WORKFARE VS. WELFARE: INCENTIVE ARGUMENTS FOR WORK REQUIREMENTS IN POVERTY ALLEVIATION PROGRAMS

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Workfare vs. Welfare: Incentive Arguments for Work Requirements in Poverty Alleviation Programs*

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Abstract

Whether those who claim benefits should face a work requirement has been an issue of long-standing social concern. Important examples of schemes which require work are the Californian workfare program, Indian food security schemes and the English Poor Law of 1834. We present two arguments for demanding work for benefits: first, a work requirement can screen the truly needy from those who are not in need of support and second, it can provide incentives for people to invest in skills which enable them to avoid poverty. In the context of a simple model of a target population with two ability types we find conditions under which a work requirement reduces the costs of poor relief, and those when it does not. We concentrate on a case when work done in return for benefits has no social value, showing that even if this is true, work requirements may be a valuable policy tool.

1. Introduction

While many would follow Samuel Johnson in his assertion that "a decent provision for the poor is a true test of civilization", there is much debate about what form poverty relief should take. Particularly controversial is the claim that in order to provide the 'appropriate' incentives, recipients of relief should be required to work.

Historically one can find many examples of such schemes. Perhaps the most notorious example is the English system instituted after the Poor Law of 1834, in which poor relief was granted through residence in a workhouse. Workfare schemes, however, are by no means simply relics of the past. They remain popular in both developed and less developed countries today. In the U.S., for example, the Californian workfare system demands that a claimant enroll in either a training or work program in order to receive benefits. Similarly, current practice in India relies heavily on public works projects as a tool for providing food security.

It is the object of this paper to explore the incentive arguments that possibly lie behind requiring work in exchange for benefits. We identify two roles for such requirements: first as a means of screening the truly deserving from the rest; and second as a means of providing incentives to avoid poverty. Familiar notions from the economics of incentives are shown to provide insights into modern day social policy as well as historical debates.

Accepting a moral commitment to the poor, we shall ask how such a commitment might best be discharged. John Stuart Mill characterized the problem as "how to give the greatest amount of needful help, with the smallest encouragement to undue reliance on it". Mill's emphasis on needful help is important. Financial economy demands that relief be given only to those who are truly poor. It is therefore necessary to establish a system by which the truly deserving are identified and assisted. One approach sets up an administrative body which investigates the circumstances of applicants and decides whose claims should be accepted. However, if the number of potential claimants is large and the existing administrative infrastructure weak, such a solution may be prohibitively costly. In such situations, it may be better to make the system "self targeting." This can be done by laying down conditions for claiming relief which mean that only the truly needy present themselves. One possible condition is to require individuals to work.

This logic clearly lay behind British administrators reliance on public works to relieve famines in nineteenth century India, as the following excerpt from the report of the 1880 Famine commission demonstrates:

> "where limited numbers have to be dealt with, and there is a numerous and efficient staff of officials, it may be possible to ascertain by personal inquiry, the circumstances of every applicant for relief sufficiently for the purposes of admitting or rejecting his claim. But in an Indian famine, the government has to deal ... with millions of people, and the official machinery at its command ... will inevitably be inadequate to the task of accurately testing the individual necessities of so great a multitude. Some safeguards ... are best found in laying down certain broad self acting tests by which necessity may be proved which may ... entitle to relief the person who submits to them. The chief of these tests and the only one... it is ordinarily desirable to enforce, is the demand of labour... in return for a wage sufficient for the purposes of maintenance..."[1]

However, against the gains from screening must be weighed the fact that if individuals are required to work to obtain benefits, they will

1 Quoted by Drèze (1986).

not be able to work in the private sector. Thus, any contribution which they would have made towards their own sustenance will be lost. Such losses must be measured against the screening gains. This trade-off emerges precisely in our formal analysis. A further problem is the possibility that costs are made so high that the poor prefer not to participate and hence the objective of alleviating poverty is not met. Some authors have argued that this was a problem with Indian famine policy in the early nineteenth century. Ira Klein, for example, reports that "many in the last stages of desperation feared the relief works ... and took to aimless wandering .... ending as corpses littering the lanes."[2]

A second issue which pervades the social policy discussion is that of the origins of poverty. Are the poor those who early in life fail to invest in enough human capital for their later needs or those for whom bad luck of one kind or another has prevailed? If the former is true then poor relief may have an effect on an agent's behavior when choosing how much to invest for future life. Hence, to reiterate the words of Mill, the existence of poor relief might engender "undue reliance" on it.

This old idea has reemerged recently in discussions of US social policy. In his controversial analysis of the past 30 years of U.S. expenditures on poor relief, Charles Murray (1984) argues that increased expenditure on social programs has reduced the incentives to avoid poverty and thereby created a dependence on State support.

To avoid this problem, poor relief must somehow be made a relatively less attractive option. This logic was accepted by the 1834 Poor Law

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2 See Klein (1984).

Commissioners' proposal that the poor be placed in workhouses. Such institutions would provide a regime of honest toil in frugal surroundings in exchange for the satisfaction of basic needs. The idea was that "condition of the able-bodied pauper be "less-eligible" - desirable, agreeable, favorable - than that of the "lowest class" of independent labourer"[3]. The Poor Law regarded this as essential: "It is only... by making relief in all cases less agreeable than wages, that anything deserving the name of improvement can be hoped for"[4].

Hence the Poor Law Report displayed a recognition of the incentive gains from work requirements. By requiring work in exchange for relief, the incentive to avoid having to claim relief was enhanced. Below, we shall provide a detailed analysis of this argument.

Demanding work requirements may have the beneficial "side-effect" that public works are constructed which have social value. In order to focus directly on the incentive arguments, we ignore completely this aspect of workfare programs. We do not think that this consideration is unimportant. However, by ignoring it, the insights obtained can be sharper. In addition, it is clear that if work requirements are desirable without the work done being socially valuable, they are certainly desirable in more realistic cases.

The analysis undertaken in this paper relates to that in a number of other papers. Screening problems were analysed using similar techniques to those in this paper by Mussa and Rosen (1979) and Baron and Myerson (1982). Both examined aspects of monopoly. Stiglitz (1982) looks at

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3 Himmelfarb (1984) page 163.

4 Quoted by Himmelfarb (1984) page 165.

pooling and separating equilibria in optimal taxation. Our model which focuses on the incentives to acquire skills to avoid poverty, is a cousin of those in Holmstrom (1979) and Harris and Townsend (1979), who consider moral hazard problems. Policies focused on poverty alleviation have also been looked at before in Besley and Kanbur (1988) who consider food subsidies as a means of poor relief. Finally, our paper relates to a number of those which have examined redistribution in cash rather than kind. This literature shows how sorting of individuals can be achieved by offering "in kind" packages which reflect preferences of different types. Nicholls and Zeckhauser (1982), Dixit (1988) and Blackorby and Donaldson (1988) have considered these. A work requirement in poor relief trades on the fact that different individuals have different opportunity costs of leisure at the margin.

