FISCAL POLICY, ADJUSTMENT COSTS, AND ENDOGENOUS GROWTH*

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1. Introduction

The last several years has seen an explosion of literature dealing with the issue of endogenous growth; see e.g. Romer (1986), Lucas (1988), Barro (1990), Rebelo (1991), Jones, Manuelli, and Rossi (1991, 1993), Mulligan and Sala-i-Martin (1993), Turnovsky (1993). Virtually all of this literature treats investment as being determined residually. That is, the amount of output available for accumulation as new capital is whatever remains after the private and public sectors' consumption needs have been met. Moreover, the transformation of this new output into new capital occurs costlessly. Within this type of framework, much of the discussion has focused on fiscal issues, with both the expenditure and taxation aspects receiving attention.[1]

While this treatment of investment serves as a reasonable first approximation and indeed follows the practice of more traditional growth theory, it runs counter to much of the recent literature on investment theory which derives from the "Tobin q" theory of investment and motivates this in terms of convex costs of adjustment. This literature originated with Lucas (1967) and Gould (1968), with more recent developments being based on Hayashi (1982). This approach has been applied by several authors to analyze issues pertaining to tax policy in a Ramsey type setting for a closed economy; see e.g. Abel (1982), Turnovsky (1990). It is also a standard feature of models of capital accumulation in a small open economy with tradable capital, being necessary for such models to give rise to nondegenerate dynamics; see e.g. Brock (1988), Sen and Turnovsky (1990).

This paper introduces convex adjustment costs into an endogenous growth model of a closed economy. In so doing our focus will be on analyzing the effects of fiscal policy on the long-run growth performance of the economy. This is a particularly important application of these models, since few economists would deny that fiscal policy has important consequences for long-run economic growth. Yet the traditional Ramsey model, with its long-run growth rate being determined by demographic factors, (and independent of the usual macroeconomic policy instruments), is incapable of adressing these issues satisfactorily. We will study an economy in which the government raises taxes to finance its expenditures. These expenditures are assumed to be directed to enhancing the productive activities of the economy. In this respect, the government expenditure will play two roles.

In the first place, it will impact directly on production conditions and enhance the productivity of the existing capital stock. This effect has received increasing attention in the literature, both with respect to its empirical relevance and its theoretical consequences.

Much of the empirical research has been stimulated by Aschauer's (1989, 1990) striking findings suggesting that the output elasticity of public capital in the United State during the 1949-85 period was around 0.39 and that 80% of the decline in productivity growth since the 1970's is attributable to the decline in the growth of public capital over that same period. Subsequent studies have yielded mixed results, with some finding a negligible role for public capital in promoting productivity growth, and others finding a significant role, although not generally as large as that implied by Aschauer's results. For example, Aaron (1990) and Tatom (1991) criticize Aschauer largely on the grounds of econometric method and argue that his results are biased in favor of the productivity of public capital. A similar view is expressed by Holtz-Eakin (1992). By contrast, Lynde and Richmond (1992), and Finn (1993) address some of these criticisms and yield results that are generally supportive of the overall Aschauer view.

The theoretical aspects of the role of productive government expenditure have been studied in Ramsey type economies by various authors, beginning with Arrow and Kurz (1970), and more recently by Aschauer (1988), Baxter and King (1993), and Turnovsky and Fisher (1994). This issue has also begun to be studied in an endogenous growth context by authors such as Barro (1990) and Turnovsky (1993).[2]

But once the adjustments costs of investment are recognized, a natural second productive role for government expenditure is introduced. It now becomes plausible to suggest that the same aspects of government expenditure which enhance the productivity of the *existing* capital stock, will also reduce the costs associated with investment and thereby facilitate the accumulation of the flow of *new* capital. The model we shall develop will incorporate both these channels for government expenditure to affect the productive performance of the economy.

Following this introduction, this paper is divided into two main parts. The first derives the equilibrium in a centrally planned economy, in which the government controls all quantities directly.

First best optimal fiscal issues are discussed. The second part of the paper derives a decentralized equilibrium in which the government controls resources only indirectly, through taxation. The effects of various forms of distortionary taxes on growth and welfare are discussed. In addition, the optimal tax structure which will enable the first best equilibrium to be replicated, is characterized.

2. A Centrally Planned Economy

2.1 The Analytical Framework

We consider an economy populated by a representative agent who consumes a private consumption good C, deriving intertemporal utility represented by the isoelastic utility function

$$W \equiv \int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\rho t} dt \qquad -\infty < \gamma < 1 \tag{1a}$$

The exponent γ is related to the intertemporal elasticity of substitution, s say, by $s = 1/(1-\gamma)$, with the logarithmic utility function being equivalent to setting $\gamma = 0$.[3]

Production in the economy takes place by means of the constant returns to scale production function

$$Y = \alpha\left(\frac{E}{K}\right)K \qquad \alpha' > 0; \quad \alpha'' < 0 \tag{1b}$$

where K denotes the capital stock and E denotes the level (flow) of government expenditure. Equation (1b) embodies the assumption that the government expenditure we shall consider enhances, at a diminishing rate, the productivity of the only factor of production, capital.

The process of capital accumulation involves adjustment costs (installation costs) which we incorporate in the quadratic (convex) function

$$\Phi(I,K) = I\left(1 + \frac{h(E/K)}{2}\frac{I}{K}\right) \qquad h' < 0; \quad h'' > 0 \tag{1c}$$

where

$$I = \dot{K} \tag{1d}$$

denotes the rate of capital accumulation. Equation (1c) is an application of the familiar Hayashi (1982) cost of adjustment framework, where we assume that the adjustment costs are proportional to the *rate* of investment per unit of installed capital rather than to absolute its level.[4] The linear homogeneity of this function is necessary if a steady-state equilibrium having ongoing growth is to be sustained. The rationale for the specification of (1c) is that government expenditure on infrastructure, such as improved roads and research and development, directed at improving the current productivity of the economy, will also facilitate the accumulation of new capital. This is represented by lowering the adjustment costs $(h' < 0)$, though these reductions are assumed to occur at a declining rate $(h'' > 0)$. Thus from (1b) and (1c) we see that government expenditure impacts on the both the productivity of the existing catpial stock K, as well as the cost of acquiring new capital, I.

In due course, we shall determine the optimal tax structure to maximize (1a). In determining this we shall show that critical elements are (i): $\alpha(e), h(e)$ and the externalities these generate for the agent; and (ii) the extent to which the private agent internalizes any externalities generated by government expenditure. Our strategy will be to determine the tax structure which is able to replicate the first-best optimum achievable by a central planner having direct control over the resources in the economy.

These resources are constrained by the economy-wide constraint

$$\alpha\left(\frac{E}{K}\right)K = C + I\left(1 + \frac{h(E/K)}{2}\frac{I}{K}\right)K \tag{1e}$$

In this section, we consider the case where the government acts as a central planner and chooses C, E, I, and K, directly to maximize (1a), subject to the resource constraint (1e) and the accumulation equation (1d).

In order for an equilibrium with steady ongoing growth to be sustained, the level of government expenditure must be tied to some index of growth in the economy. While several such measures suggest themselves, in the present context, where the expenditure is on a productive input, it is natural to assume that the government ties its expenditure to the capital stock in accordance with[5]

$$e = \frac{E}{K} \qquad \alpha > e > 0 \tag{2}$$

We shall focus initially on the case where the government sets e arbitrarily, postponing until Section 2.3 below the case where e is set optimally along with C, I, and K.

