Anomalous Speculative Attacks on Fixed Exchange Rate Regimes
Possible Resolutions of the "Gold Standard Paradox"

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This paper analyzes Krugman's contention that there is a "gold standard paradox" in the speculative attack literature. The paradox occurs if when a country runs out of gold its currency appreciates or, equivalently, if a speculative attack happens only after the country would have run out of reserves in the absence of a speculative attack. We first show that this "gold standard paradox is a very general phenomenon", relevant for all price fixing or price stabilisation schemes, which does not require mean reverting processes for the fundamentals and which can be present in discrete time models as well as in continuous time models. Next we show that the explicit consideration of the presence and role of international currency arbitrageurs is one way of eliminating the paradox. If a "natural collapse" appears to occur before a speculative collapse, abritrageurs keep official reserves just above the critical threshold level until the speculative attack point is reached. When the speculative attack occurs, official reserves do not undergo any finite change. The increased non-arbitrage demand for the currency of the country that abandons the gold standard is met out of the accumulated currency holdings of private arbitrageurs.

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1. INTRODUCTION

In a recent paper Paul Krugman [1989] (revised as Krugman and Rotemberg [1990]), has shown that in the standard model of speculative attacks on the international reserves of a country participating in a fixed exchange rate regime, perverse or anomalous speculative attacks can occur. This "gold standard paradox" (a paradox pertaining to all fixed or managed exchange rate regimes when international financial markets are efficient and stocks of reserves are finite) can be characterized as follows. Reasonable specifications of the processes governing the fundamentals that drive the shadow floating exchange rate and the stock of international reserves result in anomalous behaviour: a speculative attack can occur only **after** the country has already run out of reserves without a speculative attack i.e. without a sudden run on its currency that strips the monetary authority of its remaining international reserves in instantaneous "stock—shift" fashion. Such a "natural" collapse will be associated with a jump — (discontinuous) **appreciation** of its currency and the expectation of gradual (or smooth) **appreciation** of its currency immediately following the attack and jump—appreciation.

In Krugman's words, the gold standard paradox is that ". . . it is not possible for the public actually to expect zero change in the exchange rate while it is fixed" (Krugman [1989, p. 18]). In addition "when a country runs out of gold (as a result of a "natural" collapse), its currency appreciates" (Krugman [1989, p. 20]). Finally ". . . a speculative attack, if it occurs will happen only after the country would have run out of gold in the absence of a speculative attack" (Krugman [1989, p. 21]).

In this paper we show first that Krugman's paradox is a phenomenon that is much more general than his paper suggests. It is in particular not dependent on the presence of mean reversion in the fundamentals process. To rule it out requires either that the fundamental and the shadow exchange rate follow a random walk without drift or that they be weakly monotonic over time i.e. either strictly nondecreasing or strictly nonincreasing. What must be ruled out is a kind of mean reversion in the exchange rate process: we must exclude the possibility that a high value of the shadow exchange rate (a weak currency) be associated with an expectation of future exchange rate appreciation (a strengthening of the currency) and that a low value of the shadow exchange rate be associated with expected future depreciation.

A slightly generalized version of Krugman's model is developed in Section 2.

After demonstrating the full thrust of Krugman's critique of the speculative attack literature in Section 3 for the continuous time case and in Section 4 for the discrete time case, we offer our proposals for a resolution of the paradox in Section 5.

2. THE MODEL

The model is given in equations (1) through (6).

(1) $s(t) = m(t) - m^*(t) + v(t) + \gamma E_t \dot{s}(t) \qquad \gamma > 0$

(2) $m = ln(D + R)$

(3) $m^* = ln(D^* + R^*)$

(4) $R + R^* = G$

(5a) $R > \underline{R}$

(5b) $R^* > \underline{R}^*$

(6) $G > \underline{R} + \underline{R}^*$

s is the logarithm of the spot exchange rate, the price of foreign currency in terms of domestic currency. m is the logarithm of the domestic nominal money stock and m^* the logarithm of the foreign nominal money stock. $v(t)$ measures the logarithm of foreign money demand (at a given foreign price level) relative to domestic money demand (at a given domestic price level). v is assumed to be governed by a stationary first–order Markov process. In the continuous time version of the model the sample paths of v will also be assumed to be continuous functions of time. E_t is the expectation operator conditional on information at period t. D is the nominal stock of home country domestic credit and D^* the exogenous and constant nominal stock of foreign domestic credit, both assumed to be exogenous and constant.[1] R is the home country stock of reserves and R^* the foreign stock of reserves.[2] The total world stock of reserves G is constant. Equations (5a, b) define the values of R and R^* for which the fixed exchange rate regime is viable. Each country establishes a constant reserve floor ($\underline{R}$ for the home country and $\underline{R}^*$ for the foreign country). When reserves fall below these floors the fixed exchange rate regime collapses, and a permanent free float begins. Equation (6) states that global gold reserves are sufficient to satisfy the minimal requirements for reserves of both countries simultaneously. Without loss of generality we set $\underline{R} = \underline{R}^* = 0$, so (6) becomes

(6′) $G > 0$.

We present two different structural two–country models that yield the quasi–reduced form of equation (1). The first is a model with two monies, domestic and foreign, held by private agents, and gold, which is held only by the two national monetary authorities. There is imperfect (direct) currency substitution in this model. The second model adds two fixed nominal market value, variable interest rate bonds, one denominated in home country currency and the other in foreign currency. The two bonds are perfect substitutes in private portfolios, but imperfect direct currency substitution between the two currencies is maintained. The two–monies–and–gold model is summarized in equations (7) to (9).

(7) $$m(t) - p(t) = -\frac{1}{2}\gamma E_t\dot{s}(t) + ky(t) \qquad \gamma > 0,\ k > 0$$

(8) $$m^*(t) - p^*(t) = \frac{1}{2}\gamma E_t\dot{s}(t) + ky^*(t)$$

(9) $$p(t) = p^*(t) + s(t)$$

p and p^* are the logarithms of the domestic and foreign price levels respectively, and y and y^* domestic and foreign output. Equations (7) and (8) are standard money demand functions. Equation (9) is the condition for purchasing power parity (P.P.P.). Equations (7) to (9) yield (1) with $v = k(y^* - y)$.

The two–monies–two–bonds–and–gold model is given by equations (10) through (12) and (9), repeated here for ease of reference.

$$(10)\qquad m(t) - p(t) = -\gamma i(t) + ky(t) \qquad\qquad \gamma > 0,\ k > 0$$

$$(11)\qquad m^*(t) - p^*(t) = -\gamma i^*(t) + ky^*(t)$$

$$(12)\qquad i(t) = i^*(t) + E_t\dot{s}(t)$$

$$(9)\qquad p(t) = p^*(t) + s(t)$$

i and i^* are the domestic and foreign instantaneous nominal rates of interest. Equation (12) is the condition for uncovered interest parity (U.I.P.).

Solving equation (1) forward in time and choosing the unique, continuously convergent solution we get

$$s(t) = \frac{1}{\gamma}\int_t^{\infty} e^{-\frac{1}{\gamma}(s-t)} E_t[m(s) - m^*(s) + v(s)]\,ds\,.$$

The **dual** shadow floating exchange rate at t that will describe the economy if the home country were to run out of reserves at time t, $\hat{s}(t)$, is defined by:

$$(13)\qquad \hat{s}(t) = lnD - ln(D^* + G) + \frac{1}{\gamma}\int_t^{\infty} e^{-\frac{1}{\gamma}(s-t)} E_t v(s)\,ds\,.$$

The **dual** shadow floating exchange rate at time t that will prevail if the foreign country were to run out of reserves at time t, $\hat{s}^*(t)$, is defined by:

$$(14)\qquad \hat{s}^*(t) = ln(D + G) - lnD^* + \frac{1}{\gamma}\int_t^\infty e^{-\frac{1}{\gamma}(s-t)} E_t v(s)\,ds\,.$$

Note that

$$(15a)\qquad \hat{s}^*(t) = \hat{s}(t) + K$$

where

$$(15b)\qquad K = ln(D + G) + ln(D^* + G) - lnD - lnD^* > 0\,.$$

The world starts at $t = 0$. If at $t = t_1 \geq 0$ a successful speculative attack is launched against the home country's currency, it must be true that

$$(16a)\qquad \hat{s}(t_1) = s_0$$

and

$$(16b)\qquad \hat{s}(t) < s_0 \qquad\qquad \text{for all } t < t_1\,.$$

If at $t = t_2 \geq 0$ a successful speculative attack is launched against the foreign currency, it must be true that

$$(17a)\qquad \hat{s}^*(t_2) = s_0$$

and

(17b) $\hat{s}^{*}(t) > s_0$ for all $t < t_2$.

The range of values of the shadow floating exchange rates for which there is no risk of a speculative attack on either currency can be expressed as follows in terms of these two shadow floating exchange rates.

(18a) $\hat{s}(t) < s_0$

(18b) $\hat{s}^{*}(t) = \hat{s}(t) + K > s_0$

or

$$s_0 - K < \hat{s}(t) < s_0$$

We call (18a, b) the condition for S—viability (i.e. for speculative viability) or the **dual** survival criterion.

Let $\hat{v}$ denote the value of v for which $\hat{s} = s_0$. Similarly let $\hat{v}^{*}$ be the value of v for which $\hat{s}^{*} = s_0$. Note that $\hat{v}$ and $\hat{v}^{*}$ will depend on the nature of the stochastic process governing v.

The criterion for S—viability given in equations (18a,b) can be rewritten in terms of the fundamental v as in (19).

(19) $\hat{v}^{*} < v < \hat{v}$ **(S—viability).**

We next define $\bar{v}$ as the minimum value of v for which home country reserves are zero, conditional on the fixed exchange rate regime having survived. As long as the fixed exchange rate regime survives, the behaviour of R is governed by

$$(20) \qquad s_0 = ln(D + R(t)) - ln(D^* + G - R(t)) + v(t) + \gamma E_t \dot{s}(t)$$

With continuous time and continuous sample paths for $s(t)$, it will be true that $E_t \dot{s} = 0$ as long as the fixed exchange rate regime survives. $\bar{v}$ is therefore given by

$$(21a) \qquad \bar{v} = s_0 - (lnD - ln(D^* + G))$$

Similarly, for the foreign country we define $\bar{v}^*$ as the maximal value of v for which foreign reserves are zero, conditional on the fixed exchange rate regime having survived. It follows that

$$(21b) \qquad \bar{v}^* = s_0 - (ln(D + G) - lnD^*)$$

The fixed exchange rate system will not suffer a natural collapse (will exhibit R–viability) as long as the **primal** criterion given in equation (22) is satisfied.

$$(22) \qquad \bar{v}^* < v < \bar{v} \qquad\qquad \textbf{(R–viability)}$$

For expository purposes it is convenient to define the two primal shadow fixed exchange rates $\bar{s}$ and $\bar{s}^*$ as follows

(23a) $\bar{s}(t) = lnD - ln(D^{*} + G) + v(t)$

(23b) $\bar{s}^{*}(t) = ln(D + G) - lnD^{*} + v(t)$

$\bar{s}$ can be interpreted as the lowest value of s that can be established at time t as a viable fixed exchange rate given the actual values of D, D^{*}, G and v. $\bar{s}^{*}$ is the highest value of s that can be established at time t as a viable fixed exchange rate given the actual values of D, D^{*}, G and v. Therefore $\bar{s}(t)$ is supported by $R(t) = 0$ and $\bar{s}^{*}(t)$ is supported by $R(t) = G$. In the continuous time case, an alternative interpretation of $\bar{s}$ and $\bar{s}^{*}$ is to view them as the dual shadow floating exchange rates that would prevail under static expectations.

Note that just as the equality of the dual shadow floating exchange rates and s_0 define $\hat{v}$ and $\hat{v}^{*}$, so the equality of the primal shadow fixed exchange rates and s_0 define $\bar{v}$ and $\bar{v}^{*}$. Note also that

$$\bar{s}^{*} = \bar{s} + K$$

The R—viability criterion can be restated as

$$\bar{s} < s_0 < \bar{s}^{*}$$

or

$$\bar{s} < s_0 < \bar{s} + K$$

The combined criteria for the system not to suffer either speculative or natural attacks is therefore given by

$$max(\bar{v}^*, \hat{v}^*) < v < min(\hat{v}, \bar{v}) \qquad \text{(S\&R—viability)} . \tag{24}$$

While the fixed exchange rate regime endures, i.e. right up to the instant at which either a natural collapse occurs or a successful speculative attack is launched, the expected rate depreciation of the exchange rate is zero. This is so because v, the exogenous forcing variable, is assumed have continuous sample paths (such as the sample paths generated by Brownian motion). If the stock of reserves exceeds by any finite amount, however small, the larger of zero and the value of the reserve stock that would be withdrawn in speculative stock—shift fashion in the case of a successful speculative attack, then the instantaneous probability of a collapse is zero and the exchange rate is expected to remain constant this instant.

