Do Unions Reduce Discrimination?

A Model of Nash Bargaining Between a Union and an Employer with Discriminatory Tastes.

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Abstract: Whilst there is a significant empirical literature on the effects of unions on pay discrimination, there is little by way of a rigorous theoretical treatment of this important topic. This is particularly surprising given the many recent developments in the economic theory of the trade union. This paper offers a theoretical framework which integrates models of union-firm bargaining with the analysis of employer discrimination. Within the class of right-to-manage models of union-firm bargaining, we consider the bargain between a rent-maximising union and a utility-maximising employer with discriminatory tastes. Our main conclusion is that only weak conditions have to be satisfied for the presence of a union with bargaining power over the wage rates paid by a discriminating firm to reduce the wage gap between the different worker groups and, in the right-to-manage model, for the wage gap to fall monotonically as union bargaining power increases. Amongst other results, we also find that as employer discrimination increases, the monopoly union bargains a higher wage for the group against which the firm is discriminating.

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I. Introduction

There is a sizeable empirical literature on the effects of trade unions on the differences in pay which arise out of labour market discrimination. Sloane (1985) offers a summary of results obtained from studies carried out in Britain, Canada and the USA. His conclusion is that on balance unions appear to, "...change the relative wage differential in favour of black workers and against women in the USA (though in favour of women in Canada and Britain)." He adds that the overall impact is, however, relatively small. Freeman and Medoff (1984), using 1979 Current Population Survey data for the USA, find that the union wage effect is larger for nonwhites than for whites. Noting the variation in this effect across different studies, however, they prefer the conclusion that, "... unions raise the wages of blacks by about as much as they raise the wages of whites." In the UK, Main and Reilly (1990) have summarised results on the union wage effect by gender. They find that the union wage mark-up is greater for women than for men. Lewis (1986) investigates the variation in the union wage gap across the US workforce. He finds that estimates of the male-minus-female wage gap difference are numerically small and of ambiguous sign. The estimated wage gap difference is the partial derivative of the wage gap with respect to a dummy variable for the relevant characteristic.With respect to differences by colour, Lewis concludes that there is some strong evidence to suggest a positive nonwhite-minus-white wage gap difference, implying that the union mark-up for non-whites exceeds that for whites.

Taken together, the main thrust of these results suggests that the presence of unions in the economy causes a narrowing of wage differentials between white and nonwhite workers, at least for the USA. This conclusion, of course, puts to one side the counterfactual problem of the impossibility of observing the wage pattern in the absence of unions. The focus is on wage gaps rather than on wage gains. For the UK, at least, unions seem to reduce pay differentials by gender - a result consistent with the more general finding that unions tend to narrow the overall dispersion of pay.

A number of issues are concealed, however, in looking at estimates obtained, as is typically the case, from macro or individual worker level studies. The results are consistent with unions having conflicting effects on overall wage gap differences. It might be, for example, that union behaviour causes a narrowing of pay differences by gender or colour within establishments, but causes a widening across establishments. This could be the case if union power in all-white establishments was higher than that in all-nonwhite establishments, which might follow from a variety of reasons including, for example, higher disagreement payoffs to white workers who, on average, have greater wealth with which to sustain a work stoppage. If such effects occur then we might observe little aggregate effect of unions on pay differences by colour even if within establishments unions cause reductions in such differences. The focus of the current paper is on this latter aspect, as we are looking at bargaining between a single union and a single firm. Therefore, a proper test of our predictions would require disaggregated firm level data rather than a mere appeal to union effects in the aggregate.

Against this backdrop of an extensive empirical literature there is a contrasting paucity of theoretical work focussing on the impact of unions on pay differences by discriminated group. As Lundahl and Wadensjo (1984) have observed, discrimination theories typically view wages as set by the market rather than through bargaining. In non-rigorous models unions can play either of two polar roles. Either they are bastions of white male workers controlling hiring decisions and thereby reducing employment opportunities to workers with unlike characteristics, or else they are progressive institutions motivated by the desire to equalise wages across workers with different characteristics. These descriptions are never incorporated into formal models and accordingly have never given rise to testable predictions.

The main aim of this paper is to develop a formal framework within which there is an integration of the theoretical work on employer discrimination with that on union-firm bargaining. Work in the latter area has progressed rapidly over the last decade (see Ulph and Ulph (1989) and Oswald (1985) for surveys) whilst there has been relatively little development of theoretical models of discrimination since Becker (1957) and Arrow (1973). The latters' utility approach to employer discrimination continues to represent the standard treatment within mainstream economic analysis. The current paper focuses on a deficiency within the utility approach: that of the absence of any union with which the firm must bargain before wage and employment levels are determined. In this paper we attempt to correct for this omission by adapting the conventional union-firm bargaining model to accommodate the case in which the firm has Becker-like tastes for discrimination. The union, on the other hand, is assumed to exhibit non-discriminatory preferences. Thus, if we find that the presence of a union affects the relative wages paid by a discriminatory employer, we can conclude that this is not the artefact of arbitrary assumptions about union attitudes to discrimination. Unions are also likely to have effects on various other dimensions of discrimination such as occupational and statistical discrimination, but our focus is solely on pay discriminaation.

We consider both right-to-manage and monopoly union models of union-firm bargaining. Our main conclusion is that only weak conditions have to be satisfied for the presence of a union with bargaining power over the wage rates paid by a discriminating firm to reduce the wage gap between the different worker groups and, in the right-to-manage model, for increases in union bargaining power to reduce the wage gap monotonically. Amongst other results, we also find that as employer discrimination increases, the monopoly union bargains a higher wage for the group against which the firm is discriminating.

The next section begins the description of the formal model, considers the case of the unconstrained firm and derives facts required for later use. Section III presents the monopoly union outcome, offering both a specific example and a treatment of the general case. Section IV then analyses a more general Nash bargaining solution to the right-to-manage model and presents the main results of the paper, deriving both analytical and numerical results. Section V closes the paper with conclusions and suggestions for further work.

II. The Unconstrained Firm

The firm: objectives and unconstrained behaviour

We adopt the standard Becker utility-based approach to employer discrimination in which the firm is assumed to possess discriminatory tastes towards one of the two groups of workers in its (potential) employment. The two groups of workers are labelled A and B. The firm is characterised as maximising a strictly concave utility function, U, such that

$$U = U(\Pi, B), \tag{1}$$

where Π represents profits, B the number of group B workers employed and $U_\Pi > 0$, $U_B < 0$. The negativity of U_B embodies the notion of discrimination against the group B workers. Utility is dependent on the number of group A workers only through their effect on profits.

