# Culture, Community and Segregation

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**Abstract**

We use economic theory to examine the possible outcomes facing a culturally diverse population. We ask which cultures emerge and persist and what happens to social cohesion. We find three possible outcomes for a community: assimilation, where the community integrates and one group conforms to the culture of another; segregation, where groups socially segregate and retain their own culture; and multiculturalism, where groups integrate but individuals still retain their own culture. We further show that new cultures can emerge in diverse communities: multiculturalism requires a shared culture to develop alongside diverse cultures, while segregation can result in further differentiation of culture.

## 1 Introduction

Recent media commentary and political debate has centered around the challenge faced by European countries as they open up to higher levels of immigration. European countries, many of which have remained for a long time relatively homogeneous, now find themselves with significant sized immigrant groups. With this change comes the novel challenge of incorporating immigrant groups with new cultures into populations with well-established cultures. A challenge that is all too clear in recent politics and headlines: in France, in 2010 the senate voted $246-1$ to ban the full Islamic veil with French President Sarkozy arguing that the burka is not consistent with French identity; in Germany, in 2010 the German Chancellor claimed 'the tendency had been to say, "let's adopt the multicultural concept and live happily side by side, and be happy to be living with each other," but this concept has failed, and failed utterly'; while in Britain, in 2011 a speech by the British Prime Minister criticized a lack of British identity saying 'Under the doctrine of state multiculturalism, we have encouraged different cultures to live separate lives...we've even tolerated these segregated communities

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behaving in ways that run completely counter to our values.'[1]

The prospects for a more culturally diverse Europe are not clear. In recent years, political rhetoric has moved towards promoting assimilation (whereby immigrants conform to existing European cultures and socially integrate). Assimilation is promoted as an alternative to immigrant groups living culturally and physically separate lives from native populations. In the past however, countries such as Britain and the Netherlands promoted a third alternative: multiculturalism. Multiculturalism supposes that those with different cultures can both integrate into society and keep their own languages and customs.[2]

What are the possible outcomes facing a culturally diverse population? Can multiculturalism actually exist? When are we likely see one outcome over another? Generally, what outcomes are better, for whom and when? If multiculturalism does exist, is it to the detriment or benefit of society? This paper uses economic theory to formalize these questions on cross-cultural interaction.

Take a population consisting of groups with given diverse cultures. When groups with different cultures operate in the same environment, an individual faces two major decisions: one, to what extent to retain his or her own cultural practices, and two, to what extent to participate with other groups in society.[3] In other words, individuals choose what to do and who to interact with.

Everyday individuals make choices about what to do: what sports to play, what television programmes to watch, how much work to do, whether to attend religious events, whether to drink alcohol or not, how to dress, how to behave towards others. Some of these choices are related to an individual's culture and some are not. Thus we can divide an individual's activity choices into two groups: cultural practices and other activities. Individuals can retain their own cultural practices or adopt the cultural practices of another group; however, there is a cost to adopting cultural practices other than one's own.[4] For other activities (unrelated to an individual's given culture), the individual faces choices between activities but there are no such costs associated with the choice made.

Individuals also make choices about who to interact with. Individuals value social interaction because social interaction can provide information, economic opportunities, economic support, risk sharing, or can simply be enjoyable in itself. However, social interaction requires having things in common. Naturally, the more activities, practices, and behaviors two individuals have in common (think of language, sports, hobbies, eating habits, things to talk

[1] Taken from articles in the Guardian, New York Times, and BBC, and British Prime Minister's speech from www.number10.gov.uk. Quote by President Sarkozy 'We cannot accept to have in our country women who are prisoners behind netting'.

[2] Multiculturalism has and has had various definitions. We retain this definition throughout the paper.

[3] Indeed influential psychology literature on cross-cultural interaction (Berry, 1997) supports this view.

[4] We detail examples when setting out the model.

about in common, attending the same place of worship every week, marriage practices), the stronger the opportunity for social interaction between them.

The set-up is akin to a battle-of-the-sexes game between groups: because social interaction requires having things in common, individuals wish to coordinate on activities rather than mis-coordinate; however, members of one cultural group get a higher value from coordinating on one set of activities while members of another cultural group get higher value from coordinating on a different set of activities. An important feature of the game is that individuals can choose with whom they play the coordination game. That is, individuals form social ties and a coordination game is only played between pairs of individuals who have a social tie. Thus individuals have to balance the cost of adopting different cultural practices with the benefits of forming stronger social ties with more people.

We find that three outcomes can arise in equilibrium: assimilation, where all individuals adopt the same cultural practices and individuals form social ties within and across groups; separation, where groups retain distinct cultural practices and individuals form social ties only within groups; and multiculturalism, where individuals retain distinct cultural practices but at the same time form social ties both within and across groups.[5]

What kind of environments sustain these three outcomes? When a cultural group is relatively small, assimilation occurs. Intuitively, restricting social interaction to such a small group is not desirable and the group does better by adopting the cultural practices of a larger group and being absorbed into that larger group. When the population is more mixed, such that a minority group is relatively large, then multiculturalism and separation are both possible outcomes. This is because when groups are more equal in size, the cost to a minority individual of giving up his culture can be larger than the benefit of increased social interaction that assimilation brings. In addition to requirements on group size, multiculturalism is only possible if culture does not 'dominate' everyday life. That is, individuals must be able to retain cultural practices but also have meaningful relationships with those who adopt different cultural practices by interacting on other (non-cultural) activities.

What are the best outcomes and for whom? When a group is sufficiently small, assimilation is both the best thing for members of that group as well as being the outcome that maximizes social welfare. When groups are more equal in size multiculturalism maximizes social welfare. Multiculturalism is not only an ideology: in this model it is both an equilibrium and under certain conditions it is the welfare-maximizing outcome. Nevertheless, even when multiculturalism is welfare-maximizing, assimilation is always the payoff-maximizing outcome for members of the dominant group (the group whose culture is maintained under assimilation). When thinking in terms of immigration, it is then no surprise that we see attempts in Europe

[5]Again these outcomes are consistent with psychology literature on cross-cultural interaction, Berry (1997)

to manoeuvre rhetoric and policy towards assimilation.

To this point we have discussed whether a group retains its own cultural practices or not and whether members will integrate into wider society. Answering yes or no to these two possibilities results in the three equilibria above. However, psychology, anthropology and sociology literature on cross-cultural interaction (Berry (1997), Social-Science-Research-Council (1954)) documents that, when diverse cultural groups come into continuous first-hand contact, the cultures that emerge are more complex than the binary decision of retaining one's own culture or not. Indeed, this is exactly what we find. While individuals have some ex-ante given culture, the culture that emerges within a community is represented by the choice of cultural practices and the choice of other (non-cultural) activities that individuals make in equilibrium. In the assimilation equilibrium, groups conform completely on cultural practices and other activities and so a single culture emerges in equilibrium. When multiculturalism occurs, groups retain their diverse cultures but the population as a whole adopts the same other non-cultural activities. Thus alongside differences in cultural practices, the community develops a common shared culture which is necessary to sustain multiculturalism. At the other end of the spectrum, when diverse cultural groups share the same environment but choose to segregate, increased cultural polarization can occur. In this case, diverse groups further differentiate their behaviors and practices beyond given cultural differences, meaning that activities that are not ex-ante related to culture become associated with a particular cultural group in equilibrium. This increased cultural polarization occurs in order to raise the cost to an individual of interacting with the other group so that separation can be sustained.

Previous work examines solely choice of culture *or* choice of social interaction. Bringing these two choices into the same model produces a range of results on culture and interaction in diverse populations that do not appear in previous economic analysis. Literature on choice of culture is predominantly based on models of cultural transmission.[6] The seminal paper, Bisin and Verdier (2000), analyzes a choice by agents of how much effort to put into finding a culturally homogamous marriage and how much effort to put into passing on their cultural traits to their children, both of which increase the probability the child adopts the cultural trait of the parent (as desired). The key outcome in cultural transmission models is whether an individual adopts one culture or the other.[7] Building on this question, our aim is to examine the wider situation arising from the meeting of different cultural groups: as well as which cultures will persist we want to know whether the community will be cohesive or divided. To comment on welfare or policy, the analysis of social cohesion is crucial. Further, it is only by

[6]The present paper and other papers on culture choice are of course also related to the introduction of identity into economic modeling, the seminal paper Akerlof and Kranton (2000).

[7]A second particularly related paper, (Bisin, Patacchini, Verdier, and Zenou, 2011) examines cultural transmission as above but allows for individuals to choose the intensity with which they subscribe to a trait.

analysing both culture choice and a measure of social cohesion is multiculturalism a possible outcome.[8] Literature on social interaction across different groups (Currarini, Jackson, and Pin, 2009; Bramoulle and Rogers, 2010; Currarini and Vega-Redondo, 2011) examines when individuals of the same type will segregate, to what degree, and the segregation patterns we should expect to see. Individuals are placed in a search model setting and assumed to have a preference or bias towards interacting with people of the same type. Our results on the relationship between group size and segregation are consistent with the results in the above papers. In addition, by allowing for action choices (culture choice) alongside social interaction, we are able to be specific about the different types of segregation and interaction that can occur as well as why segregation occurs. Further we find that social interaction and culture are inextricably linked in the sense that segregation and integration drive cultural choices and vice versa.

Theoretically the model contributes to a literature on network formation where games are played across networks (Goyal and Vega-Redondo, 2005; Hojman and Szeidl, 2008; Jackson and Watts, 2002). We introduce heterogeneity into the population which drives the variety of results and allows segregated networks and different behaviors by individuals in the same population. We also allow individuals to play more than one game across links which allows for some natural and novel results.

Section 2 presents the baseline case to explain the key features of the model and intuition, before moving onto the full model and results in section 3. In section 4 we examine welfare and policy implications.

## 2 When Culture Matters

In this section we present the baseline model and results to introduce the key features and intuition.

### 2.1 The Model

#### 2.1.1 Culture

A population consists of a set $N = \{1, 2, ..., n\}$ of $n \in \mathbb{N}$ individuals. Individuals have a given type, A or B. There are $n_A \in \mathbb{N}$ individuals of type A and $n_B \in \mathbb{N}$ individuals of type B, where $n_A, n_B \geq 2$ and $n = n_A + n_B$. Type is common knowledge. We refer collectively to all individuals of a given type as a *cultural group*, or simply a *group*. Culture itself is defined as the shared set of practices, attitudes, and behaviors which characterize a cultural group.

[8] We are unaware of any analysis of multiculturalism in previous economic literature.

Let action $x^A$ be the culture associated with type A's and action $x^B$ be the culture associated with type B's.

To illustrate, consider the population to be a neighborhood consisting of an immigrant group and a native group, where the immigrant group moves into the neighborhood having different cultural practices from the native group. Alternatively, consider the population to be a school where students at the school live in different neighborhoods in which different cultures are prevalent (for example, Black, White or Hispanic neighborhoods in the US). Different cultural practices (represented by $x^A$ and $x^B$) in such settings could, for example, be different attitudes and behaviors concerning education, gender, aspirations, family, or marriage. Different cultural practices could also simply be different activities such as type of music or sport played, food eaten, or language spoken.