A "correct" speculative attack is a speculative attack in the right direction. A correct speculative attack against the home currency when $\hat{s} = s_0$ for the first time requires that the expected rate of depreciation of the exchange rate at the moment of the attack should increase from zero (which is the rational expectation while the fixed exchange rate regime prevails) to some positive value. Only then will there be the stock—shift reduction in the relative demand for home country money that, with D and D^* given, achieves the stock—shift reduction in home country reserves to its critical threshold (zero in our model). If the (rationally) expected rate of depreciation remains zero after the collapse, there is no (stock—shift) speculative attack: natural and speculative attacks coincide.

Analogously a correct speculative attack against the foreign currency requires that at the moment of the attack and collapse the expected proportional rate of depreciation of the home currency falls from zero to some negative value. Again, if the (rationally) expected rate of depreciation remains at zero, there is no (stock–shift) speculative attack.

A moment's reflection will confirm that the speculative attack at the upper bound of the S&R viable range will be correct (i.e. involve a speculative run against the home currency which strips the home country monetary authorities of their remaining reserves) if and only if

(25a) $\hat{v} \leq \bar{v}$ **(correct attack at upper boundary)** .

The speculative attack at the upper boundary should occur at a value of v no greater than than the value of v at which a natural attack occurs. Loosely speaking this can be rephrased as "the speculative attack should occur before the natural attack".

Similarly the criterion for a correct speculative attack at the lower boundary (a speculative run stripping the foreign monetary authority of its remaining reserves) is

(25b) $\bar{v}^{*} \leq \hat{v}^{*}$ **(correct attack at lower boundary)** .

The relative money demand term $v(t)$ is assumed to be governed by the following stochastic process:

(26) $$dv = \mu dt - \rho(v - v_0)dt + \sigma dz \qquad \sigma \geq 0 .$$

$z(t)$ is standardized Brownian motion i.e. the increments dz are identically, independently, and normally distributed with zero mean and unit variance. Equation (26) is a slight generalization of Krugman's equation because a drift or trend term μ is included. Values of $\rho > 0$ indicate mean reversion in the autoregressive component of (26); $\rho < 0$ indicates nonstationary behaviour of the autoregressive component of (26).

Given (1) and assuming $1 + \gamma\rho > 0$ which is required for convergence of the integrals in equations (13) and (14) when v is governed by (26), the two shadow floating exchange rates and the two shadow fixed exchange rates are given by:

(27a) $$\hat{s} = lnD - ln(D^{*} + G) + \frac{\gamma}{1 + \gamma\rho}(\rho v_0 + \mu) + \frac{1}{(1 + \gamma\rho)}v_t$$

with

(27b) $$E_t d\hat{s}(t) = \frac{1}{1 + \gamma\rho}[\mu - \rho(v(t) - v_0)]dt$$

(28a) $$\hat{s}^{*} = ln(D + G) - lnD^{*} + \frac{\gamma}{(1 + \gamma\rho)}(\rho v_0 + \mu) + \frac{1}{1 + \gamma\rho}v_t$$

with

(28b) $$E_t d\hat{s}^{*}(t) = \frac{1}{1 + \gamma\rho}[\mu - \rho(v(t) - v_0)]dt$$

(29) $$\tilde{s}(t) = lnD - ln(D^{*} + G) + v(t)$$

(30) $$\tilde{s}^{*}(t) = ln(D + G) - lnD^{*} + v(t)$$

We now have the information to determine $\hat{v}$ and $\hat{v}^{*}$. ($\tilde{v}$ and $\tilde{v}^{*}$ are always given by (21a, b)).

(31a) $$\hat{v} = [1 + \gamma\rho][s_0 - (lnD - ln(D^{*} + G))] - \gamma\rho(v_0 + \frac{\mu}{\rho})$$

(31b) $\hat{v}^{*} = [1 + \gamma\rho][s_0 - (ln(D + G) - lnD^{*})] - \gamma\rho(v_0 + \frac{\mu}{\rho})$

Note from (31a, b), (21a, b) and (15b) that

$$\hat{v} = \hat{v}^{*} + [1 + \gamma\rho]K$$

$$\tilde{v} = \tilde{v}^{*} + K$$

Therefore we have, since $[1 + \gamma\rho] > 0$ and $K > 0$,

$$\hat{v} > \hat{v}^{*}$$

$$\tilde{v} > \tilde{v}^{*}$$

The criterion for a correct speculative attack at $\hat{v}$, the upper boundary of the S—viable range, can be restated for this particular v process as:

(32a) $E_t\dot{s}\Big|_{v=\hat{v}} = (1 + \gamma\rho)^{-1}[\mu - \rho(\hat{v} - v_0)] \geq 0\,.$

The criterion for a correct speculative attack at $\hat{v}^{*}$, the lower boundary of the S—viable range can be restated as:

(32b) $E_t\dot{s}\Big|_{v=\hat{v}^{*}} = (1 + \gamma\rho)^{-1}[\mu - \rho(\hat{v}^{*} - v_0)] \leq 0\,.$

In Table 1 we summarize the various viability and correctness conditions. When an economy has a non—zero S&R viable range of v

values and when the speculative attacks at the upper and lower bounds are in the correct directions, the economy has achieved Viability as defined in equation (33).

$$(33) \qquad \bar{v}^* \leq \hat{v}^* < v < \hat{v} \leq \bar{v} \qquad \text{(Viability).}$$

3. THE PARADOX STATED AND ILLUSTRATED

v is a random walk without drift

Consider the case when $\mu = \rho = 0$. In this case the fundamental v, the shadow floating exchange rates $\hat{s}$ and $\hat{s}^*$ and the shadow fixed exchange rates $\bar{s}$ and $\bar{s}^*$ all will be (continuous time) random walks without drift.

$$(34) \qquad \hat{s} = \bar{s} = lnD - ln(D^* + G) + v$$

$$(35) \qquad \hat{s}^* = \bar{s}^* = ln(D + G) - lnD^* + v$$

Also,

$$(36) \qquad \hat{v} = \bar{v} = s_0 - (lnD - ln(D^* + G))$$

and

$$(37) \qquad \hat{v}^* = \bar{v}^* = s_0 - (ln(D + G) - lnD^*) .$$

Figure 1 shows the characterization of this economy when v follows a random walk without drift. $\hat{s}$ and $\bar{s}$ coincide as do $\hat{s}^*$ and $\bar{s}^*$,

$\hat{v}$ and $\bar{v}$ as well as $\hat{v}^*$ and $\bar{v}^*$. The S&R—viable range of v is between $\hat{v}^* (= \bar{v}^*)$ and $\hat{v}(= \bar{v})$. It has length K and is independent of s_0. As $\hat{v}$ is approached from below, the fixed exchange rate regime collapses as the home country runs out of reserves. Since the postcollapse expected rate of exchange rate depreciation is zero (see (32a)), there is no stock—shift loss of reserves when the collapse occurs. Speculative and natural collapses coincide.

As $\hat{v}^*$ is approached from above, the foreign country runs out of reserves, again without a stock—shift speculative attack. Note that the S&R—viability criterion is satisfied (equation (24)) as well as the two criteria for correct attacks at the upper and lower boundaries.

v is a random walk with drift

Now consider the case where $\rho = 0$, and $\mu > 0$ [3] shown in Figure 2. In this case we have

(38) $$\hat{s} = lnD - ln(D^* + G) + v + \gamma\mu$$

(39) $$\hat{s}^* = ln(D + G) - lnD^* + v + \gamma\mu$$

(40) $$E_t\dot{s}(\hat{v}) = \mu$$

(41) $$E_t\dot{s}(\hat{v}^*) = \mu$$

$\bar{s}$ and $\bar{s}^*$ are always as in (29) and (30).

Note that the S—viable range of v between $\hat{v}^*$ and $\hat{v}$ has length K. The S&R viable range is between $\bar{v}^*$ and $\hat{v}$, and has length $K - \gamma\mu$.

Clearly μ can be so large that $K < \gamma\mu$. In that case there is no S&R viable range since $\bar{v}^* > \hat{v}$. In what follows we assume $K - \gamma\mu > 0$. The criterion for a correct speculative attack at the upper boundary is satisfied: $\hat{v} < \bar{v}$ or $E_t\dot{s}(\hat{v}) = \mu > 0$. This speculative attack will involve a stock—shift loss of reserves for the home country. The criterion for a correct speculative attack at the lower boundary fails, however: $\hat{v}^* < \bar{v}^*$ or $E_t\dot{s}(\hat{v}^*) = \mu > 0$. It does not make sense to launch a speculative attack against the foreign country currency through a stock—shift **increase** in relative demand for foreign money and consequently a stock—shift **inflow** of reserves into the foreign country.

Note that the failure of the fixed exchange rate regime to make sense at the lower boundary has nothing to do with mean reversion in the process governing the fundamental. The relative money demand process is nonstationary in our example.

If the v process is deterministic ($\sigma = 0$), any v process starting above $\bar{v}^*$ (and below $\hat{v}$) would result in a finite life for the fixed exchange rate regime and a correct collapse at the upper boundary. If the v process had nonnegative increments, there also would be no risk of running into the incorrect attack problem at the lower boundary even if the increments were stochastic (e.g exponentially distributed).

With $\sigma > 0$ however, there is a positive probability that, with v governed by Brownian motion (i.e. normal, identically distributed independent increments), any process starting off at v' with $\bar{v}^* < v' < \hat{v}$ will reach $\bar{v}^*$ in finite time. (Note that since $\mu > 0$, the probability that v will reach any **lower** bound in finite time is strictly less than 1).

What happens when v falls to $\bar{v}^*$? If agents are myopic and had static expectations after the natural collapse (that is they continue to expect $E_t\dot{s} = 0$, even for $v \leq \bar{v}^*$), the economy would move along the shadow fixed exchange rate curve $\bar{s}^*$ after v reaches $\bar{v}^*$ for the first time. There is no speculative attack. Private agents are singularly uninformed but satisfied with their money holdings.

Suppose instead that, when v reaches $\bar{v}^*$ from above, private agents correctly realize that following the collapse (for whatever reason) of the fixed exchange rate regime there will be a free float with a positive expected rate of depreciation of the home country's currency. In this case there will, when $v = \bar{v}^*$, be a stock–shift increase in the demand for foreign money and a stock–shift reduction in the demand for home country money. There would be a stock–shift rush of reserves into the foreign country. Another way to look at this is that for $\hat{v}^* < v \leq \bar{v}^*$ in Figure 2, $\hat{s}^* > s_0$. The postnatural collapse exchange rate represents a finite jump depreciation of the home country's exchange rate. This makes foreign currency a great investment, so reserves rush into the foreign country. This "collapse scenario" therefore makes no sense. No equilibrium exists at $\bar{v}^*$.

v is a nonstationary first–order autoregression without drift

When $\mu = 0$ but $\rho \neq 0$, v follows what is sometimes called the "Ornstein–Uhlenbeck (O.U.)" process. With $\rho < 0$, the process is nonstationary. We have

(42) $$\hat{s} = lnD - ln(D^* + G) + v_0 + \frac{1}{1 + \gamma\rho}(v - v_0)$$

(43) $$E_t\dot{s}\Big|_{v=\hat{v}} = \frac{-\rho}{1 + \gamma\rho}(\hat{v} - v_0)$$

$$(44) \qquad \hat{s}^* = ln(D + G) - lnD^* + v_0 + \frac{1}{1 + \gamma\rho}(v - v_0)$$

$$(45) \qquad E_t\dot{s}\Big|_{v=\hat{v}^*} = \frac{-\rho}{1 + \gamma\rho}(\hat{v}^* - v_0)$$

$\bar{s}$ and $\bar{s}^*$ are given in (29) and (30).

With $\rho < 0$ but $\frac{1}{1 + \gamma\rho} > 0$, the $\hat{s}$ schedule and $\hat{s}^*$ schedule have a common slope in v,s space $\left[\frac{1}{1 + \gamma\rho}\right]$ which exceeds the unitary slopes of the $\bar{s}$ and $\bar{s}^*$ curves. The $\hat{s}$ and $\bar{s}$ curves intersect at $v = v_0$. So too do the $\hat{s}^*$ and the $\bar{s}^*$ curves. As always, the $\hat{s}^*$ curve lies a vertical distance K above the $\hat{s}$ curve, and the $\bar{s}^*$ curve lies a vertical distance K above the $\bar{s}$ curve.