Taking e to be set arbitrarily, the central planner's problem is to choose the rate of C, the rate of investment I, and the rate of capital accumulation to maximize (1a), subject to (1d), (1e), together with (2). The present value Hamiltonian for this optimization is given by

$$\mathrm{H} \equiv \frac{1}{\gamma}C^{\gamma}e^{-\rho t} + \lambda e^{-\rho t}\left[\alpha(e)K - C - I\left(1 + \frac{h(e)}{2}\frac{I}{K}\right) - eK\right] + q'e^{-\rho t}\left[I - \dot{K}\right] \tag{3}$$

where λ is the shadow value (marginal utility) of wealth in the form of new output and q' is the shadow value of the agent's capital stock. Analysis of the model is simplified by using the shadow value of wealth as numeraire. Consequently $q \equiv q'/\lambda$ is defined to be the market value of capital in terms of the (unitary) price of foreign bonds.

The optimality conditions with respect to C and I are respectively

$$C^{\gamma-1} = \lambda \tag{4a}$$

$$1 + h(e)\frac{I}{K} = q \tag{4b}$$

The first of these conditions equates the marginal utility of consumption to the shadow value of wealth, while the latter equates the marginal cost of an additional unit of investment, which is inclusive of the marginal installation cost $h(e)(I/K)$, to the market value of capital. Equation (4b) may be immediately solved to yield the following expression for the rate of growth of capital:

$$\frac{I}{K} = \frac{\dot{K}}{K} = \frac{q-1}{h} \equiv \phi \tag{5}$$

Applying the standard optimality conditions with respect to *K*, (while noting the relationship $q \equiv q'/\lambda$) implies the arbitrage relationship

$$\frac{\alpha(e)-e}{q}+\frac{\dot{q}}{q}+\frac{(q-1)^2}{2hq}=\rho-\frac{\dot{\lambda}}{\lambda} \tag{6}$$

Equation (6) is a standard Keynes-Ramsey consumption rule, equating the marginal return on consumption, given by the right hand side of (6) to the rate of return on acquiring a unit of physical capital. The latter consists of three components. The first is the output (net of government expenditure) per unit of installed capital (valued at the price *q*), while the second is the rate of capital gain. The third element, which is less familiar, is equal to $(qI-\Phi)/qK$. This measures the rate of return arising from the difference in the valuation of the new capital *qI* and the value of the resources its utilizes Φ, per unit of installed capital. Seen another way, this third component reflects the fact that an additional source of benefits of a higher capital stock is to reduce the installation costs (which depend upon I/K) associated with new investment.

Finally, in order to ensure that the economy's intertemporal budget constraint is met, the following transversality condition must be imposed:

$$\lim_{t\to\infty} q'Ke^{-\rho t}=0 \tag{7}$$

2.2 Macroeconomic Equilibrium

We shall look for an equilibrium in which the consumption-capital ratio is constant, i.e. $C/K=\mu$, say, so that $\dot{C}/C=\dot{K}/K$. Differentiating (4a) with respect to *t*, and using (5), we obtain

$$\frac{\dot{\lambda}}{\lambda}=-(1-\gamma)\frac{\dot{K}}{K}=-(1-\gamma)\frac{(q-1)}{h}=-(1-\gamma)\phi \tag{8}$$

Substituting for $\dot{\lambda}/\lambda$ from (8) into (6) leads to the following nonlinear differential equation in the shadow value *q*:

$$\dot{q} = \frac{(1-2\gamma)}{2h}q^2 + \left(\rho + \frac{\gamma}{h}\right)q - \left(\alpha(e) - e + \frac{1}{2h}\right) \tag{9}$$

In order for the economy ultimately to follow a path of steady growth (or decline) the stationary solution to this equation, attained when $\dot{q} = 0$ must have (at least) one *real* solution. Setting $\dot{q} = 0$ in (9), implies that the steady state value of q, $\tilde{q}$ say, must be a solution to the quadratic equation

$$\frac{(1-2\gamma)}{2h(e)}\tilde{q}^2 + \left(\rho + \frac{\gamma}{h(e)}\right)\tilde{q} - \left(\alpha(e) - e + \frac{1}{2h(e)}\right) = 0 \tag{10}$$

As long as $\gamma < 1/2$, the existence of real roots to this equation is assured. However, for $\gamma > 1/2$, it is possible for the returns to capital to dominate sufficiently the returns to consumption, irrespective of the price of capital, so that no long-run balanced equilibrium can exist where the returns to these two activities are brought into equality.

Equation (9) indicates that q satisfies a dynamic efficiency condition. Fig. 1 illustrates the phase diagram in the case where (10) has real solutions, so that a steady-state asymptotic growth path for capital does indeed exist. The two cases $\gamma < 1/2; \gamma > 1/2$ are illustrated in Parts A and B of this figure, respectively. Assuming real roots, the solutions to the quadratic equation (10) are:

$$\tilde{q}_1 = \frac{-\left(\rho + \frac{\gamma}{h}\right) - \sqrt{\left(\rho + \frac{\gamma}{h}\right)^2 + \frac{2(1-2\gamma)}{h}\left(\alpha - e + \frac{1}{2h}\right)}}{\frac{1-2\gamma}{h}} \tag{11a}$$

$$\tilde{q}_2 = \frac{-\left(\rho + \frac{\gamma}{h}\right) + \sqrt{\left(\rho + \frac{\gamma}{h}\right)^2 + \frac{2(1-2\gamma)}{h}\left(\alpha - e + \frac{1}{2h}\right)}}{\frac{1-2\gamma}{h}} \tag{11b}$$

suggesting the potential existence of two steady-state equilibrium growth paths.

However, in both cases, the negative root $\tilde{q}_1$ can be ruled out. In the case where $\gamma < 1/2$ illustrated in Fig. 1.A, it is represented by the stable equilibrium point B, which however, implies a negative shadow price of capital and is therefore of no economic interest. By contrast $\tilde{q}_2 > 0$ and is represented by the unstable equilibrium point A in Fig. 1.A. Thus, unless q is always at the stationary point A, it will ultimately converge to an equilibrium in which it maintains a negative value. Moreover, any time path for q needs to satisfy the transversality condition (7). To evaluate this, observe that

$$\lim_{t\to\infty} q'Ke^{-\rho t} = \lim_{t\to\infty} q\lambda Ke^{-\rho t}$$

Solving equations (8) implies $K(t) = K_o e^{\phi t}$; $\lambda(t) = \lambda(o)e^{-(1-\gamma)\phi t}$, where K_o is the given initial stock of capital and $\lambda(o)$ is the endogenously determined initial marginal utility, so that $\lim_{t\to\infty} q'Ke^{-\rho t}$, $= \lim_{t\to\infty} q\lambda(o)K_o e^{(\gamma\phi-\rho)t} = 0$ if and only if

$$\gamma\phi \equiv \gamma\left(\frac{q-1}{h}\right) < \rho \qquad (12)$$

Substituting the positive root $\tilde{q}_2$ from (11b) it can be shown that the transversality condition (12) is indeed met.

Case B where $\gamma > 1/2$ is analogous and is illustrated in Fig. 1.B. In this case both solutions (11a), (11b) are positive. The point C which corresponds to (11a) is a stable solution, while D, given by (11b) is unstable. However, the former can be shown to be in violation of the transversality condition (12) and can therefore be ruled out. By contrast, (11b satisfies (12).

In summary, the only solution for $q(t)$ which is consistent with both the transversality condition and satisfies the nonnegativity condition is that q always be at the steady-state solution $\tilde{q}_2$ given by the positive root (11b); i.e. $q(t) \equiv \tilde{q}_2$. *Consequently, there are no transitional dynamics in the shadow price q.* Following any disturbance, q immediately jumps to its new equilibrium value. It then follows from (5) that the economy is always on its steady-state growth path.

