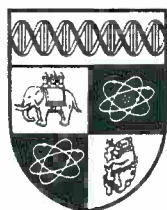


WELFARE MAXIMISING BALANCED-BUDGET PROVISION OF CONGESTIBLE
AND EXCLUDABLE JOINTLY-CONSUMED GOODS: SEPARATING ALLOCATIVE
EFFICIENCY FROM DISTRIBUTION

Clive D Fraser

No.409

WARWICK ECONOMIC RESEARCH PAPERS



DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

WELFARE MAXIMISING BALANCED-BUDGET PROVISION OF CONGESTIBLE
AND EXCLUDABLE JOINTLY-CONSUMED GOODS: SEPARATING ALLOCATIVE
EFFICIENCY FROM DISTRIBUTION

Clive D Fraser
Department of Economics
University of Warwick
Coventry CV4 7AL
England

No.409

May 1993

This paper is circulated for discussion purposes only and its
contents should be considered preliminary.

WELFARE MAXIMISING BALANCED-BUDGET PROVISION OF CONGESTIBLE AND
EXCLUDABLE JOINTLY-CONSUMED GOODS: Separating Allocative Efficiency from
Distribution*

by

Clive D. Fraser

University of Warwick

ABSTRACT. Developing Musgrave's suggestion, Bergstrom and Cornes and Bergstrom and Varian obtained necessary and sufficient conditions for separating allocation from distribution in Pareto optimal public provision and Nash equilibrium private provision of pure public goods. Assuming the most commonly analysed congestion function applies, we obtain necessary and sufficient conditions for, alternatively, the price and quality, and the price, quality, facility size and total utilisation of excludable, congestible goods to be independent of the distribution of self-selecting households' characteristics in a Stackelberg equilibrium, extending the scope of Musgrave's suggestion. When separation is possible, utility functions and optimal decisions take simple, intuitive forms.

*This paper is an outgrowth of work started at the 1991 Warwick Economics Summer Research Workshop on The Role of the Public Sector in a Mixed Economy, which was generously sponsored by SPES (EC), Contract No. SPECT91-0056 and the ESRC (UK), Contract No. W10026-1035-01. I thank Dr. John Coleman, Coventry University, for a useful suggestion. Errors are mine alone.

Clive D. Fraser
Department of Economics
Warwick University
United Kingdom.

May 1993
Gibbet Hill Road
Coventry CV4 7AL

**WELFARE MAXIMISING BALANCED-BUDGET PROVISION OF CONGESTIBLE
AND EXCLUDABLE JOINTLY-CONSUMED GOODS: Separating Allocative Efficiency
from Distribution***

ABSTRACT. Developing Musgrave's suggestion, Bergstrom and Cornes and Bergstrom and Varian obtained necessary and sufficient conditions for separating allocation from distribution in Pareto optimal public provision and Nash equilibrium private provision of pure public goods. Assuming the most commonly analysed congestion function applies, we obtain necessary and sufficient conditions for, alternatively, the price and quality, and the price, quality, facility size and total utilisation of excludable, congestible goods to be independent of the distribution of self-selecting households' characteristics in a Stackelberg equilibrium, extending the scope of Musgrave's suggestion. When separation is possible, utility functions and optimal decisions take simple, intuitive forms.

Suggested Running Head: "Separating Efficiency from Distribution."

Keywords: public goods, clubs, externalities, efficient allocation, distribution.

JEL Classifications: D11, D62, D71, H40

Welfare Maximising Balanced-Budget Provision of Congestible and Excludable Jointly- Consumed Goods: Separating Allocative Efficiency from Distribution.

This paper unites two strands of the literature on externalities by establishing necessary and sufficient conditions for separating allocative efficiency from distribution in club theory. The first strand in the literature examines the nature of the congestion or environmental quality function and the relationship between this function's properties and whether or not full financing of optimal provision of the club good will be achieved. This has been discussed by, e.g.: Serge-Christophe Kolm (1974); Finn Forsund (1974); Herbert Mohring and Mitchell Harwitz (1962); Eitan Berglas and David Pines (1981, 1984) and Todd Sandler and John Tschirhart (1984). The second strand considers when a Pareto optimal or Nash equilibrium provision level of a public good is independent of the distribution of agents' characteristics, normally taken to be income. See, e.g., Ted Bergstrom and Richard Cornes (1981 and 1983) on this. Bergstrom, Lawrence Blume and Hal Varian (1986), Douglas Bernheim (1986) and Peter Warr (1983) discuss several such "invariance" results for public goods; more generally, Bergstrom and Varian (1985a) and (1985b), respectively, discuss when Nash equilibria are independent of the distribution of agents' characteristics and market games have transferable utility.

Most jointly consumed goods seem to be congestible and/or excludable. These features have spawned a vast literature (see, e.g., Dagobert Brito and William Oakland, 1980, Geoffrey Brennan and Cliff Walsh, 1981, Michael Burns and Walsh, 1981, Cornes and Sandler, 1986, Sandler and Tschirhart, 1980, and David Starrett, 1988, for original contributions and surveys). This literature extends considerably the original insights of Erik Lindahl (1910), Howard Bowen (1943), Charles Tiebout (1956), Paul Samuelson (1954, 1955), James Buchanan (1965), and Mancur Olson (1965) on jointly consumed goods.