These examples lead to an observation: when immigrants move to a new country they can, if they wish, adopt the culture of their host country; similarly, when students from different neighborhoods share the same school environment, a child from one neighborhood may choose to adopt the behavior associated with another neighborhood. We refer to this as switching culture and assume that, while it is possible to switch culture, there is a cost to doing so. Thus in the model individuals have a given type (A or B) which assigns them an associated culture ($x^A$ and $x^B$ respectively); however, at a cost $c$, they can switch and adopt the culture of the other group (i.e. at cost $c$ a type A can adopt culture $x^B$). Formally, the cost of switching culture is denoted as follows: individual $i \in n$ of type $k \in \{A, B\}$ who chooses culture $x_i \in \{x^A, x^B\}$ faces a cost

$$c_k(x_i) = \begin{cases} 0 & \text{if } x_i = x^k \\ c & \text{if } x_i \neq x^k \end{cases}$$

Paying a cost to switch culture could arise for a variety of reasons: culture can be so deeply entrenched that individuals find it psychologically costly to adhere to behaviors or attitudes that differ from the culture one has grown up with; there may be penalties from within the group for deviating from cultural practices; if different cultures listen to and play different types of music or sport, for example, then participating in the activities of another group can be costly because it is unfamiliar; alternatively there may be fixed costs involved, such as learning a new language. We reduce these different possibilities to a single cost $c$ in line with previous work on culture choice (Akerlof and Kranton, 2000; Bisin and Verdier, 2000).[9]

[9] This is consistent with the introduction of identity into economic modeling, Akerlof and Kranton (2000). A simple version of their model of identity assumes that individuals are partitioned into groups or categories, where each category has a set of associated characteristics seen as the 'ideal' characteristics for someone of that category. Deviation by an individual from the characteristics assigned to her group can result in a loss in utility to that individual. Akerlof and Kranton (2000) detail examples. The definition of culture and a cultural group presented in the model below is consistent with this modeling of identity: categories

#### 2.1.2 Social Interaction

Were this the end of the model, individuals would never choose to pay the cost and switch culture, however, there is another side to the problem: individuals value social interaction. That is, we assume the $n$ individuals who cohabit the population wish to form social ties and interact with each other (irrespective of type). Social interaction is valuable to individuals because it can provide information, economic opportunities, economic support, risk sharing, or can simply be enjoyable in itself.

Importantly, social ties require common ground. If individuals have no common interests or activities then there is little possibility for a social tie to develop. Think back to the previous examples of different cultural practices. Most obviously, without a common language social interaction is very difficult (it is difficult to exchange information, discuss and agree on economic exchange, and any activity that involves conversation or communication is limited). Consider individuals who partake in different activities such as sports and music; not only do they spend time apart when they undertake the different activities but even when they do meet they are unable to exchange information relevant to them both and there are fewer common interests to enhance conversation. As for more subtle things such as attitudes towards education, individuals may want to associate with like minded others in order to exchange relevant information and share relevant opportunities and activities. To model the necessity of common ground in social interaction, we suppose that any two individuals can form a social tie at a cost $L$ each, and that the social tie has value 1 (to each of them) if individuals have cultural practices in common and 0 if not.

#### 2.1.3 Strategies and Payoffs

Each individual in the population chooses a strategy along two dimensions: a choice of social ties with the other individuals in the population, and a choice of whether to retain one's own culture or switch to the other group's culture. An individual does better the more social ties he forms, but only provided he coordinates on the same culture as his social ties, which may come at a cost.

Formally, individual $i$ chooses whether or not to form a social tie with the other $n-1$ individuals in the population. If individual $i$ forms a social tie with individual $j$, we denote this by $g_{ij} = 1$. If individual i does not form a social tie with j we denote this by $g_{ij} = 0$. Player $i$'s choice of social ties can be represented by a vector of 0's and 1's, $g_i = (g_{i1}, g_{i2}, ..., g_{in})$, where $g_{ii} = 0$. Individual $i$ also chooses a culture $x_i$ from the set $\{x^A, x^B\}$. An individual's strategy is represented by the vector $s_i = (x_i, g_i) \in S_i$.

are cultural groups and the characteristics associated with each group are cultural practices. Adopting the cultural practices of another group results in a loss of utility.

Individual $i$ pays a cost $L$, where $1 > L > 0$, for each social tie that he forms (for each $j$ such that $g_{ij} = 1$). He also receives a payoff from each social tie formed, where the value $i$ receives from a social tie with $j$ is denoted $\pi(x_i, x_j)$ and equal to 1 if $i$ and $j$ coordinate on the same action $x$ and 0 if not:

$$\pi(x_i, x_j) = \begin{cases} 1 & \text{if } x_i = x_j \\ 0 & \text{if } x_i \neq x_j \end{cases}$$

The total value $i$ receives from a social tie with $j$ is then $\pi(x_i, x_j) - L$ and the total value of all of individual $i$'s social ties is the sum of the values of each individual social tie

$$\sum_j (\pi(x_i, x_j) - L) g_{ij}.$$

Note three things. One, essentially individual $i$ plays a $2 \times 2$ symmetric coordination game in strategic form with each of his social ties. Two, individual $i$ can only adopt one culture, that is $i$ must choose the same action $x_i$ in each of the coordination games with his different social ties. He cannot differentiate his choice of action by social tie. Intuitively, this requires that an individual be 'consistent' in their behavior within the given population.[10] For example, if $x^A$ and $x^B$ represent required practices and activities associated with different religions, it is not possible to adopt both religious practices as these activities might take place at the same time, the different practices might be contradictory, it could be too time-consuming to do both, or it might be made impossible because of hostility from others if an individual tries to adopt both religious activities. Three, for simplicity social ties are one-sided. If $i$ forms a social tie with $j$ then the value of the social tie accrues to individual $i$ but not to individual $j$ unless individual $j$ forms a social tie with $i$.

The total utility individual $i$ receives if he is type $k \in \{A, B\}$ is then

$$u_k(s_i, s_{-i}) = v(\sum_j (\pi(x_i, x_j) - L) g_{ij}, c_k(x_i)).$$

where the first argument of the function $v(\cdot, \cdot)$ is the value of $i$'s social ties, and the second argument is the cost of switching culture. The function $v(\cdot, \cdot)$ is strictly increasing in the first argument and strictly decreasing in the second: the individual's utility is increasing for each social tie that he coordinates with, but decreasing with a switch in culture. The set-up presented describes an $n$ player game where each player $i \in N$ chooses a strategy $s_i \in S_i$ and faces a payoff $u_k(s_i, s_{-i})$. The strategy profile $(s_1^*, s_2^*, ..., s_n^*)$ is a Nash equilibrium if $u_k(s_i^*, s_{-i}^*) \geq u_k(s_i, s_{-i}^*)$ $\forall i \in N$ and $\forall s_i \in S_i$.

[10] A population consists of $n$ individuals who each have the opportunity to interact with all the others. Thus a population refers to a workplace, a neighborhood, a school etc. An individual may interact in multiple populations and play different strategies in different populations.

#### 2.1.4 Examples of the payoff function

**Example 1**

$$u_k(s_i, s_{-i}) = \sum_j (\pi(x_i, x_j) - L)g_{ij} - \theta c_k(x_i).$$

The first term states that for each social tie $j$ (i.e. $g_{ij} = 1$), individual $i$ receives a contribution of $\pi(x_i, x_j) - L$ to his utility. The second term states that if he adopts the culture associated with the other group he faces a fixed cost of $\theta c$. This could be considered as the cost of learning a new language or new interests.

**Example 2**

$$u_k(s_i, s_{-i}) = \sum_j (\pi(x_i, x_j) - L - \phi c_k(x_i))g_{ij} - \theta c_k(x_i).$$

The difference with example 1 is that adopting the culture associated with the other group results in not only a fixed cost of $\theta c$ but also reduces the value of each social tie by $\phi c$. Example 2 represents the case where switching culture reduces the value of social interaction. For example, partaking in activities one is not familiar with might be less enjoyable or it might be psychologically costly each time an individual behaves differently to the expectations he has grown up with.

### 2.2 Population Outcomes

What are the possible outcomes for a population with diverse cultures? Given the n-player game described in section 2.1, proposition 1 describes the set of Nash equilibria.

**Proposition 1** *The set of Nash equilibria are as follows:*

*(i). Each individual forms a social tie to all other individuals and all individuals adopt the same culture, $x \in \{x^A, x^B\}$.*

*(ii). Each individual forms a social tie to all other individuals of the same type as him but does not form a social tie to individuals of a different type. All the type A's adopt culture $x^A$ and all the type B's adopt culture $x^B$.*

In the equilibrium described in $(i)$, each individual in the population interacts with all other individuals in the population no matter what type they are. All individuals adopt the same culture, either $x^A$ or $x^B$. In this equilibrium, one of the cultural groups adopts the cultural activities associated with the other group. I label this equilibrium *assimilation*. I refer to the group that adopts the other group's cultural activities as the *assimilating group*, and say that this group *assimilates*. In this equilibrium, social interaction across groups comes at a cost


Figure 1: The graphs represent assimilation and separation respectively. Population size n=6, type A's are drawn as circles and type B's as squares. The culture adopted is denoted inside each node.

to the group who assimilates, that is they must pay the cost of adopting the other group's culture.

In the equilibrium described in (*ii*), each individual interacts only with his own type and adopts the cultural activities associated with his type (type $A$'s adopt action $x^A$ and type $B$'s adopt action $x^B$). I label this equilibrium *separation*. These outcomes are illustrated in figure 2.2.

Under the baseline model, assimilation and separation are the only outcomes possible for a population comprising different cultural groups. Why is this the case? First, in equilibrium individuals of a given type always undertake the same strategy.[11] This is because individuals of the same type have identical utility functions, if two individuals of the same type play different strategies then the individual who is doing weakly worse can do better by mimicking the strategy of the other individual.[12] Second, in equilibrium two individuals form a social tie if and only if they adopt the same cultural activities. This arises by the assumption that the value of a social tie is positive if and only if the two individuals coordinate on the same action. Finally note that when individuals do not interact across groups in equilibrium, each type must adopt their own culture. Suppose this was not the case. Then since we are in equilibrium a type A must be doing weakly better by playing $x^B$ than $x^A$, but then a type B does strictly better by playing $x^B$. There are thus two options: segregate and adopt one's own culture, or interact across groups and adopt the same culture.

Proposition 1 states that in a culturally diverse community we will see either assimilation or segregation. We now detail the conditions under which these are equilibria and remark on the kind of environments in which we will see assimilation versus separation. From this point

[11] The only difference in strategy being that they will form a link to each other but not to themselves.

[12] This way the individual who does weakly worse now does strictly better as she receives the same payoff as the other individual and forms an additional social tie with him.

on we assume that

$$v((1-L)(n-1), c) > v(0,0). \tag{1}$$

This rules out the uninteresting case where under no circumstances is an individual willing to adopt a culture other than his own.[13] The following proposition provides a full characterization of when the different outcomes will occur. Let $n_m \in \{n_A, n_B\}$ denote the size of the minority group.

