Truthtelling in Matching Markets*

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Abstract

We analyze a dynamic search and matching model with asymmetric information. Matched agents go through an evaluation phase before they discover each other's types and decide to marry or not. Before deciding to enter this phase, agents can send a cheap-talk message to their partner. We study whether and how this opportunity of communication shapes search strategies and, ultimately, the matching that arises in a stationary equilibrium. We show that credible information revelation is possible and depends on whether agents can stick to their initial choices once true types are revealed. A full characterization of the matching configurations emerging in equilibrium with or without truthful communication is provided. Information revelation can modify the final matching in that it affects how picky agents are. Whether agents benefit or not from such changes is examined.

Keywords: Credibility of Communication; Frictions; Information Transmission; Marriage; Matching; Search.

JEL Classification:

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"Only those who will risk going too far can possibly find out how far one can go." T.S. Eliot

1 Introduction

Final decisions about matching are taken most of the time after an *evaluation period*: employers use probation period to screen workers; men and women go through several 'dates' before they consider themselves officially 'in a relationship'; researchers take time before they start working together, etc. It is therefore not surprising that people make great efforts convincing potential partners to start this evaluation period: in the labor market, people spend hours polishing their CVs and preparing for a first interview; in the academic market, candidates learn how to behave well in talks and dinners; in the marriage market, dozens of websites give advice on how to write a profile on a dating website and what to do on a first date; etc

Competition for being matched with the "best" partners makes *information transmission* critical in the preliminary stage of a partnership: while everyone is willing to make a good first impression, the more desired partners also want to discourage the less gifted or talented ones. This raises the question of how much information can be transmitted through cheap talk communication – that is, unverifiable and costless announcement – before the decision to start an evaluation period is taken. Intuitively, informative communication is possible if one can credibly announce that he will reject unwanted partners once he has more information. For instance, if an employer is believed to offer a job that requires high qualifications, there is no interest for a lower-qualified applicant to overreport his abilities. Indeed, even if such a lie might help the applicant to get a trial period, he will never get the job once the truth is known. On the contrary, imagine that the applicant has some chances that, once the evaluation time has elapsed, the employer considers him for the job because it would take too long to find another candidate. In that case, entering an evaluation period can be worth a lie about abilities.

How far one can stick to his announcement once he has learnt all relevant information to take an informed decision? This paper aims at explaining why cheap talk communication may be informative in a matching market and how it shapes search strategies and, ultimately, the matching between employers and workers/men and women/etc. The analysis builds on a dynamic search and matching model with asymmetric information, agents who all prefer higher partners and non-transferable utility. In order to fix terminology, we use marriage as a metaphor for the matching problem analyzed in this article. We depart from the existing literature (see the discussion below) in one important way: we assume that discovering a partner's type is time consuming. Precisely, before they can enjoy the gain from matching, partners must go through the following process: first, they are randomly matched into pairs; second, they communicate bilaterally (communication strategy); third, they decide whether or not to start an evaluation phase (trial strategy) during which they cannot search for another partner; last, at the end of the evaluation phase, types are revealed and they decide whether to marry or not (marriage strategy).

The model captures the idea that communication can be informative when it allows agents to avoid spending time in evaluation phases at the end of which matched agents would have not marry anyway. More precisely, informative communication arises in two situations: first, when the higher types or the lower types can credibly announce that they will reject the lower types at the end of an evaluation phase; second, when no one cares about telling the truth because this information has no impact on final marriage decisions. In the first situation, the matching is *assortative* when the higher types manage to discourage the lower types for pretending being of high type, and the matching is *upward* when the lower types coordinate on rejecting each other so as to search for a more profitable match. In the second situation, everyone accepts everyone: the matching is said to be *random*.

When communication is informative, everything is as if the final decision about marriage were made as soon as the two agents meet: given that the other agent announces his true type, an agent accepts to start an evaluation phase only if he is willing to marry the other agent. By advancing the marriage decision, truthful communication has two effects. First, it induces search externalities at the trial decision stage. For instance, if a low type agent refuses to start an evaluation phase with another low type agent, he does not internalize that his decision increases the proportion of low type in the singles' population and, therefore, adversely affects the probability of meeting a high type for everyone. In other words, under communication, through the trial strategies, marriage strategies have a feedback effect on the steady state composition of the singles' population, and conversely. Second, everything else being equal, the opportunity cost of continuing to search is lower under informative communication because agents no longer lose time in unfruitful evaluation phases. In other words, communication provides *everyone* with stronger incentives to reject lower types agents. When theses incentives are the strongest for the higher (lower) types, the matching is assortative (upward).

The same matching outcomes (assortative, upward or random) are possible with or without informative communication. However, moving from a situation where no information is transmitted between agents to a situation where they truthfully communicate, not all transitions are possible. More precisely, we show that the only possible transitions are such that the higher types become more picky – meaning that they are more likely to reject the lower types –, while the lower types become pickier only if the high types do not. From a welfare point of view, we show that: (i) the higher types are better off under communication iff the lower types accept each other; (ii) the lower types are better off under communication iff communication makes them become more picky than it does for the higher types.

Related literature Our paper is related to the literature on search and matching models. The literature separates into two strands depending on whether utility is assumed to be transferable (Sattinger (1995), Shimer and Smith (2000), Atakan (2006)) or non-transferable (McNamara and Collins (1990), Bloch and Ryder (2000), Eeckhout (1996), Burdett and Coles (1997) and Smith (2006)). A common assumption in search and matching models is that all relevant information for the matching is revealed right after the two agents met. Putting it differently, there is no evaluation period and, therefore, there is no room for communication. As already mentioned, we assume that discovering a partner's type is time consuming.

The focus of the literature has been on finding conditions under which the matching that arises in the steady state is "similar" to the matching that would arise absent any search frictions. For instance, under NTU and identical ordinal preferences, it is well known that a perfectly assortative matching arises under complete information. Smith (2006) shows that a similar outcome obtains when there are search frictions if the surplus function is log-supermodular in the partners' types. Our approach is different. We would like to compare the matching when no information is transmitted to the matching under truthful communication. Therefore, we also need to characterize situations where a matching different from the assortative one arises in equilibrium.

An alternative approach to information transmission in matching markets is to assume that information transmission takes place *before* the matching occurs. Agents may be matched according to costly signals (in the sense of Spence (1973)). This issue has been analyzed extensively by the litterature on matching tournaments (see, e.g., Peters and Siow (2002), Chiappori et al. (2009), Hoppe et al. (2009), Mailath et al. (2011)) which studies how pre-marital investment or investment before trading shape the matching between men and women/buyers and sellers/etc. The informativeness of cheap talk communication has also be investigated in directed search models (Menzio (2007), Kim and Kircher (2011)). Last, Lee and Schwarz (2007) look at communication to all the agents of the other side of the market before the interviews in a market design perspective.

The paper is organized as follows. Section 2 describes the model. Then, we characterize the equilibria when there is no communication (Section 3) and when communication is informative (Section 4). Section 5 compares the matching that arises under no-communication and informative communication. In Section 6, we investigate whether agents benefit or not from being able to communicate. Section 7 discusses some extensions to our framework. Section 8 concludes.

2 Model

Matching Environment and Preferences. We consider one population of an atomless continuum of agents searching for a marriage partner.[1] The size of the population is normalized to one. It is made up of heterogeneous agents: a proportion $\lambda_h = \lambda \in (0, 1)$ of agents are of high type h and a proportion $\lambda_l = 1 - \lambda$ are of low type l. Type is time-invariant and private information to each agent.

Time is discrete and the horizon is infinite. At the beginning of each period, every agent in the market is either *single* or in an *evaluation phase*. An evaluation phase consists of a stochastic number of periods within which two agents stay matched with each other, while both are still unmarried. In every period of the evaluation phase, there is a probability $\beta \in (0, 1)$ that the evaluation phase ends before the end of the current period; with probability $1 - \beta$ the evaluation phase goes on to the next period. The marriage decisions take place at the end of the evaluation phase. When two agents marry, they leave the market and are immediately replaced by singles of the same types.[2]

When a type-i agent and a type-j agent, $(i, j) \in \{l, h\}^2$, get married, the flow payoff is $u_{ij} > 0$ to type i and $u_{ji} > 0$ to type j. All agents have identical ordinal preferences: they get a higher payoff when married with a high type rather than with a low type agent, i.e., $u_{ih} > u_{il}$, for all $i \in \{l, h\}$. The per-period payoff of an unmarried agent, either in an evaluation phase or single, is assumed to be the same regardless of his type, and is normalized to 0. Agents maximize their discounted expected payoffs at the common rate $0 < \delta < 1$.

Communication and Trial Strategies. At each date t, single agents are randomly matched into pairs. When meeting, they do not immediately observe each other's type.

[1] Our conclusions apply equally in a two populations model as long as the surplus function is symmetric. Most papers in the literature assume symmetry of the surplus function.

[2] This is known in the literature as a "cloning assumption". It greatly simplifies the analysis by limiting the number of possible states in which an agent might be. See, e.g., McNamara and Collins (1990), Bloch and Ryder (2000), Adachi (2003) and the discussion in Smith (2011).

Instead, within the period where their match occurred, every pair of agents can strategically communicate through direct cheap talk before deciding whether or not to enter an evaluation phase with each other.

Communication takes the following form: each of the two types of agents sends a costless and non-verifiable message from the set $\{h, l\}$ to his partner. Precisely, an agent's steady state *communication strategy* is a mapping from the set of his possible types $\{h, l\}$ into the set of available messages $\{h, l\}$.[3] The strategy is babbling if the same message is sent by the two types of agents. If all the agents of a given type always send a different message from all agents of the other type, then the communication strategy is fully revealing. Without loss of generality, we consider fully-revealing strategies where all type-h agents send message h and type-l agents send message l.

After having sent a cheap-talk message and received one from his partner, each agent takes a decision about whether or not to enter an evaluation phase. For each agent at that step, a (steady-state) *trial strategy* is a mapping from his type and the cheap-talk message he received at that period to a probability to enter or not. We denote by $\tau_{ij} \in [0, 1]$ the trial strategy of a type-i agent who got message j from his partner: τ_{ij} is the probability that type i accepts to enter the evaluation phase after receiving message j from his partner.[4] Both consents to enter the evaluation phase are required for the matched agents to enter the evaluation phase. In case of a refusal, both agents go back to the market as singles. Two agents who have been matched, have exchanged messages and agreed to give a trial to each other finally enter an evaluation phase which ends within that same period with probability β. It is only at the end of this evaluation phase that both agents learn each other's types. No information is released as long as the evaluation phase goes on to a subsequent period.

Marriage Strategies. In each period, agents whose evaluation phase ends finally discover their partner's type and then decide simultaneously whether or not to marry. For each player, a (steady-state) *marriage strategy* then maps his type and the type of his partner into a probability to accept the marriage. We denote by $\mu_{ij} \in [0, 1]$ the marriage strategy of a type-i agent who observes the type j of his partner: μ_{ij} is the probability that type i agrees to marry type j at the end of an evaluation phase. When one agent refuses to marry, the marriage cannot occur and both agents go back to the market as singles. When both agents mutually agree to marry, they leave the market and are immediately replaced by singles of

[3] We only study equilibria in pure communication strategies.

[4] Since there is a continuum of agents of each type, this is equivalent to the situation in which a proportion τ_{ij} of type-i agents accept to enter the evaluation phase after receiving message j from their partner.

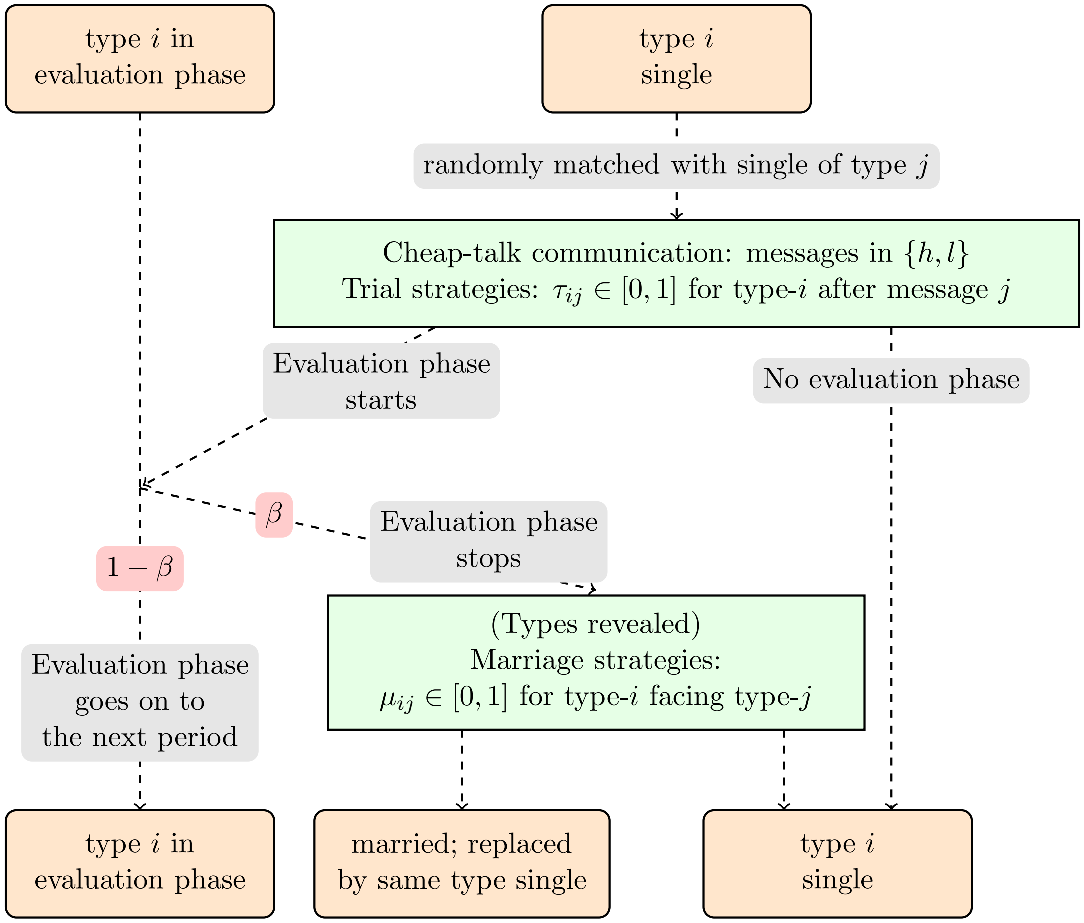

Figure 1: Timing of a Typical Period.

the same types. Note that it is a weakly dominated strategy for each type to refuse the marriage with a type-h agents, i.e., $\mu_{ih} = 1$ for all i.[5]

The timing of the game in a typical period is summarized in Figure 1.