The structure of our paper is as follows. In the next section, we set up our basic model. Section three examines the first best poverty alleviation program. In section four we consider the case in which individuals' types, which we characterize generically as their abilities, are unobservable. We show when work requirements can lower the costs of poor relief even when the work has no direct social value. Section five looks at a different aspect of the problem. Abilities are now assumed observable, but are chosen endogenously by individuals who face different costs of investing in skills. Here, work requirements might also play a role by encouraging skill formation. Section six sets our analysis in context by considering how it relates to other views of poverty generation and welfare dependence.

2. The Model:

Consider a population of n individuals each of which has some ability $a \in \{ a_L, a_H \}$ where $0 \leq a_L < a_H$ and 'H' stand for high and 'L' stands for low. Let γ denote the fraction of the population with ability a_L.

As in the literature on optimal income taxation problems, we shall take ability to be perfectly reflected in an individual's wage rate. Hence, without loss of generality, a_i can be taken to be the wage rate of an individual of type i ($i \epsilon \{L, H\}$). Each individual has the same preferences over income y and work given by u(y, l). In addition, each individual has an identical endowment T, of leisure. We shall work thoughout with a case in which the utility function has a special form: $u(y,l) = y - h(l)$ where h(.) is increasing and strictly convex. This simplification greatly sharpens the insights available from the model without affecting the character of the results. We discuss more general specifications of preferences in section 6 below.

The government is concerned to provide both ability types with a minimum income denoted by z. Giving all of its citizens at least this income level will be regarded as a binding constraint upon social policy. We shall ignore the revenue raising implications of the budget required to effect this end in order to focus directly on poverty alleviation issues. We have in mind therefore, a world in which the two types of individuals considered here form a target population; expenditures on whom are financed by taxation of the remainder of the population or aid flows.

A poverty alleviation program (PAP) is a pair $\{b_i, c_i\}$ for each ability type where b_i denotes a lump sum payment to individuals of ability a_i and c_i a cost in terms of forgone leisure (i.e. a work requirement) needed to obtain this payment. The cost of this program is

$$n[\gamma b_L + (1-\gamma)b_H]. \qquad (2.1)$$

where n is the size of the target population. The government's objective will be to minimize this cost subject to the constraint that each individual obtains at least an income of z.

Individuals of ability a_i must choose whether or not to accept the $[b_i, c_i]$ pair intended for them. Even if they do accept it, they may continue to supply labor to the private sector. Let $l(b, c, a)$ denote the labor supply function of an individual with ability a who accepts the pair (b, c). It is defined by:

$$l(b, c, a) = \arg\max \{ u(b + al, l + c): l \in [0, T-c]\} \qquad (2.2)$$

where we have omitted the argument T for convenience. Given this supply, total income from private sector earnings is given by

$$y(b, c, a) = al(b, c, a) \qquad (2.3)$$

and the individual's indirect utility is given by

$$v(b, c, a) = u(y(b, c, a) + b, l(b, c, a) + c). \qquad (2.4)$$

Clearly, an individual will accept the pair [b, c] if and only if he is better off doing so i.e. if and only if $v(b, c, a) \geq v(0, 0, a)$.

The special utility function that we have adopted has some implications for the optimal labor supply and indirect utility functions which we spell out below. Define $\bar{l}(a)$ to be the amount of labor an individual of type a would supply to the private sector if he did not participate in the program[5], i.e.

$$a = h'(\bar{l}(a)). \tag{2.5}$$

It is easy to verify that

$$l(b, c, a) = \begin{cases} \bar{l}(a) - c & \text{if } c \leq \bar{l}(a) \\ 0 & \text{otherwise.} \end{cases} \tag{2.6}$$

Hence, an individual would cease to undertake any private sector work if he were required to supply more than $\bar{l}(a)$ units of labor in the PAP. Given our special form for the utility function, labor supply and hence private sector earnings are independent of b, i.e. there is no income effect. The indirect utility function can therefore be written as:

$$v(b, c, a) = b + y(c, a) - \psi(c, a) \tag{2.7}$$

where $\psi(c, a) \equiv h(l(c, a))$.

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5 This assumes that $a_H < h'(T)$.

In order to make the problem interesting, we shall make an assumption to ensure that in the absence of intervention there is only one group (viz. the low ability types) which is poor. Hence

$$\underline{\text{A1}}: \quad y(0, a_H) > z > y(0, a_L) \tag{2.8}$$

This completes the description of the basic model. Before we turn to a characterization of the optimal poverty alleviation program with observable and exogenous abilities, two important features of the model should be noted. First it takes a particular view of how poverty is generated. Individuals are poor because they are deficient in earnings generating ability. This is not the only possible story. Individuals may, for example, have identical abilities but differ in their preferences between income and leisure. This is discussed further in section 6 below. Second, we have taken a non-welfarist definition of poverty. To be poor in the sense of this model is to have insufficient income. We do not define poverty as having too little utility. It follows that the need to obtain a certain income level, rather than a utility level, is the binding social policy constraint. This, we believe, best characterizes what most policy makers mean when they talk about giving relief from poverty.

3. Observable and Exogenous Abilities

When a_i is observable to the policy maker then it is straightforward to make sure that each type of individual receives the benefit package intended for him. The policy maker does not, however, have complete freedom in his choice of a PAP. As policy makers in 19th century Britain and India found to their cost, if the PAP is made too arduous even the very poor may choose not to participate. Thus the PAP must offer individuals at least their no-intervention utility levels i.e. participation must be voluntary.

The policy maker's problem is therefore to choose a $\{b_i, c_i\}$ pair for each type, which minimizes the costs of poor relief and satisfies two constraints. First, individuals must be willing to participate and second, those with ability a_L must escape poverty. These constraints are denoted below by VP_i (standing for "voluntary participation by type i") and PA (poverty alleviation). More formally, we have to solve the following problem, referred to hereafter as Problem I.

$$\min_{\{b_i,\ c_i\}\ i=H,L} \quad n[\gamma b_L + (1-\gamma)b_H]$$

$$\text{s.t.} \quad v(b_i, c_i, a_i) \geq v(0, 0, a_i) \quad i\varepsilon\{L, H\} \qquad VP_i$$

$$b_L + y(c_L, a_L) \geq z \qquad PA$$

Note that since VP_H implies that high ability individuals get an income level at least as big as their no intervention income, we can ignore the constraint that high types get at least z[6].

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6 $\quad VP_H \Rightarrow b_H + y(c_H, a_H) \geq z$

<u>Proof</u>: $VP_H \Rightarrow b_H + y(c_H, a_H) - \psi(c_H, a_H) \geq y(0, a_H) - \psi(0, a_H)$

$$\Rightarrow b_H + y(c_H, a_H) \geq y(0, a_H) + \psi(c_H, a_H) - \psi(0, a_H)$$

$$\geq y(0, a_H) \geq z \qquad \square$$

The solution to Problem I is obvious. High types should be offered no benefits. Low types should be offered just enough income to get them to the poverty line. They should not be required to engage in public work, for this will reduce their private sector earnings. We have, therefore, the following proposition:

<u>Proposition 1</u>: Let $\{b_i^*, c_i^*\}_{i=\{L, H\}}$ solve Problem I. Then

$$b_H^* = c_H^* = c_L^* = 0$$

and

$$b_L^* = z - y(0, a_L).$$

While this proposition is quite obvious, it is useful to state it as a benchmark for the subsequent analysis. It shows that if earnings abilities are both observable and exogenous then schemes with work requirements would be unnecessary. Of course, if the labor time taken by the State in the form of c_i were socially more productive than the shadow value of lost labor time in the hands of private agents, then there might be a case for collecting "payment" for poor relief in the form of "work for benefits" schemes. If this is not true, however, then there is no case for making poor relief costly to the claimants. The object of the next two sections is to show when this conclusion can be overturned. In the first instance, the ability levels of the two parties will be assumed unobservable. Thereafter, we shall examine a case in which abilities are determined endogenously by a process of skill investment which is unobservable to the policy maker.

