INCENTIVES TO SUPPORT THE PUBLIC SECTOR UNDER OLIGOPOLY

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Abstract: We analyse a simple economy with a consumption good, an intermediate good and a public good, where there are capitalists, shop-keepers and workers. It follows that the capitalists and the shop-keepers would always vote for a lower tax rate than the workers although preferences are identical. No group would support a larger public sector than in the first-best solution under free entry and exit among the shop-keepers. A low tax rate would increase the number of supporters of low taxes and *vice versa*.

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1. Introduction

At present, the public sector is under attack in most countries. In a first-best economy, the optimal amount of public goods would be obtained by equalising the marginal rates of substitution between the private goods and a public good with the marginal rates of transformation, as shown formally by Samuelson (1954).[1] The practical importance of this condition may be limited, but an opinion about the size of the public sector requires some definition of optimality. Our aim is to compare endogeneous attitudes towards the public sector within different groups in a distorted economy and to see whether it is likely to be larger than in the first-best solution.

Several arguments for why the public sector might be too large have been used. In the literature, the most important reason for why even the first-best level may be excessive is distortionary taxation (see, for example, Wilson, 1991a). On the other hand, it is well known that tax distortions may also increase the optimal provision from its first-best level (see, for example Atkinson and Stern, 1974 and Wilson, 1991b). Above all, it is far from certain that the excess burden of taxation is large enough to be a matter of concern.[2]

There may also be a failure in the decision making process. There is a widespread belief that the political system and the bureaucracy are biased towards excessive public sector growth. For example, in a well known article, Tullock (1959) claims that majority voting leads to over-sizedgovernment budgets. Other authors, such as Brennan and Buchanan (1977) emphasise government revenue maximisation and warn against giving 'Leviathan' access to revenue

[1] Public goods do not necessarily constitute the most important component in the growth of the public sector. However, a focus on public goods provision can be justified by the observation that the recent tendency to cut expenditures is directed not only towards transfer payments and bureaucracy but public goods and services as well.

[2] As Stiglitz (1986, p. 376) points out, there is no agreement in the literature about the whether the excess burden of taxation is in general large or small. In a well known paper by Hausman (1981), the welfare losses appear to be large, but that result has been corrected by Haveman, Gabay and Andreoni (1987). According to Ilmakunnas (1992), losses are small in Finland and she argues that models which ignore hours constraints in the labour supply tend to over-estimate the tax distortion. Stuart (1984) and Ballard (1991), like most contributions, find that the marginal efficiency costs of tax-financed government spending tend to be small.

generating instruments. Therefore, a fiscal constitution should restrict the power of the elected bodies.[3] However, these critical claims do not dominate the literature. For example, Shultze (1992) argues that there is no bias towards excessive spending in the U.S political system. According to Downs (1961) and Musgrave (1985), majority voting does not lead to over-spending; Downs (1960) claims that there may even be a tendency towards too little spending under majority rule.

At present, the notion that the public sector is too large has become part of the right-wing orthodoxy. Mainstream Economics, on the other hand, does not unanimously support the radical cuts that are fashionable at present. For example, Cullis and Jones (1987) apply modern microeconomics when criticising the tendency to roll back the frontiers of the state. There exist noteworthy empirical studies on this topic as well. A survey by Hockley and Harbour (1983) indicates that a majority of people may support more government spending rather than tax cuts if a choice is made possible.

If only few economists would claim that the public sector is too large, why then is there in many countries a pressure towards cuts? To answer this question we analyse the levels of public sector activity that different groups would prefer and relate them to the first-best optimum. More precisely, we construct a model in which the (imperfectly competitive) market structure creates three groups, namely workers, capitalists living on profits and self-employed workers. The latter are called shop-keepers. Resources have to be allocated between an intermediate good, a consumption good and a public good. The sizes of the groups depend on entry into or exit

[3] For example, Niskanen (1992), recommends an upper limit for the public debt which can be increased only by a majority of two-thirds of both Houses of the U.S. congress. The same rule should apply for new taxes or increases of the rate and base of an existing tax. He mentions Finnish experiences in support for such supermajority rules. Ironically enough, the present right-wing government in Finland has implemented simple majority rules because the earlier system allowed a minority of one-third of the Parliament to block cuts in public services.

from self-employment, unlike in the Schwab and Oates model (1991) in which, likewise, the characteristics of the residents of a community affect the level of public sector production but people vote with their feet.[4]

The public good is equally valued by all individuals. Nevertheless, it turns out that workers are always prepared to pay a higher tax rate than capitalists and shop-keepers but no group is prepared to pay as much as in the first-best allocation. A worker who owns shares becomes less willing to support the public sector. Our model of the labour market allows for no tax distortions; there is a pressure against the public sector because the opportunity cost of a given increase in its size is larger for profits than for wages, even if the tax system treats all incomes uniformly. It also turns out that a low tax rate increases the number of supporters for a low-tax policy and *vice versa*. However, without an analysis of the political system, it appears that voters would favour a low-tax party only if the economy is dominated by small firms.

