Meritocracy, Egalitarianism and the Stability of Majoritarian Organizations*

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Abstract

Egalitarianism and meritocracy are two competing principles to distribute the joint benefits of cooperation. We examine the consequences of letting members of society decide by vote among those two principles, in a context where groups of a certain size must be formed, in order for individuals to become productive. Our setup induces a hedonic game of coalition formation. We study the existence of core stable partitions (organizational structures) of this game. For societies with three types of agents we provide necessary and sufficient conditions under which core stable partitions will exist, and we identify the types of stable organizational structures that will arise. We conclude that the inability of voters to commit to one distributional rule or another is a potential source of instability. But we also prove that, when stable organizational structures exist, they may be rich in form, and quite different than those predicted by alternative models of group formation. In particular, non-segregated groups may arise within core stable structures. Stability is also compatible with the coexistence of meritocratic and egalitarian groups. We also remark that changes in society can alter their distributional regimes and influence their ability to compete.

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1. Introduction

Egalitarianism and meritocracy are two competing principles to distribute the joint benefits from cooperation. Though one could debate their relative merits and side for one or the other, we take the opposite route. We analyze the consequences of not taking sides between these two principles, and letting different organizations choose by vote among them, in a context when this choice is part of group formation decisions. What we get is a rich picture, where the lack of ability to commit "a priori" on one specific distributional criterion may lead to organizational structures and consequences that would not arise in more classical frameworks.

Specifically, we consider societies within which organizations must form to perform a certain task, and need to have a minimal size to be productive. The pool of eligible candidates to form them, as well as their minimal size, may be exogenously determined by nature, government, the market or technology, depending on our interpretations. But reward systems are determined by internal vote.

We examine the consequences of this form of governance on the size, stability and composition of organizations, on their endogenous choice of rewards, on their ability to compete for talent, and their ability to keep a competitive edge under changes in their definitional parameters.

Our highly stylized model allows for different interpretations. In all its apparent simplicity, it allows us to touch upon a variety of topics that are at the forefront of today's economic research. What we offer is a very compact view of the forces that may drive the different members of the same society to stick to one group and to dissociate from others.

An interpretation of our model approaches it to the work on country formation and secessions (Alesina and Spolaore, 1997, Le Breton et al. 2004), except that we abstract from geography and instead highlight the role of distributional issues as a driving force that shapes different types of societies. Stability issues will be central in our case, as they are in that related literature: one of our goals is to characterize core stable country configurations (organizational structures). Under this country formation interpretation, our model is suggestive of different phenomena that have been recently highlighted in the literature, regarding the differences in characteristics among advanced societies. We prove that different countries may adopt different distributional criteria and still coexist as in the literature on "varieties of capitalism" (Acemoglu et al (2012), Hall and Soskice (2001)). Of course, our static analysis cannot fully encompass all the dynamic and incentive aspects of a more complex setup, but it is significant that this important stylized fact arise from such a simple model as ours. Moreover, we can provide a highly suggestive comparative static analysis pointing at incentive issues. We present examples where changes in the population and/or in the threshold size required for organizations to become productive have consequences on the country's ability to compete for highly qualified individuals. Another relevant implication of our model is that the choice of distributional arrangements does not depend on the countries' average income. A sophisticated literature (Benabou (2000), Benabou and Tirole (2006) has analyzed how these different distributional options may give rise to different cultural attitudes regarding effort, and be reciprocally reinforced by these attitudes. In our simple case, the median voter's values do prevail, providing a simple and reasonable first approximation to the fact that the dominant group imposes its will opinion in societies, this time through majority voting.

Other interpretations of our model would reflect on the formation of organizations or jurisdictions within a country. Leading interpretations include the establishment of decentralized regions, public university systems, cooperative firms or partnerships within regulated professions. In that respect, our contribution is close to the literature on the endogenous formation of institutions (Caplin and Nalebuff, 1994). But again, our emphasis on distribution as the driving force is novel. Another major conclusion of our work is that stable organizations may be non-segregated, and formed by a variety of agents that differ in productivities. This is in contrast with the conclusion of different strands of literature, where stability pushes different groups of agents to join into segregated groups. Conspicuous examples of this tendency to partition societies into homogeneous groups are found in the literature on local public goods (Tiebout, 1959), on clubs (Schelling (1969), Ellickson et al (1999)) or sorting (Damiano et al (2010)). Our model is one where segregation may or may not arise, and the structural characteristics of the distributions leading to non-segregation can be clearly traced to the fundamentals of the productivity distribution.

Our model is deliberately very simple. Our mention of the preceding references is only made to suggest that some basic features in the formation of societies can be extracted from a very basic, stark formulation of the forces underlying cooperation. Of course, more detailed models that take into account additional features of society will be able to deepen the understanding of additional features. We just submit that through our formulation a lot can be obtained from very little.

In fact, when it comes to analyze stability, the apparent simplicity of the model is quite deceptive. The interactions among agents give rise to hedonic games. (Drèze and Greenberg, 1980); that is, games of coalition formation where each agent's preferences on coalitions are completely determined and constitute the only variable to take into account. One of our aims is to characterize societies whose associated games allow for the existence of core stable partitions. General core existence results in hedonic games are sparse: our model is general enough that many difficulties persist. One of our main contributions is to identify a rich and interesting class of societies, those that we call three-way polarized, which are specific enough to admit a full characterization of their core stable organizational structures.

The paper contains a detailed analysis of the hedonic games that arise in our simple societies, and a variety of examples showing how our model can be used to understand the basic features of coalition formation driven by distribution concerns subject to vote. After this Introduction, and a formal presentation of the model in Section 2, the paper can be read in two sequences. Sections 3, 4 and 5 contain the analysis of the hedonic games that our agents will face, and concentrate on the characterization of core stable organizational structures. Section 6 contains examples and applications of the model. The general reader may want to jump to Section 6 after Section 3, being reassured that all the examples we present there fall into the categories that we have studied, from a more technical point of view, along the intermediate Sections 4 to 6. We close the paper in Section 7, where we discuss extensions and variants of the model, and the kind of questions we'd like to keep addressing with them.

2. The model. Organizational Structures

Let $N = \{1, 2, ..., n\}$ be a set of n individuals characterized by their individual potential productivities $\lambda = (\lambda_1, ..., \lambda_n)$ with $\lambda_1 \geq ... \geq \lambda_n \geq 0$. Subsets of N are called *groups* or *organizations* interchangeably. Individuals can only become productive if they work within a group $G \subseteq N$ of size at least v. Groups of smaller size produce nothing, while groups of size v or larger produce the sum of their members' productivities. A *society* is represented by a triple (N, λ, v).

We refer to a group of cardinality less than v as being *unproductive*.

We denote the average productivity of a group $G \subseteq N$ by $\bar{\lambda}_G$, and by λ_G the vector of productivities of the agents in G.