But it is possible for $\tilde{q}_2 < 1$, so that the equilibrium is one of steady decline rather than positive growth. Since one of our points of focus will be the equilibrium growth rate, it is convenient to use (5) to transform (10) to a quadratic equation in the equilibrium growth rate $\tilde{\phi}$:

$$(1-2\gamma)\frac{h(e)}{2}\tilde{\phi}^2 + (1-\gamma+\rho h(e))\tilde{\phi} + \rho - (\alpha(e) - e) = 0 \qquad (13)$$

with the relevant equilibrium corresponding to the positive root to this equation. In the absence of adjustment costs (i.e. as $h \to 0$) the equilibrium growth rate reduces to the standard expression

$$\tilde{\phi} = \frac{(\alpha(e) - e) - \rho}{1 - \gamma} \tag{13'}$$

and the equilibrium will be one of positive growth if and only if the net productivity of capital $\alpha(e) - e$ exceeds the rate of time preference ρ; see e.g. Turnovsky (1993). The same condition can be shown to ensure positive equilibrium growth in the presence of adjustment costs.

To complete the equilibrium, we need to derive the consumption-capital ratio, which we have assumed to be constant. Dividing (1e) by K and using (5), we obtain

$$\tilde{\mu} = \alpha(e) - e - \frac{\tilde{q}^2 - 1}{2h(e)} = \alpha(e) - e - \tilde{\phi}\left(1 + \frac{h(e)}{2}\tilde{\phi}\right) \tag{14}$$

With e and q both being constants, this equation ensures that the equilibrium consumption-capital ratio is constant, as assumed. Moreover, from the relationship (5) and (13) the equilibrium consumption-capital ratio in the centrally planned economy can be expressed in the more convenient form

$$\frac{\tilde{C}}{K} = (1 + h(e))\left(\rho - \gamma\tilde{\phi}\right) \qquad \text{or} \qquad \frac{\tilde{C}}{\tilde{q}K} = \rho - \gamma\tilde{\phi} \tag{15}$$

Taken in conjunction with (12), we see that the transversality condition is equivalent to requiring a positive consumption to capital ratio.[6] Observe also that in the absence of adjustment costs ($h \to 0, q \to 1$), (15) reduces to the more standard expression

$$\frac{\tilde{C}}{K} = \rho - \gamma\tilde{\phi} = \frac{\rho - \gamma(\alpha(e) - e)}{1 - \gamma} \tag{15'}$$

Given the equilibrium growth rate, equations (14) or (15) yield corresponding equivalent expressions for the first-best optimal consumption-capital ratio.

2.3 Increase in Rate of Government Expenditure

Of particular concern is the impact of an increase in productive government expenditure *e* on the equilibrium growth rate. Differentiating (13) with respect to *e* implies

$$\frac{d\tilde{\phi}}{de} = \frac{(\alpha' - 1) - h'\tilde{\phi}\left[\frac{(1-2\gamma)}{2}\tilde{\phi} + \rho\right]}{(1-2\gamma)h\tilde{\phi} + (1-\gamma+\rho h)} \tag{16}$$

With the positive root of (13) being the relevant one, this ensures that the denominator of this equation is strictly positive. Equation (16) identifies a change in government expenditure as having three sources of impact on the equilibrium growth rate. First, to the extent that it utilizes current resources which would otherwise be accumulated as capital, it reduces the growth rate (the term -1 in the numerator). Secondly, to the extent that it enhances the productivity of existing capital, it increases the growth rate (the term α' in the numerator). Thirdly, to the extent that it reduces the cost of installing new capital (i.e. $h' < 0$), it will increase an already positive growth rate. The growth-maximizing rate of government expenditure involves balancing off these three effects. In the case where $h' = 0$, the growth rate maximizing ratio of government expenditure, is determined where $\alpha'(e) = 1$.

2.4 Welfare Maximizing Government Expenditure

Thus far, we have assumed that the level of government expenditure has been set arbitrarily. The solution for $\tilde{\phi}$ from (13) can be viewed as characterizing the response of the equilibrium growth rate to changes in the arbitrarily set level of government expenditure *e*, with the slope being described by (16). In the case that $h'(e) = 0$, so that government expenditure has no effect on marginal investment costs, this locus is illustrated by AA in Fig. 2, which reaches a maximum at $\alpha'(e^*) = 1$. If in addition $h'(e) < 0$, this curve is modified to CC in Fig. 3, which has the same general characteristics.

Suppose now that the central planner chooses to set the level of expenditure optimality. Setting $\partial H/\partial e = 0$ in (3) and noting (5) leads to the additional optimality condition

$$\alpha'(e) - \frac{h'(e)}{2}\left(\frac{q-1}{h}\right)^2 = 1,$$

or in terms of the growth rate

$$\alpha'(e) - \frac{h'(e)}{2}\phi^2 = 1 \tag{17}$$

This equation has a simple interpretation. The left hand side measures the welfare benefits of a unit increase in government expenditure. These include: (i) the marginal benefits to the productivity of existing capital, and (ii) the marginal benefits resulting from reducing the costs associated with installing new capital. At the optimum, these marginal benefits must equal the unit resource cost they involve.

Equation (17) can be viewed as the optimal response function of the central planner in setting e, to changes in the equilibrium growth rate. Differentiating (17) implies

$$\frac{de}{d\phi} = \frac{h'\phi}{\alpha'' - (h''/2)\phi^2} \tag{18}$$

and is positively (negatively) sloped according to whether the growth rate is positive (negative). Suppose $h' < 0$ and the equilibrium is one of positive growth. The central planner will have an incentive to increase his level of expenditures, in reponse to any further increase in the growth rate, in order to take advantage of the larger welfare gains associated with reducing investment costs in a rapidly growing economy (with its rapidly increasing adjustment costs).

In the case where government expenditure has no impact on investment costs, (17) reduces to $\alpha'(e) = 1$. In this case, the *welfare maximizing* level of government expenditure coincides with its *growth rate maximizing* level. In other words, maximizing the growth rate of the economy will also be optimal from an intertemporal welfare viewpoint. The locus (17) in this case is illustrated by the vertical line BB in Fig. 2. The intersection point P represents the welfare (and growth rate) maximizing level of expenditure and the corresponding optimized growth rate.

The central planner's reaction function in the more general case where $h' < 0$ is illustrated by the positively sloped locus DD in Fig. 3. While there is some ambiguity about its curvature, it is definitely more positively sloped than CC at the point of intersection Q.[7] It follows, therefore, that the growth maximizing level of government expenditure e^*, *exceeds* the welfare maximizing level $\tilde{e}$. Expressed differently, starting from e^*, a reduction in the rate of government expenditure, although reducing growth, will nevertheless be initially welfare improving.

A further aspect concerns the intertemporal distribution of the welfare gains. To see this, consider the expression for the welfare function along the optimized path, namely

$$\tilde{W} = \frac{1}{\gamma}\int_o^\infty \tilde{C}^\gamma e^{-\rho t} dt \equiv \frac{1}{\gamma}\int_o^\infty \tilde{Z} e^{-\rho t} dt \tag{19}$$

where $Z \equiv C^\gamma$ denotes the instantaneous level of utility and the tilde denotes the equilibrium value corresponding to the first best optimum. Substituting for $\tilde{C}$ from (15) and from $\tilde{K}$ from (5), leads to the expressions

$$\tilde{W} = \frac{K_o^\gamma}{\gamma}\frac{\tilde{\mu}^\gamma}{\rho - \gamma\tilde{\phi}} = \frac{K_o^\gamma}{\gamma}\frac{\left(1 + h(e)\tilde{\phi}\right)^\gamma}{\left(\rho - \gamma\tilde{\phi}\right)^{1-\gamma}} \tag{20}$$

It can be verified that the two conditions (16) and (17) do indeed maximize intertemporal welfare $\hat{W}$. Furthermore, in order for intertemporal welfare to be maximized it can be further established that at the intertemporal optimum, $\partial Z(o)/\partial e < 0$, $\partial\phi/\partial e > 0$. In other words, the *intertemporal* welfare maximizing government expenditure policy will involve giving up *initial* consumption (and welfare), in order to increase the growth rate and raise consumption over time.

