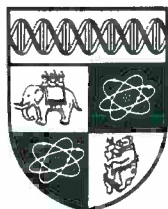


ENDOGENOUS GROWTH AND OVERLAPPING GENERATIONS  
IN A MODEL OF MONOPOLISTIC COMPETITION WITH DETERRED ENTRY

Gianluca Femminis

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ENDOGENOUS GROWTH AND OVERLAPPING GENERATIONS  
IN A MODEL OF MONOPOLISTIC COMPETITION WITH DETERRED ENTRY

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Endogenous Growth and Overlapping Generations  
in a Model of Monopolistic Competition with Deterred Entry.

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Abstract

This paper uses a model with deterred entry to study the relationships between the growth rate, which is endogenous, and the degree of imperfect competition. In an infinitely lived representative agent framework they are shown to be independent, while with Blanchard's type overlapping generations, this "neutrality results" does not hold, resulting in more complex policy problems. It is also verified that dynamic inefficiency, in such a model, is impossible.

Keywords: Overlapping generations, endogenous growth, monopolistic competition.

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## 1) Introduction

In recent years a considerable number of contributions have endeavoured to relate the rate of growth of an economic system to its "fundamentals": preferences, technologies and market structures. In particular, great attention has been devoted to the analysis of the conditions that allow for an (at least asymptotically) constant per capita growth rate.

Under standard hypotheses about the utility function of a representative consumer, high interest rates are critical in fostering the accumulation of productive factors. Therefore, a sufficiently high and non decreasing marginal product of capital can be considered as the basic requisite for growth.<sup>1</sup>

Thus, we might attempt to classify the existing endogenous growth literature depending on the reasons set forth to justify the use of a productive structure fulfilling this necessary condition. Three broad groups can be identified: the first one, which includes Romer's (1986) seminal contribution, just assumes constant or increasing returns to the accumulable inputs in (at least) one sector of the economic system, often advocating Marshallian externalities to justify this hypothesis; a second group makes explicit the possibility of accumulation of human capital, while the third one relies on the effects of an increasing stock of knowledge. Here again externalities may play an important role, knowledge being an (at least partially) public good.

Most aggregate models belonging to the first category are simpler than the others: to obtain a constant growth rate, the production function is assumed to be linear in capital. This hypothesis causes a lack of transitional dynamics: consumption immediately jumps to a level compatible with the common income and capital growth rate, and then increases at this constant rate.<sup>2</sup> Simple versions of these models, thanks to these features, have quickly become the bases for further developments: they have been used to explore the consequences of the finiteness of agents' lifetimes for endogenous growth, and to analyze politico-economic equilibria in a growth framework.<sup>3</sup>

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(1) Jones and Manuelli (1990a) forcefully illustrate this point, building a model where the production function is constrained to display marginal returns to capital bounded above zero.

(2) For an example, see Sala-i-Martin, (1990), pp. 7-8.

(3) Buiter (1991) surveys some recent developments in the former research stream, while Bertola, (1991) is a particularly clear example of the latter.

However, it seems that the (relative) lack of microfoundations which characterizes these models can justify the attempt to develop more sophisticated frameworks with which to appraise the above questions.

Finitely-lived agents are hardly appropriate for human capital accumulation models. As Lucas himself points out (1988, p.19), in such models the hypothesis of infinite lives is important. If we drop it, to obtain a growth process, we have to assume that human capital is inherited by newly born individuals; which does not seem completely satisfying.<sup>4</sup>

Thus, it seems natural to make an attempt to fit finitely-lived agents into a research-based growth model. As to this point, some elements of imperfect competition seem important: in a highly competitive environment the whole firm's revenue has to be used to reward productive factors and no room for research funds is left. Grossman and Helpman model imperfect competition using Dixit and Stiglitz (1977) preferences to grant a monopolistic power to the producer of each good; their models are then characterized by the assumption of free entry in the industrial sector, so that all the profits accrued by producers exactly balance the outlays in research they have to bear in order to enter into the market. In this framework, an higher monopolistic power resolves into larger research expenditures and, hence, into faster growth. In what follows, the free-entry hypothesis is dropped and an environment of fixed product variety is considered. A first result consist on the fact that the growth rate, with an economic system populated by infinitely lived representative agents, becomes independent of the degree of monopolistic competition. This contrasts with the existing literature and it is due to a "macroeconomic externality". In a Blanchard-Yaari framework, it turns out that the relation between the growth rate and the competition level of the system may even be reversed. As the agents' wealth is enhanced by the stream of future profits, their saving decreases and this may reduce the growth rate. It will also be shown that the ricardian debt-neutrality proposition does not hold in the sense that an increase in the debt/output ratio reduces the growth rate, as already suggested by various contributions, which used endogenous growth models of the first type.

The remainder of this paper is organized into six sections

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(4) However various exaples follow this approach. In a two generations framework, Azariadis and Drazen (1990, pp. 509-510) assume that the productivity of the labour supplied by each generation depends positively on the level of efficiency reached by the previous one. Similarly, Buitier and Kletzer (1991, pp.12-13) introduce an externality in the process of human capital formation: they assume that every new cohort starts with a human capital endowment equal to the average level achieved by previous generations. This can be considered as a form of involuntary heritage.

Existing models of monopolistic competition and growth are quickly surveyed in section 2; imperfect competition with deterred entry is then introduced (section 3) and the solution for the case of an infinitely lived representative individual is found (section 4); a simple application of the Yaari-Blanchard approach to overlapping generations is then fitted into this framework; section 7 concludes.

## 2) Leading models of imperfect competition and growth.

The links between growth and increasing product variety have been explored by Romer (1987,1990) and Grossman and Helpman (1989, 1991b, ch. 3 and 5). Along their lines, a simple specification can be described as follows. National product,  $Y(t)$ , is regarded as a homogeneous "final" good, obtained in a competitive setting, via a number of intermediate goods,  $x(t)_i$ . A formulation imposing a constant elasticity of substitution between every pair of these goods is adopted:

$$Y(t) = \left[ \int_0^{n(t)} x(t)_i^\mu di \right]^{1/\mu} \quad 0 < \mu < 1 \quad (1)$$

where  $n(t)$  is the number of intermediates actually producible at time  $t$ .

The production function above makes the demand for every intermediate good negatively sloped, so that the producer of each  $x(t)_i$  enjoys a partial monopolistic power and, in this sector, a pure profit emerges. To produce each single  $x(t)_i$  a blueprint has to be purchased, and free entry into the intermediate good sector implies that the initial outlays for the blueprint equals the discounted stream of subsequent profits. (Therefore, in equilibrium, the value of any firm in the intermediate sector is driven to its initial outlay.)

Research is carried out using labour as unique input<sup>5</sup>, and it is aimed at providing new blueprints. The research sector is characterized by free entry as well.

It is intuitive that the effect of imperfect competition on growth is positive: the higher the monopolistic profits for the intermediate good producer, the higher the amount of investment in research.<sup>6</sup>

(5) Alternative specifications are possible: see, for example, Romer and Rivera Batiz (1991)

(6) This result does not come up very clearly in Romer's model because he considers a Cobb-Douglas

The results of the Grossman and Helpman model of increasing product quality (1991a, 1991b, ch. 4 and 5) are identical, as far as the growth rate determinants are concerned, to those which emerge from the increasing variety model. Free entry in the product market again plays an important role: at the system level, the volume of profits equals the investment in research. The monopoly power, which in this model is acquired via the introduction of innovations, therefore has beneficial effects on growth.

The Schupeterian approach of these models is apparent: firms devote resources to research and development in order to secure a stream of monopoly profits; the free entry assumptions imply that this stream just covers the outlays in research.

### 3) Monopolistic competition with deterred entry.

In what follows, national product is regarded, as before, as a homogeneous final good, produced in a competitive setting, via a production function imposing a constant elasticity of substitution between every pair of the intermediate goods. The hypotheses about the variety of the intermediates, which is taken as given, and about the access to the market of each differentiated product, which is impeded, distinguish the model presented here by the Grossman and Helpman one. As each producer enjoys a partial monopolistic power, he obtains some pure profit, and determines which share of it to invest in research according to an (intertemporally) optimal plan. The result of this effort is a (continuous) stream of process innovations. Each firm manages its own research, so that the problem of splitting profits between producers and researchers is avoided.

Hindrance to new competition may be due to the possibility, for each single firm, of appropriating "product specific information". Hence, a share of firm's technological know-how has to remain private, while research provides a "general knowledge" effect. This distinction could be based on the possibility of obtaining patents, as in the models by Grossman and

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specification for the aggregate output:

$$Y(t) = \left[ \int_0^{n(t)} x(t)_i^\alpha di \right] l(t)^{1-\alpha} \quad 0 < \alpha < 1$$

(where  $l(t)$  is labour used for the final good production) In this model the same parameter,  $\alpha$ , determines both the monopoly level and labour productivity, and the two effects cannot be disentangled.



