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The Size and the Power of Unit Root Tests Against Fractional Alternatives: A Monte Carlo Investigation

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Abstract

This paper investigates the size and power of a number of unit root tests, currently in use in both applied macro and financial economics, when the data generating process is fractionally integrated. The long persistence characteristic of fractionally integrated processes lowers the power of unit root tests, when compared with their power against stationary alternatives. The performance of the unit root tests is investigated extensively in a range of experiments, which permit the fractional process to have normal white noise errors, non-normal white noise errors, heteroscedastic errors and serially correlated errors. Power is not seriously affected by non-normality, but can be adversely affected by heteroscedastic and serially correlated errors.

1. Introduction

The low power of the standard Dickey-Fuller (DF) unit root test when the data generating process (DGP) is a stationary process with a root approaching the unit circle is well documented see, for example, Schwert (1989). The power performance of the DF unit root test when the DGP is a fractionally integrated process is worse, see Diebold and Rudebusch (1991). The theoretical point, that unit root tests will have low power when the process generating the data is a fractionally integrated process was made by Sowell (1990). In their conclusion, Diebold and Rudebusch (1991) recommend a "more appropriate testing procedure ... before drawing conclusions about the presence of a unit root".

In line with the recommendation of Diebold and Rudebusch (1991), to aid the ability of researchers ability to detect a mean-reverting or mean-averting process against a random-walk, this paper analyses the performance of a variety of unit root tests when the DGP is a fractionally integrated series. This study is of particular importance in view of recent findings that certain macro and financial time series, such as, GNP (Diebold and Rudebusch (1989)), exchange rates (Cheung (1993)) and interest rates (Shea (1989)) can be represented as fractional processes. The tests used in this paper include a number of unit root tests used in both applied macro economics and financial economics. The tests considered are the variance ratio test of Cochrane (1988), the extension of this test by Lo and MacKinlay (1988) (assuming both the homoscedastic and the heteroscedastic error increments), the long autocorrelation test of Fama and French (1988), as modified according to Jegadeesh (1990) and the Augmented Dickey-Fuller (ADF) test

2. Fractionally Integrated Models

Mainstream time series analysis has, until recently, considered the dichotomous world, which distinguishes between;

(i) Stationary ARMA(p_1, p_2) series, represented as

$$A(L)y_t = B(L)\varepsilon_t, \quad t=1,..,T \tag{1}$$

where, $A(L) = 1 - \phi_1 L - \phi_2 L^2 - .. - \phi_{p_1} L^{p_1}$ and $B(L) = 1 + \theta_1 L + \theta_2 L^2 + .. + \theta_{p_2} L^{p_2}$. This y_t series is said to be integrated of order zero, that is, $y_t \sim I(0)$, and is characterised as having a finite memory, an autocorrelation function which decays to zero, the roots of A(L) lying outside the unit circle, and a finite spectral density at the origin and

(ii) Nonstationary ARIMA(p_1, d, p_2) series, represented as

$$A(L)y_t \equiv (1-L)^d A^*(L)y_t = B(L)\varepsilon_t, \quad t=1,..,T \tag{2}$$

where, d is an integer number, often taken to be unity. This y_t series is said to be integrated of order, d, that is, $y_t \sim I(d)$, and is characterised by having infinite memory, an autocorrelation function which does not decay, at least one of the roots of $A(L)$ on the unit circle, and an infinite spike in the spectral density at the origin. A non-stationary series, y_t, as represented in equation (2), is transformed into a stationary series by appropriately differencing y_t d times.

Following the paper by Granger and Joyeux (1980) the range of models available were expanded by introducing fractionally integrated series. Fractionally integrated series broke the dichotomy between I(0) and I(1) series, by permitting the integration parameter, d, in equation (2), to take on non-integer values. For non-integer values of d equation (2) is an ARFIMA(p_1,d,p_2) model.

For -1.5 < d < 0.5, the series, y_t, is stationary with an invertible ARMA representation. This ARFIMA model exhibits more persistence with the autocorrelation function decaying much slower than for the corresponding I(0) series with similar ARMA parameters. For d > 0.0 the spectral density has an infinite spike at the origin, while for d < 0.0 the value of the spectral density at the origin is zero.

For 0.5 < d < 1.0, the series, y_t, is nonstationary with a non-invertible ARMA representation. However, even though the series is non-stationary, the autocorrelation function will still decay to zero, implying that the memory of the process is finite and that given a shock the process will tend to revert to its mean, that is mean-reverting. For d > 1.0 the series is mean-averting, and a shock to the process will cause the series to deviate away from its original starting point.

Fractionally integrated processes have autocorrelation functions that decays at a much slower rate than for the corresponding I(0) series. Diebold and Rudebusch (1989) report the autocorrelation function values for a series fractionally integrated with d = 0.3 and for an AR(1) model with a coefficient $\phi_1 = 0.5$. To reiterate the point made there Figure 1 plots the autocorrelation function for two fractionally integrated series with integration parameters, d = 0.2 and d = -0.2 and two AR(1) series with coefficient $\phi_1 = 0.7, -0.5$. The persistence in these series implies that unit root tests, which attempt to estimate the extent of persistence, lose power quickly as the fractional parameter, d, approaches unity. This point is demonstrated in Diebold and Rudebusch (1991).

For a simple ARFIMA(0,d,0) process, equation (2) is written as

$$(1-L)^d y_t \equiv [1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + ...]y_t = \varepsilon_t, \quad t = 1,..,T \tag{3}$$

where $\varepsilon_t \sim \in(0, \sigma^2)$, the corresponding autocorrelation coefficients are calculated as

$$\rho_\tau = \frac{\Gamma(1-d)\Gamma(\tau+d)}{\Gamma(d)\Gamma(\tau+1-d)}, \quad \tau = \pm 1, \pm 2, \ldots, \tag{4}$$

and autocovariances as,

$$\gamma_\tau = \frac{\sigma_\varepsilon^2 \Gamma(1-2d)\Gamma(\tau+d)}{\Gamma(d)\Gamma(1-d)\Gamma(\tau+1-d)}, \ \tau = \pm 1, \pm 2, .. \quad \gamma_0 = \frac{\sigma_\varepsilon^2 \Gamma(1-2d)}{\Gamma(1-d)\Gamma(1-d)}. \tag{5}$$

3. Tests of Unit Roots

This paper uses five tests that are currently widely used in the applied literature to test the null hypothesis, at conventional significance levels, of a unit root against a suitable alternative hypothesis, when a fractionally integrated process is the underlying DGP. Whilst well-known, and well-used, the five tests are presented here for notational purposes:

(i) Augmented Dickey-Fuller (ADF) test: This is a regression based test and uses the following model:

$$\Delta y_t = \mu + \alpha y_{t-1} + \sum_{j=1}^{q} \gamma_j \Delta y_{t-j} + \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma_\varepsilon^2), \quad t = 1, .., T \tag{6}$$

where Δ is the first difference operator. The ADF test, tests the null hypothesis that the series is I(1), that is, $H_0: \alpha = 0$, against three alternative hypotheses; the series is not I(1), that is, $H_{11}: \alpha \neq 0$, the series is mean reverting, that is, $H_{12}: \alpha < 0$, and the series is mean averting, that is, $H_{13}: \alpha > 0$.[1] While Diebold and Rudebusch (1991) use both the t-ratio on α, and the actual coefficient on α (using the statistic $T.\alpha$) to test for the presence of a unit root, we only use the t-ratio. The appropriate critical values for the tests are taken from Fuller (1976).

(ii) Lo and MacKinlay (Homoscedastic) test: This test uses a variance ratio statistic, z(q), calculated as:

$$z(q) = \sqrt{T}\,\bar{M}_r(q)(2(2q-1)(q-1)/3q)^{-1/2} \sim N(0,1) \quad (7)$$

where, $\bar{M}_r(q) \equiv \dfrac{\bar{\sigma}_c^2(q)}{\bar{\sigma}_a^2} - 1$,

and $\bar{\sigma}_a^2 = \dfrac{1}{T-1}\sum_{k=1}^{T}(y_k - y_{k-1} - \hat{\mu})^2$, $\bar{\sigma}_c^2(q) = \dfrac{1}{m}\sum_{k=q}^{T}(y_k - y_{k-q} - q\hat{\mu})^2$, $m = q(T-q+1)(1-\frac{q}{T})$, and

T is the sample size. The proof that this statistic follows a normal distribution, as reported in equation (7), is an asymptotic result and requires $T/q \rightarrow \infty$. Therefore, following the suggestion of Lo and MacKinlay (1989) we use the empirical critical values calculated in their paper, and these are reported in Appendix A (Table A1) for convenience.

The test statistic z(q), tests the null hypothesis of a random walk, that is, $H_0: z(q) = 1$, against the three alternative hypotheses, that the series does not follow a random walk, that is, $H_{11}: z(q) \neq 1$, that the series is mean-reverting, that is, $H_{12}: z(q) < 1$, and that the series is mean-averting, that is, $H_{13}: z(q) > 1$.

[1]The choice of q in this test, and in each of the four subsequent tests, will be discussed in the section describing the actual experiments undertaken.

(iii) Lo and MacKinlay (Heteroscedastic) test: This is of the same form as the Lo and MacKinlay (Homoscedastic) test, except that it allows for heteroscedastic increments in the error process, and is calculated as

$$z^*(q) \equiv \frac{\sqrt{T}\,\bar{M}_r(q)}{\sqrt{\hat{\theta}}} \sim N(0,1) \qquad \textbf{(8)}$$

where, $\bar{M}_r(q)$ is as defined for the Lo and MacKinlay (Homoscedastic) test above and

$$\hat{\theta}(q) = \sum_{j=1}^{q-1}\left[\frac{2(q-j)}{q}\right]\hat{\delta}(j) \text{ and } \hat{\delta}(j) = \frac{T\sum_{k=j+1}^{T}(y_k - y_{k-1} - \hat{\mu})^2(y_{k-j} - y_{k-j-1} - \hat{\mu})^2}{\left[\sum_{k=1}^{T}(y_k - y_{k-1} - \hat{\mu})^2\right]^2}.$$

The null and alternative hypotheses are as described for the Lo and MacKinlay (Homoscedastic) test. Again the proof that the statistic $z^*(q)$ follows a normal distribution is an asymptotic result. Consequently empirical critical values obtained by simulation are used and these are reported in Table A2 in Appendix A. The simulation results to derive the critical values for this test and the two subsequent tests are based on 10,000 replications of a white noise process $\varepsilon_t = y_t - y_{t-1}$, $t = 1,..,T$, (NAG routine G05DDF is used to generate the psuedo random numbers) for each of the alternative sample sizes, T, considered and for a range of possible parameter values of q.

(iv) Cochrane test: This is a variance ratio test, formed as a weighted average of the sample autocorrelation coefficients, $\hat{\rho}(j)$ and is written:

$$VR(q) = 1 + 2\sum_{j=1}^{q-1} \left[\frac{q-j}{q}\right] \hat{\rho}(j) \tag{9}$$

This tests the null hypothesis of no mean aversion, that is, $H_0: VR(q) = 1$, against the three alternatives hypotheses; $H_{11}: VR(q) \neq 1$, $H_{12}: VR(q) < 1$ and $H_{13}: VR(q) > 1$, for no unit root, mean-reversion and mean aversion, respectively. The critical values for this statistic are obtained by simulation and are reported in Table A3 in Appendix A.

(v) Jegadeesh test: This test is of the form used by Fama and French (1988); however, Jegadeesh (1990) showed that a model with a dependent variable as in equation (10) was more powerful, than the one which considered $\sum_{i=1}^{J} \Delta y_{t-1+i}$, which was used by Fama and French (1988), the test is based on the following regression specification:

$$\Delta y_t = \alpha_k + \beta_q \sum_{i=1}^{q} \Delta y_{t-i} + u_t, \quad u_t \sim iid(0, \sigma_u^2), \tag{10}$$

This tests the null hypothesis of no mean-reversion, that is, $H_0: \beta_q = 0$, against the three alternative hypotheses, $H_{11}: \beta_q \neq 0$, $H_{12}: \beta_q < 0$, $H_{13}: \beta_q > 1$, corresponding to a non-unit rootm mean-reversion and mean-aversion. The critical values are reported in Table A4 in Appendix A. Jegadeesh (1990) noted that the errors from this process are likely to be heteroscedastic in nature if stock returns over different measurement intervals are used as the dependent and independent variables. Consequently, Jegadeesh (1990) utilised White's (1980) Heteroscedastic Consistent Standard Errors (HCSE) to alleviate potential problems.

4. Monte Carlo Simulations

We consider fractionally integrated DGPs for the series y_t, for a range of integration parameters, d = 0.7, 0.8, 0.9, 0.95, 1.00, 1.05, 1.1, 1.2, and 1.3. In order to generate the series y_t for these values of d, we actually generate $(1-L)y_t = \Delta y_t$ using the fractionally integrated parameters, $\tilde{d} = d - 1$ = -0.3, -0.2, -0.1, -0.05, 0.0, 0.05, 0.1, 0.2, 0.3, and then use a starting value of $y_0 = 0.0$ to construct the series $(y_1, y_2, .., y_T)$. The series $(1-L)y_t$ is constructed along the lines used by Diebold (1990) and Sowell (1990). A vector ε_t of T N(0, 1) deviates is constructed using the NAG routine G05DDF. Using equation (5) we calculate the desired $T \times T$ autocovariance matrix, Σ . Using NAG routine F07FDF a Choleski decomposition of Σ, $\Sigma = PP'$ is performed, where P is lower triangular. Finally,$(1-L)y_t$ is formed as $(1-L)y = P\varepsilon$. As Diebold (1989) and Granger and Joyeux (1980) note, the attraction of this method for generating the series is that the series y is formed without dependence on startup values. The number of replications undertaken for the power results is N = 1000 and for the size results is N = 10,000.

The test statistics are calculated for a range of different sample sizes and a range of q. Following Lo and MacKinlay (1989) our sample sizes are taken as T = 32, 64, 128, 256, 512, and 1024. For the two tests of Lo and MacKinlay q is allowed to take on values up to 50% of the sample size, q = 2, 4, 8, .., (1/2)T. For the tests of Cochrane and Jegadeesh q is allowed to be as large as 25% of the sample size, that is, q = 2, 4, .., (1/4)T, the smaller choice of q is because Kim, Nelson and Startz (1991) use a smaller number for q compared with Lo and MacKinlay (1989), when calculating the Cochrane statistic. The

ADF test has the lag length q which is $O(T^{1/3})$, in fact, q = 0, 1, 2,...

4.1 Results

Tables 1 through to 5 report the power of each of the five tests used in the paper, when the fractional integration parameter, d = 1.1. For each Table power results are reported at the 1%, 5% and 10% significance levels. For each significance level three power statistics are reported. The first is the power against the two-sided alternative, the second is the power against the one-sided alternative of mean reversion, and the third is the power against the one-sided alternative of mean-aversion.[2] Figures 1 - 5 show how power varies with T, q and d, against an appropriately specified 5% one-sided alternative hypothesis.[3] One-sided results are reported to avoid biases in the power statistic when using a two-sided test, which arise due to the rejection of the incorrect null hypothesis on the wrong side of the distribution. However, it must be pointed out that graphs for the two-sided alternative hypothesis are qualitatively very similar as are the graphs at the 1% and 10% significance levels.[4] Figure 6 gives a graph of the relative performance of the five alternative tests for d = 0.9 for the 5% one-sided tests over T and q.

The results from each of the separate tests will now be discussed in turn:

Lo and MacKinlay (Homoscedastic): Figure 1 shows that regardless of the sample size or the size of the fractional integration parameter, d, power initially increases q up to around

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[2] For d = 1.1, and an infinite sample size there ought to be no occasions on which the null hypothesis is rejected against the alternative of mean-reversion.