The paper is organised as follows. Section 2 presents the basic model, describing the actors of the economy (2.1), the first-best allocation (2.2) and the market allocation given the tax rate (2.3). Section 3 considers attitudes towards the public sector so that 3.1 compares tax rates favoured by different groups, 3.2 analyses the influence of share ownership on the attitudes of the workers and 3.3 asks whether the public sector is 'too large'. Section 4 is devoted to some likely outcomes of a majority rule. Replicating the number of industries in 4.1 facilitates an assessment of the conditions under which the low-tax party is likely to get a majority (4.2) and an analysis of changes in the electorate as a response to the tax rate (4.3). Finally, there are some concluding remarks in Section 5.

[4] Cuts of public expenditures are not only based on arguments about excessive size relative to individual preferences. They are often motivated by the claim that a relatively heavy tax burden would be a competitive disadvantage given the policy of other countries. This may partly explain how the tendency to restrict the public sector has spread from countries with low average tax rates to Scandinavia and Finland. As Sinn (1990) has shown, using low tax rates when competing for investments or market shares is likely to lead to a suboptimal allocation. We hope to be able to address this issue in another paper.

2. The basic model

2.1. A description of the economy

The economy has three commodities, x_1, x_2, and x_3 respectively. We assume that x_1 is not consumed but used as an input for producing x_2 which is a private good. The third commodity is a public good which is provided by the state and it is financed by taxing all incomes. We measure the public good in units of labour time. N_1 and N_2 persons respectively are employed to produce the private goods and N_3 to produce the public good. Note that we write n_2 instead of N_2 if the producers of good 2 are self-employed in analogy with the notation used for firms. All individuals provide one unit of labour.

Each unit of x_2 requires a units of x_1. The labour input coefficients are l_1 and l_2 respectively. However, Industry 2 can be organised either so that the employees are wage earners or entrepreneurs. In the former case they get the same wage as in Industry 1. In the latter case each individual is a self-employed worker (shop-keeper) living on the residual after the inputs from Industry 1 have been paid.

Suppose that all individuals value x_2 and x_3 identically. In particular, their preferences with respect to the private and the public good do not depend on whether they are capitalists, workers or independent shopkeepers. Each worker supplies one unit of labour per period. The utility functions are of a Cobb-Douglas type. Let A and B be positive parameters such that $A + B = 1$. The utility for the j:th individual in an N-person economy can then be written:

$$U = A \log x_{2j} + B \log x_3 \tag{1}$$

Thus, if both goods were private, all individuals would divide their budgets in the proportions A and B. As we shall see in 2.2, when one good is public, society would, under ideal conditions, allocate its resources between private and public goods in those same proportions. However, since the individual cannot decide the amount of the public good, the second term to the right in (1) is a constant. Therefore, the individual demand functions depend on their budget restriction only.

2.2. The first-best allocation

As a point of comparison, consider the first-best allocation in which the prices p_1 and p_2 of x_1 and x_2 equal marginal costs. If all workers earn the same wage rate w, the price of x_1 is:

$$p_1 = l_1 w \tag{2}$$

Marginal costs in Industry 2 are $ap_1 + l_2 w$; making use of (2) yields:

$$p_2 = al_1 w + l_2 w \tag{3}$$

The public good is financed by a lump sum tax T but we could equally well define the first-best allocation in terms of T/w because leisure does not enter in the utility functions. The public sector's budget restriction implies the condition $NT = N_3 w$.

Each individual j:s consumption follows straightforwardly from the budget restriction because the public good is provided by the state; aggregating and using (3) yields:

$$x_2 = \frac{(w-T)N}{al_1 w + l_2 w} \tag{4}$$

The output of x_1 follows from the input coefficient a:

$$x_1 = \frac{a(w-T)N}{al_1 w + l_2 w} \tag{5}$$

The output of the public good follows from the state's budget restriction and from the assumption that each unit of output requires one unit of work:

$$x_3 = \frac{T}{w} N \tag{6}$$

Substitute (5) and (6) for x_2 and x_3 in (1) to get the indirect utility function in terms of T and collect all constants in a term U_0:

$$U = U_0 + A \log (w - T) + B \log T \tag{7}$$

Maximising with respect to T and using the condition $A+B = 1$ yields:

$$T^* = Bw \tag{8}$$

Inserting T^* in (5) and (6) and using the definitions $N_1 = x_1 l_1$ and $N_2 = x_2 l_2$ shows that it is optimal to allocate the N workers so that $N_1 + N_2 = AN$ and $N_3 = BN$. In that sense, B denotes the optimal share of the public sector.