If a productive group is formed, its total production must be distributed among the agents of the group. Agents prefer to get a higher than a lower pay. Lexicographically, if they must choose among organizations that will pay them the same, they prefer those whose average productivity is higher. [1]

Productive groups internally decide, by majority voting, whether to distribute their product in an egalitarian or in a meritocratic manner. That is, whether all agents in the organization G get the same reward, $\bar{\lambda}_G$, or each one is rewarded by its productivity, λ_i. There is no way to commit a priori to any of these two principles. A majority in group G will favor meritocracy if the agent who is ranked median in the order of productivities is more productive than the average of the group. Otherwise, the majority will be for egalitarianism. Ties are broken in the following way: if there are more than one median agent, ties are broken in favor of the agent with the highest productivity. If the productivity of the median agent is equal to the mean productivity, we consider that the group is meritocratic.

Since agents know the rules and also the productivities of all others, they can anticipate what rewards they will get from joining any given group. They will thus play a hedonic game (Drèze and Greenberg (1980)), where outcomes are partitions of agents into groups. A natural prediction is that stable partitions will arise from playing these games. The following definitions formalize the stability concept that we use in this paper.

Definition 1. *Given a society (N, λ, v), an organizational structure is a partition of N denoted by π. Two organizational structures, π and π', are equivalent if for all $G \in \pi$ there is $G' \in \pi'$ such that $\lambda_G = \lambda_{G'}$ and viceversa. A group G is segregated if given i and j in G with $\lambda_i < \lambda_j$, and $k \in N$ such that $\lambda_i \leq \lambda_k \leq \lambda_j$, $k \in G$. An organizational structure is segregated if all the groups in the partition are segregated.*

Definition 2. *An organizational structure is blocked by a group T if all members in T are strictly better off in T than in the group they are assigned in the organizational structure. An organizational structure is core stable if there are no groups that block it.*

In our context, core stability may not always be possible to get, as shown in the following example.

Example 1. *A society with no core stable organizational structures.*
Let $N = \{1, 2, 3, 4, 5\}$, $v = 3$, and $\lambda = (100, 84, 84, 84, 60)$. Let us see first that in any core stable

[1] We adopt this lexicographic specification of preferences as the simplest way to represent the fact that, in addition to the material reward, individuals may also value other dimensions of the participation in a group, like prestige. Other specifications that reflect richer trade-offs between individual rewards and the "quality" are possible, but make the model less tractable.

organizational structure the high productivity agent can not belong to a productive meritocratic group. This is because in this example the grand coalition is the highest mean meritocratic group containing the high productivity agent. But the organizational structure composed by the grand coalition is not core stable because the medium type agents on their own can form a meritocratic group with a higher average productivity. Now, let us see that in any core stable organizational structure the high productivity agent can not belong to a productive egalitarian group. Like before, we only need to check that the organizational structure where the productive group is the egalitarian group with the highest mean productivity is blocked. This productive group is the one formed by the high type agent with two of the medium type agents. This organizational structure is blocked by the meritocratic group formed by the high type agent, the third medium type agent and the low type agent. Finally, note that the high type agent can not belong to an unproductive group either, because the productive one formed by the high type and two medium types blocks any organizational structure where the productive group does not contain the high type. Thus, there is no core stable organizational structure.

Our next example shows two important and independent points. The first one is that in a core stable organizational structure different reward systems may coexist. The second one is that a core stable organizational structure may be non-segregated.

Example 2. *A society with stable non-segregated organizations where different reward systems coexist.*

Let $N = \{1,2,3,4,5,6,7,8,9,10\}$, $v = 5$, *and* $\lambda = (100,100,75,75,75,75,75,75,75,45)$. *Let* $G_1 = \{1,3,4,5,10\}$ *and* $G_2 = \{2,6,7,8,9\}$. *Note that* G_1 *is meritocratic and* G_2 *is egalitarian. Let us see that the organizational structure* $\pi = (G_1, G_2)$ *is core stable. Note that the medium type agents in* G_2 *can only improve if a high type is added to the group or if a medium type is substituted by a high type. But since the other high type not in* G_2 *is already in a meritocratic group, he does not have incentives to form the potential blocking group. The two high types can not be together in a meritocratic group, and any other agent need high types to improve. That implies that* π *is a core stable organizational structure. Note that high and medium productivity agents are split between the two groups. Any other core stable organizational structure is equivalent to this one.*

At this point, it is worth comparing the consequences of our model with those that one would obtain under alternative criteria to choose the distributional rules. For ease of comparisons, we assume that $n = kv$, for any integer k. If agents could freely bargain about the payoffs they will get when joining a coalition, the core stable organizational structure would be the segregated one where the v most productive agents get together, the next v most productive agents form a second group, and so on, with all groups adopting the meritocratic distribution. If, on the other extreme, agents were forced to adopt a fixed distributional rule (either meritocracy or egalitarianism), a core stable organizational structure would also always exist and coincide with the segregated partition into groups of size v.

Hence, the ability of societies to choose by vote between our two distributional criteria is what gives rise to the differential traits of our results. In our societies stability becomes an issue, that we must carefully discuss. And it is also the case that rich, non-segregated organizational structures can arise. And this, in turn, allows for a richer analysis of societies where voting has distributional consequences.

Our paper now takes two complementary routes. Readers who are mostly interested in the economic implications of the model may want to turn to Section 6. Those who prefer to wait and first look into the stability issues can start with sections 3, 4 and 5 (to be written).

3. Core stable organizational structures and the weak top group property

The analysis of hedonic games is never a trivial task. In particular, finding out when a hedonic game may give rise to stable coalitional structures requires a careful understanding of the interactions between the interests of agents and their ability to sustain the groups they belongs to, or to challenge those they wish to join. Our model restricts the class of games that we need to consider, thus allowing for more specific results than those available in the literature on general hedonic games. In this and the next two sections we provide characterizations of societies for which core stable organizational structures exist. We begin here by considering a condition that is sufficient (but typically not necessary) for the existence of core stable organizational structures in general hedonic games: *the weak top group property* (Banerjee et al, 2001). We prove (Proposition 1) that identifying weak top groups in our model, when they exist, is an easy task. In addition to its intrinsic interest, this result is used in subsequent sections, when searching for potential candidates to form core stable organizational structures. Then we turn to a particular case of our model, one simple enough to generate interesting intuitions: societies where only one productive group can be formed. For these simple societies, the weak top group property has additional bite. It allows us to provide a full characterization of those "small societies" for which there are core stable organizational structures (Proposition 2), and to identify those cases where existence is guaranteed (Proposition 3).

Let us first describe the conditions that allow to define when a hedonic game satisfies the weak top group property.

