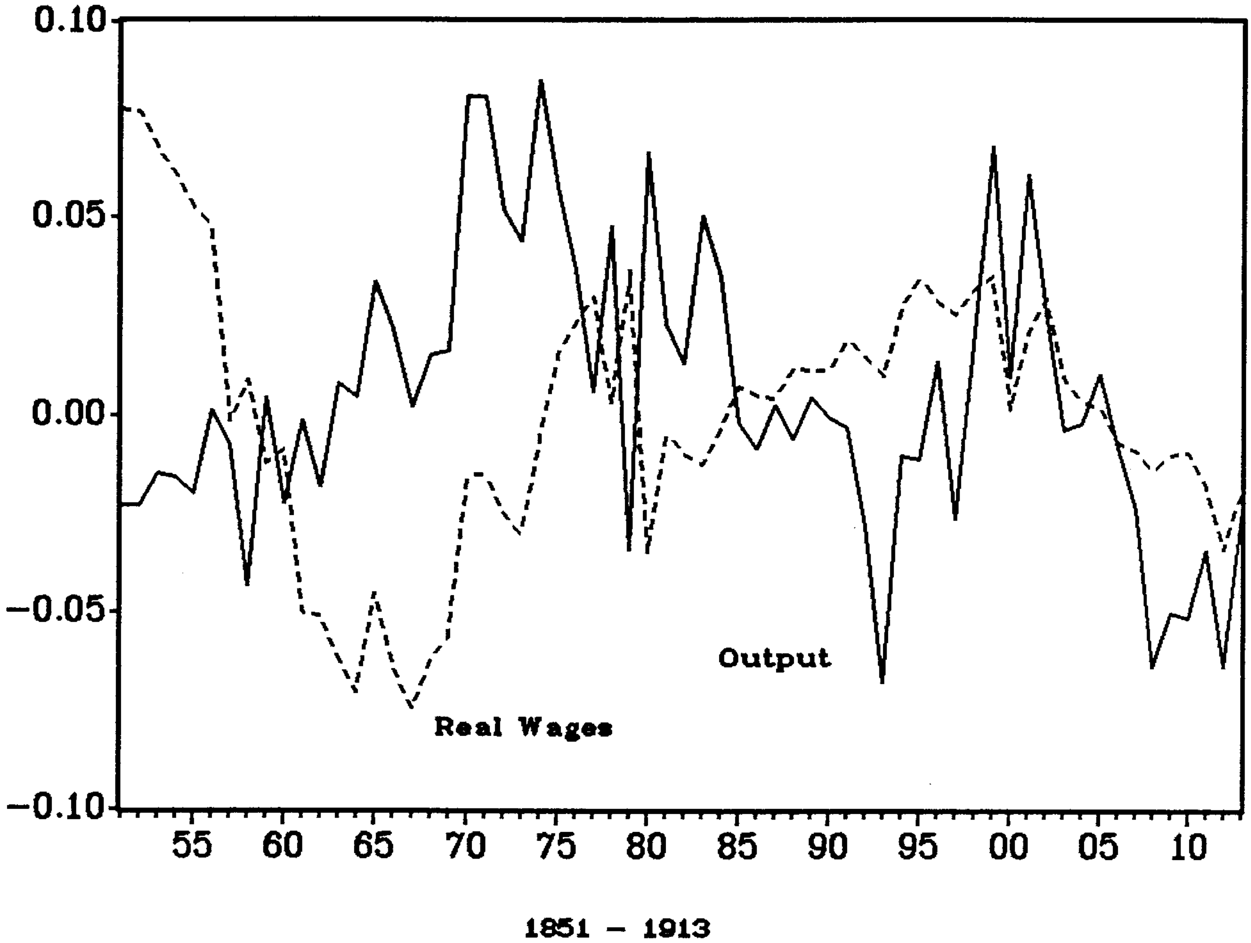
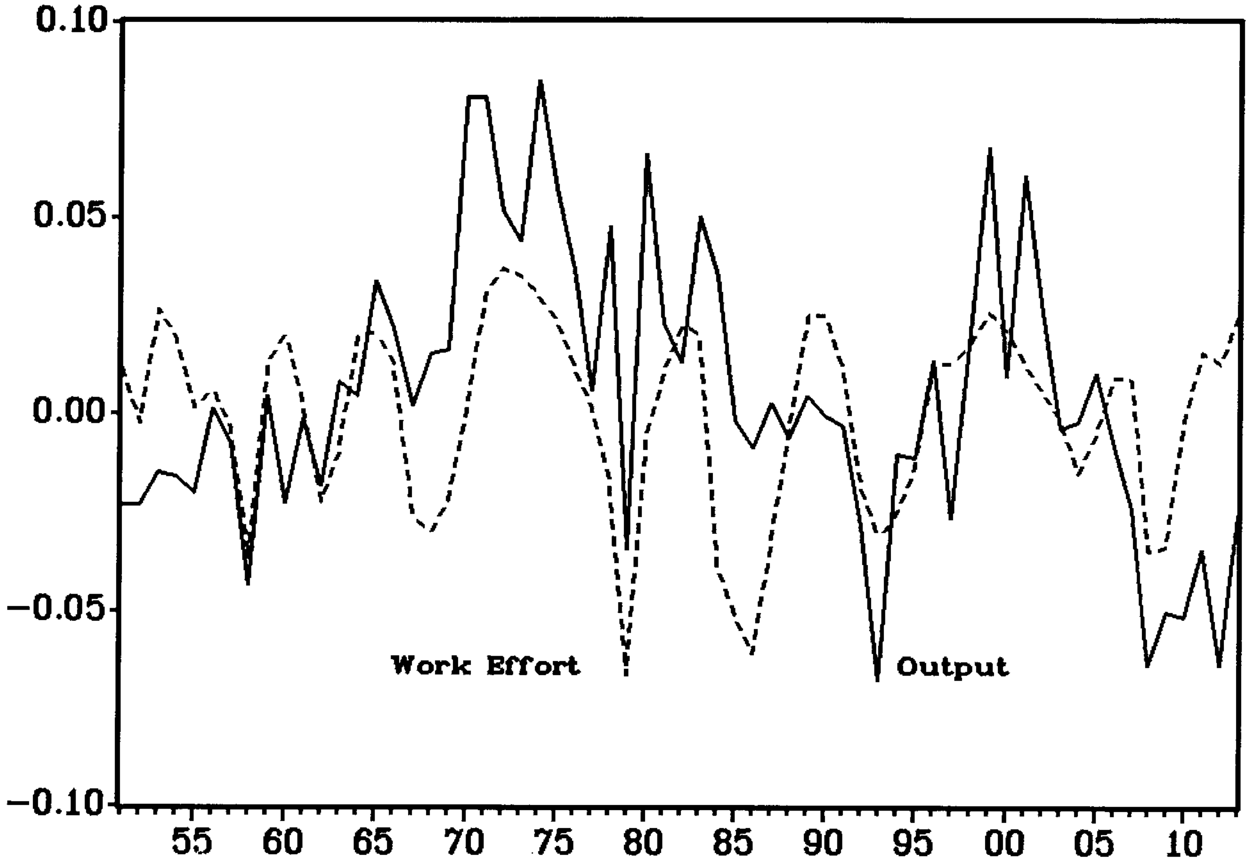