Helpman and by Romer.<sup>7</sup> Otherwise, if we accept Bertrand competition within each market, a lump sum cost to be paid starting to produce any good is enough to justify the assumed market structure.

According to the assumptions above, national product is regarded as a flow of output produced, in a competitive settings, by means of a fixed measure of "intermediate" goods. An equal and constant elasticity of substitution between every pair of these goods is imposed at any time:

$$Y(t) = C(t) + I(t) + G(t) = \left( \int_0^1 x_i(t)^{\mu} di \right)^{1/\mu} \quad 0 < \mu < 1 \quad (2)$$

Monopolistic competition can also be introduced assuming a time separable utility function characterized by a sub-utility of the Dixit and Stiglitz (1977) type. In this case the parameter  $\mu$  summarizes a taste for diversity. If we accept this approach, however, to keep the model tractable we need to constrain the government and investment demand functions for the various final goods to have the same elasticity displayed by private consumption demands. This implies, moreover, that the production function needs to be restrict accordingly. (The same problem, in a different context, is faced in Kiyotaki, 1987, p. 700)

### 3.1) The demand for a single intermediate good.

Considering (2) as a production function, final goods producers can solve the (time-separable) cost minimisation problem:

$$\begin{aligned} \min \quad & \int_0^1 p_i x_i di \\ \text{s.t.} \quad & \left( \int_0^1 x_i^{\mu} di \right)^{1/\mu} = Y \end{aligned} \quad (3)$$

Using standard techniques, (See, for example, Grossman and Helpman, 1991, pp. 45-47) one can obtain the following system of conditional demand

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(7) A more sophisticated reasoning could rely on the distinction between research and imitation (as in Rustichini and Schmitz, 1991): one could argue that to obtain all the informations about product specific technologies can be very costly. A precise formalization of this point would make the model much more complex.

functions:

$$x_i = Y \left( \frac{p_i}{P} \right)^{1/(\mu-1)} \quad (4)$$

where

$$P = \left( \int_0^1 p_i^{\mu/(\mu-1)} di \right)^{(\mu-1)/\mu}$$

is both an index of intermediate inputs prices and the aggregate price level. <sup>8</sup>

### 3.2) Firm's intertemporal optimization

The intermediate goods sector of the model is composed of a continuum of firms, which are modelled as lying, equally spaced from one another, on a circle of unit length; they are indexed by their position  $i \in [0,1]$  on the circle. It is assumed that the technologies used to produce the intermediate goods can be characterized by a degree of "similarity". The firms (and hence the goods) are positioned on the circle depending on their technological characteristic: the closer are their position, the more similar are their production processes.

The representative firm, managing research, faces an explicit intertemporal problem. Acting in a deterministic environment, it maximises the discounted stream of its cash flows, i.e. it solves, at time  $t$ , the problem:

$$\max_t \int_t^{\infty} (p_i x_i - w(l_{1t} + l_{2t}) - PI) \exp \left[ - \int_t^{\tau} r(z) dz \right] d\tau \quad (5)$$

where  $x_i$  is output of firm  $i$ ;  $l_{1t}$  and  $l_{2t}$  are the labour quantities employed to produce goods and in the research activity, respectively,  $w$  the wage paid to them,  $I$  is investment and  $r$  is the interest rate, all considered at time  $\tau$ , so that the time suffix can be suppressed.

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(8) Note that  $\int_0^1 p_i x_i di = PY$  if  $x_i$  is given by (4)

It is necessary to specify two dynamic constraints; the first one is provided by the technology of acquisition for new knowledge. The simple specification chosen by Lucas for human capital (1988, pp. 18-19), is used here at the single firm level:

$$\dot{S}_i = \kappa S_i^{\tau} \quad (6)$$

where  $S_i$  is the stock of knowledge available for the representative firm at time  $\tau$ ,  $S_i^{\tau}$  is the stock of knowledge relevant for it and  $\kappa$  is the exogenous productivity parameter in research laboratories.

$S_i^{\tau}$  can be defined as the integral sum of the publicly available results obtained by all the firms, weighted by a function taking account of the similarities between the technology of firm  $i$  and those of the other firms. This function,  $\delta(i,j)$ , depends negatively on the distance, measured on the unit circle, between firm  $i$  and the generic firm  $j$ . As the measure of each firm is zero, we can write:

$$S_i^{\tau} = 2 \int_i^{i+1/2} S_j^P \delta(i,j) dj$$

where  $S_j^P$  is the share of knowledge obtained by firm  $j$  which is publicly available.

To obtain a more tractable formulation, several assumptions are needed. We introduce, first, the hypothesis that the publicly available fraction of knowledge is exogenous and equal for every firm, i.e. that  $S_j^P = \omega S_j$ ,  $\omega \in (0,1]$ . (Were  $\omega=0$ , there would not be an externality problem) Therefore, also the possibility that firms try to prevent actively the diffusion of their private knowledge is ruled out<sup>9</sup>. Moreover, we need to specify an "inverse distance" function. Defining  $\Delta = j-i$ , a simple example is:  $\delta(i,j) = 1 - \psi \Delta$ , if  $\psi \Delta < 1$ ,  $\delta(i,j) = 0$  otherwise. With  $\psi < 2$ , we get:

$$S_i^{\tau} = 2 \int_0^{1/2} \omega S_j (1 - \psi \Delta) d\Delta$$

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(9) Since each firm has zero measure, it perceives that its research activity has negligible effects on the other agents' stock of knowledge; therefore the representative firm has no incentive in using resources to avoid the diffusion of its private scientific results.

then, using firms symmetry and the unit-measure hypothesis for firms:  $S_i^f = \omega S(1-\psi/4)$

where  $S$  is the stock of knowledge for the whole economic system.

Hence, setting  $\phi = \kappa\omega(1-\psi/4)$ :

$$dS_i/dt = \phi S \quad (6')$$

The accumulation of knowledge, therefore, depends positively on the productivity in research and on the share of results which cannot be prevented from becoming public; it depends negatively on the technological diversities among firms.<sup>10</sup>

Notice that equation (6') does not necessarily imply that the single enterprise has at its disposal the entire stock of scientific knowledge; notice, moreover, that the hypothesis of barriers to entry caused by firm specific knowledge can coexist with that of a positive externality induced by research.

As in the Lucas formulation, nondecreasing returns to the accumulation of knowledge are crucial to get an equilibrium with constant per capita growth; with diminishing returns the rate of increase of knowledge would converge to zero, no matter which share of labour is devoted to research<sup>11</sup>.

The second dynamic constraint, in absence of adjustment costs, is simply:

$$dK_i/dt = I_i$$

where  $K_i$  is capital.

The firm faces the following production function:

$$x_i = K_i^\alpha l_{i1}^{1-\alpha} S_i^\beta \quad (7)$$

which is assumed to display constant returns to scale in the "rival" factors, capital and labour, as in Romer (1990)<sup>12</sup>. The Cobb-Douglas specification is crucial to get an easy formulation for the economy's common growth rate.

The last constraint that must be taken into account is the inverse

(10) The parameter  $\psi$  can reflect also the communication infrastructure of the economic system. (Schmitz 1989)

(11) Decreasing returns to labour can, on the contrary, be fitted into the model, but at the price of a heavier notation.

(12) The hypothesis of increasing returns to scale is not crucial for the model, in the sense that it is possible to obtain steady growth with a production function linear in all the productive factors.

demand function for  $x_i$ :

$$p_i = x_i^{\mu-1} Y^{1-\mu} P$$

To simplify the problem, we can normalize total labour supply to unity<sup>13</sup>; as the total measure of firms is also unity, by imposing equilibrium on the labour market, we get the following current-value hamiltonian, for the typical intermediate goods producer:

$$\mathcal{H}_i = PY^{1-\mu}(K_i^\alpha l_{1i}^{1-\alpha} S_i^\beta)^\mu - w - PI + \lambda_1 \phi S(1-l_{1i}) + \lambda_2 I$$

where  $\lambda_1$  and  $\lambda_2$  are the costate variables. Using the final goods as numeraire, therefore normalizing  $P$  to unity, we can get some slight further simplifications, obtaining:

$$\mathcal{H}_i = Y^{1-\mu}(K_i^\alpha l_{1i}^{1-\alpha} S_i^\beta)^\mu - w - I + \lambda_1 \phi S(1-l_{1i}) + \lambda_2 I$$

The first-order (necessary) conditions are:

$$\mathcal{H}_{l_1} = \mu(1-\alpha) Y^{1-\mu} K_i^{\mu\alpha} l_{1i}^{\mu(1-\alpha)-1} S_i^{\beta\mu} - \lambda_1 \phi S = 0 \quad (8)$$

$$\mathcal{H}_I = \lambda_2 = 1 \quad (9)$$

$$\mathcal{H}_K = \mu\alpha Y^{1-\mu} K_i^{\mu\alpha-1} l_{1i}^{\mu(1-\alpha)} S_i^{\beta\mu} = -d\lambda_2/dt + r\lambda_2 \quad (10)$$

$$\mathcal{H}_S = \mu\beta Y^{1-\mu} K_i^{\mu\alpha} l_{1i}^{\mu(1-\alpha)} S_i^{\beta\mu-1} = -d\lambda_1/dt + r\lambda_1 \quad (11)$$

In a symmetric equilibrium, given the hypothesis concerning the total measure of the firm, aggregate output of final goods is equal to the production of a single intermediate firm. It is easy to verify, using (7), that the set of necessary conditions can be expressed as:

$$\mu(1-\alpha) Y/l_{1i} - \lambda_1 \phi S = 0 \quad (8')$$

$$\mu\alpha Y/K_i = r \quad (10')$$

$$\mu\beta Y/S_i = -d\lambda_1/dt + r\lambda_1 \quad (11')$$

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(13) This conceals the dependence of the growth rate on the size of population, which characterizes Romer (1987) and Grossman Helpman (1991b) models.

