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ABSTRACT

This paper examines the implications of increasing returns-to-scale evasion technologies for the optimal structure of commodity taxes. We find that, in the presence of evasion, tax design should aim at inducing uniform marginal evasion responses across commodities. This objective may dominate the concerns over inter-commodity distortions stressed by the traditional optimal commodity taxation literature. The resulting optimal tax structure can thus be more, or less, uniform than the one prescribed in the absence of evasion, even when uniform commodity taxation is feasible. In particular, our results imply that the presence of evasion may yield an optimal tax structure which features relatively low tax rates on commodities that have relatively low price elasticities of demand, if the demand for those commodities is relatively large. On the other hand, when all transactions are of similar size, the presence of evasion may provide a rationale for broad-based uniform taxation.

KEYWORDS: Optimal Taxation, Tax Evasion.

JEL CLASSIFICATION: H21, H26.

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1 Introduction

Optimal taxation theory has traditionally been concerned with the distortions induced by taxes on consumption or production choices.[1] Regrettably, the predictive and normative power of this type of analysis has not fulfilled expectations.[2]

A possible explanation for the failure of the traditional theory to provide a convincing framework for the analysis of real-world tax systems is its neglect of tax-induced real costs other than those associated with traditional consumption distortions. In particular, recent literature on optimal tax design has pointed to compliance costs (Slemrod and Sorum [1984]), and administrative costs (Heller and Shell [1974], and Yitzhaki [1978]). Another element that is missing from traditional analyses is evasion activity. This can generate private evasion costs, which, although beneficial from a private point of view, are socially wasteful and thus add to the excess burden of taxation. Available evidence suggests that these costs could be substantial.[3]

A few studies have analyzed the implications of tax evasion (Usher [1986]), and of evasion-related government activities such as enforcement (Kaplow [1990]) and auditing (Cremer and Gahvari [1993]). These studies, however, make rather restrictive assumptions

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[1] See the seminal contribution of Diamond and Mirrlees [1971].

[2] One standard result of the theory, for example, is the equal percentage change rule, whereby tax rates should be adjusted so as to produce equal proportional changes in compensated demands. This rule provides an argument against broad-based, uniform taxation (except in the special case where preferences exhibit implicit separability between taxed and untaxed goods). Yet, uniform commodity taxation appears to be the structure of choice for most developed economies. Thus, the precepts of optimal taxation theory appear to be disregarded by policy makers in the implementation of commodity taxation.

[3] Mirus and Smith [1981], for example, estimate that the size of Canada's irregular economy ranges from 5% to 20% of *total* economic activity. Evasion activity is estimated to be even higher in other developed countries; Pedone (in Aaron [1981]), for instance, estimated that evasion of indirect taxes in Italy in 1977 reduced value-added tax receipts by as much as two-thirds in some sectors. For developing countries, anecdotal evidence suggests extremely high evasion.

about evasion technologies and behaviour. Specifically, private evasion decisions are treated as separable from demand decisions. Usher, in a two commodity model that examines the marginal cost of public funds, incorporates evasion costs that are convex in the amount of evasion, but does not allow for concealment costs to depend on the level of demand. Kaplow examines the optimal commodity tax structure when there are two groups of consumers; one group that is monitored and one group that is not. He includes administration, enforcement and evasion costs into his model specification, but does not allow for the possibility of partial evasion. Cremer and Gahvari examine how costly audits and real evasion costs affect the optimal commodity tax structure; however, their specification of evasion costs implies that there is separability between demand decisions and tax evasion decisions.

In this paper we argue that it is important to account for the interdependence of commodity demand and evasion choices in the design of an optimal commodity tax structure. We incorporate commodity tax evasion and private evasion costs into the traditional optimal commodity tax problem by postulating that any reduction in the statutory base resulting from evasion activity depends both upon the amount of evasion effort undertaken and the level of demand. Allowing for this interaction between demand and evasion effort results in a particular form of increasing returns-to-scale in evasion effort which has significant implications for the optimal structure of commodity taxes.

Our analysis indicates that the structure of commodity taxes that minimizes excess burden is one that generates equi-proportional changes in the compensated responses of *net-of-evasion* tax bases for all commodities. This evasion-modified rule implies that, *ceteris paribus*, tax design should reflect two distinct objectives. On the one hand, the structure of commodity taxes should be aimed at minimizing the traditional distortions caused by distortionary taxation, while at the same time it should be chosen so as to maximize the spread of marginal evasion responses to tax changes across commodities. Thus, commodities with relatively large marginal evasion responses should be taxed at relatively lower rates. At the same time, since the marginal benefit to evasion effort is increasing in the level of demand, those commodities that represent a large proportion of total expenditures should be taxed at relatively lower rates.