We say that $W \subseteq G \subseteq N, W \neq \emptyset$, is a *weak top group* of G if it has an ordered partition $(S_1, ..., S_l)$ such that (i) any agent in S_1 weakly prefers W to any subset of G, and (ii) for any $k > 1$, any agent in S_k needs cooperation of at least one agent in $\cup_{m<k} S_m$ in order to form a strictly better coalition than W. A game satisfies the *weak top group property* if for any group $G \subseteq N, G \neq \emptyset$, there exists a weak top group W of G.

If the weak top group property is satisfied, a core stable organizational structure, $(G_1, ..., G_m)$ always exists and can be constructed by sequentially selecting weak top groups from the population: G_1 is the weak top group of N, G_2 the weak top group of N/G_1, and so on. [2]

We can show that, given the structure of our model, weak top groups, if they exist, must have a very specific form.

Proposition 1. *Let $G \subseteq N$. Any WT group of G either belongs to $M_+(G)$ or to $E_+(G)$, where $M_+(G)$ is the set of G's meritocratic subgroups with the greatest mean and $E_+(G)$ is the set of G's egalitarian subgroups with the greatest mean.*

Proof. First we show that if G has a WT group, W, then $\bar{\lambda}_W \geq \bar{\lambda}_S$ for all $S \subseteq G \backslash W$. Suppose on the contrary that there is a group $S \subseteq W \backslash T$ such that $\bar{\lambda}_W < \bar{\lambda}_S$. Let $i \in W$ be such that $\bar{\lambda}_T \leq \lambda_i < \bar{\lambda}_S$. Let $S' = S \cup \{i\}$. Since $\lambda_i < \bar{\lambda}_S$, the mean productivity of group S' will be bigger than the productivity of i, $\lambda_i \leq \bar{\lambda}_{S'}$. Thus, agent i, independently of the regime will be better off in S' than in W, in contradiction with W being a WT group.

Suppose now that the WT group is meritocratic but does not belong to $M_+(V)$. Note first that $W \cap M = \emptyset$ for all $M \in M_+(G)$, because otherwise, all agents in $W \cap M$ would strictly prefer M to W contradicting that W is a WT group. Since $W \cap M = \emptyset$, our previous reasoning applies, and therefore $\bar{\lambda}_W \geq \bar{\lambda}_M$. But then $W \in M_+(G)$, a contradiction. The same argument applies if W is an egalitarian group. ■

[2] Stronger conditions can be found in the literature that guarantee core stable organizational structures. For example, the Top Group Property (TGP), requires that any group G of agents contains a subgroup that is the best subset of G for all of its members (Banerjee et al, 2001). The TGP is a relaxation of the common ranking property introduced by Farrell and Scotchmer (1988). Under those conditions the core is not empty and it has a unique element.

This result is helpful to know in order to explore potential core stable organizational structures, as we do in the following section. Moreover, it allows us already to establish two simple results regarding "small" societies. One is a characterization result: when v is such that the population admits only one productive group, the weak top group property is necessary and sufficient for the existence of core stable organizational structures. The other shows existence of core stable organizational structures for a particular case.

Proposition 2. *A society composed of* $n < 2v$ *individuals has core stable organizational structures if and only if* N *has a* WT *group.*

Proof. Sufficiency is clear: just partition the society into the weak top group of N and leave the other agents together in an unproductive group.

Necessity follows from the fact that if a partition, $(P, N \backslash P)$ is in the core it can not be blocked. Therefore, no subgroup in the productive element, P, of the partition can gain from joining any agents from the unproductive element, $N \backslash P$. This is just the definition of P being a weak top group of N. ■

A direct application of the above result is the following proposition.

Proposition 3. *In societies where* $n < 3v/2$*, a weak top group of* N *always exists. Therefore, there are always core stable organizational structures.*

Proof. Let $T = \{1, ..., v\}$. If T is meritocratic, it is trivially a weak top group of N and thus the core is not empty. Let us see that if T is egalitarian it is also a weak top group of N. Note first that all agents with productivity below the mean are in their best group. Only agents above the mean could improve. But, since the group is egalitarian, the mean is above the median and thus the group that can improve has a cardinality smaller than $v/2$. But the unproductive group $I = \{v + 1, .., n\}$ also has a cardinality smaller that $v/2$. Thus, there is no way of forming a group that can improve upon T. ■

Notice, however, that existence of core stable organizational structures is not guaranteed when $3v/2 < n < 2v$, as we have shown in example 1, where neither the meritocratic group with the greatest mean (the group of the medium productivity agents), nor any of the egalitarian groups with the greatest mean (the high productivity plus two medium productivity agents) are weak top groups of N.

Finally, let us make clear that when $n \geq 2v$, the weak top property is not a necessary condition for the existence of core stable organizational structures as the following example shows. This remark leads us to analyze, in the next section, societies where $n \geq 2v$.

Example 3. $N = 6$, $\lambda = (100, 100, 84, 84, 84, 60)$, $v = 3$. *The meritocratic group with the greatest mean is the group formed by the medium productivity agents. This group is not a weak top group because its members would be better off in the egalitarian group with a high productivity agent. The egalitarian group with the greatest mean is the one composed by a high productivity agent and two medium productivity agents. This group is not weak top either because this high productivity agent will be better off in the meritocratic group formed by the remaining medium and low productivity agents. However,* $\pi = (G_1, G_2)$, *where* G_1 *is an egalitarian group with the highest mean (one high and two medium productivity agents), and* G_2 *is a meritocratic group with one high one medium and one low type is a core stable organizational structure.*

We now turn to examine populations where productivities are limited to no more that three types.

4. Societies with three types of agents

We have seen in the preceding section that the weak top property has specific implications for our model, but not strong enough to provide a full characterization of core stability, except for small societies where only one productive group can be formed. In this section and the next we provide full characterization for societies of any size. These results are obtained at the prize of restricting the domain of societies under consideration. In the present section, we study societies where N is composed of three types of agents, who differ in productivities: the high, the medium and the low. This three-way partition of society is certainly limitative, but also a reference case. It is not only here, where types refer to productivities, but in many other contexts. People are classified by social status into the elite, the middle and the lower class; countries are classified into developed, developing and less developed, etc... In fact, all the basic intuitions that one gets from the model can be grasped through the analysis of this three-type case. Better than that, our analysis in the present Section can be extended in the following Section 5 to the much wider family of societies where non-identical agents can be divided into three clusters, provided members in the same cluster satisfy a number of defining characteristics. Hence, the present section is presented in a double spirit, both as the study of an important class of benchmark societies, and as providing the building blocks for a considerable extension to three-way clustered societies.