4. Work Requirements when Abilities are Unobservable

The "first best" solution could not be implemented if abilities were unobservable. High ability individuals would simply masquerade as low ability types and claim the benefit. One way around this problem would be to audit individuals' incomes randomly and punish those found to have both claimed the benefit and enjoyed private sector income in excess of $y(0, a_L)$. In practice such schemes are employed. However in some contexts, especially in LDC's, income tests are administratively impossible and even in developed countries they are subject to difficulties: if "moonlighting" cannot be monitored, agents might claim benefits and then continue to work in the informal sector. Here, we shall focus on another solution to the problem via work requirements. In LDC's, it may be the only way of sorting the "types" of individuals, but even in developed countries, it may be regarded as a possible supplementary tool to random auditing of individuals. We shall, however, focus on the case where income is completely unobservable in order to home in directly on the argument for work requirements.

When abilities are unobservable we must impose the requirement that the PAP be incentive compatible, so that each agent chooses the pair $\{c_i, b_i\}$ intended for his type. Formally, the additional constraints are

$$v(b_H, c_H, a_H) \geq v(b_L, c_L, a_H) \qquad (IC_H)$$

and

$$v(b_L, c_L, a_L) \geq v(b_H, c_H, a_L) \qquad (IC_L)$$

i.e. that neither agent finds it worthwhile to claim the package intended for the other.

Thus the policy problem is now to choose a PAP to minimize costs subject to VP_H, VP_L, PA, IC_H and IC_L. We shall refer to this as Problem II. The addition of the two incentive constraints greatly complicates the problem and before we can solve it, we must simplify it. As was mentioned in the introduction, there is now a large technical literature on the solution of screening problems. The problem which we will solve here, however, differs from the standard screening problem as outlined, for example, in Matthews and Moore (1987). First, the standard problem has only incentive compatability and voluntary participation constraints. In our problem we also have the poverty alleviation constraint to worry about. Second, in the standard problem each individual's reservation utility is identical. In Problem II, however, high ability types obtain higher utility levels than low types in the absence of intervention. These two differences generate some additional complications which we deal with below.

We begin our analysis of Problem II by noting the following result.

<u>Lemma 1</u>: Let $\{b_i^*, c_i^*\}_{i=L,H}$ solve Problem II

then $c_H^* = 0$.

<u>Proof</u>: See Appendix

This says that the optimal PAP does not require high types to do any work. This makes good intuitive sense. The problem which arises when types are unobservable is that high earners masquerade as low earners.

Since high earners have a high marginal value of leisure (due to their high wage rate) separation of the types can be achieved by attaching work requirements to low earners. Little, however, would be achieved by making high earners work.

Lemma 1 permits two immediate simplifications. First we can reduce the number of choice variables by one and second we can replace VP_H with the simple constraint that b_H is non-negative. Our next result permits a further simplification.

Lemma 2: Let $(b_L, c_L, b_H, 0)$ satisfy IC_H with equality and $b_H \geq 0$, then it satisfies both IC_L and VP_L.

Proof: See Appendix.

Thus if high types are just indifferent between collecting b_H and masquerading as low types, low types must weakly prefer their own pair (b_L, c_L) to $(b_H, 0)$. This is a standard result for this type of screening problem.

The above result suggests consideration of a 'relaxed' problem of the form, minimize costs subject to IC_H, PA and b_H non-negative. By Lemma 2, provided that solutions to this problem satisfy IC_H with equality, they will be feasible for Problem II. Unfortunately, however, there may exist solutions to this 'relaxed' problem which do not satisfy IC_H with equality and hence do not necessarily satisfy IC_L and VP_L. We must therefore depart a little from standard procedures.

It is easy to verify that if IC_H does not hold with equality in the 'relaxed' problem, b_H is necessarily zero. When b_H is zero, IC_L is equivalent to VP_L. This suggests that we should try to come up with an additional constraint on the relaxed problem which guarantees that solutions always satisfy VP_L. To this end define $\bar{c}_L$ to be that work requirement which would make a low ability type indifferent between participating in the scheme and receiving z and not participating, i.e. $v(0, 0, a_L) = v(z, \bar{c}_L, a_L)$. We now have:

Lemma 3: Let (b_L, c_L) satisfy PA and let $c_L \leq \bar{c}_L$, then (b_L, c_L) satisfies VP_L.

Proof: See Appendix

We shall define Problem III to be the relaxed problem subject to the additional constraint $c_L \leq \bar{c}_L$, i.e.

$$\min_{(b_L,\, c_L,\, b_H)} \quad n[\gamma b_L + (1-\gamma) b_H]$$

$$\begin{aligned}
\text{subject to} \quad & v(b_H, 0, a_H) \geq v(b_L, c_L, a_H) && (IC_H) \\
& b_L + y(c_L, a_L) \geq z && (PA) \\
& c_L \leq \bar{c}_L && \\
& b_H \geq 0 &&
\end{aligned}$$

Finally, we have

Lemma 4: $(b_L^* \;\; c_L^* \;\; b_H^*)$ solves Problem II if and only if it solves

Problem III.

Proof: See Appendix.

Lemma 4 tells us that to characterize the solution to Problem II, it is sufficient to analyze the solution to the much simpler Problem III. It is to this which we now turn. A cursory examination of Problem III reveals that PA must hold with equality. Otherwise, the costs of poverty alleviation could be reduced by lowering b_L without affecting the other constraints. Hence the equation

$$b_L = z - y(c_L, a_L) \tag{4.1}$$

uniquely defines b_L, given c_L. Now define $\hat{c}_L$ to be that work requirement which makes high types just indifferent between the status quo (i.e. (0, 0)) and receiving $z - y(\hat{c}_L, a_L)$, that is $v(0, 0, a_H) \equiv v(z - y(\hat{c}_L, a_L), \hat{c}_L, a_H)$. Note that $\hat{c}_L$ can be greater or less than $\bar{l}(a_L)$ but is necessarily less than $\bar{c}_L$. It is clear that if $c_L < \hat{c}_L$, IC_H demands that $b_H > 0$. However, as we noted above, if the high types receive a positive benefit i.e. $(b_H > 0)$ then IC_H is satisfied with equality. If, on the other hand, $c_L \geq \hat{c}_L$ then the high type's payment need not be positive and hence optimally it should be set equal to zero. Thus

$$b_H = \begin{cases} v(z - y(c_L, a_H), c_L, a_H) - v(0, 0, a_H) & \text{if } c_L < \hat{c}_L \\ 0 & \text{otherwise.} \end{cases} \tag{4.2}$$

Equation (4.2) uniquely defines b_H for any given c_L.