It is also possible to show that each firm, at any point of time, acts so as to equalize the fraction  $\mu$  of the marginal productivity of labour to the market wage rate.

The set of necessary conditions can be used to work out the proportional growth solution.

Equation (8) can be expressed as:

$$Y^{1-\mu} K_i^{\mu\alpha} l_{1i}^{\mu(1-\alpha)} S_i^{\beta\mu-1} = \frac{\lambda_1 \phi l_1}{\mu(1-\alpha)}$$

hence, substituting it into (11), we get:

$$\frac{\beta}{1-\alpha} \lambda_1 \phi l_1 = -d\lambda_1/dt + r\lambda_1.$$

Thus,

$$\frac{d\lambda_1/dt}{\lambda_1} = r - \frac{\beta}{1-\alpha} \phi l_1 \quad (12)$$

Differentiating (8), we get:

$$\frac{d\lambda_1/dt}{\lambda_1} + \frac{dS/dt}{S} = (1-\mu) \frac{dY/dt}{Y} + \mu\alpha \frac{dK_i/dt}{K_i} + \mu\beta \frac{dS_i/dt}{S_i} \quad (13)$$

(as  $dl_1/dt=0$  in the long run)

Then, from equation (7):

$$\frac{dx_i/dt}{x_i} = \alpha \frac{dK_i/dt}{K_i} + \beta \frac{dS_i/dt}{S_i} \quad (14)$$

Hence, defining  $(dS_i/dt)/S_i$  as  $v_i$ , we can obtain,  $g_i$ , the rate of growth common to output and capital in the  $i$ -th sector:

$$g_i = \frac{\beta}{1-\alpha} v_i \quad (15)$$

In a symmetric equilibrium  $g_i=g$  and  $v_i=v$ ,  $i \in [0,1]$ , therefore equation

(13) becomes:

$$\begin{aligned} \frac{d\lambda_1/dt}{\lambda_1} &= (1-\mu) \frac{dY/dt}{Y} + \mu \left[ \alpha \frac{dK_i/dt}{K_i} + \beta \frac{dS_i/dt}{S_i} \right] - \frac{dS/dt}{S} = \\ &= [1-\mu(1-\alpha)]g - (\mu\beta-1)v = \end{aligned} \quad (16)$$

using (15) and symmetric equilibrium,

$$= \left[ 1 - \frac{1-\alpha}{\beta} \right] g.$$

Equating this expression to (12) and using (6') to substitute out  $l_2$ , we get:

$$\left[ 1 - \frac{1-\alpha}{\beta} \right] g = r - \frac{\beta}{1-\alpha} \phi (1-l_2) = r - \frac{\beta}{1-\alpha} \phi \left( 1 - \frac{y}{\phi} \right)$$

Hence, substituting  $v$  and solving for  $g$ :

$$g = \frac{\beta}{1-\alpha} \left[ \frac{\beta}{1-\alpha} \phi - r \right] \quad (17)$$

The growth rate is a positive function of  $\phi$ , the exogenous parameter which characterizes the "research" sector, and is negatively related with the interest rate.

It is interesting to note that the partial solution for the growth rate, in contrast with the existing literature, is not affected by the degree of monopolistic competition.

To understand this result, we can reformulate equation (16), by use of (14):

$$\frac{d\lambda_1/dt}{\lambda_1} = (1-\mu) \frac{dY/dt}{Y} + \mu \frac{dx_i/dt}{x_i} - \frac{dS/dt}{S}$$

The relation between the growth rate of knowledge marginal value and the growth rate of firm  $i$ 's output is weakened by monopolistic competition (since  $\mu < 1$ ). However, this market structure implies a relation between  $\lambda_1$  and aggregate output which is not present when markets are perfectly competitive (the case  $\mu = 1$ ). In the symmetric setting this "macroeconomic

externality" exactly offsets the former negative effect.

Another distinctive feature of this model consists in the fact that the representative firm, in general, does not invest all the profit gained exploiting its (partial) monopolistic power. For example, we can compute the value of the firm in a steady state equilibrium, under the simplifying assumption that it borrows the whole of physical capital from families<sup>14</sup>, showing the existence of pure profit.

The value of the representative firm, under this hypothesis concerning its financial structure, is the sum of discounted pure profits and of the capital used in production, and it turns out to be:

$$V_i(t) = \max_t \int_t^{\infty} (x_i(\tau) - w(\tau) - r(\tau)K_i(\tau)) \exp \left[ - \int_t^{\tau} r(z) dz \right] d\tau + K_i(t)$$

Using the first order conditions 10' and the fact that the wage is a fraction  $\mu$  of the labour marginal productivity, we get:

$$V_i^*(t) = \int_t^{\infty} \left[ 1 - \frac{(1-\alpha)\mu}{l_1(\tau)} - \alpha\mu \right] Y(\tau) \exp \left[ - \int_t^{\tau} r(z) dz \right] d\tau + \frac{\mu\alpha Y(t)}{r(t)}$$

In a steady growth equilibrium this expression becomes noticeably simpler: the interest and the growth rate are constant, the labour share used to produce goods is also constant and equal to  $(\phi-v)/\phi$ , hence integrating we get:

$$V_i^*(t) = \left[ 1 - \frac{(1-\alpha)\mu\phi}{\phi-v} - \alpha\mu \right] \frac{Y(t)}{r-g} + \frac{\mu\alpha Y(t)}{r}. \quad (18)$$

The first addendum in (18) represents the discounted stream of pure profits; it is immediate to see that, in a steady growth equilibrium, it will disappear only if:

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(14) As we act in a deterministic environment, it is immaterial whether investment is financed from borrowing or from issuing equities.



$$\left[ 1 - \mu(1-\alpha) \frac{\phi}{\phi-v} \right] = \mu\alpha$$

i.e. if:

$$\mu^* = \frac{\phi - v}{\phi - \alpha v}$$

Hence, with  $\mu < \mu^*$  we would expect the emergence of pure profit in the long run; on the contrary, with  $\mu > \mu^*$ , the model collapses. In this case, in fact, firms, even if acting optimally, are not able to grant the payment of the rental price of capital<sup>15</sup>.

#### 4) Consumer behaviour and optimal growth with infinite lives.

##### 4.1) Intertemporal behaviour of the representative agent.

Accepting the standard hypothesis of a time-separable, constant elasticity of substitution, utility function, the household's maximization problem can be written as follows:

$$\max_t \int_t^{\infty} [c(\tau)^{1-\sigma}/(1-\sigma)] \exp[-\theta(\tau-t)] d\tau$$

$$\text{s.t. } \frac{da(t)}{dt} = r(t)a(t) + w(t) - l(t) - c(t)$$

where  $a(t)$  is the non human wealth at time  $t$ ,  $w(t)$  the labour income,  $l(t)$  is the lump-sum tax and  $\sigma$  is the reciprocal of the constant elasticity of intertemporal substitution. To avoid an explosive accumulation of debt, the representative consumer is required to take account also of the "no-Ponzi game" condition:

$$\lim_{\tau \rightarrow \infty} a(\tau) \exp \left\{ \int_t^{\tau} r(z) dz \right\} = 0 \quad (19)$$

---

(15) One can consider  $\mu^*$  the maximum level of competition compatible with a decentralized economy and with the growth rate  $g = (\beta/1-\alpha)v$ .

The necessary conditions for this problem are:

$$c(\tau)^{-\sigma} = \lambda_3$$

$$r\lambda_3 = -d\lambda_3/dt + \theta\lambda_3$$

where  $\lambda_3$  is the costate variable associated with individual assets.