It is assumed that members of the two groups are perfect substitutes in production so that revenue is a function, R(L), of employment of the two groups: L = A + B. Profits can be written as

$$\Pi = R(A + B) - w_A A - w_B B, \tag{2}$$

where, initially, w_A and w_B - the respective wage rates of A and B - are exogenously given to the firm. The present hypothesis being that either a competitive market or a bargaining process have determined w_A and w_B, which the firm treats as parametric. The firm then selects A and B to maximise utility. We shall assume initially that $w_A > w_B$, capturing discrimination in the wider labour market. Substituting the expression for profits into the utility function yields

$$U = U[R(A + B) - w_A A - w_B B\ , B]. \tag{3}$$

Maximising (3) generates the following first-order conditions that describe the firm's behaviour

$$\frac{dU}{dA} = [R'(A + B) - w_A]U_\Pi + \mu_1 = 0, \tag{4}$$

$$\frac{dU}{dB} = [R'(A + B) - w_B]U_\Pi + U_B + \mu_2 = 0, \tag{5}$$

and

$$\mu_1 A = 0, \mu_2 B = 0, \mu_1 \geq 0, \mu_2 \geq 0. \tag{6}$$

Hence

$$w_A = R'(A + B) + \frac{\mu_1}{U_\Pi}, \tag{7}$$

and

$$w_B - U_B/U_\Pi = R'(A + B) + \frac{\mu_2}{U_\Pi}. \tag{8}$$

In the case that both A and B are positive, so that $\mu_1 = \mu_2 = 0$, it follows that

$$w_B + d_B = R'(A + B) = w_A, \tag{9}$$

where $d_B = (- U_B/U_\Pi)$ is the marginal rate of substitution between Π and B for the firm. From the structure of utility it follows that $d_B > 0$, and hence that $w_B + d_B > w_B$. Essentially, the discriminating employer treats the marginal cost of employing workers from group A as equal to their market wage, w_A, whilst regarding the marginal cost of employing an additional group B worker as the market wage, w_B, *plus* the discrimination coefficient, d_B, i.e.,

$$MCA = w_A,$$
$$MCB = w_B + d_B.$$

Optimal employment levels of the firm are then chosen to equate w_A to $w_B + d_B$.

In the typical models in the literature, for instance Becker (1957), d_B is taken as a constant for each firm, implying a segregation between group A and group B workers across firms. This follows because if d_B is such that $d_B + w_B < w_A$, then $\mu_1 > 0$ and only group B workers will be employed. If the inequality sign is reversed, $\mu_2 > 0$ and only group A workers will be employed. In this situation a (random) mix of workers can occur only when there is strict equality between the two subjective prices of the two worker groups. Because of this segregation property, the Becker model is often seen as more relevant to the study of white-nonwhite discrimination than of male-female discrimination, the latter being seen as based more on occupation than on establishment segregation.

In contrast, from our assumptions, d_B is increasing in B[1]. To illustrate the consequences of this, let w_A and w_B be fixed exogenously at their competitive levels which we denote w_A^0 and w_B^0. The model can then be represented diagrammatically as in Figure 1 below where we impose the assumption that $w_A^0 > w_B^0$.

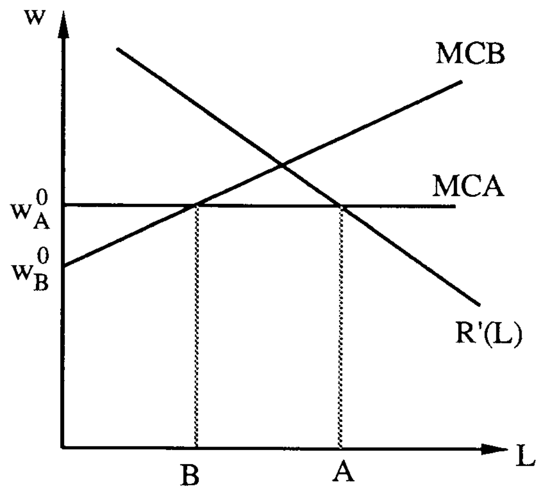

Figure 1

Knowledge of R'(L), w_A^0, w_B^0 and d_B is enough to determine A, B and, therefore, L. We are not aware of this having been shown previously in the literature. It is implicit in Figure 1 that the values of R'(L), w_A^0, w_B^0 and the behaviour of d_B generate non-zero values of each of A and B. If, conversely, either w_B^0 were lower relative to w_A^0 or the MCB increased more slowly in B, then we could observe A = 0. Total employment would then be determined by the equality of R'(L) and MCB. Otherwise, total employment is determined by the equality of R'(L) and w_A^0, as in Figure 1. Within such

1. $\frac{\partial d_B}{\partial B} = \frac{\Pi_B[U_B U_{\Pi\Pi} - U_{B\Pi} U_\Pi]}{U_\Pi^2} + \frac{[U_B U_{B\Pi} - U_{BB} U_\Pi]}{U_\Pi^2}$. The assumed strict concavity of the utility function implies strict quasiconcavity, for which the necessary condition is

$$U_B[U_{B\Pi} U_\Pi - U_B U_{\Pi\Pi}] + U_\Pi[U_B U_{B\Pi} - U_{BB} U_\Pi] > 0. \qquad \text{(i)}$$

When $\mu_2 = 0$, (5) implies

$$U_\Pi \Pi_B + U_B = 0. \qquad \text{(ii)}$$

From (i) and (ii), $\frac{\partial d_B}{\partial B} > 0$.

a regime as that depicted in the figure we obtain some strong results:

(i) Changes in w_B^0 affect the distribution of employment between the two groups, but do not affect total employment, L, in the firm.

(ii) For $w_A^0 > w_B^0$, B > 0. I.e., some B workers are employed - so long as MCB is not vertical.

(iii) Total employment, L, is unaffected by differences in the discrimination coefficient, d_B, which keep the solution within the regime. This result is similar to (i), above.

(iv) An increase in labour demand will be absorbed entirely by an increase in employment of group A workers. Similarly, a reduction in demand, if sufficiently small, will have no effect on the employment of B workers.

Further results, and facts needed for later use, can now be derived by analysing the maximisation for the firm. To simplify the analysis, the objective of the firm is assumed to be separable in Π and B and we transform the function further by taking the composition of the original utility function and the inverse of the separable profit component. Since U_Π is positive, this is simply a monotonic transformation of our original function, which is now additive in profit, and the transformation does not alter the firm's optimal choices[2]. The objective of the firm is therefore

$$\max_{\{A,\,B\}} U = R(A+B) - w_A A - w_B B - d(B), \tag{10}$$

where d(B) is termed the discrimination function. To ensure that this is a strictly concave problem, the following assumption is maintained throughout:

Assumption 1: $R'(A+B) > 0$, $R''(A+B) < 0$, $d'(B) > 0$, $d''(B) > 0$.