Substituting (4.1) and (4.2) into the objective function we obtain the cost of poverty alleviation function

$$C(c_L) = \begin{cases} n[\gamma(z - y(c_L, a_L)) + (1-\gamma)(v(z - y(c_L, a_H), c_L, a_H) \\ \qquad -v(0, 0, a_H))] & \text{if } c_L < \hat{c}_L \\ n\gamma(z - y(c_L, a_L)) & \text{otherwise.} \end{cases} \tag{4.3}$$

The optimal work requirement minimizes $C(c_L)$ subject to the constraint that $c_L \leq \tilde{c}_L$. The optimal values of b_L and b_H can then be discerned from equations (4.1) and (4.2) respectively.

Differentiating (4.3) and making use of the fact that $v(.)$ is linear in its first argument we find

$$C'(c_L) = \begin{cases} n[-y_c(c_L, a_L) + (1-\gamma)v_c(0, c_L, a_H)] & \text{if } c_L < \hat{c}_L \\ -n\gamma y_c(c_L, a_L) & \text{if } \hat{c}_L \leq c_L \leq \tilde{c}_L. \end{cases} \tag{4.4}$$

Equation (4.4) is the key equation of this section. It makes the relevant tradeoffs in workfare vs welfare precise. If $c_L < \hat{c}_L$, then there are two effects of raising c_L. First, low ability types reduce their private sector earnings (i.e. $y_c(c_L, a_L) \geq 0$). This requires an increase in b_L equivalent to the reduction in private earnings in order to maintain low ability types' earnings at the poverty line. This increase in b_L in turn requires an increase in b_H in order to maintain incentive compatibility. This increase in costs is given by $-ny_c(c_L, a_L)$. Raising c_L reduces the incentives of high ability types to masquerade (i.e. $v_c(0, c_L, a_H) < 0$). This allows b_H to be reduced, resulting in a reduction in costs of $-(1-\gamma)v_c(0, c_L, a_H)$. The overall effect on costs depends upon the relative size of these competing effects. If, on the other hand, $c_L > \hat{c}_L$, then there is only one effect of raising c_L; namely b_L may have to be raised to maintain the low ability types at the poverty line. There is no effect on b_H since it is already zero.

Further insights can be gained by actually calculating y_c and v_c. It is necessary in doing this to distinguish between two cases. The first is where $\hat{c}_L < \bar{I}(a_L)$ i.e. the work requirement which would make the high ability types indifferent between the status quo and $z - y(\hat{c}_L, a_L)$ is less than the amount of work which low ability types would do in the status quo. In this case it is straightforward to verify that

$$C'(c_L) = \begin{cases} n[a_L - (1-\gamma)a_H] & \text{if } c_L < \hat{c}_L \\ n\gamma a_L I\{c_L \leq \bar{I}\ (a_L)\} & \text{if } \hat{c}_L \leq c_L \leq \bar{c}_L \end{cases} \tag{4.5}$$

where I denotes the 'indicator function'. It is clear from (4.5) that the optimal work requirement is $\hat{c}_L$ if a_L is less than $(1-\gamma)a_H$ and zero otherwise. Using (4.1) and (4.2) we therefore obtain the following result:

<u>Proposition 2(i)</u> Let $\{b_i^*, c_i^*\}_{i = L, H}$ solve Problem II and let $\hat{c}_L < \bar{I}(a_L)$, then (a) if $a_L < (1-\gamma)a_H$

$$(b_L^*, c_L^*, b_H^*, c_H^*) = (z - y(\hat{c}_L, a_L), \hat{c}_L, 0, 0)$$

(b) if $a_L > (1-\gamma)a_H$

$$(b_L^*, c_L^*, b_H^*, c_H^*) = (z - y(0, a_L), 0, z - y(0, a_L), 0)$$

It can also be shown in this case that[7]

$$z - y(\hat{c}_L, a_L) = a_H\hat{c}_L. \tag{4.6}$$

Hence the government pays low ability types at the high ability type wage. This makes good intuitive sense. If the government paid a wage higher than a_H then, since $\hat{c}_L < \bar{I}(a_H)$, high types would choose to work on the program and claim benefits. On the other hand, if the wage were any less than a_H, it would take more work on public projects than was strictly necessary to get low earnings ability individuals up to the poverty line.

Consider next the case where $\hat{c}_L$ exceeds $\bar{I}(a_L)$. In this case

$$C'(c_L) = \begin{cases} n[a_L I_{\{c_L \leq \bar{I}(a_L)\}} - (1-\gamma)a_H] & \text{if } c_L < \hat{c}_L \\ 0 & \text{if } \hat{c}_L \leq c_L \leq \tilde{c}_L \end{cases} \tag{4.7}$$

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7 <u>Proof</u>: $\hat{c}_L < \bar{I}(a_L) \Rightarrow y(\hat{c}_L, a_L) = a_L(\bar{I}(a_L) - \hat{c}_L)$

$$\Rightarrow z - a_L(\bar{I}(a_L) - \hat{c}_L) + a_H(\bar{I}(a_H) - \hat{c}_L) - \psi(\bar{I}(a_H))$$
$$= a_H\,\bar{I}\,(a_H) - \psi(\bar{I}(a_H))$$
$$\Rightarrow z - a_L(\bar{I}(a_L) - \hat{c}_L) - a_H\hat{c}_L = 0$$
$$\Rightarrow z - y(\hat{c}_L, a_L) = a_H\hat{c}_L \qquad \square$$

It is clear from (4.7) that the optimal work requirement lies in the interval $[\hat{c}_L, \bar{c}_L]$ if a_L is less than $(1-\gamma)a_H$. If, on the other hand, a_L exceeds $(1-\gamma)a_H$, it is less clear what should be done. Costs will be increasing in c_L until the point that $c_L = \bar{I}(a_L)$. Thereafter, they will be decreasing. This is because there will be no reduction in the low earners' private sector earnings from this point on. To find the optimal work requirement, we need to compare C(0) and $C(\hat{c}_L)$. It is easy to check that

$$C(0) - C(\hat{c}_L) = n[(1-\gamma)z - y(0, a_L)]. \tag{4.8}$$

Thus the optimal requirement will be zero if $(1-\gamma)z$ is less than $y(0, a_L)$ and will be in the interval $(\hat{c}_L, \bar{c}_L]$ otherwise. Using (4.1) and (4.2) we therefore obtain the following:

<u>Proposition 2(ii)</u> Let $\{b_i^*, c_i^*\}_{i = L, H}$ solve Problem II and let $\hat{c}_L > \bar{I}(a_L)$, then

(a) If $a_L < (1-\gamma)a_H$ or if $y(0, a_L) < (1-\gamma)z$

$$(b_L^*, b_H^*, c_H^*) = (z, 0, 0) \text{ and } c_L^* \in [\hat{c}_L, \bar{c}_L]$$

(b) If $a_L > (1-\gamma) a_H$ and $y(0, a_L) > (1-\gamma)z$

$$(b_L^*, c_L^*, b_H^*, c_H^*) = (z-y(0, a_L), 0, z-y(0, a_L), 0).$$

Together Propositions 2(i) and 2(ii) provide a complete characterization of the optimal PAP when abilities are unobservable.