The configuration drawn in Figure 3 exhibits Viability. There is a finite S&R viable range $(\hat{v}^*, \hat{v})$, and $\bar{v}^* < \hat{v}^*$ while $\hat{v} < \bar{v}$.

While this case is in some ways rather like that of a random walk with positive drift, the difference here is that there is a correct speculative attack at the lower boundary. Since $v_0 > \hat{v}^*$, at the lower boundary $\hat{v}^*$ the informed speculator, knowing that $Edv = -\rho(v - v_0)$, expects v to fall further (the model is unstable). The expected rate of change of s at $\hat{v}^*$ is therefore negative, and a speculative attack is launched against the foreign currency at $\hat{v}^*$.

There are however many other configurations. They can be characterized graphically by moving the fixed exchange rate s_0 up or down. When s_0 is above $\bar{s}$ (the value of s at which the $\hat{s}^*$ and $\bar{s}^*$ curves intersect), we lose the correct speculative attack at the lower boundary: $\hat{v}^* < \bar{v}^*$, and $v_0 < \hat{v}^*$. When s_0 rises to or above $\bar{\bar{s}}$ (the value of s at which the $\hat{s}$ and $\bar{s}^*$ curves intersect) the S&R viable range vanishes altogether.

When s_0 is below $\underline{s}$ (the value of s at which the $\hat{s}$ and $\bar{s}$ curves intersect), we lose the correct speculative attack at the upper boundary. When s_0 is at or below $\underline{\underline{s}}$ (the value of s at which the $\hat{s}^*$ and the $\bar{s}$ curves intersect), the S&R viable range again vanishes altogether.

v is a stationary first–order autoregression without drift

The case $\mu = 0$, $\rho > 0$ is the stationary (or mean–reverting) first–order AR process for the fundamental analyzed by Krugman. The equations for $\hat{s}$, $\hat{s}^*$, $E_t\dot{s}(\hat{v})$ and $E_t\dot{s}(\hat{v}^*)$ are as in equations (42)–(45). With $\rho > 0$, the $\hat{s}^*$ and $\hat{s}$ curves have less than unitary slopes. The configuration analyzed by Krugman is shown in Figure 4. While there is a finite S&R viable range $(\bar{v}^*, \bar{v})$, we have incorrect speculative attacks both at the upper and at the lower boundary: $\hat{v} > \bar{v}$, and $\hat{v}^* < \bar{v}^*$. With $\hat{v}^* < \bar{v}^* < v_0 < \bar{v} < \hat{v}$, the expected rate of change of s is negative at $\bar{v}$ (and a fortiori at $\hat{v}$) and positive at $\bar{v}^*$ (and a fortiori at $\hat{v}^*$). When v is large private agents expect it to decline towards v_0 , and when s is high private agents expect it to fall. When v is low, private agents expect it to rise towards v_0. And when s is low, private agents expect it to rise. Thus mean reversion in the endogenous variable, the exchange rate, at the boundaries of the S&R–viable zone implies anomalous speculative attacks. This does not, however, require mean–reverting behaviour of the exogenous fundamental process, v.

Raising s_0 above $\bar{s}$, the value of s at which the $\bar{s}^*$ and $\hat{s}^*$ curves intersect, eliminates the anomalous attack at the lower boundary. We now have $v_0 < \bar{v}^* < \hat{v}$. Raising s_0 further above $\bar{\bar{s}}$, the value of s at which the $\hat{s}^*$ and $\bar{s}$ curves intersect causes the S&R viable range to vanish.

Lowering s_0 below $\underline{s}$, the value of s at which the $\hat{s}$ and $\bar{s}$ curves intersect, eliminates the anomalous speculative attack at the upper boundary with $\hat{v} < \bar{v} < v_0$. Reversion to v_0 now means that when v is large (but still less than v_0) private economic agents expect a further rise. When s is high private economic agents expect a further increase. The speculative attack at the upper bound of the S&R viable range ($\hat{v}$) is correct: a stock–shift loss of reserves for the home country.

When s_0 is at or below $\bar{\bar{s}}$, the value of s at which the $\bar{s}^*$ and $\hat{s}$ curves intersect, the S&R viable range again vanishes. With $\rho > 0$, there is therefore no value of s_0 for which Viability prevails.

4. THE GOLD STANDARD PARADOX IN A DISCRETE TIME MODEL.

In order to confirm that the paradox is not an artifact of continuous time models driven by Brownian motion, we reformulate in equations (46) through (50) the model of equations (1) through (6) as a discrete time model. This will also facilitate the interpretation of our resolution of the paradox in Section 5.

(46) $$s_t = m_t - m_t^* + v_t + \gamma E_t(s_{t+1} - s_t) \qquad \gamma > 0$$

(47a) $$v_t = \mu + \rho v_0 + (1 - \rho)v_{t-1} + z_t$$

(47b) $Ez_t = 0$

$$Ez_t z_s = 0 \text{ if } t \neq s$$
$$= \sigma^2 \geq 0 \text{ if } t = s$$

(48a) $m_t = ln(D + R_t)$

(48b) $m_t^* = ln(D^* + R_t^*)$

(49a) $R_t + R_t^* = G$

(49b) $G > 0$

(50a) $R_t > 0$

(50b) $R_t^* > 0$

The last two equations again define the conditions under which the fixed exchange rate regime will survive.

We define the following variables:

$$\Delta = lnD - ln(D^* + G)$$

$$\Delta^* = ln(D + G) + lnD^*$$

Note that

$$K = \Delta^* - \Delta$$

The two "dual" shadow floating exchange rates $\hat{s}$ and $\hat{s}^*$ are given by

(51a) $$\hat{s}_t = \Delta + \frac{\gamma}{(1 + \gamma\rho)}(\rho v_0 + \mu) + \frac{1}{(1 + \gamma\rho)}v_t$$

(51b) $$\hat{s}^*_t = \Delta^* + \frac{\gamma}{(1 + \gamma\rho)}(\rho v_0 + \mu) + \frac{1}{(1 + \gamma\rho)}v_t$$

Also

(51c) $$E_t s_{t+1} - s_t = \frac{1}{(1 + \gamma\rho)}(\rho v_0 + \mu) - \frac{\rho}{(1 + \gamma\rho)}v_t$$

Just as in the continuous time case, $\hat{s}_t$ is the exchange rate that prevails in period t if the gold standard collapses that period because the home country runs out of reserves. $\hat{s}^*_t$ is the exchange rate that prevails in period t if the gold standard collapses in that period because the foreign authority runs out of reserves. During a period in which a collapse occurs, reserves can be bought and sold at two potentially distinct prices, s_0 and $\hat{s}_0$, or s_0 and $\hat{s}^*_t$. Thus for a collapse to occur through the home country authority running out of reserves it is necessary and sufficient that $\hat{s}_t \geq s_0$. If $\hat{s}_t < s_0$ and the home authorities nevertheless were about to run out of reserves for "natural" (that is non–speculative) reasons (say through a sequence of increasing values of v), private agents pursuing pure arbitrage profits would sell reserves to the domestic authority in exchange for home currency at s_0

and would instantaneously sell the home currency thus acquired at the postcollapse price of foreign exchange $\hat{s}_0$. Any private agent with access to gold or foreign currency could engage in this profitable set of riskless transactions, say by buying gold from the foreign authority in exchange for foreign currency and presenting the gold thus obtained to the home authority in exchange for domestic currency at the fixed rate s_0.

Any incipient exhaustion of the home currency stock of reserves if $\hat{s}_t < s_0$ would therefore be reversed *before it could materialize* through arbitrage—induced private portfolio transactions. Home country reserves would be replenished instantaneously and the collapse would be avoided. The same holds <u>mutatis mutandis</u> for incipient "natural" reserve exhaustion in the foreign country when $\hat{s}^*_t > s_0$.

As before we define $\hat{v}$ as the minimal value of v consistent with $\hat{s} \geq s_0$. Similarly, $\hat{v}^*$ is defined as the maximal value of v consistent with $\hat{s}^* \leq s_0$. Therefore we have

(52a) $\hat{v} = (1 + \gamma\rho)(s_0 - \Delta) - \gamma(\rho v_0 + \mu)$

(52b) $\hat{v}^* = (1 + \gamma\rho)(s_0 - \Delta^*) - \gamma(\rho v_0 + \mu)$

Note that

$$\hat{s}^* = \hat{s} + K$$

and

$$\hat{v} = \hat{v}^* + (1 + \gamma\rho)K$$

In order to have convergent solutions we require $1 + \gamma\rho > 0$.

Speculative attack viability or S—viability again requires that

(53) $$\hat{v}^* < v < \hat{v} \qquad \text{(S—viability)}.$$

While the fixed exchange rate regime survives the behaviour of reserves is governed by

$$s_0 = ln(D + R_t) - ln(D^* + G - R_t) + v_t + \gamma E_t(s_{t+1} - s_0)$$

In order to have $R_t > 0$ it is therefore necessary and sufficient that

$$v_t < -\Delta + s_0 - \gamma E_t(s_{t+1} - s_0)$$

In order to have $R^*_t > 0$ it is necessary and sufficient that

$$v_t > -\Delta^* + s_0 - \gamma E_t(s_{t+1} - s_0)$$

The minimal value of v_t for which $R_t = 0$ is given by

(54a) $$\bar{v}_t = s_0 - \Delta - \gamma E_t(s_{t+1} - s_0)$$

The maximal value of v_t for which $R^*_t = 0$ is given by

(54b) $\tilde{v}^*_t = s_0 - \Delta^* - \gamma E_t(s_{t+1} - s_0)$

Reserve viability or R—viability therefore requires

(55) $\tilde{v}^* < v < \tilde{v}$ **(R—viability)**

Note that as before

$$\tilde{v} = \tilde{v}^* + K$$

Let $\pi_t \equiv Probability(\hat{s}_{t+1} \geq s_0 \mid \hat{s}_t < s_0)$;

$$\pi^*_t \equiv Probability(\hat{s}^*_{t+1} \leq s_0 \mid \hat{s}^*_t > s_0)$$

$$E_t\hat{s}_{t+1} \equiv E_t(\hat{s}_{t+1} \mid \hat{s}_{t+1} \geq s_0, \hat{s}_t < s_0)$$

and

$$E_t\hat{s}^*_{t+1} \equiv E_t(\hat{s}^*_{t+1} \mid \hat{s}^*_{t+1} \leq s_0, \hat{s}^*_t > s_0)$$

It follows that the <u>unconditional</u> future expected exchange rate, E_ts_{t+1} is given by

(56) $E_ts_{t+1} = \pi_tE_t\hat{s}_{t+1} + \pi^*_tE_t\hat{s}^*_{t+1} + (1 - \pi_t - \pi^*_t)s_0$

We now specialize the stochastic process z_t given in (47b) as follows:

(57) $z_{t+1} = \delta$ with probability 0.5

$= -\delta$ with probability 0.5

$\delta \geq 0$

The variance of z, σ^2 in (47b) is δ^2 in this case.
We define

(58a) $\eta_t = (1 + \gamma\rho)(s_0 - \Delta) - (1 + \gamma)(\rho v_0 + \mu) - (1 - \rho)v_t$

(58b) $\eta_t^* = (1 + \gamma\rho)(s_0 - \Delta^*) - (1 + \gamma)(\rho v_0 + \mu) - (1 - \rho)v_t$

Note that

(59) $\eta = \eta^* + (1 + \gamma\rho)K$

By inspection of (51a), (47a) and (58a) (and of (51b), (47a) and (58b)) it follows that $\pi_t = Probability(z_{t+1} \geq \eta_t)$ and $\pi_t^* = Probability(z_{t+1} \leq \eta_t^*)$. We therefore can establish the following:

(60a) $\pi_t = 0$ if $\eta_t > \delta$

(60b) $\pi_t = 0.5$ if $-\delta < \eta_t \leq \delta$

(60c) $\pi_t = 1$ if $\eta_t < -\delta$

(60d) $\pi_t^* = 0$ if $\eta_t^* < -\delta$ i.e. if $\eta_t < -\delta + (1 + \gamma\rho)K$

(60e) $\pi_t^* = 0.5$ if $-\delta \leq \eta_t^* < \delta$

i.e. if $-\delta + (1 + \gamma\rho)K \leq \eta_t < \delta + (1 + \gamma\rho)K$

(60f) $\pi_t^* = 1$ if $\eta_t^* \geq \delta$ i.e. if $\eta_t \geq \delta + (1 + \gamma\rho)K$