Problems involving jointly consumed goods which are both congestible and excludable are usually very complex. This complexity arises because of the need to specify: first, how congestion affects utility; second, the way in which the interactions of agents simultaneously produce the level of congestion which they individually respond to; third,

the mechanism by which exclusion is achieved, which usually requires some incentive compatibility or self-selection constraints being incorporated in the problem. Because of this complexity, authors usually treat the congestion and exclusion problems separately. Thus in club theory, which is where congestible jointly consumed goods are usually treated, it is normally assumed that all households are segregated into homogeneous, type-specific clubs. Conversely, the literature on excludable public goods, exemplified by Brito and Oakland, does not consider congestion externalities¹.

Given this complexity, it is tempting to ask: are there circumstances in which, despite households' taste and/or income heterogeneity, in an heterogeneous club setting, decisions about the pricing and the level of provision of club facilities can be essentially separated from that of membership of the club? We will show when optimal balanced-budget provision of a congestible and excludable jointly consumed good is independent of the distribution of households' characteristics and, thus, the answer to this question is in the affirmative. We will use "club good" and "excludable and congestible jointly consumed good" synonymously.

We will consider two types of separation of efficient allocation from distribution. First, a *partial separation* in which the optimal price and quality provision of the club good, the utility-relevant magnitudes for households, are independent of distribution; second, *total separation* in which the price, quality provision, facility size and total utilisation of the jointly-consumed good are all independent of distribution. We show that, if the club congestion or quality function takes the form assumed most commonly in the literature, partial separation will always be possible if and only if club quality and visits interact in one of two intuitively appealing ways in households' utility functions. This extends to a many-clubs setting. Moreover, if these conditions hold, the decision rule for the visit price and quality provision for welfare maximising break-even provision of the club good takes appealingly simple forms, such as: "maximise the bangs per buck!" Total separation is possible under more restrictive conditions. These reduce to requiring that club quality and visits interact in utility functions in the ways noted above *and* all households' club demand

functions are identical and linear in income, analogous to the usual conditions for aggregation with private goods.

We structure the paper as follows. Section I introduces the economic environment and characterises the optimal decisions by the provider and consumers of the excludable, congestible good. In section II we discuss the outcome of their joint maximisation in a special case. We present and prove our first theorem on partial separation in Section III and discuss why it holds in Section IV. Section V discusses extensions of our partial separation result and total separation. VI concludes.

I. The Model

A natural way to model equilibrium in an economy with a congestible, or club, good under full information is as the outcome of a Stackelberg game wherein the supplier of the good, the leader, sets its price and provision level. The potential consumers or club members, the followers, respond to this price, provision level, and the conjectured club congestion level or quality which will emerge with this pair, by self-selecting themselves to membership, or otherwise, in a Nash manner. In the resulting Nash equilibrium among the followers, the membership and aggregate level of visits to the club are determined for the given price and provision level. The leader, knowing the features of this equilibrium, uses that information to set the price and provision level to start with to meet its objectives².

Let there be an arbitrary but finite number of different types of households, the types being indexed by i , $i = 1, \dots, n$. We make the following assumptions on the types:

A1) All individuals or households of a given type, i , have the same utility function, U^i , but can differ in incomes, denoted M , which lie in an interval $M \equiv [\underline{M}, \overline{M}]$. The incomes of each type i have atomless distribution and density functions $F^i(M)$ and $f^i(M)$, respectively. $M(h)$ denotes the income of an h -household of all types.

A2) U^i is defined over quantities of an aggregate private consumption commodity, x , visits to a club or consumption of a club good, v , and the quality of the club good, c . It is strictly quasi-concave and differentiable in all its arguments; its first-order partial derivatives are denoted U_j^i , $j = x, v, c$, or $1, 2, 3$, as is convenient.

Thus, the utility an h -household of type i obtains is $U^i(x^{hi}, v^{hi}, c)$ if it consumes a bundle with specification (x^{hi}, v^{hi}, c) . Assuming club non-members experience no congestion externality, the utility of an h -household of type i which does not consume the club good is $U^i(M(h), 0, 0)$. The aggregate private consumption commodity is taken as the numeraire and the per visit price of the club good is denoted p . This could be the toll per trip on a toll road, say. The club quality or inverse congestion function, $c(y, V)$, depends on the level of provision of club facilities, y , and the aggregate level of visits or utilisation of the club, V . All households which are members of a given club assess its congestion or quality with the same function $c(\cdot)$, although this might enter different households' utility functions in different ways. We will specialise c in due course.

A household h of type i has the budget constraint

$$1) \quad M(h) - pv^{hi} = x^{hi}$$

It will be convenient also to make the following additional assumption on utilities:

A3) The club good is a normal good for all types i .

Satisfaction of (A3) could be ensured by strict concavity of U^i and the over-sufficient assumptions, $U_{12}^i \geq 0$, $U_{13}^i \geq 0$, $\forall i$. These latter comprise the familiar, cardinal, conditions for the "single crossing" property to hold in adverse or self-selection problems but are unnecessary for our results.

Suppose now that the agency responsible for providing the club good announces the price, p , and the specification, c , of the club good which it will maintain. Given these, households self-select themselves to membership or otherwise. Those who choose to be members will be ones whose maximised utility inside the club is no less than their utility derived from consuming all their income on the private commodity. We assume that, in choosing their levels of utilisation of the club, the atomless households act perfectly competitively, as discussed by Gerard Debreu (1991), and take the level of c as parametric.