There are three possible régimes. One régime (Proposition 2(i)(b) and 2(ii)(b)) has a zero work requirement so that there is a 'pooling equilibrium' in which high and low ability types claim a transfer equal to the shortfall of the low ability type's income from the poverty line. Pooling occurs when there are either very few poor people or low ability types have abilities close to the high ability types and hence the poverty gap is small. The remaining two régimes involve separating equilibria with positive work requirements. In the first case, Proposition 2(i)(a), the low ability types are effectively given enough work at the high ability type's wage, in order to avoid poverty. Low ability types continue to do some work in the private sector (specifically $\bar{I}(a_L) - \hat{c}_L$ units) and end up working the same amount in total as they would have done in the status quo. At the same time, high ability types are indifferent between claiming and not claiming b_L since they can earn a wage of a_H anyway.

In the second of the separating régimes, Proposition 2 (ii)(a), work requirements are set at a point where the low ability type does no work in the private sector. The transfer given is therefore equal to z, the poverty line. The work requirement is not unique but may range between two values. The first is $\hat{c}_L$, which is the point at which the high ability types are indifferent between claiming and not claiming when required to work. The upper bound is $\bar{c}_L$, the work requirement which makes low ability types indifferent between claiming z and not claiming. If $c_L^* = \hat{c}_L$, then since $z = a_H \hat{c}_L$ in this case, the government wage is a_H and the high types are indifferent between claiming and not claiming. If $c_L^* > \bar{c}_L$, the government wage is less than a_H and the incentive compatibility constraint for the high types is not strictly binding.

In this section we have seen that work requirements as screening devices may have a role in reducing the costs of providing poor relief. Our simple model has given an explicit form to the trade-off between losses of output and gains from screening. In passing, it is worth noting that if the social marginal product of work requirements is at least as great as a_L, then some kind of work requirement will always be desirable, since in effect there is no output loss involved. Our results say that work requirements may provide a gain even when this is not so. Hence even asking people to stand in line waiting for benefits might sometimes be socially beneficial. We turn next to a different kind of incentive argument which focuses upon the decision to acquire human capital.

5. Work Requirements with Endogenous Abilities

In this section, we shall revert to the case of observable abilities. However, we shall allow agents to "choose" their ability type ex ante. Each individual faces a non-monetary cost of becoming a high ability type, denoted by $\tau(\lambda)$ where λ represents a privately observed characteristic of an individual which can be thought of as his genetic makeup. Assume that $\tau' < 0$ and $\tau'' < 0$. Hence the cost of increasing one's ability is decreasing and concave in λ. The cost τ can best be thought of as the "psychic" or "effort" cost of acquiring a particular skill.

We shall continue to define a poverty alleviation program as in section 2. Note, however, that the first best program would now be the case in which λ was observable and b_i and c_i were set as functions of λ. Even in the second best when λ is unobservable it might be optimal to have an equilibrium in which agents revealed their λ's and work requirements and benefits depended upon λ. Instead of approaching the problem this way, we shall continue in the spirit of the previous section in order to provide continuity in the arguments. Furthermore, the kinds of PAP's which we are positing here resemble more closely the kind of schemes which are observed in practice.

Given the existence of a PAP of the kind discussed above a type λ individual will choose to expend the cost required to be a high ability type if and only if

$$v(b_H, c_H, a_H) - \tau(\lambda) \geq v(b_L, c_L, a_L) \qquad (5.1)$$

assuming that each individual can costlessly be a low ability type. Under our assumptions, there will be a unique λ^* which satisfies (5.2) with equality. All individuals with $\lambda < \lambda^*$ will "choose" to be low ability types whilst those with $\lambda \geq \lambda^*$ will incur the cost of becoming high ability individuals.

When (5.2) holds with equality it defines λ^* as a function of the the difference between the utility levels of high and low ability levels <u>ex post</u>. It is useful to denote that latter by χ. Hence $\lambda^* = \lambda^*(\chi)$ where

$$\chi(b_L, c_L, b_H, c_H) = v(b_H, c_H, a_H) - v(b_L, c_L, a_L) \qquad (5.2)$$

The function $\chi(.)$ represents the ex post utility difference between being a high and low ability individual. The function $\lambda^*(.)$ tells us the lowest type of individual who will choose to make the investment at ex post utility difference χ.[8]

We shall assume that λ is uniformly distributed in the population on the interval [0, 1]. The policy problem is therefore to minimize

$$n[\lambda^*(.)b_L + (1-\lambda^*(.))b_H] \qquad (5.3)$$

subject to VP_L, VP_H and PA.

We shall call this Problem IV. The key difference between this and the previous case is that the number of poor people will depend upon the PAP chosen. The introduction of the PAP defined in Proposition 1, would reduce the ex post utility difference between high and low ability types, and thereby reduce the returns to skill investment. As a result, the number of poor will increase. This makes precise the notion that poor relief can lead to the pauperization of the population.

There seem *a priori* to be two possible responses to this. The first might be to reward individuals for becoming high types (a "carrot")

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8 To ensure that $\lambda^* \varepsilon (0, 1)$ we require that the cost of investment function satisfy

$$\tau(1) = 0$$

and $$\tau(0) > v(z, 0, a_H) - v(0, 0, a_L)$$

which we shall assume throughout.

while the second would punish individuals for being low ability types (a "stick"). Our task here is to investigate which, if either, of these is desirable for incentive reasons.

Our view of poverty alleviation programs leading to increased numbers of poor must be contrasted with that taken in some of the literature on welfare dependency (see, for example, the survey by Blank (1986)). In much of this literature, it is the very act of drawing welfare benefits which lowers an agent's income earning potential. Formally this is like letting a_i depend upon b_i and c_i in the present model. Our model, instead, focuses upon the effects on exante decisions to invest in human capital. We shall discuss this further in section 6 below.

Prior to attempting a characterization of the solution to Problem IV, it will again prove helpful to simplify the problem. Lemma 5 confirms that work requirements on high ability types still have no value.

Lemma 5: Let $\{b_i^*, c_i^*\}_{i=L,H}$ solve Problem IV then $c_H^* = 0$

Proof: See Appendix.

As in section four, this result enables us to eliminate one choice variable and hence replace VP_H with the constraint that b_H is non negative. We can also use Lemma 3 to replace VP_L by the simpler constraint that $c_L \leq \bar{c}_L$. Thus, we define Problem V as

$$\min_{(b_L,\ c_L,\ b_H)} \quad n[\lambda^*(.)b_L + (1-\lambda^*(.))b_H]$$

subject to
$$b_L + y(c_L, a_L) \geq z$$
$$c_L \leq \bar{c}_L$$
$$b_H \geq 0$$

We then have the following result:

<u>Lemma 6</u>: (b_L^*, c_L^*, b_H^*) solves Problem IV if and only if it solves Problem V.

<u>Proof</u>: See Appendix.

