Economics and Football Workshop

Department of Economics
University of Warwick

7 July 2010 World Cup Semi-Final
"Germany *vs* Spain"

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Football, Footballers and Economics

Some people think football is a matter of life and death. I don't like that attitude. I can assure them it is much more serious than that.

Bill Shankly, *Sunday Times* 4 October 1981

Football is simple. But the hardest thing is to play football in a simple way.

Johan Cruyff

You know the Dutch, they're always a bit funny, some of them.

Franz Beckenbauer

I'm sure sex wouldn't be as rewarding as winning the World Cup. It's not that sex isn't good, but the World Cup is every four years and sex is not.

Ronaldo (Brasil)

My legitimate kids are Dalma and Giannina. The rest are a product of my money and mistakes.

Diego Maradona (*on the expected utility hypothesis and quantal response equilibria!*)

Germany *vs* Spain

Post scientific programme arrangements

On Penalty Kicks:
Mixed Strategies or Mixed up About Strategies?
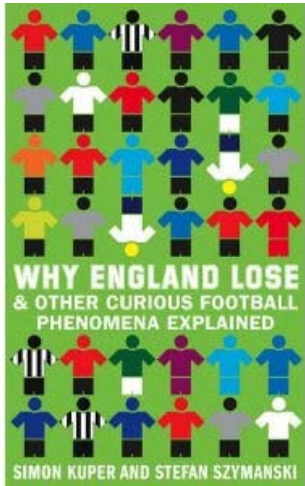
A penalty is a cowardly way to score
Pele

Whether that was a penalty or not, the referee thought otherwise
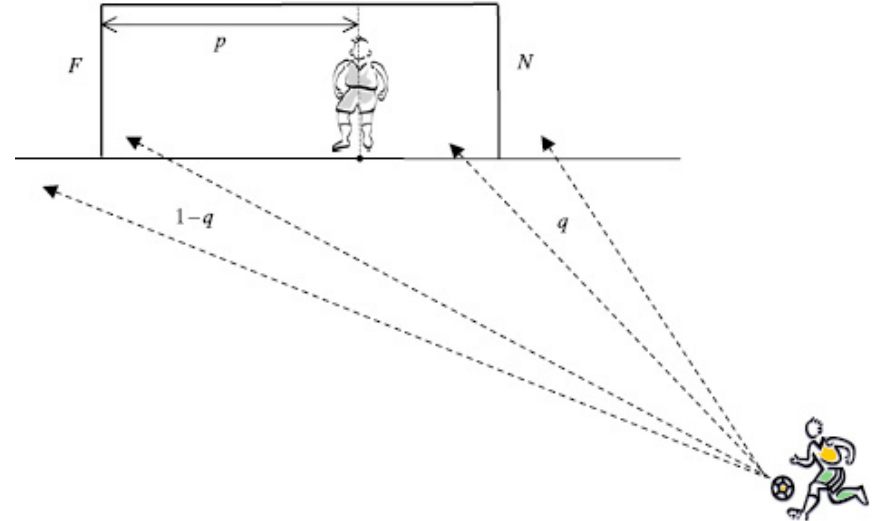John Motson, commentator

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Penalty Kicks in Football

Penalties are often dismissed as a lottery; economists tell both the kicker and goalkeeper exactly what to do. And best of all, penalties may be the best way in the known world of understanding game theory.

Kuper & Szymanski (2009), Ch. 6, p. 126

# Structure of Presentation

1. Penalty Kicks: Rules and history

2. Videos

3. World Cup shoot outs: data

4. Coaching and psychology

5. Empirical observations

6. Strategy sets

7. Payoff matrices

8. Testing for mixed strategy equilibria

9. Mixed up about strategies?

10. Conclusions

References

Some online resources

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The Penalty Kick in Football (Soccer): Rules

- Penalty kick taken from penalty spot, a spot midway between the goalposts some 12 yards (11 m) from the goal

- The penalty kick taker (**PK=Penalty Kicker**) must be clearly identified to the referee

- All players other than the defending goalkeeper (**GK=Goalkeeper**) and the PK must be outside the penalty area, behind the penalty mark, and at least ten yards (9.15 m) from the ball (i.e. outside the penalty arc) until the ball is kicked