The presence of these competing objectives may result in an optimal tax structure that is more, or less, uniform than that prescribed by traditional optimal commodity taxation theory, independently of whether or not uniform proportional taxation is feasible. For example, when transaction sizes differ for commodities that are equally complementary, the optimal tax structure will be less uniform in the presence of evasion than in the absence of evasion. On the other hand, when transactions are of similar size across goods, the optimal tax structure will be more uniform than the one obtained in the absence of evasion.

These findings may help explain why real world commodity tax systems are generally broad based and uniform, even when traditional optimal tax theory prescribes a non-uniform rate structure. In addition, our analysis provides analytical grounds for the recent trend of setting low tax rates on commodities that have very low price elasticities of demand.[4]

The plan of the paper is as follows. In Section 2 we formally discuss the characteristics of evasion technologies, while Section 3 analyzes the consumer's optimal choice in the presence of commodity taxation and evasion technologies. In Section 4, the consumers's evasion choices are embedded in a simple optimal commodity tax problem and optimal tax rules are derived and contrasted with the standard results. Section 5 summarizes our findings.

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[4]In Canada, for example, both the federal government and the provinces of Ontario and Quebec have recently eliminated excise taxes on cigarettes as a policy measure to combat illegal sales of cigarettes which had jumped dramatically in response to earlier increases in excise taxes.

2 Tax evasion technologies

In this section we will discuss the technological characteristics of evasion activities with respect to commodity taxation.[5] Since little evidence is available on evasion technologies,[6] we must adopt an axiomatic approach to their characterization. In this we depart from previous literature by postulating that the reduction in the statutory base resulting from evasion activities depends both upon the amount of evasion effort undertaken and the demand for the commodity.

This interaction between demand and evasion effort can take different forms. For example, consider a consumer who wants to purchase cigarettes through illegal channels: he must spend resources accessing the illegal market, and once he has entered this market he will be able to purchase a certain amount of cigarettes at a price below the gross-of-tax market price. In this situation the marginal benefit to the consumer of expending an additional unit of resources on evasion effort is increasing in the level of demand, i.e., there is a particular form of increasing returns-to-scale in evasion effort.[7] We formalize this situation below.

Let us begin by considering a single commodity, i, the demand for which is x_i, and denote the effective tax base as b_i. If the government levies a per unit tax at a rate τ_i, then, in the absence of evasion, we have $b_i = x_i$. In this setting, tax evasion can be described as a costly reduction in tax liabilities resulting from an abatement of the statutory tax base. In

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[5]Although it is customary to distinguish between tax *evasion*, which is realized by breaking the law, and tax *avoidance*, which is realized through lawful manipulation of tax regulations, this distinction is not essential for the purpose of our analysis. Thus, in the following, the term "evasion" will be used to denote any lawful or unlawful abatement of the statutory tax base.

[6]See Cowell [1990].

[7]We can also imagine a situation where the amount of resources spent in order to enter the illegal market for cigarettes is in proportion to the individuals demand for cigarettes. In this case the "effective" base is linear in demand, while the percentage of the effective base abated depends only upon the amount of resources spent in entering the illegal market. The resulting configuration of technology and evasion costs in this case corresponds to that analyzed in the existing literature on commodity tax evasion.

the presence of evasion, the effective tax base for good i is $b_i(\varphi_i, x_i) = x_i - E_i(\varphi_i, x_i)$, where $E_i(\varphi_i, x_i)$ is the amount of base evaded on good i.[8] We assume this to be an increasing, concave and well-behaved function of its arguments, and impose the following restrictions: $E_i(0, x_i) = 0$, and $E_i(\varphi_i, 0) = 0$. We also assume that $\partial E_i(\varphi_i, x_i)/\partial \varphi_i = \infty$ when $\varphi_i = 0$, i.e., the marginal effort required to evade on the "first dollar" of statutory base is zero.

We may thus summarize the general characteristics of evasion technologies discussed above, in terms of the effective base function, as follows:[9]

$$b_i(0, x_i) = x_i, \qquad \forall i, \tag{1}$$

$$\frac{\partial b_i(\varphi_i, x_i)}{\partial \varphi_i} \leq 0, \qquad \forall i, \tag{2}$$

$$\frac{\partial^2 b_i(\varphi_i, x_i)}{(\partial \varphi_i)^2} \geq 0, \qquad \forall i, \tag{3}$$

$$\frac{\partial b_i(\varphi_i, x_i)}{\partial \varphi_i} = -\infty, \qquad \text{for } \varphi_i = 0, \qquad \forall i, \tag{4}$$

$$b_i(\varphi_i, 0) = 0, \qquad \forall i, \tag{5}$$

$$0 < \frac{\partial b_i(\varphi_i, x_i)}{\partial x_i} \leq 1, \qquad \forall i, \tag{6}$$

$$\frac{\partial^2 b_i(\varphi_i, x_i)}{(\partial x_i)^2} \geq 0, \qquad \forall i, \tag{7}$$

$$\frac{\partial^2 b_i(\varphi_i, x_i)}{\partial \varphi_i \partial x_i} \leq 0, \qquad \forall i, \tag{8}$$

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[8] In general, if we have N commodities, we can write the effective tax base for good i as $b_i = b_i(\vec{\varphi}, \vec{x})$, and the amount of base abated on good i as $E_i(\vec{\varphi}, \vec{x}) = [x_i - b_i(\vec{\varphi}, \vec{x})]$, where $\vec{\varphi}$ and $\vec{x}$ are vectors of dimension n. Here, we simplify matters by assuming that evasion technologies exhibit a strong form of separability: the effective tax base for a certain commodity i only depends on the statutory base, the tax rate, and the evasion effort exerted against the taxes levied on commodity i, and is independent of the level of evasion effort for all other goods.