3. Decentralized Economy

We turn now to the representative agent operating in a market economy. For convenience, we normalize the size of the population to be unity. Since government bonds can be shown to have no welfare enhancing characteristics in this model, without loss of generality we can abstract from them and assume that the government raises all revenue through various forms of taxation.[8]

In determining the optimal consumption and capital accumulation decisions of the representative agent, the nature of the public good, and the relation, if any, the agent may perceive it to have to his growing level of capital is crucial. While we shall maintain the assumption that the government sets its aggregate expenditure share $e \equiv E/K$ so that the actual level of E increases linearly with K over time, it may, or may be perceived in this way by the agent in performing his optimization. That depends in part upon the nature of the public good being provided. We shall specify the representative agent's perception of government expenditure, $E^p(t)$, by:

$$E^p(t) = e\overline{K}^{1-\sigma}K(t) = e\left(K(t)/\overline{K}\right)^{\sigma}\overline{K} \qquad 0<\sigma<1 \qquad (21)$$

The exponent σ parameterizes the extent to which the agent perceives the level of public expenditure he receives as being tied to his growing personal capital stock, while $\overline{K}$ denotes some index of the average economy-wide capital stock which may grow with time, but at a rate perceived by the agent to be unrelated to his own decisions.

The specification of (21) is important and merits further discussion.[9] Two polar values of $\sigma = 0,1$, represent extreme types of public goods. If $\sigma = 1$, the agent perceives the level of public expenditure he receives as being directly proportional to his growing capital stock (and hence income). Public goods, such as city services, which tend to be proportional to size of establishment, may be thought of as being of this type. In the other polar case, $\sigma = 0$ the agent perceives the level of government expenditure he receives as being independent of his level of activity. Nonrival public goods, such as the quality of the national labor force may be put into this category.

The second equality in (21) suggests an alternative interpretation in terms of "congestion" effects. Suppose the aggregate stock of the public good is $e\overline{K}$. The term $\left(K(t)/\overline{K}\right)^{\sigma}$ represents a scaling down of the aggregate public good available to the individual, due to congestion. The absence of any congestion is represented by $\sigma = 0$, in which case the public good is fully available to the representative agent. The other polar case $\sigma = 1$ corresponds to complete congestion, with the amount of the public good available to the agent being strictly proportional to his individual capital and its implied contribution to the aggregate. As we will see, in this case the agent in effect sees the world through the eyes of the central planner and solves the same problem.

Thus we can either view (21) as characterizing an intermediate case where the public good has a mixture of the attributes we have been describing. Alternatively, we can view (21) as simply a convenient way of parameterizing the two extreme cases, $\sigma = 0,1$. From this standpoint, the formulation is a convenient way of parameterizing the contrast between the representative agent and the central planner.

The objective of the representative agent is to maximize his constant elasticity utility function (1a) subject to his capital accumulation equation (1d), his perception of government expenditure (21), and his own budget constraint. In this decentralized economy, the latter is represented by

$$\alpha\left[e\left(\frac{K}{\overline{K}}\right)^{\sigma-1}\right]K(1-\tau)=(1+\omega)C+(1-z)I\left[1+\frac{h}{2}\left[e\left(\frac{K}{\overline{K}}\right)^{\sigma-1}\right]\frac{I}{K}\right]+T \qquad (22)$$

which incorporates the following four fiscal instruments:

T = rate of lump sum taxation;

τ = rate of taxation on capital income;

ω = rate of taxation on consumption;

z = rate of credit (or taxation if negative) on investment.

As we will see below, in the absence of lump sum taxation, an income tax alone is in general incapable of yielding the first best equilibrium; other forms of distortionary taxes are also required and our formulation enables us to compare alternative forms which change the relative price of consumption and investment. One further point about the budget constraint (22) is that it involves the perception of the agents. In due course, we shall consider an equilibrium in which, with identical agents, $K = \overline{K}$. But the perceived divergence between these two quantities is important in determining the equilibrium.

In the absence of government debt, the government must maintain a continuously balanced budget, which corresponding to the above specification of taxation, is expressed as:

$$eK = \tau\alpha K + \omega C - zI\left[1 + \frac{h}{2}\frac{I}{K}\right] + T \qquad (23)$$

Combining this with (22) and (noting the equilibrium condition $K = \overline{K}$) leads to the economy-wide resource constraint which remains as set out in (1e).

3.1 Equilibrium Growth and Consumption

The macroeconomic equilibrium is derived in an analogous fashion to that carried out in Section 2 and has the same general structure as before. In particular, the only feasible solution is one in which the shadow price of capital is constant. Analogous to (5), the equilibrium growth rate of capital (which is also constant) is determined by

$$\frac{I}{K} = \frac{\dot{K}}{K} = \frac{\frac{q}{1-z} - 1}{h} = \frac{q^* - 1}{h} \equiv \phi \qquad (5')$$

where $q^* \equiv q/(1-z)$ is defined to be the shadow value of capital adjusted for the investment credit. The solution for q^* which: (i) is always positive and (ii) satisfies the transversality condition, is the positive root to

$$\frac{1-z}{2h^2}\left[(1-2\gamma)h - h'e(1-\sigma)\right]q^{*2} + \frac{1-z}{h^2}\left[h^2\rho + \gamma h + h'e(1-\sigma)\right]q^*$$

$$-\left[(1-\tau)\left[\alpha - \alpha'e(1-\sigma)\right]\right] + \frac{1-z}{2h^2}\left[h + h'e(1-\sigma)\right] = 0 \qquad (10')$$

If the solution $\hat{q}_2^*$ to (10') coincides with $\tilde{q}_2$ given in (11b), then the rate of growth in the decentralized economy will precisely replicate that of the centrally planned economy. If this occurs, it then follows from the aggregate resource constraint, that the consumption-capital ratio of the centrally planned economy is replicated as well.

The tax structures which may enable this to be accomplished will be discussed in Section 3.3 below. However, since our focus is on growth it is convenient to use (5') to transform (10') to growth terms. Thus analogous to (13), the equilibrium rate of growth in the decentralized economy is given by the positive root to the quadratic equation

$$\frac{1}{2}[(1-2\gamma)h(e)-h'(e)e(1-\sigma)]\hat{\phi}^2+[(1-\gamma)+h(e)\rho]\hat{\phi}$$

$$+\rho-\frac{(1-\tau)}{(1-z)}[\alpha(e)-\alpha'(e)e(1-\sigma)]=0 \tag{13''}$$

The corresponding equilibrium consumption-capital ratio is

$$\hat{\mu}=\alpha(e)-e-\hat{\phi}\left(1+\frac{h(e)}{2}\hat{\phi}\right) \tag{14'}$$

A key aspect of the analysis of Section 3.3 is to compare the equilibrium $(\hat{\phi},\hat{\mu})$ in the decentralized equilibrium (13"), (14') with the corresponding solutions $(\tilde{\phi},\tilde{\mu})$ in the first best equilibrium (13), (14).[10]

3.2 Effects of Fiscal Shocks

Two issues of importance concern the effects of fiscal shocks on the equilibrium growth rate and on the level of economic welfare. The former can be obtained by routine differentiation of (13"). The following conclusions can be drawn:

(i) An increase in the consumption tax ω has no effect on the growth rate. In this respect it operates like a lump sum tax; c.f. Rebelo (1991).

(ii) An increase in the tax on capital τ with the revenues being rebated in lump sum form reduces the equilibrium growth rate. The same applies if the higher tax on capital is offset by a lower tax on consumption.

(iii) An increase in the rate of investment credit z, which is financed either by a lump-sum tax or by a tax on consumption, will raise the equilibrium growth rate.