To see how the employees are optimally distributed among the industries, note the definitions $N_1 = x_1 l_1$ and $N_2 = x_2 l_2$ and that the public good is measured in terms of labour. We then get $N_1 = al_1(1-B)N/(al_1 + l_2)$, $N_2 = l_2(1-B)N/(al_1 + l_2)$ and $N_3 = BN$.

2.3. The market allocation with a given tax rate

Suppose now that we have a market economy in which x_1 and x_2 are provided by the private sector. Industry 1 has N_1 employees, but it is now characterised by imperfect competition. However, all workers in Industry 2 are self-employed shop-keepers living on what is left after having paid for inputs from Industry 1. Both industries are characterised by Cournot conjectures, but if there is oligopoly, the number n_1 of firms in Industry 1 is 'small' relative to n_2.

The public sector is financed by income taxation. More precisely, T is determined as the proportion t of all types of incomes:

$$N_3 w = [(N_1 + N_3)w + \pi_1 + \pi_2]\, t \tag{9}$$

As before, N denotes the population. However, the economy now generates groups with different sources of income. First, there are n_1 employers/capitalists and N_1 workers in Industry 1. The employers do not work. Second, there are n_2 self-employed and competing shop-keepers in Industry 2. Finally, there are N_3 employees in the public sector. The wage earners may also own assets which entitle them to a share of the profits in Industry 1.

Let θ_j stand for the proportion of the profits π_1 that individual j, $j = 1, 2,...N$ is entitled to. If j is a wage earner, the budget restriction can be written:

$$[w + \theta_j \pi_1]\,(1-t) = p_2 x_{2j} \tag{10}$$

If j is a capitalist, we obtain:

$$\theta_j \pi_1 (1-t) = p_2 x_{2j} \tag{11}$$

If j belongs to the group of self-employed workers, we obtain:

$$\frac{\pi_2(1-t)}{n_2} = p_2 x_{2j} \tag{12}$$

In the Appendix we describe the details of deriving demand, output and price given the tax rate. It turns out that the employees are allocated in the following way between producing the private and the public good:

$$N_1 = \frac{(n_1-1)(n_2-1)(N-n_1-n_2)(1-t)}{(n_1-1)(n_2-1)+(n_1+n_2-1)t} \tag{13}$$

$$N_3 = \frac{n_1 n_2 (N-n_1-n_2)t}{(n_1-1)(n_2-1)+(n_1+n_2-1)t} \tag{14}$$

Thus, the allocation of the work force depends on the market structure and the tax rate.

We get the following expressions for the profits in Industry 1 and 2:[5]

$$\pi_1 = \frac{(n_2-1)(N-n_1-n_2)w(1-t)}{(n_1-1)(n_2-1)+(n_1+n_2-1)t)w} \tag{15}$$

$$\pi_2 = \frac{n_1(N-n_1-n_2)w(1-t)}{(n_1-1)(n_2-1)+(n_1+n_2-1)t} \tag{16}$$

3. The size of the public sector

3.1. What incentives do different groups have?

What size of the public sector would different groups in the economy be prepared to support? That depends on the tax rate. Therefore, the question can be reformulated in terms of the tax rates that the groups in the economy would be prepared to pay. To analyse their attitudes towards the public sector, we differentiate their indirect utility functions with respect to t.

[5] This can be seen by inserting (13) and (14) in (A.12) and (A.13) which are derived in the Appendix.

First, consider the workers. Suppose first that they do not own shares in the firms in Industry 1. Their consumption of the private good equals the real wage after tax, that is, $(n_1-1)(n_2-1)(1-t)/al_1n_1n_2$ (use (10), (15) and (A.9) assume θ=0). Insert this and the total amount of the public good in the utility function and introduce the following abbreviation:[6]

$$\eta = \frac{(n_1-1)(n_2-1)}{n_1 n_2} \tag{17}$$

Use (17) and rearrange so that all constants are collected in V_0^w:

$$V^w = V_0^w + A \log(1-t) + B \log \frac{t}{\eta + (1-\eta)t} \tag{18}$$

Differentiating yields the following first-order condition:

$$-\frac{A}{1-t} + \frac{B}{t} - \frac{(1-\eta)B}{\eta + (1-\eta)t} = 0 \tag{19}$$

The solution can be written as follows:[7]

$$t^w = \frac{-\eta}{2(1-B)(1-\eta)} + \sqrt{\frac{\eta^2}{4(1-B)^2(1-\eta)^2} + \frac{B\eta}{(1-B)(1-\eta)}} \tag{20}$$

Next, consider the capitalists. Their consumption follows from dividing π_1/n_1 by p_2 and multiplying by (1-t). Collecting all constants in the term V_0^c implies that the indirect utility function can be written as follows:

[6] The expression η is proportional to the real wage rate and the inverse of the mark-up in Industry 2, as follows from (A.9) in the Appendix. Moreover, using (13), (14), (15) and (16) shows that the share of profits in the economy is $(1-\eta)(1-t)$.