- The GK must remain between the goalposts on the goal-line facing the ball until the ball is kicked, but may move from side to side along the goal-line. If the GK moves forward before the ball is kicked, then the penalty must be kicked again if a goal is not scored

- After the referee blows his whistle, which is the signal for the kick to be taken, the PK must kick the ball in a forward direction. The ball must be kicked after a run-up by the PK, who may slow his run but may not completely stop once the run-up has begun. If the PK scores after violating this rule, the kick must be re-taken

- Penalty introduced in England by the FA in 1891, on a line across the area, from 1902 it became a spot – first ever scored by Wolverhampton 14 September 1891

- FIFA approved penalty shoot outs in 1970, first shoot-out in 1982 World Cup

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Videos

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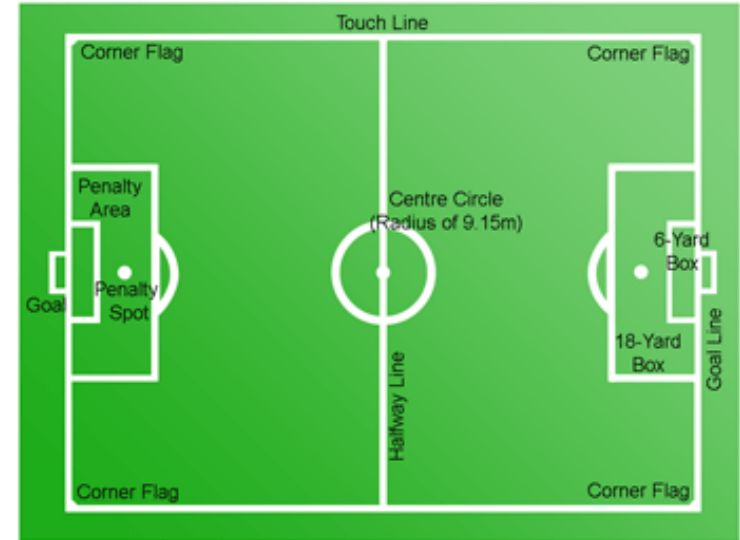
World Cup Penalty Shoot Outs
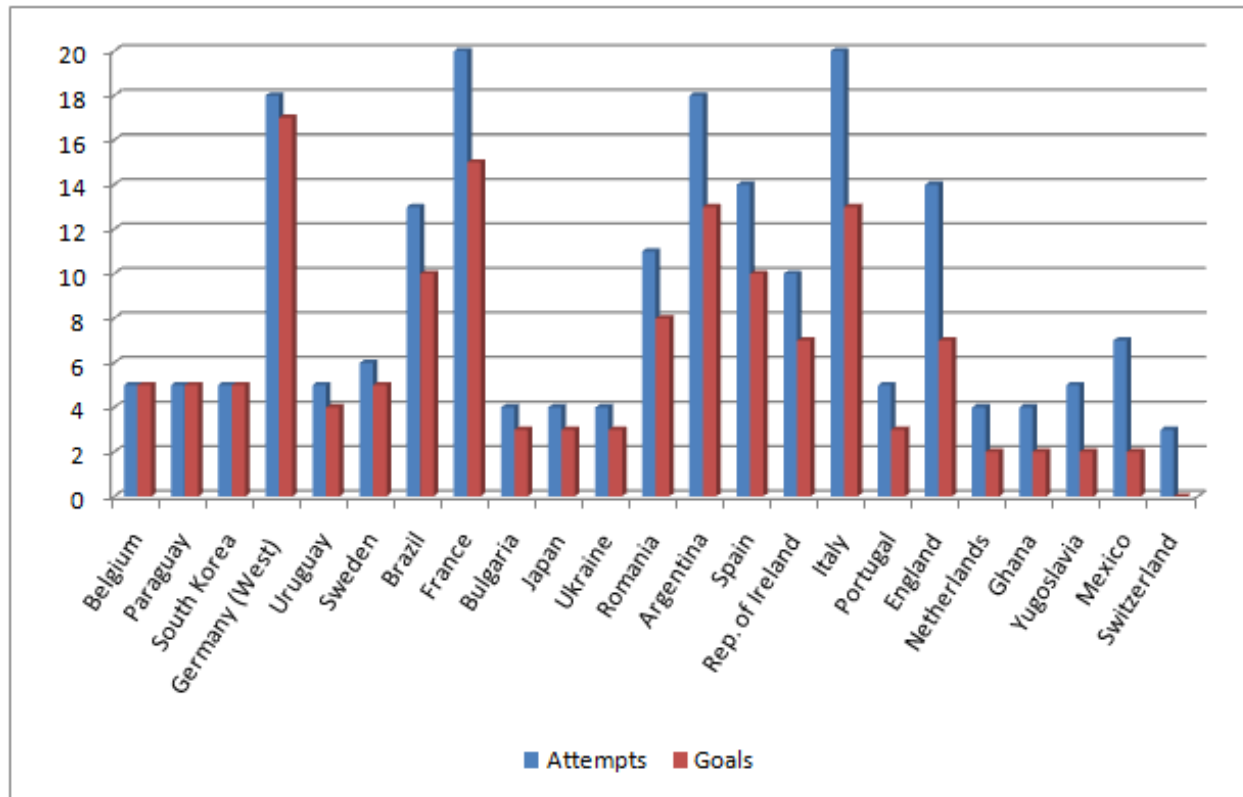
Up to 6 July 2010 (including Ghana *vs* Uruguay) there have been 204 penalties in shoot-outs, of which 144 were successful