[9] The specification of costs of evasion in Cremer and Gahvari [1992] corresponds to assuming that expression (6) is unity and that all cross effects are zero.

3 The consumer's optimal evasion problem

We now turn to the analysis of consumer behaviour in the presence of the type of evasion technology described above. We will first examine the consumer's problem, and then derive comparative static properties for the compensated case.

Let us consider a single individual consuming N private goods. His consumption levels are denoted by x_i $(i = 1, \ldots N)$. We will follow the standard convention of assuming a zero endowment point, so that x_i represents the net trade for commodity i (i.e., individual excess demand). Producer prices, q_i $(i = 1, \ldots, N)$, are fixed and equal to marginal costs (i.e., markets are assumed to be perfectly competitive). In this economy the government levies per unit taxes at rates τ_i $(i = 1, \ldots, N)$. The legal tax base for good i is simply x_i and the payable tax for good i is $\tau_i x_i$ $(i = 1, \ldots, N)$.

Suppose that the consumer is endowed with a technology for resisting tax payments, which is summarized by effective base functions $b_i(\varphi_i, x_i)$, $(i = 1, \ldots, N)$. The functions $b_i(\cdot)$, $(i = 1, \ldots, N)$. are assumed to be twice differentiable and to satisfy the restrictions (1)-(8), with the proviso that, when the statutory base is negative (denoting a net sale by the consumer), we will assume the effective base to be also negative and reverse the sign of conditions (1)-(8) accordingly. Without loss of generality, we may assume that for each commodity i $(i = 1, \ldots, N)$ evasion effort, φ_i, may be produced at a constant marginal cost equal to c_i.[10] Total evasion costs are thus equal to the sum of the costs incurred for evasion

[10]Tax evasion may produce other negative effects that directly impact on an individual's welfare: "psychic costs", stemming from moral aversion and guilt, and increased risk, resulting from the uncertain prospect of audits and punishment. Here, we only consider costs that enter directly into the individual's budget constraint and not his utility function; thus, we will neglect psychic costs. We also abstract from uncertainty by assuming that private costs include all certain costs as well as the *certainty equivalent* of the costs from increased risk faced by evaders. Some of the above private costs of evasion represent a transfer to other economic agents—for example, the expected value of the fines paid to the government, or the bribes paid to individuals to obtain their cooperation: these are excluded from our definition of evasion costs, and we also implicitly assume that private evasion benefits are net of all such transfers. The remaining part of private evasion costs will constitute a pure social waste: these are the costs that are the object of our analysis.

in all markets, i.e.

$$\sum_i c_i\varphi_i, \qquad \forall i. \tag{9}$$

The representative consumer's utility function is assumed to have the private goods as its only arguments and to be twice continuously differentiable, increasing in all of its arguments and quasiconcave. The consumer's constrained maximization problem can now be stated as follows:

$$\max_{\vec{\varphi},\vec{x}} \quad U(\vec{x})$$
$$s.t. \quad \begin{cases} M - \sum_i q_i x_i - \sum_i \tau_i b_i(\varphi_i, x_i) - \sum_i c_i\varphi_i \geq 0, \\ \varphi_i \geq 0, \qquad \forall i, \end{cases} \tag{10}$$

where M is exogenous income. Notice that the constraint is a linear combination of concave functions and is thus concave. Also, notice that a corner solution, with $\varphi_i = 0$ for some i, might occur if, when evasion efforts are close to zero on good i, the marginal cost of evading the first dollar of payable taxes is strictly positive; we rule out this possibility by assuming condition (4) holds for all goods.

Proceeding to derive the first-order conditions for an interior solution, we obtain

$$U_{x_i} - \lambda\left(q_i + \tau_i\frac{\partial b_i}{\partial x_i}\right) = 0, \qquad \forall i; \tag{11}$$

$$-\tau_i\frac{\partial b_i}{\partial \varphi_i} - c_i = 0, \qquad \forall i. \tag{12}$$

The individual's behaviour that is predicted by the above conditions agrees with intuition. Condition (11) simply says that the marginal rate of substitution between good i and good j is equated to the ratio of marginal effective prices. Condition (12) says that the marginal benefits of evasion are set equal to the marginal costs of evasion; below, we will discuss the implications of this condition in more detail. Since the second-order conditions for an optimum are satisfied, the first-order conditions will yield Marshallian commodity demand functions $x_i^m(\vec{\tau},\vec{q},\vec{c},M)$, $\forall i$, and Marshallian "evasion effort demand" functions $\varphi_i^m(\vec{\tau},\vec{q},\vec{c},M)$, $\forall i$.