In a three type society (N, λ, v) a generic type is denoted by j, $j \in \{h, m, l\}$, and productivities are $\lambda_h > \lambda_m > \lambda_l$. Let H, M, and L be the sets of all high, medium and low type agents respectively. And let v_H, v_M and v_L be the cardinality of these sets. Without loss of generality we assume agents to be ordered from 1 to n. In that order high type agents come first, then medium ones, and finally low ones. Clearly the order of individuals of the same type is arbitrary and will have no effect on our results. Under this convention, the set $T = \{1, ..., v\}$ will be the one containing the first v agents in terms of productivity. Similarly, $B = \{v+1, ..., n\}$ will stand for the $n-v$ less productive agents. Note that because of this arbitrariness, any two organizational structures which only differ in the numbering of individuals of the same type are structurally equivalent in the sense that if one is core stable the other will also be. In what follows when we refer to uniqueness of core stable organizational structures, we mean that they are all structurally equivalent.

From now on, we discuss the characteristics of societies where core stable organizational structures exist and also the form that these structures take under different conditions. We start by a remark.

Remark 1. *In degenerate cases where there are only individuals of one or two types, the existence of core stable organizational structures is guaranteed. The case with only one type is trivial. In societies composed by two types of agents, say h and l, if $v_H \geq v$, the organizational structure (H, L) is trivially core stable. If $v_H < v$, the reader may check that the organizational structure (T, B), with $T = \{1, ..., v\}$, and $B = \{v+1, ..., n\}$ is core stable.*

We now turn attention to societies with three distinct types of agents.

The following condition is crucial for the existence of core stable organizational structures.

Definition 3. *A three type society is unstructured if the following holds:*

1. N has no weak top group.

2. Where $T = \{1, .., v\}$, and $\hat{v}_M = \#((N \backslash T) \cap M))$, $\hat{v}_M < v$.

That is, after T is formed, the remaining medium type agents are too few to form a productive group.

3. Where $\hat{G}$ is any of the meritocratic groups containing the minimal number of low types such that all $i \in \hat{G} \cap T$ are better off in $\hat{G}$ than in T, and $\hat{v}_L = \#(\hat{G} \cap L)$, either $\hat{v}_M \geq v/2$; or

$v_L - (v - \hat{v}_M) \geq \hat{v}_L$.
That is, either $T^2 = \{v+1, ..., 2v\}$ is meritocratic or $\hat{v}_L$ is lower than the number of remaining low types after T and T^2 are formed.
4. Where $\widetilde{G}$ is any of the meritocratic groups that contain the maximal number of high type agents, $\#((N/\widetilde{G}) \cap (H \cup M)) < v$.
That is, after $\widetilde{G}$ is formed, the remaining high and medium types are too few to form a productive group.
A three type society is structured if it is not unstructured, that is, if some of the above conditions do not hold.

We call societies satisfying conditions 1 to 4 unstructured because, among other things, the conditions imply that the number of high types is too low to impose meritocracy among the best while the number of medium types is not small enough to be irrelevant but not large enough to impose egalitarianism to the high types.

Proposition 4. *There exist core stable organizational structures for a three type society if and only if the society is structured.*

Proof. *Part 1: Unstructured societies have no core stable organizational structures.* Assume that a core stable organization structure π exists and let $G \in \pi$ such that $G \cap H \neq \emptyset$. We show that G is not meritocratic, nor egalitarian, nor unproductive, which is a contradiction.

Assume G is meritocratic. Then G must be one of the meritocratic groups with the maximal number of high types. Since by condition 1 there is no weak top groups in N, then $v_H < v/2$, because otherwise T would be a meritocratic group and it would be a weak top group of N. Thus, if G is a meritocratic group it has to be formed by three types of agents. By condition 4, agents in $(N \backslash G) \cap (H \cup M)$ must joint a group of low type agents to be in a productive group. Apart from G, no other productive group $G' \in \pi$ with three types can be meritocratic. Otherwise the medium types in the group with lower average productivity could switch to the other and increase the average productivity while keeping meritocracy, and this new group would block π. So, if π contains another productive group G' with three types it must be egalitarian and it must contain all the high type agents in $(N \backslash G) \cap (H \cup M)$. If $\overline{\lambda}_{G'} > \lambda_m$ replacing a low type in G' by one of the medium types in G increases the average and keeps egalitarianism, and this later group blocks π. But if $\bar{\lambda}_{G'} \leq \lambda_m$ we contradict that π is core stable as well - since switching one of the medium types from G' to G increases the average in G and keeps meritocracy.

A similar argument establishes that agents in $N \backslash G$ can not be organized in two-types (high and low, or medium and low) groups either; because medium type agents in such groups, if added to G, can increase the average and keep meritocracy, blocking π. Hence, G can not be meritocratic.

Assume next that G is egalitarian. Then, since by 1 there are no weak top groups, it must be that $G = T$, and a meritocratic group exist $\bar{G}$ such that all $i \in \bar{G} \cap T$ are better off in $\bar{G}$ than in T. Conditions 2 and 3 imply that an organization structure containing T can be blocked by $\hat{G}$ no matter how the other agents are organized. To conclude, assume G is unproductive. But $h \in G$ is very welcome in any meritocratic group (even if that changes the regime), and if there are no meritocratic groups, T blocks π. Hence, there are no core stable organizational structures.

Part 2: Structured societies have core stable organizational structures.

For each condition assuring a structured society we describe how to construct a core stable organizational structure.

(i) Suppose condition 1 fails, i.e. there exist weak top groups in N. We first argue that there will always be one weak top group W such that $N \backslash W$ contains only two types. This is because

- if $v_H \geq v$, then H is weak (in fact top), and therefore $N \backslash H$ contains two types of agents, medium and low.

- if $v_H < v$, and T is meritocratic, T is weak top and $N \backslash T$ contains at most two types of agents, medium and low.

- if $v_H < v$, T is egalitarian and weak top, then $N \backslash T$ contains at most two types of agents, medium and low.

-if $v_H < v$, T is egalitarian but not weak top, then any weak top group, W, must be meritocratic with highest mean. W must contain some high type agents, because all agents in a meritocratic group without high type agents will gain from adding one high type, whether this enlarged set is egalitarian or meritocratic. In addition, W must contain all medium type agents, because if one of them was left out, adding that agent will increase the group mean while keeping meritocracy. Then $N \backslash W$ contains at most two types of agents, high and low.

Let us now construct a core stable structure. Take a weak top group W such that $N \backslash W$ contains only two types. We have just shown that this is always possible. Let W be one of the groups in the organizational structure. By Remark 1 we know that the two types society $N \backslash W$ has a core stable organizational structure. The groups in that structure plus W are core stable in our initial society.

(ii) Suppose that condition 1 holds but condition 2 fails. Condition 1 implies that $T = \{1, .., v\}$ is egalitarian, and since condition 2 fails, $\hat{v}_M > v$. Let $G_1 = T$, $G_2 = (N \backslash T) \cap M$, and $G_3 = L$. Clearly (G_1, G_2, G_3) is a core stable organizational structure.