(iv) Suppose government expenditure has no direct effect on either productivity of existing capital or on the adjustment costs (i.e. $\alpha'=\alpha''=h'=h''=0$). In this case, an increase in the rate of government expenditure e, financed either by a lump-sum tax or a tax on consumption, will leave the growth rate unchanged. All that happens is that it crowds out an equivalent amount of private consumption, consistent with Eaton (1981). By contrast, if government expenditure has a positive effects on productivity of capital while reducing the costs of investment, an increase in e financed either by T or ω will almost certainly be growth enhancing.[11]

The level of economic welfare in the decentralized equilibrium is given by an expression analogous to the first expression in (20), namely

$$\hat{W} = \frac{K_o^\gamma}{\gamma} \frac{\hat{\mu}^\gamma}{\rho - \gamma\hat{\phi}} \tag{20'}$$

where $\hat{\phi}, \hat{\mu}$ are now given by (13"), (14'). It is clear that the impact of a change in any of the distortionary tax rates on economic welfare can be analyzed through their effect on the equilibrium growth rate. Differentiating (20') and using (14'), we obtain

$$\frac{\partial \hat{W}}{\partial x} = \frac{K_o^\gamma \hat{\mu}^{\gamma-1}}{(\rho - \gamma\hat{\phi})^2}\left[(2\gamma - 1)\frac{h}{2}\hat{\phi}^2 - (1 - \gamma + \rho h)\hat{\phi} + \alpha - e - \rho\right]\frac{\partial \hat{\phi}}{\partial x} \tag{24}$$

where $x = \tau, \omega, z$, and the partial derivatives $\partial\hat{\phi}/\partial x$ can be derived from (13").

The welfare change in (24) refers to changes in the various distortionary tax rates with the resulting revenues being offset by an equivalent lump sum tax. Combining this expression with (13) enables us to assess these welfare effects in terms of the relative growth rates in the centralized and decentralized equilibria, in accordance with:

$$\frac{\partial \hat{W}}{\partial x} = \frac{K_o^\gamma \hat{\mu}^{\gamma-1}}{(\rho - \gamma\hat{\phi})^2}\left[(2\gamma - 1)\frac{h}{2}\left(\tilde{\phi} + \hat{\phi}\right) - (1 - \gamma + \rho h)\right]\left(\hat{\phi} - \tilde{\phi}\right)\frac{\partial \hat{\phi}}{\partial x} \tag{24'}$$

Equation (24') immediately implies the standard proposition, that at the first best optimum where $\hat{\phi} = \tilde{\phi}$, a marginal change in the tax rate has no impact on welfare.

Away from the first best optimum, we are in a second best situation and the welfare implications of a tax depend upon the pre-existing distortions. We consider two examples. First, suppose that $e = z = 0$ and that the only distortion is a pre-existing tax on capital $\tau > 0$. In this case, comparing (13) to (13") it is easily shown that $\hat{\phi} < \tilde{\phi}$, so that the growth rate in the decentralized equilibrium is below that of the first best optimum. Any increase in the exisiting tax on capital will increase this divergence further and impose welfare losses. On the other hand, the introduction of an investment credit will raise the growth rate and be welfare improving. By comparison, consider a second example where initially $e = \tau = 0$ and the only distortion is a pre-existing investment credit $z > 0$. In this case we find $\hat{\phi} > \tilde{\phi}$ so that the growth rate in the decentralized equilibrium is too high. Any further increase in the credit to investment will increase this growth rate further and be welfare deteriorating. In this case, welfare in the decentralized equilibrium will be improved by reducing its growth rate and this may be achieved by imposing a tax on capital. Other cases may be analyzed similarly.

3.3 Optimal Tax Structure

We turn now to the key issue, namely the determination of the structure to replicate the first best equilibrium. We shall focus on distortionary taxes and therefore abstract from lump sum taxation. In order for the decentralized economy to attain the first best outcome, two conditions need to be met. The first is that the solution for the optimal growth rate $\hat{\phi} \equiv \hat{\phi}\left(\frac{1-\tau}{1-z}, e, \sigma\right)$ in the decentralized economy obtained from (13") must coincide with that of $\tilde{\phi} \equiv \tilde{\phi}(e)$, obtained from (13) in the centrally planned economy. Expressing this equality as:

$$\hat{\phi}\left(\frac{1-\hat{\tau}}{1-\hat{z}}, e, \sigma\right) = \tilde{\phi}(e) \tag{25}$$

we see that for an arbitrarily set ratio of government expenditure e, and private sector's perception σ, this will determine an appropriate ratio involving the tax on capital $\hat{\tau}$ and the rate of investment credit $\hat{z}$. As noted before, once the two growth rates are equalized, it then follows from (14), (14') that the corresponding consumption-capital ratios must coincide as well.

The second requirement is that the tax rates in the decentralized economy must be chosen to meet the government budget constraint. Recalling (23) this requires that

$$e = \hat{\tau}\alpha(e) + \hat{\omega}\hat{\mu} - \hat{z}\hat{\phi}\left(1 + \frac{h(e)}{2}\hat{\phi}\right) \tag{26}$$

and using (14') this relationship can be written in the equivalent form

$$(1-\hat{z})e = (\hat{\tau}-\hat{z})\alpha(e) + (\hat{\omega}+\hat{z})\hat{\mu} \tag{26'}$$

Taken together, equations (25) and (26) thus impose two constraints on the three tax rates $\hat{\tau}, \hat{\omega}, \hat{z}$ which in general can be met in an infinite number of ways. The general nature of the tradeoffs among the taxes implied by these relationships is involved and insight into its nature can be obtained by focusing on special cases.

(i) $\sigma = 1$: As an initial benchmark we shall assume that the agent fully perceives the linkage between the growth of government expenditure and his own personal capital stock, in his decision making, in effect viewing the economy through the eyes of the central planner. Setting $\sigma = 1$ in (13"), and comparing to (13), we see that in order to replicate the first best growth rate, we require

$$\frac{1-\hat{\tau}}{1-\hat{z}} = \frac{\alpha - e}{\alpha} \tag{27}$$

If, further, there is no investment credit ($z = 0$), the optimal tax on capital is simply $\hat{\tau} = e/\alpha$. It then follows from the government budget constraint that $\hat{\omega} = 0$. In short, if $\sigma = 1$, the first best optimum can be attained by the following simple tax structure:

$$\hat{\tau} = \frac{e}{\alpha}; \quad \hat{\omega} = \hat{z} = 0 \tag{28}$$

This full financing of government expenditure by an income tax in effect operates like a user fee, internalizing the effects of an individual's choices on his level of public services. Individuals are essentially paying for the services they receive, so the equilibrium is Pareto optimal.

If the government chooses to set an arbitrary investment credit ($\bar{z} > 0$) then at the income tax level defined in (28), the growth rate will become too high. The tax on capital will need to be increased, in order to drive the growth rate back to its optimal level. This raises the revenue from taxation, doing so by an amount in excess of the investment credit (assuming investment to be positive), and the government generates a budget surplus. Budget balance is thus restored by subsidizing consumption. Indeed, the tax configuration

$$\hat{\tau} = \frac{e}{\alpha} + \left(\frac{\alpha - e}{\alpha}\right)\bar{z}; \quad \hat{\omega} = -\bar{z} < 0 \tag{28'}$$

will also ensure the attainment of the first best optimum.[12]

Intuitively, the reason for these results stems from the fact that the base (total output) upon which τ is levied exceeds that to which the investment credit z is applied. Accordingly, a unit increase in the latter requires a smaller offsetting increase in the former, in order to maintain an unchanged equilibrium growth rate; see (28'). The base of the capital income tax is sufficiently large relative to investment, so that despite the fact that $(d\hat{\tau} < d\bar{z})$, the additional tax revenues collected exceed the additional outlays necessary to finance the investment credit. For this reason the budget now generates a surplus and consumption should in fact be subsidized (along with investment). Other cases, where say ω is set arbitrarily and z is correspondingly optimized can be discussed similarly.[13]

The optimal tax structure changes significantly when $\sigma < 1$ so that the private agent underpredicts the link between the growth of government expenditure and the benefits they provide. We begin our treatment of this case, by considering the more familiar case where the only impact of government expenditure is on the productivity of existing capital.