For the issues that will be addressed in this paper we shall be concerned with the regime in which the firm, given the levels of w_A and w_B, chooses to employ workers from both groups; again as is depicted in Figure 1. Solving the maximisation, where it is assumed that an interior solution exists[3], gives

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2. Although it will have the effect of altering the distribution of welfare determined for a given λ by (39).

3. One sufficient assumption would be: $\lim_{B\to 0} d'(B) = 0$, $\lim_{B\to 0} d(B) = 0$, and for $\hat{B}$ such that $w_B + d'(\hat{B}) = R'(\hat{B})$, $w_B + d'(\hat{B}) > w_A$.

$$R'(A+B) = w_A, \tag{11}$$
$$R'(A+B) = w_B + d'(B), \tag{12}$$

which can be consistent only when $w_A > w_B$.

It is now possible to derive the expressions that will be required below. From (11) and (12) above,

$$\begin{bmatrix} R'' & R'' \\ R'' & R''-d'' \end{bmatrix} \begin{bmatrix} dA \\ dB \end{bmatrix} = \begin{bmatrix} dw_A \\ dw_B \end{bmatrix}. \tag{13}$$

Solving this equation

$$\frac{dA}{dw_A} = \frac{1}{R''} - \frac{1}{d''} < 0, \frac{dA}{dw_B} = \frac{1}{d''} > 0, \frac{dB}{dw_A} = \frac{1}{d''} > 0, \frac{dB}{dw_B} = -\frac{1}{d''} < 0. \tag{14}$$

From (14) it is clear why the change in w_B does not affect total employment but affects the distribution between the two groups. An increase in the wage rate of group A workers reduces total employment

$$\frac{dL}{dw_A} = \frac{1}{R''} < 0. \tag{15}$$

Now denoting the firm's maximum value function by $U(w_A, w_B)$, a further assumption is added:

Assumption 2: $U(w_A^0, w_B^0) > 0$.

This assumption simply gives the problem that we study some content. It ensures that the firm can make a positive profit at the competitive wage rates so that when bargaining with the union takes place, there is some surplus to be shared.

Employing the envelope theorem shows

$$\frac{\partial U}{\partial w_A} \equiv U_a = -A, \frac{\partial U}{\partial w_B} \equiv U_b = -B. \tag{16}$$

Using the previous relations, the second derivatives of maximum value are

$$U_{aa} = -\frac{\partial A}{\partial w_A} = \frac{1}{d''} - \frac{1}{R''}, U_{ab} = -\frac{\partial A}{\partial w_B} = -\frac{1}{d''}, U_{bb} = -\frac{\partial B}{\partial w_B} = \frac{1}{d''}. \tag{17}$$

From (17), the maximum value function is strictly convex, a factor which greatly complicates the analysis below.

To complete this section, we calculate the effect of an increase in discrimination by the firm. This can be modelled by letting d(B) increase to $(1 + \varepsilon)d(B)$, with $\varepsilon \geq 0$, and calculating the derivatives with respect to ε. Solving the resulting system[4] and evaluating at $\varepsilon = 0$, gives

$$\frac{dA}{d\varepsilon} = \frac{d'}{d''} > 0, \frac{dB}{d\varepsilon} = \frac{d'}{d''} < 0. \tag{18}$$

The increase in discrimination therefore reduces the number of group B workers and increases those from group A. As suggested after figure 1, total employment remains unchanged.

This completes the analysis of the firm for the present. We shall return to use a number of the results derived above when Nash bargaining is considered.

III. The Objectives of the Union and the Monopoly Union Model

We shall assume that a single union represents both groups of workers. To justify this we appeal to work by Horn and Wolinsky (1988) who develop a bargaining model for the case in which two groups of workers face a single employer. They derive a general principle which states that when the two types of labour are substitute factors of production, then it is in their interests to coordinate their bargaining with the employer by forming a single encompassing union. This result is relevant to our model as we are assuming that workers from each group are perfect substitutes. Were they instead complements then it would be more reasonable to assume that they would bargain separately, according to the Horn and Wolinsky result.

The bargaining structure is characterised by the right-to-manage class of models (see Nickell and Andrews (1983)) in which the union and the firm bargain over wages, leaving the firm free to choose employment levels so as to maximise firm-utility, given the bargained level of wages. There are two alternative traditions to guide us in the choice of the specification of union preferences. One approach is to assume a Utilitarian union maximising the aggregate utility of all its members, employed and unemployed. An alternative approach is to specify a modified Stone-Geary utility function of the form

$$V = [w - w^*]^a[L - L^*]^b. \tag{19}$$

This second approach is less amenable to a microeconomic interpretation than the

4. Replace the right-hand side of (13) by $\begin{bmatrix} 0 \\ d'd\varepsilon \end{bmatrix}$.

former, but has the advantage that it nests a wide range of different possible union objective functions (e.g., if a = 1, b = 0 and w* = 0, then the objective is wage-maximisation. If a = b = 1 and w* = L* = 0, this describes wage-bill maximisation.). We shall follow this approach and assume that the objective of the union is rent-maximisation[5]. Over two groups of workers this becomes:

$$V = [w_A - w_A^0]A + [w_B - w_B^0]B. \tag{20}$$

In this specification the union places an equal weight on the rents accruing from each of the two groups. It is therefore indifferent about the source of the rent and hence is described as non-discriminating. Our main concern is to discover what happens to the wage gap relative to the competitive gap, $w_A^0 - w_B^0$, when there is bargaining between the discriminating employer and the non-discriminating union. Additionally, we are interested in the impact of the union on relative employment and on how the impact of the union varies, if at all, with parameters such as the level of labour demand and the employer's taste for discrimination.

The monopoly union model is a special case within the right-to-manage framework, in which the union has total control over setting the wage levels but is subject to the labour demand curve as the firm has sovereignty over employment. In our discrimination context, the monopoly union is assumed to choose w_A and w_B in the knowledge that the firm will then set the levels of A and B to maximise utility.

The monopoly union model - an example

Before providing a general analysis, we first consider an illustrative example which we will develop further below. In the example we assume a constant-elasticity revenue function, $R(L) = L^\beta$, and a utility function for the firm given by $U = \Pi - B^\alpha$, so $U = R(L) - w_A A - w_B B - B^\alpha$, where $\alpha > 1$ is a corollary of our earlier assumptions. From this utility function it follows that $MCA = w_A$ and $MCB = w_B + \alpha B^{\alpha-1}$.