Figure IV

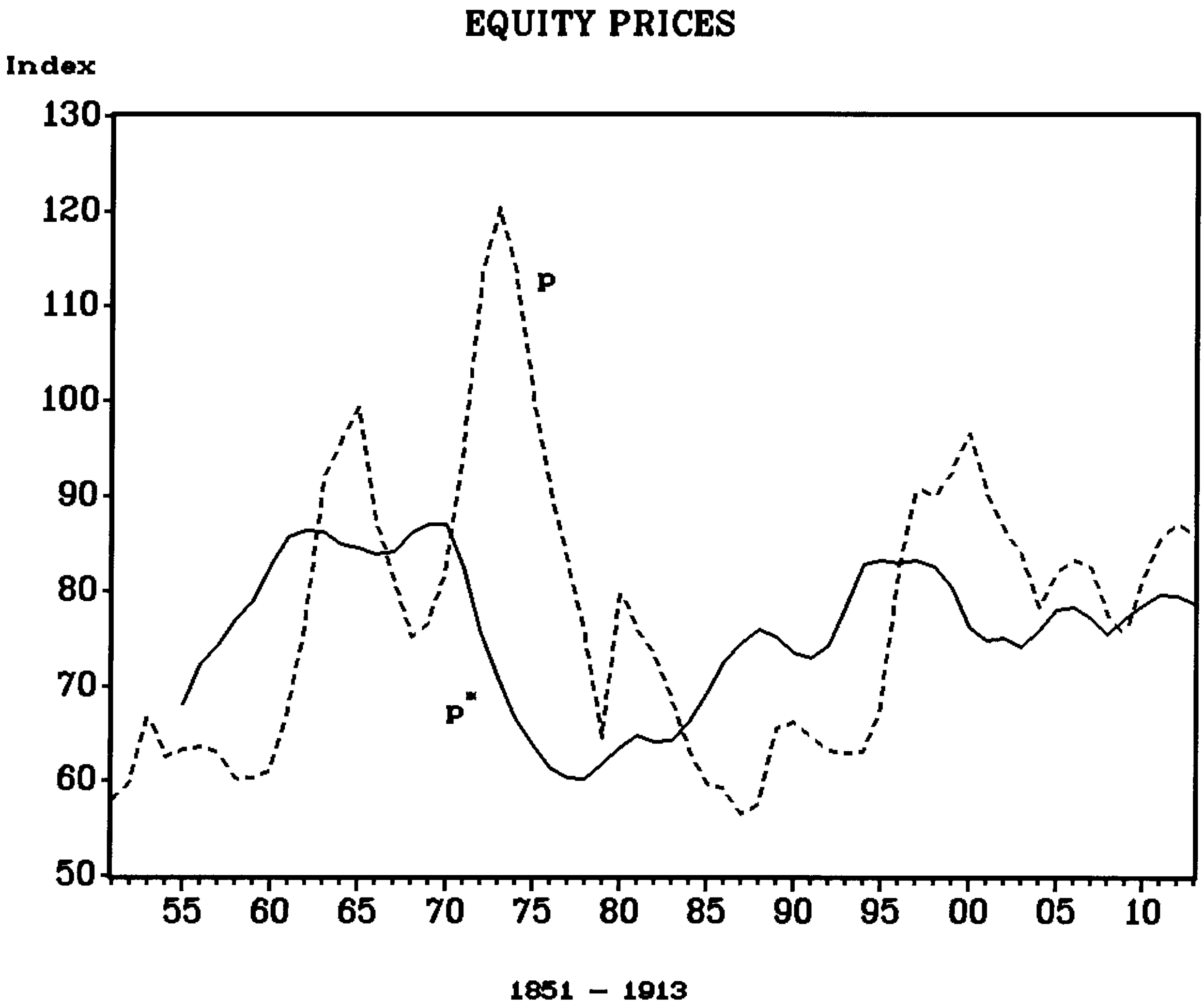
ESTIMATED DEVIATIONS FROM TREND

Output and Work Effort



1851 – 1913