**Proposition 2** *Consider any fixed $v(\cdot,\cdot)$, $c$, $n_A$, $n_B$ and $L$. When the following inequality holds*

$$v((1-L)(n_m-1), 0) \geq v((1-L)(n-n_m), c) \tag{2}$$

*then assimilation* and *separation are both equilibrium outcomes. Otherwise, assimilation is the unique outcome.*

Proposition 2 states that assimilation (note that assimilation refers to either group assimilating) is an equilibrium under all parameters and all possible functions $f(\cdot,\cdot)$. This is a result of the interplay between the group and the individual: when one group assimilates, an individual from the assimilating group cannot sustain his culture single-handedly so can do no better than assimilate.

Separation is an equilibrium when inequality (2) holds. Separation is an equilibrium when, given both groups are segregated and each group adopts its own culture, no individual wishes to leave his or her own group and instead interact with the other group and adopt the other group's culture. Inequality (2) holds when this is true for a member of the minority group. Since we assume symmetry other than in group size, it will then automatically hold for the majority group. The key trade off is represented by this inequality: an individual in the minority group can costlessly retain his own culture and interact only within the minority group, or he can pay the cost of adopting the other group's culture and interact with the larger number of people in the other group.

Corollary 1 describes when separation is an equilibrium with relation to particular parameters of the model. Let $n_m \in \{n_A, n_B\}$ be the number of individuals in the minority group.

**Corollary 1** *Separation is an equilibrium when the minority group is a large enough proportion of the population, that is when $\frac{n_m}{n} \geq \gamma^*$, where $\gamma^* \in \mathbb{R}$ depends on $v(\cdot,\cdot)$, $c$, $n$ and $L$. Further, $\gamma^*$, is decreasing in cultural cost, $c$.*

[13] If this inequality was reversed, separation would be the unique equilibrium.

Corollary 1 states that separation is an equilibrium only when the minority group is large enough; while assimilation is the unique outcome when the minority group is small.[14] A minority group individual will want to remain separated from the majority group only when the minority group is not 'too small' relative to the majority group. Intuitively, the larger the minority group relative to the majority group, the smaller the gain to a minority group member from interacting with the majority group and the less likely this gain is to outweigh the cost of conforming to majority culture. Corollary 1 also tells us that when the costs of adopting the other group's culture are higher, separation can be sustained in populations with smaller minority groups.

Figure 2 illustrates inequality (2) graphically for the utility function in example 2 : $u_k = \sum_j(\pi(x_i, x_j) - L - \phi c_k(x_i))g_{ij} - \theta c_k(x_i)$ where $\theta c < 1 - L$. The vertical axis denotes the number of type A individuals and the horizontal axis denotes the number of type B individuals. The 45° line denotes when groups are of equal size. To the left of the 45° line type A's are in the majority and to the right of the 45° line type B's are in the majority. Moving up and right on the graph denotes increasing population size. Assimilation is an equilibrium on the whole graph. Segregation is an equilibrium only in the top-right triangular region denoted. Figure 3 illustrates an increase in cost $c$.

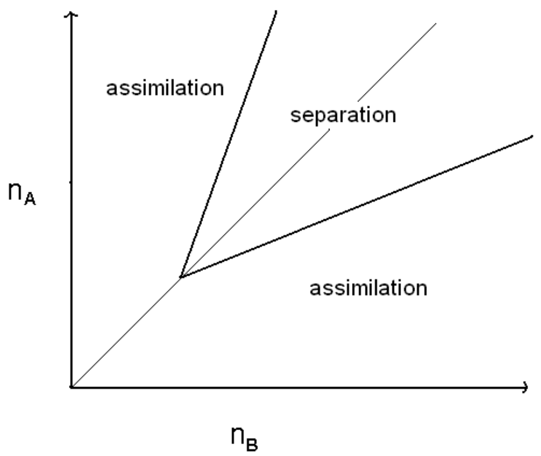


Figure 2: Separation is an equilibrium in the triangular area in the top right of the graph encompassing the 45° line.

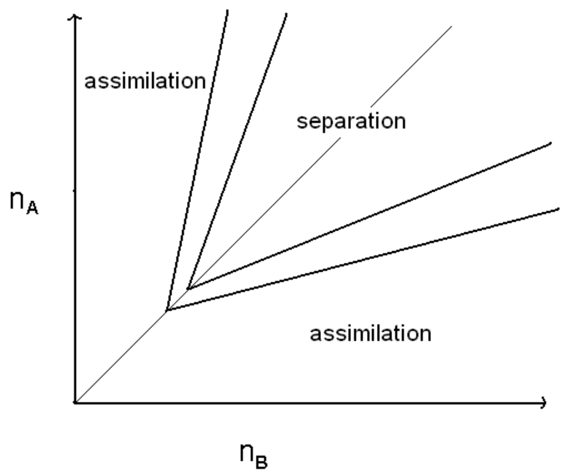


Figure 3: The larger triangular area represents the area for which separation is an equilibrium after an increase in c.

[14]Of course, since we place limited restrictions on $v(\cdot,\cdot)$, $n$, $l$ and $c$, it is possible that for some functions $v(\cdot,\cdot)$, and parameters $n$, $l$ and $c$, separation is an equilibrium for any sized minority group while for others assimilation is the unique outcome for any sized minority group.

### 2.3 Minorities Assimilate

When assimilation is an equilibrium, one group does the assimilating. Individuals in the assimilating group are worse off as they must pay the cost of adopting the other group's culture and face the loss of their own culture. For assimilation to be a Nash equilibrium it requires only that no individual wants to deviate given everyone in the community is playing the same strategy. This means that it is a Nash equilibrium both for the minority group and the majority group to assimilate. Which group is likely to assimilate and under what conditions we are likely to see the different equilibrium outcomes?

For this we use stochastic stability as an equilibrium refinement (for details see seminal papers by Kandori, Mailath, and Rob, 1993; Young, 1993). Stochastic stability has been adopted as an equilibrium refinement in related models of network formation with games played on the network (Goyal and Vega-Rendondo, 2005). To use stochastic stability, a form of dynamic adjustment within the population is assumed. We suppose the community is at some state in period $t$ where a state is given by a strategy for each individual, $(s_1^t, s_2^t, ..., s_n^t)$. This state need not be an equilibrium. The utilities for each individual in each state are given by $(u_1^t, u_2^t, ..., u_n^t)$, where $u_i^t = u_k(s_i^t, s_{-i}^t)$, as defined in section 2.1. At period $t+1$ each individual is picked with independent probability $p \in (0, 1)$ to change her strategy. If an individual is given the opportunity to change her strategy, she maximizes her payoff given the strategies of the others at period $t$,

$$s_i^{t+1} \in \arg\max_{S_i} u_k(s_i^{t+1}, s_{-i}^t).$$

When an individual has the opportunity to change her strategy, with probability $\epsilon$ she makes a mistake and chooses some other strategy randomly. This process defines a Markov chain with a unique invariant probability distribution denoted $\mu_\epsilon$. As $\epsilon \to 0$ we define $lim_{\epsilon \to 0} \mu_\epsilon = \hat{\mu}$, where any state $s$ such that $\hat{\mu}(s) > 0$ is stochastically stable. Roughly speaking, given the above dynamics, stochastic stability looks at the number of mistakes needed to persuade individuals to move from one equilibrium to another and a stochastically stable state is a strict Nash equilibrium state which takes the fewest mistakes to move to.

**Proposition 3** *Given a fixed $v(\cdot, \cdot)$, $c$, $n$ and $l$, where the proportion of the population in the minority group is $\frac{n_m}{n}$, there exists a value $\gamma^{**}$ such that:*

- *if $\frac{n_m}{n} < \gamma^{**}$ assimilation where the minority group assimilates is the unique stochastically stable state.*
- *if $\frac{n_m}{n} > \gamma^{**}$ separation is the unique stochastically stable state.*

- *if $\frac{n_m}{n} = \gamma^{**}$ both assimilation where the minority group assimilates and separation are stochastically stable.*

*When both groups are of equal size, the only change from above is that assimilation by group A and assimilation by group B are both stochastically stable whenever $\frac{n_m}{n} \leq \gamma^{**}$.*[15]

Proposition 3 states that assimilation where the minority group assimilates is the unique stochastically stable state when the minority group makes up a small proportion of the population while separation is the unique stochastically stable state in when the minority group is large. The conditions that determine the exact value of $\gamma^{**}$ for a given $v(\cdot,\cdot)$, $c$, $n$ and $l$ are detailed in the appendix.[16]

Why when we apply stochastic stability as an equilibrium refinement do we find the result that the minority group assimilates? Stochastic stability is based on assessing the number of mistakes it takes to encourage individuals to move from one equilibrium to another and deciphering which equilibrium requires the least mistakes to move to. When one group is in the majority and the other in the minority, it is more attractive to a minority group individual to interact with the majority group and adopt majority culture than it is for a majority group member to interact with the minority group and adopt the minority culture. Because of this asymmetry it takes a greater number of mistakes (where a mistake involves playing a different culture from the current equilibrium) to persuade a majority group member to conform to minority culture than the other way around.

Why is assimilation by the minority group stochastically stable when the minority group is small and separation stochastically stable when the minority group is large? The larger the proportion of the minority group, the more attractive it is for a minority member to continue interacting with his own group and the more mistakes it takes (where a mistake is assimilation by a minority member) for a minority individual to choose to assimilate. Thus as the minority group gets larger it requires an increasing number of mistakes to move from the separation equilibrium to the assimilation equilibrium.

**Corollary 2** *The value of $\gamma^{**}$ is decreasing when cultural cost, $c$, increases.*

[15]Proposition 3 has a caveat. To find stochastically stable states we examine how many mistakes $k$ are required to transit from one equilibrium to another. We have a unique stochastically stable equilibrium given any function $v$ and any parameters when we assume $k \in \mathbb{R}$. However we have discrete units of people and so $k \in \mathbb{N}$. We discuss the two assumptions $k \in \mathbb{R}$ and $k \in \mathbb{N}$ in the appendix.

[16]Briefly, how do we find $\gamma^{**}$? Assume type B's are in the minority. Denote by $k_A$ be the minimum $k$ that satisfies

$$v((1-L)(n_A+k),c) > v((1-L)(n-n_A-k-1),0) \tag{3}$$

and by $k'$ the minimum $k$ that satisfies

$$v((1-L)k,0) > f((1-L)(n-k-1),c). \tag{4}$$

Let $\hat{n}_A$ be the level of $n_A$ at which $k_A = k'$. Then $\gamma^{**} = \frac{\hat{n}_A}{n}$.

Corollary 2 states that the higher the cost of conforming to the other group's culture, the lower the level of heterogeneity at which separation is stochastically stable versus assimilation.