(iii) Suppose conditions 1 and 2 hold, but condition 3 fails. Thus, $\hat{v}_M < v/2$ and $v_L - (\hat{v}_M - v) < \hat{v}_L$. Let $G_1 = T$, $G_2 = T^2$, and $G_3 = N \backslash (G_1 \cup G_2)$, where the later is an unproductive group of low types. Again (G_1, G_2, G_3) is a core stable organizational structure.

(iv) Last, suppose that condition 1 holds but condition 4 fails. Let $\tilde{G}$ as defined in condition 4. Note first that the mean productivity of $\tilde{G}$ is below the medium type productivity, because since there is no weak top groups in N, $v_H < v/2$. Let $G_1 \subset (N \backslash \tilde{G}) \cap (HUM)$, which contains the v first agents in $(N \backslash \tilde{G}) \cap (HUM)$: note G_1 is egalitarian, or meritocratic if there is no high type agents in that intersection. Let $G_2 = \tilde{G} \cup ((N \backslash G_1) \cap M)$, and let G_3 be formed by the remaining agents (all of low type). Thus, (G_1, G_2, G_3) is a core stable organizational structure. ■

Remark 2. *Notice that when $n < 2v$, conditions 2, 3 and 4 for a society to be unstructured trivially hold, since they involve restrictions that only apply when more than one group can form. Hence, if $n < 2v$ a society is unstructured if and only if N has no weak top groups.*

This remark leads us directly to the necessary and sufficient condition for the existence of core stable organizational structures that we already discussed in Proposition 2 of the previous section.

Proposition 4 does not only provide an existence result for general three type societies. It also opens the door to study a particular case about which we know a lot, and that provides us with an important tool to generate significant examples in our next section. This is when societies contain $n = 2v$ agents, that is, just enough individuals to form two productive groups.

The first important fact about these societies is that they always have some core stable organizational structure. In fact, going further in our analysis, we can prove that the set of such stable structures is always small, containing only one or at most two of them. We can be even more precise, and establish the characteristics of the groups that can achieve stability, depending on the distribution of productivities.

In what follows we present these results in detail. But first we must introduce a property that societies may or may not satisfy, and that turns out to be determinant of the kind of stable groups that can form.

Definition 4. *A society is maximally mixed meritocratic if $v_H < v/2$, $v_L \leq v/2$, and $(\lambda_h + \lambda_M + v_L \lambda_L)/(v_L + 2) \leq \lambda_m$.*

In maximally mixed meritocratic societies we can always construct a meritocratic group of cardinality v that contains agents of all three types, all agents of the low type and the highest number of high types allowing for all the preceding characteristics to hold. We call this a *maximally mixed meritocratic group*, and denote it by $M3$. This group can be constructed as follows. Start with all v_L low types, one medium type and one high type. This starting group may not be productive, but the mean of its $\lambda's$ is below λ_m. Next start adding as many high types as possible while keeping the mean of the $\lambda's$ below λ_m. And finally, if the group is not yet productive, fill the set with medium types until reaching size v.

Remark that an organizational structure that contains a group with the characteristics of $M3$ is non-segregated. Also notice that in societies that are not maximally mixed, some group may still have the structure required in the definition of $M3$. What is characteristic of maximally mixed meritocratic societies is that they guarantee the existence of $M3$ sets which, in addition, do belong to core stable structures. We shall see this in Proposition 6.

Two other sets of agents play an important role in describing stable organizational structures. We have already defined them before, for any society: T is the set of the first v agents in the productivity ranking, and B is the set of the v last in that same ranking. In our case, (T, B) is a partition of N, since $n = 2v$, and we shall see under what conditions it is a candidate to become a stable organizational structure.

Let us now turn to the results.

Proposition 5. *Societies where $2v = n$ are always structured. Hence, they always have core stable organizational structures.*

Proof. If the society has a weak top group in N, then clearly society is structured. Suppose that a society where $n = 2v$ does not have any weak top group. Then the following conditions must hold:

a. The set of high type agents are not a majority in any group of cardinality v, that is, $v_H < v/2$. Because otherwise T would be meritocratic and consequently weak top.

b. T has to be egalitarian. Because if T is meritocratic, it is weak top.

c. There are meritocratic groups with high types. Because if not, T would be weak top.

Furthermore, since $n = 2v$, if T is formed, the remaining medium type agents are too few to form a productive group. Thus, condition 2 in the definition of unstructured societies always holds. From here we distinguish two cases, either $v_L > v/2$ or $v_L \leq v/2$.

In the first case, after T is formed, the remaining medium type agents are not a majority, $\hat{v}_M < v/2$. Thus, condition 3 in the definition of unstructured societies does not hold and therefore this society in structured. In particular, a core stable organizational structure that can be formed is (T, B), where both groups are egalitarian.

In the second case, $v_L \leq v/2$, condition 3 holds but, as we show next, condition 4 fails. Since there are meritocratic groups with high types and $v_H < v/2$, those meritocratic groups must contain three types of agents. In this case, $(\lambda_h + \lambda_m + v_L\lambda_l)/(v_L + 2) \leq \lambda_m$, because otherwise it is impossible to construct a meritocratic group with the three types. That is, we must be in a maximally mixed meritocratic society where $M3$ can be constructed. Now, since $M3$ has cardinality v and contains all low types and $n = 2v$ then $\#((N \backslash M3) \cap (H \cup M)) = v$. Thus, condition 4 does not hold, and $(M3, N \backslash M3)$ is a core stable organizational structure. ■

Our next proposition establishes that maximally mixed meritocratic societies have a unique and non segregated core stable organizational structure.

Proposition 6. *In societies with $n = 2v$ that are maximally mixed meritocratic there is a unique core stable organizational structure,* $(M3, M \backslash M3)$.

Proof. The constuctive argument in the proof of Proposition 4 shows that $(M3, N \backslash M3)$ is a core stable organizational structure. $M3$ contains three types of agents and is meritocratic. $N \backslash M3$ contains only medium type agents or a combination of medium and high types and it is egalitarian. Let us see that $(M3, N \backslash M3)$ is the unique core stable organizational structure.

We first show that no structure with only one productive group can be core stable. For this to happen, the productive group would have to be weak top. Candidates to be weak top groups are $G \in E_+(N)$, or $G \in M_+(N)$.

If $G \in E_+(N)$, G has size v and can not be part of an organizational structure with only one productive group. This is because in a maximally mixed meritocratic society, $v_H < v/2$, $v_L \leq v/2$, and consequently $v_M > v$. These conditions imply that T is egalitarian, $T \in E_+(N)$, and any other $G \in E_+(N)$ has the same structure as T.

If $G \in M_+(N)$, G only contains medium type agents. This is because any meritocratic group with high type agents has the mean below the productivity of the medium type agents, and $v_M > v$.

But groups containing only medium type agents are never weak top, because its members always prefer to add high types to their group.