[7] To see that the second-order condition will be satisfied, take the second derivative and use the first-order condition to eliminate the derivative of the third term. It then follows that the second derivative is negative for $t<2B/(1+B)$, which is larger than B. However, since the solution of the first-order condition is smaller than B, the second order condition is satisfied.

$$V^c = V_0^c + A\ \log\frac{(1-t)^2}{\eta+(1-\eta)t} + B\ \log\frac{t}{\eta+(1-\eta)t} \tag{21}$$

Repeating the exercise for the self-employed workers yields an expression that has a different constant term but is otherwise identical:

$$V^s = V_0^s + A\ \log\frac{(1-t)^2}{\eta+(1-\eta)t} + B\ \log\frac{t}{\eta+(1-\eta)t} \tag{22}$$

Both (21) and (22) yield the same first-order condition which means that capitalists and shop-keepers prefer the same tax rate:

$$-2\frac{A}{1-t} + \frac{B}{t} - \frac{1-\eta}{\eta+(1-\eta)t} = 0 \tag{23}$$

We can compare the tax rates preferred by different groups by evaluating the sign of (23) for the value of t that satisfies (19). Denote that value by t^w. Use (19) for eliminating $2A/(1-t^w)$ from (23):

$$\frac{dV^c(t^w)}{dt} = \frac{dV^s(t^w)}{dt} = -\frac{(1-B)(1-\eta)t^w + B\eta}{[\eta+(1-\eta)t^w]t^w} \tag{24}$$

This expression is unambiguously negative which means that both capitalists shop-keepers want a smaller public sector than the workers. Note that this does not depend on the size of the incomes; we might postulate that capitalists and shop-keepers earn at least as much as a worker but we have not yet made any such assumption. We summarise the findings in the following proposition:

Proposition 1: The workers will desire a higher tax rate than the capitalists and the shop-keepers.

The intuition behind the proposition is that the tax rate affects workers in only one way but capitalists and shop-keepers in two ways. All groups are affected directly as taxes reduce their disposable incomes. There is an indirect effect as well because a tax increase reduces demand which induces a greater sacrifice of consumption for capitalists and shop-keepers than for workers. This result is fairly robust and not dependent on the particular division of labour between the industries nor on the relative size of the incomes of the different groups.

3.2. How is the willingness to support the public sector affected by share ownership?

In 3.1 we assumed that the workers did not own shares in oligopolistic firms. Relaxing this restriction means that the consumption of each worker equals $(w + \theta_j\pi_1)(1\text{-}t)/p_2$. Inserting consumption of the private good (use (10), (15) and (A.9)) and the public good in the utility function and collect the constants in a term which we call V_{0j}^w. The indirect utility function for a worker endowed with shares entitling to the proportion θ_j of the profits in industry one is then:

$$V_j^w = V_{0j}^w + A \log(1-t) + \\ + A \log\left\{1 + \frac{\theta_j(N-n_1-n_2)(n_2-1)}{n_1 n_2}\frac{1-t}{\eta+(1-\eta)t}\right\} + B \log\frac{t}{\eta+(1-\eta)t} \tag{25}$$

If there exists a class of persons living on profits from Industry 1 only, their incomes are proportional but not equal to π_1/n_1. This does not change the location of the maximum of their indirect utility function as compared to 3.1. However, it makes a difference for the workers. Differentiate (25) with respect to t:

$$\frac{dV_j^w}{dt} = -\frac{A}{1-t} + \frac{B}{t} - \frac{(1-\eta)B}{\eta+(1-\eta)t} \\ - \theta_j A\frac{(N-n_1-n_2)(n_2-1)}{[\eta+(1-\eta)t]\,\{\theta_j(N-n_2)(n_2-1)(1-t)+n_1 n_2[\eta+(1-\eta)t]\}} \tag{26}$$

To see how asset ownership affects the willingness to pay taxes, insert the level t^w at which a wage earner without asset ownership reaches maximum utility. The first three terms then disappear:

$$\frac{dV_j^w(t^w)}{dt} = -\theta_j A\frac{(N-n_1-n_2)(n_2-1)}{[\eta+(1-\eta)t^w]\,\{\theta_j(N-n_2)(n_2-1)(1-t^w)+n_1 n_2[\eta+(1-\eta)t^w]\}} \tag{27}$$

The derivative is unambiguously negative in t^w which means that share owning workers are less willing to support the public sector despite being more well off. This negative term increases for higher values of θ_j which means the more the worker owns, the lower is the willingness to support the public sector:

Proposition 2. Share ownership decreases the willingness to support the public sector.

The intuition behind the result is that a given increase in the amount of public services implies a greater loss of consumption than without share ownership.