Once again, this represents a great simplification of the original problem. It is clear that the PA constraint holds with equality for the same reasons as given in the previous section. Hence

$$b_L = z - y(c_L, a_L). \tag{5.4}$$

It is now useful to define the function

$$C(c_L, b_H) = n[\lambda^*(\chi(z-y(c_L, a_L), c_L, b_H))(z-y(c_L, a_L)) + (1-\lambda^*(\chi(z-y(c_L, a_L), c_L, b_H))b_H] \tag{5.5}$$

which gives the cost of a PAP which requires c_L units of work from low ability types and pays b_H to high types. Given any particular work requirement $c_L \in [0, \bar{c}_L]$, there will be an associated optimal benefit for high types $b_H^*(c_L)$, defined as:

$$b_H^*(c_L) = \arg\min \{C(c_L, b_H): b_H \geq 0\} \tag{5.6}$$

Finally, let

$$C^*(c_L) \equiv C(c_L, b_H^*(c_L)) \tag{5.7}$$

This function gives us the minimum cost of a PAP which specifies a work requirement of c_L. The optimal work requirement c_L^* minimizes $C^*(c_L)$, subject to the constraint that $c_L \varepsilon [0, \bar{c}_L]$.

Using the envelope theorem, it can be verified that

$$\begin{aligned} C^{*\prime}(c_L) &= C_1(c_L, b_H^*(c_L)) \\ &= n[-\lambda^*(.)y_c(c_L, a_L) + \lambda^{*\prime}(.)[(\chi_2 - \chi_1 y_c(c_L, a_L))(z - y(c_L, a_L) - b_H^*(c_L))]] \end{aligned} \tag{5.8}$$

This is the key equation of this section. There are two effects from raising c_L. First, as we saw previously, low types may reduce their private sector earnings. This requires an increase in b_L equal to this loss (i.e. $-y_c(c_L, a_L)$). The total cost of this is $-n\lambda^*(.)y_c(.)$. Second, the ex post utility difference between high and low ability types is affected. The change in this utility difference is given by $\chi_2 - \chi_1 y_c(.)$. The first term, which is positive, reflects the direct effect of increasing c_L, while the second, which is non-negative, reflects the indirect effect from the compensating increase in b_L. If the utility difference increases, then the number of poor in the economy will be reduced. This results in savings in the government's budget proportional to the difference between

$z - y(c_L, a_L)$ and $b_H^*(c_L^*)$. The trade off is therefore between the loss of private sector earnings and the reduction in the number of poor.

Further progress can once again be made by actually calculating the derivatives in question and evaluating (5.8). Using (2.3), (2.6), (2.7) and (5.2), we obtain

$$C^{*\prime}(c_L) = \begin{cases} n\lambda^{*}(.)a_L & \text{if } c_L < \bar{I}(a_L) \\ n\lambda^{*\prime}(.)h'(c_L)\ (z - b_H^{*\prime}(c_L)) & \text{if } \bar{I}(a_L) < c_L < \bar{c}_L. \end{cases} \tag{5.9}$$

If $c_L < \bar{I}(a_L)$, raising c_L will not actually increase the ex post utility difference between high and low types. The term χ_2 is exactly offset by the term $\chi_1 y_c(.)$. The intuitive reason for this is straightforward. At levels of c_L less than $\bar{I}(a_L)$, raising c_L will just result in an equal reduction in private sector work and hence lead to no change in the total amount of work done by low types. The reduction in private sector earnings is exactly compensated for by raising b_L so that the utility of a low ability type is unchanged, as is the ex post utility difference.

Conversely if $c_L > \bar{I}(a_L)$, raising c_L will have no effect on poor individuals' private sector earnings but will increase the ex post utility difference. At levels of c_L above $\bar{I}(a_L)$, low ability types do no work in the private sector. Thus a rise in c_L results in no loss in private sector earnings. Hence further rises in c_L impose no further cost on the government. It does however lower the number of poor by increasing the ex post utility difference between high and low ability types.

It follows, therefore, that costs are increasing in c_L on the interval $[0, \bar{I}(a_L)]$ and decreasing on the interval $[\bar{I}(a_L), \bar{c}_L]$. This is illustrated in Figure 1. Hence, is it clear that there are only two candidates for solutions to Problem V. Either

$c_L^* = 0$ or $c_L^* = \bar{c}_L$. The former has no work requirement while the latter has low ability types just indifferent between participating and being in the status quo. This finding is summarized in:

<u>Proposition 3</u>: Let $\{b_i^*, c_i^*\}_{i=L,H}$ solve Problem IV, then

(a) if $C^*(0) < C^*(\bar{c}_L)$

$$(b_L^*, c_L^*, b_H^*, c_H^*) = (z-y(0, a_L), 0, b_H^*(0), 0)$$

(b) if $C^*(0) > C^*(\bar{c}_L)$

$$(b_L^*, c_L^*, b_H^*, c_H^*) = (z, \bar{c}_L, b_H^*(\bar{c}_L), 0)$$

We turn next to finding the conditions under which $C^*(0)$ will be greater or less than $C^*(\bar{c}_L)$. These are seen most clearly from Figure 1. If $\bar{I}(a_L)$ is close to zero, i.e. the low ability types do very little work, then it becomes more likely that workfare will be preferred. This is a case in which the foregone output from forcing people to work an amount $\bar{c}_L$ is small. If a_L were zero then $\bar{I}(a_L)$ will also be zero and workfare is always preferred. On the other hand, if $\bar{I}(a_L)$ is close to $\bar{c}_L$, it would be unlikely that a work requirement would be desirable, other things being equal. The diagram also makes clear that a higher a_L also makes workfare less desirable since output losses are greater. This is also true as λ^* increases i.e. there is a greater proportion of poor in the status quo. Both of these are reflected in the slope of the cost function in the region $[0, \bar{I}(a_L)]$.

Figure 1 also makes clear that a cost function which had a steep downward slope in the interval $[\bar{I}(a_L), \tilde{c}_L]$ would favor workfare *ceteris paribus*. Referring to equation (5.9), it is clear that this will be so, when the marginal disutility of labor is high. This is because a large work requirement would then dramatically reduce the utility of low ability types, and thereby increases the utility difference between the types. This, in turn, reduces the number of poor.

To summarize, in this section we have considered the role of work requirements as a means of providing incentives to avoid poverty. Our main result is that the optimal work requirement is either zero or at the level at which participation in the PAP is just desirable. Workfare is favored by conditions under which the private sector earnings of the poor are small, the proportion of poor in the target population is small and the impact of a work requirement on the ex post utility difference between high and low ability individuals is significant.

6. Extensions and Alternative Models.

As we have indicated throughout, the model presented in this paper takes a particular view of the poverty generation process. We believe that this view is a legitimate one. Nevertheless, it is by no means the only possible view and it is the objective of this section to compare and contrast our findings with those which might emerge under alternative assumptions.

If one took the view that the cause of poverty resided in some individuals having a greater preference for leisure, then the screening argument for work requirements would be invalid. A work requirement would discourage those with a high leisure preference from participating in a PAP, more than those with a low leisure preference. In addition, if preferences for leisure are exogenous, there would be little sense in making individuals work in order to reduce the incentives to becoming poor.