The 71% success rate is slightly below the success rate for penalties within the course of normal play, which typically ranges from 75-85% (**Bar-Eli (2009)**)

In the 1986 Mexico World Cup about 20% of the 42 kicks shot landed in a 2m wide zone at the centre of the goal (**Bar-Eli and Friedman (1988)**)

The Role of the Coach

David Moyes
Manager Everton
FC, *Sunday
Times* 27 June
2010

In a good team you win together and you lose together, the penalty shootout is the loneliest experience in football and the trick is to make participants feel they are less on their own. The players taking spot-kicks need to know that responsibility does not weigh solely on their shoulders but is shared, and the same goes for your goalkeeper. As a manager, you can take pressure off your men by making yourself accountable for success and failure.

This means that when your players walk up to the spot they — and, more importantly, you — have decided exactly what they're going to do,. When we won against Manchester United, all my lads did exactly, from the spot, what we'd said they would beforehand. I feel you take pressure away from individuals that way. You say to a player: "All I want you to do is X." If it doesn't go in, then fine. We score together, we miss together when it is penalties.

Mentally, it's important to stay calm and ignore the goalkeeper. The keeper will most likely be jumping around trying to distract you. It is a good idea to make a quick check of the keeper's position just to make sure he isn't lined up properly, but other wise don't look at him. To enter a state of flow or 'being in the zone' when taking a penalty shot you need to stop thought. Sure you can have a pre-decided idea as to where you are going to blast the ball. But thought or any self consciousness about what you are doing will just block your success. In order to be able to reach this state consistently, you have to practice under pressure. Ultimatesoccercoaching.com

Coaching and Options for the Kicker

"A well-placed ball, high to the corner, will not be stopped by the goalkeeper even if he anticipates it" says Professor Tom Riley, Liverpool John Moores University. "There is not enough time to react, so a kick placed in this area would have a 100% strike rate." "Some players blast the ball straight down the middle, assuming that the goalkeeper will move, but it's not always successful."

Conventional wisdom says to go for the side netting (lower 90), low and just down inside the post. While this is an easier strike, a keeper that guesses correctly can get to the spot and make the save.

Source: http://www.ultimatesoccercoaching.com/soccer-kick/how-to-take-a-penalty-kick.html