Within the region in which the firm employs workers from both groups, we know that:

(i) L is chosen to satisfy $w_A = R'(L) = \beta L^{\beta-1}$,

(ii) B is chosen to satisfy $w_A = w_B + \alpha B^{\alpha-1}$,

and

(iii) $A = L - B$.

5. This can also be justified within the utility-maximising approach as the special case of union risk-neutrality.

Substituting these conditions into the union objective function gives us a simple maximisation problem in w_A, w_B :

$$\max_{\{w_A,w_B\}} \left(w_A - w_A^0\right)\left(\left(\frac{w_A}{\beta}\right)^{\frac{1}{\beta-1}} - \left(\frac{w_A - w_B}{\alpha}\right)^{\frac{1}{\alpha-1}}\right) + \left(w_B - w_B^0\right)\left(\left(\frac{w_A - w_B}{\alpha}\right)^{\frac{1}{\alpha-1}}\right). \tag{21}$$

The first-order conditions for a maximum are

$$\alpha^{\frac{-1}{\alpha-1}}\left(w_A - w_B\right)^{\frac{2-\alpha}{\alpha-1}}(\alpha-1)^{-1}\left(\left(w_A^0 - w_B^0\right) - \alpha\left(w_A - w_B\right) + \beta^{-1}\left(\frac{w_A}{\beta}\right)^{\frac{2-\beta}{\beta-1}}\left(w_A\beta - w_A^0\right)\right) = 0, \tag{22}$$

$$\alpha^{\frac{-1}{\alpha-1}}\left(w_A - w_B\right)^{\frac{2-\alpha}{\alpha-1}}(\alpha-1)^{-1}\left(\alpha\left(w_A - w_B\right) - \left(w_A^0 - w_B^0\right)\right) = 0. \tag{23}$$

We can now state a number of properties of this example.

From (23), and the second-order conditions, it follows that

$$w_A - w_B = (w_A^0 - w_B^0)/\alpha. \tag{24}$$

Hence the monopoly union chooses wages in such a way that the wage gap is reduced below the competitive level, $w_A^0 - w_B^0$, by a factor $1/\alpha$. This is a property of the rent-maximising behaviour of the union - not a consequence of any explicitly egalitarian behaviour. However, the union does not reduce the wage gap to zero. If it did set equal wages for the two groups then employment of B would fall to zero, by the complementary slackness conditions in (6), and the union would fail to acquire any rents from B employment. Consider, on the other hand, a union which set wages such that the wage gap was unchanged, and hence that the rent per worker was the same for both groups. Then, in terms of Figure 1, the marginal labour cost curves would shift upwards by equal magnitudes and employment of B would be unaffected. If the union now raised w_B by a small amount total employment would not be affected but the firm would substitute some A workers for an equal number of B workers. The effect of this substitution on union rents would be zero. However, the remaining B workers would be receiving higher wages and so net rents to the union are higher. Thus, the union optimally sets a wage gap less than the initial, unconstrained wage gap. It does not pay the union to continue to raise w_B to equate it to w_A as each successively displaced B worker has a higher rent than each successively appointed A worker. The union balances these two effects when there is still a positive wage gap, but one below the initial non-union level. We note also that the reduction in the wage gap depends on α. The more discriminating is the firm, ceteris paribus, the more the union acts to reduce the wage gap. This effect is independent of β.

Substituting (24) into the first-order condition (22) gives

$$w_A = w_A^0/\beta. \tag{25}$$

Therefore, the wage mark-up for group A workers depends only on β, the elasticity of revenue with respect to labour, and is independent of the firm's taste for discrimination. The wage mark-up is greater the lower is the elasticity.

From these two results it follows that the union's impact on w_B depends on both α and β. More precisely,

$$w_B = w_A^0[1/\beta - 1/\alpha] + w_B^0[1/\alpha] \ . \tag{26}$$

It is clear that the bargained levels of w_A and w_B are unaffected by slope-preserving shifts in the labour demand curve. Thus, for example, an exogenous change in the product price does not affect the wage bargain.

Finally, substituting into (11) and (12) the values of w_A and w_B derived in (25) and (26) above, we can show that compared to the unconstrained firm outcome, the monopoly union lowers employment of B, reduces total employment and lowers the employment share of group A workers. For group B workers, denoting the employment levels at the competitive wages with a superscript 0, we have from (11) and (12) that $w_A^0 - w_B^0 = \alpha B^{0\alpha-1}$ and $w_A - w_B = \alpha B^{\alpha-1}$. From (24), this implies $\alpha B^{\alpha-1} = B^{0\alpha-1}$ so that

$$B < B^0.$$

For total employment, L, we know from (i) above that $w_A^0 = \beta L^{0\beta-1}$ and, from (26) and (i), that $w_A = w_A^0/\beta = \beta L^{\beta-1}$. Together these imply that $L = L^0/\beta^{1/(\beta-1)}$, and hence that

$$L < L^0.$$

For A/B, we can use the relationships derived above to show that $A/B - A^0/B^0 = \left[\alpha^{1/\alpha-1}/\beta^{1/\beta-1} - 1\right]L^0/B^0$. Hence, it follows that, for $\beta^{1/(\beta-1)} > \alpha^{1/(\alpha-1)}$,

$$A/B < A^0/B^0.$$

The monopoly union model - the general case

We now return to the more general specification of the monopoly union model defined by (9) and (20). It follows from earlier results that given the firm's selection of A and B conditional on the wages, it is possible to express the union's welfare level as a function of the wage rates: $V = V(w_A, w_B)$. Having done this the following derivatives are implied by previous calculations

$$\frac{\partial V}{\partial w_A} \equiv V_a = A + (w_A - w_A^0).\frac{\partial A}{\partial w_A} + (w_B - w_B^0).\frac{\partial B}{\partial w_A},$$

$$= A + \frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''}. \tag{27}$$

$$\frac{\partial V}{\partial w_B} \equiv V_b = B + (w_A - w_A^0).\frac{\partial A}{\partial w_B} + (w_B - w_B^0).\frac{\partial B}{\partial w_B},$$

$$= B + \frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''}. \tag{28}$$

Differentiating these again,

$$V_{aa} = \frac{2}{R''} - \frac{2}{d''} - \frac{(w_A^0 - w_B^0 - d').d'''}{d''^3} - \frac{(R' - w_A^0).R'''}{R''^3}, \tag{29}$$