3.3. Can the public sector become too large?

What would the popular belief that the public sector is 'too large' mean in this context? Under marginal cost pricing the optimal size is a question of the preferences for public goods or services given the production possibility frontier. A public sector which employs more than the proportion B of the work force is excessive and vice versa.

The question becomes more complicated if there is not marginal cost pricing. The economy then consists of different groups with different attitudes although their basic preferences are the same. Ignoring political failures, the public sector cannot be larger than in the first-best solution unless there is a majority with an incentive to expand the public sector beyond that level.

It follows from Proposition 1 that t^w is the highest tax rate that any group would support. It can easily be seen from (20) that $t^w < B$. Thus, neither the workers nor any other group find a higher tax rate than in the first-best allocation optimal. Differentiating (14) shows that the size of the public sector is increasing in t. Thus, the public sector in fact tends to be smaller than in the first-best solution.

How is the willingness to support the public sector affected by changes in the market structure? After some manipulations, (19) can be rewritten as follows:

$$\frac{1-B}{1-t} = \frac{B\eta}{t[\eta + (1-\eta)t]} \tag{28}$$

The expression to the left is increasing in t while the expression to the right is decreasing. An increase in n_1 and/or n_2 means an upwards shift in the latter. This means that the curves intersect at higher value of t; thus, t^w is increasing in n_1 and n_2. It can be shown in a similar way that the tax rate t^c, which satisfies (23), increases with increased competition.

We summarise these findings in the following proposition:

Proposition 3: a) No group's optimal public sector that is larger than in the first best solution; b) Decreasing competition decreases the size of the public sector that the workers prefer.[8]

4. Who gets the majority? Some simple examples.

4.1. Replicating the number of industries

In a democracy, the allocation depends on the the relative size of the different groups. Until now we have taken the number of firms and self employed workers as given. However, as the tax rate changes, people may want to change occupation. We can allow for this effect by assuming that a capitalist earns at least the same as a shop-keeper which in turn earns as much as a worker. The story of what allocation that will actually emerge is then not necessarily convincing but may throw light on some important tendencies.

Ignore worker share ownership and suppose that there is one high-tax and one low-tax party. The former reflects the views of workers and the latter those of shop-keepers and capitalists. Further, suppose that all voters are fully informed and that preferences are not mis-represented. In fact, the workers would then always have majority because the non-wage incomes vanish rapidly as n_1 and n_2 increase. In the present model version, a low-tax (philistine) majority would require the condition $n_2 > (N - 2n_1)/2$ or, alternatively, $n_1 + n_2 > N/2$. This would give capitalists and shop-keepers too small earnings.[9] Thus, a philistine majority would require a political failure.

[8] Our criterion conforms to Atkinson and Stern (1974). An alternative approach would be to use a, for example utilitarian, social tax rate welfare function to get a compromise between the conflicting views.

[9] Setting t=0 and $N = 2(n_1+n_2)-1$ in (16) and assuming that $\pi_2/n = w$ shows the value of n_2 that is consistent with equal earnings for employed and self-employed workers as a function of n_1. The largest integer value of n_1 consistent with the condition $\pi_1/n_1 \geq \pi_2/n_2$ turns out to n_2 - 1. The smallest permissible value of n_1 is 2. It follows that the small two-industry economy cannot support a majority of capitalists and self-employed workers.

To make another outcome possible at least in theory, we increase the number of industries. To keep the model simple we expand the 'small' economy by replicating its pair of industries m times. The m consumption goods are indexed by k and the public good by p. All consumption goods have the same weight $(1-B)/m$ in the utility function. Instead of (1) we obtain:

$$U = \sum_{k=1}^{m} \frac{1-B}{m} \log x_{kj} + B \log x_p \tag{29}$$

When deriving demand, we transform the utility function so that all private goods get the weight $1/m$ because the individuals cannot decide the size of x_p.

Note that the industries are pairwise connected in the sense that for each of the m consumer goods industries there is one that produces its inputs. The expressions n_1 and n_2 are now re-interpreted as the number of firms and self-employed workers respectively in each industry. The whole population is denoted N. The symbol N_o stands for N/m.