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Psychology

Savelsbergh et al. (2002) used a novel methodological approach to examine skill-based differences in anticipation and visual search behaviour during the penalty kick. Expert and novice goalkeepers were required to move a joystick in response to penalty kicks presented on film. The proportion of penalties saved was assessed, as well as the frequency and time of initiation of joystick corrections. Visual search behaviour was examined using an eye movement registration system. Expert goalkeepers were generally more accurate in predicting the direction of the penalty kick, waited longer before initiating a response and made fewer corrective movements with the joystick. The expert goalkeepers used a more efficient search strategy involving fewer fixations of longer duration to less disparate areas of the display. The novices spent longer fixating on the trunk, arms and hips, whereas the experts found the kicking leg, non-kicking leg and ball areas to be more informative, particularly as the moment of foot± ball contact approached. No differences in visual search behaviour were observed between successful and unsuccessful penalties. The results have implications for improving anticipation skill at penalty kicks.
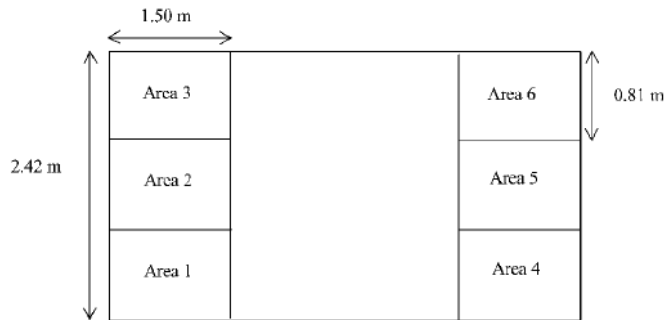

Fig. 1. The goal divided into six areas for placement of penalties and joystick movements.
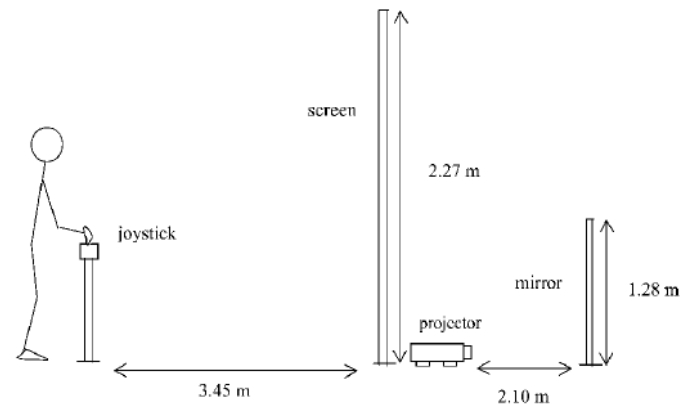
Fig. 2. A side view of the experimental set-up.

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Level-k reasoning: Osvaldo Soriano
The Longest Penalty Ever

A short story by the late Argentine writer and journalist based on a real experience in the 1950s.

Story: A match in the Argentine provinces has to be abandoned seconds before the final whistle when a corrupt referee is laid unconscious by an angry player objecting to a penalty kick awarded to the opposition. The league decides that the last 20 seconds of the game comprising the penalty kick, shall be played one week later. Everyone has a week to prepare.

Extract: At a dinner a few nights before the penalty Gato DÍaz the GK discusses:
"Constante kicks to the right'
'Always' said the president of the club.
'But he knows that I know'
'Then we're fucked'
'Yeah, but I know that he knows,' said el Gato.
'Then dive to the left and be ready,' said someone at the table. 'No. He knows that I know that he knows,' said Gato DÍaz, and he got up to go to bed.

Palacios-Huerta (2003) "[a penalty] requires unpredictability and mutual outguessing'" (p. 396)

Ch. **18 in** *The Global Game*, **edited by John Turnbull** *et al.* **(2008)**

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Empirical Observations

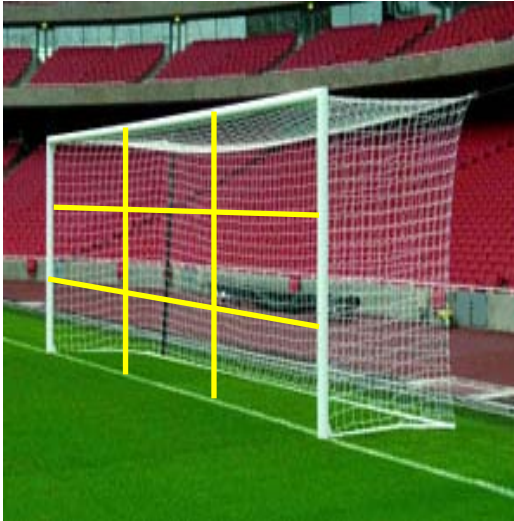- **Michael Bar-Eli and Ofer Azar (2009)** analyzed video of 286 PKs from professional leagues in Europe and South America, the European Championships and World Cup competitions. Each PK is one of three vertical (H, M or L) and horizontal (R, M or L) directions. Shots that missed the goal were excluded. GK movements are "R, L or M" and whether or not they stopped the shot.