Thus, *they not only ignore the congestion they impose on others, but also the negligible congestion which they impose on themselves* ³. The optimal level of visits for a club member, found by maximising the utility function subject to a budget constraint given by (1), then satisfies the first-order condition (FOC)(2):

$$2) \quad -pU_1^i[M(h) - pv^{hi}, v^{hi}, c(.)] + U_2^i[M(h) - pv^{hi}, v^{hi}, c(.)] = 0, \forall h, i.$$

We can invert (2) to yield $v^{hi}(M(h), p, c(.))$, the optimal level of visits as a function of income, club per visit price and quality.

As non-members experience no congestion externalities, club members satisfy

$$3) \quad U^i(M(h) - pv^{hi}, v^{hi}, c(.)) \geq U^i(M(h), 0, 0), \forall h, i,$$

where v^{hi} satisfies (2). A *marginal household of type i* will be one of that type which satisfies (3) with equality and (2). Denote the income of such a household by $M^{i*}(p, c)$. Normally, the functional dependence on p and c will be suppressed in the notation.

It can be shown to follow from (A3) that we must have :

Lemma 1. If a household of a given type is marginal, all those of that type having higher income will be club members and those of lower income will not belong to the club.

Given Lemma 1, it follows that

$$(4) \quad V = \sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} f^i(M(h)) dM$$

Naturally, V is a function of p and c , hence y , via the v^{hi} and M^{i*} .

Suppose the agency providing the club good now seeks the welfare maximising balanced-budget level of provision. It will choose p and y to solve:

5)

$$M \max_{p, y} \sum_i \int_{M^{i*}}^{\bar{M}} U^i(x^{hi}, v^{hi}, c(p)) f^i(M(h)) dM + \int_{\bar{M}}^{M^{i*}} U^i(M(h), 0, 0) f^i(M(h)) dM$$

$$6) \quad \text{s.t. } p \sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} f^i(M(h)) dM = y.$$

In solving (5) and (6), it is necessary to specify the nature of the congestion function further. We will assume that it takes the form which is employed most commonly in the literature (see, e.g., Kolm, Berglas and Pines, 1981 and 1984, Cornes and Sandler, 1986, and Barro and Romer, 1987 and 1991):⁴

$$A4) \quad c(y, V) = c(y/V); \quad c'(\cdot) \geq 0, \quad c''(\cdot) \leq 0.$$

This congestion function is homogeneous of degree zero in y and V . Berglas and Pines (1981) show that, when $c(\cdot)$ is of this form, the club good is essentially a private good and the Pareto optimal level of provision of it will involve a break-even price⁵.

Using (4), (6) and (A4), we can rewrite the congestion function as

$$7) \quad c(y, V) = c(p); \quad c'(p) \geq 0, \quad c''(p) \leq 0,$$

the partials in (7) being assumed to satisfy strict inequalities over some range of p . Thus, the club quality now depends solely on the per visit price, hence expenditure on facility provision per visit, p . For given distribution and density functions, households' choices and the identities of the marginal men are all functions of p rather than y and V separately.

The agency's maximisation problem now becomes

$$8) \quad M \max_p \sum_i \int_{M^{i*}}^{\bar{M}} U^i(x^{hi}, v^{hi}, c(p)) f^i(M(h)) dM + \int_{\bar{M}}^{M^{i*}} U^i(M(h), 0, 0) f^i(M(h)) dM$$

where the x^{hi} satisfy the budget constraint (1) and the v^{hi} satisfy the FOC (2). Let p^* denote an optimal p . Then, by Leibnitz's formula for differentiating integrals, (3) with equality for marginal men of all types, and the envelope theorem, the first-order condition for this maximisation is

$$9) \quad \frac{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} [\partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial x] f^i(M(h)) dM}{\sum_i \int_{M^{i*}}^{\bar{M}} [\partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial c] f^i(M(h)) dM} = c'(p^*)$$

For brevity, we will assume second-order conditions for the optimisation are satisfied, as will be the case in the scenarios discussed below.

II. A Special Case

As a precursor to a more general analysis of (9), consider the special case where all types have quasi-linear utility functions, $U^i \equiv u^i(x) + v^i(y/V)$. Using these, (9) becomes

$$10) \quad \frac{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} u^{i'}(x^{hi}) f^i(M(h)) dM}{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} f^i(M(h)) dM} = c'(p^*).$$

But, from the utility maximisation of each household of each type, if h is in the club, then

$$11) \quad u^{i'}(x^{hi}) = c(p)/p.$$

Inserting (11) evaluated at p^* into (9) reduces the latter to

$$12) \quad c(p^*)/p^* = c'(p^*).$$

Given (A4) and (7), (12) is precisely the condition for maximising the average quality purchased per unit of expenditure on facility provision or, as stated in our introduction, for "maximising the bangs per buck!" This is irrespective of the income distribution, which was arbitrary in the above, and irrespective of the actual level of V and y . Thus, what we termed *partial separation* of allocation from distribution in the introduction holds in this case. But, quasi-linear utility as specified, while having the advantage of resulting in empirically plausible self-exclusion from club consumption at the bottom of each type's income distribution, if at all, also has some well-known disadvantages (see, e.g., Bergstrom and Cornes, 1981 or 1983). Given that the outcome (12) has other desirable features discussed below, the question is then: to what extent does this separation result extend to more general preferences?

III. The Main Theorem

We now state and prove the first of the two central results of this paper.

Theorem 1. A Partial Separation Theorem. If the congestion function c satisfies (A.4), the welfare maximising break-even price and quality provision of the excludable, congestible good satisfies (12) for *all* income distributions for the various types if and only if all types have utility functions of the form $U^i(x, v, c) = \psi^i[x, v, c(\cdot)] \equiv \psi^i[x, v]$.