Steady State Equilibrium. We are looking for steady state (i.e., time invariant) perfect[6] Bayesian equilibria in a steady state environment. In every period, the (steady) state of the game is given by $\langle n_i, n_{ij} : (i,j) \in \{l,h\}^2 \rangle$, where n_i is the number of single type-i agents at the beginning of the period and $n_{ij} = n_{ji}$ is the number of type-i agents who are in their

[5]This is the key difference with transferable utility (TU) models. Under TU, we would also have to consider existence of stationary equilibria in which type-h agents only accept type-l and reject type-h agents. See Smith (2011) for a comparison of TU and NTU search and matching models.

[6]Compared to Nash equilibrium, perfection only plays a role in the cheap talk extension of the game: it requires that an agent who realizes at the end of an evaluation phase that his partner lied about his type still chooses an optimal marriage decision, even though this occurs off the equilibrium path (Nash equilibrium does not make any sequential rationality restriction at such information set).

evaluation phase with a type-j agent as the period starts.

Both the decisions to give a trial or to marry are taken simultaneously and require mutual consent by the two partners. It follows that if one of the agents refuses the evaluation phase or the union, the other agent is indifferent between accepting it or rejecting it. To resolve this ambiguity, we focus on equilibria in which, if matching weakly increases both agents' payoffs, then they both accept the evaluation phase and the union.

For simplicity of exposition, an equilibrium in which agents' communication strategy is fully revealing will be called an *informative communication equilibrium*,[7] and it induces the same outcome as an equilibrium of the game under complete information. An equilibrium in which the communication strategy is babbling is called a *no-communication* equilibrium since it induces the same outcome as an equilibrium of the game without communication possibilities.

3 No-Communication Equilibria

We first characterize no-communication equilibria, i.e., the different matching configurations that can occur in equilibrium when no information is ever transmitted through cheap-talk messages. In that case, a single agent always accepts to enter an evaluation phase: $\tau_{ij} = 1$ for every $i, j \in \{h, l\}$. Indeed, agents have no information on which to condition their choice of giving a try or not, and we ruled out cases in which players coordinated on the dominated equilibrium where no one never enter an evaluation phase with anyone.

3.1 Steady State

Since agents whose evaluation phase ends either return to the marriage market as singles or are replaced by single agents of the same type, the steady state $\langle n_i, n_{ij} : (i,j) \in \{l,h\}^2 \rangle$ is the same in every no-communication equilibrium. In particular the proportion of single type-i agents is identical to the overall proportion of type-i agents ($\frac{n_i}{n_l+n_h} = \lambda_i,\ i = l, h$). The number of single type-i agents is equal to the number of type-i agents whose evaluation phase ends:

$$n_i = \beta(n_{il} + n_{ih} + n_i). \tag{1}$$

The number of type-i agents matched with type-j agents in the evaluation phase is equal

[7] Our definition of "communication equilibrium" is rather narrow since we only consider simultaneous face-to-face communication in pure strategies (it does not correspond to the more general notion introduced by Myerson (1982) which allows general mediated communication).

to the number of such agents whose evaluation phase continues from a period to the next plus the number of single type-i agents that are newly matched with type-j agents:

$$n_{ij} = (1-\beta)\left(n_{ij} + n_i \frac{n_j}{n_i + n_j}\right). \tag{2}$$

For every $i \in \{l, h\}$, type-i agents in the market are either singles or in an evaluation phase with some type-j agent, so we have: $n_i + n_{il} + n_{ih} = \lambda_i$. We use this equation to rearrange (1) and (2) and get:

$$n_i = \beta\lambda_i \text{ and } n_{ij} = (1-\beta)\lambda_i\lambda_j. \tag{3}$$

3.2 Equilibria

We now look at the conditions for any of the following three possible matching configurations to occur in equilibrium: Positive Assortative Matching (PAM) when agents of similar types get married together, Random Matching (RM) when agents end up randomly married and Upward Matching (UM) when low types end up being married only with high types whereas high types marry every type. In the no-communication case, agents decide to accept or reject a given type at the end of the evaluation phase. At that point of time, agents compare the immediate gain of marriage to the continuation payoff they get when returning to the single status.

In what follows, we denote by V_i the continuation equilibrium payoff of a single type-i agent, and V_{ij} the continuation equilibrium payoff of a type-i agent who is in the evaluation phase with a type-j agent. The corresponding continuation payoffs following an equilibrium deviation by a type-i agent are respectively denoted by $\tilde{V}_i$ and $\tilde{V}_{ij}$.[8] We will also use the following notation

$$\zeta(x) := \frac{x}{1-(1-x)\delta}, \tag{4}$$

with $x \in (0,1)$, and observe that $\zeta(x) \in (0,1)$ is increasing in x and δ, and $\zeta(xy) < \zeta(x)\zeta(y)$ for all $x, y \in (0,1)$.

No-Communication PAM Equilibrium ($\mu_{hl} = 0, \mu_{ll} = 1$). In such an equilibrium, every type-l agent accepts to marry any other agent independently of his type, while every type-h agent accepts to marry only type-h ones. First, note that if $\mu_{hl} = 0$ (i.e., type-h always

[8]Notice that since we are looking at steady state equilibria and since there is a continuum of players (so that a unilateral deviation does not modify the state of the game), any unilateral deviation in a single period is profitable if and only if the corresponding deviation for all remaining periods is profitable.

reject type-l), then in equilibrium type-l agents should indeed always accept type-l agents, i.e., $\mu_{ll} = 1$, as they would obtain 0 in each period otherwise. Therefore $(\mu_{hl} = 0, \mu_{ll} = 1)$ is an equilibrium iff type-h agents are not willing to deviate. A type-h agent who accepts to marry a type-l agent receives a flow payoff u_{hl}; if he refuses to marry a type-l agent, he receives his equilibrium continuation payoff δV_h, where

$$\begin{aligned} V_h &= \lambda \underbrace{(\beta u_{hh} + (1-\beta)\delta V_{hh})}_{V_{hh}} + (1-\lambda) \underbrace{(\beta \delta V_h + (1-\beta)\delta V_{hl})}_{V_{hl}} \\ &= \lambda \frac{\beta}{1-(1-\beta)\delta} u_{hh} + (1-\lambda) \frac{\beta}{1-(1-\beta)\delta} \delta V_h \\ &= \frac{\lambda\beta}{1-(1-\beta\lambda)\delta} u_{hh} = \zeta(\lambda\beta) u_{hh} \end{aligned} \tag{5}$$

Therefore, $(\mu_{hl} = 0, \mu_{ll} = 1)$ is an equilibrium iff $u_{hl} \leq \delta V_h$, i.e.,

$$\frac{u_{hl}}{u_{hh}} \leq \delta \zeta(\lambda\beta). \tag{6}$$

No-Communication RM Equilibrium ($\boldsymbol{\mu_{hl} = 1, \mu_{ll} = 1}$). In such an equilibrium, all types of agents accept to marry any type of agent. A type-h agent who marries a type-l agent receives a flow payoff u_{hl}; if he rejects a type-l agent, he receives his continuation payoff $\delta \tilde{V}_h$. Using a similarly calculation as before, we get

$$\tilde{V}_h = \zeta(\lambda\beta) u_{hh}. \tag{7}$$

A type l who marries a type a type l receives a flow payoff u_{ll}; if he rejects a type l, he receives his continuation payoff $\delta \tilde{V}_l$, where

$$\begin{aligned} \tilde{V}_l &= \lambda \underbrace{\left(\beta u_{lh} + (1-\beta)\delta \tilde{V}_{lh}\right)}_{\tilde{V}_{lh}} + (1-\lambda) \underbrace{\left(\beta \delta \tilde{V}_l + (1-\beta)\delta \tilde{V}_{ll}\right)}_{\tilde{V}_{ll}} \\ &= \lambda \frac{\beta}{1-(1-\beta)\delta} u_{lh} + (1-\lambda) \frac{\beta}{1-(1-\beta)\delta} \delta \tilde{V}_l = \zeta(\lambda\beta) u_{lh}. \end{aligned}$$

Therefore, $(\mu_{hl} = 1, \mu_{ll} = 1)$ is an equilibrium iff $u_{hl} \geq \delta \tilde{V}_h$ and $u_{ll} \geq \delta \tilde{V}_l$, i.e.,

$$\frac{u_{hl}}{u_{hh}} \geq \delta \zeta(\lambda\beta) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \geq \delta \zeta(\lambda\beta). \tag{8}$$

No-Communication UM Equilibrium ($\boldsymbol{\mu_{hl} = 1, \mu_{ll} = 0}$). In such an equilibrium, every type-l agent accepts to marry only type-h agents while every type-h agent accepts to marry any type of agent. Following the same logic as before, this is an equilibrium iff

$$\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\lambda\beta) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \leq \delta\zeta(\lambda\beta). \tag{9}$$

No-Communication Mixed Equilibria ($\boldsymbol{\mu_{ij} \in (0,1)}$). A mixed equilibrium is an equilibrium in which marriage strategies are not the same for all agents of some type i, so every type-i agent should be indifferent between marrying a type-j agent or not. As already mentioned, in any equilibrium we have $\mu_{hh} = \mu_{lh} = 1$; and from the analysis above, $\mu_{hl} \in (0,1)$ implies $\frac{u_{hl}}{u_{hh}} = \delta\zeta(\lambda\beta)$, and $\mu_{ll} \in (0,1)$ implies $\frac{u_{ll}}{u_{lh}} = \delta\zeta(\lambda\beta)$. Hence, no-communication mixed equilibria only exist for non generic sets of parameters of the game.

The following proposition summarizes the characterization of no-communication equilibria. For every set of parameters of the game, the no-communication equilibrium is unique. The zones of existence of each of these equilibria appear are illustrated by Figure 2.

Proposition 1. *Generically, there is a unique* no-communication equilibrium*. It is such that $\tau_{ij} = 1$ for $i, j \in \{h, l\}$, and*

- *the matching is Positive Assortative if $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda\beta)$,*
- *the matching is Random if $\frac{u_{hl}}{u_{hh}} > \delta\zeta(\lambda\beta)$ and $\frac{u_{ll}}{u_{lh}} > \delta\zeta(\lambda\beta)$,*
- *the matching is Upward if $\frac{u_{hl}}{u_{hh}} > \delta\zeta(\lambda\beta) > \frac{u_{ll}}{u_{lh}}$.*

For each matching configuration, incentives of players are affected by their continuation value of being single when they reject type-l agents, which increases with the common threshold $\delta\zeta(\lambda\beta)$. Hence, when the threshold $\delta\zeta(\lambda\beta)$ increases, incentives to reject type-l agents are stronger. This threshold is the same whatever the matching configuration because the proportion of single type-h agents (λ) is not affected by the matching configuration. This explains why there is generically a unique no-communication equilibrium, a feature that will be lost in informative communication equilibria in which the proportion of single type-h agents will be affected by the matching configuration.

The threshold $\delta\zeta(\lambda\beta)$ is increasing in the parameters δ, λ and β and their effects are easy to understand. First, as the present value of a future marriage increases with δ, the continuation value of being single increases with δ. Next, it also increases with β which determines the expected length of the next evaluation period: the larger is β, the shorter is the expected time to the next potential gain of marriage for a single. In that sense, a large β makes being single relatively more attractive than getting an immediate gain of marriage. Finally, we have that $\delta\zeta(\lambda\beta)$ increases with λ. It is clear that, because every agent accept type-h agents, the continuation payoff of any single agent who rejects type-l agents increases with the proportion of single type-h agents available in the steady state.

One can state more generally this *Proportion Effect*, which tells how incentives are affected by changes in the steady-state proportions of singles: when the steady-state proportion of singles of a given type increases, the continuation payoff of agents accepting this type, in equilibrium or as a result of a deviation, increases. Consequently, incentives to reject these agents are smaller and incentives to deviate and accept them are greater. The reverse is true for the type of agents for which the steady-state proportion decreases. In a UM equilibrium for instance, an increase in the steady-state proportion of single type-h, λ, makes the rejection of type-l agents by type-h more attractive and the ratios $\frac{u_{hl}}{u_{hh}}$ preventing this deviation have to be higher. Such an increase makes the acceptation of type-l agents by type-l less attractive and the ratios $\frac{u_{hl}}{u_{hh}}$ preventing this deviation can include smaller values.