The first order conditions can be summarized by:

$$dc(\tau)/dt = (1/\sigma)(r-\theta)c(\tau) \quad (20)$$

Equation (12) implies that consumption can grow at a steady rate only if:

$$r = \theta + \sigma g. \quad (21)$$

#### 4.2) Determination of the growth rate.

Solving the system composed of equation (17) and (21) we get the steady state growth rate for the economy:

$$g = \frac{\beta^2 \phi - \beta(1-\alpha) \theta}{(1-\alpha)(1-\alpha + \sigma\beta)} \quad (22)$$

The growth rate displays the usual negative dependence on the intertemporal preference and on the reciprocal of the intertemporal elasticity of substitution: the higher these parameters are, the less willing is the representative consumer to substitute present for future consumption. Also the positive relation of the growth rate to the exogenous parameter characterising research is quite obvious.

More interestingly, equation (22) does not show any relation between the growth rate and the degree of competition. Intuitively, such a relation should exist, since the presence of monopoly reduces, *coeteris paribus*, the interest rate, and this affects both consumers and firms behaviour.

Formally, this "neutrality" result is due to the fact that the competition parameter and the marginal productivity of capital do not enter separately into equations (17) and (21), which so determine only the interest rate; if  $\mu$  varies, the capital stock, in the long run, adjusts to keep the interest rate

unchanged. Therefore, the fact that  $\mu$  does not explicitly appear into (17) has deep consequences.

Moreover, let  $\pi$  be the pure profit to output ratio and recall, from (18), that, in a steady state equilibrium,  $\pi = 1 - \frac{(1-\alpha)\mu\phi}{\phi-v} - \alpha\mu$ . Hence, by use of (22) we can compute:

$$\frac{\partial\pi}{\partial\phi} = \frac{(1-\alpha)\mu}{(\phi-v)^2} \left[ v - \phi \frac{\partial v}{\partial\phi} \right] = \frac{(1-\alpha)\mu}{(\phi-v)^2} \frac{-(1-\alpha)\theta}{(1-\alpha + \sigma\beta)} < 0$$

and

$$\frac{\partial\pi}{\partial\theta} = \frac{(1-\alpha)\mu\phi}{(\phi-v)^2} \left[ - \frac{\partial v}{\partial\theta} \right] = \frac{(1-\alpha)\mu\phi}{(\phi-v)^2} \frac{(1-\alpha)}{(1-\alpha + \sigma\beta)} > 0$$

Therefore, we can conclude that the model unambiguously predicts a negative steady state relation between the growth rate and the share of pure profit.

If we set  $\beta=1-\alpha$  (i.e. if we introduce the hypothesis that technical progress is labour enhancing, so that  $Y = K_i^\alpha (l_{i1} S_i)^{1-\alpha}$ )<sup>16</sup> the expression for the growth rate can be simplified to:

$$g = \frac{\phi-\theta}{1+\sigma} \tag{22'}$$

In this case, the growth rate becomes independent from the factors' marginal productivities; therefore the growth rate gains independence also from income distribution. The special case of the above equation (22') also allows the growth rate of knowledge to be equal to that for physical quantities, permitting some simplifications in the analysis of steady growth.

#### 4.3) The command optimum

Suppose that a planner aims at maximising welfare at time  $t$ : facing a representative consumer, he has simply to solve the following problem:

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(16) With Cobb-Douglas production functions any form of technical progress is Harrod neutral, so the hypothesis has been qualified as above.

$$\max_t \int_t^{\infty} [c(\tau)^{1-\sigma}/(1-\sigma)] \exp[-\theta(\tau-t)] d\tau$$

subject to:

$$dK(\tau)/dt = K(\tau)^\alpha l_1(\tau)^{1-\alpha} S(\tau)^\beta - c(\tau)$$

and

$$dS(\tau)/dt = \phi S(\tau) (1 - l_1(\tau)),$$

(where, for aggregate output, use has been made of the hypothesis of unit measure for firms)

After some simplification, and dropping the time indexes, the set of necessary conditions can be expressed as:

$$c^{-\sigma} = \xi_1$$

$$dc/dt = (1/\sigma)(\alpha Y/K - \theta) c$$

$$(1-\alpha) Y/l_1 - \left( \frac{\xi_2}{\xi_1} \right) \phi S = 0$$

$$\beta Y/S + \left( \frac{\xi_2}{\xi_1} \right) \phi (1-l_1) = - \left( \frac{d\xi_2/dt}{\xi_1} \right) + \theta \left( \frac{\xi_2}{\xi_1} \right)$$

where  $\xi_2$  is the costate variable associated with the stock of knowledge and  $\xi_1$  is the one associated with that of capital. If we define  $\xi$  as the ratio between the two costate variables<sup>17</sup>, using the fact that  $d\xi/dt = d\xi_2/dt/\xi_1 - [(d\xi_1/dt)/\xi_1]\xi$ , the system can be simplified further to get:

$$dc/dt = (1/\sigma)(\alpha Y/K - \theta) c \tag{23}$$

$$(1-\alpha) Y/l_1 - \xi \phi S = 0 \tag{24}$$

$$\beta Y/S = - d\xi/dt - [(d\xi_1/dt)/\xi_1]\xi + \xi [\theta - \phi(1-l_1)] \tag{25}$$

or, as  $c^{-\sigma} = \xi_1$ ,

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(17)  $\xi$  can be interpreted as the ratio between the marginal values, at time  $\tau$ , of capital and knowledge.

$$\beta Y/S = - d\xi/dt + \xi [\theta + \sigma (dc/dt)/c - \phi(1-l_1)]$$

Following the same procedure used above we can work out the optimal steady state growth rate, which is:

$$g^* = \frac{\beta\phi - (1 - \alpha)\theta}{(1 - \alpha)\sigma} \quad \left[ = \frac{\phi - \theta}{\sigma} \quad \text{if } \beta = (1 - \alpha) \right]$$

The externality problem embedded in the model, which is apparent in the addendum  $-\xi\phi(1-l_1)$  in equation (25), causes the "command" optimum growth rate,  $g^*$ , to be higher than the market one. (Compare with equation (22))

Consider, now, the system composed of the two sets of first order conditions, derived from the "decentralized" maximization pursued by families and firms, acting atomistically. After some simplification, it can be expressed as:

$$dc/dt = (1/\sigma)(\mu\alpha Y/K - \theta) c$$

$$\mu(1-\alpha) Y/l_1 - \lambda_1\phi S = 0$$

$$\mu\beta Y/S = - d\lambda_1/dt + \mu\alpha Y/K\lambda_1$$

and

$$\mu\alpha Y/K = r$$

Inspection of the two sets of conditions shows that, to reach the optimum, factors' marginal productivities must be equalized to their marginal valuations. Thus, a planner wishing to decentralize decisions has to offset the static distortion caused by monopolistic competition; this can be obtained subsidizing production of every commodity at the rate  $(1/\mu)-1$ . The central planner has also to correct the distortion due to the presence of the externality in research. The private marginal value of knowledge,  $\lambda_1$ , must be reduced to  $\xi$ . This can be implemented via a subsidy to "research and development" expenditure<sup>18</sup>, which must be time-varying, while the

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(18) This result could be obtained transferring to firms a share of the wage bill for research staff.

economic system is approaching the steady state growth rate.

A government implementing such a policy can balance its budget by levying lump-sum taxes on the firms or on the families; however public debt is neutral in this model, as wealth does not affect the growth rate of consumption (equation (20)).

## 5) Consumer behaviour and optimal growth with finite lives.

It has already been pointed out that, in general, the market structure considered in this paper entails the presence of pure profit. Within the infinite lives case, one does not need to consider the way in which such income is distributed to the household, insofar as this distribution is egalitarian. In the Ramsey case, consumption evolution depends only on the difference between the market interest rate and the subjective time preference rate, so that, to keep everything as simple as possible, one can consider pure profit as a lump-sum transfer from firms to the representative agents. Similarly, in various papers on endogenous growth and overlapping generations it is assumed that profits, arising as a consequence of externalities, are handed over to consumers regardless of their age. (Alogoskoufis and Van del Ploeg, 1990a, p. 6)

In Blanchard's framework, which will be used in this section, people have a potentially infinite horizon, but face, at each instant of time, a constant probability of death. As shown by Blanchard (1985, pp. 227-9 in particular), this limited uncertainty affects the relation between consumption and wealth. Therefore the way in which income is distributed becomes relevant, as it may influence the assets' total value. For this reason, a partial modification to the consumer budget constraint seems necessary. Accepting the hypothesis that profits are distributed to shareholders, as equities and physical capital in this nonstochastic framework are perfect substitutes, it seems sensible to assume that the private sector's overall assets are equal to the firm's total value<sup>19</sup>.

### 5.1) The consumer problem: a restricted version

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(19) See Rankin and Scalera, 1991, pp. 13 ff., for a similar approach in a different context.