Proof. Sufficiency. If $U^i(x, v, c) = \psi^i[x, v, c(\cdot)] \equiv \psi^i[x, v]$ and c satisfies (7), the optimum choice of p^* is characterised by the following analogue of (9):

$$13) \quad \frac{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} \frac{\partial \psi^i[x^{hi}, v^{hi} c(p^*)]}{\partial x} f^i(M(h)) dM}{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} \frac{\partial \psi^i[x^{hi}, v^{hi} c(p^*)]}{\partial v} f^i(M(h)) dM} = c'(p^*).$$

Now, utility maximisation by club members at the optimal p^* satisfies the following analogue of (2):

$$14) \quad -p^* \frac{\partial \psi^i[x^{hi}, v^{hi}c(p^*)]}{\partial x} + c(p^*) \frac{\partial \psi^i[x^{hi}, v^{hi}c(p^*)]}{\partial v} = 0.$$

Thus,

$$p^* v^{hi} \frac{\partial \psi^i[x^{hi}, v^{hi}c(p^*)]}{\partial x} = c(p^*) v^{hi} \frac{\partial \psi^i[x^{hi}, v^{hi}c(p^*)]}{\partial v}$$

or

$$15) \quad v^{hi} \frac{\partial \psi^i[x^{hi}, v^{hi}c(p^*)]}{\partial x} = \frac{c(p^*)}{p^*} v^{hi} \frac{\partial \psi^i[x^{hi}, v^{hi}c(p^*)]}{\partial v}.$$

Integrating (15) over all type i households which belong to the club and then summing over i yields

$$\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} \frac{\partial \psi^i[x^{hi}, v^{hi}c(p^*)]}{\partial x} f^i dM = \frac{c(p^*)}{p^*} \sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} \frac{\partial \psi^i[x^{hi}, v^{hi}c(p^*)]}{\partial v} f^i dM.$$

Substituting this last expression into (13) yields, finally:

$$16) \quad c(p^*)/p^* = c'(p^*).$$

Necessity: For our claim to be true we need, from (9),

$$17) \quad \frac{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} [\partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial x] f^i(M(h)) dM}{\sum_i \int_{M^{i*}}^{\bar{M}} [\partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial c] f^i(M(h)) dM} = \frac{c(p^*)}{p^*}.$$

If (17) holds, then we must have

$$18) \frac{p^* \sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} [\partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial x] f^i(M(h)) dM}{c(p^*) \sum_i \int_{M^{i*}}^{\bar{M}} [\partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial c] f^i(M(h)) dM} = 1.$$

Or, using the first-order condition for individual household maximisation ((2)), we need:

$$19) \frac{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} [\partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial v] f^i(M(h)) dM}{\sum_i \int_{M^{i*}}^{\bar{M}} c(p^*) [\partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial c] f^i(M(h)) dM} = 1.$$

Now, for (19) to be true for *all* income distributions, it is necessary that

$$20) \quad v^{hi} \partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial v = c(p^*) \partial U^i(x^{hi}, v^{hi}, c(p^*)) / \partial c \quad \text{a.e., } \forall i, h.$$

If (20) does *not* hold everywhere except on a set of measure zero, we can use a standard route to find a distribution f^i , for some i , which leads to a violation of (19). E.g., suppose $v \partial U^i / \partial v \neq c \partial U^i / \partial c$ on some interval, $[M^-, M^+]$, for type i households which belong to the club. W.l.o.g., suppose $v \partial U^i / \partial v > c \partial U^i / \partial c$ on this interval but $v \partial U^i / \partial v = c \partial U^i / \partial c$ elsewhere and $v \partial U^j / \partial v = c \partial U^j / \partial c$ everywhere, $\forall j \neq i$. Then, if the densities f^i are uniform, the LHS of (19) exceeds 1. Thus we require (20) to hold almost everywhere.

(20) is a simple linear partial differential equation (see William Williams, 1980). Its general solution is the family of weakly separable functions of the form

$$21) \quad U^i(x, v, c) = \psi^i[x, v c(\cdot)] \quad \text{Q.E.D.}$$

IV. Discussion

Why does this theorem hold? It holds because, for all households with the preferences (21), the price p^* and the associated $c(p^*)$ satisfying (16) maximise their

willingness to pay for the congestible collectively-consumed good, irrespective of income. This can be seen by noting that everyone with such preferences has elasticity of demand for the club good = -1 at the pair $(p^*, c(p^*))$.

Theorem 2. If $U^i(x, v, c) = \psi^i[x, v, c(\cdot)]$, then $(\partial v^{hi}/\partial p)(p/v^{hi}) = -1$ at $(p^*, c(p^*))$ satisfying (16) for all households satisfying (2).