$$V_{bb} = -\frac{2}{d''} - \frac{(w_A^0 - w_B^0 - d').d'''}{d''^3}, \tag{30}$$

$$V_{ab} = \frac{(w_A^0 - w_B^0 - d').d'''}{d''^3}. \tag{31}$$

Considering the general problem of the union, it can be expressed as

$$\max_{\{w_A, w_B\}} V(w_A, w_B). \tag{32}$$

This maximisation has first-order conditions

$$V_a = 0, V_b = 0. \tag{33}$$

From this can be derived a result linking the wage differential to the employment levels of the two groups. The wage gap prior to introducing the union is $w_A^0 - w_B^0$, with the union it is equal to d'. Since the optimum implies $V_a = V_b$, the terms derived above lead to

$$w_A^0 - w_B^0 - d' > 0 \Rightarrow B > A. \tag{34}$$

This statement can be phrased as saying that if the wage gap is reduced, the union comes to represent more B workers. Note that this is a strongly sufficient condition.

Next, consider the effect of changes in the competitive wage levels and a shift in the discrimination function from d(B) to $(1+\varepsilon)d(B)$, $\varepsilon > 0$, which is the increase in discrimination introduced above (18). Expanding the first-order conditions gives the equation system

$$\begin{bmatrix} V_{aa} & V_{ab} \\ V_{ab} & V_{bb} \end{bmatrix}\begin{bmatrix} dw_A \\ dw_B \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{R''} - \frac{1}{d''}\right)dw_A^o + \left(\frac{1}{d''}\right)dw_B^0 + \left(\frac{w_A^o - w_B^o}{d''}\right)d\varepsilon \\ \left(\frac{1}{d''}\right)dw_A^o - \left(\frac{1}{d''}\right)dw_B^0 - \left(\frac{w_A^o - w_B^o}{d''}\right)d\varepsilon \end{bmatrix}. \tag{35}$$

To analyse (35) we assume for present purposes

$$V_{aa}V_{bb} - V_{ab}^2 > 0. \tag{36}$$

Although not a strong restriction, this condition is not imposed in the following section. It then follows from solving the system that

$$\frac{dw_A}{dw_A^0} > 0, \frac{dw_A}{dw_B^0} < 0, \frac{dw_A}{d\varepsilon} < 0, \tag{37}$$

and

$$\frac{dw_B}{dw_A^0} < 0, \frac{dw_B}{dw_B^0} > 0, \frac{dw_B}{d\varepsilon} > 0. \tag{38}$$

The union therefore reacts to an increase in the competitive wage of one group by raising the bargained wage of that group but at the cost of a fall in the wage of the other group. Thus if, for example, anti-discrimination legislation produces an exogenous increase in w_B, the wage gap in the unionised firm, anyway less than that where unions do not set wages, is reduced further. More interestingly, the union counters an increase in discrimination within the firm by raising the wage of the group that is the object of the discrimination.

IV The Nash Bargaining Solution

We now collect together the analyses developed above and place the union and firm into a general Nash bargaining model. This model is then studied from both an analytical and numerical perspective. The aim of the analytical approach is to characterise sufficient conditions for the presence of the union to reduce the wage gap and therefore counter the discrimination. The numerical results are intended to illustrate the formal reasoning and to provide some evidence for the possible size of the effects we discuss.

The standard Nash bargaining solution is extended by the inclusion of a parameter, λ, that measures the relative weights given to the union and firm. This approach is adopted since it permits the analysis of variations in union influence on the bargained outcome. Using the maximium value functions, the modified Nash bargaining solution can be expressed as

$$\max_{\{w_A, w_B\}} V(w_A, w_B)^{\lambda} U(w_A, w_B)^{1-\lambda}. \tag{39}$$

The specification in (39) collapses to the unconstrained firm when $\lambda = 0$ and to the monopoly union when $\lambda = 1$. By varying λ it is then possible to move continuously between these extremes. It is implicit in (39) that the no-agreement utility level of the firm is set at zero and that the inclusion of competitive wage levels in (20) captures the no-agreement outcome for the union.

Analytical Results

There are two major difficulties in analysing the solution to (39). Firstly, the solution will be discontinuous at $\lambda = 0$: at this value of λ the objective function is unbounded as w_A and w_B decrease without limit. This implies that it will be necessary to restrict λ to an open set not including 0. The second difficulty is that the objective will be convex at $\lambda = 0$ and is likely to be concave at $\lambda = 1$, although the latter cannot be guaranteed as inspection of (29) - (31) makes clear. If this is the case, the principal minors of its Hessian must change signs at least once and there may be values of λ for which the determinant of the Hessian is negative. In addition, this lack of concavity in the objective may result in non-unique solutions. From this it follows that there will be few global results on the solution to (39) and that we must take care to consider possible corner solutions. Some of these difficulties could be overcome by imposing constraints on the wage rates and using Kuhn-Tucker conditions. However, the presence of the multipliers would considerably complicate the analysis of the system[6] and we prefer to adopt a more direct route.

Proceeding with the analysis, from the maximisation in (39) are generated the following first-order conditions

$$\lambda V_a U + (1-\lambda) V U_a = 0, \tag{40}$$

and

$$\lambda V_b U + (1-\lambda) V U_b = 0. \tag{41}$$

6. Consider adding two further rows to the matrix in the proof of lemma 6.

The first lemma is a global result that applies whenever (40) and (41) chararacterise a maximum. It follows from noting that these imply

$$V_b U_a = V_a U_b, \text{ for } 0 < \lambda < 1. \tag{42}$$

We can now state Lemma 1 which relates relative employment levels to the bargained wage gap.

Lemma 1

a) If $w_A^0 - w_B^0 - d' > 0$, the Nash bargain results in B > A.

b) If $w_A^0 - w_B^0 - d' < 0$, the bargain results in A > B.

Proof

By substitution from (16), (27) and (28). See appendix for details.

For the following lemmata we are concerned with the dependence of the solution of the Nash bargain upon λ. For this purpose, denote the solutions of (40) and (41) conditional upon λ as $w_A(\lambda)$, $w_B(\lambda)$. The second lemma rules out the possiblity that the wage rates can both remain at the competitive levels for positive λ, that is for $\lambda \in (0, 1]$. Thus for all λ in the open set (0, 1), both U and V are positive.

Lemma 2

(i) For all $\lambda \in (0, 1]$, $w_A(\lambda) = w_A^0$, $w_B(\lambda) = w_B^0$ cannot be a solution;
and

(ii) For all $\lambda \in (0, 1)$, U > 0 and V > 0.