Solution of the dual problem yields Hicksian compensated demand functions, $x_i^c(\vec{\tau},\vec{q},\vec{c},U)$,

$\forall i$, and $\varphi_i^c(\vec{\tau}, \vec{q}, \vec{c}, U)$, $\forall i$. (For the sake of simplicity, from now on producer prices, $\vec{q}$, and the marginal cost of evasion effort, $\vec{c}$, will be dropped from the list of arguments.)

By substituting the uncompensated and compensated demand functions into the effective base functions, we may derive uncompensated and compensated "effective base reaction functions" defined as follows:

$$B_i^m(\vec{\tau}, M) = b_i\left[\varphi^m(\vec{\tau}, M), x_i^m(\vec{\tau}, M)\right], \quad \forall i, \tag{13}$$

$$B_i^c(\vec{\tau}, U) = b_i\left[\varphi^c(\vec{\tau}, U), x_i^c(\vec{\tau}, U)\right], \qquad \forall i. \tag{14}$$

3.1 Comparative statics for the compensated case

In this section we examine the comparative statics of the dual to the consumer's utility maximization problem. To formalize these responses, we will begin by showing how the price of the commodity is affected by tax changes. Let us denote by P_i the effective marginal price of good i, i.e.

$$P_i(q_i, \vec{\tau}) = g_i[q_i, \tau_i, x_i(\vec{\tau}), \varphi_i(\vec{\tau})] = q_i + \tau_i \frac{\partial b_i}{\partial x_i}, \qquad \forall i. \tag{15}$$

If we differentiate this price with respect to τ_i and φ_i together with the i-th equation of (12), we can show that

$$\frac{\partial P_i}{\partial \tau_i} < 1. \tag{16}$$

The intuition behind this result is that a marginal tax increase on commodity i will proportionately raise the gross-of-tax price to the consumer, but since the marginal benefit to undertaking more evasion increases with the tax rate, evasion effort on the commodity will increase, which manifests itself as a reduction in the effective price to the consumer. Notice that, in principle, it is possible for $\partial P_i/\partial \tau_i$ to be negative, implying that an increase in the tax rate on good i could have the "perverse" effect of reducing the marginal price of good i.

It can also be shown that the tax effects on compensated demands and compensated evasion efforts are simply

$$S_{ij}^x = S_{ij} \frac{\partial P_j}{\partial \tau_j}, \qquad \forall i, j; \tag{17}$$

$$S_{ii}^{\varphi} = \frac{\partial \varphi_i^F}{\partial \tau_i} + \frac{\partial \varphi_i}{\partial x_i} S_{ii} \frac{\partial g_i}{\partial \tau_i}, \qquad \forall i; \tag{18}$$

$$S_{ij}^{\varphi} = \frac{\partial \varphi_i}{\partial x_i} S_{ij} \frac{\partial g_j}{\partial \tau_j}, \qquad \forall i \neq j, \tag{19}$$

where

$$\frac{\partial \varphi_i}{\partial x_i} = -\frac{\partial^2 b_i / \partial x_i \partial \varphi_i}{\partial^2 b_i / \partial \varphi_i^2} > 0, \qquad \forall i. \tag{20}$$

$$\frac{\partial g_i}{\partial \tau_i} = \frac{\partial b_i}{\partial x_i} > 0, \qquad \forall i, \tag{21}$$

$$\frac{\partial \varphi_i^F}{\partial \tau_i} = -\frac{\partial b_i}{\partial \varphi_i} / \tau_i \frac{\partial^2 b_i}{\partial \varphi_i^2} > 0, \qquad \forall i. \tag{22}$$

The term S_{ij} denotes the standard substitution effect between good i and good j. If we rule out the "perverse" case where $\partial P_i / \partial \tau_i < 0$,[11] we can conclude that the signs of the compensated effects on demand agree with the signs of the standard Slutsky terms; their size, however, will generally be smaller since $\partial P_i / \partial \tau_i < 1$.

As to the own compensated effects on evasion, the expression S_{ii}^{φ} has an ambiguous sign. The reason for this is two-fold. On the one hand, the increase in the tax rate on good i reduces the own compensated demand for good i, which in turn reduces the marginal benefit to evasion effort on good i, causing evasion effort to decline.[12] On the other hand, the increase in the tax rate on good i raises the marginal benefit to evasion effort on good i, causing evasion effort to increase.[13] The overall sign is therefore ambiguous. The cross effects include only one term, which represents an indirect effect whose sign is positive for substitutes and negative for complements.[14]

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[11] The possibility that $\partial P_i / \partial \tau_i < 0$ has been traditionally linked with the disincentive effects produced by taxes on market activities (see Cowell [1991]).