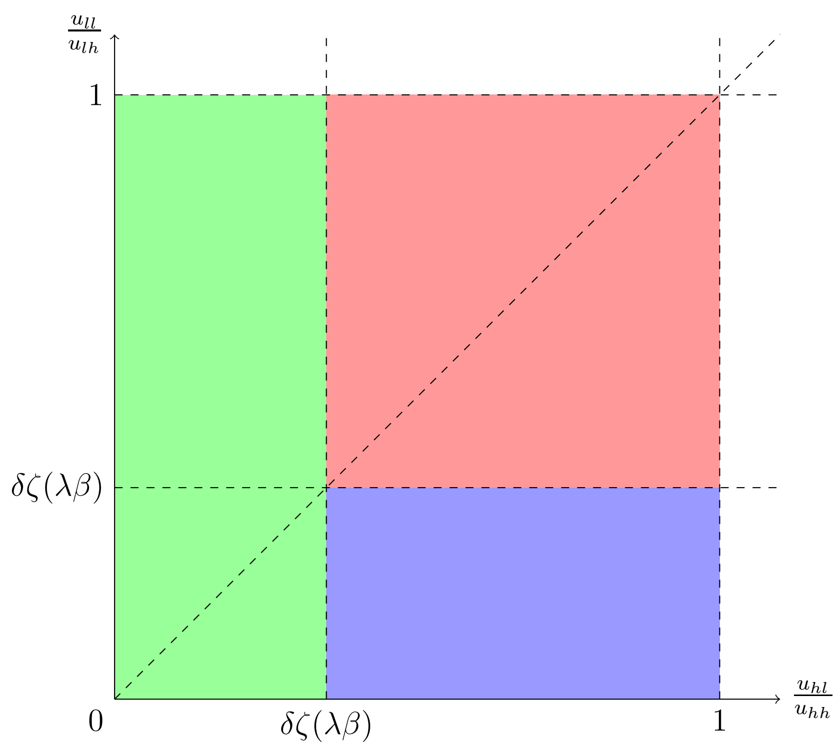

Figure 2: No-communication equilibria of the game: PAM equilibria (green), RM equilibria (red), UM equilibria (blue).

4 Communication and Incentives for Truthtelling

Before characterizing equilibria under informative communication, we study the equilibrium matching configurations when each agent has learned his potential partner's type before deciding whether or not to initiate an evaluation phase. That is, we first characterize trial and marriage equilibrium strategies under complete information, as if the communication strategies were exogeneously assumed to be fully revealing. Then, we study whether agents indeed have an incentive to reveal their true type to the partner they are randomly matched with, before deciding whether to enter the evaluation phase with him or not.

4.1 Steady State

We first describe the dynamics of the complete information game as a function of agents' strategies. Note that it is without loss of generality (in terms of equilibrium outcomes) to consider trial strategies such that each type of agent always accepts to start an evaluation phase with a type-h agent, i.e. $\tau_{ih} = 1$ for all i (recall that both consents are required to enter an evaluation phase). Notice also that if a pair of agents (i, j) takes the decision to enter the evaluation phase once the real type of each partner has been revealed, then they will decide to marry with probability one at the end of the evaluation phase: $\tau_{ij} > 0 \Rightarrow \mu_{ij} = 1$; and if pairs of agents (i, j) never start an evaluation phase ($\tau_{ij} = 0$), then, they never have the opportunity to get married: μ_{ij} is irrelevant. A steady state under complete information is therefore fully determined by the trial strategies τ_{ij}, $(i, j) \in \{l, h\}^2$.

The steady state number of single type-i agents in any period is equal to the number of type-i agents whose evaluation phase ended in the previous period plus the number of single type-i agents from the previous period who did not enter an evaluation phase because they rejected (or were rejected by) the agent to whom they had been matched (see Figure 4.1 for an illustration of transitions probabilities at the steady state):

$$n_h = \beta(n_{hl} + n_{hh}) + n_h \left(\frac{n_l}{n_l + n_h}(1 - (1-\beta)\tau_{hl}) + \frac{n_h}{n_l + n_h}\beta \right), \quad (10)$$

$$n_l = \beta(n_{ll} + n_{lh}) + n_l \left(\frac{n_l}{n_l + n_h}(1 - (1-\beta)\tau_{ll}^2) + \frac{n_h}{n_l + n_h}(1 - (1-\beta)\tau_{hl}) \right). \quad (11)$$

The number of type-i agents matched with type-j agents in the evaluation phase is equal to the number of such agents from the previous period who continue their evaluation phase plus the number of single type-i agents from the previous period who entered an evaluation

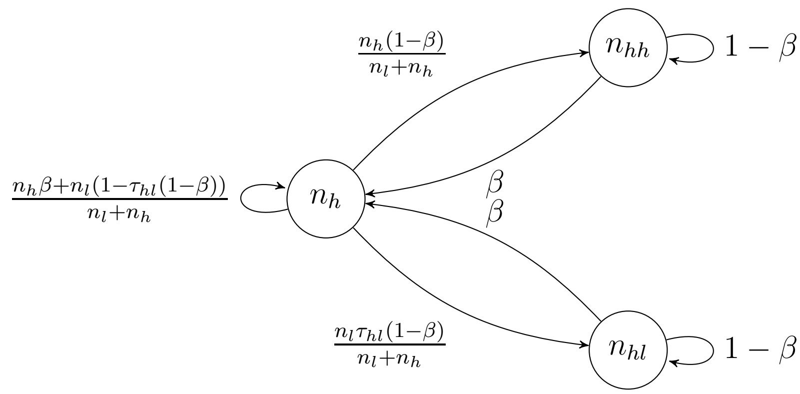

Figure 3: Steady state transitions probabilities for type-h agents

phase with a type-j agent:

$$n_{ij} = (1-\beta)\left(n_{ij} + n_i \frac{n_j}{n_l+n_h}\tau_{ij}\tau_{ji}\right). \tag{12}$$

For every $i \in \{l, h\}$, a type-i agent in the market is either single or in an evaluation phase with some type-j agent, so we have: $n_i + n_{il} + n_{ih} = \lambda_i$. We use this equation to rearrange (10), (11) and (12) and get:

$$n_h = \beta(\lambda_h - n_h) + n_h\left(\frac{n_h}{n_l+n_h}\beta + \frac{n_l}{n_l+n_h}(1-(1-\beta)\tau_{hl})\right), \tag{13}$$

$$n_l = \beta(\lambda_l - n_l) + n_l\left(\frac{n_l}{n_l+n_h}(1-(1-\beta)\tau_{ll}^2) + \frac{n_h}{n_l+n_h}(1-(1-\beta)\tau_{hl})\right). \tag{14}$$

The proportion of single type-h agents now depends on the strategies τ_{hl} and τ_{ll} and on parameters λ and β; it is given by

$$\hat{\lambda}(\tau_{hl}, \tau_{ll}) := \frac{n_h}{n_l+n_h}, \tag{15}$$

where (n_h, n_l) solves (13)–(14).[9] For the case of complete information random matching, we get in particular that $\hat{\lambda}(1,1) = \lambda$. Under positive assortative matching, we denote $\hat{\lambda}(0,1) = \hat{\lambda}_{PAM}$ and have:

$$\hat{\lambda}_{PAM} = \frac{2\lambda}{2\lambda + \beta(1-2\lambda) + \sqrt{\beta^2(1-2\lambda)^2 + 4\lambda(1-\lambda)}},$$

[9]Simplifying yields $\hat{\lambda}(\tau_{hl}, \tau_{ll}) = \frac{\beta+(1-\beta)(\tau_{hl}(1-2\lambda)+2\tau_{ll}^2\lambda)+\sqrt{(\beta+(1-\beta)\tau_{hl})^2+4(1-\beta)\lambda(1-\lambda)(\beta+\tau_{ll}^2-\tau_{hl}^2(1-\beta)-2\tau_{hl}\beta)}}{2(1-\beta)(1-\lambda-\tau_{hl}(1-2\lambda)-\tau_{ll}^2\lambda)}$ when $(\tau_{hl}, \tau_{ll}) \neq (1,1)$.

Under upward matching, we denote $\hat{\lambda}(1,0) = \hat{\lambda}_{UM}$ and have:

$$\hat{\lambda}_{UM} = \frac{2(1-\beta)\lambda - 1 + \sqrt{1 - 4(1-\beta)\lambda(1-\lambda)}}{2(1-\beta)\lambda}.$$

Figure 4 illustrates the values of the complete information proportions of single type-h under PAM and UM as a function of the proportion of type-h agents (λ).

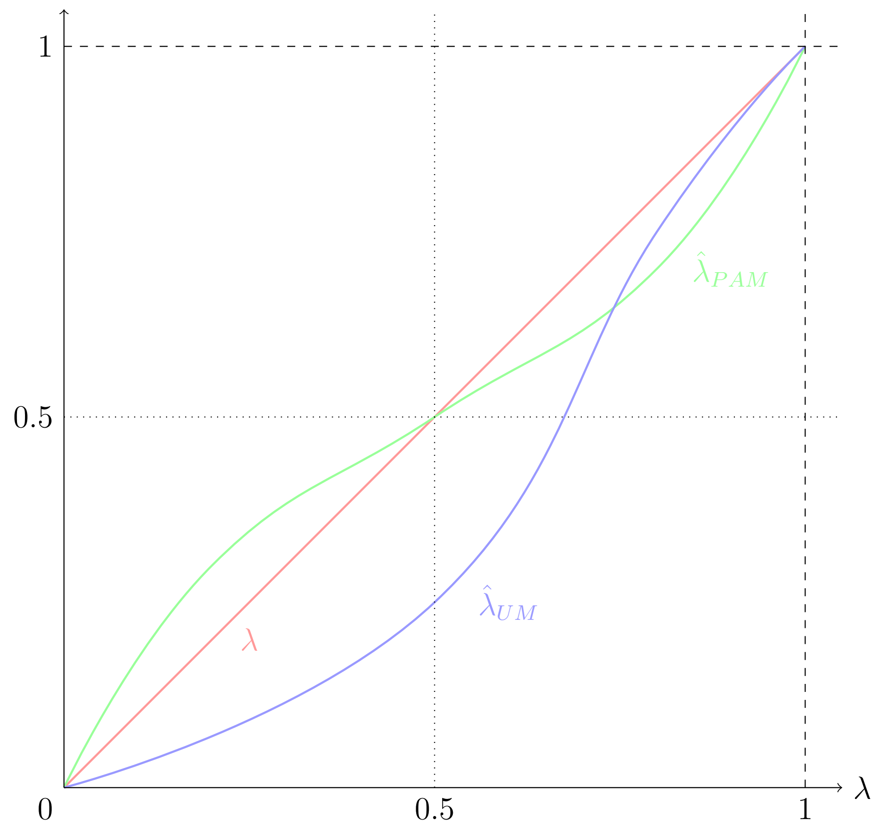

Figure 4: Complete information proportions of single type-h agents under positive assortative matching ($\hat{\lambda}_{PAM}$) and under upward matching ($\hat{\lambda}_{UM}$) as a function of the proportion of type-h agents (λ).

General properties of these steady state proportions $\hat{\lambda}(\tau_{hl}, \tau_{ll})$ of single type-h agents are summarized by Lemma 1 below.

Lemma 1. *The steady state proportion $\hat{\lambda}(\tau_{hl}, \tau_{ll})$ of single type-h agents has the following properties:*

1. $\hat{\lambda}_{PAM} \geq \lambda$ *iff* $\lambda \leq 1/2$;

2. $\hat{\lambda}_{UM} \leq \lambda$ *for all* λ;

3. $\hat{\lambda}(\tau_{hl}, \tau_{ll})$ *is increasing in* λ.

Proof. See Appendix A.1. ☐

The difference between the steady-state proportions of singles in the no-communication and complete information cases is due to the fact that, in the absence of communication, some pairs of agents enter the evaluation phase while they will reject each other once their types will be discovered; on the contrary, in the complete information case, such pairs never enter an evaluation phase.

Under UM, since type-h agents get married with any other type, the number of type-h singles is not affected by the information being discovered before the evaluation phase might start. Indeed, both in the no-communication and the complete information cases, the type-h agents directly enter an evaluation phase when they are single. However, the total number of single type-l agents—and hence, the total number of singles—is higher under complete information because contrary to the case of no-communication, a type-l agent is never in an evaluation phase with another type-l. These two effects lead unambiguously to the proportion of single type-h agents being smaller under upward matching with complete information than under upward matching without communication, and we always have $\hat{\lambda}_{UM} \leq \lambda$.

Under positive assortative matching, the proportion of single type-h agents with complete information, $\hat{\lambda}_{PAM}$, is smaller than λ when $\lambda \leq 1/2$, but it is higher than λ when $\lambda > 1/2$. The intuition of this property is less straightforward, but could be understood as follows. Under positive assortative matching with complete information, both the number of single type-h and type-l are higher compared to a no-communication situation because the evaluation phases involving type l and type h agents no longer occur under complete information. When λ is small ($\lambda < 1/2$), the rise in number of single type-h agents is more pronounced than on number of single type-l agents because, first, there are fewer type h agents than type l agents and, second, single type-h agents search only for type h agents. The opposite logic applies when $\lambda > 1/2$.

4.2 Complete Information Equilibria

We characterize below all possible pure and mixed equilibrium trial strategies under complete information. Notice that ($\tau_{hl} = 0, \tau_{ll} = 0$) is never an equilibrium since type-l agents always have a strict incentive to start an evaluation phase and get married together when they are rejected by type-h agents.