[3] The alternative hypothesis is on of mean-reversion for d < 1.0 and mean-aversion for d > 1.0.

[4] Due to space limitations graphs for d = 0.7 and d = 1.3 are not reported, but follow the same pattern as the rest of the graphs.

q = 4, then declines markedly for larger values of q. In fact, across all values of d and all sample sizes, a value of q = 4 or, less frequently, 8, seems to yield highest power. A small optimal value of q in these fractionally integrated models is in contrast with the much larger optimal values found by Lo and MacKinlay (1989). There is a small loss in this power from choosing small, but non-optimal values of q; however, for larger values of q power falls quite dramatically. For example, from Table 1, for T = 1024, d = 1.1 and q = 2, power is 74.9% at the 10% significance level, and this rises to 87.3% at q = 4, and then falls to 37.7% at q = 128 and further still to 15.6% at q = 512.

This test exhibits high power and correctly rejects the null hypothesis of a random walk up to 97.9% of the time using the two-sided 10% significance level at q =4 (and 98.9% of the time for a one-sided 10% significance level) when d= 1.1. This power falls markedly, when we consider d = 1.05 at the 10% two-sided significance level power at q = 4 is 57.1%. Power also falls with sample size, T, and is only 37.5% for T = 512 and 16.0% for T = 128 when d = 1.05.

This test performs slightly asymmetrically, for d < 1 and d > 1, see Figure 1. There is evidence that power does not fall quite as quickly for d > 1.0 with increases in q, compared with the results for d < 1.0. Overall, it seems this test is capable of detecting mean aversion better than mean reversion, although this distinction between d > 1.0 and d < 1.0 is less noticeable as T increases. In fact as $T/q \rightarrow \infty$ this asymmetry disappears entirely. In addition, there is a tendency to reject the null hypothesis on the incorrect side of the distribution for d > 1.0, compared with d < 1.0. For example, for d = 0.9 and T = 128 at the 5% significance level there are only 0.2% or 0.3% of cases (depending on q) when the null hypothesis is rejected against an alternative hypothesis of mean aversion; however, for d = 1.1 and T = 128 at the 5% significance level there are up to 1.8% of rejections of the null by mean reversion. It should also be noted that this bias tends to increase with q. The nature of this result is that one ought to be a bit wary of the better power properties for $d > 1.0$, when using two-sided alternative hypotheses.

Lo and MacKinlay (Heteroscedastic): Table 2 and Figure 2 shows that this test has similar results to that of the Lo and MacKinlay (Homoscedastic) test. However, this test is somewhat less powerful (which would be expected when the errors are homoscedastically distributed).

Cochrane test: Figure 3, indicates that the optimal value of q appears to be around either $q = 4$ or 8, with a slight tendency for higher values of q compared with the Lo and MacKinlay tests. The shape of the power function is quadratic in q, and there is a large cost in choosing too large a value for q, with power falling rapidly for values of q bigger than $q = 8$. The power results for this test are slightly better than those for the Lo and MacKinlay test, although the difference between the two is small (see Figure 6). The asymmetry between $d < 1.0$ and $d > 1.0$ exists, with this test better able to detect mean aversion than mean reversion, a fact which becomes slightly more prominent for larger values of q. For this test the tendency to reject in the wrong direction is less evident.

Jegadeesh test: The results from this test (Figure 4 and Table 4) suggest that the best lag length is $q = 2$, with power quickly falling after $q = 4$. Figure 6 shows that this test is less powerful than either of the Lo and MacKinlay tests or the Cochrane tests for $d < 1.0$, presumably due to the use of the inefficient HCSE. Surprisingly, this test has comparable power results to those of Lo and MacKinlay and Cochrane for $d > 1.0$. The relative power performance of the test does, however, fall more steeply as q increases.

ADF: Figure 5 shows that the power of this test declines geometrically as the lag length q is increased above zero (Table 5 confirms this finding). However, this is surprising due to the significance of the estimated $\hat{\gamma}_j$ parameters in equation (6), Table 6 reports the choice of the lag length, q, using the significance of the γ_j parameter as the criteria. While the optimal lag length, q, is never large, values of q = 3 do arise for d = 0.7, 1.3 for T = 1024, contradicting the maximum power results which choose q = 0 as the best lag length.

Figure 6 shows that this test is substantially less powerful than all of the other tests considered in this paper across all values of d and its relative performance deteriorates with increases in T.

Table 6: Optimal Choice for the Lag length q, for ADF Model

		T					
		32	64	128	256	512	1024
d	-0.3	0	0	0	1	2	3
	-0.2	0	0	0	1	1	2
	-0.1	0	0	0	0	0	1
	-0.05	0	0	0	0	0	0
	0.05	0	0	0	0	0	0
	0.1	0	0	0	0	1	1
	0.2	0	0	1	1	2	2
	0.3	0	1	1	1	2	3

From Figure 5, the asymmetry between d < 1.0 and d > 1.0 observed in the previous tests, is also evident here, but is more marked, and, more importantly, shows that the ADF test is more capable of detecting mean reversion than mean aversion. This finding is not consistent for all q and as q increases the relative power of mean aversion to mean reversion improves. For small T, T = 32, the power of the mean aversion test is actually greater than that of mean reversion. An additional peculiarity of the ADF test is that for d

> 1.0, there are a number of cases when the null hypothesis is rejected on the incorrect side of the distribution. For example, at the 5% significance level for d = 1.3 and T = 128 the null hypothesis is rejected 37.0% of times against an alternative of d > 1.0 and 9.7% of times against an alternative of d < 1.0, for a 10% two-sided test this yields a much larger power statistic than ought to be observed of 46.7%. Compare this results with that when d = 0.7 and T = 128. In this case, there are no instances of the null being rejected by values of d > 1.0 and 77.9% of times when it is rejected by d < 1.0, yielding a 10% two-sided power of 77.9%. In part, therefore the two-sided power results for d > 1.0 are overly optimistic.

4.2 Non-Normality

To investigate the effects of non-normality on the size and power of these test statistics, following Diebold (1990) the series y_t is formed as $\Delta y_t = (1-L)^{d-1}\ [sign(\varepsilon_i)\,\varepsilon_i^2]$, which has leptokurtic, but symmetric innovations.

Lo and MacKinlay (Homoscedastic) test: Empirical size results are reported in Table 7. For each of the significance levels, 1%, 5% and 10%, three size results are reported. The first reflects the 2-sided rejection of the null hypothesis, the second, is the number of one-sided rejections of the null hypothesis by the alternative of d < 1.0 and the third, the number of one-sided rejections of the null hypothesis by the alternative hypothesis, d > 1.0.

These size results lie close to their corresponding theoretical sizes for large T. For smaller T, the empirical sizes are too small, and exhibit an asymmetry, with the alternative d > 1.0, being rejected too infrequently. For a 5% test, with N = 10,000 replications, the 95% confidence interval for a 5% significance level test is 4.6% - 5.4%. There are nineteen occasions on which the rejection frequency lay outside this confidence interval, of which thirteen were against the alternative d > 1.0, and all thirteen times had rejection frequencies below 4.6%. Tables 15 and 16 report the power performance of this test for the optimal choice of q, over all values of T, for d = 1.1 and d = 0.9, respectively. The power results, show that non-normality does not seriously affect the power of this test across all values of q. Surprisingly, despite the under-rejection of the null hypothesis for d > 1.0, the power of the test to reject d > 1.0 is greater than its power to reject d < 1.0.

Lo & MacKinlay (Heteroscedastic): This test does not perform as well as the homoscedastic test. In particular, the empirical size results (Table 8) shows a tendency to over-reject the true null hypothesis at the 5% and 10% significance levels. These size problems diminish as T increases. The asymmetry in the size results between d > 1.0 and d < 1.0 exist; however, there is an excessive rejection by the alternative d < 1.0. Again the power properties of this test (see Tables 17 and 18) are similar to those reported obtained under normality.

Cochrane test: The size results for this test (Table 9) are smaller than their theoretical levels for T < 256, but there is little eveidence of the asymmetry between d > 1.0 and d < 1.0. The power results for this test, for the optimal value of q, are similar to those shown in Table 4, see Tables 19 and 20. However, for non-optimal values of q the power results are poorer. As T any effects from non-normality disappear.

Jegadeesh test: The size results (Table 10) are different from those of the other three tests with a tendency for d < 1.0 to be under-rejected compared to d > 1.0. Consequently, the power of this test to reject d > 1.0 is markedly greater than its ability to reject d < 1.0 (see Tables 21 and 22).

ADF test: This test is not affected by non-normality, with the size results close to their theoretical levels. This test still has the lowest power of all the tests considered, see Table

23 and 24.

Non-normality does not distort the size results or the power of any of the tests considered. To investigate this further a highly non-normal U-shaped distribution was generated for the error process. The results across all tests are very similar in nature to those reported here, although there was less evidence of asymmetry in the empirical sizes (see Tables 15-24 for the power results across all tests for d = 1.1 and d = 0.9).

4.3 ARCH

As two of our tests are designed to allow for heteroscedastic errors, we now report the finding when we assume a near integrated ARCH model of the form $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$. The empirical size results for the Lo & MacKinlay (Homoscedastic) test and the Cochrane test are poor with a tendency to over-reject the correct null hypothesis for each sample size T for small q, (see Tables 11 and 13, respectively). For these two tests there are two offsetting results which need to be considered when evaluating the results, firstly, size approaches its theoretical value at T/q falls, whilst power falls as T/q falls.

The size results of the Lo and MacKinlay (Heteroscedastic) test are close to their theoretical values (Table 12). With, if anything, a tendency to under-reject the null hypothesis, especially for small q. The power performance of this test is dramatically worse with the introduction of the ARCH errors (see Tables 17 and 18). For example, the one-sided 5% significance level, for d = 0.8, T = 128, and q = 4, has power of 57.3%, and this falls to 38.9% for the ARCH model. The optimal value of q appears to grown with an increase in persistence in the errors from q = 4 to q = 8. There now appears to be less of a cost to over-specifying q, as power is less variable as a function of q.

The Jegadeesh test performs well, with the empirical size results (Table 14) close to their theoretical values. The power of this test (Table 21 and 22) is smaller compared with the results when the errors are white noise errors, and is smaller than that of the Lo and MacKinlay (Heteroscedastic) test, although the extent of the difference in power is not as great as previously noted. The optimal value of q has now increased to be either q = 2 or 4, again due to the increase in persistence in the error.

The size results for the ADF test are not affected by the presence of ARCH errors. and the power performance of this test (Tables 23 and 24) is not seriously harmed by the presence of the ARCH errors, although its performance relative to the other tests continues to be poor.

4.4 AR and MA Parameters

Finally, results are reported for an ARFIMA model which permits the existence of both Autoregressive (AR) and Moving Average parameters (MA) in equation (2). The ARFIMA model is assumed to be either ARFIMA(1, d, 0) or ARFIMA(0, d, 1) and only two parameter values are considered for both the AR parameter, $\phi_1 = 0.2, -0.2$, and the MA parameter, $\theta_1 = 0.2, -0.2$. Sowell (1992), gives the form of the autocovariance function for ARFIMA models of this type, which is markedly more complicated than that for no ARMA parameters.

The results show that the AR and MA parameter can either help or hinder the ability of the unit root tests to determine whether the underlying series has a unit root. Tables 25-28 below reports the power of the various unit root tests at rejecting the existence of a unit root for d = 0.8, 1.2 for the four ARFIMA models. To save on space the results are reported for a sample size, T = 128. The results across all sample sizes are qulitatively similar, although there is evidence to suggest that the power performance deteriorates relative to the ARFIMA (0 , d, 0) model for small sample sizes.

The results show that the presence of either a positive MA or AR parameter markedly increases the power of the various unit root tests to reject the null hypothesis of a random walk model for d = 1.2. While a negative AR or MA parameter reduces the power of these tests to reject the false null hypothesis, for example for $\theta = 0.2$ and d = 0.8, the power of the Lo and MacKinlay test falls from is 4.2% for a 2-sided test at the 1% significance level, compared with 32.4% when $\theta = 0.0$. For $\theta = -0.2$ power is 93.4%. Across all test procedures the presence of an AR or MA parameter has a similar effect on the power results. For the ARFIMA model with d = 1.2 and $\phi = 0.2$, the autocorrelations of the first difference of the series, the beyond the first lag, will be more positve than for the corresponding series with a positive MA parameter. However, for d = 0.8 the positive AR parameter will actually offset the negative autocorrelations for the first difference of the series beyond the first by more than the model with a postive MA parameter. Consequently, the AR models will have a larger number of rejections relative to the MA model for $\tilde{d} = d - 1 = 0.2$, and a smaller number for $\tilde{d} = d - 1 = -0.2$. When $\phi = -0.2$, as the autocorrelations alternate in sign one expects the reverse finding to that noted above. A comparison across the alternative test statistics shows a overall pattern of results similar to that noted elsewhere in this paper, in particular, the Lo and MacKinlay and Cochrane tests perform similarly, the Jegadeesh performs slightly worse for d < 1.0 and the ADF test performs markedly worse.

Inceasing the AR or MA parameter even more has the effect of increasing rejections of the unit root null hypothesis. Although in those cases where the AR or MA parameter offsets the fractional parameter, that is for those models when ϕ, $\theta > 0$ (< 0) and $\tilde{d} < 0$ (> 0) these rejections occur on the wrong side of the distribution.

5. Concluding Remarks

This paper looked at the performance of alternative tests to investigate the performance of unit root tests against fractional alternatives. Whether or not the errors are normally distributed has little effect on the performance of the test. The two tests of Lo and MacKinlay and the Cochrane test all perform similarly, with high power against fractional alternatives. Jegadeesh's test performs slightly worse, presumably due to the ineffient standard errors used, and the ADF test is worst.

The superior power properties of the variance ratio test compared with regression based test against non-fractional, that is, ARIMA, alternatives is well documented in Poterba and Summers (1988). Our results show that the same conclusion carries over to the case of fractional, that is, ARFIMA, alternatives.

All tests, except the ADF test, detect mean aversion with slightly higher power than mean reversion. For the ADF test this result is reversed with a markedly better performance at detecting mean reversion compared to mean aversion. Rejection of the null hypothesis on the wrong side of the distribution by the ADF tests makes the use of two-sided tests less desirable.

For all tests the optimal lag length q, was small, q = 0 for the ADF, q = 2 for Jegadeesh and q = 4 for the Lo and MacKinlay test and the test of Cochran. This lag length seems shorter than those traditionally used in the empirical literature.

The introduction of ARCH errors slightly increases the optimal lag length, q, in all tests, due to greater persistence. The Lo and MacKinlay (Homoscedastic) test and the Cochrane test now have substantial size bias and are therefore unreliable. The remaining three tests showed no sign of size bias, although there was a marked fall in power for all three tests, with the Lo and MacKinlay test performing best, followed by the test of Jegadeesh, and the ADF test the least powerful.

The introduction of AR or MA processes into the fractionally integrated model, either accentuates or reduces the ability of the unit root test to detect the presence of a unit root, depending upon whether the autocorrelation coefficents in the series in first differences are made more or less non-zero by the AR or MA parameters.