[12] This is represented by the second term in expression (18).

[13] This is represented by the first term in expression (18).

[14] The cross effects on evasion are simplified because of our separability assumption on evasion technologies.

When we examine uncompensated responses, we have to take into account income effects. If the compensated effects are negative, then own uncompensated tax effects on x_i will be unambiguously negative. Uncompensated cross effects on x_i and φ_i will also be unambiguously negative when x_i and φ_i are substitutes. The sign of the uncompensated cross effects on x_i and φ_i for complements, however, is ambiguous, as is the uncompensated own tax effect on φ_i. Fortunately, as will be shown in the next section, the optimal tax problem for the government can be completely characterized in terms of compensated effects.

4 The government's optimal tax problem

We will now move on to the derivation of an evasion-modified optimal commodity tax rule. We will then compare this rule and the optimal commodity tax structure with the traditional no-evasion rule. We will first examine the case where all transactions may be taxed; then we will examine the situation where some transaction are untaxed.

4.1 No untaxed commodities

Consider an economy with N commodities and one representative consumer. The government's objective is assumed to be the maximization of the representative consumer's welfare subject to a revenue constraint. The government is assumed to have full information on preferences and on evasion technologies.[15] Formally, the government chooses tax rates τ_i $(i = 1, \ldots, N)$ so as to maximize the representative agent's indirect utility, $V(\vec{\tau}, M)$, subject to a revenue requirement, $\overline{R}$:

$$\max_{\vec{\tau}} \; V(\vec{\tau}, M)$$

[15]This information is sufficient to recover all of the variables in the consumer's problem as well as indirect utility. A more reasonable hypothesis would be to only allow the government information on indirect utility and the effective base function. This would imply that individual choices could not be recovered and a situation where there is asymmetric information would exist. This game would be a natural extension to the framework we analyze here; in this paper we abstract from such informational restrictions.

$$s.t. \quad \sum_i \tau_i \, B_i^m(\vec{\tau}, M) - \overline{R} \geq 0. \tag{23}$$

Assuming that the second-order conditions for a maximum are satisfied, the first-order conditions for a solution to this problem yield the following optimality conditions (the arguments of the demand functions are dropped for notational brevity):

$$\frac{\partial V}{\partial \tau_i} + \alpha \, B_i^m + \alpha \sum_k \tau_k \frac{\partial B_k^m}{\partial \tau_i} = 0, \qquad \forall i. \tag{24}$$

It can be shown that, for a given set of taxes, $\vec{\tau}$, the following equality holds:[16]

$$\frac{\partial V}{\partial \tau_i} = -\lambda \, B_i^m, \qquad \forall i. \tag{25}$$

Combining (24) and (25) with (13) and (14), and rearranging terms, we obtain the evasion-modified Ramsey rule in terms of compensated effects:

$$-\sum_k \tau_k \frac{\partial B_k^c}{\partial \tau i} = \frac{1}{\alpha} \overline{B_i} \left\{ \alpha - \lambda - \alpha \sum_k \tau_k \frac{\partial B_k^m}{\partial M} \right\}, \qquad \forall i. \tag{26}$$

In order to interpret condition (26), we can follow the methodology suggested by Auerbach [1985]. The net-of-evasion marginal social utility of income, μ, can be defined as

$$\mu = \lambda + \alpha \sum_k \tau_k \frac{\partial B_k^m}{\partial M}. \tag{27}$$

where

$$\frac{\partial B_k^m}{\partial M} = \frac{\partial b_k}{\partial x_k} \frac{\partial x_k^m}{\partial M} + \frac{\partial b_k}{\partial \varphi_k} \frac{\partial \varphi_k^m}{\partial M}, \qquad \forall k. \tag{28}$$

Using the above definitions, the evasion-modified Ramsey rule, (26), becomes

$$-\sum_k \tau_k \frac{\partial B_k^c}{\partial \tau_i} = \overline{B_i} \frac{\alpha - \mu}{\alpha}, \qquad \forall i. \tag{29}$$

The expression $(\alpha - \mu)/\alpha$ represents the difference between raising a dollar of revenue at the actual margin and raising it through a direct taking of income from the consumer, i.e., the marginal excess burden of the tax. Furthermore, since it is independent from i, it is constant across all commodities.

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[16]This is simply Roy's identity in the presence of evasion.