The details of deriving the allocation of the work force is derived in the Appendix. Let the number of workers producing the public good be denoted by N_p. We then get:

$$N_1 = \frac{m(N_0 - n_1 - n_2)(n_1 - 1)(n_2 - 1)(1-t)}{(n_1 - 1)(n_2 - 1) + (n_1 + n_2 - 1)t} \tag{30}$$

$$N_p = \frac{m(N_o - n_1 - n_2)n_1 n_2 t}{(n_1 - 1)(n_2 - 1) + (n_1 + n_2 - 1)t} \tag{31}$$

Note that instead of $N_1+N_3 = N-n_1-n_2$ as in Section 2.3 we get $N_o-n_1-n_2$. The profits in an industry of type 1 and 2 respectively are then:

$$\pi_{1k} = \frac{(n_2 - 1)(1-t)(N_o - n_1 - n_2)w}{(n_1 - 1)(n_2 - 1) + (n_1 + n_2 - 1)t} \tag{32}$$

$$\pi_{2k} = \frac{n_1(1-t)(N_o - n_1 - n_2)w}{(n_1 - 1)(n_2 - 1) + (n_1 + n_2 - 1)t} \tag{33}$$

4.2. When can the low-tax party get a stable majority?

A majority for capitalists and shop-keepers requires the condition $m(N_o\text{-}n_1\text{-}n_2) > m(n_1+n_2)$; the smallest possible majority means $mN_o = 2m(n_1 + n_2) - 1$. Assuming that m is large allows us to set N_o is equal to $2(n_1+n_2)$.[10]

The incomes of the self employed workers in each industry are a decreasing function of both n_2 and t. The upper bound of n_2 consistent with the condition $\pi_{2k}/n_2 = w$ and a majority for the low tax party is obtained by setting t=0 and $N_o = 2(n_1+n_2)$ in (33):

$$n_2 = \frac{2n_1 - 1}{2(n_1 - 1)} + \sqrt{\frac{(2n_1 - 1)^2}{4(n_1 - 1)^2} + \frac{n_1^2}{(n_1 - 1)}} \tag{34}$$

Under free entry, new firms are established until π_1/n_1 approaches π_2/n_2. Equations (32) and (33) would then yield:

$$n_2^2 - n_2 - n_1 n_2 = 0 \tag{35}$$

This implies that the smallest integer value of n_1 which ensures that a shop-keeper does not earn more than a capitalist is n_2-1. Substituting n_2-1 for n_1 in (34) shows that this condition is inconsistent with a majority for the low-tax party if n_1 is larger than 2. Equation (33) in turn implies that n_2 cannot be larger than 3 because otherwise the tax rate would have to be negative to ensure equal earnings for workers and shop-keepers. On the other hand, as can be seen from the expressions for the prices and quantities (see the Appendix), there does not exist any equilibrium for values of n_1 and n_2 less than 2. Thus, the only possible majority for the low-tax party requires $n_1 = 2$ and $n_2 = 3$ in each pair of industries.[11] Firms would have to be small, with on average

[10] Strictly speaking, a majority of one requires that we make an exception because all pairs of industries are homogeneous. Without such an exception, the smallest majority must equal m. The low-tax party has the smallest possible majority if $(m\text{-}1)N_o + (N_o\text{-}1) = 2m(n_1+n_2)\text{-}1$. Divide this equality by m. As m becomes very large, we get $N_o \approx 2(n_1+n_2)$.

[11] With $n_1 = 2$ and $n_2 = 3$ we can assume that the population consists of approximately $10m$ persons because 10 is the limit of $[10(m\text{-}1) + (10\text{-}1)]/m$ as m gets large.

only 3 employees. The economy would consist of $10(m\text{-}1) + 9$ individuals. The reason for this result is that unitary elastic demand implies that profits decrease rapidly with the number of competitors.[12]

Inserting $n_1 = 2$ and $n_2 = 3$ in (33) implies the tax rate must lie in the interval $(0, 0.182]$ to ensure that $\pi_{2k}/n_2 = w$. To see which preferences that make the groups accept tax rates within this interval, we turn to their indirect utility functions. Substituting consumption of the private goods and the public good in the worker utility function, rearranging and collecting the constants in V_o^w yields an expression which is identical to (18) with the exception that the constant term has a different interpretation. The same is true about the other groups. The first order conditions are therefore represented by (19) and (23) respectively.

According to (16), the condition $\pi_2/n_2 = w$ implies:[13]

$$(n_1 - 1)(n_2 - 1) + (n_1 + n_2 - 1)t = \frac{n_1(N - n_2)(1 - t)}{n_2} \tag{36}$$

Use (36) to re-write (19) and set $N_o = 2(n_1 + n_2)$ to ensure that the low-tax party has the majority:

$$\frac{B - t}{t} - \frac{B(n_1 + n_2 - 1)n_2}{(n_1 + n_2)n_1} = 0 \tag{37}$$

Let t^w denote the tax rate that satisfies (37):

$$t^w = \frac{B}{1 + [Bn_2(n_1 + n_2 - 1)]/n_1(n_1 + n_2)} \tag{38}$$

[12] This is not necessarily the case under more general conditions. Further research is needed before we know whether the economy can maintain a majority of low-tax party supporters under less extreme circumstances.

[13] This assumption can be justified by the observation that workers are prepared to change their status from self-employed to employees and *vice versa* if the gap in earnings becomes too large; the utility function (1) ignores any preferences for being self-employed which means that the earnings would have to be exactly equal under free entry.