# 3 When More than Culture Matters

In the baseline model we assume that each individual chooses an action $x \in \{x^A, x^B\}$, which we interpret as a choice of culture and assume there is a cost to choosing the culture associated with the other group. In this section we allow for the possibility that other practices and activities take place in the population besides those related to culture. To illustrate, think of a neighborhood with two different religious communities where individuals decide which religious activities to attend at the weekend. On weekdays however, there are no religious activities and individuals face a choice between joining the neighborhood soccer team or joining the neighborhood art class, where playing soccer and doing art are not related to religion or to an individual's type. Thus I now allow for a more comprehensive model where individuals face two action choices: which cultural practices to adopt, and which other activities (unrelated to an individual's type) to adopt.

To introduce this into the model, I allow for a second action choice $y$ from the set $\{y^C, y^D\}$. This represents the choice over activities unrelated to culture. Each individual $i$ now chooses two actions $x_i \in \{x^A, x^B\}$ and $y_i \in \{y^C, y^D\}$. Individual $i$'s strategy is then represented by the vector $s_i = (x_i, y_i, g_i)$. As in the baseline model, the individual faces a cost $c$ from adopting the other group's culture. Formally as above, individual $i$ of type type, $k \in \{A, B\}$, who chooses culture $x_i \in \{x^A, x^B\}$ faces a cost

$$c_k(x_i) = \begin{cases} 0 & \text{if } x_i = x^k \\ c & \text{if } x_i \neq x^k \end{cases}$$

There is no such cost related to the choice of action $y_i$.

The value individual $i$ receives from a social tie with $j$, changes slightly from the baseline model. Individual $i$ still pays a cost $L$, where $1 > L > 0$, for each social tie that he forms but we now assume each individual $i$ receives a payoff of $(1-\alpha)\pi_1(x_i, x_j) + \alpha\pi_2(x_i, x_j)$ from a social tie with $j$ where:

$$\pi_1(x_i, x_j) = \begin{cases} 1 & \text{if } x_i = x_j \\ 0 & \text{if } x_i \neq x_j \end{cases}$$

and

$$\pi_2(y_i, y_j) = \begin{cases} 1 & \text{if } y_i = y_j \\ 0 & \text{if } y_i \neq y_j \end{cases}$$

Thus the value of a social tie depends on whether individuals $i$ and $j$ coordinate on culture or not and whether they coordinate on other activities or not, where the importance of culture to the value of the social tie is weighted by $1-\alpha$.[17] It follows that if individuals $i$ and $j$ coordinate on both $x$ and $y$ the value of the social tie is $1-L$, if they coordinate on $x$ but not on $y$ it is $1-\alpha-L$, if they coordinate on $y$ but not on $x$ it is $\alpha-L$, if they coordinate on neither $x$ nor $y$ it is $-L$. The total value $i$ receives from all his social ties is now $\sum_j((1-\alpha)\pi_1(x_i,x_j)+\alpha\pi_2(x_i,x_j)-L)g_{ij}$. The total utility individual $i$ receives if he is type $k\in\{A,B\}$ is

$$u_k(s_i,s_{-i})=v(\sum_j((1-\alpha)\pi_1(x_i,x_j)+\alpha\pi_2(x_i,x_j)-L)g_{ij},c_k(x_i))$$

where $v(\cdot,\cdot)$ is strictly increasing in the first argument and strictly decreasing in the second, as in the baseline model.

The only change from the baseline case is the addition of a second action choice $y_i\in\{y^C,y^D\}$. The payoff from individual $i$'s choice of action, $y_i$, is independent of his type and we refer to $y$ as the 'type-independent' action. As with choice of culture however, the payoff from choice of action, $y_i$, depends on whether or not $i$ coordinates with his social ties. If individual $i$ and $j$ coordinate on the same action $y_i=y_j$, then the value of $i$'s social tie with $j$ increases by $\alpha$. The parameter $\alpha$ provides a measure of the weight of non-cultural activities relative to cultural activities in the population. When $\alpha=0.5$ cultural activities and non-cultural activities have equal weight. We return to the baseline model when $\alpha=0$.

### 3.1 Population Outcomes

What are the possible outcomes for a population with diverse cultures in the full model? Following the game described in section 3, proposition 4 describes the possible Nash equilibria.

**Proposition 4** *The set of Nash equilibria are as follows:*

*(i). Each individual forms a social tie with all other individuals and all individuals adopt same action set $(y,x)$, where $x\in\{x^A,x^B\}$ and $y\in\{y^C,y^D\}$.*

*(ii). Each individual forms a link to all other individuals of the same type as him but does not form a link to individuals of a different type. Type $A$'s adopt action $x^A$ and type $B$'s adopt action $x^B$.*

[17] Essentially now individual $i$ plays two different $2\times 2$ symmetric coordination games with each social tie, $j$. Individuals $i$ and $j$ each choose actions from common action sets: for the first game $x_i,x_j\in\{x^A,x^B\}$ and for the second game $y_i,y_j\in\{y^C,y^D\}$. In each game payoffs are 1 if individuals $i$ and $j$ coordinate and 0 if not.

*(iii). Each individual forms a social tie with all other individuals. Type A's adopt action $x^A$, type B's adopt action $x^B$ and all individuals adopt the same action $y \in \{y^C, y^D\}$.*

In the outcome described in proposition 4$(i)$, one group adopts the culture of the other group and groups integrate. This outcome is referred to as assimilation, consistent with proposition 1$(i)$, the only difference from the baseline model being that individuals also adopt an additional common action $y$. In the outcome described in proposition 4$(ii)$ groups adopt different cultures and segregate. This is referred to as segregation, consistent with proposition 1$(ii)$. Again the only difference from the baseline is the addition of a choice of action $y_i$ and it is possible that both groups adopt the same $y \in \{y^C, y^D\}$ or alternatively both groups adopt different $y \in \{y^C, y^D\}$. We return to this below. In the baseline model there was no equilibrium where individuals could retain their own culture and socially interact with individuals who adopt a different culture. Proposition 4 reveals a third outcome of this form. In the outcome described in $(iii)$ agents form social ties between and across groups but also retain their respective cultural activities. This outcome is made possible now that agents choose two actions: one action, $x$, where the payoffs from $x$ depend on an individual's type and one action $y$ where the payoffs from $y$ do not depend on an individual's type. The value of a social tie can be positive if individuals coordinate on $y$ without coordinating on $x$, which allows for this third type of equilibrium. We refer to this equilibrium as *multiculturalism.*

We now describe exactly when each of these outcomes will occur and make some remarks about the kind of environments that engender the different outcomes. The following proposition provides a full characterization of when assimilation, separation and multiculturalism will occur. As above we rule out the uninteresting case where an individual is willing to maintain his own culture single-handedly.[18] Let $n_m \in \{n_A, n_B\}$ denote the size of the minority group. Proposition 4 uses the following three conditions:

A. $\quad v((1-L)(n_m-1), 0) \geq v((1-L)(n-n_m), c)$

B. $\quad v((1-L)(n_m-1) + (\alpha - L)(n-n_m), 0) \geq v((\alpha - L)(n_m - 1) + (1-L)(n-n_m), c)$

C. $\quad n_m - 1 \geq \frac{\alpha - L}{1-\alpha}(n - n_m) \quad \text{if} \quad 1 - \alpha - L > 0.$

[18] This requires the inequality

$$v((1-L)x, c) > v((\alpha - L)x, 0)$$

for all $0 < x \leq n-1$. This says that given the individual has $x$ social ties, he does better by coordinating with them on both activities and switching culture than by coordinating only on non-cultural activities and being the only one playing his culture. This is simply saying that culture is a social phenomenon, it does not have value without interaction with others. Of course if $\alpha = 1$ this inequality cannot hold, but if $\alpha = 1$ then culture becomes irrelevant anyway.

**Proposition 5** *Consider any fixed $v(\cdot,\cdot)$, $c$, $n_m$, $n$ and $L$. When $\alpha - L < 0$ then assimilation and separation are Nash equilibrium outcomes if A holds, otherwise assimilation is the unique outcome. When $\alpha - L > 0$ :*

*(i). If A, B and C hold then assimilation, separation and multiculturalism are all Nash equilibrium outcomes;*

*(ii). If B holds but A or C do not hold then assimilation and multiculturalism are Nash equilibrium outcomes;*

*(iii). If A and C hold but B does not hold then assimilation and separation are Nash equilibrium outcomes;*

*(iv). Otherwise assimilation is the unique outcome.*

As in Proposition 2, assimilation is an equilibrium under all parameters and all possible functions $v(\cdot,\cdot)$. Separation is an equilibrium under the same condition given in Proposition 2, the only difference is that here individuals also choose non-cultural actions and so we need the extra condition $C$ to ensure separation is a Nash equilibrium alongside multiculturalism. Multiculturalism was not a possible outcome in Proposition 1, so it remains to explain the conditions under which multiculturalism is an equilibrium. First, multiculturalism requires $\alpha - L \geq 0$. This ensures cultural activities are not all consuming in everyday life such that a meaningful social tie can be sustained when individuals have only non-cultural activities in common. Second for multiculturalism to be an equilibrium $B$ must be satisfied. Condition $B$ states that when both types adopt their own culture but share the same action $y \in \{y^C, y^D\}$ then a minority individual must prefer to retain his own culture and interact with all individuals in the population rather than adopt the majority culture and interact with all individuals in the population. This is much the same trade-off as before: the minority individual still faces a choice between having cultural activities in common only with a small group of people or adopting majority culture and having cultural activities in common with a larger group of people.

The following corollary places three arguably natural restrictions on the function $v$. One, $v$ is continuous. Two, $v$ is weakly concave in the first argument. This implies that the marginal utility of social interaction is nonincreasing: the first friend is at least as valuable as a second and so on. Three, $v_1(x,0) \geq v_1(x,c)$, which implies that the marginal value of an extra social tie cannot be higher when the individual adopts the other group's culture rather than his own.

**Corollary 3** *Multiculturalism is an equilibrium when the minority group is a large enough proportion of the population, that is when $\frac{n_m}{n} \geq \eta^*$, where $\eta^* \in \mathbb{R}$ depends on $v(\cdot,\cdot)$, $c$, $n$ and $L$. Further when $v(\cdot,\cdot)$ is continuous and concave in the first argument and $v_1(x,0) \geq v_1(x,c)$*

*(i). $\eta^*$ is decreasing in cultural cost, $c$;*

*(ii). $\eta^*$ is decreasing in the importance of non-cultural activities, $\alpha$;*

*(iii). multiculturalism is an equilibrium whenever separation is an equilibrium, $\eta^* \leq \gamma^*$.*

Corollary 3 says that multiculturalism occurs only when the proportion of the minority group in a population is large enough. Even when there is cross-cultural interaction and coordination on non-cultural activities $y$, adopting majority culture and coordinating on culture with the majority group raises the value of social ties with majority individuals relative to the value of social ties with minority individuals. Thus adopting majority culture could be optimal if the majority group is large. Corollary 3(i) and (ii) states that when the cultural costs are higher or the importance of non-cultural activities relative to cultural activities is higher, then multiculturalism can be sustained in populations with smaller minority groups. It is important to note the difference between c, which represents the cost of adopting the other group's culture, and $\alpha$ which represents the importance of non-cultural activities relative to cultural activities in the value of a social tie. A large $\alpha$ could represent a population (say a workplace) where most of the activities taking place are unrelated to type and culture. When $\alpha$ is larger and so the importance of non-cultural activities relative to cultural activities increases, then two things happen: one, social ties formed with those who adopt a different culture are more valuable; and two, the loss from adopting a different culture is smaller. Thus when culture becomes relatively less important it is easier to sustain multiculturalism. Finally, corollary 3(iii) states that multiculturalism can be sustained as an equilibrium whenever separation is an equilibrium.