(ii) $\boldsymbol{\sigma < 1, \alpha' > 0, h' = 0}$**:** In this case, the optimal combination of τ and z which will attain the first best optimal growth rate is[14]

$$\frac{1-\hat{\tau}}{1-\hat{z}} = \frac{\alpha - e}{\alpha - \alpha'(1-\sigma)} \tag{29}$$

which for any arbitrarily set value of the investment tax credit implies

$$\hat{\tau} = \frac{e[1-\alpha'(1-\sigma)]}{\alpha - \alpha' e(1-\sigma)} + \left(\frac{\alpha - e}{\alpha - \alpha' e(1-\sigma)}\right)\bar{z} \tag{30a}$$

Substituting this solution into the government budget constraint (26'), the corresponding value of the optimal consumption tax must satisfy

$$\hat{\omega}\hat{\mu} = \frac{\alpha'(1-\sigma)(\alpha - e)}{\alpha - \alpha' e(1-\sigma)}(1-\bar{z}) - \hat{\mu}\bar{z} \tag{30b}$$

Suppose for the moment that $\bar{z} = 0$. As long as $\sigma < 1$, the tax on capital will generate insufficient revenue to meet expenditures and will need to be supplemented by a strictly positive consumption tax in order to balance the budget. This is because by underestimating the linkage between his own investment and the growing level of government expenditures, the agent overconsumes and underinvests. Reducing the tax on capital (relative to Case (i)) and taxing consumption corrects for this distortion. The appropriate tax mix to offset this distortion depends not only upon σ, but also upon how far the actual level (e) of government expenditure deviates from the first best optimum ($\tilde{e}$).

As before, the introduction of an investment credit $\bar{z}$ will raise the growth rate and will therefore require a higher tax on capital income for the optimal growth rate to be sustained. As noted above, if $\bar{z}$ is sufficiently large, because of the differential tax base, this will generate surplus government revenue, enabling consumption eventually to be subsidized rather than taxed.

It is possible to use the explicit results just discussed in Case (ii) to extrapolate to the more general case, where in addition government expenditure reduces the costs to investment ($h' < 0$). For the given tax structure determined in (30), if $h' < 0, h'' > 0$, the equilibrium growth rate in the decentralized economy $\hat{\phi}$, will now be less than that of the first best optimum, $\tilde{\phi}$. The growth rate in the former economy will need to be raised and for given $\bar{z}$, this will require τ to be reduced below the rate given in (30a). Accordingly, the revenues from capital income taxation will be lower, requiring the consumption tax to be raised above that indicated in (30b). Intuitively, the lower investment costs reflected in $h' < 0$, provide an added benefit to investment and increase the optimal growth rate. The reduction in the capital tax together with the higher consumption tax is necessary in order to induce the appropriate reallocation of output in the decentralized economy.

(iii) $\alpha' > 0, h' < 0, e$ set optimally: As a third example, we turn to the most important case where the fiscal authority sets its expenditure optimally ($\tilde{e}$ in accordance with (17). Imposing this condition on (13"), leads to

$$\frac{1}{2}(1-2\gamma)h(\tilde{e})\hat{\phi}^2 + \left[(1-\gamma)+h(\tilde{e})\rho\right]\hat{\phi}$$

$$+\rho - \frac{(1-\tau)}{(1-z)}\left[\alpha(\tilde{e}) - \alpha'(\tilde{e})\tilde{e}(1-\sigma)\right] + \left[1-\alpha'(\tilde{e})\right]\tilde{e}(1-\sigma) = 0 \qquad (13''')$$

Comparing this with (13) it is readily seen that the capital income tax rate and rate of credit on investment will now replicate the growth rate in the first-best equilibrium if and only if

$$\frac{1-\hat{\tau}}{1-\hat{z}} = \frac{\left(\alpha(\tilde{e})-\tilde{e}\right)+\left(1-\alpha'(\tilde{e})\right)\tilde{e}(1-\sigma)}{\alpha(\tilde{e})-\alpha'(\tilde{e})\tilde{e}(1-\sigma)} \qquad (31)$$

Solving for $\hat{\tau}$ and recalling the government budget constraint (26), the optimal integrated fiscal policy (for given investment tax credit $\bar{z}$) is summarized by

$$\hat{\tau} = \frac{\tilde{e}\sigma + \bar{z}\left[(\alpha(\tilde{e}) - \tilde{e} + (1-\alpha'(\tilde{e})\tilde{e}(1-\sigma)\right]}{\alpha(\tilde{e}) - \alpha'(\tilde{e})\tilde{e}(1-\sigma)} \qquad (32a)$$

$$\hat{\omega}\hat{\mu} = \frac{\tilde{e}\left[(\alpha(\tilde{e}) - \alpha'(\tilde{e})\tilde{e}\right](1-\sigma)}{\alpha(\tilde{e}) - \alpha'(\tilde{e})\tilde{e}(1-\sigma)}(1-\bar{z}) - \bar{z}\hat{\mu} \qquad (32b)$$

where

$$\alpha'(\tilde{e}) - \frac{h'(\tilde{e})}{2}\tilde{\phi}^2 = 1 \qquad (32c)$$

Beginning with a case of a zero investment credit ($z = 0$), there is a clear tradeoff between the capital income tax $\hat{\tau}$ and consumption tax $\hat{\omega}$, which depends critically upon the agent's perception of government expenditure as parameterized by σ. By setting e optimally, the fiscal authority is in effect taking full account of all productivity externalities. In the case where $\sigma = 1$, (Case (i) discussed previously), the agent sees the world "correctly" and the optimal expenditure should be fully financed by a tax on capital income alone. At the other extreme, where $\sigma = 0$ and the agent perceives no linkage between his own decisions and the growth of government expenditure, the distortion is a pure consumption distortion and should should be corrected by a tax on consumption alone. For reasons discussed previously, these results are subject to some modification, in the case where the government chooses to impose an arbitrary investment credit.