Noting that the partial derivative on the left-hand side of (29) is simply the change in the effective tax base for good k from an increase in the tax rate on good i, and employing the fact that the Hessian of the expenditure function is symmetric, allows us to rewrite (29) as

$$-\frac{\sum_k \tau_k \, \partial B_i^c / \partial \tau_k}{\overline{B}_i} = \frac{\alpha - \mu}{\alpha}, \qquad \forall i. \tag{30}$$

Now dropping the superscripts, we can note that condition (30) yields the standard Ramsey rule when $\varphi_i = 0$, $\forall i$, and $S_{ik}^{\varphi} = 0$, $\forall i, k$:

$$-\frac{\sum_k \tau_k \, S_{ik}}{x_i} = \frac{\alpha - \mu}{\alpha}, \qquad \forall i. \tag{31}$$

Examining (30) and (31) allows us to make a direct comparison between the no evasion Ramsey rule and the evasion-modified Ramsey rule. Equation (31) says that tax rates should be chosen so as to induce equal percentage changes in compensated demands for all goods at the margin. An equivalent interpretation in light of our model is that tax rates should be chosen so as to induce equal percentage changes in the tax base, x_i, for all commodities. This terminology may be extended directly to the evasion-modified rule (30): tax rates should be set so as to cause equal percentage changes in the effective tax base, B_i, for all commodities.

In the standard optimal commodity tax model, if exogenous income, M, is non-zero, uniform taxation is feasible and optimal. In fact, if $M \neq 0$, a set of uniform rates, $\tau_i = \gamma p_i$, $\forall i$ (where $p_i = 1 + \tau_i$), will satisfy the rule given by (31) if

$$-\gamma \sum_k p_k \, S_{ik} = x_i \, \frac{\alpha - \mu}{\alpha}, \qquad \forall i. \tag{32}$$

Since, by Euler's theorem,[17] the left-hand side of (32) is zero, we obtain

$$\alpha - \mu = 0. \tag{33}$$

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[17]The expenditure function is homogeneous of degree one in prices, and compensated demands are homogeneous of degree zero in prices.

Thus, uniform taxation involves a zero marginal excess burden. In contrast, when evasion is present, the Ramsey rule fails to collapse into a simple rule, even when exogenous income is positive, and uniform taxation will generally not be optimal. For example, when evasion effort technologies are identical across commodities, if compensated demands and net-of-tax prices are also equal across commodities, uniform proportional taxation will clearly be optimal, but these are only sufficient, not necessary conditions for optimality. Condition (30) implies that the presence of evasion modifies the optimal tax rule even when the government has a full set of instruments with which to meet the revenue requirement.[18]

Unfortunately, (30) is much too general too provide a more in depth understanding of how evasion affects the optimal tax structure. In order to gain some insight, we will next examine a two-commodity model with non-zero exogenous income. We will restrict our analysis to interior solutions (i.e., with non-zero taxation on both commodities).

In a two-commodity environment, condition (30) allows us to solve for the optimal ratio of tax rates on commodity 1 and 2:

$$\frac{\tau_1}{\tau_2} = \frac{-\left(\partial B_2^c/\partial\tau_2\right)/B_2 + \left(\partial B_1^c/\partial\tau_2\right)/B_1}{-\left(\partial B_1^c/\partial\tau_1\right)/B_1 + \left(\partial B_2^c/\partial\tau_1\right)/B_2}. \tag{34}$$

Notice that, in the absence of evasion, condition (34) simplifies into

$$\frac{\tau_1}{\tau_2} = \frac{-S_{22}/x_2 + S_{12}/x_1}{-S_{11}/x_1 + S_{21}/x_2}, \tag{35}$$

where the Slutsky terms simply represent compensated price effects. In this case, both the numerator and denominator on the left-hand side become zero and the optimal tax rule degenerates into uniform proportional taxation, as explained above. Noting that the changes in the effective base are

$$\frac{\partial B_i^c}{\partial\tau_j} = \frac{\partial b_i}{\partial x_i} S_{ij}^x + \frac{\partial b_i}{\partial\varphi_i} S_{ij}^\varphi, \quad \forall i,j, \tag{36}$$

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[18]It trivially follows that a lump-sum tax on exogenous income is no longer equivalent to uniform proportional taxation.

allows us to rewrite equation (34) in terms of the compensated tax effects on evasion effort and demand, as follows:

$$\frac{\tau_1}{\tau_2} = \frac{-\left(\frac{\partial b_2}{\partial x_2} S_{22}^x\right)/B_2 + \left(\frac{\partial b_1}{\partial x_1} S_{12}^x\right)/B_1 - \left(\frac{\partial b_2}{\partial \varphi_2} S_{22}^\varphi\right)/B_2 + \left(\frac{\partial b_1}{\partial \varphi_1} S_{12}^\varphi\right)/B_1}{-\left(\frac{\partial b_1}{\partial x_1} S_{11}^x\right)/B_1 + \left(\frac{\partial b_2}{\partial x_2} S_{21}^x\right)/B_2 - \left(\frac{\partial b_1}{\partial \varphi_1} S_{11}^\varphi\right)/B_1 + \left(\frac{\partial b_2}{\partial \varphi_2} S_{21}^\varphi\right)/B_2}. \tag{37}$$

Inspecting condition (37), we can see that evasion activity affects the optimal structure of tax rates through two different channels. First, the tax effects on compensated demands, S_{ij}^x, are affected by the reaction of evasion activity to tax changes through the price. Recall from expression (17) that $S_{ij}^x = S_{ij}(\partial P_j/\partial \tau_j)$: since $\partial P_j/\partial \tau_j < 1$, the responsiveness of the price to changes in evasion activities at the margin produces a reduction in the magnitude of all compensated effects on x_i. This reduction in the compensated effects will be further magnified by the reaction of the effective base to changes in the statutory base: the less the effective base increases when the statutory base increases, the more the reduction in compensated effects will be enhanced. Thus, commodities that have a large impact on the statutory base should be taxed at lower rates, while commodities for which tax increases induce a relatively large reduction in the marginal price should be taxed at relatively higher rates, *ceteris paribus*.