Nevertheless, if one were to look behind the reasons for leisure preferences, then the arguments of this paper may still have some purchase. To see this, suppose that having children induces an increased preference for leisure. Turning first to the model of section four, the problem with a work requirement is that it would discourage those with children more than those without, and the poverty alleviation constraint is unlikely to be met for poor families with children. In the spirit of this paper, one might therefore propose introducing free child care in addition to a work requirement. The argument from section five of the paper can also be modified appropriately. One can think of work requirements here as affecting the incentives to have children as well as the decision to invest in skills.

Taking the view of welfare dependency outlined in the previous section, in which earnings ability is eroded by claiming benefits, there still might be a role for work requirements in reducing poverty. The character of the argument, however, is changed. In the previous section work requirements helped by increasing the incentives to avoid poverty. In a world of welfare dependence, work requirements could help by lessening the erosion of individuals' earnings ability. Furthermore, a clear rationale for requiring job training rather than menial labor would emerge from such considerations.

Finally, we turn to the implications of income effects in labor supply for our arguments. There is a large literature analyzing the effects of welfare programs on the incentives to supply labor (see Danziger, Haveman and Plotnick (1981)). First note that income effects do not provide a separate argument from work requirements. Work requirements will not prevent individuals from reducing their private sector labor supply upon receiving benefits. On the contrary, imposing work requirements will lead to even larger losses in private sector earnings. It follows, therefore, that Proposition 1 remains valid in the presence of income effects. Moreover, while increasing the complexity of the analysis, income effects do not fundamentally change the logic of the two arguments for work requirements that we have presented. With or without income effects, work requirements can screen types, as in section four and encourage skill formation, as in section five.

7. Conclusion.

The question of whether there should be work requirements for receiving benefits in poverty alleviation is one of considerable practical interest. This paper focuses on the incentive case for such requirements and, using a simple model, we have drawn a distinction between two different arguments. The purpose of this paper is not to persuade policy makers that either workfare or welfare is better. In particular, the practical considerations upon which policies are based, account for broader factors than those considered here. Our hope is rather that the model has illuminated some factors of interest which might serve as a focus for the policy debate. We believe that a piecemeal approach such as this, can play a valuable role in policy analysis.

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APPENDIX

Lemma 1: Let $(b_i^*, c_i^*)_{i=L,H}$ solve Problem II, then $c_H^* = 0$

Proof: Suppose that, on the contrary, $c_H^* > 0$. Choose $\bar{b}_H \varepsilon (0, b_H^*)$ and $\bar{c}_H \varepsilon (0, c_H^*)$ such that

$$v(\bar{b}_H, \bar{c}_H, a_H) = v(b_H^*, c_H^*, a_H).$$

Now consider the PAP $(b_L^*, c_L^*, \bar{b}_H, \bar{c}_H)$. Clearly this program is less costly than $\{b_i^*, c_i^*\}_{i=L,H}$. We also claim that it is feasible.

It is obvious that $(b_L^*, c_L^*, \bar{b}_H, \bar{c}_H)$ satisfies VP_L, VP_H, IC_H and PA. It suffices to show, therefore, that it satisfies IC_L. Since $\{b_i^*, c_i^*\}_{i=L,H}$ satisfies IC_L we know that

$$v(b_L^*, c_L^*, a_L) \geq v(b_H^*, c_H^*, a_L).$$

It therefore suffices to show that

$$v(b_H^*, c_H^*, a_L) \geq v(\bar{b}_H, \bar{c}_H, a_L)$$

Using the linearity of v in b, and the definition of $(\bar{b}_H, \bar{c}_H)$ we have that

$$v(b_H^*, c_H^*, a_L) - v(\bar{b}_H, \bar{c}_H, a_L)$$

$$= (b_H^* - \bar{b}_H) + v(0, c_H^*, a_L) - v(0, \bar{c}_H, a_L)$$

$$= [v(0, \bar{c}_H, a_H) - v(0, c_H^*, a_H)] + [v(0, c_H^*, a_L) - v(0, \bar{c}_H, a_L)]$$

By the Fundamental Theorem of Calculus we may rewrite this as

$$v(b_H^*, c_H^*, a_L) - v(\bar{b}_H, \bar{c}_H, a_L) = -\int_{\bar{c}_H}^{c_H^*} v_c(0, c, a_H)dc$$

$$+ \int_{\bar{c}_H}^{c_H^*} v_c(0, c, a_L)dc$$

$$= \int_{\bar{c}_H}^{c_H^*} (v_c(0, c, a_L) - v_c(0, c, a_H))dc$$

Using (2.3), (2.6) and (2.7) it is simple to verify that $v_c(0, c, a_L) \geq v_c(0, c, a_H)$ for all c which implies that

$$v(b_H^*, c_H^*, a_L) - v(\bar{b}_H, \bar{c}_H, a_L) \geq 0.$$

Since $(b_L^*, c_L^*, \bar{b}_H, \bar{c}_H)$ is feasible and less costly than $\{b_i^*, c_i^*\}_{i=L,H}$, the latter PAP cannot solve Problem II - a contradiction □

<u>Lemma 2</u>: Let $(b_L, c_L, b_H, 0)$ satisfy IC_H with equality and $b_H \geq 0$, then it satisfies both IC_L and VP_L.

<u>Proof</u>: (i) $(b_L, c_L, b_H, 0)$ satisfies IC_L. Using the linearity of v in b, we may write

$$v(b_L, c_L, a_L) - v(b_H, 0, a_L)$$
$$= (b_L - b_H) + [v(0, c_L, a_L) - v(0, 0, a_L)].$$

Since IC_H is satisfied with equality, this becomes

$$v(b_L, c_L, a_L) - v(b_H, 0, a_L) = [v(0, 0, a_H) - v(0, c_L, a_H)]$$

$$+ [v(0, c_L, a_L) - v(0, 0, a_L)].$$

Using the same argument as that used in Lemma 1, the right hand side can be shown to be non-negative.

(ü) $(b_L, c_L, b_H, 0)$ satisfies VP_L.

We know by (i) that

$v(b_L, c_L, a_L) \geq v(b_H, 0, a_L)$

and since $b_H \geq 0$,

$v(b_H, 0, a_L) \geq v(0, 0, a_L)$. □

<u>Lemma 3</u>: Let (b_L, c_L) satisfy PA and let $c_L \leq \tilde{c}_L$, then (b_L, c_L) satisfies VP_L.

<u>Proof</u>: By PA

$$\begin{aligned} v(b_L, c_L, a_L) &\equiv b_L + y(c_L, a_L) - \psi(c_L, a_L) \\ &\geq z - \psi(c_L, a_L) \end{aligned}$$

But since $c_L \leq \tilde{c}_L$

$$\begin{aligned} z - \psi(c_L, a_L) &\geq z - \psi(\tilde{c}_L, a_L) \\ &= z - \psi(\tilde{c}_L) \\ &= v(0, 0, a_L) \end{aligned}$$ □

<u>Lemma 4</u>: (b_L^*, c_L^*, b_H^*) solves Problem II if and only if it solves Problem III.

<u>Proof</u>: We prove only that if (b_L^*, c_L^*, b_H^*) solves Problem III it

solves Problem II. The proof of the converse is similar.