4. Conclusions

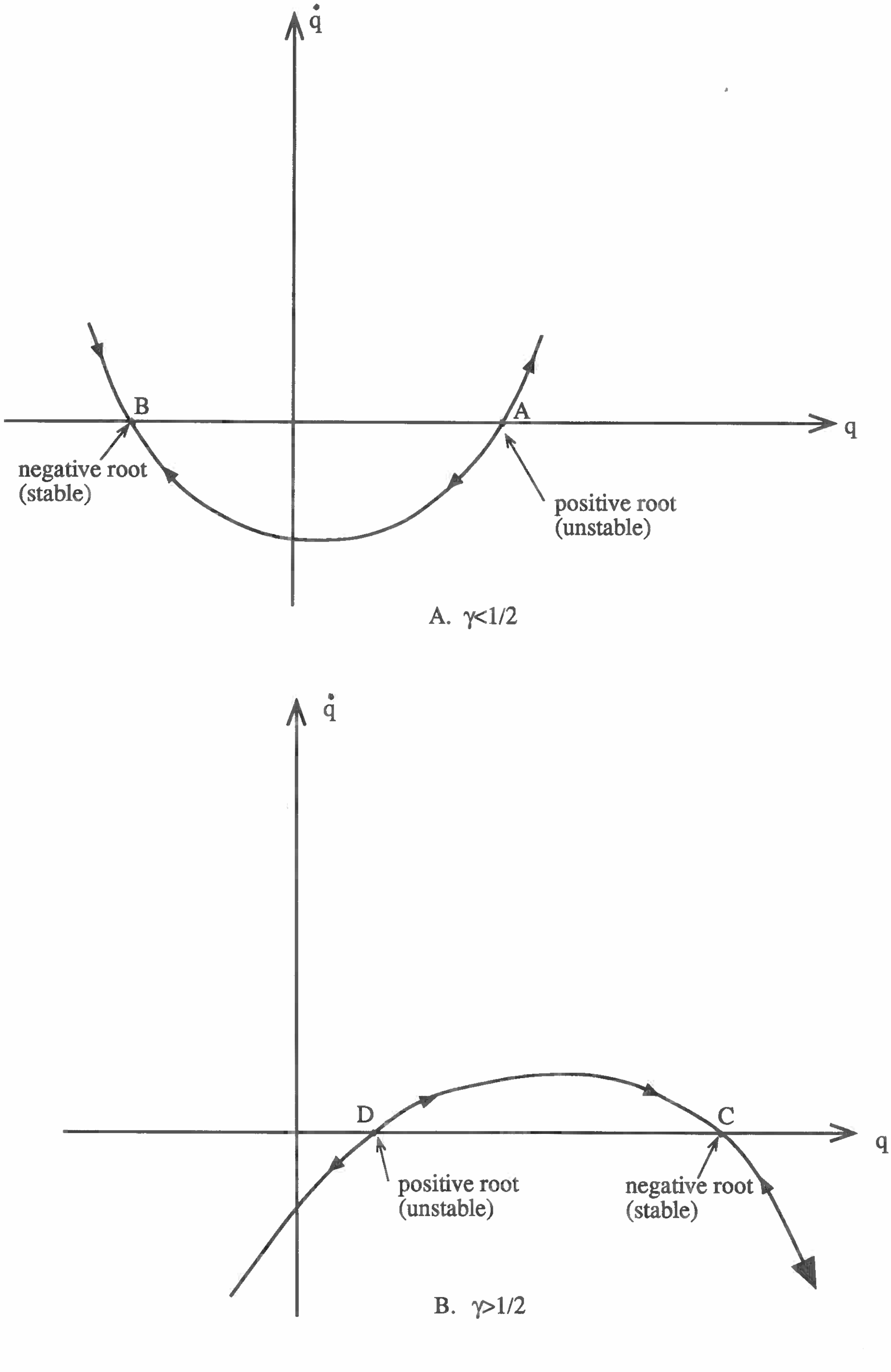
This paper has developed a one-sector endogenous growth model embodying the Tobin q theory of investment. This has been done by postulating investment to take place by means of a convex cost of adjustment function, rather than being determined residually as previous models have assumed. One consequence of this is that despite the linear technology, it is possible for no steady growth path to exist. This may occur if $\gamma > 1/2$, or equivalently if the intertemporal elasticity of substitution exceeds two. In this case the possibility arises for the returns to capital to sufficiently dominate the returns to consumption, irrespective of the price of capital, so that no long-run balanced growth equilibrium can exist where these two activities yield the same rate of return.

Using this framework we have focused on fiscal issues, in particular analyzing circumstances where the government uses its tax revenues to finance productive public expenditures. In contrast to previous models, these expenditures have been assumed to impact on both the productivity of the existing capital stock, as well the costs of installing new capital. The detailed results with respect to (i) the impact of government expenditure; (ii) the effects of alternative tax changes and (iii) the optimal tax configuration are discussed in the text and need not be repeated. Instead, we shall conclude by addressing two issues.

First, how important is the introduction of costs of adjustment in analyzing the effects of fiscal policy? This paper suggests that they do play an important role. In the first best equilibrium of the central planner, it is no longer the case that maximizing the growth rate is equivalent to maximizing welfare, as it is when capital may be accumulated costlessly; see Barro (1990), Turnovsky (1993). The welfare-maximizing level of government expenditure will now typically be less than the growth-maximizing level. This in turn has implications for the optimal structure of taxation, both with respect to the absolute level and the relative mix between the alternative sources of revenue.

Secondly, our discussion bears upon the more specific question of the optimal taxation of capital, an issue which has received extensive discussion, particularly in the context of a Ramsey economy. One important proposition is the result due to Chamley (1986) which argues that asymptotically the optimal tax on capital in such an economy should converge to zero.[15] The present framework differs from this traditional analysis in two key respects. First, the Chamley analysis did not consider any externalities from government expenditure. But setting $\alpha' = h' = 0$, we still find it is optimal to finance expenditure fully by a tax on capital, in accordance with (28) or (28'). This reflects the second, and more important difference in our formulation, namely in the specification of government expenditure. In the standard neoclassical growth model, such as that assumed by Chamley (1986), the *level* of government expenditure is assumed to be set exogenously. By contrast, by specifying government expenditure as a fraction of output, in the present analysis its level is no longer exogenous, but instead, is proportional to the size of the growing capital stock. The decision to accumulate capital by the private sector leads to an increase in the supply of future public goods. In general, if the private sector treats government spending as independent of its investment decision (when in fact it is not), a tax on capital is necessary to correct this distortion and thereby internalize the externality.

An important exception to this arises, however, when the government: (i) sets its expenditure ratio optimally, and (ii) the private agent perceives no link between its investment decision and the rate of government spending. Provided there is no investment credit, all expenditure should be financed solely by a tax on consumption, with capital being completely untaxed. In that case a Chamley type result again obtains.[16]



q̇
B
A
q
negative root
(stable)
positive root
(unstable)
A. γ<1/2
q̇
D
C
q
positive root
(unstable)
negative root
(stable)
B. γ>1/2