- 85% of the penalty shots placed on goal were successful. 57% of shots taken were placed in the lower one-third of the goal. These low attempts were successful around 80% of the time. Only 13% of shots were placed in the upper third of the goal. However, all of these efforts resulted in a goal scored (100% success).

- Slightly more shots were placed to the GK's right side compared to the centre or left. Of these three directions, PKs were most successful when shooting at the centre of the goal. Shots aimed at the centre of the goal were successful 87% of the time compared to an 83% success rate for shots placed at the outer thirds of the goal.

- Based on these numbers, professional penalty kick takers most often place the ball at the lower right corner of the goal (40% of attempts). However, they are far more successful when shooting at the upper portion.

- The most successful strategy for the PK is to place the ball in the upper third of the goal area rather than the lower portion. Assuming the shot is on target, placing the shot in the upper region of the goal will almost insure a successful attempt.

- Why is this action not done so much in practice? Maybe PKs players prefer to take the risk that the GK will stop the ball rather than the risk that their kick will miss the goal, because in the former case it will not be perceived as being entirely their fault that a goal is not scored, while in the latter case it will be **NOTE players are part of a larger game.**

Strategy Sets

Table 1. Distribution of kicks.

		Horizontal direction			
		Left	Centre	Right	Total
Vertical direction	High	17 (5.9%)	10 (3.5%)	10 (3.5%)	37 (12.9%)
	Middle	25 (8.7%)	24 (8.4%)	38 (13.3%)	87 (30.4%)
	Low	50 (17.5%)	48 (16.8%)	64 (22.4%)	162 (56.6%)
	Total	92 (32.2%)	82 (28.7%)	112 (39.2%)	286 (100%)

Table 2. Stopping probabilities.

		Horizontal direction			
		Left	Centre	Right	All
Vertical direction	High	0.000	0.000	0.000	0.000
	Middle	0.160	0.083	0.132	0.126
	Low	0.240	0.188	0.172	0.198
	All	0.174	0.134	0.143	0.150

Note: The numbers under 'All' are computed as the total number of kicks stopped in all the associated areas divided by the total number of kicks to these areas; the result is therefore the weighted average of the stopping probabilities of the corresponding kick directions, and not their simple average.

Action bias, what if Shooting high is 90% Successful but low 80%? **Bar-Eli *et al.* (2007)**

***Note :* 18 from original 311 penalty Kicks were misses, thus 94.2% of Penalties on target**

Source of tables: **Michael Bar-Eli and Ofer Azar (2009)**

Strategy Sets

- A PK is likely to have 2 or 3 'strong' (measured in terms of success) penalty kicks out of the nine zones and these will be repeatedly practised

- A GK has essentially three actions (L,C,R) and chooses a direction based on prior research and anticipation prior to the action choice of the PK

- The decision which action to use for each player will likely be discussed with the coach prior to a match and agreed upon with various contingencies

- The GK knows that a PK will have made up his/her mind which action to play before the match

- The penalty subgame is a simultaneous move one-shot game where each player treats the other as part of the environment (probability distribution over states (strategies)), reducing the game to a complex individual decision problem, as described above

- At the highest level of football, 'pressure' from spectators and the media (TV, newspapers) will play a part in affecting a player's composure, leading to possible *action bias (*similar to *ambiguity aversion* where players are uncomfortable with unkowns*)*

  - This proposition may be testable by looking at lower league matches with less coverage and comparing against top league matches

Penalty Payoff Matrices

		GK		
		L	*C*	*R*
PK	*Off*	-1,1	-1,1	-1,1
	LL	2*w*-1, 1-2*w*	1,-1	1,-1
	LM	2*x*-1, 1-2*x*	1,-1	1,-1
	LH	2*y*-1, 1-2*y*	1,-1	1,-1
	CL	1,-1	2*w*-1, 1-2*w*	1,-1
	CM	1,-1	2*x*-1, 1-2*x*	1,-1
	CH	1,-1	2*y*-1, 1-2*y*	1,-1
	RL	1,-1	1,-1	2*w*-1, 1-2*w*
	RM	1,-1	1,-1	2*x*-1, 1-2*x*
	RT	1,-1	1,-1	2*y*-1, 1-2*y*

The 10x3 payoff matrix assumes certainty in action choice and common knowledge about the probabilities of stopping. No paper has appears to have examined the penalty game to this level of detail.