Figure V


EQUITY PRICES
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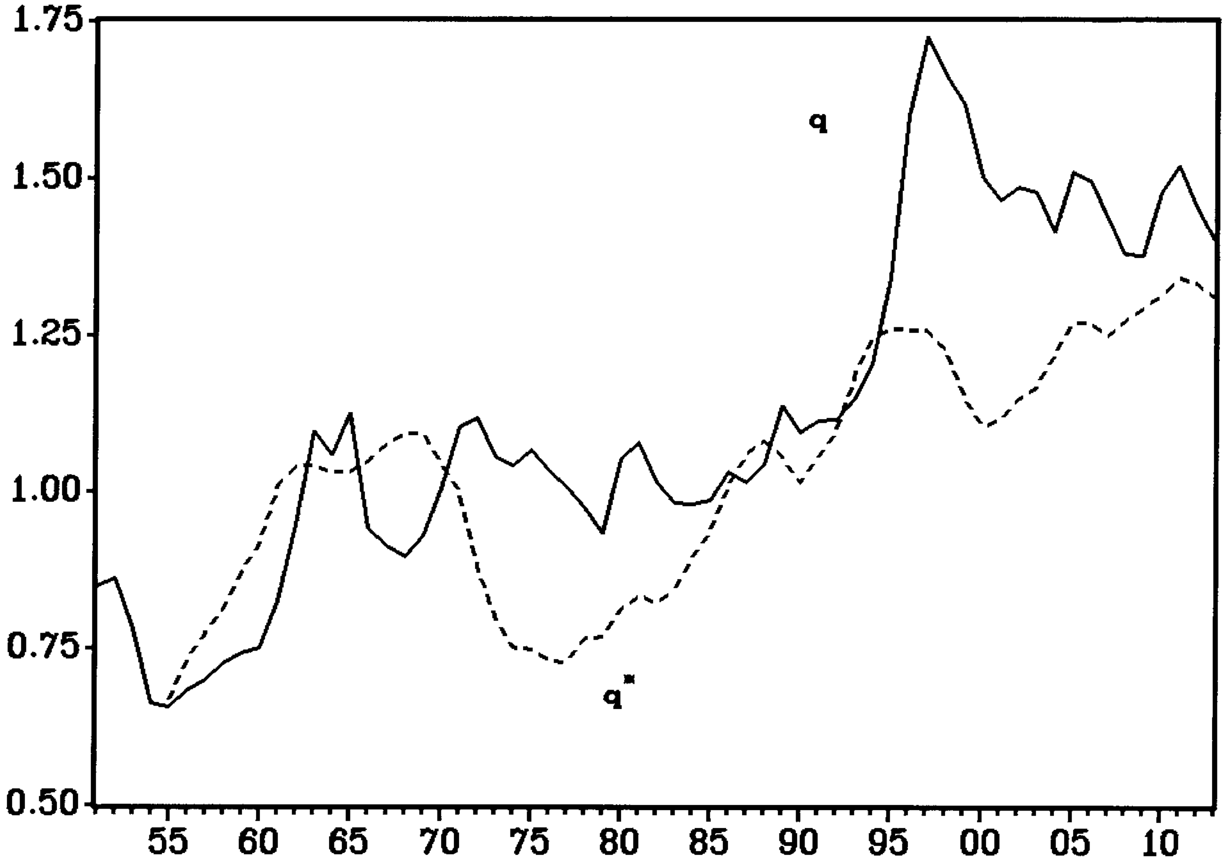
1851 – 1913