Table 1 Power Results for Lo & McKinley (Homoscedastic) Test: d= 1.1

T	q	1%			5%			10%		
32	2	1.6	0.3	3.2	7.1	1.6	11.1	12.7	3.9	20.8
32	4	1.6	0.3	2.5	7.2	1.9	11.7	13.6	5.3	19.8
32	8	1.6	0.4	2.6	6.3	1.8	9.5	11.3	4.5	18.7
32	16	1.8	0.6	2.8	6.2	2.5	9.4	11.9	5.7	17.9
64	2	2.5	0.1	4.3	9.8	1.3	16.9	18.2	2.6	28.7
64	4	2.8	0.1	5.6	11.5	1.5	18.8	20.3	2.6	28.9
64	8	2.4	0.1	4.5	9.5	1.3	16.2	17.5	3.6	27.8
64	16	2.3	0.3	4.0	10.9	1.9	15.5	17.4	4.0	23.9
64	32	3.7	0.9	4.8	11.1	3.4	14.0	17.4	6.9	22.8
128	2	6.6	0.0	10.9	18.5	0.2	27.8	28.0	1.1	43.0
128	4	8.7	0.0	12.5	22.2	0.3	32.7	33.0	1.0	45.2
128	8	9.1	0.1	12.4	21.0	1.0	29.6	30.6	1.4	42.1
128	16	7.2	0.1	11.0	19.6	1.0	28.1	29.1	2.7	39.6
128	32	6.1	0.2	8.9	15.8	1.8	21.3	23.1	2.8	31.2
128	64	5.4	0.6	8.0	14.7	1.7	19.4	21.1	3.4	28.1
256	2	19.2	0.0	25.6	35.9	0.0	49.2	49.2	0.0	62.9
256	4	23.5	0.0	32.1	44.1	0.0	56.6	56.6	0.1	68.0
256	8	22.3	0.0	29.1	43.4	0.1	53.7	53.8	0.3	65.3
256	16	20.5	0.1	26.9	38.2	0.2	47.4	47.6	1.1	59.1
256	32	16.4	0.1	20.4	29.1	0.5	39.8	40.3	1.4	49.2
256	64	10.8	0.1	14.8	22.1	0.8	30.7	31.5	1.8	40.5
256	128	8.9	0.2	12.5	19.3	1.1	26.1	27.2	3.0	36.7
512	2	43.0	0.0	51.9	63.5	0.0	75.0	75.0	0.0	84.5
512	4	53.5	0.0	62.6	73.7	0.0	82.3	82.3	0.0	89.7
512	8	53.1	0.0	62.8	74.2	0.0	80.5	80.5	0.0	87.8
512	16	46.2	0.0	53.8	65.0	0.0	74.0	74.0	0.1	82.3
512	32	33.0	0.1	41.3	53.1	0.1	62.2	62.3	0.2	72.3
512	64	21.7	0.1	27.4	38.7	0.2	49.2	49.4	0.5	59.7
512	128	12.8	0.0	17.3	26.3	0.5	35.1	35.6	1.3	46.8
512	256	10.4	0.0	13.2	19.3	0.3	28.0	28.3	1.5	36.8
1024	2	74.9	0.0	82.6	89.8	0.0	94.4	94.4	0.0	97.4
1024	4	87.3	0.0	90.9	95.4	0.0	97.9	97.9	0.0	98.9
1024	8	85.2	0.0	89.4	94.6	0.0	96.7	96.7	0.0	98.6
1024	16	77.2	0.0	83.3	89.5	0.0	93.2	93.2	0.0	96.6
1024	32	64.3	0.0	71.3	79.0	0.1	86.5	86.6	0.2	92.3
1024	64	48.8	0.0	54.9	64.9	0.0	74.9	74.9	0.2	84.0
1024	128	31.7	0.0	37.7	48.5	0.1	56.4	56.5	0.2	69.5
1024	256	18.9	0.0	25.4	33.4	0.4	42.0	42.4	0.5	54.9
1024	512	15.6	0.0	21.5	28.4	0.4	36.3	36.7	0.6	47.3

Table 2: Power Results for Lo & McKinley (Heteroscedastic) Test: d = 1.1

T	q	1%			5%			10%		
32	2	1.7	0.6	3.0	7.6	2.4	12.0	14.4	4.6	19.8
32	4	2.1	0.5	2.5	8.1	2.1	12.4	14.5	5.2	21.0
32	8	1.7	0.2	3.2	6.6	1.3	10.2	11.5	4.4	19.1
32	16	2.3	0.5	3.3	6.2	2.3	10.6	12.9	5.2	17.9
64	2	2.3	0.1	4.1	8.4	1.3	17.3	18.6	2.3	29.5
64	4	3.3	0.3	4.9	11.4	1.6	19.5	21.1	2.4	29.5
64	8	2.7	0.0	4.6	10.2	0.9	17.5	18.4	3.5	28.9
64	16	2.5	0.3	5.0	11.2	2.0	15.7	17.7	4.0	24.6
64	32	4.0	0.8	4.8	11.8	3.1	14.3	17.4	6.5	23.0
128	2	6.3	0.0	8.9	17.4	0.4	27.6	28.0	0.8	42.7
128	4	9.3	0.0	13.0	23.3	0.2	32.6	32.8	1.0	44.8
128	8	10.4	0.1	12.7	21.7	0.9	30.3	31.2	1.4	42.1
128	16	7.7	0.1	11.3	19.8	0.8	29.0	29.8	2.6	40.2
128	32	7.0	0.2	9.1	16.8	1.8	22.0	23.8	3.0	31.3
128	64	6.2	0.6	8.4	14.7	1.9	19.8	21.7	4.1	27.4
256	2	19.4	0.0	25.4	37.1	0.0	49.9	49.9	0.0	62.8
256	4	25.7	0.0	34.9	45.9	0.0	55.9	55.9	0.2	68.0
256	8	25.5	0.0	32.2	44.4	0.1	53.9	54.0	0.4	66.0
256	16	22.5	0.1	29.8	39.1	0.3	47.7	48.0	1.1	59.5
256	32	17.0	0.1	21.3	30.6	0.5	39.8	40.3	1.4	49.3
256	64	11.2	0.2	15.1	22.3	0.7	31.0	31.7	1.9	40.9
256	128	8.3	0.1	12.9	19.1	1.0	26.4	27.4	3.2	36.8
512	2	42.7	0.0	52.2	63.0	0.0	74.7	74.7	0.0	84.8
512	4	53.2	0.0	63.0	73.1	0.0	82.4	82.4	0.0	89.7
512	8	53.9	0.0	63.0	74.1	0.0	80.8	80.8	0.0	88.1
512	16	48.7	0.0	54.2	64.4	0.0	73.6	73.6	0.1	82.2
512	32	33.6	0.1	41.2	53.7	0.1	62.3	62.4	0.2	72.4
512	64	20.0	0.1	27.5	39.0	0.2	50.3	50.5	0.5	60.3
512	128	12.2	0.0	16.1	26.3	0.4	35.7	36.1	1.3	47.2
512	256	9.5	0.0	13.1	19.2	0.2	28.2	28.4	1.7	37.3
1024	2	74.1	0.0	82.4	90.2	0.0	94.6	94.6	0.0	97.5
1024	4	88.0	0.0	90.9	95.3	0.0	98.0	98.0	0.0	98.9
1024	8	86.4	0.0	90.5	94.5	0.0	96.7	96.7	0.0	98.7
1024	16	78.3	0.0	83.3	89.9	0.0	93.2	93.2	0.0	96.6
1024	32	63.9	0.0	71.5	79.5	0.1	86.4	86.5	0.2	92.6
1024	64	46.8	0.0	51.9	64.5	0.0	74.5	74.5	0.2	84.6
1024	128	31.5	0.0	37.8	47.6	0.1	55.8	55.9	0.2	69.6
1024	256	21.8	0.1	25.7	33.5	0.4	42.1	42.5	0.5	54.8
1024	512	15.7	0.0	21.0	28.5	0.4	36.1	36.5	0.9	46.1

Table 3: Power Results for Cochran Test: d = 1.1

T	q	1%			5%			10%		
32	2	1.9	0.2	4.1	7.5	1.5	12.4	13.9	3.7	21.6
32	4	1.7	0.1	3.0	8.4	1.5	12.8	14.3	4.5	21.3
32	8	1.9	0.3	3.2	7.7	2.1	12.0	14.1	4.3	19.1
64	2	1.6	0.1	4.4	9.7	1.2	16.7	17.9	2.6	30.6
64	4	2.4	0.1	4.9	11.3	1.2	20.1	21.3	2.7	29.6
64	8	3.4	0.3	5.6	10.9	1.2	18.4	19.6	2.9	29.4
64	16	3.8	0.5	5.8	12.2	1.6	17.7	19.3	3.4	26.8
128	2	6.3	0.0	9.9	19.1	0.3	27.9	28.2	1.0	43.0
128	4	9.3	0.0	12.9	23.2	0.3	34.0	34.3	1.0	45.2
128	8	10.7	0.2	14.6	21.1	0.8	31.1	31.9	1.3	43.7
128	16	9.4	0.1	13.9	23.0	0.9	30.6	31.5	2.7	41.9
128	32	8.8	0.3	10.6	19.0	1.2	24.7	25.9	3.0	34.6
256	2	20.5	0.0	25.9	37.2	0.0	50.9	50.9	0.0	62.0
256	4	27.5	0.0	35.0	46.0	0.0	56.2	56.2	0.2	67.6
256	8	24.5	0.0	32.1	45.5	0.1	55.8	55.9	0.3	66.7
256	16	24.2	0.0	29.6	40.6	0.5	48.8	49.3	1.3	61.7
256	32	18.4	0.0	23.0	33.3	0.8	40.5	41.3	1.6	51.8
256	64	12.6	0.0	17.5	26.6	0.6	33.5	34.1	1.6	44.7
512	2	43.4	0.0	52.2	62.8	0.0	75.1	75.1	0.0	84.6
512	4	52.6	0.0	62.9	73.4	0.0	82.7	82.7	0.0	90.4
512	8	54.3	0.0	62.4	75.0	0.0	81.5	81.5	0.0	89.0
512	16	48.8	0.0	54.4	66.0	0.0	73.9	73.9	0.1	82.2
512	32	35.5	0.1	42.6	55.7	0.1	64.1	64.2	0.1	73.6
512	64	22.7	0.1	29.2	42.6	0.1	52.6	52.7	0.3	63.0
512	128	14.2	0.0	18.1	29.6	0.3	39.0	39.3	0.7	51.8
1024	2	76.0	0.0	83.2	90.9	0.0	94.6	94.6	0.0	97.5
1024	4	88.1	0.0	91.1	95.7	0.0	98.3	98.3	0.0	99.0
1024	8	87.0	0.0	90.0	94.7	0.0	96.7	96.7	0.0	98.8
1024	16	78.7	0.0	84.0	90.0	0.0	93.2	93.2	0.0	96.8
1024	32	64.7	0.0	72.5	80.5	0.1	87.7	87.8	0.1	92.8
1024	64	49.4	0.0	55.8	66.6	0.0	76.5	76.5	0.1	85.6
1024	128	34.5	0.0	41.1	51.3	0.1	60.5	60.6	0.1	72.5
1024	256	24.8	0.0	29.5	39.4	0.2	47.0	47.2	0.5	58.4

Table 4: Power Results for Jegadeesh Test: d = 1.1

T	q	1%			5%			10%		
32	2	2.1	0.3	3.5	7.3	2.3	11.4	13.7	4.2	20.2
32	4	1.9	0.3	3.6	7.6	2.1	9.8	11.9	5.0	18.9
32	8	2.5	0.4	3.0	7.6	3.5	11.1	14.6	7.1	18.9
64	2	2.9	0.1	4.5	10.3	1.7	17.6	19.3	2.8	27.0
64	4	3.6	0.3	5.2	10.2	1.6	15.3	16.9	3.3	27.1
64	8	2.4	0.5	4.0	10.4	2.8	14.3	17.1	6.6	24.5
64	16	3.3	0.8	4.2	9.0	3.6	11.9	15.5	7.0	20.9
128	2	7.4	0.0	11.1	18.4	0.3	29.4	29.7	0.8	41.6
128	4	7.8	0.2	11.6	20.0	0.4	27.0	27.4	1.5	38.5
128	8	7.8	0.3	10.3	19.3	1.1	28.1	29.2	2.6	37.8
128	16	7.3	0.2	9.3	15.8	2.3	21.3	23.6	4.2	32.0
128	32	4.1	0.5	5.5	11.7	3.2	15.5	18.7	6.3	22.4
256	2	22.5	0.0	28.5	40.3	0.0	51.5	51.5	0.2	63.2
256	4	20.7	0.0	26.8	38.3	0.1	50.0	50.1	0.5	62.8
256	8	17.8	0.0	25.9	36.0	0.7	43.8	44.5	1.5	54.8
256	16	14.7	0.2	19.0	28.8	1.2	36.4	37.6	2.1	46.4
256	32	8.0	0.4	13.5	22.0	1.9	27.2	29.1	4.3	38.0
256	64	5.1	0.8	7.7	15.4	2.6	20.2	22.8	5.4	30.4
512	2	44.5	0.0	56.0	68.8	0.0	77.3	77.3	0.0	86.3
512	4	46.7	0.0	53.9	66.0	0.0	76.4	76.4	0.0	84.3
512	8	39.1	0.0	48.7	59.2	0.1	67.4	67.5	0.3	77.6
512	16	30.1	0.1	36.6	49.9	0.1	57.4	57.5	0.4	67.7
512	32	17.3	0.1	23.2	33.8	0.4	45.7	46.1	1.3	55.3
512	64	10.3	0.4	15.5	23.5	1.1	31.2	32.3	2.9	42.6
512	128	7.1	0.5	8.9	16.0	3.2	21.3	24.5	5.3	29.3
1024	2	82.1	0.0	86.6	92.3	0.0	96.4	96.4	0.0	98.5
1024	4	78.0	0.0	84.0	90.5	0.0	95.2	95.2	0.0	97.7
1024	8	69.7	0.0	76.0	84.0	0.0	90.0	90.0	0.1	94.1
1024	16	54.8	0.0	63.9	74.5	0.1	81.2	81.3	0.2	88.2
1024	32	38.9	0.1	46.1	57.1	0.1	67.2	67.3	0.1	77.9
1024	64	26.3	0.0	32.3	43.3	0.4	52.6	53.0	1.3	64.6
1024	128	17.6	0.2	22.7	31.4	1.4	37.3	38.7	3.0	47.5
1024	256	11.5	0.8	15.2	22.0	2.8	27.0	29.8	4.3	34.8

Table 5: Power Results for ADF Test: d = 1.1

T	q	1%		5%			10%		
32	0	0.7	3.4	8.1	3.8	10.9	14.7	7.0	18.4
32	1	0.6	2.7	7.8	4.4	8.9	13.3	9.4	15.5
32	2	0.9	2.9	8.0	4.1	10.5	14.6	9.2	17.6
64	0	0.6	4.6	10.0	3.0	14.2	17.2	6.4	21.3
64	1	0.8	3.7	8.7	3.0	11.6	14.6	7.4	18.3
64	2	0.6	3.7	9.2	3.1	11.6	14.7	6.9	17.9
64	3	0.9	3.7	9.0	3.4	11.6	15.0	7.2	17.4
128	0	0.7	6.6	11.2	2.7	13.8	16.5	5.4	21.5
128	1	0.5	4.2	9.9	3.2	12.2	15.4	5.8	18.6
128	2	0.6	3.5	9.2	2.7	11.1	13.8	6.0	17.9
128	3	0.5	3.2	9.0	3.1	10.6	13.7	6.4	17.4
128	4	0.6	3.2	8.2	2.8	10.4	13.2	5.7	17.1
256	0	0.8	7.3	13.7	3.1	16.8	19.9	5.1	26.1
256	1	0.6	5.2	10.8	2.9	14.3	17.2	5.5	21.7
256	2	0.6	4.0	10.2	2.8	13.0	15.8	5.5	19.8
256	3	0.5	3.6	9.3	2.7	12.6	15.3	6.1	19.4
256	4	0.6	3.5	9.4	2.5	12.3	14.8	5.9	18.9
256	5	0.6	3.3	8.7	2.7	11.6	14.3	5.9	18.5
512	0	0.7	9.0	15.3	2.6	20.7	23.3	4.4	30.0
512	1	0.4	6.6	12.1	2.0	17.0	19.0	4.4	26.0
512	2	0.3	6.1	10.9	2.1	15.9	18.0	4.9	24.0
512	3	0.4	5.6	10.0	2.2	15.6	17.8	4.5	22.6
512	4	0.4	5.0	10.0	2.6	13.9	16.5	4.4	22.1
512	5	0.5	4.8	9.2	2.4	13.8	16.2	4.4	21.7
512	6	0.4	4.8	9.1	2.3	13.2	15.5	4.8	20.7
1024	0	1.1	9.4	17.0	4.0	19.7	23.7	5.4	29.8
1024	1	0.8	7.7	13.2	3.2	17.2	20.4	5.2	26.3
1024	2	0.8	6.2	12.1	3.1	15.8	18.9	5.2	24.9
1024	3	0.8	5.5	10.9	3.0	14.6	17.6	5.8	23.4
1024	4	0.8	5.1	10.3	2.9	14.1	17.0	6.1	22.9
1024	5	0.9	4.5	9.7	2.8	13.7	16.5	6.2	22.3
1024	6	0.9	4.3	9.7	3.0	13.0	16.0	6.0	21.7
1024	7	0.8	4.1	9.6	3.0	12.4	15.4	6.1	21.2