Proof

Let $w_A(\lambda) = w_A^0$, $w_B(\lambda) = w_B^0$ and $\lambda \in (0, 1]$. Evaluating V at these values gives V = 0 and hence $V^{\lambda}U^{1-\lambda} = 0$. This cannot be optimal given Assumption 2. Hence at least one of the wage rates must be above the competitive level which proves (i). (ii) follows as a corollary.

We next add an assumption that permits the knowledge that the determinant of the Hessian of U is positive to be extended to the objective function in (39) in a neighbourhood around $\lambda = 0$.

Assumption 3: The derivatives of U and V are bounded.

From this follows Lemma 3.

Lemma 3

There exists $\lambda^* > 0$ such that for $\lambda \in [0, \lambda^*)$, the principal minors of $V^{\lambda}U^{1-\lambda}$ are positive and the Hessian has a positive determinant.

Proof

The claim is clearly true at $\lambda = 0$ considering the definitions in (17). Calculating the Hessian shows that, given Lemma 2, this will be true for a connected set of positive λ when the derivatives are bounded. Taking λ^* as the supremum of λ in this set provides the result.

For the next result we restrict our attention to values of λ in the open set $(0, \lambda^*)$. Concerning variations in λ, we next note

Lemma 4

Assume that V and U are of class C^r, $r \geq 2$. Then $w_A(\lambda)$ and $w_B(\lambda)$ are continuous and at least once differentiable for $\lambda \in (0, \lambda^*)$.

Proof

Lemma 3 guarantees that the Jacobian of the mapping defined by (40) and (41) is non-zero for all λ in the range defined. Application of the implicit function theorem then gives the result.

The non-concavity of the objective makes it possible for there to be multiple solutions to (39). The typical nature of these will be that one is a low wage outcome which provides a high return to the firm and another will have high wages and thus provide a high pay-off to the union. Our response to this is to focus, for values of λ close to zero, upon the low wage outcome. This is justified by appealing to the fact that in the absence of the union the firm will pay the competitive wage levels and that adding an appropriately small amount of union power should not disturb this equilibrium too far. In a sense, we are imposing a degree of continuity upon the solution[7].

Lemma 5

$\lim_{\lambda \to 0} w_A(\lambda) = w_A^0$ and $\lim_{\lambda \to 0} w_B(\lambda) = w_B^0$.

Proof

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7. If we had added constraints $w_A \geq w_A^0$ and $w_B \geq w_B^0$ to (39), it is clear these would bind at $\lambda = 0$ and that a solution path would exist from this point that was continuous in λ. The present assumption is intended to capture this fact.

It is clear that $\lim_{\lambda \to 0} w_A(\lambda) < w_A^0$ and $\lim_{\lambda \to 0} w_B(\lambda) < w_B^0$ can be easily ruled out since, by continuity, this would give a negative value to the union for positive λ close to 0. This cannot be a solution, so $\lim_{\lambda \to 0} w_A(\lambda) \geq w_A^0$ and $\lim_{\lambda \to 0} w_B(\lambda) \geq w_B^0$.

Now assume that the optimal solution is such that $\lim_{\lambda \to 0} w_A(\lambda) > w_A^0$ and $\lim_{\lambda \to 0} w_B(\lambda) > w_B^0$ Now take any pair of paths $w_A'(\lambda), w_B'(\lambda)$ with the properties that for any λ such that $0 < \lambda < \varepsilon$, $w_A^0 < w_A'(\lambda) < w_A(\lambda)$, $w_B^0 < w_B'(\lambda) < w_B(\lambda)$ and $w_A^0 < \lim_{\lambda \to 0} w_A'(\lambda) < \lim_{\lambda \to 0} w_A(\lambda)$ and $w_B^0 < \lim_{\lambda \to 0} w_B'(\lambda) < \lim_{\lambda \to 0} w_B(\lambda)$. Since U is decreasing in w_A and w_B it follows that it is possible to find $\hat{\lambda}$ sufficiently small that $U(w_A'(\hat{\lambda}), w_B'(\hat{\lambda}))^{1-\hat{\lambda}} V(w_A'(\hat{\lambda}), w_B'(\hat{\lambda}))^{\hat{\lambda}} > U(w_A(\hat{\lambda}), w_B(\hat{\lambda}))^{1-\hat{\lambda}} V(w_A(\hat{\lambda}), w_B(\hat{\lambda}))^{\hat{\lambda}}$. This inequality contradicts the optimality of the proposed solution. Therefore $\lim_{\lambda \to 0} w_A(\lambda) = w_A^0$ and $\lim_{\lambda \to 0} w_B(\lambda) = w_B^0$, as was to be proved.

The sixth result concerns the effect of changing the share parameter λ in the Nash bargain and provides a sufficient condition for increased weight on the union to reduce the wage differential. It is more than a local result since it applies whenever the Hessian of the objective function is positive.

Lemma 6

If the Hessian of $V^{\lambda}U^{1-\lambda}$ is positive, $R''' > 0$ and $w_A^0 - w_B^0 - d' > 0$, then $\frac{dw_B}{d\lambda} > \frac{dw_A}{d\lambda}$.

That is, shifting the weight of the bargain in favour of the union results in a fall in the wage gap between the two groups.

Proof

By direct calculation. See appendix for details.

The content of this lemma can be expressed in an alternative form by noting that lemma 4 implies $w_A^0 - w_B^0$ is the wage differential as λ tends to 0 and, from (11) and (12), that d' measures the differential for all λ. Whenever the determinant of the Hessian is positive, the lemma therefore says that if the bargaining ever reduces the wage gap, further weight on the union's utility will result in the wage gap being reduced further. Note from the final inequality, (A.1), of the proof that the condition given is strongly sufficient and the necessary condition would be much weaker.

Having now derived conditions under which the wage gap is always reduced further by extra union weight, the next step is to identify a necessary and sufficient condition for a move from the independent firm to a "little" union power to reduce the gap. This is the content of theorem 1.

Theorem 1

If $B\left[\frac{dA}{dw_A} - \frac{dA}{dw_B}\right] + A\left[\frac{dB}{dw_A} - \frac{dB}{dw_B}\right] < 0$ when evaluated at w_A^0, w_B^0, then $\frac{dw_B}{d\lambda} > \frac{dw_A}{d\lambda}$ as $\lambda \to 0$ from above. The wage gap therefore falls monotonically with λ for $\lambda \in (0, \lambda^*)$.