Through this channel, the rule may prescribe a structure of commodity taxes that is more or less uniform than the one prescribed by the traditional literature. We may think of an example with identical compensated own and cross price effects, and suppose that the technology available to the consumer for evading on good 1 (say, lawn care services) is more effective than the technology for evading on good 2 (say, postal services), i.e., for a given evasion effort, evasion activity on lawn care services results in a larger reduction in the price of lawn care services than evasion activity on postal services does for the price of postal services. Through this channel, the optimal tax rule implies that the tax rate on postal services should be relatively lower than the tax rate on lawn care services.

The second channel through which the presence of evasion impacts on the optimal tax structure is presented by the Slutsky terms for evasion activity in the numerator and denominator on the right-hand side of equation (37). Focussing on the numerator of (37), we may recall from the comparative statics of the consumer's compensated problem that the own compensated evasion response to tax changes, S_{22}^{φ} (as defined in (18)), contains two terms: a direct response—which is positive, increasing in the level of demand and decreasing in the marginal cost of evasion—and an indirect response that is some multiple of the compensated tax effect on demand, S_{22}. Also recall that the cross compensated tax change, S_{12}^{φ} (as defined in (19)), contains only the indirect response. Finally, both own and cross effects are weighted by the responsiveness of the effective base to the statutory base.

Since the direct tax effect on evasion effort is directly proportional to the level of demand, the optimal tax rate on good 2 will be inversely proportional to the level of demand for good 2 and directly proportional to the level of demand for good 1.[19] The rationale behind this result is that, since the marginal benefits from evasion effort are increasing in the level of demand, the deadweight loss from evasion activities will be reduced if evasion efforts become more spread across commodities instead of being concentrated on selected commodities. Consequently, the optimal tax structure will be one that tends to induce uniform evasion levels across commodities by levying higher taxes on those goods whose share in demand is comparatively small. Hence, *ceteris paribus*, the optimal tax structure will diverge from uniformity if demand shares are not uniform.

The indirect tax effect on evasion effort causes the ordinary price effects on demand to be further reduced by the price reduction to the consumer that results from adjusting evasion effort in response to tax changes: the more effective the consumer is in reducing the tax effect on the price of the commodity, the more the compensated price effects on demand will be reduced. This further reduction in the compensated demand effects is also offset by the reaction of the effective base to changes in the statutory base as discussed above.

The combined impacts of these effects may result in a rate structure that diverges from uniformity, which is in contrast with the prescriptions of the traditional theory. Referring to our previous example, assume that lawn care services and postal services have identical demand shares. Then, as discussed above, the optimal tax rule prescribes that, through

[19]When the level of demand for good 2 increases, the term $(-\partial b_2/\partial \varphi_2)$ becomes larger which implies that τ_2 should be relatively smaller than τ_1.

the price channel and the indirect evasion channel, lawn care services should be taxed at a relatively higher rate, but, through the direct evasion channel, the same rule prescribes that lawn care services should be taxed relatively more lightly. On the other hand, if the demand share for lawn care services is smaller than the demand share for postal services, then, *ceteris paribus*, the tax rate on lawn care services should be relatively higher.

To summarize, even when it is feasible for the government to levy uniform taxes, they may not be the optimal choice. There are several reasons for this result. First, taxes increase the marginal benefit of evasion effort through the first-order conditions for the consumer problem; this causes an increase in evasion effort that manifests itself as a reduction in the tax effect on the price of the commodity, which dampens all compensated price effects on demand. *Ceteris paribus*, those commodities that exhibit a relatively larger decrease in price should be taxed relatively more heavily. Second, commodities that induce relatively larger increases in the effective tax base should be taxed at relatively higher rates. The rationale behind this is that excess burden is minimized when commodities that add relatively more to revenue are taxed more heavily relative to commodities that have a lesser impact on revenue. Third, commodities for which there are relatively higher levels of demand should be taxed at relatively lower rates so as to maximize the spread of marginal evasion effort across commodities, while commodities that have large reductions in evasion activity when the marginal cost of evasion increases should be taxed relatively more heavily.