Let us now prove that for all other organizational structures with two productive groups $(G_1, G_2) \neq (M3, N \backslash M3)$ there is always a group that blocks (G_1, G_2).

a. If G_1 and G_2 are both meritocratic, both groups have three types of agents or one of them three types and the other two types, medium and low. In any case, adding the medium type agents to the group with greater mean constitutes a meritocratic group with increased mean that blocks (G_1, G_2).

b. If G_1 and G_2 are both egalitarian then none of them is T, because $N \backslash T = B$ is meritocratic. Thus, T blocks (G_1, G_2).

c. If G_1 is meritocratic and G_2 is egalitarian, $G_2 \neq T$. Because otherwise, $G_1 = B$ and then $M3$ blocks (T, B). G_2 can not have three types of agents, because by replacing low types in G_2 by medium types, the mean increases while keeping egalitarianism. This new group will block (G_1, G_2). Thus, G_2 can only contain two types of agents. Since $v_H < v/2$, $v_L \leq v/2$, G_2 contains only high and medium types. Since G_2 is different from $N \backslash M3$ it must contain more high type agents. But then, given the construction of $M3$, we can replace medium type agents in G_1 by high type agents while keeping meritocracy and increasing the mean, and this new group will block (G_1, G_2).

Thus, $(M3, M \backslash M3)$ is the unique core stable organizational structure. ■

Although for non maximally mixed meritocratic societies we can not guarantee uniqueness of the core stable organizational structures, we prove that there are at most two core stable organizational structures.

Proposition 7. *In societies with $n = 2v$ that are not maximally mixed meritocratic, (T, B) is always a core stable organizational structure and there exists at most another core stable one.*

The proof is presented in the Appendix.

We close the section by recapitulating what we have learned about the case $n = 2v$,and highlighting some of the main findings that hold in these societies but also basically extend to more general cases.

One first lesson refers to segregation. For societies that are maximally mixed meritocratic, stability implies non-segregation, as proven in Propositions 5 and 6. For societies that are not, we can assert for sure that stability holds for the segregated structure (T, B), but this is sometimes compatible with the existence of a second stable structure which may be not segregated.

A second set of remarks refer to the combinations of reward schemes that are compatible within core stable organizational structures. In societies that are maximally mixed meritocratic, at least one of the groups in a stable structure must be meritocratic, while the second group may adopt meritocracy or egalitarianism. In societies where (T, B) is stable, each one of the two sets can adopt any of the two distributional criteria. Moreover, notice that in this case the resulting distributional criteria are determined by the number of agents of each type that belong to each of the two sets, and not on the exact values of their productivities.

All of these facts are exploited in our next section, where we illustrate the implications of our model through examples, some of which involve a beginning of comparative static remarks. The (almost) uniqueness results in the present section provide the grounds for the use of comparative statics, which could be blurred in cases where multiple stable structures could arise.

5. Three-way polarized societies

To be written

6. On the size and competitiveness of stable organizations

This section continues with the presentation of examples that, along with those already proposed in Section 2, give us a measure of the different issues that we can tackle within our present model. Further hopes for extensions and additional results are collected in the conclusions.

Recall that our examples 1 and 2 already show that the choice of reward schemes by majority voting creates the possibility that different regimes coexist in stable arrangements, and that the choice of egalitarianism or meritocracy is not tied to the average productivity level of the different groups that coexist. We have seen more on this in Section 4 for the case of three agent types. This conclusion is interesting under several possible interpretations of the model. In particular, when interpreted as one of country formation, it generates an important stylized fact in the debate on varieties of capitalism: countries with a similar level of productivity do not need to share the same distributional principles.

We have also already remarked that instability may arise, as a consequence of the inability of individuals to commit to a given distributional principle. And that the groups that may form within stable organizational structures can sometimes be segregated, and at other times non segregated. Again, these remarks apply to any possible interpretation of the model, but may be particularly relevant when our model is interpreted as one where communities are formed, or individuals are sorted.

In this section we provide new examples that emphasize the role of the parameters defining our societies in shaping stable organizational structures, and the consequences of changing these parameters. In that case, our remarks, even if they apply in general, may be of special relevance when we think of groups as being different institutions within a society whose government may try to influence the group formation process. We do not have a formal model of government here, but we assume that what it can control is the minimal size of organizations and the total number of agents who are eligible to form them. For example, if we think of a university system within a country, the government may determine who is qualified to become a professor, and what is the minimal size of the faculty to form a university. Similar requirements on group size and qualifications apply if we interpret the groups to be formed as partnerships in regulated professions.

Our first remark concerns the lack of ability on the part of the government to control the effective size of emerging organizations, even if they may decide on the parameters v and n. Given a minimal size v and a total number of agents $n = kv$, which is therefore sufficient to form k groups, it is possible that no organizational structure with groups of size v can achieve stability, even in contexts where other core stable structures exist. Stability may require larger units. This is shown in the following example.

Example 4. *A case where $n = kv$, and yet no partition of agents into groups of of size v can achieve stability.*

Let $N = \{1, 2, 3, 4, 5, 6\}$, $v = 3$, and $\lambda = (50, 40, 40, 35, 25, 10)$. Let (P, U) be an organizational structure where $P = \{1, 2, 3, 5\}$ and it is meritocratic and $U = \{4, 6\}$ and it is an unproductive group. (P, U) can not be blocked because P is the meritocratic group with the highest mean and the only agent that could improve without using anyone from P is agent 4 but $\{4, 5, 6\}$ is meritocratic. The egalitarian group with the greatest mean in $E = \{1, 2, 3\}$, $N \backslash E$ is meritocratic. The organization $(E, N \backslash E)$ is blocked by $G = \{1, 4, 6\}$ which is a meritocratic group with a greatest mean than $N \backslash E$. Any organization with two meritocratic groups or one meritocratic and one *unproductive group is blocked by P, any organization with two egalitarian groups or one egalitarian and one unproductive group is blocked by E. It can be checked that any other organization is blocked by P. Thus, (P, U) is the unique core stable organizational structure.*

Now we turn to a remark regarding the potential consequences of changing v, the minimal size of organizations. In particular, we note that in contexts where stability is not guaranteed, changes in v can be either stabilizing or de-stabilizing.

Example 5. *Changes in v can be either stabilizing or de-stabilizing.*
Let $N = \{1, 2, ..., 7\}$ and $\lambda = (100, 84, 84, 84, 84, 60, 60)$. Suppose that initially $v = 4$.
Note that medium type agents can form a group by their own with a payoff of 84. The egalitarian with the greatest mean is blocked by the meritocratic with the high, one medium and two low type agents. Any meritocratic G with the high type is blocked by the four medium agents together. No organizational structure is stable.
But, if $v = 3$, (G_1, G_2, U), with $G_1 = \{1, 2, 6\}$, a meritocratic group, $G_2 = \{3, 4, 5\}$ also a meritocratic group and $U = \{7\}$ and unproductive group with a low type is a core stable organizational structure, basically because the high type is in a meritocratic group and he can not increase the mean above 84 keeping meritocracy.