Table 7: Size Results for Lo & McKinley (Homoscedastic) Test (Non-normality) $(1-L)\,y_t = (1-L)^{d-1} sign(\varepsilon_t)\,\varepsilon_t^2$

T	q	1%			5%			10%		
32	2	1.0	1.0	0.7	4.2	4.2	3.9	8.0	8.4	8.3
32	4	0.8	0.9	0.7	3.8	4.5	3.8	8.4	9.0	8.3
32	8	0.8	1.0	0.4	4.0	4.5	3.8	8.3	9.0	8.9
32	16	0.5	0.7	0.4	3.5	4.2	3.7	7.9	9.5	8.2
64	2	1.3	1.2	0.8	4.5	4.7	4.2	9.0	9.6	8.3
64	4	0.9	1.1	0.7	4.0	5.2	4.0	9.2	9.8	8.6
64	8	0.7	0.8	0.6	4.0	4.9	4.2	9.1	9.9	8.9
64	16	0.7	1.1	0.5	4.4	4.8	4.3	9.1	9.4	9.0
64	32	0.9	1.2	0.6	4.4	4.9	4.1	9.0	9.5	9.3
128	2	1.2	1.0	0.9	4.5	4.6	4.7	9.3	9.7	9.4
128	4	1.0	1.1	0.8	4.3	4.9	4.2	9.1	9.6	9.2
128	8	0.9	0.9	0.7	4.5	4.8	4.6	9.4	9.6	9.1
128	16	1.0	1.1	0.8	4.8	5.0	4.4	9.4	9.9	9.0
128	32	0.9	0.9	0.8	4.3	5.0	4.4	9.4	9.6	9.5
128	64	0.8	0.8	0.6	4.1	4.5	4.4	8.9	9.1	9.8
256	2	1.1	1.1	1.0	4.9	4.8	4.8	9.7	9.7	9.6
256	4	1.1	1.2	0.9	4.7	5.0	4.9	9.9	9.9	10.0
256	8	0.8	1.1	0.7	4.6	5.1	4.8	9.9	9.7	9.7
256	16	0.7	0.9	0.8	4.6	5.0	4.7	9.7	10.0	9.6
256	32	0.9	1.1	0.9	4.6	4.8	4.7	9.5	9.9	10.1
256	64	0.9	0.7	0.9	4.4	4.6	4.9	9.5	9.5	9.7
256	128	1.0	1.2	0.8	5.0	5.0	5.2	10.2	9.4	10.0
512	2	1.2	1.1	1.2	5.0	4.7	5.0	9.7	9.5	10.1
512	4	1.2	1.2	1.2	5.1	4.8	5.0	9.8	9.5	9.7
512	8	1.1	1.0	1.0	4.9	4.9	5.0	9.9	10.2	9.9
512	16	1.0	1.1	0.8	5.3	5.4	5.3	10.7	10.8	10.1
512	32	1.1	1.2	0.9	5.3	5.4	4.9	10.3	9.7	10.1
512	64	1.1	1.1	1.1	5.1	5.0	4.8	9.7	9.9	10.1
512	128	1.2	1.1	1.0	5.2	5.3	4.8	10.1	10.1	9.8
512	256	1.1	1.0	1.1	4.7	4.9	4.7	9.6	9.3	9.9
1024	2	1.1	1.2	0.8	4.8	5.2	4.7	9.8	10.2	9.5
1024	4	1.0	1.2	1.0	5.1	5.3	4.6	9.9	9.9	9.2
1024	8	1.0	1.2	0.8	5.3	5.6	4.9	10.6	10.3	9.4
1024	16	1.3	1.3	0.9	4.9	5.5	4.3	9.8	10.2	9.3
1024	32	1.2	1.2	1.0	4.9	5.4	4.4	9.8	10.6	9.3
1024	64	1.3	1.2	1.2	4.9	5.2	4.8	10.0	10.0	9.5
1024	128	0.8	1.2	1.1	5.2	5.0	5.0	10.1	10.0	9.9
1024	256	0.9	1.0	1.0	5.2	5.1	4.6	9.7	10.0	10.1
1024	512	1.0	1.1	1.1	4.8	5.0	4.9	9.9	9.6	10.2

Table 8: Size Results for Lo & McKinley (Heteroscedastic) Test (Non-normality) $(1-L)\,y_t = (1-L)^{d-1}\,sign(\varepsilon_i)\,\varepsilon_i^2$

T	q	1%			5%			10%		
32	2	2.2	1.8	1.3	6.5	5.3	6.8	12.0	10.6	13.2
32	4	2.0	2.2	1.0	6.4	6.0	5.8	11.9	10.6	11.5
32	8	2.4	2.8	1.1	7.3	7.2	5.6	12.8	11.8	10.8
32	16	3.3	4.0	0.9	8.1	8.1	5.2	13.3	12.9	9.9
64	2	1.0	1.2	0.8	4.8	5.1	5.5	10.6	10.3	12.0
64	4	1.3	1.4	0.6	5.5	5.6	5.3	10.9	10.8	10.8
64	8	1.6	2.2	0.8	6.2	6.2	5.3	11.5	10.9	10.7
64	16	2.2	2.8	1.0	7.1	6.7	5.0	11.8	11.3	10.4
64	32	2.8	3.0	0.8	7.2	7.5	5.0	12.5	12.6	10.3
128	2	0.8	1.0	0.8	4.9	5.0	5.4	10.3	10.4	10.7
128	4	1.0	1.1	0.9	4.9	5.1	4.9	9.9	10.9	10.0
128	8	1.0	1.1	0.9	5.6	5.7	5.2	10.9	10.8	10.2
128	16	1.2	1.6	0.9	5.6	5.7	5.3	10.9	10.6	10.1
128	32	1.4	1.7	1.0	6.1	6.2	5.2	11.4	10.8	10.0
128	64	1.8	2.0	0.8	6.3	6.2	5.0	11.2	11.6	9.9
256	2	0.8	0.8	0.8	5.0	5.0	5.1	10.2	10.8	10.2
256	4	1.1	1.1	1.1	5.2	5.2	5.2	10.4	10.5	10.6
256	8	1.1	1.3	0.9	5.2	5.5	5.2	10.7	10.3	10.7
256	16	0.9	1.1	1.1	5.2	5.5	5.0	10.6	10.5	10.5
256	32	1.2	1.3	1.0	5.3	5.3	5.2	10.6	10.4	10.3
256	64	1.4	1.4	1.1	5.5	5.3	5.1	10.4	10.8	10.3
256	128	1.5	1.7	1.0	5.7	5.7	5.5	11.2	10.7	10.1
512	2	0.7	0.9	0.9	4.6	4.8	4.9	9.8	10.0	10.6
512	4	0.9	0.9	1.0	4.9	5.0	5.2	10.2	10.5	10.6
512	8	1.1	1.1	0.9	4.8	5.0	5.4	10.4	10.5	10.3
512	16	1.2	1.2	0.9	5.4	5.3	5.2	10.5	10.1	10.5
512	32	1.2	1.1	1.0	5.4	5.5	5.2	10.7	9.8	10.3
512	64	1.1	1.3	1.1	5.5	5.3	5.3	10.6	10.4	10.4
512	128	1.1	1.4	1.0	5.6	5.8	5.1	10.9	10.9	10.1
512	256	1.6	1.4	1.0	5.2	5.3	4.9	10.2	10.2	10.3
1024	2	0.7	0.9	0.8	4.6	5.3	5.1	10.4	10.4	10.2
1024	4	0.9	1.0	0.9	4.8	4.9	5.0	9.9	10.3	9.9
1024	8	0.8	0.9	1.1	5.0	5.6	4.7	10.3	10.3	10.1
1024	16	0.9	1.0	1.0	5.0	5.5	4.7	10.1	10.5	9.8
1024	32	1.0	0.9	1.0	4.8	5.2	4.5	9.7	10.4	9.8
1024	64	1.1	1.1	1.0	4.8	4.7	4.8	9.5	10.1	10.0
1024	128	1.1	1.3	1.1	5.0	5.3	4.8	10.2	10.0	10.1
1024	256	1.3	1.3	1.0	5.4	5.5	4.7	10.2	10.2	10.0
1024	512	1.3	1.3	1.1	5.2	5.3	4.9	10.2	10.5	9.8

Table 9: Size Results for Cochran Test

(Non-normality) $(1-L)\, y_t = (1-L)^{d-1} sign(\varepsilon_t)\, \varepsilon_t^2$

T	q	1%			5%			10%		
32	2	0.9	1.0	0.9	4.0	3.8	4.0	7.8	8.2	8.0
32	4	0.8	0.9	0.6	3.8	4.0	3.6	7.6	7.8	8.2
32	8	0.8	0.9	0.6	3.9	4.4	3.9	8.3	8.5	8.5
64	2	1.1	1.1	0.8	4.1	4.1	4.2	8.3	8.5	8.5
64	4	0.8	0.8	0.6	3.7	4.1	4.0	8.2	8.9	8.1
64	8	0.6	0.9	0.6	3.7	4.3	4.3	8.6	8.7	9.2
64	16	0.7	1.1	0.6	4.2	4.4	4.4	8.8	8.9	9.0
128	2	1.1	1.1	0.8	4.4	4.3	4.5	8.7	9.0	9.0
128	4	1.0	0.9	0.8	4.3	4.7	4.1	8.7	9.3	8.6
128	8	0.9	0.9	0.8	4.3	4.6	4.6	9.2	9.5	9.1
128	16	0.8	0.9	0.7	4.4	4.4	4.5	8.9	9.4	9.3
128	32	0.8	0.8	0.7	4.3	4.6	4.4	9.0	9.6	9.6
256	2	1.2	1.1	1.1	4.9	4.5	4.8	9.3	9.6	9.3
256	4	1.4	1.2	1.1	5.1	5.0	4.8	9.7	9.5	9.6
256	8	1.0	1.1	0.8	4.9	5.0	4.9	9.8	9.9	9.8
256	16	0.9	1.0	0.9	4.5	5.0	4.9	9.8	9.9	10.1
256	32	0.9	0.9	0.9	4.5	5.0	4.9	9.9	10.0	9.9
256	64	0.7	0.8	0.9	4.6	4.7	5.2	9.8	9.8	10.0
512	2	1.2	1.0	1.2	4.7	4.5	5.0	9.4	9.8	10.1
512	4	1.1	1.1	1.1	4.9	4.8	5.1	9.9	9.9	10.0
512	8	1.3	1.0	0.9	4.8	4.9	5.3	10.2	10.2	10.1
512	16	1.2	1.1	0.9	5.0	5.1	5.0	10.1	10.1	10.3
512	32	1.0	0.9	0.9	4.8	5.3	5.2	10.5	9.7	10.2
512	64	1.0	1.2	0.9	5.3	5.2	5.0	10.2	9.9	10.1
512	128	1.1	1.2	0.9	5.1	5.3	4.9	10.2	9.9	10.1
1024	2	0.9	0.9	0.9	4.9	4.9	4.9	9.7	9.4	10.4
1024	4	1.1	1.0	1.1	4.7	4.9	5.0	9.9	9.6	9.8
1024	8	0.9	0.9	1.0	4.9	5.4	4.7	10.2	10.0	10.0
1024	16	1.1	0.9	1.0	4.8	5.3	4.6	9.9	10.0	9.9
1024	32	1.0	0.8	1.1	4.8	5.2	4.5	9.7	10.2	10.0
1024	64	1.0	0.9	1.0	4.8	4.9	4.6	9.5	9.8	9.7
1024	128	0.8	1.0	1.0	5.4	5.2	4.7	9.8	9.9	10.1
1024	256	0.9	1.0	0.9	5.4	5.3	4.8	10.1	10.6	9.8

Table 10: Size Results for Jegadeesh Test

(Non-normality) $(1-L)\, y_t = (1-L)^{d-1}\, sign(\varepsilon_i)\, \varepsilon_i^2$

T	q	1%			5%			10%		
32	2	0.5	0.4	0.8	3.5	3.6	4.7	8.3	8.8	10.4
32	4	0.4	0.3	1.1	3.6	2.7	5.0	7.7	7.3	10.7
32	8	0.6	0.2	1.0	3.5	1.9	5.6	7.5	6.0	10.4
64	2	0.5	0.5	0.8	4.1	4.5	5.1	9.6	10.2	10.4
64	4	0.5	0.3	0.6	4.0	3.7	5.2	8.9	9.4	11.1
64	8	0.5	0.3	1.0	3.8	3.0	5.5	8.5	8.2	10.3
64	16	0.6	0.3	0.9	3.7	2.6	5.6	8.2	6.7	10.6
128	2	0.7	0.8	0.8	4.5	5.1	4.8	9.9	10.2	10.2
128	4	0.6	0.7	0.8	4.6	4.7	5.0	9.7	10.2	10.4
128	8	0.7	0.6	0.8	4.5	4.5	5.5	10.0	9.7	10.3
128	16	0.7	0.4	0.9	4.4	3.9	5.0	8.9	8.7	10.6
128	32	0.6	0.4	1.0	4.2	3.2	5.3	8.6	8.0	9.7
256	2	1.0	1.1	1.0	5.2	4.7	5.2	9.9	10.4	10.3
256	4	1.0	1.0	1.0	5.0	5.0	5.2	10.2	10.2	10.2
256	8	0.9	0.7	1.1	5.1	5.1	5.3	10.4	10.1	10.2
256	16	0.7	0.7	0.9	4.6	5.0	5.4	10.4	9.9	10.9
256	32	0.8	0.7	1.1	4.6	4.4	5.3	9.7	9.4	10.7
256	64	0.8	0.5	1.1	4.3	4.1	5.7	9.8	8.5	10.6
512	2	0.8	0.9	1.1	5.2	5.2	5.2	10.4	10.6	10.4
512	4	0.8	0.8	0.9	4.7	4.8	5.3	10.1	10.0	10.6
512	8	0.8	0.9	1.0	5.2	5.2	5.0	10.1	10.2	10.6
512	16	1.1	1.0	1.2	5.2	5.0	5.4	10.5	10.4	10.6
512	32	0.9	0.7	1.0	4.8	4.9	5.5	10.4	9.9	10.6
512	64	0.9	0.9	1.0	4.7	4.4	5.3	9.7	9.9	10.7
512	128	1.0	0.8	1.1	4.9	4.9	5.3	10.2	9.4	10.0
1024	2	0.9	0.9	1.0	5.0	4.9	5.2	10.2	10.0	10.2
1024	4	0.9	0.9	0.9	4.7	5.0	4.8	9.9	10.1	10.0
1024	8	1.1	1.1	1.0	4.7	5.1	4.8	9.9	9.8	10.0
1024	16	0.9	0.9	1.2	5.0	5.1	5.0	10.1	10.1	10.5
1024	32	1.0	0.9	1.0	4.7	4.5	4.8	9.3	10.0	9.9
1024	64	0.9	0.8	1.1	4.7	4.8	5.1	10.0	9.7	10.2
1024	128	0.9	0.8	1.1	5.1	4.9	5.0	9.8	9.5	10.3
1024	256	0.8	1.0	1.1	5.2	4.9	5.1	10.1	9.7	10.4