To simplify the analysis, a logarithmic specification for the time separable utility function is used. Thus the representative individual born at time  $s$  maximises, at time  $t$ , the functional:

$$\max_{\{c(\tau,s)\}} E \left[ \int_t^{\infty} \ln(c(\tau,s)) \exp[-\theta(\tau-t)] d\tau \mid \Omega_t \right]$$

$$\text{s.t. } \frac{da(t,s)}{dt} = r(t)a(t,s) + w(t) - l(t) - c(t,s)$$

Where  $\Omega_t$  is the information set at time  $t$ ; a "no-Ponzi" game condition also applies.

Following the usual methods it is possible to show that the aggregate behaviour can be summarized by the system:

$$\frac{dC}{dt} = (r - \theta)C - p(p + \theta)(V+D) \quad (26)$$

$$\frac{dK}{dt} = Y - C - G \quad (27)$$

where  $p$  is the instant probability of death; the substitution of  $V$  for  $K$  comes from our hypothesis concerning the distribution of profits; the time index has been suppressed. As equation (26) and (27) are non autonomous, they can now be "deflated" by using income, so that the system can be rewritten as:

$$\frac{dz}{dt} + gz = (r - \theta)z - p(p + \theta)(v+d)$$

$$\frac{dx}{dt} + gx = 1 - z - f$$

where  $z$ ,  $x$ ,  $d$  and  $f$  are the ratio of consumption, capital, public debt and government expenditure to income, respectively;  $v$  is the ratio between total value of the firm and income.

## 5.2) A steady state solution for the model.

We now follow again the existing literature (e.g. Lucas 1988, p. 21, Romer 1990, p. 1020) in restricting the analysis to the proportional growth solution for the system.

A further limitation is also introduced: the case  $\beta=1-\alpha$ , which permits a briefer parametrization, is considered. In a steady state equilibrium, we can substitute out  $v^{20}$ ; then, recalling from (17) that in this restricted case  $r=\phi-g$ , we get:

$$\frac{dz}{dt} = (r - \theta - g) z - p(p+\theta) \left[ \frac{\mu\alpha}{\phi - g} + \frac{(1 - \mu) \phi - (1 - \mu\alpha) g}{(\phi - g) (\phi - 2g)} + d \right] \quad (28)$$

$$\frac{dx}{dt} = 1 - z - \frac{g \mu\alpha}{\phi - g} - f \quad (29)$$

To obtain the possible steady state solutions for the model, we have now to set to zero  $dx/dt$  and  $dz/dt$  in the previous equations (28) and (29) and then solve for the growth rate the resulting expression:

$$(\phi - \theta - 2g) \left[ 1 - f - \mu \frac{\alpha g}{\phi - g} \right] = p(p + \theta) \left[ \frac{(1 - \mu (1 - \alpha)) \phi - (1 + \mu\alpha) g}{(\phi - g) (\phi - 2g)} + d \right] \quad (30)$$

To help in looking for solutions, we define:

$$A(g) = (\phi - \theta - 2g) \left[ 1 - f - \mu \frac{\alpha g}{\phi - g} \right]$$

which is the left hand side of (30), and

$$B(g) = p(p + \theta) \left[ \frac{(1 - \mu (1 - \alpha)) \phi - (1 + \mu\alpha) g}{(\phi - g) (\phi - 2g)} + d \right]$$

which is the right hand side of (30), and study separately these two main addenda.

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(20) Dividing (18) by  $Y(t)$  one gets the long run equilibrium value for  $v$ , i.e.:

$$v = \left[ 1 - \frac{(1-\alpha)\mu\phi}{\phi-v} - \alpha\mu \right] \frac{1}{r-g} + \frac{\mu\alpha}{r}$$



The properties of  $A(g)$  in the interval  $[0, \phi)$  are represented in figure 1.  $A(g)$  displays a vertical asymptote at  $g = \phi$ ; at a zero growth rate it assumes the value:

$$A(0) = (1 - f)(\phi - \theta) > 0;$$

when  $g$  approaches  $r$  (so that  $g = \phi/2$ ), the corresponding value is:

$$A\left(\frac{\phi}{2}\right) = \theta(\alpha\mu + f - 1),$$

which is less than zero if:

$$f < 1 - \alpha\mu. \tag{31}$$

This condition has a precise economic meaning: it requires the consumption/output ratio to be positive even when  $g$  reaches  $\phi/2$ , the value at which the growth and the interest rate are equal.

Setting  $A(g) = 0$ , we get:

$$g_1 = \frac{\phi - \theta}{2},$$

which corresponds to the "Ramsey" solution (see equation (22')), and:

$$g_2 = \frac{\phi(1 - f)}{1 - f + \alpha\mu}$$

If condition (31) holds, it is possible to show that  $g_2 > \phi/2$ , and hence also that  $g_1 < g_2$ . Notice also that, for  $g > g_2$ , the consumption/income ratio is negative; in this interval no sensible long run solution is therefore possible.

Consider now the first derivative of  $A(g)$ ,

$$\frac{\partial A}{\partial g} = - \frac{2g^2(1 - f + \alpha\mu) - 4\phi g(1 - f + \alpha\mu) + \phi(\alpha\mu(\phi - \theta) - 2\phi(f - 1))}{(\phi - g)^2}$$

and notice that it is positive between  $g_{\min}$  and  $g_{\max}$ , where:

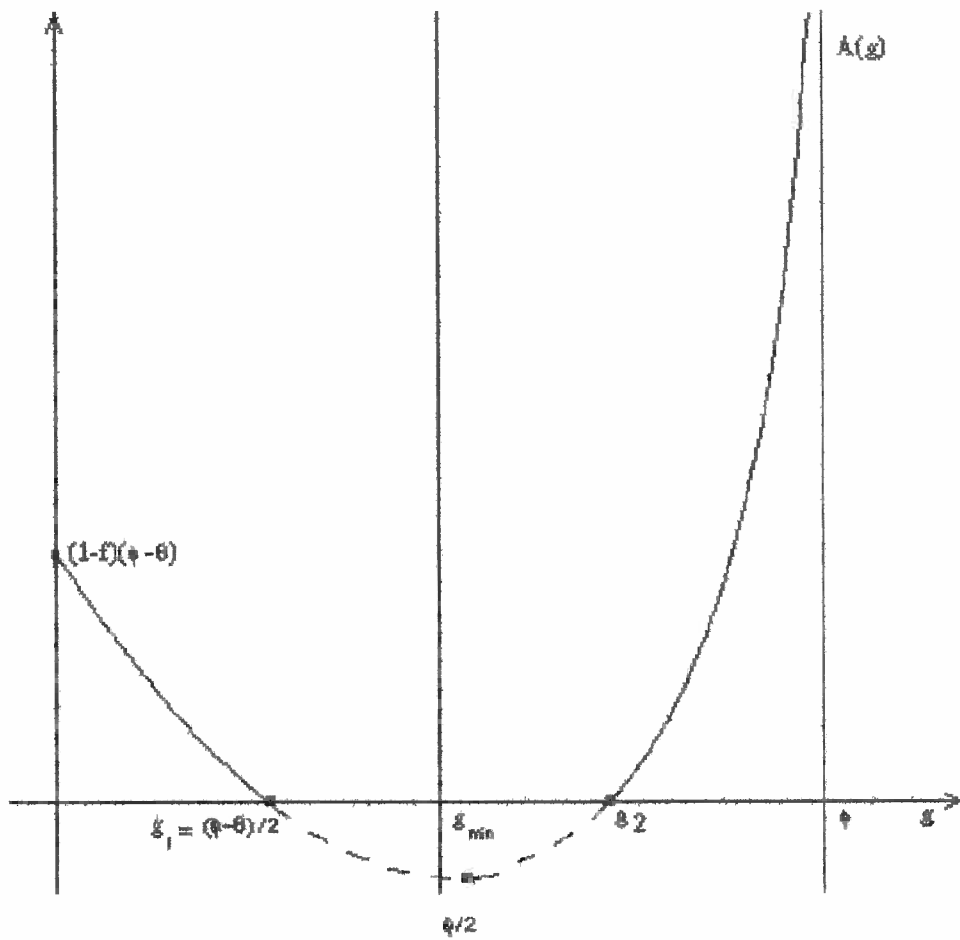


Fig. 1 : behaviour of  $A(g)$ .

$$g_{\min} = \phi - \left( \frac{\alpha\mu\phi(\phi + \theta)}{2(\alpha\mu + 1 - f)} \right)^{1/2}$$

$$g_{\max} = \phi + \left( \frac{\alpha\mu\phi(\phi + \theta)}{2(\alpha\mu + 1 - f)} \right)^{1/2}$$

Finally, notice that  $g_{\min}$  is bigger than  $(\phi - \theta)/2$  if:

$$f < 1 - \alpha\mu \left( \frac{\phi - \theta}{\phi + \theta} \right)$$

and that this condition is encompassed by (31).