Figure VI

TOBIN'S 'Q'



1851 – 1913

Figure VII

INVESTMENT AND Q

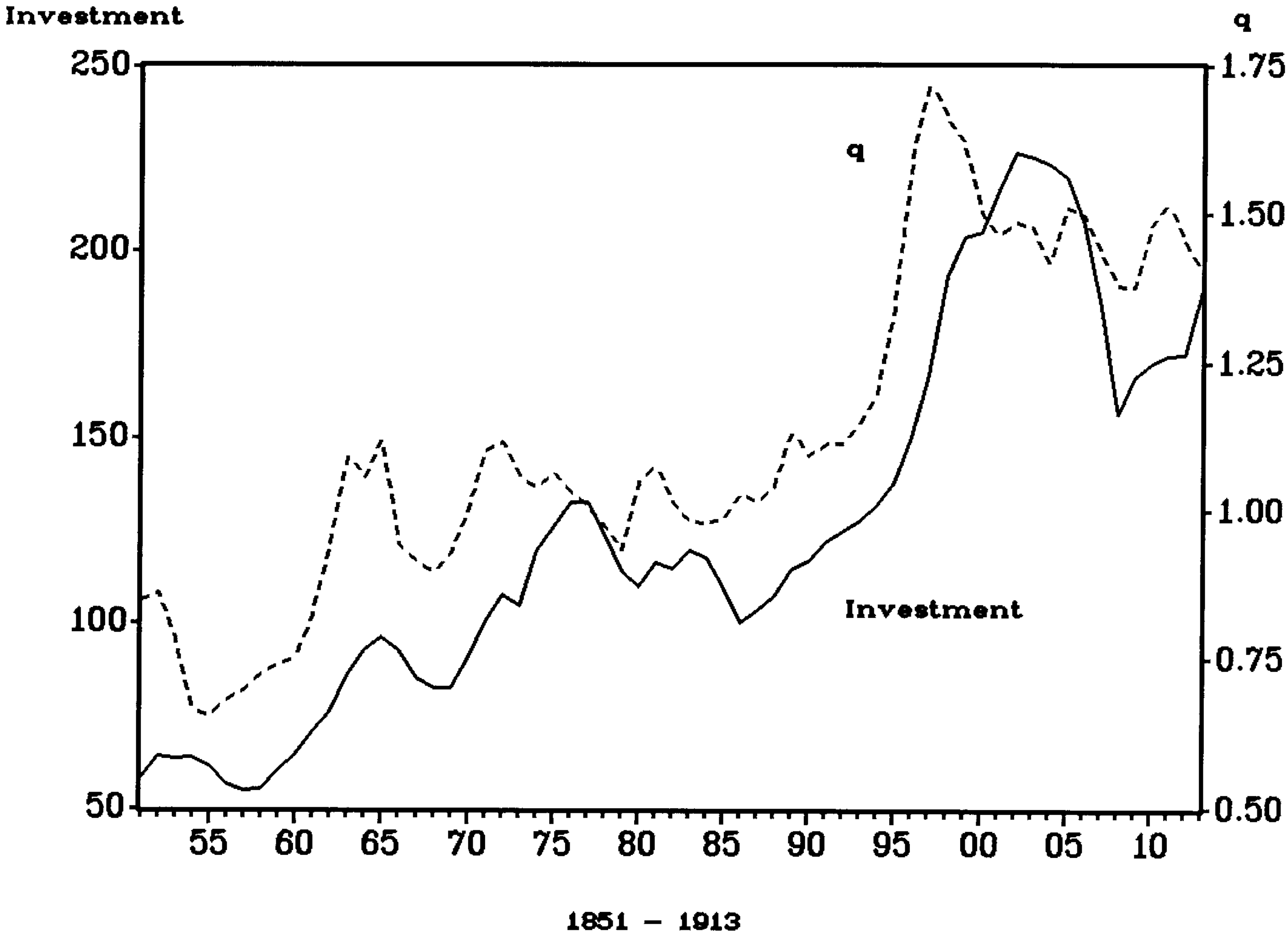
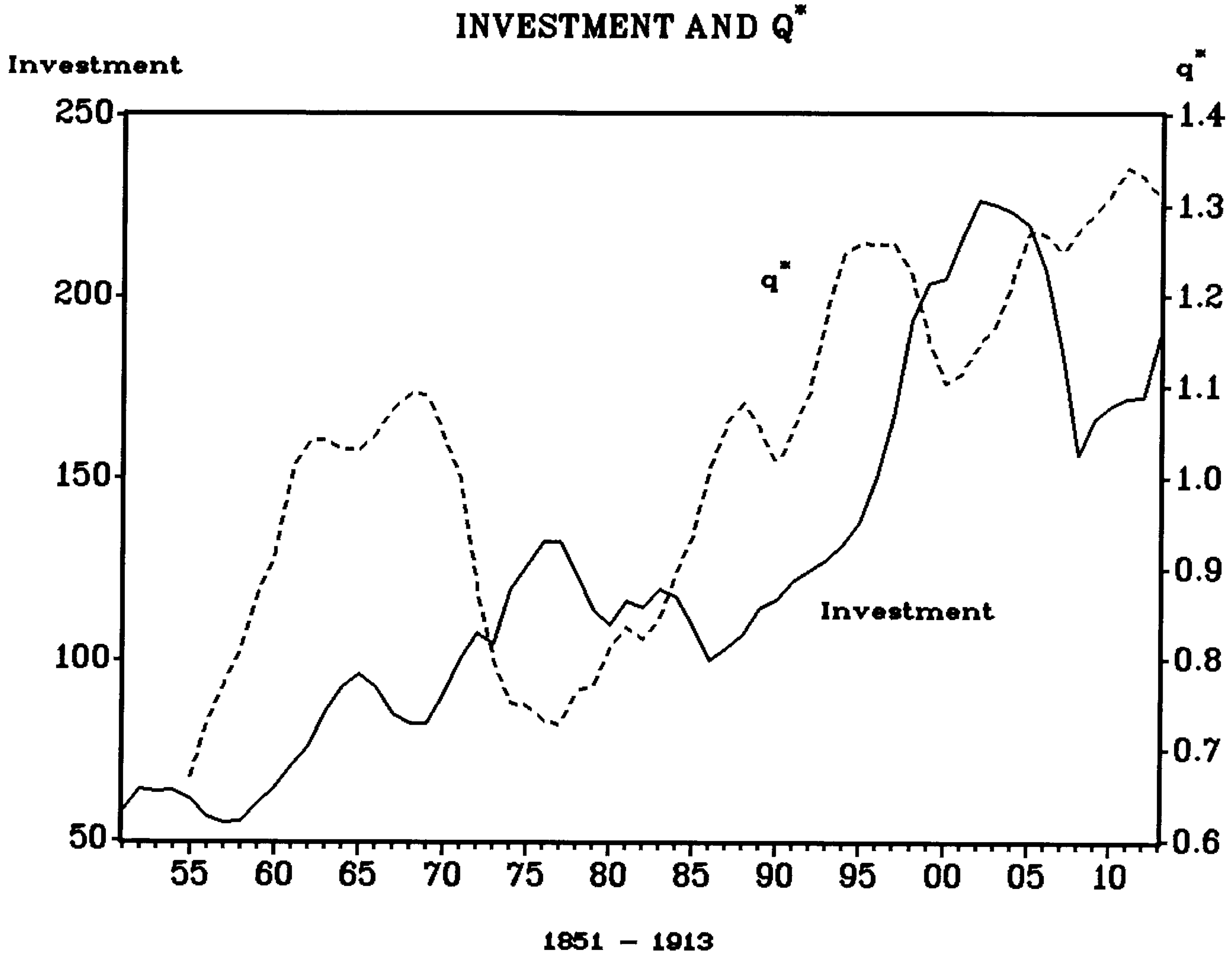


Figure VIII



INVESTMENT AND Q*
Investment
q*
250
200
150
100
50
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
q*
Investment
55
60
65
70
75
80
85
90
95
00
05
10
1851 – 1913

Figure IX