Table 11: Size Results for Lo & McKinley (Homoscedastic) Test (ARCH = 0.95)

32	4	2.4	2.3	1.9	9.0	8.2	7.3	15.5	14.1	12.2
32	8	1.6	1.7	1.2	6.8	6.8	5.9	12.7	12.7	11.0
32	16	1.2	1.1	1.1	5.7	6.1	5.2	11.3	11.7	9.7
64	2	4.7	3.5	3.9	13.6	10.8	10.3	21.0	16.8	16.1
64	4	3.2	2.8	2.6	10.0	9.1	8.0	17.1	15.1	13.5
64	8	1.7	1.6	1.6	7.3	7.3	6.0	13.3	13.1	11.3
64	16	1.3	1.5	1.1	5.8	6.1	5.2	11.3	11.9	10.4
64	32	1.2	1.6	0.9	5.9	6.6	4.9	11.6	11.3	9.9
128	2	5.4	3.9	4.4	14.3	10.7	11.7	22.4	17.0	18.0
128	4	3.6	2.8	3.1	11.3	9.4	8.9	18.3	15.0	14.6
128	8	2.2	2.0	1.9	7.8	7.3	6.7	14.1	13.0	12.1
128	16	1.7	1.5	1.4	6.3	6.4	5.7	12.1	11.5	10.7
128	32	1.4	1.4	1.1	5.4	6.0	5.0	10.9	10.5	10.3
128	64	1.0	1.1	1.0	5.0	5.1	4.6	9.7	10.3	10.0
256	2	6.4	4.5	4.8	16.1	11.8	12.0	23.8	17.4	17.9
256	4	4.0	3.1	3.1	11.6	9.7	9.6	19.3	15.5	15.2
256	8	2.3	2.1	2.0	8.4	7.5	7.2	14.7	12.7	12.7
256	16	1.4	1.5	1.3	6.6	6.3	6.1	12.4	11.6	11.1
256	32	1.3	1.3	1.1	5.7	5.9	5.5	11.4	11.1	10.9
256	64	1.2	1.2	1.2	5.5	5.3	5.0	10.4	10.5	10.0
256	128	1.2	1.2	1.0	5.3	5.2	5.2	10.5	9.9	10.3
512	2	6.8	4.9	5.1	15.9	11.5	12.2	23.7	17.4	18.3
512	4	4.2	3.2	3.5	11.8	9.4	9.4	18.8	15.1	15.0
512	8	2.3	2.0	2.0	8.6	7.3	7.3	14.6	12.8	13.0
512	16	1.7	1.4	1.6	7.1	6.4	6.8	13.2	12.2	11.6
512	32	1.4	1.3	1.2	5.8	5.5	5.5	11.0	10.9	10.8
512	64	1.4	1.1	1.1	5.1	5.2	5.2	10.4	9.9	10.2
512	128	1.1	1.0	1.1	5.1	4.8	4.9	9.8	9.6	10.2
512	256	1.0	1.1	1.0	5.1	5.0	5.1	10.2	9.7	10.0
1024	2	6.8	5.5	4.7	16.5	12.6	11.9	24.5	18.2	17.9
1024	4	4.2	3.6	3.3	12.4	10.0	9.6	19.6	15.5	15.0
1024	8	2.5	2.2	2.0	9.4	7.7	7.8	15.6	13.3	12.7
1024	16	2.0	1.9	1.6	6.7	6.4	6.0	12.4	11.4	11.2
1024	32	1.7	1.5	1.3	6.0	5.6	5.7	11.3	11.0	10.6
1024	64	1.6	1.2	1.4	5.5	5.5	5.5	11.0	10.1	10.0
1024	128	1.0	1.0	1.1	5.1	5.0	5.4	10.4	10.4	10.2
1024	256	1.0	1.1	1.0	5.2	4.8	5.1	9.9	9.9	10.2
1024	512	1.1	1.0	1.0	4.8	5.3	5.1	10.4	9.6	10.8

Table 12: Size Results for Lo & McKinley (Heteroscedastic) Test (ARCH = 0.95)

32	4	0.9	0.4	1.2	4.3	3.4	5.7	9.2	8.2	10.3
32	8	0.6	0.4	1.2	4.1	3.5	5.4	8.9	8.5	10.4
32	16	0.9	0.8	1.2	4.3	4.0	5.2	9.2	8.4	9.7
64	2	0.8	0.7	0.8	4.5	4.6	5.3	9.9	9.6	10.4
64	4	0.9	0.5	1.0	4.4	4.0	5.4	9.4	8.9	10.6
64	8	0.7	0.6	1.1	4.2	3.8	5.2	9.0	8.6	10.1
64	16	0.8	0.7	1.1	4.4	3.9	4.8	8.7	8.8	10.2
64	32	1.0	0.9	1.0	4.8	4.5	4.9	9.4	9.9	9.7
128	2	0.8	0.7	0.9	4.6	4.7	5.0	9.7	9.7	10.3
128	4	0.8	0.6	1.2	4.8	4.1	5.4	9.5	9.7	10.4
128	8	0.9	0.7	1.1	4.4	4.4	5.4	9.7	9.4	10.3
128	16	0.7	0.7	1.1	4.3	4.1	5.3	9.4	9.2	10.3
128	32	1.0	0.8	1.1	4.6	4.6	5.0	9.6	9.3	10.1
128	64	1.0	1.0	1.0	5.0	4.8	4.8	9.6	10.0	9.7
256	2	0.9	0.9	0.9	5.0	4.9	5.3	10.2	10.4	10.3
256	4	1.0	0.8	1.2	5.0	4.5	5.4	9.9	10.1	10.6
256	8	1.0	0.9	1.1	5.0	4.6	5.2	9.8	9.4	10.6
256	16	0.9	0.8	1.1	4.7	4.5	5.1	9.6	9.7	10.1
256	32	1.0	0.7	1.1	4.8	4.9	5.2	10.1	9.7	10.1
256	64	1.1	1.0	1.1	5.0	4.9	5.0	9.8	10.2	10.2
256	128	1.1	0.9	1.1	4.9	4.8	5.3	10.1	9.8	10.2
512	2	1.1	1.2	1.0	4.9	5.1	5.1	10.2	9.8	10.4
512	4	0.9	0.7	1.2	4.8	4.5	5.2	9.7	10.0	10.1
512	8	1.1	0.8	1.2	4.6	4.3	5.3	9.6	9.4	10.2
512	16	1.1	0.8	1.1	4.7	4.5	5.3	9.8	9.2	10.2
512	32	1.0	0.8	1.0	4.7	4.5	5.2	9.7	9.3	10.1
512	64	1.1	0.9	1.0	4.8	4.7	5.4	10.1	9.6	10.1
512	128	0.8	0.8	1.0	4.9	4.7	5.1	9.8	9.4	10.4
512	256	1.0	1.0	1.0	4.9	4.8	5.3	10.1	9.7	10.2
1024	2	0.9	1.0	0.9	5.0	5.4	5.3	10.7	10.4	10.3
1024	4	0.9	0.9	1.0	4.7	4.4	5.2	9.6	10.0	10.3
1024	8	0.9	0.7	1.2	4.8	4.7	5.2	9.8	9.3	10.1
1024	16	0.9	0.8	1.0	4.9	4.5	5.2	9.7	10.0	10.2
1024	32	0.9	0.9	1.1	4.6	4.5	5.2	9.7	9.7	10.3
1024	64	1.1	0.9	1.0	4.8	4.8	5.1	10.0	9.6	10.0
1024	128	1.0	0.9	1.1	4.7	5.0	5.0	9.9	9.8	10.2
1024	256	1.2	1.1	1.0	5.2	4.9	5.1	10.0	9.7	10.0
1024	512	1.1	1.0	0.9	4.8	5.2	5.0	10.3	10.1	10.2

Table 13: Size Results for Cochran Test (ARCH = 0.95)

32	4	2.9	2.5	2.2	9.7	8.6	7.2	15.8	14.5	12.6
32	8	1.7	1.9	1.3	7.5	7.5	6.2	13.7	12.8	10.9
64	2	4.8	3.7	3.8	13.6	10.8	10.2	21.1	16.6	16.4
64	4	3.3	2.6	2.4	10.0	8.9	8.2	17.1	15.1	13.5
64	8	2.0	2.0	1.7	7.4	7.2	6.5	13.7	12.7	11.7
64	16	1.4	1.8	1.4	6.4	6.1	5.7	11.8	11.5	10.5
128	2	5.9	4.6	4.3	14.5	11.1	11.2	22.3	17.1	17.5
128	4	3.6	2.7	3.0	11.6	9.6	8.6	18.2	15.4	14.2
128	8	2.4	1.9	2.1	8.0	7.4	6.9	14.3	13.2	12.1
128	16	1.5	1.5	1.4	6.6	5.8	5.9	11.7	11.5	10.9
128	32	1.4	1.3	1.2	5.7	5.5	5.3	10.8	10.6	10.3
256	2	6.9	4.9	5.0	16.4	11.9	11.9	23.9	17.6	17.5
256	4	4.4	3.5	3.6	12.6	9.9	9.4	19.4	15.4	14.9
256	8	2.7	2.2	2.2	8.6	7.7	7.4	15.1	13.4	12.7
256	16	1.7	1.5	1.4	7.1	6.2	6.1	12.3	11.9	11.7
256	32	1.4	1.2	1.3	5.9	5.8	5.7	11.6	11.6	10.4
256	64	1.2	1.3	1.1	5.7	5.5	5.4	10.9	10.9	10.2
512	2	6.8	4.6	5.2	15.4	11.6	11.9	23.5	17.9	18.3
512	4	3.8	3.1	3.5	11.7	9.4	9.2	18.6	15.5	15.4
512	8	2.7	2.2	2.0	8.2	7.2	7.4	14.6	13.1	13.1
512	16	1.9	1.4	1.7	6.7	6.2	6.4	12.6	11.8	11.7
512	32	1.4	1.1	1.2	5.6	5.5	5.7	11.2	10.9	10.8
512	64	1.3	1.3	1.1	5.2	5.3	5.2	10.5	10.2	10.3
512	128	1.0	1.1	1.0	5.2	5.1	4.9	10.0	10.1	10.6
1024	2	6.5	4.7	4.7	16.8	12.2	12.2	24.3	17.7	18.7
1024	4	4.5	3.4	3.3	12.1	9.7	10.1	19.8	15.3	15.6
1024	8	2.5	1.9	2.4	8.9	7.5	7.7	15.2	12.9	13.0
1024	16	1.5	1.4	1.6	6.7	6.3	6.4	12.7	11.4	11.8
1024	32	1.3	1.2	1.3	5.7	5.3	5.7	11.1	10.8	11.2
1024	64	1.3	1.1	1.2	5.3	5.1	5.4	10.5	10.0	10.4
1024	128	1.1	1.0	1.2	5.4	5.1	5.1	10.1	9.9	10.4
1024	256	1.1	1.2	1.0	5.6	5.0	4.9	9.9	10.5	10.2

Table 14: Size Results for Jegadeesh Test (ARCH = 0.95)

32	4	1.1	0.9	1.3	5.2	5.0	5.2	10.2	10.2	10.3
32	8	0.9	0.8	1.1	4.9	4.9	5.5	10.5	9.7	10.4
64	2	1.3	1.2	1.4	5.7	5.3	5.7	11.0	10.6	10.4
64	4	1.2	1.0	1.0	5.3	5.0	5.0	10.1	10.1	10.6
64	8	1.0	0.9	1.0	5.1	4.6	5.0	9.6	10.2	10.1
64	16	1.0	0.9	1.0	4.9	4.6	5.2	9.8	9.8	10.3
128	2	1.0	1.0	1.3	5.5	5.6	5.6	11.2	10.6	10.2
128	4	1.1	1.1	1.0	5.4	4.9	5.1	10.0	10.2	9.9
128	8	1.0	0.9	0.9	5.2	4.9	5.3	10.2	10.2	10.1
128	16	1.1	0.7	1.1	4.9	4.8	5.1	9.9	9.8	10.1
128	32	1.0	0.8	1.0	4.7	4.5	5.0	9.5	9.4	10.0
256	2	1.2	1.2	1.3	5.7	5.0	5.3	10.3	10.6	10.4
256	4	1.1	1.2	1.0	5.5	5.2	5.2	10.3	10.4	10.2
256	8	0.9	0.8	1.1	5.1	5.2	5.0	10.2	10.0	10.1
256	16	1.0	0.8	1.1	5.1	5.2	5.1	10.3	10.7	9.9
256	32	0.9	0.8	1.0	5.1	4.8	5.2	10.0	9.8	10.1
256	64	1.1	1.0	0.9	4.8	4.8	5.1	9.9	9.4	10.3
512	2	0.9	1.0	1.1	5.1	5.2	5.0	10.2	10.3	10.2
512	4	1.1	0.9	1.0	5.0	5.1	5.0	10.1	10.1	10.0
512	8	0.8	1.1	1.1	4.9	4.8	4.8	9.7	9.8	10.4
512	16	1.1	0.9	1.1	4.9	5.0	5.1	10.2	10.1	10.2
512	32	0.9	0.8	1.0	4.8	5.1	4.9	10.1	9.8	10.0
512	64	1.0	1.0	1.0	5.0	4.7	5.0	9.7	9.7	10.7
512	128	1.0	0.9	1.0	4.7	4.7	5.2	9.9	9.3	10.0
1024	2	0.9	0.9	1.0	5.0	4.8	5.1	9.9	9.9	10.9
1024	4	0.8	0.9	1.0	4.7	4.7	5.0	9.7	9.9	9.8
1024	8	0.9	1.1	1.0	5.0	4.9	5.2	10.1	9.5	10.1
1024	16	0.9	1.0	1.0	4.8	4.9	5.1	10.0	9.7	10.2
1024	32	1.0	1.0	1.1	4.9	4.8	5.0	9.8	9.8	10.1
1024	64	1.0	0.9	1.1	5.1	4.8	5.0	9.8	9.9	10.3
1024	128	1.0	0.8	1.1	5.3	4.7	5.0	9.6	9.8	9.9
1024	256	1.0	1.2	1.1	4.9	5.0	4.9	9.9	10.2	10.4