The results from proposition 5 and corollary 3 are illustrated in figure 4 for the same utility function as figure 2: $u_k(s_i, s_{-i}) = \sum_j (\pi(x_i, x_j) - l - \phi c_k(x_i))g_{ij} - \theta c_k(x_i)$.

### 3.2 Emergent Cultures

Assuming individuals can interact on both cultural activities and non-cultural activities not only brings into play a third outcome, multiculturalism, but also allows us to discuss the choices of $x$ and $y$ that emerge in equilibrium. While $x^A$ and $x^B$ are the ex-ante given cultures, we can think of the choices of $x$ and $y$ in equilibrium as the ex-post culture that emerges in a diverse community.

In the multiculturalism equilibrium we can view the adoption of a common $y \in \{y^C, y^D\}$ by both groups as the emergence of a shared 'community culture.' This shared culture allows the different cultural groups to interact while at the same time groups retain their respective cultural activities $x^A$ and $x^B$. In the assimilation equilibrium one group conforms to the other

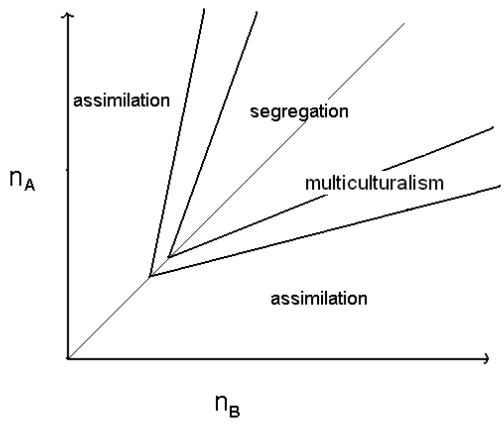


Figure 4: Multiculturalism is an equilibrium under the same qualitative conditions as separation but can be sustained as an equilibrium with a smaller minority group.

group's cultural activities $x$ and both groups conform on action $y$. Once both groups conform to the same action $x$ and each individual interacts with all others, choice of action $y$ behaves like a generic coordination game where the whole community will adopt the same action. Thus there is a single culture $(x, y)$ that arises in equilibrium. In the separation equilibrium, further polarization of activities emerges in equilibrium beyond the different ex-ante given cultures $x^A$ and $x^B$. This is described in corollary 4.

**Corollary 4** *When $\alpha - L \geq 0$ separation is an equilibrium if and only if types A adopt culture $x^A$, type B's adopt culture $x^B$, and the two types adopt different non-cultural activities (that is type A's adopt action $y^C$ and B's adopt action $y^D$ or vice versa).*

While the payoffs an individual receives from his choice of action $y$ are independent of his type, in the separation equilibrium described in corollary 4 action $y^C$ is adopted by one group and action $y^D$ by the other group. Thus action $y^C$ becomes associated with a particular cultural group in equilibrium and $y^D$ with the other cultural group. Think back to our previous example of a neighborhood with two different religious communities. Individuals decide which religious activities to attend at the weekend but on weekdays individuals face a choice between joining the neighborhood soccer team or art class, where playing soccer and doing art are not related to religion or type. In the equilibrium described in corollary 4, in equilibrium both groups attend their respective religious activities and, in addition, soccer becomes associated with one religion while art becomes associated with the other religion.

The cultures that emerge in equilibrium are more polarized in the sense that the diverse groups further differentiate their respective activities when they come together in a single community, beyond their given cultural differences and beyond those that would exist were they interacting in separate communities.

In the equilibrium described in corollary 4, the two groups endogenously take on different actions for $y$, which raises the cost to an individual of interacting with the other group so much so that separation can be sustained. This is one explanation for why some segregated groups seem to have many differences in activities and behavioral norms, and why these differences can sometimes seem arbitrary.[19]

The idea that different cultural groups, when they come into contact, sometimes further differentiate their respective behaviors and activities has been long acknowledged. A Social Science Research Council paper (1954) describes reactions of some native groups to cultural interaction saying 'native forms are reaffirmed and re-enforced by renewed commitment to them.' More recently Harris (2009) describes some minority groups as holding 'secondary cultural differences...that emerge after two groups have been in continuous contact'. This also relates to a phenomenon described as 'Oppositional Culture', where a minority group's view of how people should behave is opposite to majority group behavior. We find that, under certain parameters, segregation is sustained in equilibrium only because different groups adopt behaviors at odds with each other. This appears to be a novel explanation of oppositional culture and we find it to be a positive signal of a model purporting to explain cross-cultural interaction that there is an equilibrium admitting this possibility.[20]

# 4 Welfare and Policy Implications

This section conducts a welfare analysis of the three equilibrium outcomes: assimilation, separation and multiculturalism. For simplicity I restrict assimilation to the equilibrium where the minority group assimilates. Aggregate welfare is assessed by summing individual

[19]Note that the condition $\alpha - L \geq 0$ implies that multiculturalism is also a feasible equilibrium. When $\alpha - L < 0$ multiculturalism is not a feasible equilibrium and corollary 4 no longer holds; in fact there are no restrictions on the actions $y$ chosen in equilibrium when $\alpha - L < 0$.

[20]More complex explanations of oppositional culture exist but it is this key feature to which our results relate. The seminal economic paper on oppositional culture is that of Austen-Smith and Fryer (2005). Their model can explain, for example, why Black students might deride working hard at school as 'acting white' and act in opposition to this. They show that in a two-audience signaling model those of middle ability will adopt a significantly low level of education in order to signal to their peer group that they are worthy of acceptance. See also Akerlof and Kranton (2002), who suppose that when a child's characteristics are far from the ideals proposed by the school (for example if the school promotes typical White middle-class behavior) then the child may lose utility and so may react by adopting an identity with different ideal characteristics from that of the school which are easier for her to obtain. Also Bisin, Patacchini, Verdier, and Zenou (2011) who assume that an individual reduces the psychological cost of interacting with someone who adopts a different culture or behaviors by increasing identification with his own culture.

utilities. To be able to make some statements about welfare I also assume the function $v(\cdot, \cdot)$ is continuous and concave in the first argument, as discussed above. Proposition 6 characterizes the welfare-maximizing equilibria:

**Proposition 6** *(i). When $\alpha - L < 0$, there exists a $\gamma^W$ which depends on $v(\cdot, \cdot)$, $c$, $n$ and $L$, such that assimilation is welfare-maximizing for all $\frac{n_m}{n} \leq \gamma^W$ and separation is welfare maximizing for all $\frac{n_m}{n} \geq \gamma^W$. Also $\gamma^W > \gamma^*$.*

*(ii). When $\alpha - L \geq 0$, then separation is never welfare-maximizing. There exists a $\eta^W$ which depends on $v(\cdot, \cdot)$, $c$, $n$ and $L$, such that assimilation is welfare maximizing for all $\frac{n_m}{n} \leq \eta^W$ and multiculturalism is welfare maximizing for all $\frac{n_m}{n} \geq \eta^W$. Also $\eta^W > \eta^*$.*

Proposition 6$(i)$ states that when multiculturalism is not a feasible equilibrium, separation maximizes social welfare when the minority group is large and assimilation maximizes social welfare when the minority group is small. Segregation allows the minority group to avoid the cost of losing their culture. If this loss is large enough then segregation is social-welfare maximizing, that is the aggregate gain to the minority group from separating outweighs the aggregate loss to majority group from moving from assimilation to separation.[21]

Under the parameters in 6$(ii)$, multiculturalism is welfare-maximizing when the minority group is large and assimilation is again welfare-maximizing when the minority group is small. When multiculturalism is a Nash equilibrium separation is never welfare-maximizing. This is because moving from separation to multiculturalism all individuals do strictly better: individuals retain their respective cultures and interact with the whole population instead of interacting only within group.

Let us now compare welfare-maximizing outcomes with Pareto optimal ones. Proposition 7 characterizes the Pareto optimal outcomes.

**Proposition 7** *(i). Assimilation is always Pareto optimal.*

*(ii). When $\alpha - L < 0$, there exists a $\gamma^P$ which depends on $v(\cdot, \cdot)$, $c$, $n$ and $L$, such that separation is Pareto optimal when $\frac{n_m}{n} \geq \gamma^P$. Also $\gamma^W > \gamma^P > \gamma^*$.*

*(iii). When $\alpha - L \geq 0$, separation is never Pareto optimal. There exists an $\eta^P$ which depends on $v(\cdot, \cdot)$, $c$, $n$ and $L$, such that multiculturalism is Pareto optimal for all $\frac{n_m}{n} \geq \eta^P$. Also $\eta^W > \eta^P > \eta^*$.*

First of all, note that assimilation is always Pareto optimal. This is because the majority group can do no better: they retain their own culture and they maximize social interaction.

[21] In assimilation the majority group adopt their own culture and interact with the whole population whereas in separation they adopt their own culture but interact only with their own group.

It is only the group that assimilates who faces a cost. This suggests that, when a group is able to do so they will put pressure on other groups to assimilate. In the case of immigration, the native group might feel they have the power to call for assimilation by the immigrant group to native culture (as recent calls by European politicians suggest). Similarly, if one group dominates another economically that group might promote assimilation by the economically inferior group.

Propositions 6 and 7 together uncover a tension between segregation maximizing social welfare, segregation being Pareto optimal and segregation being a Nash equilibrium. Segregation becomes Pareto optimal when the minority group is larger than that at which segregation becomes an equilibrium. The minority group segregates in equilibrium before it is Pareto optimal to do so. This is because an individual from the minority group compares whether they are better off interacting with the less numerous minority group but keeping their own culture or joining the majority group and adopting its culture. They do not compare whether the minority group as a whole might be better off assimilating to the majority group. We also see that segregation maximizes social welfare at a higher level of heterogeneity than that at which segregation is Pareto optimal. This is because social welfare maximization also takes into account how segregation reduces the utility of the majority group relative to assimilation, whereas Pareto optimality only takes into account the utility of the minority group. This, combined with the fact that separation is never pareto optimal or welfare-maximizing when multiculturalism is a feasible equilibrium suggests there may be an argument for policies directed at reducing separation between groups.

## 5 Conclusion

We examine a population consisting of groups with diverse cultures. We examine what happens to culture and social interaction in diverse communities. Individuals choose who to interact with from within the population and whether to retain their own culture or not. We find three equilibria: assimilation, separation and multiculturalism. Assimilation is the unique equilibrium when the minority group is small, while separation and multiculturalism can be equilibria for populations with larger minority groups.