Proof. If households' utility functions satisfy (21), at $(p^*, c(p^*))$ (2) takes the form

$$(22) \quad -p^* \psi_1^i(x^{hi}, v^{hi}, c(p^*)) + \psi_2^i(x^{hi}, v^{hi}, c(p^*)) c(p^*) = 0.$$

Thus (suppressing functional arguments), differentiating through (22) w.r.t. p^* yields

$$(23) \quad (\partial v^{hi}/\partial p)[p^{*2} \psi_{11}^i - 2p^* c(p^*) \psi_{12}^i + \psi_{22}^i c(p^*)^2] \\ = \psi_1^i - c'(p^*) \psi_2^i + p^* c'(p^*) \psi_{12}^i v^{hi} + c(p^*) \psi_{12}^i v^{hi} - c'(p^*) c(p^*) \psi_{22}^i v^{hi} - p \psi_{11}^i v^{hi} \\ = (v^{hi}/p^*) [p^{*2} c'(p^*) \psi_{12}^i + p^* c(p^*) \psi_{12}^i - p^{*2} \psi_{11}^i - p^* c'(p^*) c(p^*) \psi_{22}^i] \\ = - (v^{hi}/p^*) [p^{*2} \psi_{11}^i - 2p^* c(p^*) \psi_{12}^i + \psi_{22}^i c(p^*)^2]$$

after using (22) and, repeatedly, (16). Thus, $(p^*/v^{hi})(\partial v^{hi}/\partial p) = -1, \forall i, h$. Q.E.D.

Thus, the agency supplying the club good with the price-quality specification $(p^*, c(p^*))$ is not only breaking even and "maximising the bangs per buck," but also, it is maximising the level of provision, y , given by (6).

Of course, although Theorem 1 indicates that the agency can determine the optimal per visit price and hence quality of provision independent of the distribution of households, this does not mean that the actual level of y is independent in this way. (6) indicates that the agency would still need to know the aggregate level of visits which would result at $(p^*, c(p^*))$ were it to supply the Stackelberg equilibrium level of y ex ante. The situation is closely analogous to that which occurs with two-stage budgeting in consumer maximisation

with weakly separable utility functions. Here, the weakly separable form of the utility functions (21) enables the agency determining p^* and $c(p^*)$ to separate this decision from those relating to the x^{hi} s and thus y .

An issue which has received much attention in the literature is whether households should be segregated into type-specific clubs, offering different menus of price and quality, where this is possible (e.g., see Berglas & Pines, 1981, 1984, Sandler and Tschirhart, 1984, and Suzanne Scotchmer and Myrna Wooders, 1987). A corollary of Theorem 1 is that, provided the congestion or quality function is identical across households and visits and quality enter the utility function as in (21), this segregation will be irrelevant. The break-even welfare maximising $(p, c(p))$ is the same for all types and can be provided in one club or a system of clubs, assuming clubs are replicable because they do not each possess a unique attribute⁶. It is also clear that this outcome could be decentralised in a free entry equilibrium where firms would have to offer the (common) club price-quality specification which maximised the welfare of individual members to attract any members at all.

It is important to note that utility functions of the family (21) can encompass a wide range of behaviour. E.g., consider two types i and j with respective utility functions $U^i(x, v, c) = x^\alpha (vc)^{(1-\alpha)}$ and $U^j(x, v, c) = (x - \underline{x})^\alpha (vc + 1)^{(1-\alpha)}$, where $\underline{x} > 0$ is a scalar. The second utility function is a quasi-homothetic extension of the *linear expenditure system* to allow for zero consumption of one good, the club good. It is easy to see that, for $c > 0$, all households of type i will be club members, irrespective of the level of their (positive) income, but for type j only households with incomes $M(h) > (\alpha / (1-\alpha)) (p/c) + \underline{x}$ will be club members.

Utility functions in the family (21) allow an appealing, and familiar, interpretation of the interaction between club visits and the quality of those visits. This interpretation is in terms of the “effective” number of visits, or of visits in “efficiency” or “quality-adjusted units”. In the Stone-Geary example, what then matters to households determining their demand for the club good is its quality-adjusted price. Conversely, the Cobb-Douglas example has the peculiarity that household demand for the club good, and hence for the

private good, is actually independent of the club quality, which enters utility only as a multiplicative constant when households regard quality as parametric in their optimisation.

V. Extensions

Utility functions of the family (21) yield a particular $(p, c(p))$ outcome which is independent of the distribution of households' characteristics and has certain desirable features. But, this is not the only family which yields $(p, c(p))$ outcomes which are independent of the distribution of households' characteristics. The next theorem gives the family which yields particular $(p, c(p))$ outcomes, independently of the distribution of households' characteristics, which differ from $(p^*, c(p^*))$ as identified by (16).

Theorem 3. If all types i have utility functions of the form $U^i(x, v, c) = \phi(x, ve^{c/k})$, where $k > 0$ is a scalar and c satisfies (7), then the optimal price and quality satisfy $k = pc'(p)$, independently of the distribution of households' characteristics.

Proof. The sufficiency of this family of utility functions can be proven very easily as before by using (9). To prove the necessity of this family for the stated result, note from (9) and (2) that the optimum satisfies

$$24) \quad \frac{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} \frac{\partial U^i(x^{hi}, v^{hi}, c(p))}{\partial v} f^i(M(h)) dM}{\sum_i \int_{M^{i*}}^{\bar{M}} p \frac{\partial U^i(x^{hi}, v^{hi}, c(p))}{\partial c} f^i(M(h)) dM} = c'(p).$$