Table 15: Power Results for Lo & McKinley (Homoscedastic): d = 1.1

T	q	Ex	1%			5%			10%		
32	4		1.6	0.3	2.5	7.2	1.9	11.7	13.6	5.3	19.8
64	4		2.8	0.1	5.6	11.5	1.5	18.8	20.3	2.6	28.9
128	4	1	8.7	0.0	12.5	22.2	0.3	32.7	33.0	1.0	45.2
256	4		23.5	0.0	32.1	44.1	0.0	56.6	56.6	0.1	68.0
512	8		53.1	0.0	62.8	74.2	0.0	80.5	80.5	0.0	87.8
1024	4		87.3	0.0	90.9	95.4	0.0	97.9	97.9	0.0	98.9
32	4		1.4	0.7	1.8	6.3	1.7	10.3	12.0	3.7	20.5
64	4		2.5	0.2	4.7	10.2	0.6	18.1	18.7	1.4	29.5
128	4	2	9.9	0.1	13.6	22.7	0.5	31.9	32.4	1.2	46.5
256	4		24.8	0.0	32.4	45.2	0.0	56.4	56.4	0.1	68.6
512	4		52.9	0.0	62.9	73.3	0.0	81.5	81.5	0.1	88.6
1024	4		88.3	0.0	91.9	95.9	0.0	97.6	97.6	0.0	98.9
32	2		5.4	1.4	6.9	13.8	4.6	16.9	21.5	6.9	25.6
64	2		7.2	1.6	9.6	16.7	3.9	22.8	26.7	7.4	33.0
128	2	3	13.7	0.6	18.0	26.4	2.3	34.8	37.1	4.6	43.9
256	4		27.0	0.0	35.1	44.0	0.2	52.9	53.1	0.7	63.8
512	8		52.6	0.0	60.4	69.9	0.0	78.4	78.4	0.0	85.6
1024	8		83.7	0.0	88.2	92.9	0.0	95.2	95.2	0.0	97.1
32	2		1.9	0.8	3.2	9.6	3.1	13.1	16.2	5.2	21.4
64	4		3.7	0.1	6.4	13.1	2.1	19.4	21.5	3.4	31.6
128	8	4	8.5	0.0	12.7	22.5	0.7	31.0	31.7	1.1	44.2
256	4		22.0	0.0	31.8	44.2	0.0	56.2	56.2	0.2	69.4
512	4		52.8	0.0	62.3	75.5	0.0	84.8	84.8	0.0	90.5
1024	4		87.9	0.0	91.5	95.4	0.0	97.0	97.0	0.0	98.9

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon_t^2 * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 16: Power Results for Lo & McKinley (Homoscedastic): d = 0.9

T	q	Ex	1%			5%			10%		
32	4		1.2	2.4	0.1	6.5	10.3	0.8	11.1	18.9	2.3
64	4		2.6	4.7	0.0	9.1	17.2	0.2	17.4	29.8	0.6
128	4	1	5.6	10.8	0.0	21.6	31.3	0.1	31.4	46.0	0.4
256	4		18.2	23.3	0.0	38.5	53.1	0.1	53.2	68.1	0.1
512	8		37.8	51.6	0.0	68.5	79.2	0.0	79.2	89.4	0.0
1024	8		83.1	88.7	0.0	95.0	97.9	0.0	97.9	99.6	0.0
32	4		1.1	2.0	0.0	4.6	8.4	0.4	8.8	17.9	1.7
64	4		1.4	3.4	0.0	8.6	16.1	0.3	16.4	28.7	0.7
128	4	2	6.7	10.0	0.0	18.7	28.8	0.1	28.9	43.9	0.4
256	8		15.6	22.4	0.0	38.2	50.9	0.0	50.9	64.9	0.0
512	8		40.8	51.9	0.0	68.4	79.0	0.0	79.0	89.5	0.0
1024	8		82.7	89.5	0.0	95.3	98.6	0.0	98.6	99.5	0.0
32	2		3.4	4.3	1.0	11.3	12.5	4.6	17.1	20.7	8.3
64	2		6.0	9.3	0.6	16.6	21.4	2.2	23.6	29.7	4.5
128	4	3	11.5	16.3	0.3	28.2	35.9	0.7	36.6	46.8	1.2
256	4		24.2	30.5	0.1	40.5	53.3	0.4	53.7	65.3	0.4
512	8		41.0	50.7	0.0	66.1	75.6	0.0	75.6	86.3	0.0
1024	8		78.9	85.1	0.0	91.9	95.1	0.0	95.1	98.0	0.0
32	4		1.7	2.8	0.1	7.8	11.0	0.9	11.9	20.8	2.8
64	4		3.2	5.6	0.0	10.3	18.6	0.6	19.2	31.0	1.1
128	4	4	6.1	10.8	0.0	22.2	32.2	0.2	32.4	44.8	0.4
256	8		14.3	23.1	0.0	37.8	51.2	0.1	51.3	66.2	0.1
512	4		41.8	51.3	0.0	69.0	81.0	0.0	81.0	90.0	0.0
1024	8		83.6	89.7	0.0	95.5	97.9	0.0	97.9	99.8	0.0

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon_t^2 * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 17: Power Results for Lo & McKinley (Heteroscedastic): d = 1.1

T	q	Ex	1%			5%			10%		
32	4		2.1	0.5	2.5	8.1	2.1	12.4	14.5	5.2	21.0
64	4		3.3	0.3	4.9	11.4	1.6	19.5	21.1	2.4	29.5
128	4	1	9.3	0.0	13.0	23.3	0.2	32.6	32.8	1.0	44.8
256	4		25.7	0.0	34.9	45.9	0.0	55.9	55.9	0.2	68.0
512	8		53.9	0.0	63.0	74.1	0.0	80.8	80.8	0.0	88.1
1024	4		88.0	0.0	90.9	95.3	0.0	98.0	98.0	0.0	98.9
32	4		3.4	1.2	4.9	11.4	1.7	16.9	18.6	4.0	28.7
64	4		4.4	0.3	6.5	16.2	0.6	23.4	24.0	1.5	34.5
128	4	2	10.8	0.0	16.8	26.2	0.2	35.4	35.6	0.9	48.1
256	4		29.0	0.0	38.3	48.9	0.0	58.2	58.2	0.0	71.2
512	8		52.2	0.0	60.7	72.7	0.0	82.4	82.4	0.0	89.2
1024	8		87.1	0.0	91.0	94.9	0.0	96.3	96.3	0.0	98.5
32	4		1.9	0.1	3.3	7.9	1.8	11.2	13.0	4.8	18.4
64	8		3.0	0.2	4.7	10.3	0.9	15.4	16.3	2.7	24.1
128	8	3	8.7	0.0	11.4	19.2	0.4	27.1	27.5	1.2	36.8
256	8		20.6	0.0	26.9	37.1	0.1	46.5	46.6	0.3	59.4
512	8		43.0	0.0	51.2	64.3	0.0	73.8	73.8	0.0	82.6
1024	8		76.9	0.0	83.0	90.0	0.0	93.7	93.7	0.0	96.2
32	2		2.7	1.3	2.9	9.3	3.3	12.3	15.6	5.5	19.5
64	8		3.4	0.3	5.7	12.9	1.5	18.1	19.6	2.9	27.9
128	8	4	9.3	0.0	13.3	22.8	0.7	31.9	32.6	1.1	44.3
256	8		25.6	0.0	34.6	45.1	0.1	55.8	55.9	0.2	69.6
512	4		53.0	0.0	63.0	75.3	0.0	84.7	84.7	0.0	90.9
1024	4		88.4	0.0	91.4	95.7	0.0	97.1	97.1	0.0	99.0

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon_t^2 * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 18: Power Results for Lo & McKinley (Heteroscedastic): d = 0.9

T	q	Ex	1%			5%			10%		
32	4		1.7	2.8	0.1	6.2	9.6	1.2	10.8	17.0	2.3
64	8		2.4	4.0	0.0	8.6	15.0	0.1	15.1	26.0	0.7
128	4	1	6.0	10.1	0.0	19.9	29.9	0.1	30.0	45.6	0.4
256	4		17.4	23.3	0.0	40.9	53.4	0.1	53.5	68.0	0.1
512	8		40.7	51.7	0.0	66.4	78.9	0.0	78.9	89.6	0.0
1024	8		79.0	87.5	0.0	94.5	97.7	0.0	97.7	99.6	0.0
32	6		3.5	4.6	0.1	7.7	11.4	0.8	12.2	17.6	2.8
64	8		4.3	6.0	0.0	10.8	16.7	0.1	16.8	29.6	0.7
128	4	2	8.1	11.6	0.0	21.5	31.2	0.1	31.3	46.4	0.2
256	4		20.8	27.0	0.0	40.7	53.0	0.0	53.0	68.2	0.0
512	8		43.2	52.6	0.0	66.9	79.2	0.0	79.2	88.8	0.0
1024	8		76.7	87.4	0.0	94.0	97.6	0.0	97.6	99.2	0.0
32	2		0.9	1.2	0.2	5.1	6.5	2.2	8.7	14.1	4.8
64	2		0.7	1.4	0.0	5.6	12.1	0.7	12.8	19.6	2.5
128	4	3	3.1	5.7	0.1	13.9	23.0	0.5	23.5	36.0	0.7
256	8		9.2	15.0	0.1	27.8	41.2	0.1	41.3	60.1	0.2
512	8		27.7	37.0	0.0	51.6	66.2	0.0	66.2	81.0	0.0
1024	8		59.8	73.4	0.0	85.5	91.4	0.0	91.4	96.2	0.0
32	4		1.5	2.4	0.1	6.9	10.4	0.9	11.3	19.0	2.7
64	8		2.1	4.0	0.0	8.4	15.1	0.2	15.3	26.5	1.0
128	4	4	6.6	10.5	0.0	21.3	30.0	0.2	30.2	44.6	0.4
256	8		15.2	24.3	0.0	37.4	51.0	0.1	51.1	66.2	0.1
512	4		41.0	50.8	0.0	67.8	81.0	0.0	81.0	90.8	0.0
1024	8		77.5	88.2	0.0	94.9	97.8	0.0	97.8	99.8	0.0

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon^2_t * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 19: Power Results for Cochran Test: d = 1.1

T	q	Ex	1%			5%			10%		
32	4		1.7	0.1	3.0	8.4	1.5	12.8	14.3	4.5	21.3
64	4		2.4	0.1	4.9	11.3	1.2	20.1	21.3	2.7	29.6
128	4	1	9.3	0.0	12.9	23.2	0.3	34.0	34.3	1.0	45.2
256	4		27.5	0.0	35.0	46.0	0.0	56.2	56.2	0.2	67.6
512	8		54.3	0.0	62.4	75.0	0.0	81.5	81.5	0.0	89.0
1024	4		88.1	0.0	91.1	95.7	0.0	98.3	98.3	0.0	99.0
32	8		1.1	0.5	2.5	8.1	2.2	11.4	13.6	4.2	20.1
64	8		2.9	0.2	4.9	11.6	0.6	19.1	19.7	1.7	31.0
128	4	2	8.6	0.1	13.7	24.4	0.6	33.2	33.8	1.3	46.7
256	8		25.6	0.0	33.7	46.9	0.2	56.6	56.8	0.2	67.3
512	4		51.7	0.0	63.3	73.3	0.0	81.6	81.6	0.1	89.7
1024	4		89.1	0.0	92.2	96.2	0.0	97.9	97.9	0.0	99.0
32	2		5.9	1.5	7.6	15.3	4.9	18.0	22.9	7.8	26.3
64	2		6.4	1.7	9.2	17.2	3.8	22.6	26.4	7.5	32.4
128	4	3	12.3	0.2	17.2	27.1	1.2	33.4	34.6	2.9	44.2
256	4		30.4	0.0	37.3	44.9	0.3	53.1	53.4	0.6	63.3
512	8		54.0	0.0	59.0	70.9	0.0	79.1	79.1	0.0	85.9
1024	8		85.3	0.0	89.3	93.1	0.0	95.0	95.0	0.0	97.1
32	2		2.7	0.9	4.8	10.5	3.0	14.4	17.4	5.7	21.6
64	4		3.4	0.2	7.1	13.9	1.7	21.1	22.8	3.6	32.1
128	4	4	9.2	0.0	14.3	23.8	0.6	32.7	33.3	1.2	46.5
256	8		25.8	0.0	33.4	45.6	0.0	55.5	55.5	0.3	68.1
512	4		52.1	0.0	62.2	75.5	0.0	84.9	84.9	0.0	90.9
1024	8		87.4	0.0	90.9	94.7	0.0	97.1	97.1	0.0	98.2

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon_t^2 * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 20: Power Results for Cochran Test: d = 0.9

T	q	Ex	1%			5%			10%		
32	4		1.4	2.2	0.1	6.5	11.1	0.8	11.9	18.4	2.4
64	4		2.8	4.4	0.0	9.4	16.9	0.5	17.4	29.3	0.8
128	4	1	6.8	10.8	0.0	20.8	31.2	0.1	31.3	46.1	0.4
256	4		17.6	25.3	0.0	41.9	53.1	0.1	53.2	67.7	0.1
512	8		42.0	52.6	0.0	66.1	78.8	0.0	78.8	90.0	0.0
1024	8		79.5	87.0	0.0	94.6	98.0	0.0	98.0	99.6	0.0
32	8		1.4	2.3	0.1	5.2	8.7	0.6	9.3	14.5	1.8
64	4		1.8	2.6	0.0	8.7	15.7	0.3	16.0	28.5	0.7
128	4	2	6.5	9.1	0.0	18.0	29.0	0.1	29.1	45.4	0.2
256	4		18.7	25.6	0.0	39.0	52.5	0.0	52.5	67.2	0.0
512	4		39.4	50.7	0.0	66.3	79.0	0.0	79.0	89.0	0.0
1024	8		78.3	87.2	0.0	94.6	98.3	0.0	98.3	99.4	0.0
32	2		4.2	6.0	1.2	11.4	14.3	5.1	19.4	23.4	8.1
64	2		7.3	9.6	0.5	16.9	21.0	2.1	23.1	29.4	4.4
128	4	3	12.8	15.3	0.4	28.1	35.4	0.7	36.1	47.4	1.1
256	4		24.4	32.3	0.1	44.7	54.1	0.4	54.5	65.7	0.5
512	4		41.8	52.2	0.0	63.6	73.0	0.0	73.0	82.6	0.0
1024	8		74.9	83.6	0.0	91.1	94.8	0.0	94.8	98.0	0.0
32	2		2.4	3.5	0.2	7.5	10.8	1.9	12.7	18.9	4.5
64	4		3.5	4.6	0.1	10.3	18.9	0.6	19.5	30.5	1.3
128	4	4	6.9	9.5	0.0	21.6	31.5	0.2	31.7	44.8	0.4
256	4		17.5	25.4	0.0	40.3	52.2	0.1	52.3	67.6	0.1
512	4		38.8	51.8	0.0	68.8	80.6	0.0	80.6	90.6	0.0
1024	8		78.4	88.0	0.0	95.0	97.8	0.0	97.8	99.6	0.0

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon_t^2 * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 21: Power Results for Jegadeesh Test: d = 1.1

T	q	Ex	1%			5%			10%		
32	8		2.5	0.4	3.0	7.6	3.5	11.1	14.6	7.1	18.9
64	8		2.4	0.5	4.0	10.4	2.8	14.3	17.1	6.6	24.5
128	4	1	7.8	0.2	11.6	20.0	0.4	27.0	27.4	1.5	38.5
256	2		22.5	0.0	28.5	40.3	0.0	51.5	51.5	0.2	63.2
512	2		44.5	0.0	56.0	68.8	0.0	77.3	77.3	0.0	86.3
1024	2		82.1	0.0	86.6	92.3	0.0	96.4	96.4	0.0	98.5
32	2		2.8	0.0	4.5	10.1	1.4	15.2	16.6	2.9	24.9
64	4		5.6	0.1	7.3	14.3	0.4	20.9	21.3	0.8	31.5
128	2	2	11.1	0.0	16.6	23.8	0.2	34.5	34.7	1.0	47.1
256	2		28.2	0.0	36.2	45.1	0.0	55.4	55.4	0.2	67.0
512	2		47.8	0.0	57.9	69.3	0.0	77.4	77.4	0.0	86.1
1024	2		82.9	0.0	87.7	93.4	0.0	95.6	95.6	0.0	97.9
32	2		2.9	0.7	4.8	9.1	2.8	11.6	14.4	6.4	19.9
64	8		2.8	0.5	5.0	10.7	2.0	14.4	16.4	5.7	22.7
128	4	3	6.8	0.3	9.4	19.3	0.7	26.3	27.0	2.0	35.6
256	4		17.7	0.0	22.0	35.0	0.3	43.9	44.2	0.6	57.7
512	4		39.6	0.0	48.3	61.3	0.0	71.4	71.4	0.0	80.0
1024	4		70.7	0.0	77.6	86.0	0.0	92.4	92.4	0.0	95.5
32	4		2.7	1.8	2.8	8.8	5.3	9.4	14.7	9.1	17.4
64	4		3.5	0.9	4.7	12.8	2.8	16.6	19.4	4.8	27.5
128	4	4	7.0	0.2	10.2	19.7	0.7	27.3	28.0	2.6	40.3
256	2		20.8	0.0	27.8	38.7	0.2	50.8	51.0	0.4	65.7
512	2		42.9	0.0	55.9	68.6	0.0	79.6	79.6	0.0	87.9
1024	2		82.2	0.0	86.8	92.6	0.0	96.0	96.0	0.0	98.1