The above remarks have referred to changes in v while keeping the eligible population constant. Now we consider the case where v is reduced, and the total number of available agents is also reduced, so as to keep fixed the maximum number of productive institutions. This is relevant to analyze, in a simple way, some possible consequences of budget cuts. Before we present examples of the type of conclusions we can reach, let us define a second form of stability, other than the one associated to the core.

Remark that in order to define a hedonic game we fix the set of potential players who may be part of the groups that eventually form. In our university story, these would be the people in the country who can eventually join a university, because they meet all the credentials. Typically, these people may have outside options. We shall assume that the best agents in our set are highly demanded by other instances (other jobs, other countries), and that they will only stay for long in the group they join if they are paid at least their productivity. It is natural, in the presence of outside options, to assume that the best will only stabilize in one of our organizations, if they join a meritocratic group. This suggests a second (external) stability requirement, that we can call competitive stability: the best (however we define them) must be rewarded meritocratically.

Our comparative static analysis proves that even societies that enjoy core stability may easily shift from being competitively stable to competitively unstable, due to changes in our basic parameters. To the extent that these changes may be generated by or controlled through public policies, keeping the best agents within the system may require finely tuned actions, in order to guarantee that competitive stability remains when circumstances do change.

To illustrate our point, consider the following example.

Example 6. *Competitive stability*

Let $N = \{1, ..., 14\}$, $v = 7$, $\lambda = (10, 10, 7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1, 1)$.

A possible arrangement, which is both core and competitive stable, because the best university would be meritocratic, consists of having one group with all the high and medium types, plus two lows, and the other containing low types only. Assume now that society must reduce the size of universities by two faculty members each. It would seem natural to fire the two worse people of each group. The groups resulting from these actions (two high and three medium types in the best university, all lows in the other), would still be in the core of the corresponding game with 10 *candidates and* $v = 5$*. Yet, this coalition structure is no longer competitively stable, since now the worst university will adopt egalitarianism as a norm! In that case, firing two medium agents, rather than two low ones, would preserve competitive stability.*

Similar and apparently anomalous phenomena would arise as the potential result of other parametric changes. For example, if the low type members would upgrade their qualifications close to the medium type, say from 1 to 6, competitive stability would also be lost.

7. Concluding Remarks

We have presented a very simple model of group formation where people are driven to cooperate by a minimal size requirement, and choose their reward schemes by majority. This very simple model is able to generate a variety of interesting stylized facts that are under examination in different strands of literature, through more complex formulations. We do not claim that the features of our model can be immediately transposed to reality. But they certainly show that one can get a head start in explaining several intriguing phenomena with a minimal set of tools.

We also want to emphasize that the model is simple to describe, but complex to analyze. Our existence and characterization results are hard to get, even after restricting attention to a subclass of societies with three types of agents. Hedonic games are difficult to handle, and our treatment contributes to the literature on coalition formation by providing several fresh, positive results.

Our model admits many potential extensions, on which we intend to keep working ad hope some readers may also provide.

One natural extension is to model the externality resulting from cooperation with other agents in a richer way. Here the monetary reward is only supplemented lexicographically with some preference to belong to a group with the highest mean, given the same payment. But we could think of stronger impacts to be received from cooperating with others, ones where the prestige of working along with highly productive agents may lead to accept lower pays that the ones one can get in less productive groups. Exploring the combinations between material and subjective rewards would certainly be a next step in understanding the interaction between group formation and the choice of distributional criteria.

Another extension of our model would enrich the choices of distributional regimes. We only allow here for a dichotomic choice between egalitarianism and meritocracy. But there could be intermediate choices. For example, many societies try to guarantee some minimal level of salary for all workers, while still allowing for a partly meritocratic reward system. There are different ways to model the kind of implicit taxation schemes involved in this compromise between meritocracy and egalitarianism. For each one of them, one could again analyze the choice of taxes by agents who are at the same time considering with whom to cooperate.

Other feature of our model that could be extended are those relating to the technology of individual and of joint production. Here agents either contribute zero, when they belong to small groups, or a fixed productivity in groups larger than v. In a larger picture, one could incorporate complementarities among agents, or returns to size. A related issue that we do not model here relates to incentives to exert effort. If we had modeled the possibility of contributing more or less to production depending on rewards, the resulting model would have richer, and certainly harder to deal with!

All of these extensions seem promising, and none of them appears to be trivial.

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8. Appendix

Proof of Proposition 7

Case 1. Assume $v_H \geq v/2$.

In this case, $T \in M_+(N)$, and therefore is a weak top group. Thus, (T, B) is a core stable organizational structure. Furthermore, we first show that there may exist a second core stable structure if (i) $G \in E_+(N)$, $\#G > v$, and G is a weak top group, or (ii) $G \in E_+(N)$, $\#G = v$, and $N\backslash G$ is also egalitarian[3].

If (i), since G is a weak top group, $(G, N\backslash G)$ is core stable and only G is productive.

If (ii), since both G and $N\backslash G$ are egalitarian, $(G, N\backslash G)$ is a core stable organizational structure with two productive groups. To see that, note that no group can block $(G, N\backslash G)$ because such group would have to be meritocratic and thus form by agents that are receiving less than their productivity in $(G, N\backslash G)$. Given that both group in $(G, N\backslash G)$ are egalitarian, those agents are the ones whose productivity is above the mean of the group, and since the mean is above the median, they are less than $v/2$ in each group. Hence they can not form a productive group blocking $(G, N\backslash G)$.

Let us see that, apart from this possible second core stable structure, there can be no other.

In structures $(P, N\backslash P)$ where only P is productive, if P is not weak top, there will exist a productive group G such that all $i \in P \cap G$ will be better off in G than in P. Since all $i \in (N\backslash P) \cap G$ are getting zero in $N\backslash P$, they will also be better off in G. Thus, G will block $(P, N\backslash P)$. Hence, P has to be weak top, and the unique candidate in this case is the one described in (i).

Finally, let us show that any structure (G_1, G_2) with two productive groups different from (T, B) and the one considered in case (ii) will be unstable.

(a) If G_1 and G_2 are meritocratic, it is blocked by T which is also meritocratic.

(b) If G_1 is meritocratic and G_2 is egalitarian we distinguish two cases.