Focusing our attention on  $B(g)$ , we notice that it is just a multiple of the ratio between total assets and income. As  $p(p+\theta)$  can never be negative, for  $g_1 < g < g_2$  no sensible long run equilibrium is possible: a negative  $B(g)$  is not acceptable. Thus it is sufficient to study  $B(g)$  in the interval  $[0, \phi/2]$ . The value for  $B(g)$  at a zero growth rate is:

$$B(0) = p(p+\theta) \left[ \frac{1 - \mu(1 - \alpha)}{\phi} + d \right]$$

At  $\phi/2$ ,  $B(g)$  has an asymptote because the growth rate approaches the interest rate, causing the explosion of the firm value. The limit for  $g$  approaching  $\phi/2$  depends on the value of the firm: if it is always positive, as previously required, the limit must approach plus infinity.

As the properties of the derivative of  $B(g)$  are the same of those of the derivative of the firm's steady state value, consider:

$$v(g) = \left[ \frac{(1 - \mu(1 - \alpha))\phi - (1 + \mu\alpha)g}{(\phi - g)(\phi - 2g)} \right] = \frac{N(g)}{D(g)}$$

The numerator of this expression,  $N(g)$ , is positive if  $g < \underline{g} = \frac{1 - \mu(1 - \alpha)}{1 + \mu\alpha}\phi$ ; some simple algebra shows that  $\underline{g} > \phi/2$  if:

$$\mu < 1/(2 - \alpha) \tag{32}$$

The denominator of  $v(g)$ ,  $D(g)$ , is always positive, except for the interval  $[\phi/2, \phi]$ . To establish the behaviour of the derivative of  $v(g)$  for

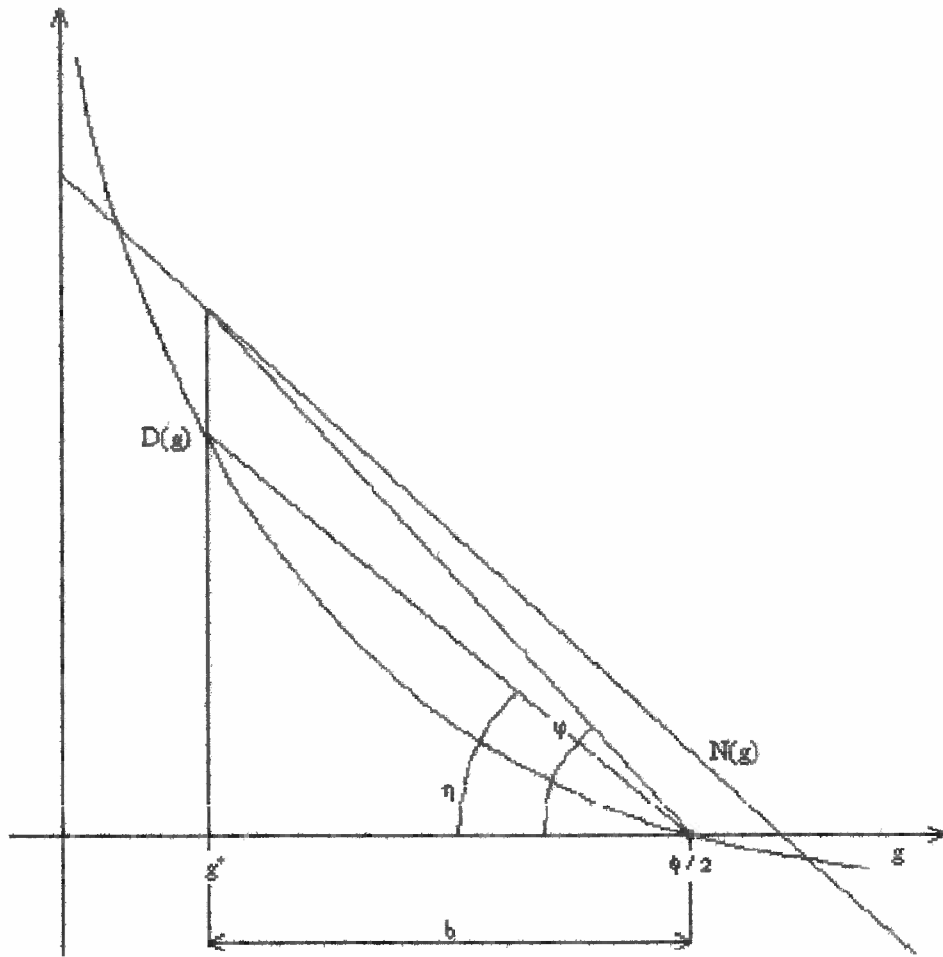


Fig. 2: as  $g^*$  approaches  $\phi/2$ , the ratio of  $N(g^*)$  to  $D(g^*)$  increases.

$g \in [0, \phi/2)$ , consider figure 2.  $N(g)$  can be expressed as  $btg(\varphi)$  and  $D(g)$  as  $btg(\eta)$ . When  $g$  increases,  $tg(\varphi)$  increases, while  $tg(\eta)$  decreases; therefore the ratio  $D(g)/N(g)$  is always increasing and the derivative of  $\varkappa(g)$  is positive.<sup>21</sup>

Notice that condition (32) implies both a positive value for the firm and a positive derivative of this value with respect to the growth rate.

It is now possible to draw figure 3, which depicts  $A(g)$  and  $B(g)$  in the interval  $[0, \phi/2)$ ; this is helpful also to recognise that, to have a non negative solution for  $g$ , a third condition is required. In fact, we need:

$$(1-f)(\phi-\theta) > p(p+\theta) \left[ \frac{1 - \mu(1-\alpha)}{\phi} + d \right] \quad (33)$$

This condition implies that, if the probability of death or the asset/income ratio are too high the growth process can not take off, because the steady state saving is too low. For the same reason, the higher is the ratio between government consumption and national product, the lower will be the growth rate.

### 5.3) Two non-neutrality results.

The effect on the growth rate of an increase of the debt/income ratio, which is considered as a policy instrument, can be seen analysing equation (30).  $A(g)$  is clearly unaffected, while  $B(g)$  is shifted upwards. Therefore, with a positive probability of death, an increase in  $d$  unambiguously reduces the growth rate. (See fig. 4) In fact, such a policy action, raising consumption, increases the interest rates making research more costly and lowering the equilibrium capital/output ratio.

The debt non neutrality has already been pointed out in endogenous growth models where the capital/output ratio is exogenous and the "engine for growth" is provided by externalities only, without any need for research. (Alogoskoufis and Van der Ploeg, 1990a, 1990b; Saint Paul 1991; Buiter 1991) A more complex formulation for the production side of the economic system does not seem, therefore, to affect this result.

In particular, in these models, dynamic inefficiency can never occur because the linearity in capital which characterizes the aggregate technology prevents the marginal productivity of capital from diverging from the

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(21) Notice that:  $(\partial N(\phi/2)/\partial g)|_{\phi/2=\phi}$  and that  $(\partial D(\phi/2)/\partial g)|_{\phi/2=\phi} = -(1+\mu\alpha)$ ; therefore, even if  $\mu=1/(2-\alpha)$ , the derivative of  $\varkappa(g)$  is strictly positive.

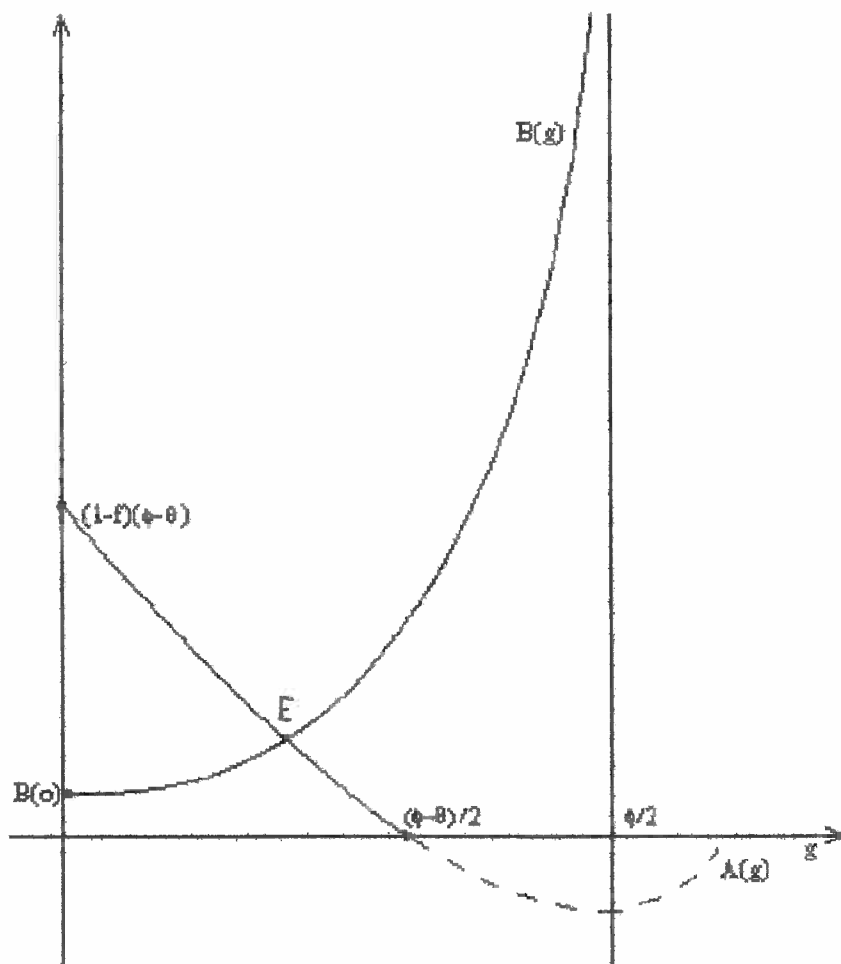


Fig. 3: the steady state equilibrium (E).