If $(1+\gamma\rho)K \leq 2\delta$ we have:

(60g) $1 - \pi - \pi^* = 0.5$ if $-\delta < \eta < -\delta + (1 + \gamma\rho)K$

(60h) $1 - \pi - \pi^* = 0.5$ if $\delta < \eta < \delta + (1 + \gamma\rho)K$

(60i) $1 - \pi - \pi^* = 0$ if $-\delta + (1 + \gamma\rho)K < \eta \leq \delta$

(60j) $1 - \pi - \pi^* = 0$ if $\eta < -\delta$

(60k) $1 - \pi - \pi^* = 0$ if $\eta > \delta + (1 + \gamma\rho)K$

If $(1 + \gamma\rho)K > 2\delta$ we have:

(60ℓ) $1 - \pi - \pi^* = 1$ if $\delta < \eta < -\delta + (1 + \gamma\rho)K$

(60m) $1 - \pi - \pi^* = 0.5$ if $-\delta < \eta < \delta$

(60n) $1 - \pi - \pi^* = 0.5$ if $-\delta + (1 + \gamma\rho)K < \eta < \delta + (1 + \gamma\rho)K$

(60o) $1 - \pi - \pi^* = 0$ if $\eta < -\delta$

(60p) $1 - \pi - \pi^* = 0$ if $\eta > \delta + (1 + \gamma\rho) K$

Figure 5 illustrates these probabilities. Note that there is no range of values of η_t for which the survival until the next period of the fixed exchange rate regime is certain if the world stock of reserves isn't large enough ($(1 + \gamma\rho)K \leq 2\delta$). This case is drawn in Figure 5a. Figure 5b has a range of η_t values ($\delta < \eta_t < -\delta + (1 + \gamma\rho)K$) for which the gold standard is certain to survive until the next period. This requires $(1 + \gamma\rho)K > 2\delta$.

Note that, unless $\eta_t \leq -\delta$, only a positive realization ($+\delta$) can push z_{t+1} beyond η_t for the first time and that, unless $\eta_t^* \geq \delta$ (that is unless $\eta_t \geq \delta + (1 + \gamma\rho)K)$, only a negative $(-\delta)$ realization can push z below η^* for the first time. We therefore have:

$$(61a)\quad E_t\hat{s}_{t+1} = \Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t + \delta] \quad \text{if } \eta_t > -\delta$$

$$= \Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t \quad \text{if } \eta_t \leq -\delta$$

and

$$(61b)\quad E_t\hat{s}^*_{t+1} = \Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t - \delta] \quad \text{if } \eta^*_t < \delta$$

$$= \Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t] \quad \text{if } \eta^*_t \geq \delta$$

From (56), (60) and (61) we finally obtain $E_t s_{t+1}$. First consider the case illustrated in Figure 5a where $(1 + \gamma\rho)K \leq 2\delta$.

$$(62a)\quad E_t s_{t+1} = 0.5s_0 + 0.5\{\Delta + (1+\gamma\rho)^{-1}[(1+\gamma)(\rho v_0+\mu) + (1-\rho)v_t+\delta]\} \quad \text{if } -\delta < \eta_t < -\delta + (1 + \gamma\rho)K.$$

In this case there is a fifty percent chance of a collapse of the home currency in the next period.

$$(62b)\quad E_t s_{t+1} = 0.5s_0 + 0.5\{\Delta^* + (1+\gamma\rho)^{-1}[(1+\gamma)(\rho v_0+\mu) + (1-\rho)v_t-\delta]\}$$

if $\delta < \eta_t < \delta + (1 + \gamma\rho)K$.

In this case there is a fifty percent chance of a collapse of the foreign currency in the next period.

(62c) $$E_t s_{t+1} = 0.5\{\Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t + \delta]\}$$
$$0.5\{\Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t - \delta]\}$$

if $-\delta + (1 + \gamma\rho)K < \eta_t < \delta$.

In this case a collapse in the next period is certain and it is equally likely to be a collapse of the home currency as a collapse of the foreign currency.

(62d) $$E_t s_{t+1} = \Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t]$$

if $\eta_t \leq -\delta$.

In this case there is a certain collapse of the home country currency in the next period.

(62e) $$E_t s_{t+1} = \Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t]$$

if $\eta_t \geq \delta + (1 + \gamma\rho)K$.

In this case there is a certain collapse in the next period of the foreign currency.

Next consider the case illustrated in Figure 5b where $(1 + \gamma\rho)K > 2\delta$.

(63a) $$E_t s_{t+1} = s_0$$

if $\delta < \eta_t < -\delta + (1 + \gamma\rho)K$.

In this case the fixed exchange rate regime is certain to survive until the next period.

(63b) $E_t s_{t+1} = 0.5 s_0 + 0.5\{\Delta + (1+\gamma\rho)^{-1}[(1+\gamma)(\rho v_0+\mu) + (1-\rho)v_t+\delta]\}$

if $-\delta < \eta_t < \delta$.

There is a fifty percent chance of a collapse of the home currency during the next period.

(63c) $E_t s_{t+1} = 0.5 s_0 + 0.5\{\Delta^* + (1+\gamma\rho)^{-1}[(1+\gamma)(\rho v_0+\mu)+(1-\rho)v_t-\delta]\}$

if $-\delta + (1 + \gamma\rho)K < \eta_t < \delta + (1 + \gamma\rho)K$.

There is a fifty percent chance of a collapse of the foreign currency next period.

(63d) $E_t s_{t+1} = \Delta + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t]$

if $\eta_t \leq -\delta$.

There is a certain collapse of the home currency in the next period.

(63e) $E_t s_{t+1} = \Delta^* + (1 + \gamma\rho)^{-1}[(1 + \gamma)(\rho v_0 + \mu) + (1 - \rho)v_t]$

if $\eta_t \geq \delta + (1 + \gamma\rho)K$.

There is a certain collapse of the foreign currency in the next period.

Now that we know $E_t s_{t+1}$ from equations (62) and (63) we can calculate $\bar{v}_t$ and $\bar{v}^*_t$ from equations (54a,b). With $\hat{v}$ and $\hat{v}^*$ given in equations (52a,b) we can establish whether the gold standard paradox arises in this model. There ia s paradox if $\bar{v} < \hat{v}$ or if $\bar{v}^* > \hat{v}^*$. In that case a country will "run out of reserves" without a speculative attack even though the economy possesses a speculative collapse point. For brevity's sake we focus on $\bar{v} < \hat{v}$. The $\bar{v}^* > \hat{v}^*$ case is symmetric.

The easiest example that proves that the paradox can occur is the special case of the discrete time model in which it replicates exactly the key features of the continuous time model of Section 3. This is the case where the survival of the gold standard into the next period is guaranteed, i.e. the case with $E_t s_{t+1} = s_0$ given in equation (63a). It requires $(1 + \gamma\rho)K > 2\delta$ and $\delta < \eta_t < -\delta + (1 + \gamma\rho)K$. In this case $\bar{v} = s_0 - \Delta$ and since we always have $\hat{v} = (1+\gamma\rho)(s_0-\Delta)-\gamma(\rho v_0+\mu)$ the analysis of Section 3 is directly transferable.

For instance, we have $\bar{v} < \hat{v}$ if $\rho = 0$ and $\mu < 0$. If v follows a random walk with negative drift there will be a natural collapse at the upper boundary (the home country runs out of reserves) before a speculative attack occurs. Alternatively, if $\mu = 0$ and $0 < 1 - \rho < 1$ (v is a first order stationary autoregression without drift) then $\bar{v} < \hat{v}$ if $s_0 > v_0 + \Delta$.

The occurrence of the paradox is not dependent on the special feature of the previous examples that $E_t s_{t+1} = s_0$. One further illustration should suffice to make this point. Consider the case where there is a fifty percent chance of a collapse of the foreign currency in the next period and no chance at all of a home currency collapse. We choose $(1 + \gamma\rho)K < 2\delta$ so the relevant equation for $E_t s_{t+1}$ is (62b) with

$\delta < \eta_t < \delta + (1 + \gamma\rho)K$. For simplicity consider the case where v follows a random walk with drift ($\rho = 0$). In this case $\hat{v} = s_0 - \Delta - \gamma\mu$ and $\bar{v}$ is solved from equations (62b) and (54a). This yields

$$\bar{v} = s_0 - (1 + 0.5\gamma)^{-1}\{\Delta + 0.5\gamma[\Delta^* + (1 + \gamma)\mu - \delta]\}$$

It follows that

$$\hat{v} - \bar{v} = 0.5\gamma(1 + 0.5\gamma)^{-1}[K - (\mu + \delta)] > 0 \text{ if } K > \mu + \delta$$

Apart from demonstrating the analytical advantages of working with continuous time models, Section 4 proves that the gold standard paradox is not an artifact of the special class of continuous time processes studied by Krugman and in Sections 2 and 3 of this paper. It also supplies the convenience of a finite length unit period which may facilitate the interpretation of some of the resolutions of the paradox we propose in Section 5.

5. THE PARADOX RESOLVED.

5.1. The scope of the paradox.

The reason for the occurrence of the paradox should by now be clear: if a low value of the currency is associated with expectations of appreciation (a high value of $\hat{s}$ is associated with $E_t\dot{s}(t) < 0$ following the collapse in the continuous time model or a high value of $\hat{s}$ is associated with a lower value of $E_t(s_{t+1} - s_t)$ following the collapse in the discrete time model), there will be an anomalous speculative attack at the upper boundary of the S&R viable zone. If a high value of the currency is associated with expectations of depreciation (a low value of $\hat{s}$ is associated with $E_t\dot{s}(t) > 0$ following the collapse or a low value of $\hat{s}$ is associated with a higher value of $E_t(s_{t+1} - s_t)$ following the collapse), there will be an anomalous speculative attack at the lower boundary of the S&R viable zone. This means there will never be any trouble when the shadow exchange rates are weakly monotonic over time. In that case the movement of the actual shadow exchange rate is always in the same direction as the change in the expected rate of change of the exchange rate at the moment a collapse occurs: if the actual and expected shadow exchange rate always rise or remain constant (fall or remain constant), there will always be a correct speculative attack at the upper (lower) boundary of the S&R viable zone. If we start above (below) the lower (upper) boundary, the system will never descend (rise) towards it.

Deterministic models with a constant rate of change of the fundamental ($\rho = 0$, $\sigma = 0$) therefore never present any problems. If the drift is zero ($\mu = 0$), the fixed exchange rate regime will survive forever provided the initial value of v is in the interior of the S&R viable zone. With $\sigma = 0$, positive drift ($\mu > 0$) means a certain collapse at the upper boundary (a selling attack against the home country currency). Negative drift means a certain collapse at the lower boundary (a selling attack against the foreign currency). Such models were considered in Krugman [1979], Flood and Garber [1984], and Buiter [1987].

Stochastic models in which the increments in v are always positive (say because they are drawings from an exponential distribution) will also display a monotonically nondecreasing shadow exchange rate over time. The only possible collapse is a correct collapse at the upper boundary. Such models were considered by Flood and Garber [1984], and Buiter[1987].

The random walk without drift ($\mu = \rho = 0$) is trouble free not because its actual movement over time is monotonic but because for every value of $v(t)$, $E_t dv(t) = 0$, and for every value of $s(t)$, $E_t ds(t) = 0$. There are never any (stock–shift) speculative attacks in that model.

For all other models in the class of v processes given in (26) there are parameter configurations with anomalous attacks at the lower boundary and/or at the upper boundary. Examples are Grilli [1986], Obstfeld [1986] and Buiter [1989]. An S&R viable range may also fail to exist when $\mu \neq 0$ and/or $\rho \neq 0$, but that poses no paradox. The paradox is the existence of an S&R feasible range with anomalous attacks at one or both boundaries. In other words, the paradox is the failure of a well–defined equilibrium to exist for some range of values of the fundamentals.[4]

Krugman proposes to resolve the paradox by concluding that, except in the special cases just referred to, there can be no nonparadoxical fixed exchange rate regime or gold standard. In Krugman and Rotemberg [1990] it is proposed that the "two–sided" fixed exchange rate regime be abandoned, and replaced with the analysis of an exchange rate target zone in which limited reserves are used to "regulate" the fundamental at the edges or boundaries of the target zone. These interventions stop the exchange rate from rising above the upper boundary and from falling below the lower boundary, as long as the reserves exceed their exogenously given threshold levels. Changes in the unregulated fundamental that move the exchange rate from either of its boundaries back into the interior of the target zone are not counteracted. A fixed exchange rate is approximated when the upper and lower boundaries of the target zones are brought very close together.