Complete Information PAM Equilibrium ($\tau_{hl} = 0, \tau_{ll} = 1$). Positive assortative matching is an equilibrium under complete information iff type-h agents are not willing to deviate by entering an evaluation phase with type-l agents (and hence accept to marry at the end of the evaluation phase). A type-h agent who accepts a type-l agent gets

$$\tilde{V}_{hl} = \beta u_{hl} + \delta(1-\beta)\tilde{V}_{hl} = \zeta(\beta) u_{hl},$$

while if he rejects a type-l agent[10] he receives his continuation payoff δV_h, where

$$\begin{aligned} V_h &= \hat{\lambda}_{PAM} \underbrace{(\beta u_{hh} + (1-\beta)\delta V_{hh})}_{V_{hh}} + (1-\hat{\lambda}_{PAM})\delta V_h \\ &= \frac{\hat{\lambda}_{PAM}\beta}{(1-(1-\hat{\lambda}_{PAM})\delta)(1-(1-\beta)\delta)} u_{hh} = \zeta(\hat{\lambda}_{PAM})\zeta(\beta) u_{hh}. \end{aligned} \tag{16}$$

Therefore, a positive assortative matching equilibrium exists iff $\delta V_h \geq \zeta(\beta) u_{hl}$, i.e.,

$$\frac{u_{hl}}{u_{hh}} \leq \delta \zeta(\hat{\lambda}_{PAM}). \tag{17}$$

Complete Information RM Equilibrium, ($\tau_{hl} = 1, \tau_{ll} = 1$). In this case, the dynamic of the game is the same as in the no-communication equilibrium: the steady state proportion of type-h agents in the population of singles is $\hat{\lambda}(1,1) = \lambda$, and the equilibrium conditions simplify to

$$\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\lambda) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \geq \delta\zeta(\lambda). \tag{18}$$

Complete Information UM Equilibrium ($\tau_{hl} = 1, \tau_{ll} = 0$). Similarly, the conditions under which ($\tau_{hl} = 1, \tau_{ll} = 0$) is a steady state equilibrium are given by

$$\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}_{UM}) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \leq \delta\zeta(\hat{\lambda}_{UM}). \tag{19}$$

Complete Information Mixed Strategy Equilibria In a mixed strategy equilibrium, if $\tau_{hl} \in (0,1)$ then a type-h agent should be indifferent between entering an evaluation phase with a type-l agent or not. Following the same logic as for the PAM and RM equilibria above, the indifference condition is

$$\frac{u_{hl}}{u_{hh}} = \delta\zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll})). \tag{20}$$

[10] Note that compared to the no-communication equilibrium of the previous section, if a type-h agent refuses to marry a type-l agent then he never starts an evaluation phase with a type-l agent.

If $\tau_{ll} \in (0,1)$ then a type-l agent should be indifferent between entering an evaluation phase with a type-l agent or not. The logic is slightly more difficult than the equilibrium conditions above for two reasons. First, τ_{hl} now affects the steady state $\hat{\lambda}(\tau_{hl}, \tau_{ll})$. Second, τ_{hl} also appears in V_l because τ_{hl} determines the probability that a single type-l agent starts an evaluation phase when he meets a type-h agent. A type-l agent who accepts a type-l agent gets

$$V_{ll} = \beta u_{ll} + (1-\beta)\delta V_{ll} = \zeta(\beta) u_{ll},$$

while if he rejects a type-l agent he gets δV_l, where

$$\begin{aligned} V_l &= \hat{\lambda}\left(\tau_{hl} \underbrace{(\beta u_{lh} + (1-\beta)\delta V_{lh})}_{V_{lh}} + (1-\tau_{hl})\delta V_l\right) + (1-\hat{\lambda})\delta V_l \\ &= \frac{\tau_{hl}\hat{\lambda}\beta}{(1-(1-\tau_{hl}\hat{\lambda})\delta)(1-(1-\beta)\delta)} u_{lh}. \end{aligned} \tag{21}$$

Therefore, the indifference condition for $\tau_{ll} \in (0,1)$ is

$$\frac{u_{ll}}{u_{lh}} = \delta\zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, \tau_{ll})). \tag{22}$$

Potentially, there may be an equilibrium in which both type-h and type-l agents are indifferent between entering the evaluation with a type-h or a type-l agent ($\tau_{hl} \in (0,1)$ and $\tau_{ll} \in (0,1)$), when there is a solution to the system (20)-(22).

The other equilibrium conditions follow the same logic as before. The following proposition summarizes the characterization of all pure and mixed strategy equilibria under complete information.

Proposition 2. *There exists a pure strategy* complete information equilibrium *such that*

- *the matching is Positive Assortative iff* $\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\hat{\lambda}_{PAM})$,
- *the matching is Random iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\lambda)$ *and* $\frac{u_{ll}}{u_{lh}} \geq \delta\zeta(\lambda)$,
- *the matching is Upward iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}_{UM}) \geq \frac{u_{ll}}{u_{lh}}$,

There exists a mixed strategy complete information equilibrium *such that*

- $(\tau_{hl} \in (0,1), \tau_{ll} = 1)$ *iff* $\frac{u_{hl}}{u_{hh}} = \delta\zeta(\hat{\lambda}(\tau_{hl}, 1))$ *and* $\delta\zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, 1)) \leq \frac{u_{ll}}{u_{lh}}$,
- $(\tau_{hl} \in (0,1), \tau_{ll} \in (0,1))$ *iff* $\frac{u_{hl}}{u_{hh}} = \delta\zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll}))$ *and* $\delta\zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, \tau_{ll})) = \frac{u_{ll}}{u_{lh}}$,

- $(\tau_{hl} = 1, \tau_{ll} \in (0,1))$ *iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}(1, \tau_{ll})) = \frac{u_{ll}}{u_{lh}}$,
- $(\tau_{hl} \in (0,1), \tau_{ll} = 0)$ *iff* $\frac{u_{hl}}{u_{hh}} = \delta\zeta(\hat{\lambda}(\tau_{hl}, 0))$ *and* $\delta\zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, 0)) \geq \frac{u_{ll}}{u_{lh}}$.

In addition all equilibria are such that $\tau_{ij} > 0 \Rightarrow \mu_{ij} = 1$, *and there is no other equilibrium than the ones described above.*

The ratios $\frac{u_{hl}}{u_{hh}}$ and $\frac{u_{ll}}{u_{lh}}$ for which a pure equilibrium with complete information exists are now given by the threshold $\delta\zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll}))$ while they were given by $\delta\zeta(\beta\lambda)$ in Proposition 1. First, it is interesting to note that the thresholds now depend on β only through its effect on the steady-state proportions of singles. This difference is due to the fact that, in the complete information case, choices to accept or reject an agent are made before the evaluation phase starts. An agent therefore compares returning to a single situation to entering an evaluation phase. In both cases, any potential gain of marriage occur after an evaluation period, which expected length depends on β.

The thresholds determining existence of the different matching conditions under complete information depend on different steady state proportions of singles. It naturally follows that, under complete information, several equilibrium configurations can co-exist. In particular, there are some zones where both a PAM and a UM equilibria co-exist, and some zones where both a PAM and a RM equilibria co-exist. However, since we always have that $\zeta(\hat{\lambda}_{UM}) < \zeta(\lambda)$, the zones of $\frac{u_{hl}}{u_{hh}}$ and $\frac{u_{lh}}{u_{ll}}$ for which UM and RM exist never overlap. Overall, three distinct orderings of the thresholds $\delta\zeta(\hat{\lambda}_{PAM})$, $\delta\zeta(\hat{\lambda}_{UM})$ and $\delta\zeta(\lambda)$ are possible. The corresponding pure strategies equilibrium outcomes configurations are illustrated by Figure 5 (for $1/2 < \hat{\lambda}_{PAM} < \hat{\lambda}_{UM} < \lambda$), Figure 6 (for $\hat{\lambda}_{UM} < 1/2 < \hat{\lambda}_{PAM} < \lambda$) and Figure 7 (for $\hat{\lambda}_{UM} < \lambda < \hat{\lambda}_{PAM} < 1/2$).

4.3 Informative Communication Equilibria

When communication is strategic, we still have $\tau_{ij} > 0 \Rightarrow \mu_{ij} = 1$ as in the complete information case; however, if a pair of agents (i, j) do not agree to enter the evaluation phase ($\tau_{ij} = 0$) then they might have to decide, off the equilibrium path, whether they would like to marry or not after an evaluation phase if one of them lied about its type before starting the evaluation phase. That is, even if $\tau_{ij} = 0$, the equilibrium conditions will depend on whether $\mu_{ij} = 0$ or $\mu_{ij} = 1$ due to possible deviations at the cheap talk stage.

Consider first the positive assortative matching outcome $(\tau_{hl} = 0, \tau_{ll} = 1)$ (and hence $\mu_{ll} = 1$). If a type-h deviates from truthful communication and reveals that his type is l instead of h, then he will not start an evaluation phase if he is matched with another type-h

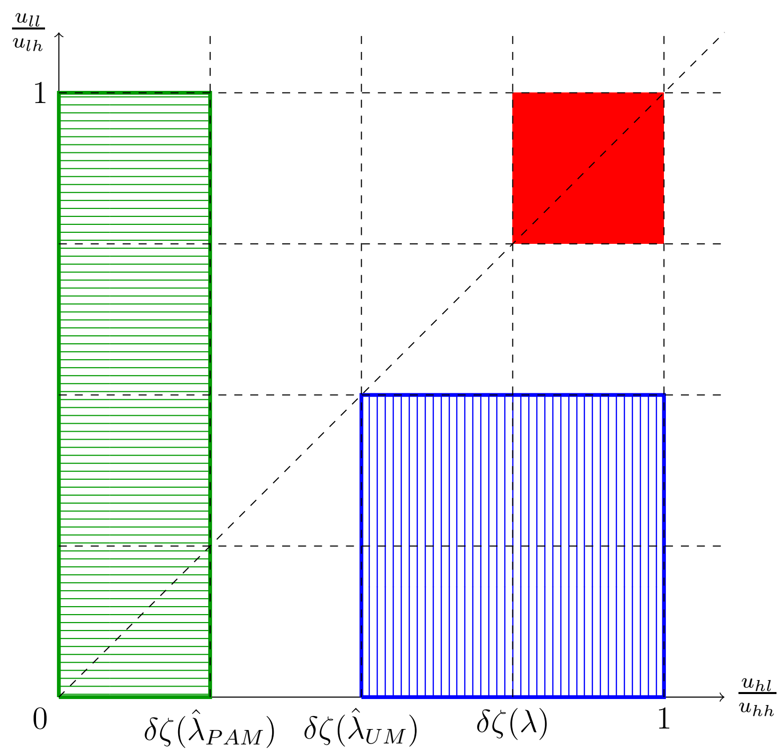

Figure 5: Pure strategy equilibria under complete information when $1/2 < \hat{\lambda}_{PAM} < \hat{\lambda}_{UM} < \lambda$. Green: PAM equilibria; Red: RM equilibria; Blue: UM equilibria.

agent (who will reject him), and he may start an evaluation phase if he is matched with a type-l agent. In the first case he his clearly worse off; in the second case the deviation is payoff-relevant only if he deviates from his trial strategy $\tau_{hl} = 0$ to $\tau_{hl} = 1$, which is not profitable under the complete information equilibrium conditions of the positive assortative matching outcome. Now consider a deviation by a type-l agent, who reveals that his type is h instead of l. If $\mu_{hl} = 1$ then this deviation is profitable because he will then start an evaluation also with a type-h agent who will accept to marry him at the end of the evaluation phase even though he would have rejected a type-l agent before starting the evaluation phase. Hence, to have truthful information transmission with a positive assortative matching outcome, a type-h agent should prefer to reject a type-l agent at the end of the evaluation phase, i.e., we must have $u_{hl} \leq \delta V_h$, where V_h is the same continuation payoff as in (16): $V_h = \zeta(\hat{\lambda}_{PAM})\zeta(\beta)u_{hh}$. Therefore, there is a fully revealing equilibrium with positive assortative matching iff

$$\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\hat{\lambda}_{PAM})\zeta(\beta). \tag{23}$$

Notice that this condition is strictly stronger than the condition for a positive assortative

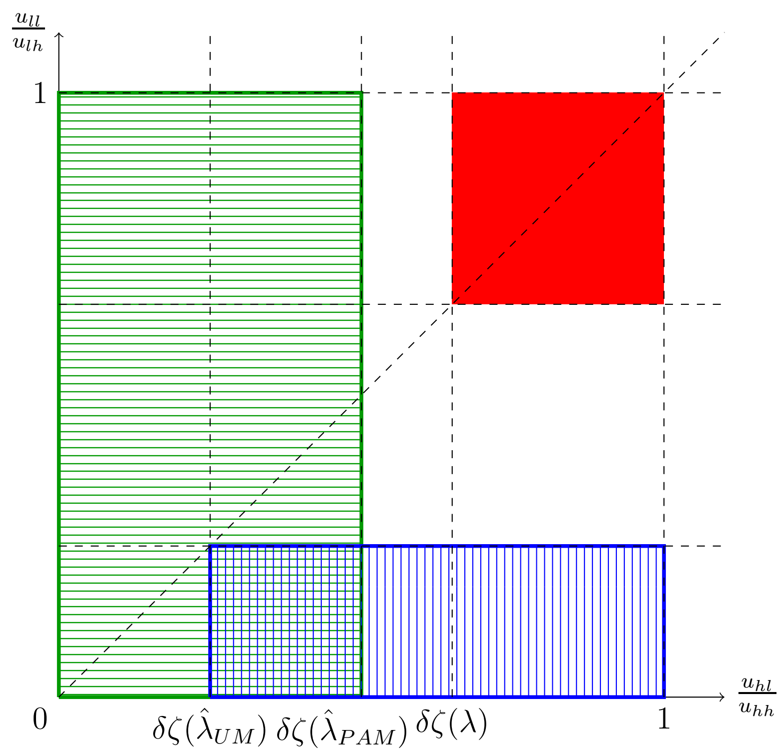

Figure 6: Pure strategy equilibria under complete information when $\hat{\lambda}_{UM} < 1/2 < \hat{\lambda}_{PAM} < \lambda$. Green: PAM equilibria; Red: RM equilibria; Blue: UM equilibria.

matching equilibrium to exist under complete information (see Condition (17)) since

$$\delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) < \delta\zeta(\hat{\lambda}_{PAM}),$$

when $\beta < 1$.

Consider now the random matching outcome $\tau_{hl} = \tau_{ll} = 1$ (and hence $\mu_{hl} = \mu_{ll} = 1$). Since the trial and marriages do not depend on information, truthful information is clearly incentive compatible since it is not influential.

Next, consider the upward matching outcome $(\tau_{hl} = 1, \tau_{ll} = 0)$ (and hence $\mu_{hl} = 1$). Here, the conditions for full information transmission are implied by the equilibrium conditions under complete information: if a type-h agent pretends to be a type-l agent he gets the same payoff as if he chooses $\tau_{hl} = 0$ instead of $\tau_{hl} = 1$, and if a type-l agent pretends to be a type-h agent he gets the same payoff as if chooses $\tau_{ll} = 1$ instead of $\tau_{ll} = 0$. Therefore, there is a fully revealing UM equilibrium iff there is an UM equilibrium under complete information. Exactly the same reasoning applies to the mixed UM equilibrium outcome $(\tau_{hl} = 1, \tau_{ll} \in (0, 1))$.