		GK		
		L	*C*	*R*
PK	*L*	0.704	1	1
	C	0.902	0.4	0.968
	R	1	1	0.746

The 3x3 payoff matrix shows expected payoffs for the PK, using the **Bar-Eli and Azar (2007)** dataset. **Azar and Bar-Eli (2010)** show a MSNE performs best against alternatives like matching probabilities. First analysed by **Chiappori *et al.* (2002).**

		GK	
		L (NNS)	*R*
PK	*L (NNS)*	0.583	0.949
	R	0.921	0.699

Payoff matrix **Palacios-Huerta (2003)**

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Game Theory and the Penalty Subgame

- Complete Information game of imperfect information (simultaneous move)

- Zero sum game

- Matching Pennies game is obvious analogy – no pure strategy Nash Equilibrium (NE)

- For the symmetric case there is a unique mixed strategy NE (MSNE) e.g. **Gibbons (1992)** (or a pure strategy Bayesian NE, the **Harsanyi (1973)** purification idea)*

- Prediction from game theory is that the solution to the penalty subgame is a MSNE

- Do players really play according to randomisation?

- Looking at field data is likened to a natural experiment, but lacks the control available in a laboratory and is usually too complex a setting

- Penalty kicks are simple, hence their appeal to test MSNE (minimax)

  * Based on idea that the payoff (or the type) of the opponent is not known with certainty

Mixed Strategy Nash Equilibrium

1. In finite zero sum games, where an equilibrium does not exist in pure strategies, there exists a MSNE (**von Neumann & Morgenstern (1944)**)

2. If the equilibrium is unique (as in Matching Pennies) and non-degenerate, penalty shoot-outs could be a 'real life laboratory'

3. It seems unlikely a PK will determine action at the time of the kick according to a probability distribution', but more likely the coach advises prior to the game and chooses according to a mixed equilibrium

4. Research on penalties (in economics) has focussed largely on tests for MSNE, while methodologically having to deal with the issue of aggregation (**Chiappori *et al.* (2002)**)

5. Research results have not rejected the hypothesis of MSNE, and alternative hypotheses (such as matching probabilities) perform less well (**Azar and Bar-Eli (2010)**)

Testing for mixed strategies

O'Neil (1987) conducted experiment with 25 pairs of subjects over 105 rounds (5 cents payoff)

		Player 2					
		Ace	*2*	*3*	*J*	NE	Empirical Frequencies
Player 1	*Ace*	-5	5	5	-5	0.20	0.221
	2	5	-5	5	-5	0.20	0.215
	3	5	5	-5	-5	0.20	0.203
	J	-5	-5	-5	5	0.40	0.362
NE		0.20	0.20	0.20	0.40		
Empirical Frequencies		0.226	0.179	0.169	0.426		

Brown and Rosenthal *Econometrica* **(1990)** re-examined the O'Neil game and rejected minimax play in both frequencies and serial independence, and observed an Ace bias