This approach is logically coherent and interesting, and ties in neatly with the by now vast literature on exchange rate target zones (see e.g. Dixit [1988], Krugman [1987,1988,1989], Miller and Weller [1988(a,b),1989], Flood and Garber [1989], Froot and Obstfeld [1989(a,b)] and Dumas [1989].)

We believe, however, that interest continues to attach to the analysis of the traditional two–sided fixed exchange rate regime under perfect capital mobility, and therefore do not wish to throw the fixed exchange rate regime baby with the anomalous speculative attack bath water. The argument that even the historical gold standard was not a truly fixed exchange rate regime but rather a (very narrow) target zone because of the existence of the "gold points" misses the point. The gold points wedge reflected the real cost of shipping gold (mainly between the U.S. and Britain) and does not represent distinct floor and ceiling prices ,net of transportation costs, of foreign exchange . These transportation and transactions costs are anyway surely negligible today. Charles de Gaulle may have insisted on physical shipment of gold from Fort Knox to Paris, but efficient business practice today means the (virtually costless) exchange of paper ownership claims to gold rather than physical transshipment. Most foreign exchange reserves today are paper claims rather than heavy physical objects in any case. Reserve gains and losses are bookkeeping entries that can be effected virtually instantaneously and at negligible cost.

Clearly a truly fixed exchange rate regime is an abstraction that very closely approximates some historical international monetary arrangements as well as some prospective future arrangements (e.g. those that are emerging for the E.C.). In the next two subsections we present alternative ways of guaranteeing the existence of an equilibrium without abandoning the assumption of a fixed exchange rate regime.

5.2. The missing equation and the missing money holdings of arbitrageurs in the two–monies–and–gold model.

Unlike the exposition of the gold standard paradox in the previous two Sections, its resolution is a brief affair. It is implicit in our discussion at the beginning of Section 4 of what happens during the period in which (at the instant at which) a country runs out of official reserves and a floating exchange rate is adopted. During that period, currencies can be sold in exchange for reserves and repurchased instantaneously at two potentially distinct prices: s_0 and $\hat{s}$ in the case of a collapse of the home currency; s_0 and $\hat{s}^*$ in the case of a collapse of the foreign currency. This possibility of risk–free arbitrage profits, or rather the market response that eliminates this possibility, is not part of the formal structure of either the continuous time model (equations (1) through (6)) or the discrete time model (equations (46) through (50)). Inclusion of this missing equation (given as equation (66a,b) below) and inclusion of the missing economic agents (currency arbitrageurs) dissolves the paradox and confirms the validity of the formal analyses of the conventional approach.

Without loss of generality we focus in what follows on a threatening home currency collapse in period t with $\bar{v}_t < \hat{v}_t$. If $\bar{v}_t < \hat{v}_t$ it follows that when $v_t = \hat{v}_t$ we have $\hat{s}_t < s_0$.

The money demand functions represented in equations (1) and (46) only represent the demands for home and foreign currency **excluding** any demand reflecting international currency arbitrage. Let m and m^* denote, as before, the stocks of home and foreign currency. Money holdings of international currency arbitrageurs are denoted m^a and m^{*a}. The monetary equilibrium condition including the money holdings of arbitrageurs is, in discrete time

(65a) $s_t = m_t - m^*_t - (m^a_t - m^{*a}_t) + v_t + \gamma E_t(s(t+1) - s(t))$

The nonarbitrage demand for home currency (relative to foreign currency), $m^n - m^{*n}$ is of course given by

(65b) $m^n_t - m^{*n}_t - s_t = -v_t - \gamma E_t(s(t+1) - s(t))$

The presence of efficient arbitrageurs ready to avail themselves of opportunities for riskless profit means that we can impose the "no arbitrage profits" assumption given in equations (66a,b).

Assumption 1: No Abrbitrage

(66a) $\hat{s}_t - s_0 \geq 0$ i.f.f. $R_t = 0$

and

(66b) $\hat{s}^*_t - s_0 \leq 0$ i.f.f. $R^*_t = 0$

To remove a major indeterminacy from the model we assume that if there are no pure arbitrage profits to be earned, arbitrageurs will reduce $m^a - m^{*a}$, their relative holdings of home currency to foreign currency (henceforth to be referred to as relative currency holdings) to zero . The problem of determining the *gross* asset holdings of arbitrageurs (private agents ready to exploit risk–free opportunities for pure profit by constructing zero net worth portfolios that will yield a strictly positive return in at least one state of nature and non–negative returns in all states of nature) when there are no pure profit opportunities to be had, is a general one.

Assumption 2: Minimal Efficient Size Arbitrage Portfolios

(67) If $m_t^a - m_t^{*a} = 0$ implies $R(t) > 0$ and $R^*(t) > 0$ then

$$m_t^a - m_t^{*a} = 0$$

If $R_t = 0$ and $\hat{s}_t \geq s_0$ then $m_t^a - m_t^{*a} = 0$

If $R_t^* = 0$ and $\hat{s}_t^* \leq s_0$ then $m_t^a - m_t^{*a} = 0$

Second order costs of managing any portfolio other than $m^a - m^{*a} = 0$ could be used to rationalize (67).

While it would be interesting to consider a model in which arbitrageurs hold gold as well as home and foreign currency, we wish to modify the standard model in as few ways as possible. We therefore assume that gold is only used by arbitrageurs to switch between home and foreign currency and that their gold holdings are zero. Equations (4) and (49a) are therefore maintained.

There will be pure arbitrage profit opportunities whenever $R_t = 0$ (the home country abandons the fixed exchange rate standard) and $\hat{s}_t < s_0$. By using reserves to purchase home currency at s_0 and instantaneously reselling that home currency at $\hat{s}$, arbitrageurs can earn riskless positive profits. The same holds when the foreign country abandons the fixed exchange rate regime ($R^*_t = 0$) and $\hat{s}^*_t > s_0$.

When $R_t = 0$ and $\hat{s}_t < s_0$, the opportunity cost variable governing non–arbitrage relative money demands $m^n_t - m^{*n}_t - s_t$ is still the *intertemporal relative price* $-E_t ds(t)$ (or $-E_t(s_{t+1} - s_0)$ in the discrete time case). The relevant opportunity cost variable governing relative arbitrage demand for money $m^a - m^{*a}$ is the *instantaneous or static relative price* $-(\hat{s}_t - s_0)$ and the sensitivity of relative arbitrage demand for money to this opportunity cost variable is infinite. Since the market cannot eliminate this pure arbitrage profit opportunity once the contingency triggering it has occurred (i.e. once $R_t = 0$) and the home country abandons the fixed exchange rate regime with $s_t = \hat{s}_t$, the market instead prevents the contingency that triggers the pure arbitrage opportunity from occurring: it removes the threat of a home country collapse with $\hat{s} < s_0$ by ensuring that home country official reserves stay above the critical threshold level of zero as long as $\hat{s} < s_0$. The mechanism that brings this about is that arbitrageurs replenish home country official reserves when $v \geq \bar{v}$ (and $v < \hat{v}$), or equivalently that for $\bar{v} \leq v < \hat{v}$, changes in v are absorbed into equivalent changes in relative money holdings of arbitrageurs.

What this means is that when $\bar{v} \leq v < \hat{v}$, the stock of international reserves R is no longer governed by v. Reserves are kept just above the minimal threshold level by the missing actors in the account of the Gold Standard paradox: the international currency arbitrageurs.

It is important to note that the arbitrageurs' reverse flow of gold to the home country authorities (their absorption of relative home currency) is never more nor less that the amount required to restore reserves to a positive level. That it is never less follows from equations (66a,b). That it is never more follows from equation (67). In the continuous time case where dz is an infinitesimal, the arbitrageurs' response at each instant will have the same infinitesimal magnitude. As soon as $R_t > 0$, the arbitrageurs' incentive to sell reserves in exchange for home country currency vanishes as the probability of an immediate collapse with $\hat{s} < s_0$ disappears. If, however, equation (67) did not hold, then arbitrageurs, indifferent between holding home and foreign currency when $R > 0$ and $R^* > 0$ could arbitrarily set reserves at any level by choosing arbitrary values of $m^a - m^{*a}$.

Recapitulating, if $R_t = 0$ and $\hat{s}_t < s_0$, arbitrageurs would buy up the entire domestic money stock at the fixed exchange rate s_0 in order to get rid of it again that same instant (in the same period) at $\hat{s}$. The fact that if $\hat{s}_t < s_0$ a large (stock—shift) inflow of reserves, driven by arbitrage, would take place bounds R_t away from (above) zero. This will occur for as long as $\hat{s} < s_0$. What this means is that for values of v such that $\bar{v} \leq v < \hat{v}$, arbitrageurs will increase or reduce their money holdings according to $d(m^a - m^{*a}) = dv$. This will preserve money market equilibrium at the fixed exchange rate since with $s = s_0$ and $E_t ds(t) = 0$ (in the continuous time case; the discrete time case is slightly more complex) we have $d(m^n - m^{*n}) = -dv$

Thus, in Figure 4, when $\bar{v} \leq v < \hat{v}$, as v increases from $\bar{v}$ we move horizontally to the right along the s_0 schedule from Ω_1 to Ω_3. At Ω_3 there is a collapse of the home currency with $R = 0$, but since $s_0 = \hat{s}$ this creates no problem for the conventional analysis. As the fixed exchange rate regime collapses at Ω_3 because $R = 0$, the expected rate of depreciation becomes negative (after being equal to zero in the continuous time model; after being equal to some higher (possibly non—zero) rate in the discrete time model). There is a stock—shift increase in the relative non—arbitrage demand for home currency.

Where does the money come from that satisfies the increased (relative) non—arbitrage demand for home currency when the expected rate of depreciation falls following a collapse? Not out of domestic credit: by assumption D and D^* are constant. Not out of the official reserves of the home country, which are given at zero. It comes out of money balances released by arbitrageurs who (by equation (67)), since $R = 0$ and $s_0 \geq \hat{s}$) now reduce their relative money demand to zero. The (relative) home country currency accumulated by arbitrageurs at $\hat{v}$ (at Ω_3) in Figure 4 is $m^a - m^{*a} = \hat{v} - \tilde{v}$. This will be shown to also be equal to the stock—shift increase in the relative demand for home currency by non—arbitrageurs at $\hat{v}$.

In the continuous time model, for any v satisfying $\tilde{v} \leq v < \hat{v}$, the relative demand for home currency by non—arbitrageurs is given by

(68a) $$m^n - m^{*n} = s_0 - v$$

For the same range of values of v, the relative holdings of home currency by arbitrageurs are given by

(68b) $m^a - m^{*a} = v - \bar{v}$

When $v = \hat{v}$ the relative demand for home currency by arbitrageurs falls to zero, i.e is reduced by $\hat{v} - \bar{v}$. The relative demand for home currency by non—arbitrageurs increases by $-\gamma E_t \dot{s}\Big|_{v=\hat{v}}$.

From equations (31a), (21a) and (27a,b) it follows that:

(69) $$\hat{v} - \bar{v} = -\gamma E_t \dot{s}\Big|_{v=\hat{v}} = \gamma\rho[s_0 - (lnD - ln(D^* + G)) - (v_0 + \frac{\mu}{\rho})]$$

Thus the increase in the relative demand for home country currency by non—arbitrageurs at $\hat{v}$ can be met and is met exactly out of the money holdings released by arbitrageurs who no longer have any riskless profit motive for holding on to home currency.

The gold standard therefore collapses in Figure 4 at Ω_3, as the traditional analysis asserted. Unlike what was suggested by the traditional analysis, however, there is no speculative attack on the remaining home country reserves at Ω_3. Instead the increased non—arbitrage demand for relative home country currency associated with the fall in the expected depreciation rate at Ω_3 is met out of the accumulated money balances of arbitrageurs who maintained the gold standard between Ω_1 and Ω_3.

In a similar manner, in Figure 4 to the left of Ω_2 (for $\hat{v}^* < v \leq \bar{v}^*$) arbitrageurs will be building up relative holdings of foreign currency, thus preventing foreign reserves R^* from falling to zero. At Ω_4 where $v = \hat{v}^*$ there is is a collapse of the gold standard as R^* falls to zero (after being infinitesimally above zero for all v between $\hat{v}^*$ and $\bar{v}^*$). There is a stock–shift increase in the relative non–arbitrage demand for foreign currency at $\hat{v}^*$, which is associated with the increase in the expected rate of depreciation of the home currency. This increase in the relative non–arbitrage demand for foreign currency is met out of the the now redundant holdings of relative foreign currency by arbitrageurs. From Ω_2 to Ω_4 the exchange rate stays at s_0. Once Ω_4 is reached, $\hat{s}^*$ takes over. Exactly the same story can be told about the behaviour of the economy between Ω_2 and Ω_4 in Figure 2.