Finally, consider a mixed equilibrium in which $\tau_{hl} \in (0, 1)$; such an equilibrium cannot

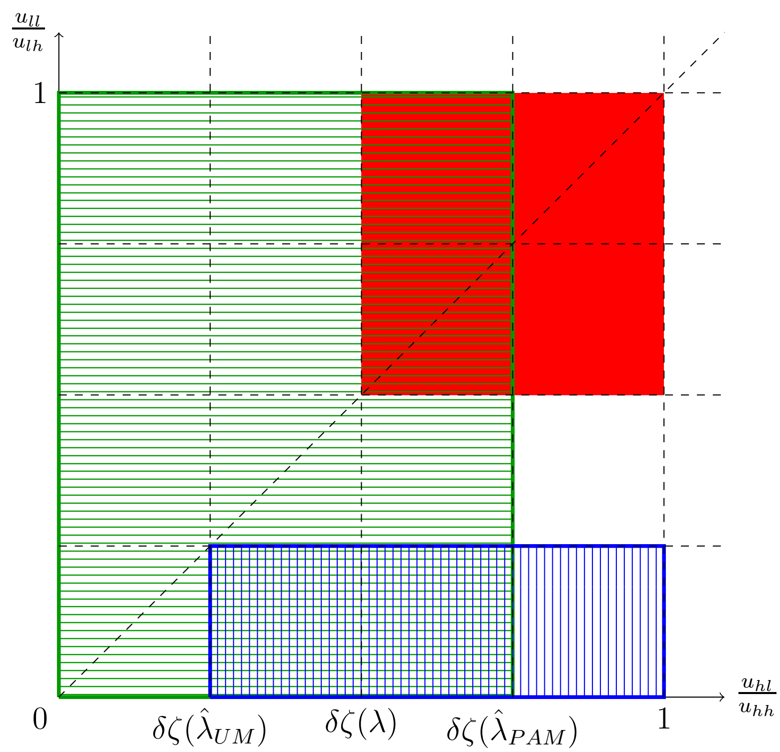

Figure 7: Pure strategy equilibria under complete information when $\hat{\lambda}_{UM} < \lambda < \hat{\lambda}_{PAM} < 1/2$. Green: PAM equilibria; Red: RM equilibria; Blue: UM equilibria.

be implemented with cheap talk because $\tau_{hl} \in (0,1) \Rightarrow \mu_{hl} = 1$, so a type-$l$ agent will always pretend to be a type-h agent to be able to start an evaluation phase with a type-h agent, who will then accept the type-l agent at the end of the evaluation phase. Indeed, if a type-h agent is indifferent between starting an evaluation phase with a a type-l agent ($\tau_{hl} \in (0,1)$) then he will strictly prefer to marry him at the end of the evaluation phase ($\mu_{hl} = 1$).

The following proposition summarizes the equilibrium outcomes of the matching game when communication is informative, i.e., the complete information outcomes that could be implemented with truthful communication strategies. For the positive assortative matching equilibrium we can have truthful information transmission even by type-l agents because type-h agents may reject them when they discover their type at the end of the evaluation phase. For the UM equilibrium, the incentives for type-l agents to reveal the truth come on the contrary from other type-l agents: since they are only looking for type-h agents, they can accelerate their search for a type-h agent by skipping any evaluation period with another type-l agent.

Proposition 3. *There exist an* informative communication equilibria *such that*

- *the matching is* Positive Assortative *iff* $\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$,

- *the matching is* Random *iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\lambda)$ *and* $\frac{u_{ll}}{u_{lh}} \geq \delta\zeta(\lambda)$,
- *the matching is* Upward *iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}_{UM}) \geq \frac{u_{ll}}{u_{lh}}$,
- *the matching is* Mixed Upward $(\tau_{hl} = 1, \tau_{ll} \in (0,1))$ *iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}(1, \tau_{ll})) = \frac{u_{ll}}{u_{lh}}$.

In addition there is no other informative communication equilibrium than the ones described above.

5 Informative Communication vs. No-Communication

The objective of the section is to globally examine how equilibrium matching configurations are affected by informative communication. We start by having a close look at the sustainability of any given matching outcome when communication becomes truthful. We then describe the possible changes in the matching configurations induced by such communication. Finally, we derive general conclusions about the effect of communication on equilibrium behaviors. In the rest of the paper, all terms with a "hat" will refer to informative communication equilibria, and those without to no-communication equilibria.

5.1 Sustainability of Each Matching Configuration

To start with, note two main differences between the situations of informative communication and no communication. First, as mentioned earlier, the steady-state proportions of single agents, and therefore their continuation payoffs, depend on the matching configuration in the case of informative communication. The effect of this change on agents' incentives to deviate from a given trial strategy corresponds to the *Proportion Effect (PE)* explained after Proposition 1. Second, the informed choices of accepting or rejecting a given type are not made at the same time when communication is informative or not. In the babbling case, agents decide at the end of the evaluation phase by comparing an immediate gain of marriage to a return to a single situation. Under informative communication, informed decisions are taken before an evaluation period may start as the types are directly revealed. So, when comparing informative-communication and no-communication equilibria, a *Deviation Time Effect (henceforth DTE)* is at play. Incentives to accept a type (resp. reject) are stronger (resp. weaker) when such a deviation occur after the evaluation phase than before.[11] Indeed,

[11]Put differently, a type-i rejects a type-j under no-communication if $\delta V_i \geq u_{ij}$. If this holds, it implies that i also rejects j under informative communication as $\delta V_i \geq u_{ij}$ implies $\delta V_i \geq \beta u_{ij}$. One the contrary, it can be that type-i accepts type-j under no-communication but not under informative communication as we can have $\beta u_{ij} \geq \delta V_i \geq u_{ij}$.

accepting an agent before the evaluation phase starts always induces a delay to the moment where the marriage could effectively be consumed. This delay is absent once the evaluation phase has elapsed. The larger is β, the weaker is this effect as the expected length of the evaluation phase is shorter.

It is the interplay of these two effects that makes it relatively harder or easier to sustain a given matching configuration in an informative communication equilibrium than in a no-communication equilibrium. A precise comparison requires linkening thresholds given in Proposition 1 and Proposition 3. The following Lemma proves useful:

Lemma 2. *$\hat{\lambda}_{UM} \geq \beta\lambda$ and $\hat{\lambda}_{PAM} \geq \beta\lambda$.*

Proof. See Appendix A.2. □

For RM equilibria, the statement is straightforward: the conditions for a RM equilibrium to exist are stronger under informative communication than under no-communication. It formally relies on $\delta\zeta(\lambda\beta) < \delta\zeta(\lambda)$ but is very intuitive. In the case of RM, only the DTE is at play as the proportions of singles are the same for the cases of no-communication and informative communication. The fact that types of agents are discovered before the evaluation phase makes the rejection of type-l by any type harder to prevent when communication is informative. In the case of UM, we can show that the conditions for a UM to exist are weaker for type-l agents and stronger for type-h agents with informative communication than without. It formally relies on the fact that $\delta\zeta(\hat{\lambda}_{UM}) \geq \delta\zeta(\beta\lambda)$ by Lemma 2. Again, the intuition is direct: true types being revealed before a potential evaluation phase makes it harder for type-h to accept type-l than in the absence of communication. The reverse is true for type-l agents. Since $\hat{\lambda}_{UM} < \lambda$, the PE is however playing in the opposite direction but is weaker than DTE.

For the existence of no-communication PAM equilibria and informative communication PAM equilibria however, we need to compare $\zeta(\lambda\beta)$ to $\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$ which is ambiguous. If $\hat{\lambda}_{PAM} > \lambda$, we have $\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) > \zeta(\lambda\beta)$.[12] In that case, a PAM equilibrium is easier to sustain under informative communication. Part of the intuition works as follows: since the proportion of single type-h is higher in the steady-state with informative communication than without, type-h are less eager to deviate and accept type-l under informative communication. Regarding the DTE effect, it does not play as clearly here as for RM or UM. Indeed, existence of a PAM informative communication equilibrium requires that type-l agents are rejected even if their true type is only discovered at the end of the evaluation phase. β therefore plays

[12] Formally, we use Lemma 1 and the property that $\zeta(xy) < \zeta(x)\zeta(y)$ for all $x, y \in (0, 1)$.

a direct role on both thresholds $\zeta(\lambda\beta)$ and $\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$. In case $\hat{\lambda}_{PAM} < \lambda$, β determines whether PAM is easier or harder to sustain than $P\hat{A}M$. Both are possible but the intuition is less clear.

Overall, independently of whether existence conditions are weaker or stronger under informative communication, the following result holds:

Lemma 3. *For every possible matching configuration, there exists a generic set of parameters such that this configuration emerges in equilibrium both under informative communication and under no-communication.*

Proof. See Appendix A.3. □

In the case of RM for instance, the ratios $\frac{u_{hl}}{u_{hh}}$ and $\frac{u_{ll}}{u_{lh}}$ ensuring existence of $R\hat{M}$ also ensure existence of RM. $\hat{\lambda}_{UM} \geq \beta\lambda$, equilibrium exists with and without communication if $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}_{UM})$ and $\frac{u_{ll}}{u_{lh}} \leq \delta\zeta(\lambda\beta)$.

5.2 Changes in Matching Configurations and General Conclusions

Next, we naturally wonder whether it is possible that, for a given set of parameters, the equilibrium matching configuration changes when communication becomes informative. It is a simple but rather tedious comparison of above mentioned thresholds that lead to the following answer:

Lemma 4.

1. *There exists a generic set of parameters such that RM is an equilibrium under no communication and UM is an equilibrium under informative communication.*

2. *There exists a generic set of parameters such that RM is an equilibrium under no communication and PAM is an equilibrium under informative communication.*

3. *There exists a generic set of parameters such that UM is an equilibrium under no communication and PAM is an equilibrium under informative communication.*

Proof. See Appendix A.4. □

Clearly, not all possible changes in matching configurations are possible when communication becomes informative. We precisely end up with six possible changes that communication can induce: from RM to $R\hat{M}$, from PAM to $P\hat{A}M$, from UM to $U\hat{M}$, from RM to $U\hat{M}$, from

RM to $P\hat{A}M$ and finally from UM to $P\hat{A}M$. These possible switches in matching configurations are a direct consequence of the following general effect that informative communication has on agents' equilibrium behaviors:

Proposition 4.

1. *High type agents are pickier under informative communication:*

$$\hat{\mu}_{hl} \leq \mu_{hl}$$

2. *If informative communication makes high type agents strictly pickier, then it makes low type agents less picky:*

$$\hat{\mu}_{hl} < \mu_{hl} \;\Rightarrow\; \hat{\mu}_{ll} \geq \mu_{ll}.$$

Proof. See Appendix A.5. □

This proposition sheds light on a very clear asymmetry between the low and high types in the population. As type-h agents are accepted by every type of agent, they are the ones whose decision affect their match in equilibrium. It follows that the introduction of informative communication can only make them pickier. It is the Deviation Time Effect that gives the best intuition of why it is so: since type-h discover the type of their partner earlier with informative communication than without, they keep on rejecting types they rejected without communication but may now even reject some types they were accepting. This explains, for instance, why it is impossible to switch from a PAM no-communication equilibrium where type-l agents are rejected by type-h agents to any other matching configuration under informative communication.

Regarding type-l, the second part of the proposition clearly show their dependance on type-h agents' "pickiness". If type-h are strictly pickier with informative communication, it means that type-l are then rejected by type-h. Type-l therefore have no other choice than accepting each other, that is, being less picky. Equivalently, it is only in cases where type-h are not strictly pickier that type-l can be so. For example, in the case of a switch from RM to $\hat{UM}$, type-l are given some room to be strictly pickier under informative communication by the fact type-h always accept to marry every type.

	no com.		informative com.	Type-h agents $\hat{U}_h - U_h$	Type-l agents $\hat{U}_l - U_l$
Fixed Matching	PAM	$\rightarrow$	$\widehat{\text{PAM}}$	+	+
	UM	$\rightarrow$	$\widehat{\text{UM}}$	−	+
	RM	$\rightarrow$	$\widehat{\text{RM}}$	=	=
Changing Matching	RM	$\rightarrow$	$\widehat{\text{PAM}}$	+	−
	UM	$\rightarrow$	$\widehat{\text{PAM}}$	+	−
	RM	$\rightarrow$	$\widehat{\text{UM}}$	−	+

Table 1: Welfare Comparisons.

6 Welfare Effects of Informative Communication

In this section we study the effect of strategic communication on agents' welfare. We denote by

$$U_i := n_i V_i + n_{il} V_{il} + n_{ih} V_{ih}, \tag{24}$$

the total expected welfare of all type-i agents at a stationary equilibrium. Table 1 summarizes the comparisons of the total expected welfare for each type of agent and for each possible equilibrium matching configuration when one moves from a no-communication equilibrium to an informative communication equilibrium. Appendix A.6 contains the detailed computations.