Thus, for the result to hold we require:

$$25) \quad \frac{\sum_i \int_{M^{i*}}^{\bar{M}} v^{hi} \frac{\partial U^i(x^{hi}, v^{hi}, c(p))}{\partial v} f^i(M(h)) dM}{\sum_i \int_{M^{i*}}^{\bar{M}} \frac{\partial U^i(x^{hi}, v^{hi}, c(p))}{\partial c} f^i(M(h)) dM} = k = pc'(p)$$

for some scalar $k > 0$. By a similar route as before, for the inner equality in (25) to hold for all distributions, we require $v^{hi} \partial U^i(x^{hi}, v^{hi}, c) / \partial v = k \partial U^i(x^{hi}, v^{hi}, c) / \partial c$. This is again a first-order linear partial differential equation. It has the unique family of solutions

$$(26) \quad U^i(x, v, c) = \phi(x, ve^{c/k}). \text{ Q.E.D.}$$

Unlike the case in Theorem 1, a solution satisfying the FOC $k = pc'(p)$ is not necessarily unique. This can be seen by noting that $pc'(p)$ is not necessarily monotonic even if $c'(p) > 0$, $c''(p) < 0$ everywhere⁷. Multiple solutions satisfying the FOC would have to be separated by appealing to the second-order conditions. It can be shown, by using (25), that the second-order necessary condition for the welfare maximisation reduces to $pc''(p) + c'(p) < 0$ - i.e., to $d[pc'(p)]/dp < 0$. Thus, the optimum will occur on a falling portion of the $pc'(p)$ schedule.

Theorem 2 suggests that, if all households have utility of the form (26) with a common k , the allocation satisfying (24) or (25) might have the following feature. At the optimum, all households have unitary demand price elasticity for club visits; thus, this allocation has households contributing their maximum willingness to pay for the provision of the club good. This is indeed the case, as the following corollary to Theorem 3 confirms.

Theorem 4. If all households have utility functions of the family (26) with a common k , at the optimal allocation satisfying (25) and (2) for all club members, all households have price elasticity of demand for club visits equal to minus unity.

Proof. Retrace the steps in proving Theorem 2, using $p\phi_1(x, ve^{c/k}) = e^{c/k}\phi_2(x, ve^{c/k})$ for (2) and $k = pc'(p)$. Q.E.D.

We have already remarked in the context of Theorem 1 that segregation of types into type-specific homogeneous clubs will be irrelevant if all types have utility functions of the form (21) and if the club congestion or environmental quality function satisfies (A3). Moreover, we saw then that the constrained welfare maximising outcome could be

decentralised in a free entry equilibrium. Clearly, the same results extend to the case where all types have utility functions of the family (26) with an identical k . However, this family of utility functions also provides a convenient way of indexing the difference between types.

Evidently, if types differ in k , and if clubs are replicable, it is optimal for segregation to occur according to k . Indeed, if a menu of price-quality combinations satisfying $k^i = p^i c'(p^i)$, $\forall i$, were offered, individuals of the various types would self-select themselves to clubs offering the appropriate $(p^i, c(p^i))$ combination. It is then a trivial exercise to show that the optimal price and hence quality in these type-specific clubs will be inversely related to the k^i 's which characterise their clienteles⁸.

This is to be expected. With this utility function, the modulus of a type's MRS between the quality of club visits and its use intensity of the club is proportional to its k^i at any given level of visits, irrespective of a given household's income. I.e., in transparent notation, $(\partial U^{hi}/\partial v)/(\partial U^{hi}/\partial c) = k^i/v^{hi}$ for each h of type i . Thus, all else equal, types with higher k^i will pay less for quality.

The utility-relevant magnitudes for households are p and $c(p)$. Our results so far apply to the invariance of these magnitudes for *any* population and *any* distribution of *any* aggregate income. For the overall allocation $(p, c(p), y, V)$ to be independent of distribution, called *total separation* in the introduction, we must restrict ourselves to environments with a fixed population and a fixed aggregate income as well as requiring all utility functions to satisfy either (21) or (26). Otherwise, the essentially private nature of club visits would prevent us achieving a fixed level of y and V even were p and $c(p)$ fixed. It is then well-known from the theory of aggregation (see H. A. John Green, 1964) and Bergstrom and Cornes and Bergstrom and Varian (1985a) that, for independence of V , hence y from distribution, households' utility functions must generate demand functions for the club good which are linear in income and have an income coefficient which is the same for all households. Moreover, it is also clear that there cannot be a threshold income below which households of any type will not make club visits⁹.

If all households must have identical linear-in-income demand functions for club visits going through the origin, then, in our two-good world, they must also have identical

linear-in-income demand functions for the private good. But this means that they must have identical preferences yielding linear Engel curves through the origin - i.e., homothetic preferences. Thus there can be only one type with identical homothetic preferences although possibly different incomes. If there *are* more than one type according to utility functions and there are type-specific clubs, we can obtain a more restrictive form of independence of the overall allocation from the distribution of aggregate income if the distribution of income across types is constant. I.e., if all redistributions preserve the aggregate income of any given type. We summarise these observations as Theorem 5.

Theorem 5. Total Separation. If the congestion function satisfies (A.4), the overall allocation $(p, c(p), y, V)$ is independent of the distribution of a given aggregate income among a fixed population if and only if all households have identical homothetic preferences satisfying either (21) or (26).

Finally, it is of interest to relate our analysis of invariance of the optimal allocation with respect to the distributions of households' characteristics in a club context with Bergstrom and Cornes' (BC) for pure public goods. Unlike BC, we do not require the standard Samuelson condition, equivalent to assuming the availability of unrestricted lump-sum taxation, to hold¹⁰. This is because the club good is essentially private in our formulation where the congestion function takes a particularly accommodating form. Moreover, in the BC model, households are united by their common consumption level of the public good(s). In our model, rather, they are united by the common quality of visits consumed in a particular club. Thus, the latter plays for us a role analogous, but not identical to the common consumption level for the public good(s) in BC. Visits, and thus the levels of club consumption, are allowed to vary with incomes and between households, unlike the case with pure public goods. Self-exclusion from club consumption at some income levels is a possibility. Thus we require a stronger restriction on preferences than do BC, homotheticity instead of quasi-homotheticity, for total separation of allocation from distribution to hold.