5.3 The Resolution of the Gold Standard Paradox when Bond Arbitrage Occurs.

In the case where arbitrage is conducted through interest–bearing bonds denominated in different currencies, the argument of Section 5.2 is applicable only if it can be shown that this bond arbitrage spills over into the monetary equilibrium condition. That is, it must affect either the relative money demands or the relative stocks of domestic credit.

The reason why the introduction of interest–bearing capital–value–certain assets with positive nominal interest rates threatens the existence of a proper speculative attack equilibrium is the following.

Consider the case where the the nominal interest rates on the two bonds are positive (and equal, because of U.I.P.) while the fixed exchange rate regime survives ($R > 0$ and $R^* > 0$). Arbitrageurs will prefer holding bonds to holding money in this case. Therefore, at a point like Ω_1 in Figure 4, where the price of domestic currency is about to undergo a discontinuous increase, there are only would–be sellers of foreign–currency–denominated debt in the private sector. Every arbitrageur will be attempting to be short in (borrow by issuing) foreign–currency–denominated debt instruments and to be long in (lend by purchasing) domestic–currency–denominated debt instruments. With D and D^* exogenous, there also are no public sector purchasers of foreign currency denominated debt or sellers of domestic currency denominated debt. If would–be bond arbitrageurs can find no takers for their offers to sell foreign currency denominated debt, one might think they would have no choice but to switch their arbitrage operations to the other foreign currency denominated asset (non–interest–bearing foreign currency itself) for which there is a public sector taker at the fixed rate of exchange. Thus the frustrated would–be bond arbitrage spills over into the currency markets, home country official foreign exchange reserves are replenished and the threats to the fixed exchange rate regime to the bond market are simultaneously canceled.

The problem with this argument is that, as soon as domestic currency reserves are restored above their minimal threshold level and the threat of a discrete appreciation of the home country currency is thereby eliminated, arbitrageurs will once again wish to switch out of home country currency into bonds with a positive nominal rate of interest. Home country reserves are threatened again. No equilibrium exists in this case.

While an exhaustive analysis of the two monies, two bonds and gold model remains to be done, we can point to a number of instances in which a well–defined equilibrium can exist even in this case.

(1) *There is no equilibrium with interest–bearing assets.*

In two—period overlapping generations (OLG) models of a closed economy with each generation consisting of identical individuals and without firms whose "life span" exceeds that of individuals, there can be no private lending or borrowing. If in addition there is no interest—bearing public debt outstanding and there are no real assets, but there is a noninterest—bearing "outside" stock of money, we have a single country money—only model. There are no equilibria with debt of any kind. In a two—country version of this model, further restrictions on the international uses of national currencies would have to be imposed to obtain limited substitutability between domestic and foreign currency and a determinate exchange rate (see Kareken and Wallace [1981]).

(2) *Interest is paid on currency.*

If interest is paid on currency and if the currency interest rates mimic the bond interest rates, we again would have direct currency arbitrage.

(3) *Money demand and bond demand are segmented.*

A subset of private agents may be constrained not to hold bonds but can hold the two currencies. The single—country version of such a model can be found in Sargent and Wallace's "Unpleasant Monetarist Arithmetic" OLG model (Sargent and Wallace [1981]). Small bills—type arguments and the assumption that the poor cannot pool resources to invest in interest—bearing assets that can be acquired only in large denominations lead to an equilibrium in which the poor hold only money and (if the nominal interest rate is positive) the rich only hold bonds. An obvious two—country extension results in one set of agents (the poor) holding only the two currencies while the rich hold the two bonds. The poor would supply the direct currency arbitrage that would make the argument of the previous subsection applicable. To obtain limited direct currency substitution in the non—arbitrage demands for the two currencies, some further restrictions on the uses of the national currencies would have to be imposed.

(4) *Saving the bond market.*

Where the previous three arguments had the bond arbitrageurs' demand spill over directly into the relative demands for currency, our final argument assumes an impact of bond arbitrage on the stocks of domestic credit, D and D^*.

Assume that v reaches $\bar{v} < \hat{v}$ from the left, at a point such as Ω_1 in Figure 4. and that a natural collapse of the home currency occurs. In that case there would, with D and D^* constant, be a collapse of the foreign currency denominated bond market: every arbitrageur would try to sell foreign currency denominated debt (and indeed go short in it) and buy home currency denominated debt. If the authorities respond to this threatened collapse of the foreign bond market by undertaking the minimal open—market operations required to prevent a bond market collapse, they would choose D and D^* such that

$$(70) \qquad (D^* + G)^{-1}dD - D^{-1}dD = dv$$

If $dv > 0$, this could involve the foreign country's government switching from borrowing from the private sector to borrowing from its central bank. In addition (or instead) the home government could move in the opposite direction and engage in an open—market sale. If the assumption of exogenous domestic credit were replaced by (70) when $\bar{v} \leq v < \hat{v}$, then the two governments would effectively take over from the private currency arbitrageurs described in the previous subsection. The prevention of a collapse of the fixed exchange rate regime would be a by—product of policies aimed at preventing a bond market collapse. At $\hat{v}$ there would be a stock—shift reversal of the cumulative flow open market operations that brought the economy from $\bar{v}$ to $\hat{v}$. There would be an open market purchase by the home government and/or an open market sale by the foreign government to provide private agents with the relatively larger stocks of home money demanded at the negative postcollapse expected rate of depreciation of the exchange rate.

This argument can be clarified with the help of Figure 4. Consider e.g. the central banks' behaviour at the upper end of the S&R viable range. The policy rule has two parts:

(I) Any time there is a transition to a floating exchange rate, the stocks of domestic credit will revert to the original levels of D and D^*, that is the values of the stocks of domestic credit that prevailed when v first reached $\bar{v}$ from the left. This fixes the $\hat{s}$ and $\hat{s}^*$ schedules (which depend on the postattack stocks of domestic credit) in the same positions that they had when domestic credit was assumed to be exogenous throughout.

(II) In order to prevent the natural collapse, the authorities engage for values of v such that $\bar{v} \leq v < \hat{v}$ in the minimal size open market operations required to keep home country reserves positive. Sine the $\bar{s}$ and $\bar{s}^*$ schedules are defined for the actual current stocks of domestic credit, they will shift with the actual values of v (keeping the same slopes) between $\bar{v}$ and $\hat{v}$. When v reaches $\hat{v}$, $\bar{s}$ and $\hat{s}$ intersect at Ω_3 where $\bar{s} = \hat{s} = s_0$, and the natural and speculative collapses coincide.

It is hardly surprising that (relative) domestic credit expansion policies can be used to stabilize reserves in the face of exogenous relative money demand shifts. It is nevertheless interesting that such a policy can be rationalized as a response to a threatened bond market collapse. If government debt is denominated in the national currency, a forward–looking foreign government may well have a strong incentive to prevent a collapse in the market for its debt.

The "discretionary" behaviour of the authorities when v is between $\bar{v}$ and $\hat{v}$, characterized above, can be made automatic if we extend the governments' exchange rate commitment to include sales or purchases at the official parity of all their nominal assets, that is both currency and interest–bearing debt. This would restore the private (bond) arbitrageur to the position of the only active agent, with the authorities merely accommodating incipient private excess demand and supply at the fixed parity. Historical evidence exists that is consistent with our interpretation —— for example the U.S. gold standard crisis during the 1890s. As analyzed in Grilli[1989], between 1893 and 1896 there was a widespread fear that the U.S. Treasury would run out of gold reserves and that the U.S. would have to abandon the gold standard. During this period the financial markets were very unstable, and this instability reached a peak with the panic of 1893. To ensure the viability of the fold standard, on four occasions the U.S. Treasury issued bonds. These operations were exactly of the kind illustrated above: a swap of gold for domestic bonds. In one instance (the issue of February 1895), in addition to issuing bonds the Treasury also "subsidized" speculators (the Belmont–Morgan syndicate) in order for them to hold domestic currency. In this way they transformed money into an interest–bearing asset —— a measure which would also provide a remedy to the crisis, as illustrated in Section 5.2 above.

(5) *Other approaches to direct currency arbitrage in the presence of debt with positive nominla interest rates.*

In all of Section 5, and especially in Section 5.3, the importance has become apparent of the details of the specification of the choice problem that generates the demands for the pecuniary rate of return dominated assets home and foreign currency. Without explicit microfoundations of the demands for the two national currencies, the case for and against direct currency arbitrage in the presence of assets (capital—certain bonds with positive nominal interest rates) that dominate currency in rate of return cannot be resolved conclusively. It seems likely, however, that it will be possible to generate plausible conditions under which private agents who hold various national currencies for transactions or precautionary motives can be induced to depart from their normal cash holdings in response to opportunities for pure arbitrage profits.

6. CONCLUSION

The gold standard paradox turns out not to be a gold standard contradiction or inconsistency. Krugman's critical probing of the speculative attack literature has brought out serious weaknesses in the way in which these models were interpreted and described, but not in the formal analyses of when and under what circumstances fixed exchange rate regimes collapse or of how the post—collapse exchange rate behaves.

The explicit recognition in speculative attack models of the role of arbitrageurs faced with the prospect of an imminent collapse of the fixed exchange rate regime and the associated possibility of riskless profits permits us to rule out the possibility of a natural collapse occurring before the speculative attack takes place. Speculative collapses occur where and when the traditional literature says they will.

The (stock–shift) changes in non–arbitrage money demand associated with collapses are however in the paradoxical cases identified by Krugman not accommodated by (stock–shift) changes in official international reserves but instead come out of the accumulated money balances of arbitrageurs who release them when a conventional collapse (which eliminates their opportunity for riskless profits) occurs. Between a paradoxical natural collapse point and the proper speculative collapse point private arbitrageurs keep the government whose reserves are about to be exhausted supplied with the minimal amount of reserves required for the survival of the fixed exchange rate regime. If the fundamentals drive the shadow exchange rate to the conventional speculative attack point, the arbitrageurs release the money holdings they had accumulated while they kept the threatened government supplied with reserves and thus satisfy the increased non–arbitrage demand for money. A conventional transition to a free float (or to some other post–collapse regime not analyzed in this paper) then takes place.

Table 1

Summary of Viability and Correctness Criteria

S–viability:

(21) $\hat{v}^* < v < \hat{v}$

R–viability:

(23) $\bar{v}^* < v < \bar{v}$

S&R–viability

(24) $max(\bar{v}^*, \hat{v}^*) < v < min(\hat{v}, \bar{v})$

Correct attack at upper boundary:

(25a) $\hat{v} \leq \bar{v}$

or

(32a) $E_t\dot{s}(\hat{v}) = (1 + \gamma\rho)^{-1}[\mu - \rho(\hat{v} - v_0)] \geq 0$

Correct attack at lower boundary:

(25b) $\bar{v}^* \leq \hat{v}^*$

or

(32b) $E_t\dot{s}(\hat{v}^*) = (1 + \gamma\rho)^{-1}[\mu - \rho(\hat{v}^* - v_0)] \leq 0$.

Viability (S&R viability and correct speculative attacks at both boundaries):

(33) $\bar{v}^* \leq \hat{v}^* < v < \hat{v} \leq \bar{v}$

NOTES

[1]A nonconstant but exogenous $D(t)$ or $D^*(t)$ can be subsumed under $v(t)$.

[2]The purist will note some untidiness as regards the composition of the stock of reserves. If reserves are gold, let p^G be the domestic currency price of gold and p^{*G} the foreign currency price of gold. Then $m = ln(D + p^G R)$ and $m^* = ln(D^* + p^{*G} R^*)$. The exchange rate $S \equiv e^s = \frac{p^G}{p^{*G}}$. Without loss of generality we can choose units such that $p^G = p^{*G} = 1$, but in that case we must also set $s_0 = 1$.

[3]The case $\mu < 0$ is conceptually identical.

[4]Note that if we consider arbitrary (nonlinear) v processes the shadow exchange rate curves may intersect the s_0 line more than once, and we could get several S&R viable ranges with different combinations of correct and incorrect speculative attacks at the boundaries!