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon_t^2 * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 22: Power Results for Jegadeesh Test: d = 0.9

T	q	Ex	1%			5%			10%		
32	2		1.2	2.0	0.4	5.3	9.0	1.3	10.3	17.9	3.0
64	2		2.0	3.6	0.1	8.4	14.8	0.4	15.2	27.5	1.2
128	2	1	4.3	7.6	0.0	17.0	27.2	0.0	27.2	40.1	0.3
256	2		12.8	18.7	0.0	33.5	45.9	0.0	45.9	61.8	0.1
512	2		29.6	42.1	0.0	57.7	70.1	0.0	70.1	84.3	0.0
1024	2		70.3	78.2	0.0	87.7	93.5	0.0	93.5	97.0	0.0
32	2		0.5	1.5	0.1	5.4	11.4	0.9	12.3	21.8	1.8
64	2		2.5	5.0	0.0	10.4	20.3	0.1	20.4	32.7	0.6
128	2	2	6.8	11.9	0.0	20.7	32.3	0.1	32.4	45.8	0.4
256	2		16.9	23.8	0.0	39.0	49.5	0.0	49.5	62.6	0.1
512	2		33.5	44.2	0.0	60.4	71.9	0.0	71.9	84.4	0.0
1024	2		70.7	77.9	0.0	87.4	93.5	0.0	93.5	97.0	0.0
32	2		1.1	2.4	0.4	5.1	8.5	2.1	10.6	16.6	4.4
64	4		1.4	2.9	0.2	8.4	13.1	0.8	13.9	25.4	1.7
128	4	3	3.2	6.5	0.1	15.2	24.4	0.4	24.8	38.0	1.1
256	2		8.2	12.4	0.0	25.1	36.1	0.1	36.2	52.9	0.4
512	4		22.4	30.2	0.0	44.9	57.4	0.0	57.4	73.3	0.1
1024	4		54.4	65.0	0.0	77.3	87.3	0.0	87.3	92.7	0.0
32	4		3.2	4.5	0.3	7.8	10.9	1.1	12.0	17.3	3.4
64	4		2.8	4.3	0.0	8.2	12.7	0.7	13.4	23.2	2.2
128	4	4	4.4	7.4	0.1	14.8	22.8	0.2	23.0	38.5	0.6
256	2		11.6	17.6	0.0	32.1	42.1	0.0	42.1	61.1	0.0
512	2		27.7	39.9	0.0	59.8	71.6	0.0	71.6	85.5	0.0
1024	2		71.1	77.1	0.0	88.5	94.7	0.0	94.7	97.7	0.0

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon^2_t * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 23: Power Results for ADF Test: d = 1.1

T	q	Ex	1%		5%			10%		
32	0		0.7	3.4	8.1	3.8	10.9	14.7	7.0	18.4
64	0		0.6	4.6	10.0	3.0	14.2	17.2	6.4	21.3
128	0	1	0.7	6.6	11.2	2.7	13.8	16.5	5.4	21.5
256	0		0.8	7.3	13.7	3.1	16.8	19.9	5.1	26.1
512	0		0.7	9.0	15.3	2.6	20.7	23.3	4.4	30.0
1024	0		1.1	9.4	17.0	4.0	19.7	23.7	5.4	29.8
32	0		1.5	3.5	9.8	4.7	12.2	16.9	7.3	18.8
64	0		1.0	5.2	11.2	4.0	13.4	17.4	6.3	21.9
128	0	2	0.9	5.5	11.2	2.5	15.0	17.5	5.8	21.8
256	0		0.6	7.4	13.2	3.3	17.3	20.6	5.9	26.1
512	0		0.7	10.0	17.8	2.7	20.7	23.4	5.0	28.9
1024	0		1.0	9.9	16.7	2.9	21.1	24.0	5.4	29.6
32	0		0.5	4.0	9.5	3.8	11.5	15.3	7.6	18.9
64	0		0.9	4.3	10.0	3.2	13.5	16.7	7.2	21.5
128	0	3	0.8	6.1	12.6	3.2	14.4	17.6	5.9	22.9
256	0		1.0	7.4	13.5	3.9	17.7	21.6	6.2	25.7
512	0		0.8	9.3	16.6	3.2	22.3	25.5	5.2	29.0
1024	0		1.4	9.3	17.7	2.9	21.1	24.0	6.0	30.1
32	0		0.5	3.7	8.5	3.4	11.6	15.0	7.1	18.6
64	0		0.1	5.6	10.8	3.4	12.4	15.8	6.1	21.3
128	0	4	0.8	5.9	10.4	3.0	13.9	16.9	6.4	20.3
256	0		0.8	6.9	13.5	2.5	17.0	19.5	5.6	24.2
512	0		1.3	8.0	15.1	3.5	19.2	22.7	6.4	29.6
1024	0		1.1	9.6	16.1	3.5	19.6	23.1	5.7	28.6

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon_t^2 * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 24: Power Results for ADF Test: d = 0.9

T	q	Ex	1%		5%			10%		
32	0		2.0	0.3	5.6	8.9	1.5	10.4	16.9	4.5
64	0		2.9	0.1	7.2	12.5	0.9	13.4	21.4	2.7
128	0	1	4.7	0.0	9.0	16.4	0.5	16.9	27.0	2.5
256	0		6.6	0.1	13.2	19.9	0.7	20.6	32.3	1.4
512	0		10.1	0.0	17.1	24.8	0.5	25.3	37.3	0.8
1024	0		15.1	0.0	22.6	32.9	0.1	33.0	44.7	0.4
32	0		1.7	0.4	4.5	8.6	1.3	9.9	15.5	3.9
64	0		3.0	0.1	7.8	12.8	0.8	13.6	21.4	3.0
128	0	2	4.5	0.0	9.6	17.4	0.8	18.2	26.6	2.6
256	0		8.4	0.0	15.1	21.0	0.7	21.7	30.8	1.7
512	0		9.1	0.0	16.6	25.4	0.2	25.6	38.2	1.0
1024	0		13.4	0.0	23.5	32.3	0.0	32.3	44.7	0.3
32	0		2.4	0.4	5.6	9.6	2.3	11.9	18.1	4.5
64	0		3.9	0.1	9.0	13.5	0.9	14.4	23.1	2.8
128	0	3	5.3	0.2	10.4	17.5	0.9	18.4	27.1	2.8
256	0		8.4	0.1	15.3	21.9	1.0	22.9	32.1	1.6
512	0		10.2	0.0	17.3	26.1	0.2	26.3	38.1	1.0
1024	0		15.6	0.0	24.0	33.4	0.0	33.4	44.3	0.4
32	0		2.1	0.2	5.0	8.8	1.3	10.1	17.0	4.3
64	0		3.1	0.0	7.2	11.3	1.4	12.7	19.9	3.6
128	0	4	5.3	0.1	10.1	16.8	0.8	17.6	26.9	2.0
256	0		6.1	0.1	12.4	19.6	0.1	19.7	31.6	1.5
512	0		10.2	0.1	17.1	25.7	0.2	25.9	37.2	1.0
1024	0		14.4	0.0	22.4	33.5	0.1	33.6	44.1	0.6

Note: Ex - Refers to the experiment type

1 - $\varepsilon_t \sim IN(0, \sigma^2)$

2 - $u_t = \varepsilon^2_t * sign(\varepsilon_t)$, $\varepsilon_t \sim IN(0, \sigma^2)$

3 - $\varepsilon_t \sim N(0, h_t)$, $h_t = 0.95 h_{t-1} + \eta_t$

4 - ε_t U-shaped distribution

Table 25: Power Results for Lo & MacKinlay (Homoscedastic) Test for ARFIMA Models

Coefficients	d	1%			5%			10%		
$\phi=.0,\theta=.0$	0.8	32.4	43.9	0.0	59.4	72.9	0.0	72.9	85.7	0.0
$\phi=.0,\theta=-.2$	0.8	93.4	96.7	0.0	99.1	99.7	0.0	99.7	99.9	0.0
$\phi=.0,\theta=.2$	0.8	4.2	7.3	0.0	14.8	23.5	0.0	23.5	38.5	0.0
$\phi=-.2,\theta=.0$	0.8	90.8	95.1	0.0	98.4	99.5	0.0	99.5	99.7	0.0
$\phi=.2,\theta=.0$	0.8	2.8	5.0	0.0	11.4	18.8	0.0	18.8	31.5	0.5
$\phi=.0,\theta=.0$	1.2	49.9	0.0	58.0	69.2	0.0	77.7	77.7	0.0	85.8
$\phi=.0,\theta=-.2$	1.2	10.1	0.2	14.9	23.5	1.2	31.1	32.3	2.4	42.1
$\phi=.0,\theta=.2$	1.2	96.8	0.0	98.0	99.3	0.0	99.7	99.7	0.0	99.9
$\phi=-.2,\theta=.0$	1.2	13.5	0.2	19.1	27.6	0.8	36.3	37.1	1.8	47.5
$\phi=.2,\theta=.0$	1.2	97.5	0.0	98.5	99.4	0.0	99.7	99.7	0.0	100

Table 26: Power Results for Cochrane Test for ARFIMA Models

Coefficients	d	1%			5%			10%		
$\phi=.0,\theta=.0$	0.8	31.6	41.4	0.0	60.5	74.7	0.0	74.7	88.2	0.0
$\phi=.0,\theta=-.2$	0.8	94.0	96.4	0.0	99.2	99.7	0.0	99.7	99.8	0.0
$\phi=.0,\theta=.2$	0.8	3.4	7.2	0.0	14.3	23.1	0.0	23.1	38.9	0.2
$\phi=-.2,\theta=.0$	0.8	92.0	94.9	0.0	98.3	99.5	0.0	99.5	99.7	0.0
$\phi=.2,\theta=.0$	0.8	2.3	5.2	0.0	11.1	18.2	0.1	18.3	32.2	0.4
$\phi=.0,\theta=.0$	1.2	50.3	0.0	57.7	71.2	0.0	78.0	78.0	0.0	85.6
$\phi=.0,\theta=-.2$	1.2	11.5	0.2	16.8	25.9	1.0	33.5	34.5	2.4	44.8
$\phi=.0,\theta=.2$	1.2	96.8	0.0	98.0	99.3	0.0	99.7	99.7	0.0	99.9
$\phi=-.2,\theta=.0$	1.2	15.3	0.2	21.4	31.0	0.8	29.3	40.1	1.8	50.6
$\phi=.2,\theta=.0$	1.2	97.4	0.0	98.5	99.4	0.0	99.8	99.8	0.0	99.9

Table 27: Power Results for Jegadeesh Test for ARFIMA Models

Coefficients	d	1%			5%			10%		
$\phi=.0,\theta=.0$	0.8	22.1	30.7	0.0	46.6	62.2	0.0	62.2	77.0	0.0
$\phi=.0,\theta=-.2$	0.8	76.8	85.8	0.0	93.8	97.6	0.0	97.6	99.3	0.0
$\phi=.0,\theta=.2$	0.8	3.5	6.0	0.0	12.7	21.7	0.2	21.9	34.2	0.7
$\phi=-.2,\theta=.0$	0.8	68.2	78.7	0.0	89.7	95.5	0.0	95.5	98.4	0.0
$\phi=.2,\theta=.0$	0.8	2.4	4.3	0.0	9.8	18.1	0.2	18.3	30.3	0.7
$\phi=.0,\theta=.0$	1.2	41.3	0.0	52.2	63.3	0.0	75.3	75.3	0.0	83.5
$\phi=.0,\theta=-.2$	1.2	10.8	0.2	14.2	24.5	1.0	32.4	32.3	2.3	42.5
$\phi=.0,\theta=.2$	1.2	85.5	0.0	89.8	93.8	0.0	96.8	96.8	0.0	98.4
$\phi=-.2,\theta=.0$	1.2	14.1	0.2	18.3	29.1	0.7	37.4	38.1	1.8	47.6
$\phi=.2,\theta=.0$	1.2	90.7	0.0	93.8	96.8	0.0	98.4	98.4	0.0	99.3

Table 28: Power Results for ADF Test for ARFIMA Models

Coefficients	d	1%			5%			10%		
$\phi=.0,\theta=.0$	0.8	-	19.0	0.0	31.6	44.6	0.0	44.6	56.8	0.2
$\phi=.0,\theta=-.2$	0.8	-	48.9	0.0	61.5	71.8	0.0	71.8	82.0	0.0
$\phi=.0,\theta=.2$	0.8	-	5.7	0.0	11.6	20.4	0.1	20.6	33.5	0.6
$\phi=-.2,\theta=.0$	0.8	-	44.4	0.0	57.2	68.2	0.0	68.2	79.4	0.0
$\phi=.2,\theta=.0$	0.8	-	4.4	0.0	9.4	16.6	0.2	18.8	29.4	0.8
$\phi=.0,\theta=.0$	1.2	-	1.6	14.5	21.6	4.8	25.6	30.4	8.0	33.6
$\phi=.0,\theta=-.2$	1.2	-	1.0	9.9	17.3	4.1	20.9	25.0	7.1	29.5
$\phi=.0,\theta=.2$	1.2	-	0.0	98.0	99.3	0.0	99.7	99.7	0.0	99.9
$\phi=-.2,\theta=.0$	1.2	-	1.2	10.7	18.6	4.2	21.6	25.8	7.1	30.6
$\phi=.2,\theta=.0$	1.2	-	3.4	19.0	29.7	6.9	31.1	38.0	9.9	38.8

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Table A1: Lo and McKinley Crictical Values

T	q	0.005	0.010	0.0250	0.050	0.100	0.900	0.950	0.975	0.990	0.995
32	2	-2.56	-2.33	-1.99	-1.70	-1.35	1.33	1.72	2.04	2.41	2.65
32	4	-1.96	-1.86	-1.67	-1.49	-1.25	1.45	1.95	2.43	3.02	3.34
32	8	-1.52	-1.47	-1.38	-1.28	-1.14	1.50	2.22	2.87	3.72	4.28
32	16	-1.13	-1.11	-1.06	-1.01	-0.92	1.34	1.96	2.63	3.39	3.99
64	2	-2.56	-2.35	-1.97	-1.66	-1.29	1.31	1.67	1.99	2.38	2.61
64	4	-2.16	-2.01	-1.79	-1.54	-1.26	1.37	1.83	2.22	2.72	3.10
64	8	-1.85	-1.75	-1.59	-1.42	-1.21	1.43	1.99	2.51	3.17	3.67
64	16	-1.47	-1.43	-1.35	-1.26	-1.12	1.46	2.18	2.86	3.82	4.48
64	32	-1.10	-1.08	-1.04	-0.99	-0.92	1.29	1.96	2.63	3.50	4.10
128	2	-2.63	-2.36	-2.01	-1.68	-1.29	1.30	1.66	1.98	2.35	2.56
128	4	-2.30	-2.11	-1.81	-1.57	-1.27	1.33	1.76	2.15	2.56	2.85
128	8	-2.05	-1.92	-1.71	-1.50	-1.24	1.39	1.86	2.32	2.88	3.24
128	16	-1.77	-1.69	-1.54	-1.40	-1.20	1.46	2.04	2.59	3.28	3.86
128	32	-1.45	-1.41	-1.33	-1.23	-1.11	1.50	2.25	2.95	3.84	4.60
128	64	-1.10	-1.08	-1.04	-0.99	-0.91	1.33	2.07	2.76	3.67	4.36