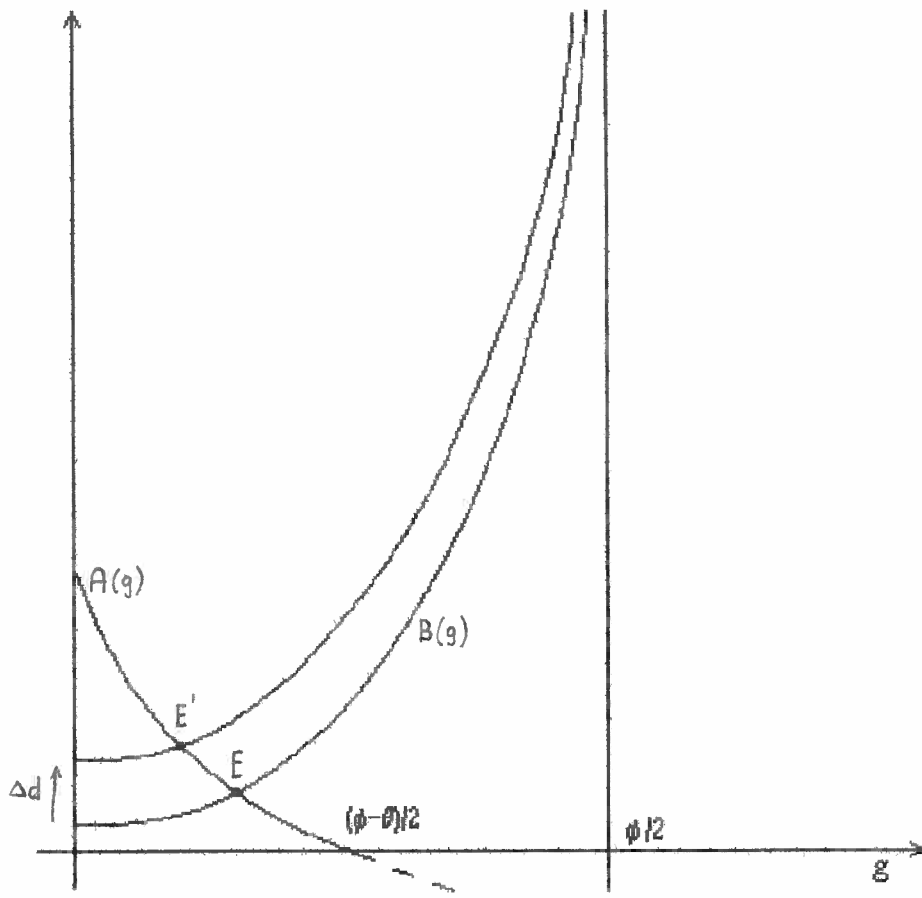


Fig. 4: an increase in the debt/output ratio.

average one, precluding the possibility of any production inefficiency. (Saint Paul, 1990, p. 7-8; Buiter 1991, p. 22-23). In the model presented here, in contrast with those quoted above, the interest rate is endogenous, so that production inefficiencies are not ruled out a priori; therefore, it becomes clearer that the absence of dynamic inefficiency is basically due to the fact that any increase in the debt/output ratio unambiguously reduces the growth rate, so that one can always find a future generation, appearing far enough from the present, which is harmed by this policy (The same point has been made by Saint Paul, 1990, p. 9-10).

In the infinite lives case the growth rate was independent also of the degree of competition. This does not hold true any more:  $\mu$  influences both equation (28) and (29). More precisely, an (exogenous) increase in  $\mu$  shifts downwards  $A(g)$  in the interval  $[0, (\phi - \theta)/2)$  and  $B(g)$  in the interval  $[0, \phi/2)$ :

$$\frac{\partial A}{\partial \mu} = - \frac{(\phi - \theta - 2g) \alpha g}{\phi - g} < 0, \text{ for } g < \frac{\phi - \theta}{2},$$

$$\frac{\partial B}{\partial \mu} = \frac{p(p + \theta) (\phi (\alpha - 1) - \alpha g)}{(\phi - 2g) (\phi - g)} < 0, \text{ for } g < \frac{\phi}{2}$$

(see fig. 5)

The effect on  $A(g)$  is due to the fact that an increase in  $\mu$  raises the equilibrium capital/output ratio, reducing the growth rate for a given volume of savings.  $B(g)$  is shifted downwards because the firm value decreases with  $\mu$ , and this, in a finite lives framework, reduces consumption. We can try to determine the sign of this effect on a comparative static basis, studying the equation:

$$\frac{\partial A}{\partial g} dg + \frac{\partial A}{\partial \mu} d\mu = \frac{\partial B}{\partial g} dg + \frac{\partial B}{\partial \mu} d\mu$$

or:

$$\frac{dg}{d\mu} = \frac{\partial B/\partial \mu - \partial A/\partial \mu}{\partial A/\partial g - \partial B/\partial g}$$

While the sign of the denominator is negative, the one of the numerator turns out to be ambiguous. Some algebra shows that, for the interval we are interested in, this sign is positive only if:



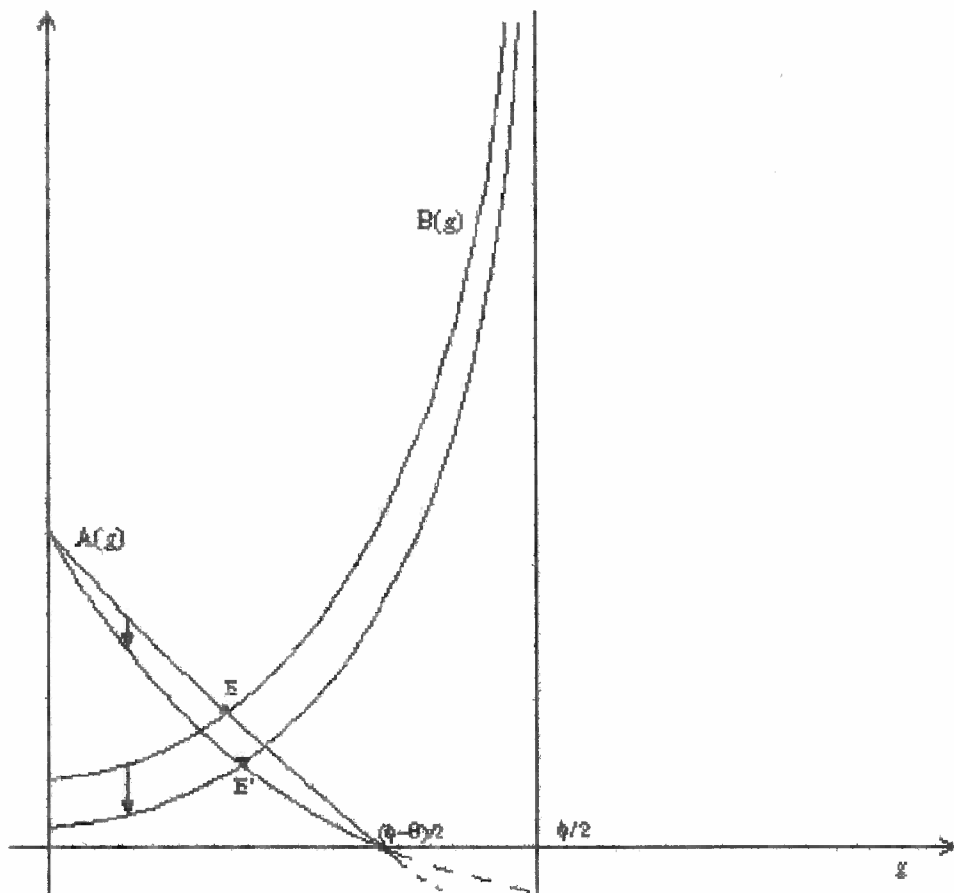


Fig. 5: an (exogenous) increase in  $\mu$ .

$$C(g) = 4\alpha g^3 + \alpha g^2(2\theta - 4\phi) + \alpha g[-p(p+\theta) + \phi^2 - \phi\theta] + p(p+\theta)\phi(\alpha - 1) > 0$$

The study of such a function does not lead to any brief condition. However, it has a local minimum when:

$$g = \frac{2\phi - \theta}{6} + \frac{(\phi^2 - \phi\theta + \theta^2 + 3p(p+\theta))^{1/2}}{6}$$

which is always larger than the "Ramsey" solution,  $(\phi - \theta)/2$ . The value for  $C((\phi - \theta)/2)$  is  $[p(p+\theta)(\alpha(\phi + \theta) - 2\phi)]/2 < 0$ .