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FIGURE 1

Speculative attacks when the fundamental follows a random walk without drift: $\delta = \mu = 0$

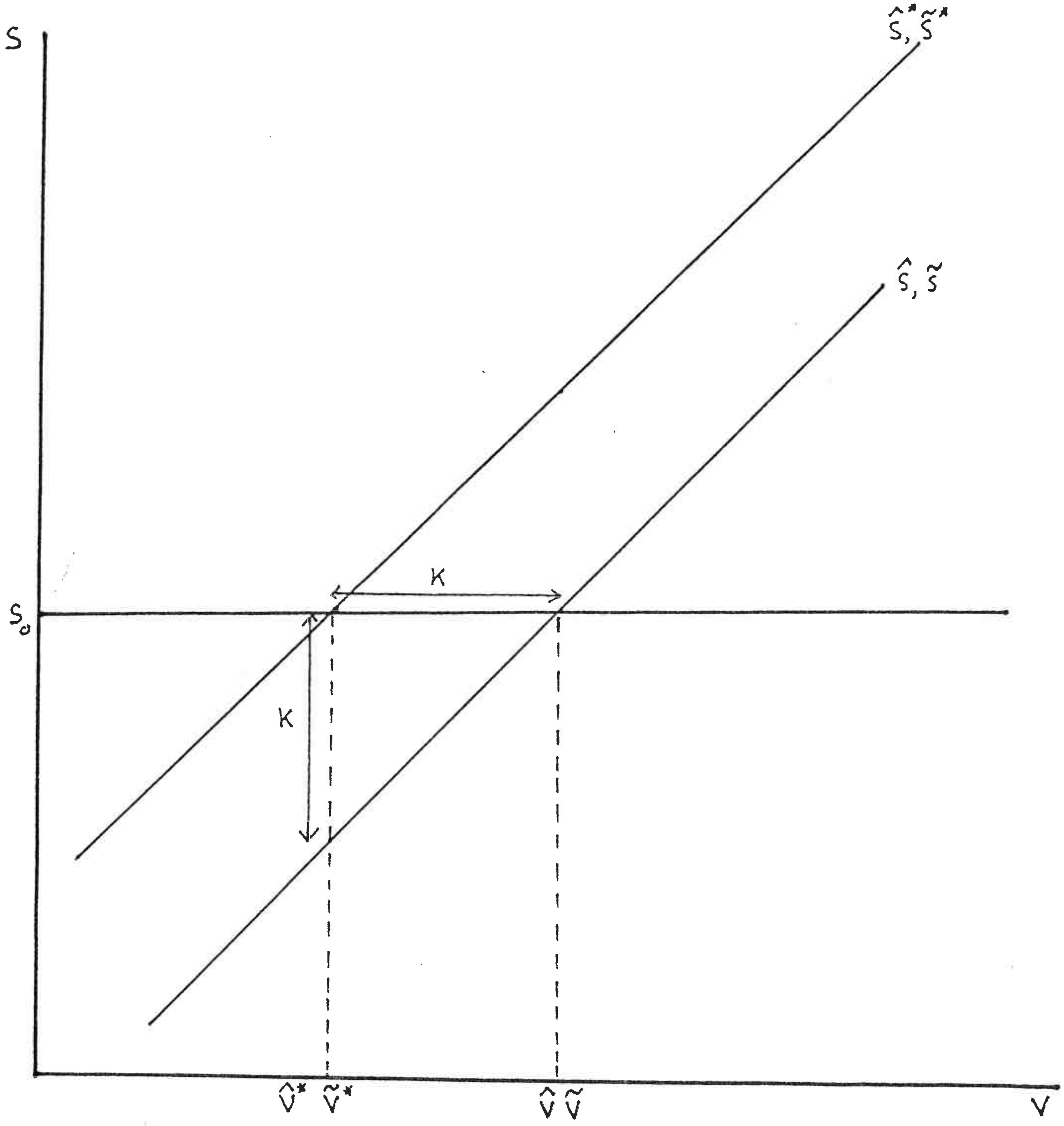


FIGURE 1

FIGURE 2

Speculative attacks when the fundamental follows a random wall with positive drift: $\delta = 0; \mu > 0$

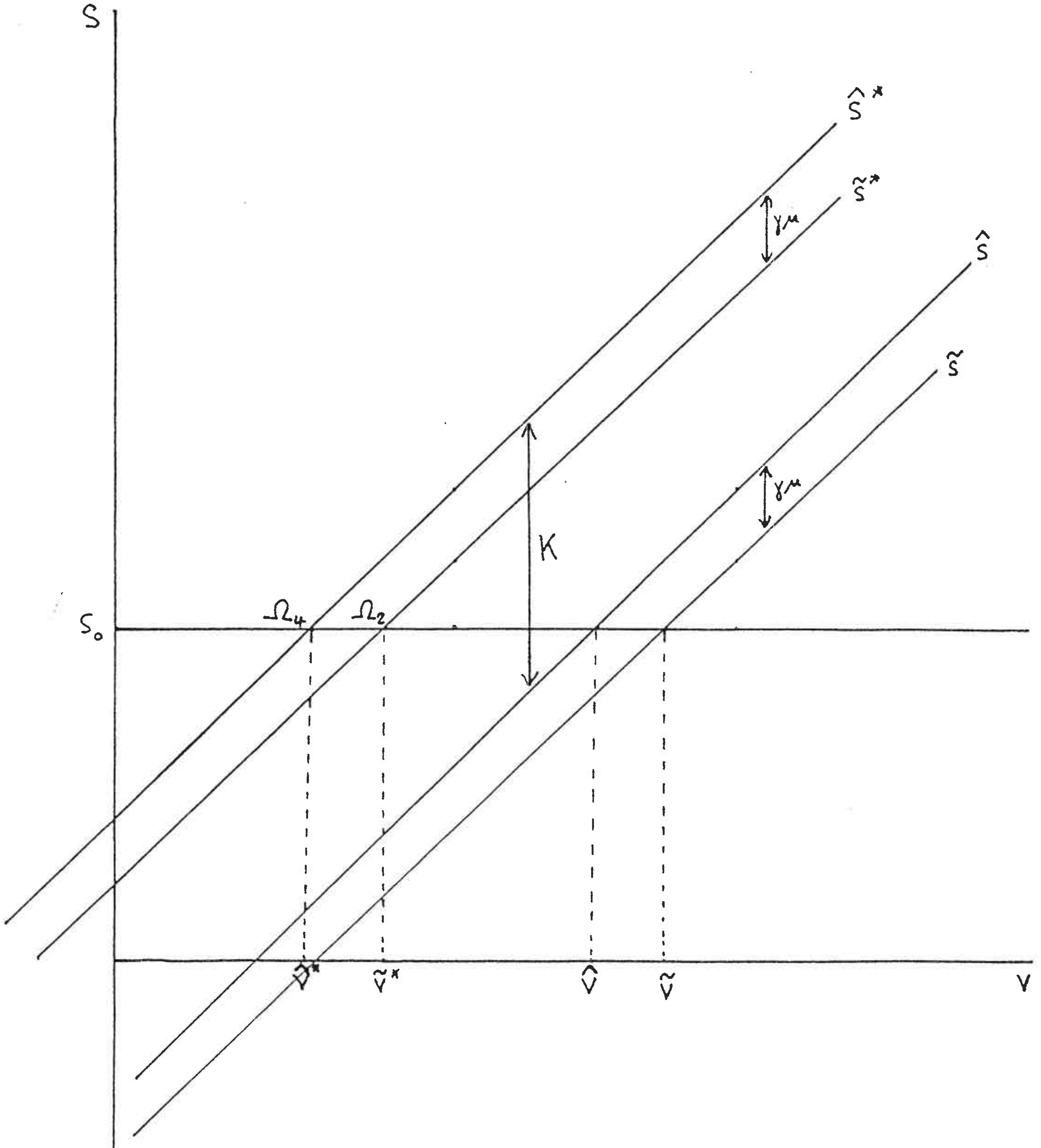

S
$\hat{S}^*$
$\tilde{S}^*$
$\hat{S}$
$\tilde{S}$
$\gamma\mu$
$\gamma\mu$
K
S_0
Ω_4
Ω_2
$\hat{V}^*$
$\tilde{V}^*$
$\hat{V}$
$\tilde{V}$
V

FIGURE 2

FIGURE 3

Speculative attacks when the fundamental follows a nonstationary first-order autoregressive process: $\mu = 0; \delta < 0$

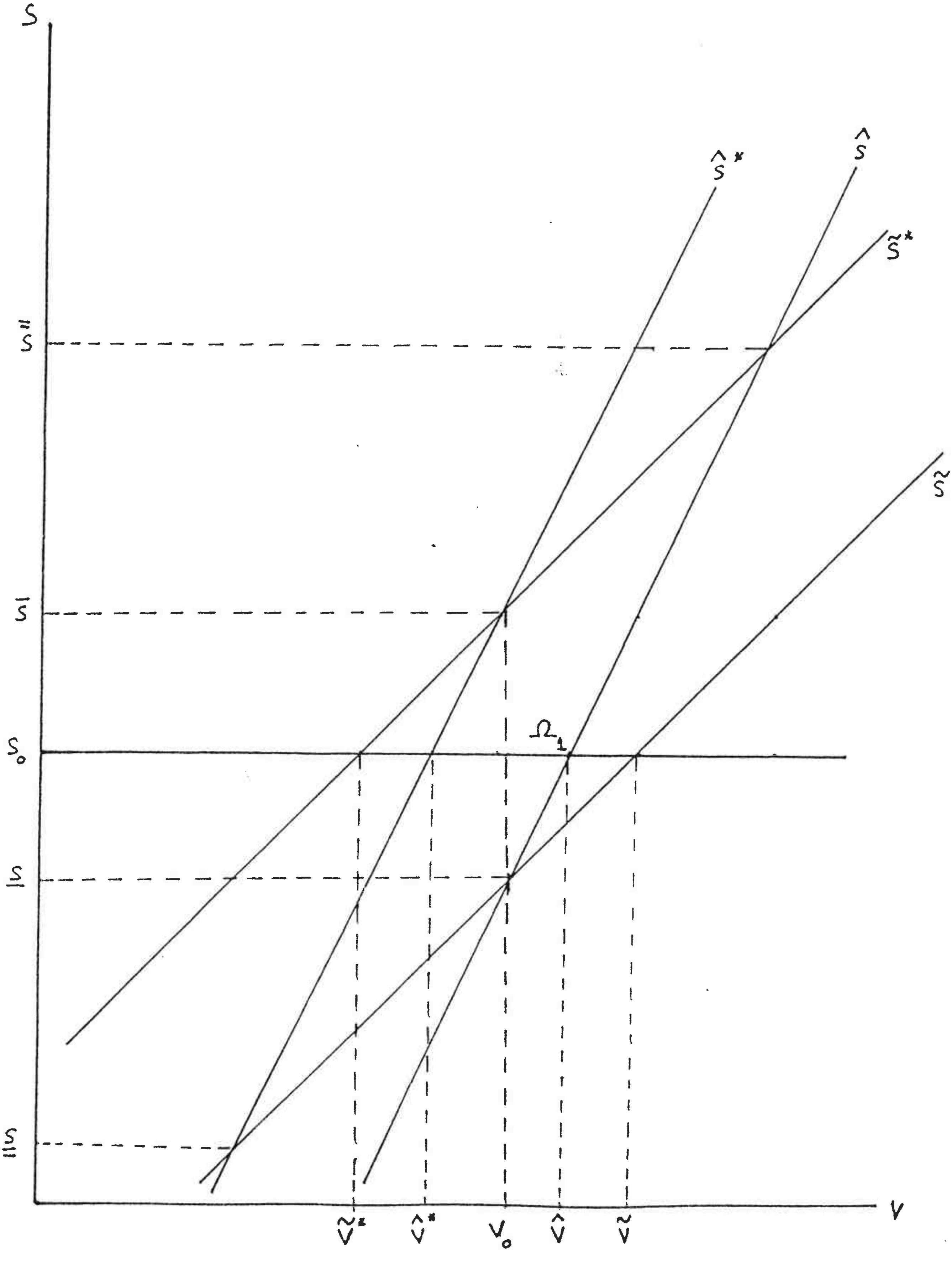
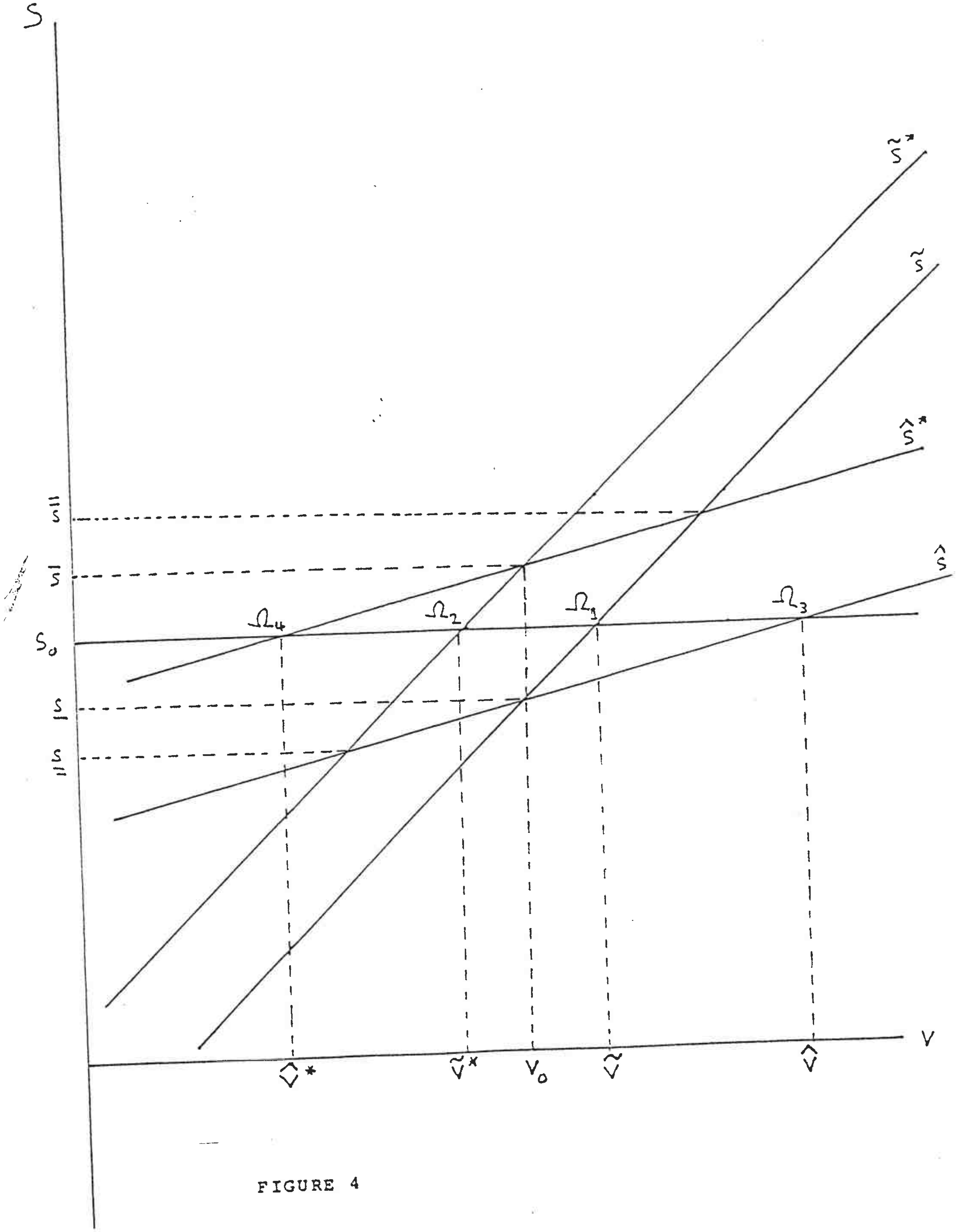