VI. Conclusions

This paper has united two strands of the theory of externalities. Using the most commonly employed form of club congestion or quality function, we have shown that the welfare maximising break-even price and quality provision of a club good is then independent of the distribution of self-selecting households' characteristics if and only if all households' utility functions belong to one or other of two families of preferences. In both of these families, the value households place on the volume of club consumption relative to the quality of that consumption is independent of their consumption of the private good, and thus of income. Moreover, we show that in one of these cases the optimal per-visit price-quality pair maximises the quality per unit of expenditure (the "bangs per buck") and, in both cases, it maximises households' willingness to pay for the club good. The homothetic members of the same families of preferences will be necessary and sufficient for the price, quality provision, facility size and the total utilisation of the club good to be independent of the distribution of income provided that there is only one type according to utility function.

The quest, notably by Bergstrom and Cornes, 1981 and 1983, for separation or invariance results of the type which we have obtained was motivated originally by the need to reduce the complexity of the analysis of environments involving public goods. This followed a suggestion of Musgrave (1969). To paraphrase Bergstrom and Cornes, 1983, Musgrave noted that although separation of allocative and distributive decisions is not strictly legitimate, it is a convenient simplification which in practice might produce better decisions than the attempt to determine allocation and distribution simultaneously. Among other things, our results show Musgrave's contention sometimes can be extended to congestible and excludable jointly consumed goods as well as the pure public goods which Bergstrom and Cornes and others consider.

FOOTNOTES

1. Sandler and Tschirhart (1981) and Cornes and Sandler (1986) do examine the Pareto optimal allocation of a private good and a club good in a single-club economy incorporating both exclusion and congestion. However, they did not explicitly consider self-selection to

membership or the separation issues which we address. Fraser and Hollander (1992) present a detailed analysis of the demand side of a single-club economy model which emphasises self-selection and exclusion.

2. This is essentially the approach of, e.g., Oakland and Brito (1980) who use a variational approach inspired by Mirrlees' optimal tax model. They do not consider a congestible good and consider only one type of households in terms of utility function. It is also essentially the approach of Scotchmer (1985) who also considers just one type of household, with all households having the same income, and does not stress exclusion.

3. This assumption is quite standard. See, e.g., Berglas (1976), Scotchmer (1985, footnote 8) and Barro and Romer's (1991) cogent statement of the essence of club models.

4. The most comprehensive discussions of the congestion function are Kolm's, Forsund's, and Berglas and Pines' (1981). The reader is referred to these papers for further details on this topic.

5. Also see Robert Barro and Paul Romer (1991) on this.

6. This line of argument could be shown to lead to a slight extension of the second part of Scotchmer and Wooders' Proposition 1. This states, essentially, that mixed clubs can at best do as well as segregated clubs. Our slight extension arises because we are allowing individuals of the same 'type' to differ in endowments and, hence, in their use intensity of their club.

7. Differentiating, $d[pc'(p)]/dp \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} 0$ as $I \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} - pc''(p)/c'(p)$.

8. To see this, note that the first-order condition for each type i satisfies $k^i = p^i c'(p^i)$.

Thus $I = (\partial p^i / \partial k^i) [p^i c''(p^i) + c'(p^i)] = (\partial p^i / \partial k^i) d[p^i c'(p^i)]/dp$. But, from the second-order condition, $d[p^i c'(p^i)]/dp < 0$. Thus, to satisfy the equalities just given, we must have $\partial p^i / \partial k^i < 0$.

9. Otherwise, we could consider two sets of households of non-zero measure with preferences all either satisfying (21) or (26), one set with members having income below the threshold and the other with members having income above the threshold, and redistribute income so that all members of the union of the two sets now have incomes below the threshold. At unchanged p and $c(p)$, this redistribution would leave unchanged the

aggregate demand for club visits by all other households but it would reduce the demand for visits by households in these two sets, and hence y and V . This rules out quasi-linear preferences of the type used in our special case in Section II and, indeed, the extension of the linear expenditure system to allow for zero consumption of the club good below some threshold income. Thus, *total separation* of allocation from distribution is only possible with the assumed congestion function if households have preferences for the excludable good which precludes self-exclusion!

10. Technically, B-C require the invariance of a sum of ratios - MRSs - while we require the invariance of the ratio of a sum of terms. Although we could convert the latter to a weighted sum of MRS terms of the sort familiar from the analysis of distortionary taxation - see, e.g., Tony Atkinson and Joe Stiglitz (1980) or John Wilson (1991) - this is not particularly illuminating here.