Proof

From lemma 5 it follows that $\lim_{\lambda\to 0} w_A^0 - w_B^0 - d' = \lim_{\lambda\to 0} R' - w_A^0 = 0$. Therefore, in the limit inequality (A.1) reduces to

$$-\frac{B}{R''} + \frac{2(B-A)}{d''} > 0.$$

Using (14) then gives the inequality in the statement of the theorem. The second statement follows from application of lemma 6 in the given open neighbourhood.

It should be noted that the value of this theorem is in giving a predictive condition that applies to the unconstrained firm and can be evaluated before any reference to the union is introduced. That it is also a fairly weak restriction can be seen by noting that from (14) $\frac{dA}{dw_A} < \frac{dB}{dw_B}$ and $\frac{dA}{dw_B} = \frac{dB}{dw_A}$. Therefore it is sufficient, though far from necessary, that $B \geq A$.

Numerical Results

The analytical results provide some strong characterisations of the solution to the Nash bargain. Unfortunately, a number of the more interesting conclusions are restricted to values of λ in open neighbourhoods, with no real insight into the size of these neighbourhoods. It is therefore worthwhile to consider some numerical results that provide an overview of possibilities. One of the major conclusions will be that, for our example at least, many of the analytical results actually hold on an open dense subset of [0,1].

The specification that we consider is that of the example of section III. To recall, this assumes a revenue function of the form $R = L^\beta$ and a discrimination function $d = B^\alpha$. The basic parameter restrictions that we employ are:

$$\beta = 0.4,\ w_A^0 = 0.4,\ w_B^0 = 0.2.$$

The competitive wage gap, without union intervention, is therefore 0.2.

The results are presented in the following tables for two values of α in the discrimination function.

λ	.001	.01	.02	.04	.06	.08	.1	.2	.4	.6	.8	1
U	.622	.616	.610	.598	.587	.576	.566	.520	.450	.398	.357	.329
V	.001	.006	.012	.023	.033	.042	.051	.084	.120	.135	.141	.142
w_A	.401	.404	.410	.420	.431	.442	.453	.511	.632	.758	.891	1
w_B	.201	.214	.225	.246	.265	.284	.302	.384	.528	.667	.807	.927
Gap	.200	.190	.185	.174	.166	.158	.151	.127	.103	.091	.084	.079

Table 1. $\alpha = 2.5$

λ	.001	.01	.02	.04	.06	.08	.1	.2	.4	.6	.8	1
U	.634	.628	.622	.610	.598	.587	.577	.529	.457	.403	.360	.332
V	.006	.006	.012	.024	.034	.044	.053	.087	.126	.142	.149	.150
w_A	.401	.404	.408	.418	.428	.438	.449	.505	.623	.750	885	1
w_B	.202	.214	.225	.247	.267	.286	.305	.388	.533	.672	.815	.933
Gap	.199	.190	.183	.171	.161	.152	.144	.117	.091	.078	.071	.066

Table 2. $\alpha = 3$

These tables illustrate sets of results that conform with the conclusions of the analytical analysis. In particular, they support our chief conclusion that the wage gap decreases as the bargaining power of the union increases. In addition, they demonstrate considerably more continuity and regularity than could be directly established. The solutions are continuous for all non-zero λ in both cases, so in this case the continuity in lemma 4 holds for [0, 1] which is open and dense in [0, 1]. All variables are also monotonic with respect to λ: wages and union utility are monotonically increasing whilst the wage gap and firm utility are decreasing.

One striking aspect of both tables is the extent to which the wage gap is reduced and wages raised by the union influence. The rate of change of wage gap with respect to λ appears to be fairly constant, though diminishing as λ approaches 1. The wage gap is reduced further when the firm is more discriminatory. This is a consequence of the union acheiving a greater increase in the wage of the group B since the wage of group A is 1 in both cases. The union also recieves a higher pay-off when there is more discrimination.

V. Conclusion.

We have shown how the standard union-firm Nash-bargaining model can be employed to analyse the previously ignored question of the effect of unions on wages and employment in the presence of employer discrimination. Our main result is to show how the presence of a union which has bargaining power over the wage causes a reduction in the wage gap between a discriminated and a non-discriminated group. This has been shown to be true in the specific example given for the monopoly union model and, by theorem 1, for the more general Nash bargaining model under weak conditions. As union bargaining increases the wage gap falls monotonically. This result arises with a union whose objective is simple rent-maximisation - we have not had to introduce assumptions about a union concern for equity across worker groups. Our other results show that, in the case of a monopoly union model, an increase in discrimination by the firm leads the union to set a higher wage for the group against which the firm is discriminating. Furthermore, an exogenous increase in the competitive wage, occurring as a result of, say, either anti-discrimination or minimum wage legislation, will produce a further narrowing of the bargained wage gap. The sensitivity of the results to the bargaining structure or to the objective functions remains to be investigated in further work.

Appendix

Proof of Lemma 1

Substitute into $V_bU_a + V_aU_b$ to give

$$\left[B + \frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''}\right][-A] = [-B]\left[A + \frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''}\right].$$

From this,

$$\frac{A}{B} = \frac{\left[\frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''}\right]}{\left[\frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''}\right]}.$$

To prove (a), assume that

(i) $\left[\frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''}\right] > 0.$

Hence if $A > B$ it follows that

$$\left[\frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''}\right] > \left[\frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''}\right],$$

which can be reduced to

$$\frac{(w_A^0 - w_B^0)}{d''} > \frac{d'}{d''} + \frac{w_A^0}{2R''} - \frac{R'}{2R''}.$$

Using the initial assumption (i), this can be replaced by

$$\frac{d'}{d''} - \frac{w_A^0}{2R''} - \frac{d'}{d''} + \frac{R'}{2R''} > 0,$$

or

$$R' - w_A^0 < 0,$$

which is false since

$$R' = w_A \geq w_A^0.$$

Now assume that

(ii) $\left[\frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''}\right] < 0.$

Hence if $A > B$ it follows that

$$\left[\frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''}\right] < \left[\frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''}\right],$$

which can be reduced to

$$0 < \frac{(w_A^0 - w_B^0)}{d''} < \frac{d'}{d''} + \frac{w_A^0}{2R''} - \frac{R'}{2R''}.$$

Using (ii), this can be replaced by

$$\frac{d'}{d''} - \frac{w_A^0}{2R''} - \frac{d'}{d''} + \frac{R'}{2R''} > 0,$$

or

$$R' - w_A^0 > 0.$$

which is clearly true.

(b) can be proved analogously.