256	2	-2.59	-2.34	-1.97	-1.65	-1.29	1.30	1.66	1.96	2.31	2.54
256	4	-2.33	-2.18	-1.89	-1.60	-1.26	1.31	1.70	2.08	2.49	2.81
256	8	-2.20	-2.03	-1.77	-1.53	-1.25	1.34	1.78	2.19	2.72	3.04
256	16	-2.00	-1.87	-1.67	-1.47	-1.22	1.40	1.88	2.33	2.91	3.39
256	32	-1.77	-1.67	-1.53	-1.38	-1.18	1.45	2.03	2.62	3.29	3.76
256	64	-1.45	-1.40	-1.32	-1.23	-1.10	1.51	2.24	2.99	3.91	4.57
256	128	-1.09	-1.07	-1.03	-0.98	-0.91	1.33	2.02	2.72	3.63	4.22
512	2	-2.57	-2.31	-1.96	-1.65	-1.29	1.28	1.65	1.98	2.33	2.58
512	4	-2.46	-2.24	-1.90	-1.61	-1.28	1.32	1.70	2.05	2.46	2.76
512	8	-2.34	-2.13	-1.83	-1.58	-1.26	1.33	1.75	2.11	2.58	2.91
512	16	-2.17	-2.02	-1.76	-1.52	-1.23	1.35	1.79	2.18	2.70	3.09
512	32	-1.97	-1.86	-1.66	-1.47	-1.22	1.39	1.92	2.39	2.93	3.39
512	64	-1.74	-1.67	-1.53	-1.38	-1.19	1.44	2.06	2.68	3.41	3.83
512	128	-1.43	-1.39	-1.31	-1.22	-1.10	1.48	2.22	2.95	3.91	4.64
512	256	-1.09	-1.07	-1.03	-0.98	-0.91	1.30	2.00	2.70	3.58	4.27
1024	2	-2.52	-2.28	-1.94	-1.63	-1.27	1.30	1.66	2.00	2.36	2.63
1024	4	-2.45	-2.19	-1.88	-1.60	-1.27	1.33	1.71	2.04	2.43	2.71
1024	8	-2.35	-2.14	-1.81	-1.56	-1.24	1.33	1.72	2.09	2.55	2.85
1024	16	-2.22	-2.05	-1.80	-1.54	-1.25	1.35	1.77	2.18	2.62	2.97
1024	32	-2.10	-1.96	-1.73	-1.51	-1.23	1.36	1.83	2.27	2.76	3.10
1024	64	-1.94	-1.84	-1.65	-1.46	-1.23	1.40	1.89	2.41	2.95	3.33
1024	128	-1.76	-1.66	-1.53	-1.38	-1.18	1.43	2.02	2.58	3.35	3.93
1024	256	-1.43	-1.39	-1.31	-1.23	-1.10	1.45	2.21	2.92	3.82	4.70
1024	512	-1.09	-1.07	-1.03	-0.98	-0.91	1.27	1.97	2.68	3.56	4.36

Table A2: Lo and McKinley (Heteroscedastic Consistent) Critical Values

T	q	0.005	0.010	0.0250	0.050	0.100	0.900	0.950	0.975	0.990	0.995
32	2	-2.490	-2.292	-1.997	-1.726	-1.364	1.407	1.774	2.062	2.445	2.652
32	4	-2.094	-1.971	-1.769	-1.582	-1.329	1.505	2.032	2.489	3.033	3.387
32	8	-1.702	-1.627	-1.504	-1.379	-1.227	1.592	2.316	3.001	3.764	4.584
32	16	-1.303	-1.257	-1.198	-1.129	-1.032	1.480	2.137	2.820	3.624	4.216
64	2	-2.608	-2.360	-2.021	-1.700	-1.363	1.329	1.711	2.032	2.413	2.660
64	4	-2.212	-2.090	-1.835	-1.599	-1.313	1.401	1.856	2.263	2.821	3.108
64	8	-1.879	-1.800	-1.650	-1.496	-1.276	1.450	1.996	2.535	3.195	3.694
64	16	-1.579	-1.524	-1.432	-1.334	-1.192	1.495	2.232	2.857	3.708	4.491
64	32	-1.223	-1.198	-1.150	-1.095	-1.004	1.401	2.105	2.760	3.699	4.375
128	2	-2.571	-2.341	-1.983	-1.689	-1.338	1.336	1.690	2.025	2.383	2.615
128	4	-2.290	-2.134	-1.852	-1.625	-1.284	1.357	1.781	2.139	2.559	2.853
128	8	-2.100	-1.963	-1.737	-1.522	-1.261	1.403	1.846	2.316	2.877	3.171
128	16	-1.843	-1.740	-1.604	-1.454	-1.242	1.449	2.019	2.610	3.331	3.909
128	32	-1.524	-1.479	-1.388	-1.294	-1.161	1.552	2.261	3.037	3.952	4.606
128	64	-1.196	-1.169	-1.118	-1.064	-0.976	1.457	2.192	2.909	3.857	4.525

256	2	-2.578	-2.337	-1.981	-1.664	-1.296	1.312	1.662	1.957	2.291	2.518
256	4	-2.339	-2.175	-1.857	-1.598	-1.265	1.312	1.727	2.056	2.393	2.697
256	8	-2.193	-2.021	-1.793	-1.541	-1.252	1.333	1.769	2.174	2.627	2.885
256	16	-2.036	-1.896	-1.692	-1.497	-1.236	1.386	1.877	2.319	2.813	3.228
256	32	-1.808	-1.721	-1.569	-1.405	-1.208	1.488	2.047	2.586	3.251	3.732
256	64	-1.498	-1.443	-1.368	-1.276	-1.135	1.531	2.297	3.041	3.965	4.638
256	128	-1.182	-1.159	-1.112	-1.057	-0.973	1.432	2.149	2.921	3.789	4.615
512	2	-2.590	-2.328	-2.001	-1.671	-1.296	1.280	1.663	2.001	2.340	2.558
512	4	-2.484	-2.259	-1.928	-1.614	-1.253	1.308	1.704	2.056	2.449	2.759
512	8	-2.288	-2.132	-1.867	-1.595	-1.258	1.318	1.735	2.115	2.592	2.889
512	16	-2.171	-2.043	-1.778	-1.551	-1.264	1.355	1.818	2.208	2.694	2.957
512	32	-2.019	-1.916	-1.694	-1.487	-1.242	1.397	1.921	2.388	2.953	3.389
512	64	-1.779	-1.697	-1.546	-1.402	-1.203	1.451	2.037	2.709	3.482	4.061
512	128	-1.493	-1.440	-1.351	-1.262	-1.130	1.512	2.239	3.058	4.113	4.895
512	256	-1.166	-1.148	-1.102	-1.053	-0.970	1.363	2.118	2.955	3.873	4.726
1024	2	-2.563	-2.348	-1.975	-1.646	-1.290	1.271	1.636	1.973	2.364	2.677
1024	4	-2.422	-2.208	-1.927	-1.642	-1.270	1.314	1.681	2.017	2.444	2.695
1024	8	-2.494	-2.200	-1.870	-1.586	-1.262	1.314	1.745	2.081	2.472	2.755
1024	16	-2.372	-2.151	-1.814	-1.559	-1.240	1.327	1.762	2.168	2.635	2.932
1024	32	-2.231	-2.038	-1.773	-1.535	-1.245	1.337	1.830	2.266	2.748	3.134
1024	64	-1.992	-1.883	-1.685	-1.497	-1.239	1.385	1.920	2.454	3.128	3.473
1024	128	-1.774	-1.689	-1.556	-1.394	-1.206	1.447	2.103	2.694	3.402	4.000
1024	256	-1.472	-1.433	-1.353	-1.266	-1.138	1.513	2.276	2.996	3.907	4.484
1024	512	-1.167	-1.143	-1.102	-1.047	-0.965	1.410	2.129	2.866	3.850	4.635

Table A3: Cochran Critical Values

T	q	0.005	0.010	0.0250	0.050	0.100	0.900	0.950	0.975	0.990	0.995
32	2	0.558	0.596	0.647	0.695	0.755	1.183	1.240	1.293	1.344	1.390
32	4	0.352	0.376	0.429	0.481	0.548	1.298	1.441	1.554	1.705	1.808
32	8	0.190	0.210	0.251	0.291	0.348	1.346	1.564	1.793	2.096	2.319
64	2	0.679	0.702	0.744	0.781	0.825	1.138	1.184	1.223	1.269	1.307
64	4	0.483	0.507	0.563	0.615	0.677	1.246	1.337	1.424	1.543	1.627
64	8	0.307	0.338	0.386	0.440	0.504	1.322	1.483	1.650	1.851	1.981
64	16	0.165	0.192	0.227	0.267	0.324	1.339	1.580	1.826	2.156	2.387
128	2	0.773	0.794	0.821	0.847	0.878	1.106	1.137	1.165	1.198	1.217
128	4	0.615	0.636	0.685	0.726	0.775	1.190	1.255	1.308	1.384	1.435
128	8	0.452	0.481	0.535	0.583	0.648	1.276	1.385	1.496	1.610	1.695
128	16	0.284	0.318	0.369	0.416	0.490	1.345	1.528	1.690	1.910	2.093
128	32	0.160	0.179	0.220	0.260	0.321	1.363	1.622	1.888	2.232	2.429
256	2	0.838	0.853	0.874	0.893	0.916	1.077	1.098	1.116	1.138	1.150
256	4	0.719	0.743	0.778	0.806	0.843	1.139	1.183	1.223	1.261	1.292
256	8	0.588	0.614	0.656	0.702	0.755	1.207	1.283	1.350	1.436	1.492
256	16	0.434	0.466	0.521	0.569	0.635	1.282	1.404	1.513	1.652	1.737
256	32	0.277	0.305	0.356	0.415	0.488	1.358	1.536	1.706	1.925	2.066
256	64	0.157	0.179	0.218	0.259	0.321	1.363	1.632	1.892	2.252	2.531

512	2	0.884	0.894	0.911	0.925	0.942	1.054	1.071	1.086	1.100	1.111
512	4	0.788	0.810	0.837	0.862	0.891	1.100	1.133	1.162	1.195	1.223
512	8	0.694	0.715	0.746	0.784	0.827	1.154	1.207	1.255	1.321	1.354
512	16	0.567	0.595	0.642	0.684	0.738	1.220	1.305	1.380	1.466	1.520
512	32	0.419	0.446	0.503	0.559	0.626	1.294	1.424	1.549	1.696	1.795
512	64	0.282	0.312	0.360	0.411	0.479	1.358	1.556	1.744	2.004	2.202
512	128	0.161	0.181	0.216	0.260	0.317	1.347	1.653	1.916	2.316	2.619
1024	2	0.918	0.925	0.937	0.947	0.958	1.038	1.050	1.059	1.072	1.080
1024	4	0.855	0.868	0.884	0.902	0.922	1.072	1.094	1.114	1.137	1.151
1024	8	0.767	0.790	0.821	0.848	0.878	1.112	1.152	1.183	1.218	1.241
1024	16	0.671	0.694	0.741	0.776	0.817	1.162	1.219	1.274	1.333	1.379
1024	32	0.545	0.580	0.634	0.678	0.732	1.221	1.316	1.395	1.487	1.559
1024	64	0.415	0.449	0.503	0.552	0.617	1.293	1.430	1.563	1.729	1.822
1024	128	0.273	0.301	0.358	0.406	0.473	1.356	1.569	1.746	1.988	2.194
1024	256	0.150	0.179	0.215	0.253	0.313	1.360	1.644	1.895	2.271	2.458

Table A4: Jagadeesh Critical Values

T	q	0.005	0.010	0.0250	0.050	0.100	0.900	0.950	0.975	0.990	0.995
32	2	-3.506	-3.153	-2.683	-2.238	-1.796	1.009	1.428	1.806	2.209	2.593
32	4	-3.689	-3.353	-2.852	-2.447	-2.017	0.786	1.214	1.558	1.925	2.349
32	8	-4.255	-3.894	-3.286	-2.805	-2.293	0.403	0.795	1.181	1.673	1.974
64	2	-3.187	-2.818	-2.378	-2.010	-1.604	1.103	1.473	1.862	2.247	2.618
64	4	-3.294	-2.975	-2.502	-2.163	-1.741	0.919	1.322	1.625	2.105	2.323
64	8	-3.512	-3.161	-2.726	-2.350	-1.918	0.713	1.073	1.375	1.805	2.114
64	16	-3.826	-3.413	-2.928	-2.566	-2.160	0.396	0.756	1.103	1.554	1.800
128	2	-2.915	-2.631	-2.213	-1.853	-1.493	1.150	1.521	1.876	2.235	2.554
128	4	-2.920	-2.656	-2.270	-1.939	-1.555	1.053	1.441	1.745	2.177	2.448
128	8	-3.157	-2.835	-2.425	-2.070	-1.684	0.904	1.250	1.574	2.017	2.234
128	16	-3.368	-3.107	-2.613	-2.275	-1.875	0.722	1.102	1.436	1.783	2.007
128	32	-3.407	-3.144	-2.815	-2.469	-2.094	0.423	0.746	1.031	1.490	1.730
256	2	-2.765	-2.520	-2.089	-1.809	-1.400	1.176	1.542	1.851	2.188	2.447
256	4	-2.739	-2.517	-2.169	-1.841	-1.469	1.116	1.484	1.787	2.179	2.390
256	8	-2.880	-2.656	-2.243	-1.907	-1.547	1.032	1.378	1.676	1.999	2.324
256	16	-3.139	-2.830	-2.417	-2.056	-1.657	0.908	1.251	1.555	1.949	2.181
256	32	-3.221	-2.924	-2.542	-2.205	-1.831	0.713	1.080	1.387	1.765	2.065
256	64	-3.311	-3.071	-2.711	-2.365	-2.034	0.384	0.748	1.053	1.448	1.699

512	2	-2.796	-2.494	-2.074	-1.747	-1.348	1.210	1.577	1.897	2.270	2.620
512	4	-2.760	-2.507	-2.121	-1.792	-1.419	1.162	1.545	1.880	2.258	2.471
512	8	-2.798	-2.517	-2.164	-1.833	-1.473	1.096	1.476	1.768	2.094	2.378
512	16	-2.899	-2.649	-2.286	-1.929	-1.561	1.008	1.368	1.691	2.056	2.321
512	32	-2.988	-2.767	-2.374	-2.030	-1.679	0.885	1.237	1.572	1.998	2.259
512	64	-3.154	-2.876	-2.537	-2.222	-1.827	0.670	1.064	1.385	1.799	2.067
512	128	-3.245	-3.027	-2.669	-2.361	-2.037	0.364	0.696	1.039	1.398	1.636
1024	2	-2.650	-2.418	-2.051	-1.728	-1.353	1.215	1.587	1.913	2.306	2.550
1024	4	-2.699	-2.431	-2.057	-1.739	-1.366	1.224	1.586	1.919	2.287	2.517
1024	8	-2.745	-2.475	-2.081	-1.782	-1.429	1.144	1.512	1.875	2.206	2.433
1024	16	-2.821	-2.524	-2.178	-1.846	-1.487	1.076	1.449	1.775	2.129	2.401
1024	32	-2.843	-2.600	-2.270	-1.971	-1.565	0.996	1.351	1.688	2.073	2.309
1024	64	-3.000	-2.786	-2.355	-2.045	-1.680	0.873	1.244	1.577	1.932	2.187
1024	128	-3.104	-2.897	-2.479	-2.183	-1.823	0.691	1.053	1.327	1.679	1.976
1024	256	-3.329	-3.025	-2.694	-2.360	-2.027	0.371	0.704	0.997	1.344	1.633