REFERENCES

- Atkinson, Anthony B. and Stiglitz, Joseph E., *Lectures on Public Economics*, New York: McGraw-Hill, 1980.
- Barro, Robert, and Romer, Paul. "Ski-lift Pricing, with Applications to Labour and Other Markets," *American Economic Review*, December 1987, 77, 875-890.
- _____ and _____ (1991), "Ski-lift Pricing, with Applications to Labour and Other Markets: Reply," *American Economic Review*, March 1991, 81, 378-80.
- Berglas, Eitan, "On the Theory of Clubs," *American Economic Review*, March 1976, 66, 116-121.
- _____ and Pines, David, "Clubs, Local Public Goods and Transportation Models: A Synthesis," *Journal of Public Economics*, April 1981, 15, 141-162.
- _____ and _____, "Resource Constraint, Replicability and Mixed Clubs: A Reply," *Journal of Public Economics*, 1984, 23, 391-7.
- Bergstrom, Theodore C., Blume, Lawrence and Varian, Hal. "On the Private Provision of Public Goods." *Journal of Public Economics*, February 1986, 29, 25-49.
- Bergstrom, Theodore C. and Cornes, Richard, C., "Gorman and Musgrave are Dual - An

- Antipodean Theorem on Public Goods,” *Economics Letters*, 1981, 7, 1983.
- _____, and _____, “Independence of Allocative Efficiency from Distribution in the Theory of Public Goods,” *Econometrica*, November 1983, 51, 1753-1765.
- _____ and Varian, Hal R., “When are Nash Equilibria Independent of the Distribution of Agents’ Characteristics?” *Review of Economic Studies*, October 1985a, 52, 715-718.
- _____ and _____, “When do Market Games have Transferable Utility?” *Journal of Economic Theory*, April 1985, 35, 222-233.
- Bernheim, B. Douglas. “On the Voluntary and Involuntary Provision of Public Goods,” *American Economic Review*, September 1986, 76, 789-793.
- Bowen, Howard, 1943, “The Interpretation of Voting in the Allocation of Economic Resources,” *Quarterly Journal of Economics*, November 1943, 58, 27-48.
- Brennan, Geoffrey. and Walsh, Cliff. “A Monopoly Model of Public Good Provision: The Uniform Pricing Case,” *American Economic Review*, March 1981, 71, 196-206.
- Brito, Dagobert. and Oakland, William H. “On the Monopolistic Provision of Excludable Public Goods,” *American Economic Review*, September 1980, 70, 691-704.
- Burns, Michael E. and Walsh, Cliff. “Market Provision of Price-Excludable Public Goods: A General Analysis,” *Journal of Political Economy*, February 1981, 89, 166-191.
- Buchanan, James .M., “An Economic Theory of Clubs,” *Economica*, February 1965, 32, 1-14.
- Cornes, Richard and Sandler, Todd, *The Theory of Externalities, Public Goods and Club Goods*, Cambridge, U.K.: Cambridge University Press, 1986.
- Debreu, Gerard, “The Mathematization of Economic Theory,” *American Economic Review*, March 1991, 81, 1-7.
- Forsund, Finn, “Discussion (of Kolm, 1974),” in Jerome Rothenberg and Ian G. Heggie, eds., *The Management of Water Quality and the Environment*, London: MacMillan, 1974, 177-188.
- Fraser, Clive D., and Hollander, Abraham, “On Membership in Clubs,” mimeo., Warwick University, July 1992.
- Green, H. A. John, *Aggregation in Economic Analysis: An Introductory Survey*, Princeton:

- Princeton University Press, 1964
- Kolm, Serge-Christophe, "Qualitative Returns to Scale and the Optimum Financing of Environmental Policies," in Jerome Rothenberg and Ian G. Heggie, eds., *The Management of Water Quality and the Environment*, London: MacMillan, 1974, 151-171.
- Lindahl, Erik, "Just Taxation: A Positive Solution," English translation of 1910 paper in Richard Musgrave and Alan Peacock, eds., *Classics in the Theory of Public Finance*, London: MacMillan, 1958.
- Mohring, Herbert and Harwitz, Mitchell, *Highway Benefits: An Analytical Framework*. Evanston, Illinois: Northwestern University Press, 1962.
- Musgrave, Richard, "Provision for Social Goods," in Julius Margolis and Henry Guitton, eds., *Public Economics*, London: Macmillan, 1969.
- Olson, Mancur, *The Logic of Collective Action*. Cambridge: Harvard University Press, 1965.
- Samuelson, Paul A., "A Pure Theory of Public Expenditure," *Review of Economics and Statistics*, November 1954, 36, 387-89.
- _____, "Diagrammatic Exposition of a Theory of Public Expenditure," *Review of Economics and Statistics*, November 1955, 37, 350-356.
- _____, "Pure Theory of Public Expenditure and Taxation," in Julius Margolis and Henry Guitton, eds., *Public Economics*, London: Macmillan, 1969.
- Sandler, Todd and Tschirhart, John, "The Economic Theory of Clubs: An Evaluative Survey," *Journal of Economic Literature*, December 1980, 18, 1481-1521.
- _____ and _____ (1984), "Mixed Clubs: Further Observations," *Journal of Public Economics*, 1984, 23, 381-9.
- Scotchmer, Suzanne, "Two-Tier Pricing of Shared Facilities in a Free-entry Equilibrium," *Bell Journal of Economics*, Winter 1985, 16, 456-472.
- _____ and Wooders, Myrna H, "Competitive Equilibrium and the Core in Club Economies with Anonymous Crowding," *Journal of Public Economics*, 1987, 34, 159-73.
- Starrett, David A., *Foundations of Public Economics*, New York: Cambridge University Press, 1988.

Tiebout, Charles M., "A Pure Theory of Local Expenditures," *Journal of Political Economy*, October 1956, 64, 416-424.

Warr, Peter, "The Private Provision of a Public Good is Independent of the Distribution of Income," *Economics Letters*, 1983, 13, 207-211.

Williams, William E., *Partial Differential Equations*, Oxford: Oxford University Press, 1980

Wilson, John D., "Optimal Public Good Provision with Limited Lump-Sum Taxation," *American Economic Review*, March 1991, 81, 153-166.