Moreover  $C(g)$  is independent of the policy parameters,  $f$  and  $d$ ; by use of (32), we can calculate a combination of these parameters such that the growth rate is nought and condition (31) is fulfilled. As  $C(0)$  is less than 0, in such a situation an increase in  $\mu$  would unambiguously increase the steady state growth rate.

Therefore, the possibility that the growth rate, in contrast to what happens in the Grossman and Helpman model, is *positively* related with the competition level can not be rejected. However, the mere presence of this relation contrasts with the infinite lives case and can have some implication for the policy analysis.

## 6) Policy intervention and static efficiency.

Considering the "command" solution for the infinitely-lived case, a subsidy  $s$  to production was introduced. The same policy measure is now examined within the perpetual youth framework.

Under the hypothesis that the representative firm is affected only by this policy measure, its intertemporal optimization problem becomes:

$$\mathcal{H} Y^{1-\mu}(K_i^\alpha l_{1i}^{1-\alpha} S_i^\beta)^\mu - w - I + \lambda_1 \phi S(1 - l_{1i}) + \lambda_2 I$$

If we set  $s = (1 - \mu)/\mu$ , to completely offset the effects of monopolistic competition, some algebra and the hypothesis of symmetric equilibrium lead to the following set of first order conditions:

$$\mathcal{H}_{l_1} \mu(1 - \alpha) Y/l_{1i} - \lambda_1 \phi S = 0 \quad (8'')$$

$$\mathcal{H}_K \mu\alpha Y/K_i = r \quad (10'')$$

$$\mathcal{H}_S \mu \beta Y/S_i = -d\lambda_1/dt + r\lambda_1 \quad (11'')$$

It is possible to show, applying the same procedure carried over in section 3, that the partial solution for the growth rate (equation 17) is unchanged.

By use of the first order conditions, in a steady state equilibrium, the value of the subsidized firm turns out to be:

$$V^{\circ}_i(t) = \left[ \frac{1}{\mu} - \frac{(1-\alpha)\phi}{\phi-v} - \alpha \right] \frac{Y^{\circ}(t)}{r-g} + \frac{\alpha Y^{\circ}(t)}{r}$$

Correspondingly, the new value/output ratio is:

$$v^{\circ} = \frac{1}{\mu} \left[ \left[ 1 - \frac{(1-\alpha)\mu\phi}{\phi-v} - \mu\alpha \right] \frac{1}{r-g} + \frac{\mu\alpha}{r} \right]$$

As  $\mu > 1$ ,  $v^{\circ} > v$ . If we imagine, now, that it is possible to set up a system of lump sum taxes, levied on consumers, such that equation 26 is not affected<sup>22</sup>, the new solution for the finite-lived cases is to be looked for studying the following system:

$$A^{\circ}(g) = (\phi - \theta - 2g) \left[ 1 - f - \frac{\alpha g}{\phi - g} \right]$$

and

$$B^{\circ}(g) = p(p + \theta) \left[ \frac{1}{\mu} \left[ \left[ 1 - \frac{(1-\alpha)\mu\phi}{\phi-v} - \mu\alpha \right] \frac{1}{r-g} + \frac{\mu\alpha}{r} \right] + d \right]$$

$A^{\circ}(g)$  is shifted downwards, with respect to  $A(g)$ , because of the increase in the capital/output ratio;  $B^{\circ}(g)$  is shifted upwards, because of a raised  $v^{\circ}$ . Therefore, as shown in fig. 6, the long run equilibrium growth rate unambiguously decreases (from E to E'). Therefore, with finite lives, it is not any more possible to cope with the static distortion problem without affecting the growth rate.

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(22) The implementation of such a policy would however be problematical: as the volume of subsidies is higher than the labour share of output, a system of lump sum taxes should be, at least partially, age dependent; this would be discounted by agents, resulting in a change in (26). For an example of such a system of lump sum taxes, see Buiters, 1991, pp. 9 ff.

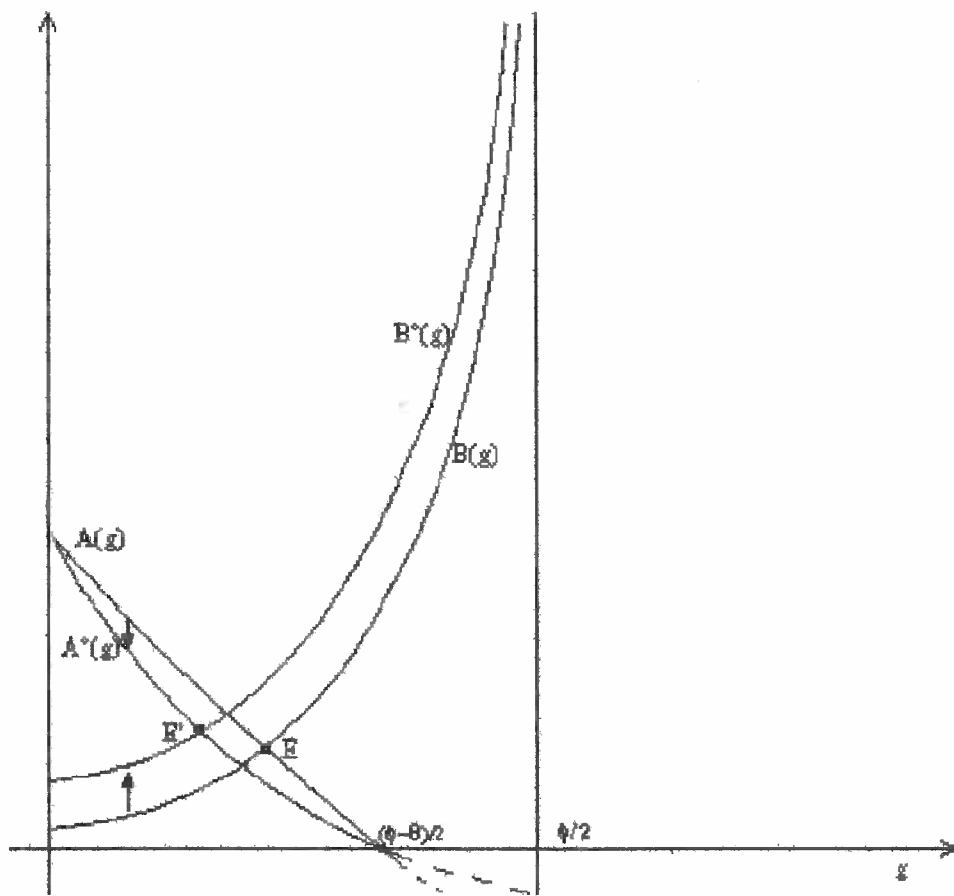


Fig. 6 : steady state effects of production subsidies.

Notice, also, that lump sum taxes levied on firms would have a beneficial effect for growth, because they reduce  $v^{\circ 23}$ . Clearly, to decide which kind of instrument should be adopted, a social welfare function has to be used. With such a tool, one can address jointly the removal of static and dynamic distortions. It seems that the planner's felicity function proposed by Calvo and Obstfeld (1988a,b) for continuous time overlapping generation models could play an important role for further developments on this point.

## 7) Concluding remarks

In this paper a model of endogenous growth with imperfect competition with deterred entry has been developed. A certain minimum degree of monopolistic power has proved to be a necessary condition for growth, because research is funded out of profits; however, provided that this condition is fulfilled, in the infinitely lived case, in contrast with the existing results, no relation has emerged between the degree of monopolistic power and the rate of growth. This result has been ascribed to the symmetry among firms and to the presence of a "macroeconomic externality" which characterizes the model.

With finite-lives agents, this neutrality result does not hold and policy actions aimed at removing the static inefficiencies caused by imperfect competition affect also the growth rate. Therefore, even for these relatively simple choices, a social welfare function proves to be necessary.

The model has been used also to check that the Ricardian debt neutrality proposition does not hold with finite lives, and some insight into the impossibility of dynamic inefficiency in endogenous growth model has been provided.

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(23) It is not possible to levy the whole of the lump sum taxes on firms, because the volume of subsidies is higher than the resulting total profits.

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