Consider first the effect of information transmission on agents' welfare when it does not affect the equilibrium matching configuration. Clearly, information transmission is irrelevant when the equilibrium is a RM equilibrium. Next, consider the case in which a no-communication equilibrium exhibits positive assortative matching and an informative communication equilibrium also exhibits a positive assortative matching. In this case, all agents (low and high types agents) are always better off in the matching game with informative communication.[13] The direct intuition of this result is simple: when agents reveal their true type before choosing whether to start an evaluation phase or not, low (high, resp.) types agents do not waste their time anymore in an evaluation phase with high (low, resp.) types agents ($\hat{n}_{ij} = 0$ while $n_{ij} > 0$, $i \neq j$). Hence, the number of type-i agents in an evaluation phase with other type-i agents is higher with informative communication than without ($\hat{n}_{hh} > n_{hh}$ and $\hat{n}_{ll} > n_{ll}$), so agents end up being married assortatively more quickly. However, the result is not so direct because this intuition does not take into account the effect of communication

[13]Notice that under this condition, the informative communication equilibrium is the unique neologism proof equilibrium under PAM (see Section 7).

on the proportions of high and low singles in the population ($\hat{\lambda}_{PAM}$ is different from λ, see Figure 4). These proportions have a strong impact on the probability that a single agent is matched with another agent of the same type. Consider for example single type-h agents. By Equations (5) and (16) we have $\hat{V}_h - V_h = (\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) - \zeta(\beta\lambda))u_{hh}$. When $\lambda \leq 1/2$, we have $\hat{\lambda}_{PAM} \geq \lambda$ (see Lemma 1), so that $\hat{V}_h - V_h \geq \zeta(\beta)\zeta(\lambda) - \zeta(\beta\lambda) \geq \zeta(\beta\lambda) - \zeta(\beta\lambda) = 0$, where the first inequality stems from the fact that the function $\zeta(x)$ in increasing in x on $[0, 1]$ and the second inequality comes from the fact that $\forall\,(x, y) \in [0, 1]^2$, $\zeta(x)\zeta(y) \geq \zeta(xy)$. But when $\lambda > 1/2$ is high, single type-h agents may be *worse off* with informative communication; more precisely, it can be computed that we have $\hat{V}_h - V_h < 0$ iff $\lambda \in (\frac{1+\beta\delta}{2-(1-\beta)(2-\delta)\delta}, 1)$. Similarly, when $\lambda \geq 1/2$, and using the analogs of Equations (5) and (16) for type-l agents we have $\hat{V}_l - V_l = (\zeta(\beta)\zeta(1 - \hat{\lambda}) - \zeta(\beta(1 - \lambda)))u_{ll} \geq 0$, but when $\lambda < 1/2$ is low enough ($\lambda \in (0, \frac{(1-\delta)(1-(1-\beta)\delta)}{2-(1-\beta)(2-\delta)\delta})$) single type-$l$ agents are worse off with informative communication. This possible negative proportion effect of communication on single agents is always compensated by the efficiency gained from avoiding non-assortative evaluation phases. But it is important to notice that our welfare comparisons take into account all type-h or type-l agents, taking together single agents and agents in an evaluation phase, so that these welfare comparisons might not be directly related to the welfare comparisons for single agents taken separately.

Next, consider the situation in which the no-communication equilibrium has upward matching and the informative communication equilibrium remains a upward matching equilibrium. In this case, low types agents are better off but high types agents are worse off in the informative communication equilibrium than in the no-communication equilibrium of the matching game.[14] The intuition for the high type agents is simple because both single high types and high types in an evaluation phase are worse off with information than without information. Indeed, we have seen before Lemma 1 that information does not affect the number of single high types under UM but it increases the number of single low types, so high types are matched with low types more often with information, resulting in a lower expected payoff both for single high types and high types in an evaluation phase, with the proportion of single high types and high types in an evaluation not being affected by information.

The following proposition summarizes the welfare effects of communication described above as a function of the pattern of the marriages strategies with and without communication.

[14] In Section 7 we show that the informative communication equilibrium UM is the unique neologism proof UM equilibrium when $\frac{u_{hl}}{u_{hh}} > \delta\zeta(\lambda)$, but both the informative communication and no-communication UM equilibria are neologism proof when $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda)$.

Proposition 5.

- *High type agents are better off under communication iff low type agents accept each other, i.e., iff*

$$\hat{\mu}_{ll} = 1;$$

- *Low type agents are better off under communication iff communication makes them more picky than it does for the high type agents, i.e., iff*

$$\mu_{ll} - \hat{\mu}_{ll} \geq \mu_{hl} - \hat{\mu}_{hl}.$$

7 Extensions

Neologism-Proof Equilibria Farrell (1993) defines the notion of "neologism-proof equilibrium" because standard equilibrium refinements do not help to make any equilibrium selection in cheap talk games (see, e.g., Sobel, 2010, for a recent discussion). While Farrell's solution concept is defined for sender-receiver games (in particular, only the receiver chooses an action, and only the sender has private information) the idea of the solution concept may be extended to our communication game. Assume the following "rich language" condition: at every equilibrium, and for every subset of types S of a player, there is a message which is not used in equilibrium by this player and has the literal meaning "my type is in S". This message is called a "neologism". Such a neologism is said to be "credible" in some equilibrium if S is self-signaling, i.e., if all types in S have a strict incentive to use it when the receiver believes that it is true, but types outside S have no strict incentive to use it when the receiver believes that it is true. An equilibrium is said to be *neologism-proof* if there is no credible neologism. Notice that a neologism-proof equilibrium may not exist and may not be unique.[15]

The following proposition characterizes the neologism-proof equilibria of our matching game with informative communication.

Proposition 6 (Neologism–Proof Equilibria)**.**

(i) Informative communication equilibria are always neologism-proof;

(ii) A no-communication PAM equilibrium is never neologism-proof;

[15] More complex criteria related to neologism-proofness have been proposed; see, for example, the announcement-proof criterion of Matthews et al. (1991). Presumably, these criteria coincide in our class of communication games.

(iii) A no-communication UM equilibrium is neologism-proof if $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda)$*, but is not neologism-proof if* $\frac{u_{hl}}{u_{hh}} > \delta\zeta(\lambda)$*;*

(iv) A no-communication RM equilibrium is neologism-proof if $\frac{u_{hl}}{u_{hh}}h, \frac{u_{ll}}{u_{lh}} > \delta\zeta(\lambda)$ *or if* $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda)$*, but is not neologism-proof if* $\frac{u_{ll}}{u_{lh}} < \delta\zeta(\lambda) < \frac{u_{hl}}{u_{hh}}$

Proof. (i) To show that the informative communication equilibria have no credible neologism it suffices to observe that when the (unused) message "my type could be either l or h" is believed by the receiver it is never strictly beneficial for any sender type. Indeed, it either does not affect the decision of the receiver, or it changes it as if the sender were sending the message h when his type is l or the message l when his type is h (which is not beneficial given the informational incentive constraints of a fully revealing equilibrium).

(ii) Consider a no-communication PAM equilibrium, so that $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda\beta)$, and consider the neologism "my type is h". If this neologism is believed by the receiver, then a type-h receiver will not change his decision (he continues to enter the evaluation phase) while a type-l receiver will change his decision and decide not to enter the evaluation phase because a type-h agent will reject him at the end of it (because $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda\beta)$). Therefore, if the neologism "my type is h" is believed by the receiver, a type-l agent is strictly worse off sending it while a type-h agent is strictly better off compared to the original no-communication equilibrium. This neologism is therefore credible, meaning that the no-communication PAM equilibrium in not neologism-proof.

(iii) Consider a no-communication UM equilibrium, so that $\frac{u_{ll}}{u_{lh}} < \delta\zeta(\lambda\beta) < \frac{u_{hl}}{u_{hh}}$. If the neologism "my type is h" is believed by the receiver, then, whatever his type, the receiver type will not change his decision (he continues to enter the evaluation phase). If the neologism "my type is l" is believed by the receiver, then a type-l receiver will change his decision and decide not to enter the evaluation phase (because a type-l agent will reject a type-l agent at the end of it since $\frac{u_{ll}}{u_{lh}} < \delta\zeta(\lambda\beta)$), while the reaction of a type-h receiver depends on the parameters. If a type-h receiver enters an evaluation phase with a type-l agent he gets

$$V_{hl}^{UM} = \zeta(\beta)u_{hl},$$

while if he does not enter he gets

$$\delta V_h^{UM} = \delta\zeta(\beta)\left(\lambda u_{hh} + (1-\lambda)u_{hl}\right).$$

Hence, a type-h receiver enters an evaluation phase when he believes that the sender's type is l if $\frac{u_{hl}}{u_{hh}} > \delta\zeta(\lambda)$. In this case, a type-$l$ agent is strictly better off by sending the neologism "my type is l" and a type-h agent is strictly worse off by doing so. Hence, when $\frac{u_{hl}}{u_{hh}} > \delta\zeta(\lambda)$ this neologism is credible, so that the no-communication UM equilibrium is not neologism-proof. On the contrary, when $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda)$, a type-$h$ receiver does not enter into an evaluation phase when he believes that the sender's type is l, and hence all types of agents are strictly worse off sending the neologism "my type is l". Therefore, when $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda)$ there is no credible neologism, so that the no-communication UM equilibrium is neologism-proof.

(iv) Consider a no-communication RM equilibrium, so that $\frac{u_{ll}}{u_{lh}}, \frac{u_{hl}}{u_{hh}} > \delta\zeta(\lambda\beta)$. First, notice that when $\frac{u_{ll}}{u_{lh}}, \frac{u_{hl}}{u_{hh}} > \delta\zeta(\lambda)$ there is still a RM equilibrium under complete information, and hence any neologism believed by a receiver will not affect his decision; thus, the equilibrium is neologism-proof. Notice also that the neologism "my type is h" is never influential under RM so it cannot be a credible neologism. Next, consider the case in which $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda)$. The neologism "my type is l" will induce a type-h agent who believes it not to enter the evaluation phase, which is strictly detrimental for the sender whatever his type; hence, there is no credible neologism when $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\lambda)$. Finally, consider the case in which $\frac{u_{ll}}{u_{lh}} < \delta\zeta(\lambda) < \frac{u_{hl}}{u_{hh}}$. The neologism "my type is l" will induce a type-h agent who believes it to still enter the evaluation phase (because $\delta\zeta(\lambda) < \frac{u_{hl}}{u_{hh}}$) but a type-$l$ agent who believes it will not enter the evaluation phase (because $\frac{u_{ll}}{u_{lh}} < \delta\zeta(\lambda)$). This is strictly beneficial for a type-l sender only, and therefore "my type is l" is a credible neologism. Hence, the babbling RM equilibrium is not neologism-proof iff $\frac{u_{ll}}{u_{lh}} < \delta\zeta(\lambda) < \frac{u_{hl}}{u_{hh}}$. □

8 Conclusion

TO BE DONE.

A Appendix

A.1 Proof of Lemma 1

1. $\hat{\lambda}_{PAM} \geq \lambda$ iff $\lambda \leq 1/2$. We have:

$$\hat{\lambda}_{PAM} - \lambda = \frac{\lambda(2 - 2\lambda - \beta(1 - 2\lambda) - \sqrt{A})}{2\lambda + \beta(1 - 2\lambda) + \sqrt{A}},$$

where $A = \beta^2(1 - 2\lambda)^2 + 4\lambda(1 - \lambda)$. Since, for all $\lambda \in [0, 1]$, $2 - 2\lambda - \beta(1 - 2\lambda) \geq 0$ we have

$$\hat{\lambda}_{PAM} \geq \lambda \;\Leftrightarrow\; (2 - 2\lambda - \beta(1 - 2\lambda))^2 \geq A.$$

Then, simple calculations show that

$$(2-2\lambda-\beta(1-2\lambda))^2-A=4(1-\beta)(1-\lambda)(1-2\lambda)$$

which has the sign of $1-2\lambda$. Therefore $\hat{\lambda}_{PAM}\geq\lambda$ iff $\lambda\leq 1/2$.

2. $\hat{\boldsymbol{\lambda}}_{\boldsymbol{UM}}\leq\boldsymbol{\lambda}$**.** Simple calculations show that:

$$\lambda-\hat{\lambda}_{UM}=\frac{1-2\lambda(1-\lambda)(1-\beta)-\sqrt{B}}{2(1-\beta)\lambda},$$

where $B=1-4\lambda(1-\lambda)(1-\beta)$. Since, for all $\lambda\in[0,1]$, $1-2\lambda(1-\lambda)(1-\beta)\geq 0$, we have

$$\lambda\geq\hat{\lambda}_{UM}\;\Leftrightarrow\;(1-2\lambda(1-\lambda)(1-\beta))^2\geq B.$$

To conclude the proof, notice that $(1-2\lambda(1-\lambda)(1-\beta))^2-B=(2\lambda(1-\lambda)(1-\beta))^2\geq 0$.

A.2 Proof of Lemma 2

(i) $\hat{\lambda}_{UM}\geq\beta\lambda$. Simple calculations show that:

$$\hat{\lambda}_{UM}-\beta\lambda=\frac{2\lambda(1-\beta\lambda)(1-\beta)-1+\sqrt{B}}{2(1-\beta)\lambda},$$

where $B=1-4\lambda(1-\lambda)(1-\beta)$. Since, for all $\lambda\in[0,1]$, $1-2\lambda(1-\lambda)(1-\beta)\geq 0$, we have

$$\hat{\lambda}_{UM}\geq\beta\lambda\;\Leftrightarrow\;B\geq(1-2\lambda(1-\beta\lambda)(1-\beta))^2.$$

To conclude the proof, notice that $B-(1-2\lambda(1-\beta\lambda)(1-\beta))^2=4\lambda^3(2-\beta\lambda)\beta(1-\beta)^2\geq 0$.

(ii) $\hat{\lambda}_{PAM}\geq\beta\lambda$. We have:

$$\hat{\lambda}_{PAM}-\beta\lambda=\frac{\lambda(2-\beta(2\lambda+\beta(1-2\lambda))-\beta\sqrt{A})}{2\lambda+\beta(1-2\lambda)+\sqrt{A}},$$

where $A=\beta^2(1-2\lambda)^2+4\lambda(1-\lambda)$. Since, for all $\lambda\in[0,1]$, $2-\beta(2\lambda+\beta(1-2\lambda))\geq 0$ we have

$$\hat{\lambda}_{PAM}\geq\beta\lambda\;\Leftrightarrow\;(2-\beta(2\lambda+\beta(1-2\lambda)))^2\geq\beta^2A.$$

Then, simple calculations show that

$$(2-\beta(2\lambda+\beta(1-2\lambda)))^2-\beta^2 A=4(1-\beta)(1-\beta(1-\beta\lambda)(2\lambda-1)).$$

Since $1-\beta(1-\beta\lambda)(2\lambda-1)\geq 0$, this conclude the proof.