If there is a stable majority for the low-tax party, the preferences of the workers' opinions do not matter. Nevertheless, setting $n_1 = 2$ and $n_2 = 3$ shows that also the workers would accept a tax rate in the interval (0, 0.182] if B is smaller than 0.233.

Next, rewrite (23) in the same way as (19):

$$\frac{B + Bt - 2t}{t} = \frac{(n_1 + n_2 - 1)n_2}{n_1(n_1 + n_2)} \tag{39}$$

Solving yields:

$$t^c(n_2) = \frac{B}{2 - B + (n_1 + n_2 - 1)n_2/(n_1 + n_2)n_1} \tag{40}$$

It follows, with $n_1 = 2$ and $n_2 = 3$, that if B is smaller than 0.492, the capitalists and the self-employed workers would prefer a tax rate smaller than 0.182. If $0.233 < B < 0.492$, the majority will force the minority to accept the small public sector associated with the tax rate 0.182.

4.3. When will the majority change?

It is possible that the majority's choice of tax rate brings about a change of status from worker to shop-keeper and *vice versa* which changes the majority and hence the tax rate. For example, since workers prefer a tax rate that is smaller than 0.182 if B is smaller than 0.233, their majority cannot be stable if N_o=10. Suppose that B is 0.204 and that n_1=2 and n_2=2. As follows from (19), t^w then equals 0.15. However, given this tax rate, a shop-keeper earns more than four times as much as a worker. With free entry we would sooner or later get n2=3 so that the low-tax party gets the majority. On the other hand, if n_2=4, shop-keepers would earn less than workers.

With the low-tax party in power, the tax rate is $t^c = 0.068$ which is obtained from (23) or, since N_o=2(n_1+n_2), from (41). This is about half of the tax rate preferred by the workers. Thus, even if all individuals would in the first best solution be prepared to pay one fifth of their incomes to support the public good, the majority will vote for a tax rate of less than 7%. It can be shown that the capitalists will get a stable majority and implement a tax rate that less than 0.079 as long as $B<0.233$.

To ensure a stable worker majority, the tax rate must be larger than 0.556 because otherwise the class of self-employed workers will grow until there is a philistine majority. It follows from (38) that such a large tax rate corresponds to a value of B larger than 0.769. For example, suppose that we start from a majority for the low tax party in a situation in which $B = 0.8$. The low-tax party would then, according to (40), choose $t^c = 0.333$. This would give the shop-keepers too small earnings so that one of them would become a worker. This change brings about a majority for the high-tax party which would set t^w=0.591. At such a high tax rate no worker would change to a self-employed status which means that their majority is stable.

What happens if $0.233<B<0.769$? The situation is then unclear because n_1 and n_2 must be integers. It may even happen that no majority is stable. Suppose that a discrepancy between π_{2k}/n_2 and w brings about a change of status but that there is always an election before there is time to experience that the discrepancy has in fact been reversed by the change. For example, suppose that B is 0.5 and that the low-tax party has majority. It would set the tax rate at 0.185 according to (40), which makes the shop-keepers earn less than the workers. Therefore, one of the shop-keepers in each pair of industries becomes a worker and that gives the high-tax party the majority. Its optimal choice is $t^w = 0.333$. However, a shop-keeper would then earn more than twice as much as a worker. This gives one of the workers an incentive to become a shop-keeper but then their earnings would become 2/3 of the workers' wages. If there is again an election before the agents have experienced that, the low-tax party gets the majority and sets the tax rate at 0.185.

If the parties know when the majorities are unstable they may choose t tactically. The ruling party may set the tax rate as close as possible to the un-constrained optimum, with the restriction that its majority should not be eroded. Thus, in this example the low tax party would prefer the tax rate 0.182 to its un-constrained optimum 0.185 and the high tax party would prefer 0.556 to its un-constrained optimum 0.333. This is the only way in which the public sector can become larger than in the first-best allocation.

The numbers in the example should not be taken too seriously. In a real economy, people rank parties according to several criteria and they are not necessarily perfectly informed, nor even necessarily rational in the admittedly narrow sense adopted in the model. Most important, under our simplifying assumptions the economy must consist of pairs of industries with small firms which are nevertheless oligopolistic. However, we learn from the exercise that there may be a tendency for the choice of tax rate to strengthen the majority of the forces behind the party in power. Further research may explain why there may be a majority for a low-tax party under more reasonable circumstances despite a high preference for the public sector in the first-best solution. In particular, it would be interesting to extend the analysis to cases with incomplete perception of the political parties' intentions.

5. Concluding remarks

Tax resistance among employers and shop-keepers is sometimes explained by tax progression and transfer payments but our model shows that there is another important reason for a hostile attitude towards public sector expansion. The intuition is that profits, unlike wages, are affected both directly and indirectly by the public sector. However, the model gives a majority of low-tax party supporters under stringent conditions only. Therefore, a philistine majority usually requires some type of political failure that is not included in our analysis.