Assimilation is both welfare-maximizing and optimal for the minority group when the minority group is small enough: when a minority group is very small, the reduction in social interaction that comes with retaining one's own culture is too costly. However, while assimilation is only optimal for the assimilating group if they form a small minority, assimilation is always the utility-maximizing equilibrium for the group who retain their own culture in assimilation. Thus it should come as no surprise to observe a push for immigrants to assimilate in European countries that have had recent increases in the level of immigration.

Multiculturalism, an ideology previously espoused by Britain and the Netherlands, is both an equilibrium and the welfare-maximizing outcome in populations with a large minority group. However, multiculturalism is only an equilibrium when cultural activities do not dominate everyday life. Indeed this suggests room for policies that aim to increase the possibility for common ground between groups through shared activities (provided these activities are 'culture free'). Only when multiculturalism is not an equilibrium and when the population has a large minority group is separation welfare-maximizing.

Finally, we show that the cultures that emerge in equilibrium can be more complex than the binary decision of retaining one's own culture or not. For example, to sustain multiculturalism as an equilibrium, groups must have a 'community culture' in common as well as retaining their diverse cultural practices. While under certain parameters separation is an equilibrium if and only if the cultures that emerge in equilibrium are further differentiated beyond the ex-ante cultural differences $x^A$ and $x^B$.

This is not a model of discrimination; however, a future step is clearly to interact the ideas of culture and social interaction presented here with a population where different cultural groups have different economic opportunities. These economic opportunities may further shape the benefits of social interaction and one may find a relationship between culture and discrimination in equilibrium.

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# 6 Appendix

## 6.1 Proof of proposition 1 and 2

First I show that these two outcomes are indeed equilibria. Suppose each individual forms a social tie to all others and all individuals play $x^A$ (the argument when all play $x^B$ is symmetric).

The payoff to a type A is $v((1-l)(n-1),0)$. A type A can do no better since the first argument is maximized and the second is minimized. The payoff to a type B is $v((1-l)(n-1),c)$. A type B can deviate by reducing the number of ties and/or changing action. A reduction in ties strictly lowers the first argument with no change in the second. A change in action gives him a negative contribution for each social tie he retains. The best he can do by deviating is thus $v(0,0)$. Thus each individual forms a social tie to all others and all individuals play $x^A$ is an equilibrium when

$$v((1-l)(n-1),c) > v(0,0).$$

This implies that an individual prefers to interact with the whole population and adopt the other group's culture than interact with no one but retain his own culture.

Now suppose the two types form separate groups where each individual of a given type forms a link to everyone else of that type and all the type $A$'s play $x^A$ and all the type $B$'s play $x^B$. An individual can deviate by changing action and/or social ties. If he changes action or social ties independently he does strictly worse. The only alternative is that he changes to the other groups action and changes social ties. The best he can do is discard his social ties and form social ties with all individuals of the other type (who are all playing the other action). Such a deviation, by either type, is not profitable when the following two conditions hold

$$v((1-l)(n_A-1),0) \geq v((1-l)n_B,c)$$
$$v((1-l)(n_B-1),0) \geq v((1-l)n_A,c).$$

Next we show that there are no other equilibria. There are three conditions that must hold in equilibrium. One, in equilibrium everyone playing the same action must have a social tie. If not, an individual can strictly increase the first argument of his utility function without changing the second by forming a social tie to someone playing the same action with whom he does not already have a social tie. This strictly increases his payoff. Two, no individual will form a social tie with another individual playing a different action since this strictly decreases the first argument of his utility function without changing the second. Three, in equilibrium no two individuals of the same type can be playing different strategies (excluding them forming a social tie with each other). If they are undertaking different strategies, one individual must be doing weakly better than the other. The player who is doing weakly worse must either be playing a different action, or have different ties, or both. Suppose he plays a different action. Then the player doing weakly better does not have a tie to him (by condition two) so the player doing weakly worse does strictly better by mimicking the player doing weakly better and forming a tie to him as well. Suppose the player doing weakly worse is playing the same action as the player doing weakly better but has different social ties. From condition one, this is not an equilibrium.

Taking these three conditions, the only possible equilibria are: 1. each individual is linked to all others and all individuals play the same action; 2. the two types form separate groups where each individual of a given type forms a link to everyone else of that type and each group plays a different action. We show that outcome described in 2 must have all the type $A$'s playing $x^A$ and all the type $B$'s playing $x^B$. The only alternative has the A's play $x^B$ and the B's play $x^A$. This is an equilibrium if both the following inequalities hold:

$$v((1-l)(n_A-1),c) \geq v((1-l)n_B,0),$$

$$v((1-l)(n_B-1),c) \geq v((1-l)n_A,0).$$

Which imply respectively $n_A - 1 > n_B$ and $n_B - 1 > n_A$. A contradiction.

### 6.2 Proof of Corollary 1

We can rewrite the inequalities for which separation is an equilibrium as

$$v((1-l)(n_A-1),0) \geq v((1-l)(n-n_A),c) \tag{5}$$

$$v((1-l)(n-n_A-1),0) \geq v((1-l)n_A,c). \tag{6}$$

Inequality (5) is strictly satisfied when $n_A - 1 \geq n - n_A$, or $n_A \geq \frac{n+1}{2}$ and inequality (6) is strictly satisfied when $n - n_A - 1 \geq n_A$, or $\frac{n-1}{2} \geq n_A$. However, for separation to be an equilibrium we need $n_A$ such that both inequalities (5) and (6) are satisfied.

The LHS of inequality (5) is increasing in $n_A$ while the RHS is decreasing in $n_A$, while for inequality (6) the LHS is decreasing in $n_A$ while the RHS is increasing in $n_A$. Since inequality (5) is strictly satisfied when $n_A \geq \frac{n+1}{2}$, there exists an $n_A^* \in \mathbb{N}$ where $n_A^* \leq \frac{n+1}{2}$ such that inequality (5) is satisfied for all $n_A \geq n_A^*$. Similarly since inequality (6) is strictly satisfied when $n_A \leq \frac{n-1}{2}$ there exists an $n_A^{**} \in \mathbb{N}$ where $n_A^{**} \geq \frac{n-1}{2}$ such that inequality (6) is satisfied for all $n_A \leq n_A^{**}$. The function $v$ and cost $c$ are the same for both types so $n_A^{**} = n - n_A^*$. Thus separation is an equilibrium when $n_A$ such that $n - n_A^* \geq n_A \geq n_A^*$. This requires $n_A^* \leq n/2$ so separation is never an equilibrium if $n_A^* \leq n/2$ does not hold. Thus separation is an equilibrium if $n_A$ is in the interval $[n_A^*, n - n_A^*]$ where $n_A^* \leq n/2$. The interval $[n_A^*, n - n_A^*]$ is thus symmetric around $n/2$. We have that, one, separation is not necessarily an equilibrium; two, separation is only an equilibrium when the minority group is large enough; three, if $[n_A^*, n - n_A^*] = [2, n-2]$ separation is always an equilibrium.

How does an increase in $c$ affect $n_A^*$? Fix $c$, the function $v$, and all other parameters. Let $n_A^* \in \mathbb{N}$ be such that for all $n_A \geq n_A^*$ inequality (5) holds. If we increase $c$ then $v$ strictly decreases; therefore, all else equal the LHS of (5) does not change, but the RHS strictly decreases. Thus inequality (5) now holds for all $n_A \geq \hat{n}_A$ where $\hat{n}_A \in \mathbb{N}$ and $\hat{n}_A \leq n_A^*$. As above, since $v$ and $c$ are symmetric for both types inequality (6) holds for all $n_A$ such that $n_A \leq n - \hat{n}_A$. So separation is now an equilibrium when $n - \hat{n}_A \geq n_A \geq \hat{n}_A$.

### 6.3 Proof of Proposition 3

Note first that only strict Nash equilibria are absorbing, so I do not address the parameters where a Nash equilibrium is weak. Second, I am going to assume continuity of $k$ where $k$ is the number of errors, and adjust for the discrete nature of $k$ when necessary. Third, note that I will look at stochastic stability when type B is in the minority. The results when identity group B makes up the majority of the population are symmetric. Finally I will consider stochastic stability when $n_A = n_B$.

We first find the minimum costs (number of mistakes) to transit from one equilibrium to another. We start with a transit from the assimilation equilibrium where all play $x^A$ to the assimilation equilibrium where all play $x^B$ . Suppose all play $x^A$. What is the minimum number of mutations needed to trigger a transit to the equilibrium where all play $x^B$? Let k be the minimum number of mutations. Note that an individual playing $x^A$ will choose only to link with those playing $x^A$, an individual playing $x^B$ will choose only to link with those playing $x^B$. An individual will choose to link to all others who are playing the same action. Thus when given the option to change strategy, an individual compares payoffs from linking with all those playing $x^A$ or all those playing $x^B$ . Since a type A individual gets a lower payoff than a type B individual when they both play action B and form the same links, we need to examine how many mutations it takes for a type A individual to wish to switch and play B. This then implies a transition to the equilibrium where all play $x^B$. The minimum number of mutations required for transition from the equilibrium where all play $x^A$ to the equilibrium where all play $x^B$ is thus the minimum number of errors (where an error is that an individual playing $x_A$ plays $x_B$ instead) such that a type A who is currently playing $x^A$ wants to switch and play $x_B$ also:

$$v((1-l)k, c) > v((1-l)(n-k-1), 0).$$

For a B to want to change we require the following weaker condition

$$v((1-l)k, 0) > v((1-l)(n-k-1), c).$$

If all type B's change to play $x_B$, a type A will want to change to play $x_B$ if

$$v((1-l)n_B, c) > v((1-l)(n_A-1), 0)$$

We assume B are a minority so $n_B \leq n_A - 1$ so this inequality does not hold. Thus the minimum number of errors required for a transit from the equilibrium where all play $x^A$ to the assimilation equilibrium where all play $x^B$ is the minimum k such that

$$v((1-l)k, c) > v((1-l)(n-k-1), 0).$$

We next look at a transit from the equilibrium when all play $x_B$ to the equilibrium where all play $x_A$. The minimum number of mutations required for a type B to choose to play $x_A$ is:

$$v((1-l)k, c) > v((1-l)(n-k-1), 0)$$

For a type A to choose to play $x^A$ the minimum number of mutations is:

$$v((1-l)k, 0) > v((1-l)(n-k-1), c)$$

If all type A's change and play $x_A$ a type $x_B$ will want to change only if

$$v((1-l)n_A, c) > v((1-l)(n_B-1), 0)$$

These parameters denote the parameters for which separation is not a Nash equilibrium when B are in the minority. Hence, under the parameters for which separation is an equilibrium, the minimum number of mutations required to trigger the equilibrium where all play $x_A$ is the minimum k such that

$$v((1-l)k, c) > v((1-l)(n-k-1), 0)$$

Whereas when we are in the parameter range where separation is not an equilibrium, the minimum number of mutations required to trigger the equilibrium where all play $x_A$ is the minimum k such that

$$v((1-l)k, 0) > v((1-l)(n-k-1), c)$$

.