Proof of Lemma 6

The first-order conditions for the maximisation of (39) can be written

$$\lambda V_a U + (1-\lambda) U_a V = 0,$$

and

$$\lambda V_b U + (1-\lambda) U_b V = 0.$$

Considering variations in w_A, w_B and λ gives the system

$$\begin{bmatrix} \lambda(V_{aa}U+V_aU_a) + (1-\lambda)(U_{aa}V+V_aU_a) & \lambda(V_{ab}U+V_aU_b) + (1-\lambda)(U_{ab}V+V_bU_a) \\ \lambda(V_{ab}U+V_bU_a) + (1-\lambda)(U_{ab}V+V_aU_b) & \lambda(V_{bb}U+V_bU_b) + (1-\lambda)(U_{bb}V+V_bU_b) \end{bmatrix} \begin{bmatrix} dw_A \\ dw_B \end{bmatrix}$$

$$= \begin{bmatrix} (U_aV - V_aU)d\lambda \\ (U_bV - V_bU)d\lambda \end{bmatrix}.$$

Denoting the matrix on the left-had side by H, the conditions of the lemma assume that $|H| > 0$. Solving the sytem gives

$$\frac{dw_A}{d\lambda} = \frac{1}{|H|}\begin{bmatrix} (U_aV - V_aU)(\lambda(V_{bb}U+V_bU_b) + (1-\lambda)(U_{bb}V+V_bU_b)) \\ - (U_bV - V_bU)(\lambda(V_{ab}U+V_aU_b) + (1-\lambda)(U_{ab}V+V_bU_a)) \end{bmatrix},$$

$$\frac{dw_B}{d\lambda} = \frac{1}{|H|}\begin{bmatrix} (U_bV - V_bU)(\lambda(V_{aa}U+V_aU_a) + (1-\lambda)(U_{aa}V+V_aU_a)) \\ - (U_aV - V_aU)(\lambda(V_{ab}U+V_aU_b) + (1-\lambda)(U_{ab}V+V_bU_a)) \end{bmatrix}.$$

Therefore $\frac{dw_B}{d\lambda} > \frac{dw_A}{d\lambda}$ if

$$\begin{bmatrix} (U_bV - V_bU)(\lambda(V_{aa}U+V_aU_a) + (1-\lambda)(U_{aa}V+V_aU_a)) \\ - (U_aV - V_aU)(\lambda(V_{ab}U+V_aU_b) + (1-\lambda)(U_{ab}V+V_bU_a)) \end{bmatrix} >$$

$$\begin{bmatrix} (U_aV - V_aU)(\lambda(V_{bb}U+V_bU_b) + (1-\lambda)(U_{bb}V+V_bU_b)) \\ - (U_bV - V_bU)(\lambda(V_{ab}U+V_aU_b) + (1-\lambda)(U_{ab}V+V_bU_a)) \end{bmatrix}.$$

Using (42), substitution into the first-order conditions yields the identities

$$(U_aV - V_aU) = \frac{1}{\lambda} U_aV = -\frac{1}{1-\lambda} V_aU < 0 ,$$

and

$$(U_bV - V_bU) = \frac{1}{\lambda} U_bV = -\frac{1}{1-\lambda} V_bU < 0 .$$

Using these in the previous inequality reduces it to

$$U_bV(V_{aa}U+V_aU_a) - V_bU(U_{aa}V+V_aU_a) - U_aV(V_{ab}U+V_bU_a) + V_aU(U_{ab}V+V_aU_b) >$$

$$U_aV(V_{bb}U+V_bU_b) - V_aU(U_{bb}V+V_bU_b) - U_bV(V_{ab}U+V_bU_a) + V_bU(U_{ab}V+V_aU_b) .$$

Cancelling terms, and using (42) again, provides the inequality

$$VU[(V_{aa}U_b-V_{ab}U_a) + (U_{aa}V_a-V_bU_{aa})] >$$

$$VU[(V_{bb}U_a-V_{ab}U_b) + (U_{aa}V_b-V_aU_{bb})] .$$

Simplifying again and collecting terms

$$U_b(V_{aa}+V_{ab}) - U_a(V_{bb}+V_{ab}) + V_a(U_{bb}+U_{ab}) - V_b(U_{aa}+U_{ab}) > 0.$$

Using (16), (17) and (27)-(31), the above inequality can be evaluated as

$$-\frac{B}{R''} + \frac{2(B-A)}{d''} + \frac{B(R'-w_A^0)R'''}{(R'')^3} - \frac{(w_A^0-w_B^0-d')}{d''R''} > 0 . \qquad \text{(A.1)}$$

Since $R' = w_A$, it follows that $R'-w_A^0 \geq 0$. Part (a) of lemma 1 then implies that if $w_A^0-w_B^0-d' > 0$ then $B > A$. The condition of the lemma is then clearly sufficient for the inequality to be satisfied.

References

Arrow, K.J. (1973). 'The Theory of Discrimination.' In *Discrimination in Labour Markets*. (O.Ashenfelter and A. Rees, eds.) Princeton, Princeton University Press.

Becker, G.S. (1957). *The Economics of Discrimination*. Chicago, University of Chicago Press.

Freeman, R.B. and Medoff, J.T. (1984). *What do Unions do?* New York, Basic Books.

Horn, H. and Wolinsky, A. (1988). 'Worker Substitutability and Patterns of Unionisation.' *Economic Journal*, vol. 98, no. 391, pp. 484-497.

Lewis, H.G. (1986). 'Union Relative Wage Effects.' In *Handbook of Labour Economics*. (O.Ashenfelter and R. Layard, eds.) Elsevier Science Publishers.

Lundahl, M. and Wadensjo, E. (1984). *Unequal Treatment: A Study in the Neoclassical Theory of Discrimination*. Beckenham, Croom Helm.

Main, B. and Reilly, B. (1990). 'Gender, Unions and Wages.' Mimeo, University of St. Andrews.

Nickell, S.J. and Andrews, M. (1983). 'Unions, Real Wages and Employment in Britain, 1951-1979.' *Oxford Economic Papers*, vol. 35, suppl., pp. 183-206.

Oswald, A.J. (1985). 'The Economic Theory of Trade Unions.' *Scandinavian Journal of Economics*, vol. 87, pp. 160-193.

Sloane, P.J. (1985). 'Discrimination in the Labour Market.' In *Labour Economics*. (D.Carline, C.Pissarides, W.Siebert and P.Sloane, eds.) Longman.

Ulph, A. and Ulph, D. (1990). 'Union Bargaining: A Survey of Recent Work.' In *Current Issues in Labour Economics*. (D.Sapsford and Z.Tzannatos, eds.) Macmillan.