Let (b_L^*, c_L^*, b_H^*) solve Problem III. We first check that (b_L^*, c_L^*, b_H^*) is feasible for Problem II. There are two cases to consider.

Case 1: $b_H^* > 0$. In this case, it is easily verified that IC_H is satisfied with equality. By Lemma 2, therefore, (b_L^*, c_L^*, b_H^*) satisfies IC_L and VP_L. Since (b_L^*, c_L^*, b_H^*) necessarily satisfies PA and VP_H, it is feasible for Problem II.

Case 2: $b_H^* = 0$. Since $(b_L^*, c_L^*, 0)$ satisfies PA and $c_L^* \leq \bar{c}_L$, it follows from Lemma 3 that it satisfies VP_L and, hence, since $b_H^* = 0$, IC_L. Thus $(b_L^*, c_L^*, 0)$ is feasible for Problem II.

Suppose now that (b_L^*, c_L^*, b_H^*) does not solve Problem II. Then if $(\hat{b}_L, \hat{c}_L, \hat{b}_L)$ is a solution to Problem II and

$n[\gamma\hat{b}_L + (1-\gamma)\hat{b}_H] < n[\gamma b_L^* + (1-\gamma)b_H^*]$.

But if $(\hat{b}_L, \hat{c}_L, \hat{b}_H)$ solves Problem II, then it must satisfy IC_H, PA and $\hat{b}_H \geq 0$. In addition, we also claim that $\hat{c}_L \leq \bar{c}_L$. To see this note that if $\hat{c}_L > \bar{c}_L$, then VP_L implies $\hat{b}_L > z$. Now consider the PAP $(z, \bar{c}_L, 0)$. It is clear that

$n\gamma z < n[\gamma\hat{b}_L + (1-\gamma)\hat{b}_H]$

and, in addition, $(z, \bar{c}_L, 0)$ is feasible for Problem II. Thus

$(\hat{b}_L, \hat{c}_L, \hat{b}_H)$ cannot be optimal - a contradiction. It follows therefore that $(\hat{b}_L, \hat{c}_L, \hat{b}_H)$ is feasible for Problem III which implies that

$n[\gamma\hat{b}_L + (1-\gamma)\hat{b}_H] \geq n[\gamma b_L^* + (1-\gamma)b_H^*]$,

a contradiction. □

Lemma 5: Let $(b_i^*, c_i^*)_{i=L,H}$ solve Problem IV.

then

$c_H^* = 0$

Proof: Suppose the converse, then $c_H^* > 0$. By VP_H this implies that $b_H^* > 0$. Now choose $\hat{b}_H \varepsilon (0, b_H^*)$ and $\hat{c}_H \varepsilon (0, c_H^*)$ such that

$v(\hat{b}_H, \hat{c}_H, a_H) = v(b_H^*, c_H^*, a_H)$

and consider the PAP $(b_L^*, c_L^*, \hat{b}_H, \hat{c}_H)$. Clearly this program satisfies VP_H, VP_L and PA. Note also that

$\chi(b_L^*, c_L^*, \hat{b}_H, \hat{c}_H) = \chi(b_L^*, c_L^*, b_H^*, c_H^*)$

so that

$\lambda^*(.)b_L^* + (1-\lambda^*(.))\hat{b}_H < \lambda^*(.)b_L^* + (1-\lambda^*(.))b_H^*$.

But this contradicts the fact that $(b_i^*, c_i^*)_{i=L,H}$ solves Problem IV. □

Lemma 6: (b_L^*, c_L^*, b_H^*) solves Problem IV if and only if it solves Problem V.

Proof: We prove only that if (b_L^*, c_L^*, b_H^*) solves Problem V it solves Problem IV. The proof of the converse is similar.

Let (b_L^*, c_L^*, b_L^*) solve Problem V. It is clear that (b_L^*, c_L^*, b_L^*) is feasible for Problem IV. If (b_L^*, c_L^*, b_H^*) does not solve Problem IV, then if $(\hat{b}_L, \hat{c}_L, \hat{b}_H)$ is a solution to Problem IV

$$n[\lambda^*(.)\hat{b}_L + (1-\lambda^*(.)\hat{b}_H] < n[\lambda^*(.)b_L^* + (1-\lambda^*(.))b_H^*].$$

But if $(\hat{b}_L, \hat{c}_L, \hat{b}_H)$ solves Problem IV it must satisfy PA and $\hat{b}_H \geq 0$. In addition, we also claim that $\hat{c}_L \leq \tilde{c}_L$. To see this note that if $\hat{c}_L > \tilde{c}_L$, VP_L implies $\hat{b}_L > z$. Now choose $\bar{b}_L \leq (z, \hat{b}_L)$ and $\bar{c}_L < \hat{c}_L$, such that

$$v(\bar{b}_L, \bar{c}_L, a_L) = v(\hat{b}_L, \hat{c}_L, a_L),$$

then $(\bar{b}_L, \bar{c}_L, \hat{b}_H)$ is less costly than (b_L^*, c_L^*, b_H^*) and is feasible for Problem IV - a contradiction.

It follows therefore that $(\hat{b}_L, \hat{c}_L, \hat{b}_H)$ is feasible for Problem V which implies that

$$n[\lambda^*(.)b_L^* + (1-\lambda^*(.))b_H^*] \leq n[\lambda^*(.)\hat{b}_L + (1-\lambda^*(.))\hat{b}_H],$$

a contradiction. □

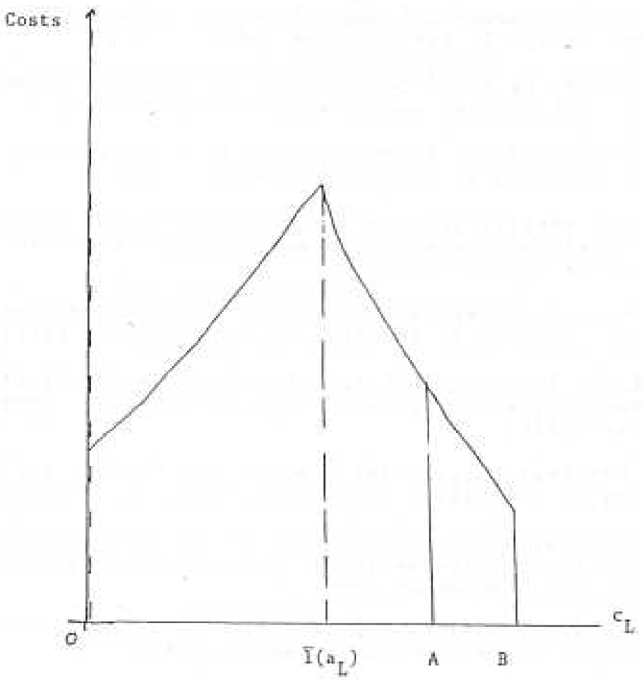

Figure 1

As c_L is raised above zero then costs increase until $\bar{I}(a_L)$ after which all work is undertaken for the government. Costs fall after $\bar{I}(a_L)$ since increments in c_L have disutility on low ability individuals thereby enhancing incentives to acquire skills. Points A and B represent possible values at which c_L is set so that participation in the PAP is just desirable. At A costs are higher than with no work requirement whilst at B they are lower.

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