We next examine the transit from the equilibrium where all play $x^A$ to the segregated equilibrium. Clearly, we restrict ourselves to the parameters for which separation is an equilibrium. To transit to the separation equilibrium we require that a type B wants to move from the group playing $x_A$ and play $x_B$ and link to those playing $x^B$ but the type A's wish to remain in the group playing $x_A$. A type B will want to switch and play $x_B$ only if playing $x_B$ and linking with those playing $x_B$ gives more utility than playing $x_A$ and linking with those playing $x_A$. Thus the minimum number of mutations required to transit to separation is the minimum number of individuals playing B required for a type B to wish to switch under best response dynamics. That is, the minimum k such that:

$$v((1-l)k, 0) > v((1-l)(n-k-1), c)$$

A type A individual will not want to play $x_B$ even when all the type B's have transited since this requires

$$v((1-l)n_B, c) > v((1-l)(n_A - 1), 0)$$

but since the B's are in the minority this cannot hold.

We next look at a transit from separation to the equilibrium where all play $x^A$ . We want to find k, the minimum number of individuals playing B who mutate and play $x^A$ such that a type B playing $x^B$ wants to play $x^A$ under best response dynamics:

$$v((1-l)(n_A + k), c) > v((1-l)(n_B - k - 1), 0)$$

Symmetrically, for a transit from the equilibrium where all play $x^B$ to separation we require k mutations where:

$$v((1-l)k, 0) > v((1-l)(n - k - 1), c)$$

A type B individual will not want to play $x_A$ even when all the type A's have transited since this requires

$$v((1-l)n_A, c) > v((1-l)(n_B - 1), 0)$$

but we are looking at the parameters for which separation is a equilibrium so this inequality does not hold. To transit from separation to the equilibrium where all play $x^B$ :

$$v((1-l)(n_B + k), c) > v((1-l)(n_A - k - 1), 0)$$

We now calculate the stochastic potential for each equilibrium.

The stochastic potential of the equilibrium where all play $x^A$, under the parameters for which separation is not an equilibrium, is the minimum k such that the following holds

$$v((1-l)k, 0) > v((1-l)(n - k - 1), c) \tag{7}$$

The stochastic potential of the equilibrium where all play $x^B$, when separation is not an equilibrium, is the minimum k such that

$$v((1-l)k, c) > v((1-l)(n - k - 1), 0) \tag{8}$$

The stochastic potential of the equilibrium where all play $x^A$ is lower, since the minimum k required to satisfy inequality (7) is smaller than that required to satisfy (8). However, if we do not take $k \in \mathbb{R}$ but instead look for the minimum integer which satisfies each of the inequalities respectively, it is possible that the minimum integer k required to satisfy (8) is equal to that required to satisfy (7). This can be the case when at the point when the individual is indifferent between $k \in \mathbb{R}$ social ties and playing his own culture and $n - k - 1$ social ties and playing the other groups culture, the individual needs less than one extra friend

$a < 1$ to make him indifferent between having $k + a$ friends and the other group's culture or $n - k - a - 1$ friends and his own culture. This can be the case when the change in utility with respect to the second argument is small relative to the change in utility with respect to the first argument. I.e. culture is unimportant relative to social ties. In this case, the equilibrium where all play $x^B$ is also stochastically stable.

When separation is an equilibrium we need the following number of mistakes to transit from one equilibrium to another: To transit from the equilibrium where all play $x^A$ to the equilibrium where all play $x^B$ requires the minimum k (which we denote by $\hat{k}$) such that

$$v((1-l)k, c) > v((1-l)(n-k-1), 0).$$

To transit from the equilibrium where all play $x^B$ to the equilibrium where all play $x^A$ requires the minimum k (which we denote by $\hat{k}$ since the inequality below is the same as above) such that

$$v((1-l)k, c) > v((1-l)(n-k-1), 0)$$

To transit from separation to the equilibrium where all play $x^A$ requires the minimum k (which we denote by $k_A$) such that

$$v((1-l)(n_A + k), c) > v((1-l)(n_B - k - 1), 0)$$

To transit from separation to the equilibrium where all play $x^B$ requires the minimum k (which we denote by $k_B$) such that

$$v((1-l)(n_B + k), c) > v((1-l)(n_A - k - 1), 0)$$

To transit from the equilibrium where all play $x^A$ to separation requires the minimum k (which we denote by $k'$) such that

$$v((1-l)k, 0) > v((1-l)(n-k-1), c)$$

To transit from the equilibrium where all play $x^B$ to separation requires the minimum k (which we denote by $k'$ also since the inequality below is the same as the inequality directly above) such that

$$v((1-l)k, 0) > v((1-l)(n-k-1), c)$$

When we do not restrict $k$ to be an integer we have $k > k'$, $k_A < k_B$ and $k_B < k$. When we restrict $k$ to be an integer these inequalities are weak rather than strict. We can use the above to calculate the tree of least resistance for each equilibrium to find the stochastic potential of each equilibrium. The stochastic potential of the equilibrium where all play $x^A$ is $k' + k_A$. The stochastic potential of the equilibrium where all play $x^B$ is either $k' + k_B$ or $k_A + k$. The stochastic potential of separation is $k' + k'$.

When we do not restrict k to be an integer, the stochastic potential of the equilibrium where all play $x^A$ is strictly less than the stochastic potential of the equilibrium where all play $x^B$. When we restrict $k$ to be an integer under some conditions the stochastic potential could be equal. In fact this follows as described previously under certain conditions when the change in utility with respect to the second argument is small relative to the change in utility with respect to the first argument. Or when groups A and B are similar in size and the change in utility with respect to the first argument is small.

We compare the stochastic potential of the equilibrium where all play $x^A$, $k' + k_A$, and the stochastic potential of separation, $k' + k'$. Thus we need to compare $k_A$ and $k'$ which are the minimum k such that the following inequalities are satisfied respectively

$$v((1-l)(n_A + k), c) > v((1-l)(n - n_A - k - 1), 0) \tag{9}$$

$$v((1-l)k, 0) > v((1-l)(n - k - 1), c). \tag{10}$$

Recall we are currently only examining stochastic stability when separation is a Nash equilibrium. That is when inequalities (2) and (**??**) hold. When $k = 0$ neither inequality holds and when $k = n_B - 1$ then both inequalities hold. Fix k equal in inequalities (9) and (10), then the on the LHS the first argument in (9) is greater than in (10) but the second argument is also greater. For the RHS, the first argument is smaller in (9) than (10) but the second argument is also smaller. It is clear that there are cases where $k_A > k'$, $k' > k_A$, and $k_A = k'$. When $k_A < k'$ the equilibrium where all play $x^A$ is uniquely stochastically stable, when $k' < k_A$ separation is uniquely stochastically stable, and when $k' = k_A$ both are stochastically stable.

Fix $v(\cdot,\cdot)$, $n$, $c$, and $l$. The minimum k required to satisfy (9) is decreasing in $n_A$ while the minimum k required to satisfy (10) is unaffected by $n_A$ since $n$ is fixed. As before, since we place limited restrictions on $v$ or $n$, $c$, and $l$, there are cases where $k_A > k'$ for all feasible $n_A$ and cases where $k_A < k'$ for all feasible $n_A$. In the intermediate case, when $n_A$ is high $k_A < k'$ and at some point as $n_A$ falls we find $k_A > k'$. It is possible that for some $n_A$, $k_A = k'$. Note that $k_A = k'$ only for a single value of $n_A$ since if we increase $n_A$ by 1 we can decrease $k_A$ by 1 so $k_A < k'$ and if we decrease $n_A$ by one we must increase $k_A$ by 1, so $k_A > k'$.

If we want to write this in terms of population heterogeneity, write $\gamma^{**} = 1 - \hat{n}_A^2 - (n - \hat{n}_A)^2$ where $\hat{n}_A$ is the level of $n_A$ where $k_A = k'$. Then for all $n_A < \hat{n}_A$ (since $n_A$ is discrete) $k_A > k'$ and for all $n_A > \hat{n}_A$, $k_A < k'$. Since $n_A$ is discrete we have that for all feasible levels of population heterogeneity greater than $\gamma^{**}$ ($n_A < \hat{n}_A$) separation is stochastically stable while for all feasible levels of population heterogeneity less than $\gamma^{**}$ ($n_A > \hat{n}_A$) the equilibrium where all play $x^A$ is stochastically stable. At $\gamma^{**}$ both are stochastically stable.

Finally we must check what happens when $n_A = n_B$. When separation is not stochastically stable (separation is stochastically stable in exactly the conditions described above) then both the equilibrium where all play $x^A$ and the equilibrium where all play $x^B$ will be stochastically stable since the stochastic potential for each is symmetric.

### 6.4 Proof of Corollary 2

Fix $v(\cdot,\cdot)$, $n$, $c$, and $l$. Find $\hat{n}_A$. Now increase $c$. The LHS of (9) decreases while there is no change in the RHS. Thus $k_A$ must weakly increase. Similarly, the RHS of (10) decreases with no change in the LHS, thus $k'$ must weakly increase. Thus with an increase in $c$ the level of $n_A$ at which $k_A = k'$ must weakly increase. Since $\gamma^{**} = 1 - \hat{n}_A^2 - (n - \hat{n}_A)^2$, and we assumed $n_A > n_B$, $\gamma^{**}$ weakly decreases with an increase in $c$.

### 6.5 Proof of Proposition 4, 5 and Corollary 4

There are three conditions that must be satisfied for a Nash equilibrium. One, if an individual has a link to a player playing a particular action set $(x, y)$ then he has a link to everyone else playing that action set. This follows by noting that in equilibrium, if an individual has a social tie with someone playing $(x, y)$ then the value of that tie must be positive, so by forming links to everyone else playing that action set the individual can strictly increase the first argument of his payoff function without changing the second. Two, all individuals of the same type must play the same strategy; if not, the player doing weakly worse can do strictly better by mimicking the player doing weakly better and also forming a social tie to him. Three, an individual will never have a social tie with someone who plays a different action from him for both $x$ and $y$, and will form a social tie with at least everyone who adopts the same action set $(x, y)$. This leaves room for three possibilities: groups A and B adopt different action sets $(x, y)$ and form ties only with their own group; groups A and B adopt different action sets $(x, y)$ and form ties with all others in the population; groups A and B adopt the same set of actions $(x, y)$ and form ties with all others in the population.

Assimilation where groups A and B adopt the same set of actions $(x, y)$ and form ties with all others in the population is an equilibrium under all parameters. This follows by noting that no individual can do better by switching culture (given the assumption $v((1-L)(n-1), c) > v((\alpha - L)(n-1), 0))$, by reducing the number of his ties, nor from any combination of the two. This is the only equilibrium outcome where all individuals of both types adopt the same culture $x$. Suppose not, then all individuals are integrated but the two groups play different actions for $y$ or the two groups are segregated and play different actions for $y$. An individual of either type A or type B must be doing weakly better than the other type in the first argument of the payoff function. An individual of the type who is doing weakly worse can mimic an individual of the type who is doing weakly better and also form/retain a link to him. Since the first argument in his utility function strictly increases and the second does not change, the individual does strictly better.