FIGURE 3

FIGURE 4

Speculative attacks when the fundamental follows a stationary first-order autoregressive process: $\mu = 0; \delta > 0$

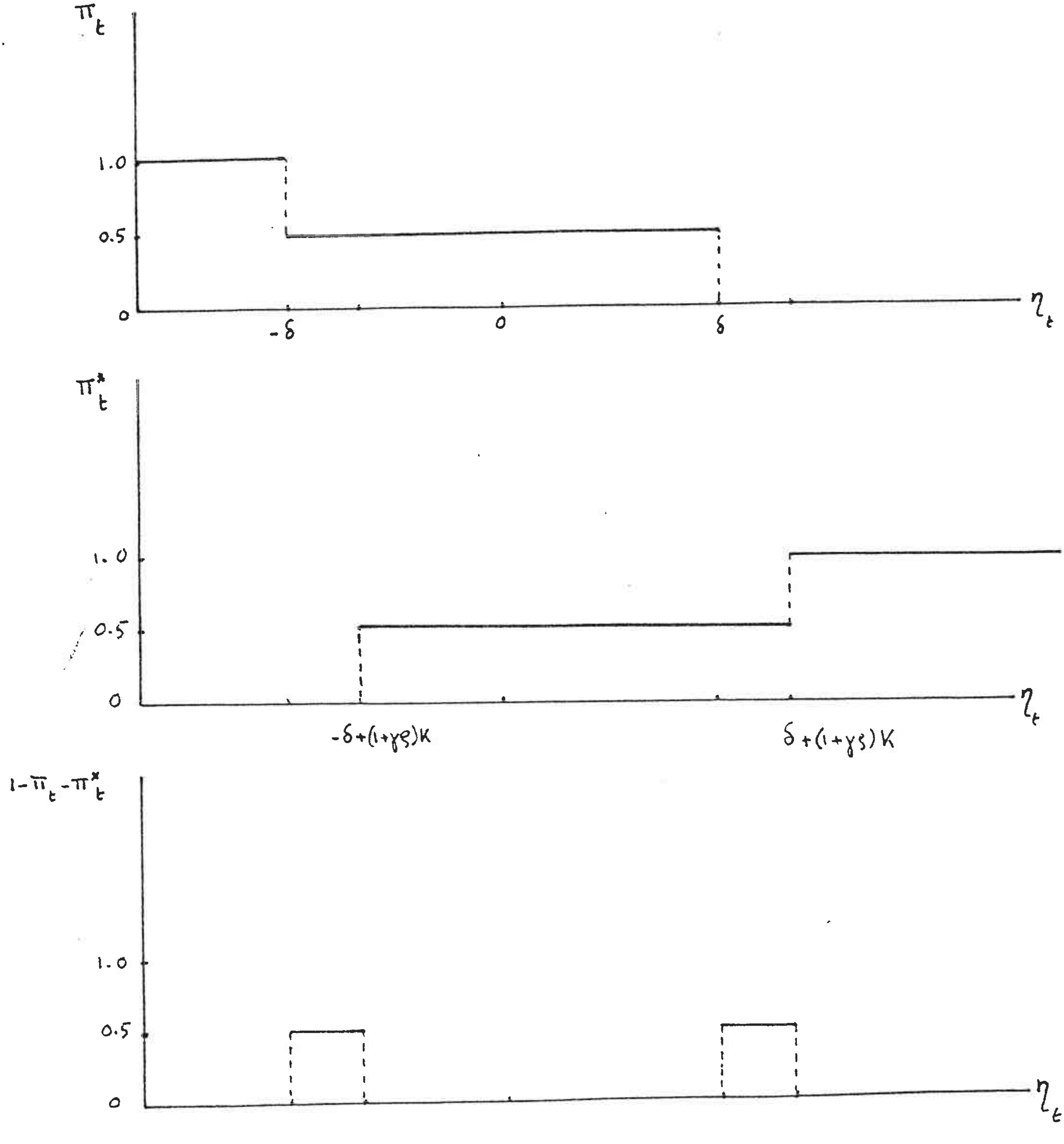


S
$\tilde{S}^*$
$\tilde{S}$
$\hat{S}^*$
$\hat{S}$
$\bar{\bar{S}}$
$\bar{S}$
S_0
$\underline{S}$
$\underline{\underline{S}}$
Ω_4
Ω_2
Ω_1
Ω_3
V
$\hat{V}^*$
$\tilde{V}^*$
V_0
$\tilde{V}$
$\hat{V}$

FIGURE 4

Figure 5a

The probability that the gold standard survives one more period when world reserves are "small" ($(1+\gamma\delta)k < 2\delta$)

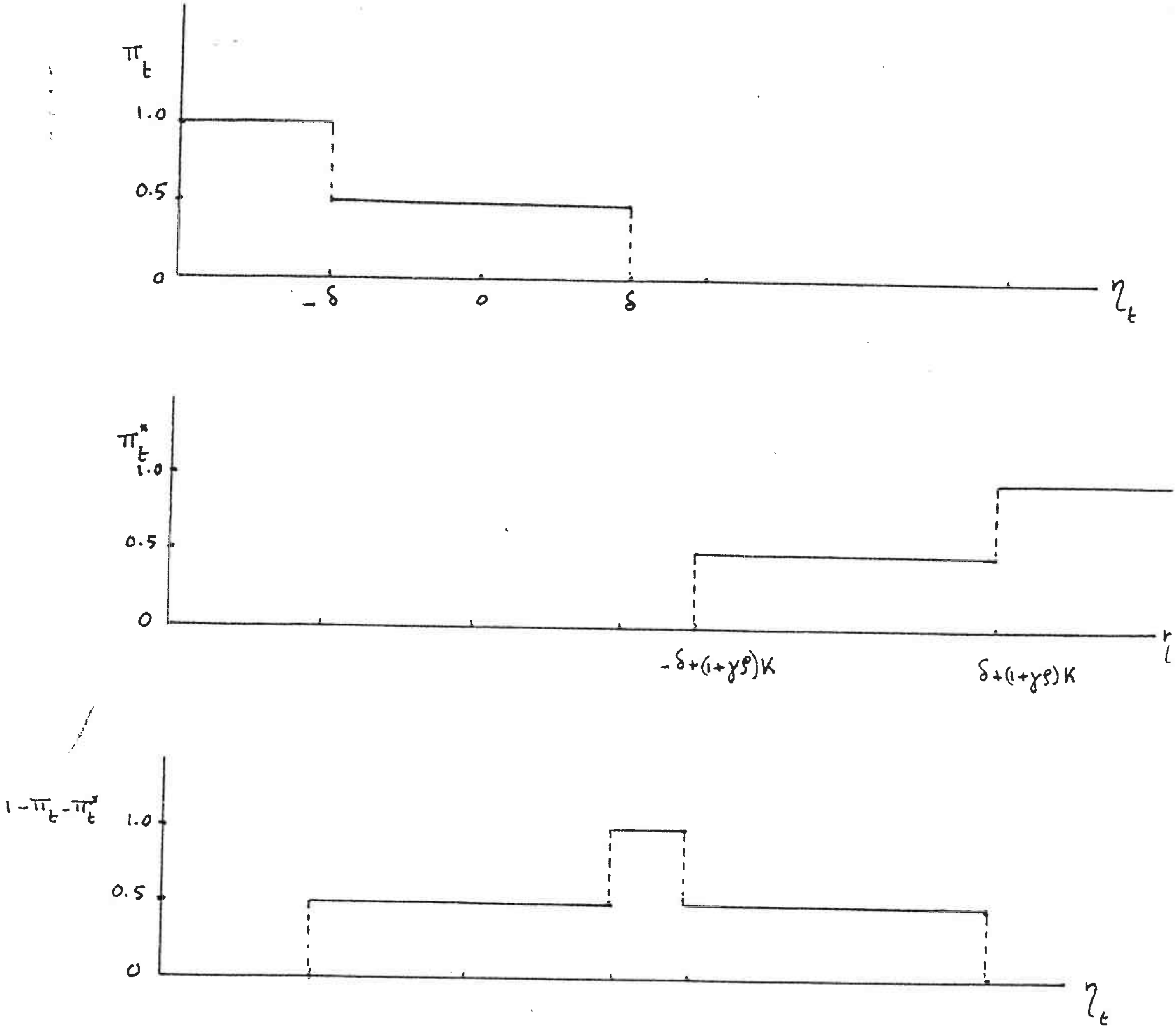


π_t
1.0
0.5
0
$-\delta$
0
δ
η_t
π_t^*
1.0
0.5
0
η_t
$-\delta+(1+\gamma\varsigma)K$
$\delta+(1+\gamma\varsigma)K$
$1-\pi_t-\pi_t^*$
1.0
0.5
0
η_t

FIGURE 5a

FIGURE 5b

The probability that the gold standard survives one more period when world reserves are "large" ($(1+\gamma\delta)k > 2\delta$)



π_t
1.0
0.5
0
$-\delta$
0
δ
η_t
π_t^*
1.0
0.5
0
$-\delta+(1+\gamma\rho)K$
$\delta+(1+\gamma\rho)K$
r_l
$1-\pi_t-\pi_t^*$
1.0
0.5
0
η_t

Figure 5b