A.3 Proof of Lemma 3

Since $\delta\zeta(\lambda\beta)<\delta\zeta(\lambda)$, the fact that $\frac{u_{hl}}{u_{hh}}\geq\delta\zeta(\lambda)$ and $\frac{u_{ll}}{u_{lh}}\geq delta\zeta(\lambda)$ ensures existence of both RM and $\hat{RM}$. Next, Lemma 2 establishes that $\hat{\lambda}_{UM}\geq\beta\lambda$. The fact that $\frac{u_{hl}}{u_{hh}}\geq\delta\zeta(\hat{\lambda}_{UM})$ and $\frac{u_{ll}}{u_{lh}}\leq\delta\zeta(\lambda\beta)$ therefore ensures existence of both UM and $\hat{UM}$. Finally, $\frac{u_{hl}}{u_{hh}}\leq \text{Min}\{\zeta(\lambda\beta),\zeta(\beta)\zeta(\hat{\lambda}_{PAM})\}$ ensures existence of both PAM and $P\hat{A}M$.

A.4 Proof of Lemma 4

Since $\zeta(\hat{\lambda}_{UM})>\zeta(\lambda\beta)$ (from Lemma 2), there are ratios $\frac{u_{hl}}{u_{hh}}$ and $\frac{u_{ll}}{u_{lh}}$ ensuring existence of both RM and a $U\hat{M}$. Such ratios satisfy $\frac{u_{ll}}{u_{lh}}\geq\delta\zeta(\lambda\beta)$ and $\frac{u_{hl}}{u_{hh}}\geq\zeta(\hat{\lambda}_{UM})$. From Lemma 2, we also have that $\lambda\beta\leq\hat{\lambda}_{PAM}$. Using the property that $\zeta(\lambda\beta)<\zeta(\beta)\zeta(\lambda)$, we end up with two other possible changes in the pure strategy matching configurations when informative communication occurs. If $\zeta(\lambda\beta)<\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$, there are ratios $\frac{u_{hl}}{u_{hh}}$ and $\frac{u_{ll}}{u_{lh}}$ ensuring existence of both RM and $P\hat{A}M$. Such ratios satisfy $\frac{u_{ll}}{u_{lh}}\geq\delta\zeta(\lambda\beta)$ and $\delta\zeta(\lambda\beta)\leq\frac{u_{hl}}{u_{hh}}\leq\delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$. If $\zeta(\lambda\beta)<\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$, there are also ratios ensuring existence of both UM and $P\hat{A}M$. Such ratios satisfy $\frac{u_{ll}}{u_{lh}}\leq\delta\zeta(\lambda\beta)\leq\frac{u_{hl}}{u_{hh}}\leq\delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$.

A.5 Proof of Proposition 4

When switching from a matching configuration under no-communication to the same configuration under informative communication, marriage equilibrium strategies do not change: $\hat{\mu}_{hl}=\mu_{hl}$ and $\hat{\mu}_{ll}=\mu_{ll}$. When switching from RM to $P\hat{A}M$, $\hat{\mu}_{hl}=0<\mu_{hl}=1$ and $\hat{\mu}_{ll}=\mu_{ll}=1$. When switching from RM to $U\hat{M}$, $\hat{\mu}_{hl}=\mu_{hl}=1$ and $\hat{\mu}_{ll}=0<\mu_{ll}=1$. When switching from UM to $P\hat{A}M$, $\hat{\mu}_{hl}=0<\mu_{hl}=1$ and $\hat{\mu}_{ll}=1<\mu_{ll}=0$.

A.6 Detailled Welfare Comparisons

Proposition 7. *The total expected welfare of both low and high type agents is higher in an informative communication PAM equilibrium than in a no-communication PAM equilibrium.*

Proof. Let $\hat{U}_i$ and U_i denote respectively the type-i agents total expected welfare in a fully revealing and in a babbling PAM equilibrium. We show that (a) $\hat{U}_h \geq U_h$ and (b) $\hat{U}_l \geq U_l$.

(a) $\hat{U}_h \geq U_h$. We have:

$$\hat{U}_h - U_h = \hat{n}_{hh}\hat{V}_{hh} + \hat{n}_h\hat{V}_h - n_{hh}V_{hh} - n_hV_h - n_{hl}V_{hl}. \tag{25}$$

Notice that $\hat{V}_{hh} = V_{hh} = \zeta(\beta)u_{hh}$. Rearranging terms in equation (25), we obtain:

$$\begin{aligned} \hat{U}_h - U_h &= (\hat{n}_{hh} - n_{hh})V_{hh} + \hat{n}_h\hat{V}_h - (n_h + n_{hl})V_h + n_{hl}(V_h - V_{hl}), \\ &= (\hat{n}_{hh} - n_{hh})(V_{hh} - \hat{V}_h) + (\lambda - n_{hh})(\hat{V}_h - V_h) + n_{hl}(V_h - V_{hl}), \end{aligned} \tag{26}$$

where the second equality stems from the fact that $n_h + n_{hl} + n_{hh} = \lambda$ and $\hat{n}_{hh} + \hat{n}_h = \lambda$.

Notice that $V_{hh} - \hat{V}_h \geq 0$, a type h is better off being engaged with a type-h agent than being single, and $V_h - V_{hl} \geq 0$, a type h is better off being single than being engaged with a type-l agent that he will reject in the end. To show that $\hat{U}_h - U_h \geq 0$ we proceed in two steps: we show that (i) $\hat{n}_{hh} - n_{hh} \geq 0$ and (ii) $(\lambda - n_{hh})(\hat{V}_h - V_h) + n_{hl}(V_h - V_{hl}) \geq 0$.

(i) Recall that

$$n_{hh} = (1-\beta)\lambda^2 \text{ and } \hat{n}_{hh} = (1-\beta)\lambda - \frac{\beta}{2(1+\beta)}\left(\sqrt{\beta^2 + 4\lambda(1-\lambda)(1-\beta)} - \beta\right),$$

so that

$$\hat{n}_{hh} - n_{hh} = \lambda(1-\lambda)(1-\beta) - \frac{\beta}{2(1+\beta)}\left(\sqrt{\beta^2 + 4\lambda(1-\lambda)(1-\beta)} - \beta\right).$$

Therefore

$$\begin{aligned} \hat{n}_{hh} - n_{hh} \geq 0 \quad &\Leftrightarrow \quad \left(\tfrac{2(1+\beta)}{\beta}\lambda(1-\lambda)(1-\beta) + \beta\right)^2 - \beta^2 - 4\lambda(1-\lambda)(1-\beta) \geq 0, \\ &\Leftrightarrow \quad \tfrac{4\lambda(1-\lambda)(1-\beta)}{\beta^2}(\beta^3 + (1-\beta)(1+\beta)^2\lambda(1-\lambda)) \geq 0, \end{aligned}$$

which concludes step (i).

(ii) Recall that $\lambda - n_{hh} = \lambda - (1-\beta)\lambda^2$ and $n_{hl} = (1-\beta)\lambda(1-\lambda)$. Tedious but straightforward calculations show that:

$$\begin{aligned}
&(\lambda - n_{hh})(\hat{V}_h - V_h) + n_{hl}(V_h - V_{hl}) \\
&= \lambda \Big((1-(1-\beta)\lambda)(\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) - \zeta(\beta\lambda)) + (1-\beta)(1-\lambda)\zeta(\beta\lambda)(1-\delta\zeta(\beta)) \Big) u_{hh}, \\
&= \lambda\zeta(\beta)\zeta(\beta\lambda)\frac{(1-\delta)}{\beta}\frac{2(1-\lambda)-\beta(1-(1-\beta)\delta)(\sqrt{A}-\beta(2\lambda-1))}{(1-\delta)(\sqrt{A}-\beta(2\lambda-1))+2\lambda} u_{hh},
\end{aligned} \tag{27}$$

where $A = \beta^2 + 4\lambda(1-\lambda)(1-\beta)$. Notice that $\sqrt{A} \geq \beta$. The term $2(1-\lambda) - \beta(1-(1-\beta)\delta)(\sqrt{A}-\beta(2\lambda-1))$ in the numerator is (linearly) increasing in δ, and is equal to $2(1-\lambda)-\beta(\sqrt{A}-\beta(2\lambda-1))$ when $\delta = 0$; it is therefore always positive since $2(1-\lambda)-\beta(\sqrt{A}-\beta(2\lambda-1)) \geq 0$ iff $4(1-\beta^2)(1-\lambda)^2 \geq 0$. Then, notice that that $\sqrt{A}-\beta(2\lambda-1) \geq \sqrt{A}-\beta \geq 0$, so that the denominator is also positive.

(b) $\hat{U}_l \geq U_l$. The proof is similar to part (a). From $V_{ll} = \hat{V}_{ll} = \zeta(\beta)u_{ll}$ and $1-\lambda = n_l + n_{lh} + n_{ll} = \hat{n}_l + \hat{n}_{ll}$, we get:

$$\hat{U}_l - U_l = (\hat{n}_{ll} - n_{ll})(V_{ll} - \hat{V}_l) + (1-\lambda-n_{ll})(\hat{V}_l - V_l) + n_{lh}(V_l - V_{lh}). \tag{28}$$

In a PAM equilibrium, a type l is never married with type-h agents, he is better off being engaged with a type l agent than being single, i.e. $V_{ll} - \hat{V}_l \geq 0$, and he is better off being single than being engaged with a type-l agent, i.e. $V_l - V_{lh} \geq 0$. In the following, we show that (i) $\hat{n}_{ll} - n_{ll} \geq 0$ and (ii) $(1-\lambda-n_{ll})(\hat{V}_l - V_l) + n_{lh}(V_l - V_{lh}) \geq 0$.

(i) We have

$$\hat{n}_{ll} - n_{ll} = \frac{2(1-\lambda)\lambda + \beta^2(1-2(1-\lambda)\lambda) - \beta\sqrt{4(1-\lambda)\lambda + \beta^2(1-2\lambda)^2}}{2(1+\beta)},$$

so $\hat{n}_{ll} - n_{ll} \geq 0 \Leftrightarrow 4(1-\beta^2)^2(1-\lambda)^2\lambda^2 \geq 0$, which is always satisfied.

(ii) Tedious but straightforward calculations show that:

$$\begin{aligned}
&(1-\lambda-n_{ll})(\hat{V}_l - V_l) + n_{lh}(V_l - V_{lh}) \\
&= (1-\lambda)\Big((1-(1-\beta)(1-\lambda))(\zeta(\beta)\zeta(1-\hat{\lambda}_{PAM}) - \zeta(\beta(1-\lambda))) + (1-\beta)\lambda\zeta(\beta(1-\lambda))(1-\delta\zeta(\beta)) \Big) u_{ll} \\
&= \lambda\zeta(\beta)\zeta(\beta(1-\lambda))\frac{(1-\delta)}{\beta}\frac{\sqrt{A}-\beta(1-2(1-\beta)\delta(1-\lambda)^2)}{\sqrt{A}-\beta(2\lambda-1)+2\lambda(1-\delta)} u_{ll}.
\end{aligned}$$

Notice that $\beta(1-2(1-\beta)\delta(1-\lambda)^2) \leq \beta$. Hence, since $\sqrt{A} \geq \beta$, the numerator is positive. Then, notice that $\sqrt{A}-\beta(2\lambda-1)+2\lambda(1-\delta) \geq \sqrt{A}-\beta \geq 0$ which shows that the denominator is positive and concludes the proof. □

Proposition 8. *The total expected welfare of high type (low type, respectively) agents is lower (higher, respectively) in an informative communication UM equilibrium than in a no-communication UM equilibrium.*

Proof. Let $\hat{U}_i$ and U_i denote respectively the type-i agents total expected welfare in a fully revealing and in a babbling UM equilibrium. We show that (a) $\hat{U}_h \leq U_h$ and (b) $\hat{U}_l \geq U_l$.

(a) $\hat{U}_h \leq U_h$. In a babbling UM equilibrium we have

$$n_h = \beta\lambda,\ n_{hh} = (1-\beta)\lambda^2,\ n_{hl} = (1-\beta)(1-\lambda)\lambda,$$

$$V_{hh} = \zeta(\beta)u_{hh},\ V_{hl} = \zeta(\beta)u_{hl},\ V_h = \zeta(\beta)\left(\lambda u_{hh} + (1-\lambda)u_{hl}\right).$$

Hence:

$$\begin{aligned} U_h &= n_h V_h + n_{hl}V_{hl} + n_{hh}V_{hh}, \\ &= \zeta(\beta)\lambda(\lambda u_{hh} + (1-\lambda)u_{hl}). \end{aligned} \tag{29}$$

Similarly, in a fully revealing UM equilibrium, we have

$$\hat{n}_h = \beta\lambda,\ \hat{n}_{hh} = (1-\beta)\lambda\hat{\lambda}_{UM},\ \hat{n}_{hl} = (1-\beta)\lambda(1-\hat{\lambda}_{UM}),$$

$$\hat{V}_{hh} = V_{hh} = \zeta(\beta)u_{hh},\ \hat{V}_{hl} = V_{hl} = \zeta(\beta)u_{hl},\ \hat{V}_h = \zeta(\beta)(\hat{\lambda}_{UM}u_{hh} + (1-\hat{\lambda}_{UM})u_{hl}).$$

Hence:

$$\begin{aligned} \hat{U}_h &= \hat{n}_h\hat{V}_h + \hat{n}_{hl}\hat{V}_{hl} + \hat{n}_{hh}\hat{V}_{hh}, \\ &= \zeta(\beta)\lambda(\hat{\lambda}_{UM}u_{hh} + (1-\hat{\lambda}_{UM})u_{hl}). \end{aligned} \tag{30}$$

By Lemma 1, we have $\hat{\lambda}_{UM} \leq \lambda$. Then, Equations (29) and (30) together yield $\hat{U}_h \leq U_h$.