The only alternative equilibria have the two types adopt different actions for $x$. In such an equilibrium the type A's adopt action $x^A$ and the type B's adopt action $x^B$. Suppose not, then the type B's adopt action $x^A$ and the type A's adopt action $x^B$. Since it is an equilibrium it must be the case that a type B does better by playing $x^A$ rather than $x^B$, but then an A must also do better by playing $x^A$. There are then two possibilities: all individuals form social ties with all others or the two types segregate.

(i). Suppose each individual forms a social tie with all others. For this to be an equilibrium the value of a social tie with the other type must be weakly positive, which requires both individuals to adopt the same action $y$ and that $\alpha - L \geq 0$. The only possibly better strategy would be to mimic the other group. No individual wishes to mimic the strategy of the other group when

$$v((1-L)(n_A-1)+(\alpha-L)n_B,0) \geq v((\alpha-L)(n_A-1)+(1-L)n_B,c) \tag{11}$$

and

$$v((1-L)(n_B-1)+(\alpha-L)n_A,0) \geq v((\alpha-L)(n_B-1)+(1-L)n_A,c). \tag{12}$$

(ii). Suppose individuals form ties only with their own group. For this to be an equilibrium an individual cannot wish to switch culture and form ties only with the other group:

$$v((1-L)(n_A-1),0) \geq v((1-L)n_B,c) \tag{13}$$

$$v((1-L)(n_B-1),0) \geq v((1-L)n_A,c). \tag{14}$$

He must not wish to keep his culture and ties and form ties with the other group. This holds if $\alpha < L$. If $\alpha > L$ then the group must adopt a different action $y$ and if $1-\alpha-L>0$ it must also be the case that

$$v((1-L)(n_A-1),0) \geq v((1-\alpha-L)(n_A-1)+(\alpha-L)n_B,0) \tag{15}$$

and

$$v((1-L)(n_B-1),0) \geq v((1-\alpha-L)(n_B-1)+(\alpha-L)n_A,0) \tag{16}$$

which requires

$$n_m - 1 \geq \frac{\alpha-L}{\alpha}(n-n_m) \tag{17}$$

where $n_m$ is the size of the minority group.

### 6.6 Proof of corollary 3

We can rewrite the inequalities for which multiculturalism is an equilibrium as

$$v((1-l)(n_A-1)+(\alpha-l)(n-n_A),0) \geq v((\alpha-l)(n_A-1)+(1-l)(n-n_A),c) \tag{18}$$

$$v((1-l)(n-n_A-1)+(\alpha-l)n_A,0) \geq v((\alpha-l)(n-n_A-1)+(1-l)n_A,c). \tag{19}$$

Inequality (18) is strictly satisfied when $n_A - 1 \geq n - n_A$, or $n_A \geq \frac{n+1}{2}$ and inequality (19) is strictly satisfied when $n - n_A - 1 \geq n_A$, or $\frac{n-1}{2} \geq n_A$. The LHS of inequality (18) is increasing in $n_A$ while the RHS is decreasing in $n_A$. For inequality (19) the LHS is decreasing in $n_A$ while the RHS is increasing in $n_A$. As before, for multiculturalism to be an equilibrium we need $n_A$ such that both inequalities are satisfied. The argument follows as in the proof of corollary 1.

Fix $\alpha$ and let $n_A^\dagger \in \mathbb{N}$ be the value such that inequality (18) holds for all $n_A \geq n_A^\dagger$. If multiculturalism is an equilibrium then $n_A^\dagger \leq n/2$. Suppose we are at an equilibrium

$$v((1-l)(n_A^\dagger-1)+(\alpha-l)(n-n_A^\dagger),0) \geq v((\alpha-l)(n_A^\dagger-1)+(1-l)(n-n_A^\dagger),c) \tag{20}$$

Take derivatives with respect to the first argument on the LHS:

$$v_1'((1-l)(n_A^\dagger-1)+(\alpha-l)(n-n_A^\dagger),0)(n-n_A^\dagger) \tag{21}$$

and RHS

$$v_1'((\alpha-l)(n_A^\dagger-1)+(1-l)(n-n_A^\dagger),c)(n_A^\dagger-1) \tag{22}$$

The first term of expression (21) is strictly greater than the first term of expression (22) since $(1-l)(n_A^\dagger-1)+(\alpha-l)(n-n_A^\dagger) < (\alpha-l)(n_A^\dagger-1)+(1-l)(n-n_A^\dagger)$, then $v_1'((1-l)(n_A^\dagger-1)+(\alpha-l)(n-n_A^\dagger),0) \geq v_1'((\alpha-l)(n_A^\dagger-1)+(1-l)(n-n_A^\dagger),0) \geq v_1'((\alpha-l)(n_A^\dagger-1)+(1-l)(n-n_A^\dagger),c)$. Since we assume $v_1''(\cdot,\cdot) \leq 0$ and $v_1'(x,0) \geq v_1'(x,c)$. The second term in expression (21) is strictly less than the second term in expression (22) since we assume multiculturalism is an equilibrium.

The rate of change of the LHS in inequality (20) is higher than the rate of change of the RHS with an increase in $\alpha$. Thus as we increase $\alpha$ the value of $n_A^\dagger$ is weakly decreasing.

Separation is an equilibrium for all $n_A \geq n_A^*$ where

$$v((1-l)(n_A^*-1),0) \geq v((1-l)(n-n_A^*),c). \tag{23}$$

Multiculturalism is an equilibrium for all $n_A \geq n_A^\dagger$ where

$$v((1-l)(n_A^\dagger-1)+(\alpha-l)(n-n_A^\dagger),0) \geq v((\alpha-l)(n_A^\dagger-1)+(1-l)(n-n_A^\dagger),c) \tag{24}$$

We see that multiculturalism happens at a lower level of heterogeneity than separation by setting $\alpha = l$ in (21) and (22), then we see that the argument above continues to hold.

### 6.7 Proof of Proposition 6

Assume type B's are in the minority where $n_B \leq n_A$ and the type B's assimilate. Separation is welfare maximizing relative to assimilation where the B group assimilates when:

$$n_B[v((1-l)(n_B-1),0)-v((1-l)(n-1),c)] \geq n_A[v((1-l)(n-1),0)-v((1-l)(n_A-1),0)] \quad (25)$$

The RHS is always positive. The LHS is positive only when a type B earns greater utility by separating than assimilating. This occurs when $n_A$ is low enough. Note than at the threshold level of heterogeneity at which separation becomes an equilibrium we have $v((1-l)(n_B-1),0) = v((1-l)n_A, c)$ inequality (26) does not hold as the LHS is negative. Thus separation is only welfare-maximizing at a higher $n_A$.

Recall $n_A + n_B = n$ so we can rewrite inequality (26) as

$$(n-n_A)[v((1-l)(n-n_A-1),0)-f((1-l)(n-1),c)] \geq n_A[v((1-l)(n-1),0)-v((1-l)(n_A-1),0)] \quad (26)$$

What really matters is the two second terms on either side. Both are decreasing in $n_A$. If the LHS term is increasing faster than the RHS term then at some $n_A$ they are equal and below that $n_A$ the LHS is greater. Since the first term on the LHS is increasing as $n_A$ falls and the first term on the RHS is decreasing as $n_A$ falls, once the LHS is greater than the RHS for some $n_A$ it is like that for all $n_A$ lower. So we need $v_1((1-l)(n-n_A-1),0) > v_1((1-l)(n_A-1),0)$ which is actually only true for $n_A$ in the majority and if $v$ concave.

Compare multiculturalism and separation:

$$\begin{gathered} n_A v((1-l)(n_A-1)+(\alpha-l)n_B,0)+n_B v((1-l)(n_B-1)+(\alpha-l)n_A,0) \geq \\ n_A v((1-l)(n_A-1),0)+n_B v((1-l)(n_B-1),0) \end{gathered} \quad (27)$$

Thus multiculturalism is welfare-maximizing relative to separation whenever it is an equilibrium, when $\alpha > L$.

Compare multiculturalism and assimilation:

$$\begin{gathered} n_A v((1-l)(n_A-1)+(\alpha-l)n_B,0)+n_B v((1-l)(n_B-1)+(\alpha-l)n_A,0) \geq \\ n_A v((1-l)(n-1),0)+n_B v((1-l)(n-1),c) \end{gathered} \quad (28)$$

$$\begin{gathered} n_B[v((1-l)(n_B-1)+(\alpha-l)n_A,0)-v((1-l)(n-1),c)] \geq \\ n_A[v((1-l)(n-1),0)-v((1-l)(n_A-1)+(\alpha-l)n_B,0)] \end{gathered} \quad (29)$$

Rewrite

$$(n-n_A)[v((1-l)(n-n_A-1)+(\alpha-l)n_A,0)-v((1-l)(n-1),c)] \geq \quad (30)$$

$$n_A[v((1-l)(n-1),0) - v((1-l)(n_A-1)+(\alpha-l)(n-n_A),0)]$$

Need to see what is happening with the second terms on either side. Suppose at some $n_A$ $[v((1-l)(n-n_A-1)+(\alpha-l)n_A,0) - v((1-l)(n-1),c)] > [v((1-l)(n-1),0) - v((1-l)(n_A-1)+(\alpha-l)(n-n_A),0)]$. Then as $n_A$ continues to fall until $n_A = n_B$ we need the LHS increasing faster than the right hand side thus the LHS decreasing faster than the RHS with a fall in $n_A$ : $-(1-\alpha)v_1((1-l)(n-n_A-1)+(\alpha-l)n_A,0) \geq -(1-\alpha)v((1-l)(n_A-1)+(\alpha-l)(n-n_A),0)$ so $v_1((1-l)(n-n_A-1)+(\alpha-l)n_A,0) \leq v((1-l)(n_A-1)+(\alpha-l)(n-n_A),0)$ which holds if $v$ is concave and since $n_A \geq n_B$. Thus multiculturalism is welfare-maximizing when the minority group is large enough otherwise assimilation is welfare-maximizing.

### 6.8 Proof of Proposition 7

Separation is Pareto optimal if (we have to examine the group who assimilates here as the group who is assimilated into can always do better with assimilation):

$$v((1-l)(n_B-1),0) \geq v((1-l)(n-1),c) \tag{31}$$

This is with a larger minority group than that at which separation is a Nash equilibrium but a smaller minority group than that at which separation is welfare maximizing.

Multiculturalism is Pareto optimal when (again we look at the group who assimilates):

$$v((1-l)(n_B-1)+(\alpha-l)n_A,0) \geq v((1-l)(n-1),c) \tag{32}$$

This is with a larger minority group than that at which multiculturalism is a Nash equilibrium but a smaller minority group than that at which multiculturalism is welfare maximizing.