Fig 1

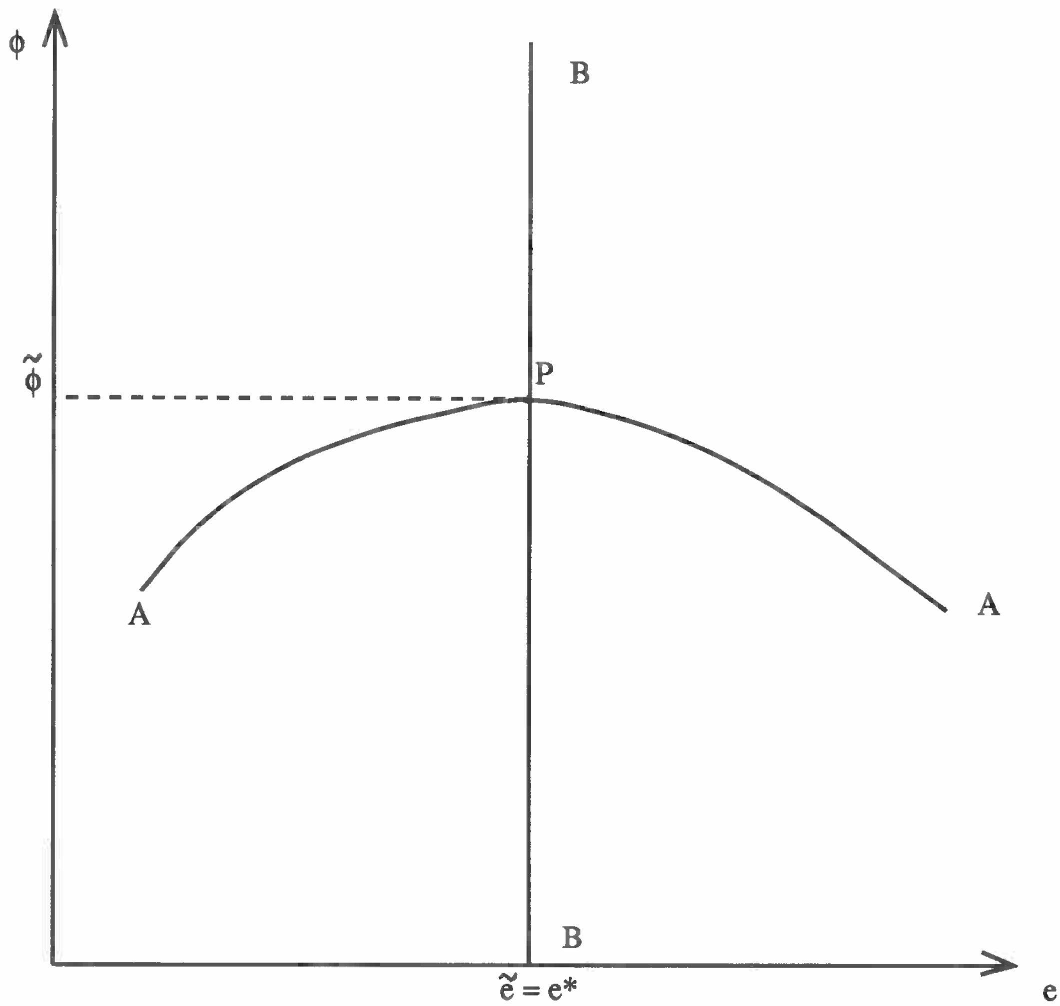


Fig 2

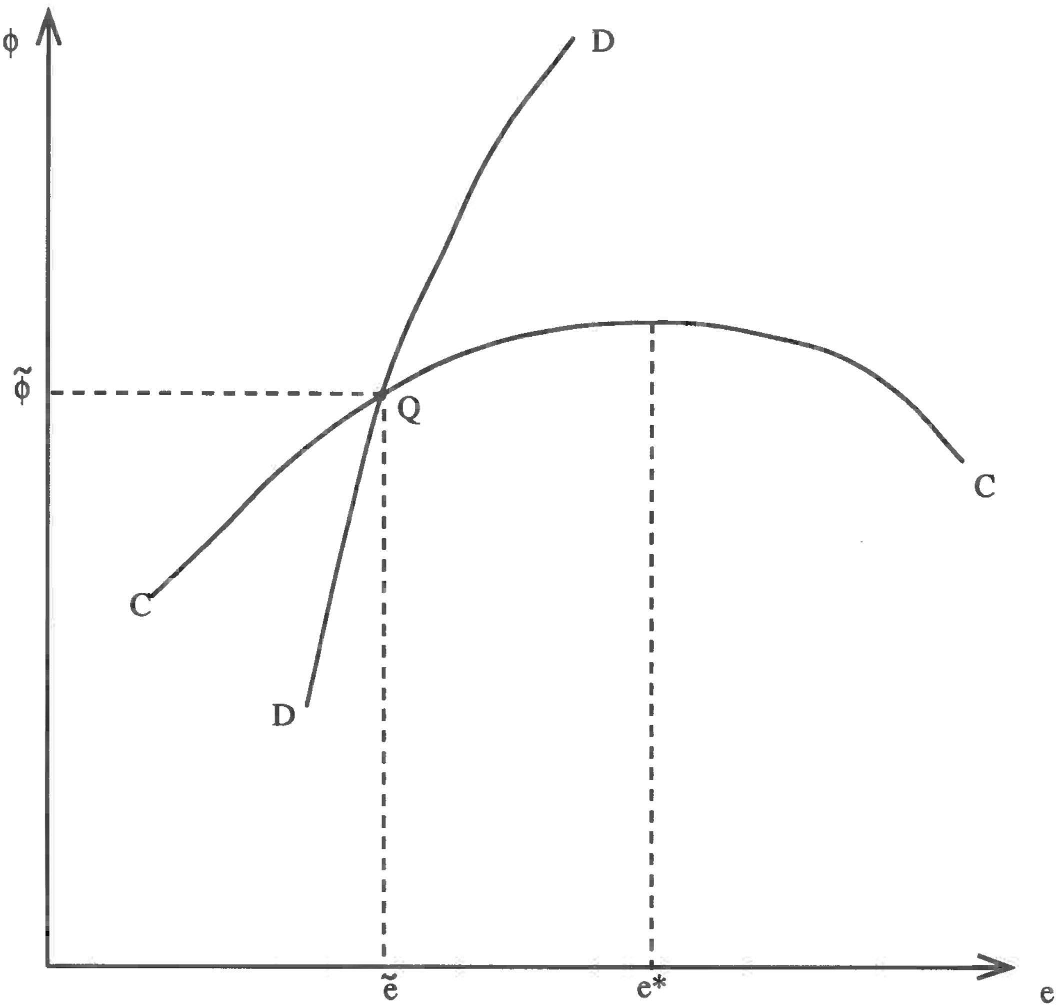


Fig 3

FOOTNOTES

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[1]All of these papers differ in terms of the specific aspects they consider. Barro (1990) focuses on the expenditure side, while Rebelo (1991) and Jones, Manuelli and Rossi (1991, 1993) address taxation. Pecorino (1993) addresses taxation issues in a two sector application of the Lucas (1988) model. Turnovsky (1993) argues that in general the expenditure and taxation decisions are interdependent and develops a more integrated approach to an overall optimal fiscal policy.

[2]Barro (1990) presents the most detailed discussion of productive government expenditure in an endogenous growth model, while Turnovsky (1993) deals more with government consumption expenditure. Both these models treat investment as being residually determined. There is also the issue of whether government expenditure on infrastructure should be modeled as stock or a flow. Both Barro and Turnovsky treat it as a flow, as do Aschauer (1988) and Turnovsky and Fisher (1994) in their analyses based on the neoclassical growth model. Arrow and Kurz (1970) and Baxter and King (1993), on the other hand, introduce government expenditure as a capital good. The distinction between these formulations is more acute in the short run, where public capital, along with private capital, is constrained to evolve gradually. The difference is much less substantive in the long run, when stocks are free to adjust along with flows. Since the focus of this analysis is on the latter, our treatment of government expenditure as a flow is satisfactory for our purposes.

[3]Strictly speaking, the logarithmic utility function emerges as $\lim_{\gamma \to 0}\left[\left(C^{\gamma} - 1/\gamma\right)\right]$. This function differs from (1a) by the subtraction of the term in the numerator and the two forms of utility function have identical implications.

[4]Note that this specification implies that in the case where disinvestment may occur $I<0$, $\Phi(I,K)<0$ for low rates of disinvestment. This may be interpreted as reflecting the revenue obtained as capital is sold off.

[5]For example in the context where government expenditure is on a consumption good, (G) specifying the ratio $g \equiv G/K$ is not unreasonable and is in general consistent with sustaining a steady-state growth path.

[6]This relationship between the transversality condition and a positive equilibrium consumption-capital ratio was originally pointed out by Merton (1969) in a stochastic growth context.

[7]The ambiguity stems from the fact that the curvature properties of the DD function involve the third derivatives, α''', h'''. The fact that DD is steeper than CC at the point Q is seen by substituting (17) into (16) and comparing the resulting slope with (18).

[8]See e.g. Turnovsky (1993).

[9]Some discussion of alternative specifications of (21) are provided by Barro (1990). In terms of our notation most of his attention is directed to assuming $\sigma=0$ although some remarks on the alternative polar case $\sigma=1$ are also given.

[10]We have written the crucial equations (13) and (13") in forms which will faciliate the comparison of their respective roots for $\tilde{\phi}, \hat{\phi}$, in the cases we shall consider.

[11]An increase in government expenditure can be easily shown to raise the equilibrium growth rate if $h'=0$. While it is less straightforward to establish this response when $h'<0$, intuitively one would expect it to continue to be so, particularly in an already growing economy.

[12]Saint-Paul (1992) discusses in some detail the role of an investment credit in an endogenous growth model.

[13]We should point out that precisely the same optimal tax policy as set out in (28') applies independent of σ, as long as $\alpha'=h'=0$ and government expenditure has no direct impact on the behavior of the private sector. This corresponds to the traditional treatment of government expenditure as representing a pure drain on the economy.

[14]We shall assume $\alpha - \alpha' e(1-\sigma) > 0$, a condition which is almost certainly assured by the assumptions we have imposed on $\alpha(e), \sigma$.

[15]This issue is discussed at further length by Lucas (1990).

[16]The following degenerate type of Chamley result may also obtain in the present endogenous growth framework. If the government continues to maintain its level of expenditures fixed in absolute terms, then if the economy follows a path of steady endogenous growth, the share of government expenditure expenditure as a fraction of output $e/\alpha \to 0$, implying that the optimal tax on capital $\hat{\tau}$ will tend to zero as well. This, however, would be an economy with a negligibly small government sector and therefore one of limited interest in a world of ongoing growth.

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