(b) $\hat{U}_l \geq U_l$. By analogy with the proof of Proposition 7, we rewrite $\hat{U}_l - U_l$ as follows:

$$\hat{U}_l - U_l = (\hat{n}_{lh} - n_{lh})(V_{lh} - \hat{V}_l) + (1-\lambda-n_{lh})(\hat{V}_l - V_l) + n_{ll}(V_l - V_{lh}). \tag{31}$$

Let us prove first that the first term in the rhm of Equation (31) is positive. Notice first that $V_{lh} \geq \hat{V}_l$: in a UM equilibrium, low types are only matched with high types, hence, a low type is better off being engaged with a high type than being single. Then, notice that

$$\begin{aligned} \hat{n}_{lh} \geq n_{lh} &\Leftrightarrow \tfrac{1}{2}(1-\sqrt{1-4\lambda(1-\lambda)(1-\beta)}) \geq \lambda(1-\lambda)(1-\beta), \\ &\Leftrightarrow (1-2\lambda(1-\lambda)(1-\beta))^2 \geq 1-4\lambda(1-\lambda)(1-\beta), \\ &\Leftrightarrow (2\lambda(1-\lambda)(1-\beta))^2 \geq 0. \end{aligned}$$

Therefore, the first term in the rhs of Equation (31) is positive: $(\hat{n}_{lh}-n_{lh})(V_{lh}-\hat{V}_l) \geq 0$.

Let us show that the sum of the second and third terms in the rhs of Equation (31) are also positive: $(1-\lambda-n_{lh})(\hat{V}_l-V_l)+n_{ll}(V_l-V_{lh}) \geq 0$. It is immediate that this is equivalent to showing that:

$$\Delta := (1-(1-\beta)\lambda)(\zeta(\beta)\zeta(\hat{\lambda}_{UM})-\zeta(\beta\lambda)) + (1-\beta)(1-\lambda)\zeta(\beta\lambda)(1-\delta\zeta(\beta)) \geq 0.$$

Denote by $B = 1-4\lambda(1-\lambda)(1-\beta)$. Replacing function $\zeta(.)$ by its expression in the above equation shows that Δ rewrites $\Delta = (1-\delta)N/D$, where

$$N(\lambda,\beta,\delta) = \beta\left(-1+\sqrt{B}-(1-\beta)\lambda\left(-3+\sqrt{B}+\lambda(2-\beta\delta(\sqrt{B}-1))\right)\right), \tag{32}$$

$$D(\lambda,\beta,\delta) = (1-(1-\beta)\delta)(1-(1-\beta\lambda)\delta)\left(2(1-\beta)\lambda-\delta(1-\sqrt{B})\right). \tag{33}$$

To conclude the proof, let us prove that (i) $D(\cdot) \geq 0$ and (ii) $N(\cdot) \geq 0$.

(i) $D(\cdot) \geq 0$. By equation (33), we have to prove that $F(\lambda) := 2(1-\beta)\lambda-\delta(1-\sqrt{B}) \geq 0$. The first and second derivatives of $F(.)$ wrt λ are given by:

$$F'(\lambda) = \frac{2(1-\beta)((2\lambda-1)\delta+\sqrt{B})}{\sqrt{B}} \text{ and } F''(\lambda) = \frac{4\beta(1-\beta)\delta}{B^{3/2}} \geq 0.$$

Therefore $F'(.)$ is nondecreasing in λ. Then, notice that $F'(0) = 2(1-\beta)(1-\delta)$ so that $F'(\lambda) \geq 0$ for all λ, i.e. $F(.)$ is nondecreasing in λ. To conclude, notice that $F(0) = 0$.

(ii) $N(\cdot) \geq 0$. Notice that $\partial N/\partial\delta = \beta(1-\beta)\lambda^2(1-\sqrt{B}) \geq 0$. Therefore, showing that $N(.) \geq 0$ is equivalent to showing $N(\lambda,\beta,\delta=0) \geq 0$. We have:

$$N(\lambda,\beta,0) = -1+\sqrt{B}-(1-\beta)\lambda(2\lambda-3+\sqrt{B}),$$

and

$$\frac{\partial N(\lambda,\beta,0)}{\partial\lambda} = \frac{1-\beta}{\sqrt{B}}(2(1-\beta)\lambda-(1-\sqrt{B}))(4\lambda-3)$$

Notice that, if $G(\lambda) := 2(1-\beta)\lambda - (1-\sqrt{B}) \geq 0$, then, $\partial N(\lambda, \beta, 0)/\partial \lambda$ has the sign of $4\lambda - 3$, i.e. $N(\lambda, \beta, 0)$ is increasing in λ on $[0, 3/4]$ and decreasing in λ on $[3/4, 1]$. Since $N(0, \beta, 0) = 0$ and $N(1, \beta, 0) = 0$, this would prove that $N(\lambda, \beta, 0) \geq 0$ and, therefore conclude the proof. Then, let us show that $G(\lambda) \geq 0$. The first and second derivatives of $G(.)$ wrt λ are given by

$$G'(\lambda) = \frac{2(1-\beta)(2\lambda - (1-\sqrt{B}))}{\sqrt{B}} \text{ and } G''(\lambda) = \frac{4\beta(1-\beta)}{B^{3/2}} \geq 0.$$

Therefore $G'(.)$ is nondecreasing in λ. Then, notice that $G'(0) = 0$ so that $G'(\lambda) \geq 0$ for all λ, i.e. $G(.)$ is nondecreasing in λ. To conclude, notice that $G(0) = 0$. □

Proposition 9. *When one moves from a no-communication RM equilibrium to an informative communication PAM equilibrium,*

(a) low type agents are worse off under informative communication: $\hat{U}_l^{PAM} - U_l^{RM} < 0$*;*

(b) high type agents are better off under informative communication: $\hat{U}_h^{PAM} - U_h^{RM} > 0$*.*

Proof. **(a)** $\hat{U}_l^{PAM} - U_l^{RM} < 0$**.** Recall that:

$$\hat{U}_l^{PAM} = \hat{n}_l \hat{V}_l + \hat{n}_{ll} V_{ll} = \hat{n}_l \zeta(\beta)\zeta(1-\hat{\lambda}_{PAM}) u_{ll} + \hat{n}_{ll}\zeta(\beta) u_{ll},$$

$$U_l^{RM} = n_l V_l + n_{lh} V_{lh} + n_{ll} V_{ll} = (1-\lambda)\zeta(\beta)(\lambda u_{lh} + (1-\lambda) u_{ll}).$$

Then, notice that $\hat{n}_{ll} = 1 - \lambda - \hat{n}_l$ and, after rearranging terms, we obtain:

$$\hat{U}_l^{PAM} - U_l^{RM} = -\zeta(\beta)\left(\hat{n}_l\left(1 - \zeta(1-\hat{\lambda}_{PAM})\right)\frac{u_{ll}}{u_{lh}} + \lambda(1-\lambda)(1-\frac{u_{ll}}{u_{lh}})\right) u_{lh} < 0.$$

(b) Sign of $\hat{U}_h^{PAM} - U_h^{RM}$. Similar calculations show that:

$$\hat{U}_h^{PAM} - U_h^{RM} = \zeta(\beta)\left(\lambda(1-\lambda)(1-\frac{u_{hl}}{u_{hh}}) - \hat{n}_h(1-\zeta(\hat{\lambda}_{PAM}))\right) u_{hh},$$

where $\frac{u_{hl}}{u_{hh}} := \frac{u_{hl}}{u_{hh}} \in (\delta\zeta(\beta\lambda), \delta\zeta(\hat{\lambda}_{PAM})))$. Since $\hat{U}_h^{PAM} - U_h^{RM}$ is decreasing in $\frac{u_{hl}}{u_{hh}}$, we would like to show that $\lambda(1-\lambda)(1-\delta\zeta(\hat{\lambda}_{PAM})) - \hat{n}_h(1-\zeta(\hat{\lambda}_{PAM})) \geq 0$.

TO BE COMPLETED

□

Proposition 10. *When one moves from a no-communication RM equilibrium to an informative communication UM equilibrium,*

(a) low type agents are better off under informative communication: $\hat{U}_l^{UM} - U_l^{RM} > 0$*;*

(b) high type agents are worse off under informative communication: $\hat{U}_h^{UM} - U_h^{RM} < 0$*.*

Proof. TO BE COMPLETED

- Similar to (b) in proof of Proposition 9
- Similar to (a) in proof of Proposition 9

□

Proposition 11 (PAM informative communication equilibrium vs. UM no-communication equilibrium)**.** *When one moves from a no-communication UM equilibrium to an informative communication PAM equilibrium,*

(a) high type agents are better off under informative communication: $\hat{U}_h^{PAM} > U_h^{UM}$*;*

(b) low type agents are worse off under informative communication: $\hat{U}_l^{PAM} < U_l^{UM}$*.*

Proof. (a) We have

$$\hat{U}_h^{PAM} = \hat{n}_h \hat{V}_h + \hat{n}_{hh} \hat{V}_{hh} = \zeta(\beta)(\lambda - \hat{n}_h(1 - \zeta(\hat{\lambda}_{PAM})))u_{hh},$$

where we used the condition $\hat{n}_h + \hat{n}_{hh} = \lambda$. From the proof of Proposition 8, Equation (29), we know that

$$U_h^{UM} = \zeta(\beta)\lambda(\lambda + (1-\lambda)\frac{u_{hl}}{u_{hh}})u_{hh}.$$

Using the conditions for a fully revealing PAM equilibrium to exist, we have $\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) \leq \delta\zeta(\hat{\lambda}_{PAM})$. Therefore, to prove the announced result, it suffices to show that

$$\lambda - \hat{n}_h(1 - \zeta(\hat{\lambda}_{PAM})) \geq \lambda(\lambda + (1-\lambda)\delta\zeta(\hat{\lambda}_{PAM})).$$

Tedious but straightforward calculations then show that

$$\begin{aligned}&\lambda - \hat{n}_h(1 - \zeta(\hat{\lambda}_{PAM})) - \lambda(\lambda + (1-\lambda)\delta\zeta(\hat{\lambda}_{PAM}))\\ =&\frac{\lambda(1-\delta)\left(\beta^2(2\lambda-1)\lambda + (2\lambda-\beta)(1-\lambda) + (1-\lambda(1+\beta))\sqrt{A}\right)}{(1+\beta)(2\lambda + (1-\delta)(\sqrt{A} - \beta(2\lambda-1)))},\end{aligned} \tag{34}$$

where $A = \beta^2(2\lambda-1)^2+4\lambda(1-\lambda)$. Since $\sqrt{A} \geq \sqrt{\beta^2(2\lambda-1)^2} \geq \beta(2\lambda-1)$, the denominator in equation (34) is positive. Therefore, it remains to show that the numerator in (34) is positive. Let $\Delta = \beta^2(2\lambda-1)\lambda+(2\lambda-\beta)(1-\lambda)+(1-\lambda(1+\beta))\sqrt{A}$. Since $A \geq \beta^2$, we have $(1-\lambda)\sqrt{A} \geq (1-\lambda)\beta$, so that

$$\Delta \geq \beta^2(2\lambda-1)\lambda+(2\lambda-\beta)(1-\lambda)+(1-\lambda)\beta-\lambda\beta\sqrt{A} := F(\lambda,\beta).$$

Let us show that $F(\lambda,\beta) \geq 0$. Notice first that, for all $\lambda \in [0,1]$, $F(\lambda,1) = 0$. Then, to conclude the proof, let us show that, for all $\lambda \in [0,1]$, $\beta \mapsto F(\lambda,\beta)$ is decreasing in β. Taking the derivative of $F(.)$ wrt β, we find

$$\frac{\partial F}{\partial \beta}(\lambda,\beta) = -\frac{(\sqrt{A}-\beta(2\lambda-1))^2}{\sqrt{A}} \leq 0.$$

(b) In a babbling UM equilibrium, the total welfare of low type agents is given by

$$\begin{aligned} U_l^{UM} &= n_l V_l + n_{lh} V_{lh} + n_{ll} V_{ll} \\ &= \beta(1-\lambda)\zeta(\lambda\beta)u_{lh} + (1-\beta)\lambda(1-\lambda)\zeta(\beta)u_{lh} + (1-\beta)(1-\lambda)^2\delta\zeta(\beta)\zeta(\lambda\beta)u_{lh} \\ &= (1-\lambda)\zeta(\lambda\beta)u_{lh}. \end{aligned} \tag{35}$$

In a fully revealing PAM equilibrium, the total welfare of low type agents is given by

$$\begin{aligned} \hat{U}_l^{PAM} &= \hat{n}_l \hat{V}_l + \hat{n}_{ll} \hat{V}_{ll} \\ &= \hat{n}_l \zeta(\beta)\zeta(1-\hat{\lambda}_{PAM})u_{ll} + \hat{n}_{ll}\zeta(\beta)u_{ll}. \end{aligned} \tag{36}$$

Hence, we have $\hat{U}_l^{PAM} < U_l^{UM}$ iff

$$\frac{u_{ll}}{u_{lh}}\zeta(\beta)(\hat{n}_l\zeta(1-\hat{\lambda}_{PAM})+\hat{n}_{ll}) < (1-\lambda)\zeta(\lambda\beta).$$

The equilibrium conditions for a babbling UM equilibrium imply $\delta\zeta(\lambda\beta) > \frac{u_{ll}}{u_{lh}}$, so it suffices to show that

$$\delta\zeta(\beta)(\hat{n}_l\zeta(1-\hat{\lambda}_{PAM})+\hat{n}_{ll}) < (1-\lambda).$$

This inequality is always satisfied since $\delta\zeta(\beta) < 1$ and $\hat{n}_l\zeta(1-\hat{\lambda}_{PAM})+\hat{n}_{ll} \leq \hat{n}_l+\hat{n}_{ll} = 1-\lambda$. □

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