- If all the high type agents are in G_1, G_2 can only contain contains medium and low types, and since it is egalitarian $\bar{\lambda}_G < \lambda_m$. But then adding a medium type agent from G_2 to G_1,creates

[3] This last situation can only happen if $v_L > v/2$. To see this, note that, since T is meritocratic, high type agents have to be distributed between G and $N\backslash G$. Furthermore, let us see that all medium type agents have to be in G. If $\bar{\lambda}_G < \lambda_m$, the median agent is a low type agent, and $N\backslash G$ has to contain three types. Adding a high, medium and a low type agent to G from $N\backslash G$ will create a new egalitarian group of higher mean, contradictin that $G \in E_+(N)$. If $\bar{\lambda}_G \geq \lambda_m$, adding a high and medium type to G from $N\backslash G$ will create a new egalitarian group of higher mean. Again, this contradicts that $G \in E_+(N)$. Thus, G contains all the medium type agents. Therefore, for $N\backslash G$ to be egalitarian, $v_L > v/2$.

a new meritocratic group of higher mean than G_1 which blocks (G_1, G_2).

- If the high type agents are split between G_1 and G_2, we can add all missing high type agents to G_1 and drop enough non high types in G_1 to create a new group of size v. This new group will still be meritocratic, have a higher mean than G_1 and blocks (G_1, G_2).

(c) If G_1 and G_2 are egalitarian, neither G_1 nor G_2 are in $E_+(N)$. Thus, any egalitarian group $G \in E_+(N)$ will block (G_1, G_2).

Case 2. Assume $v_H < v/2$ and $v_L > v/2$.

In this case, T can be either egalitarian or meritocratic. In both cases, as proved in Proposition 5, (T, B) is a core stable organizational structure.

Case 2a. Suppose first that T is meritocratic.

Since $v_H < v/2$, T has three type of agents and consequently B is the meritocratic group with just low types, which implies that $v_L > v$.

As in Case 1, a second core stable structure may exist if (i) $G \in E_+(N)$, $\#G > v$, and G is a weak top group, or (ii) $G \in E_+(N)$, $\#G = v$, and $N \backslash G$ is also egalitarian[4]. The same argument as in Case 1 applies.

Let us see that, apart from this possible second core stable structure, there can be no other.

As explained in Case 1, structures $(P, N \backslash P)$ where only P is productive are core stable if and only if P is weak top. The unique candidate in this case is the one described in (i).

Finally, let us show that any structure (G_1, G_2) with two productive groups different from (T, B) and the one considered in case (ii) will be unstable.

The arguments in (a) and (b) in Case 1 apply here.

In the case that G_1 is meritocratic and G_2 is egalitarian, G_2 has to contain at least two types of agents. If G_2 contains high type agents, replacing a low type agent in G_1 by a high type agent will create a new meritocratic group G (because T is meritocratic) of higher mean than G_1 which blocks (G_1, G_2). The same kind of argument will apply if G_2 does not contains high type agents but contains medium type agents.

Case 2b. Suppose that T is egalitarian. Since $v_H < v/2$, T has two or three type of agents and consequently B is either egalitarian with medium and low types ot meritocratic with only low type agents.

[4] Note that since T is meritocratic, this situation can only happen if G contains all the high type agents and $v - v_H$ low type agents and it should be such that adding a medium type changes the regime. This structure only exists if $v_H = v/2 - 1$ and $v_M < v/2$.

We first show that may exist a second core stable structure if (i) $G \in M_+(N)$, $\#G > v$, and G is a weak top group, or (ii) if $G \in M_+(N)$, $\#G = v$, $N \backslash G$ is egalitarian, $(N \backslash G) \cap M = \emptyset$, and the mean productivity of the group is below λ_m.

If (i), since G is a weak top group, $(G, N \backslash G)$ is core stable and only G is productive.

If (ii), since the mean productivity of $N \backslash G$ is below λ_m, $(G, N \backslash G)$ is core stable. There is no possibility of blocking because a potential blocking group should contain mediun type agents. Since they are in a meritocratic group with the greatest mean, they will only participate in an egalitarian group with mean above their productivity. But this is not possible.

Let us see that, apart from this possible second core stable structure, there can be no other.

As explained in Case 1, estructures $(P, N \backslash P)$ where only P is productive are core stable if and only if P is weak top. The unique candidate in this case is the one described in (i).

Finally, let us show that any structure (G_1, G_2) with two productive groups different from (T, B) and the one considered in case (ii) will be unstable.

(a) If G_1 and G_2 are both egalitarian, it is block by T which is also egalitarian.

(b) If G_1 and G_2 are both meritocratic, and neither G_1 nor G_2 are in $M_+(N)$, any meritocratic group $G \in M_+(N)$ will block (G_1, G_2). If one of them belongs to $M_+(N)$ (let us say $G_1 \in M_+(N)$), since $v_H < v/2$ and $v_L > v/2$, both G_1 and G_2 contains medium type agents. Suppose that $\bar{\lambda}_{G_1} \geq \bar{\lambda}_{G_2}$, then adding a medium type agent from G_2 to G_1,creates a new meritocratic group of higher mean than G_1 which blocks (G_1, G_2).

(c) If G_1 is meritocratic and G_2 is egalitarian, G_1 may contain agents of two or three types. In the first case they must be medium and low types with a majority of medium types. Thus, G_2 contains low type and high type agents and (possibly) medium types. In any case, $T \in E_+(N)$ blocks (G_1, G_2). If G_1 contains three types, G_2 can contain two or three types (with low and medium types for sure in both cases). If $\bar{\lambda}_{G_2} < \lambda_m$, adding a medium type agent from G_2 to G_1creates a new meritocratic group of higher mean than G_1 which blocks (G_1, G_2).If $\bar{\lambda}_{G_2} > \lambda_m$, replacing a low type in G_2 with a medium type from G_1 creates a new egalitarian group of higher mean than G_2 which blocks (G_1, G_2).

Case 3. Assume that $v_H < v/2$, $v_L \leq v/2$, and $(\lambda_H + \lambda_M + v_L \lambda_L)/(v_L + 2) > \lambda_M$.

In this case T only contains high types and medium type agents and it is egalitarian, B contains only medium and low type agents and is meritocratic. Condition $(\lambda_H + \lambda_M + v_L \lambda_L)/(v_L + 2) > \lambda_M$ implies that high type agents can not be part of a meritocratic group, thus T is weak top and

(T, B) is core stable.

Note that the meritocratic group with the greatest mean in this case is M, which is not a weak top group. Thus, no other organizational structure with only one productive group can be core stable.

Finally, let us show that any structure (G_1, G_2) with two productive groups different from (T, B) will be unstable.

(a) Note that G_1 and G_2 can not be both meritocratic, since there is no meritocratic group that contains high type agents.

(b) If G_1 and G_2 are both egalitarian, it is blocked by T which is also egalitarian.

(c) If G_1 is meritocratic and G_2 is egalitarian, G_1 can only contain medium and low types or only medium type agents, but since this group is different from B, G_2 must contain low type agents also. Notice that since G_2 is egalitarian and low types do not constitute a majority, $\bar{\lambda}_{G_2} > \lambda_m$.

Replacing in G_2 low type agents by medium type agents from G_1 will create a new egalitarian group of higher mean than G_2 which blocks (G_1, G_2). ■