Despite the simplicity of the model of the political system, it might be worthwile to observe that a low tax rate strengthens the support for the low-tax party and *vice versa*. This depends on the influence of the tax rate on the decision to be either a worker or a shop-keeper. Moreover, to encourage small business may create a society of philistines because a larger proportion of the population then has to live on a residual.

Appendix

1. Deriving the allocation of the work force in the three sector model

Let π stand for the sum of the profits in Industry 1 and the net incomes of the self-employed workers in Industry 2. The demand function in Industry 2 follows directly from (10) and (11);

the derived demand in Industry 1 follows from the input coefficient and we have obtain x_2 as a function of p_1 before that.

$$x_2 = \frac{(1-t)\,[(N-n_1-n_2)w+\pi]}{p_2} \tag{A.1}$$

Let us introduce the abbreviation $M = (1-t)\,[(N-n_1-n_2)w+\pi]$. The inverse demand function is then $p_2=M/x_2$. The costs in Industry 2 consist of the inputs from Industry 1 only; the self-employed workers have to live on the residual. The residual of the i:th 'firm', $i = 1, 2,...n_2$, is:

$$\pi_i = \frac{M}{x_2}x_{2i} - x_{2i}ap_1 \tag{A.2}$$

Taking the first order condition, rearranging and using symmetry yields $x_2(p_1, M)$; inserting in the inverse demand function yields $p_2(p_1)$:

$$x_2(p_1, w) = \frac{n_2-1}{n_2}\frac{M}{ap_1} \tag{A.3}$$

$$p_2(p_1) = \frac{n_2}{n_2-1}ap_1 \tag{A.4}$$

The aggregate profits in the economy are M/n_2; thus, each self-employed worker has to live on M/n_2^2.

The derived demand for the products of Industry 1 is obtained by multiplying (A.3) with the input coefficient a:

$$x_1 = \frac{n_2-1}{n_2}\frac{M}{p_1} \tag{A.5}$$

Marginal costs in Industry 1 are l_1w. Maximising profits and solving for industry output and price yield:

$$x_1(M) = \frac{n_1-1}{n_1}\frac{n_2-1}{n_2}\frac{M}{l_1w} \tag{A.6}$$

$$p_1 = \frac{n_1}{n_1 - 1} l_1 w \tag{A.7}$$

Inserting these solutions in (A.3) and (A.4) yields:

$$x_2(M) = \frac{n_1 - 1}{n_1} \frac{n_2 - 1}{n_2} \frac{M}{a l_1 w} \tag{A.8}$$

$$p_2 = \frac{n_2}{n_2 - 1} \frac{n_1}{n_1 - 1} a l_1 w \tag{A.9}$$

The profits in each industry can now be written as $(p_1 - l_1 w)x_1$ and $(p_2 - a p_1)x_2$ respectively. Further, (9) allows us to rewrite M as $N_1 w + \pi_1 + \pi_2$. This yields the following equation system:

$$\pi_1 = \frac{n_2 - 1}{n_2 n_1}(N_1 w + \pi_1 + \pi_2) \tag{A.10}$$

$$\pi_2 = \frac{1}{n_2}(N_1 w + \pi_1 + \pi_2) \tag{A.11}$$

Solving this shows that the profits can be written as functions of N_1 as follows:

$$\pi_1(N_1) = \frac{1}{n_1 - 1} N_1 w \tag{A.12}$$

$$\pi_2(N_1) = \frac{n_1}{(n_1 - 1)(n_2 - 1)} N_1 w \tag{A.13}$$

Inserting (A.12) and (A.13) in (9) yields (13) and (14); inserting them in (A.8) yields $x_2(t)$.

2. Deriving the allocation in the replicated economy

Reworking quantities and prices in the replicated case is straightforward; we omit the details. The profits in each industry equal π_1/m and π_2/m respectively. Let N_1 stand for the total number of workers in the input producing industries. The profits for each pair of industries as a function of the employed workers are then obtained from the following equation system:

$$\pi_{1k} = \frac{n_2 - 1}{n_2 n_1} \frac{N_1 w + \pi_1 + \pi_2}{m} \tag{A.14}$$

$$\pi_{2k} = \frac{1}{n_2}\frac{N_1 w + \pi_1 + \pi_2}{m} \tag{A.15}$$

We obtain:

$$\pi_{1k}(N_1) = \frac{1}{(n_1 - 1)m} N_1 w \tag{A.16}$$

$$\pi_{2k}(N_1) = \frac{n_1}{(n_1 - 1)(n_2 - 1)m} N_1 w \tag{A.17}$$

Substituting N_p for N_3 in (9), inserting (A.16) and (A.17) multiplied by m and rearranging yields (30) and (31).

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