4.2 Optimal taxation with untaxed commodities

We next examine a situation where it is not feasible for the government to levy uniform commodity taxes. Condition (37) captures the optimal tax structure for a three-commodity economy with zero exogenous income, where commodity 3 is untaxed.[20] When exogenous income is zero and there is no evasion, the optimal structure of tax rates is not unique,

[20]The coefficient matrix of the vector of taxes is the inverse of the Hessian of the expenditure function for the taxed goods. When commodity 3 is untaxed, solving for τ_1/τ_2 yields the expression on the right-hand side of (37).

and, in order to achieve normalization, the tax rate on one commodity (typically the numeraire) may be set equal to zero. In the presence of evasion, the budget constraint is not homogeneous of degree zero in gross-of-tax prices, and therefore it is not possible to arbitrarily normalize tax rates.[21] Thus, the choice of a zero tax rate on one commodity must be postulated exogenously, perhaps as an institutional constraint.

With no evasion, condition (37) is not degenerate, since the numerator and denominator on the right-hand side are not equal to zero. In this case we obtain the well known prescription that the tax rate on the commodity that is comparatively more complementary to the untaxed commodity should be relatively higher. The presence of evasion, however, may again cause the optimal tax structure to become more or less uniform than prescribed be the standard Ramsey rule. Suppose, for example, that the sum of the Slutsky tax terms for compensated demands appearing in the numerator is larger than the corresponding sum in the denominator. If the compensated demands for the taxed goods are close, and the structure of evasion technologies is identical across commodities, the sum of the last two terms in the numerator will be close to the corresponding sum in the denominator, and the optimal tax structure will tend to be more uniform than it would be in the absence of evasion.

To further strengthen our understanding of the optimal structure of tax rates for the three-commodity case, we may examine the special situation where compensated cross-price effects between commodities are zero. In this case, equation (37) may be written as

$$\frac{\tau_1}{\tau_2} = \frac{\left(\frac{\partial b_2}{\partial x_2} S^x_{22}\right)/B_2 + \left(\frac{\partial b_2}{\partial \varphi_2} S^{\varphi}_{22}\right)/B_2}{\left(\frac{\partial b_1}{\partial x_1} S^x_{11}\right)/B_1 + \left(\frac{\partial b_1}{\partial \varphi_1} S^{\varphi}_{11}\right)/B_1}. \tag{38}$$

Notice that, in the absence of evasion, equation (38) collapses to

$$\frac{\tau_1}{\tau_2} = \frac{S_{22}/x_2}{S_{11}/x_1}, \tag{39}$$

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[21] It follows that the traditional equivalence relationships between different tax structures no longer hold: for example, uniform proportional commodity taxation on all transactions except labour supply, is no longer equivalent to a tax on wages.

and we obtain a structure of tax rates that follows the well known inverse-elasticity rule: tax rates should be relatively higher on commodities that have relatively low compensated own price elasticities of demand. In contrast, when evasion is present, there will be two forces at work. First, the own compensated change in demand will be dampened by the first-order evasion effect on the marginal price. Secondly, although there are no cross effects, the own evasion effect also enters additively into both the numerator and the denominator of expression (38), implying that the inverse-elasticity rule may be dominated by evasion effects.

To summarize, the presence of evasion affects the optimal structure of commodity tax rates in directions that are conditional on the structure of evasion technologies and preferences: the evasion-modified Ramsey rule prescribes that tax rates that minimize excess burden should be set so as to cause equal percentage changes in the effective tax bases across commodities. Commodities for which the tax impact on evasion effort results in relatively lower gross-of-tax, net-of-evasion marginal prices, should be taxed at relatively lower rates; at the same time, commodities that make a relatively large contribution to marginal revenue should be taxed at relatively higher rates. Thus, the presence of evasion may cause the optimal tax structure to be more, or less uniform than the one prescribed by traditional theory.

5 Summary and conclusion

This paper has explored the implications of a particular form of increasing returns-to-scale evasion technologies for the optimal structure of commodity taxation by deriving an evasion-modified optimal tax rule. Although this rule is too general to yield simple prescriptions, we have developed some intuition as to how the presence of tax evasion might affect the design of commodity tax systems. In order to minimize the deadweight loss from evasion activity, tax structures should aim at maximizing the spread of marginal evasion activities across commodities. We have shown that this objective may potentially dominate concerns over the inter-commodity distortions stressed by the traditional optimal taxation literature.

In particular, uniform taxation is no longer optimal even if feasible. We also find that, when some transactions are excluded from taxation, the optimal structure may appear more or less uniform than the optimal tax structure prescribed when no evasion is present.

The implications of these findings for the design of indirect taxes may be important. For example, our analysis implies that the presence of evasion may yield an optimal tax structure that features relatively lower tax rates on commodities that have relatively lower price elasticities of demand. On the other hand, when all transactions are of similar size, the presence of evasion may provide a rationale for broad-based uniform taxation.

Extensions to this study should take into account informational asymmetries that exist between the government and the consumer as well as the distributional effects of tax evasion. Further research is also needed to assess the empirical relevance of our restrictions on evasion technologies, and to test the predictive power of our approach in contributing to explain the structure of real-world commodity tax systems.

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