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Camerer (2003), Fig 3.1, p. 121
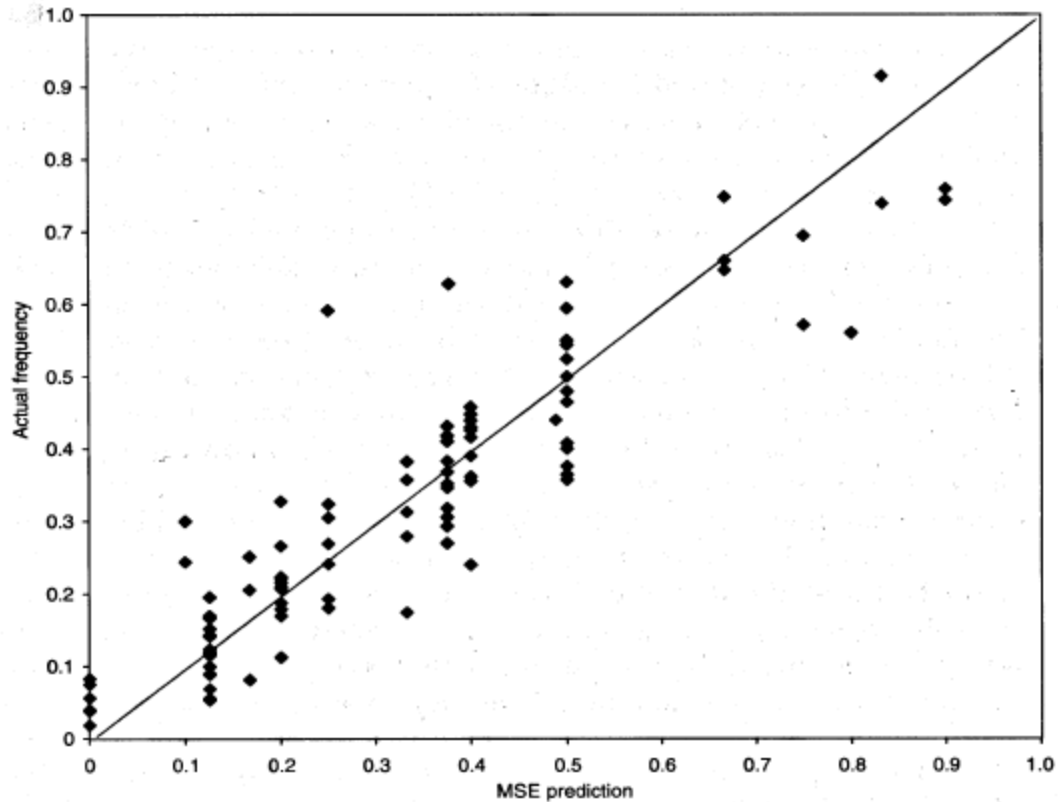
Ignacios Palacios-Huerta, *REStud* (2003)

- A test for von Neumann's Minimax Theorem in a 'natural' setting
  - 2 player one-shot zero sum game
    - **Walker and Wooders (WW)** *AER* **(2001)** is similar using the first tennis serve and data from the Wimbledon tournament
- Data on 1,417 penalty kicks in professional soccer, detailed actions and outcomes
- Results show:
  1. Winning probabilities statistically identical across strategies (also in WW)
     - Experimental data have found this difficult to obtain
  2. Players' choices serially independent (iid)
     - Experimental data in psychology and economics tends to find players 'switch strategies' too often to be consistent with random play (also found in WW with professional tennis players first serve – which were negatively serially correlated)

Palacios-Huerta (2003)

TABLE 1

Distribution of strategies and scoring rates

Score difference	#Obs.	LL	LC	LR	CL	CC	CR	RL	RC	RR	Scoring rate
0	580	16·9	1·3	21·0	4·3	0·8	5·6	19·4	0·6	27·9	81·9
1	235	19·1	0	19·1	4·2	0	2·5	28·0	0	26·8	77·8
−1	314	19·7	0·9	25·8	1·9	0	6·4	20·0	0·6	30·2	80·2
2	97	23·7	2·0	17·5	5·2	0	0	20·6	1·0	29·9	75·2
−2	114	26·3	0	25·4	3·5	0	3·5	16·6	0	24·5	78·0
3	27	14·8	0	18·5	3·7	0	11·1	22·2	0	29·6	77·7
−3	23	30·4	0	30·4	0	0	0	21·7	0	17·4	82·6
4	7	42·8	0	28·5	0	0	0	14·2	0	14·2	100
−4	12	25·0	0	25·0	0	0	16·6	16·6	0	16·6	83·3
Others	8	50·0	0	0	0	0	12·5	37·5	0	0	87·5
Penalties shot in:											
First half	558	21·1	0·8	19·8	3·9	0·3	3·5	20·0	0·3	29·7	82·9
Second half	859	18·7	0·9	23·2	3·3	0·3	3·6	22·8	0·5	26·3	78·3
Last 10 min	266	21·8	0	21·0	0·3	0	0·7	25·1	0	30·8	73·3
All penalties	1417	19·6	0·9	21·9	3·6	0·3	3·6	21·7	0·5	27·6	80·1
Scoring rate	80·1	55·2	100·0	94·2	94·1	50·0	82·3	96·4	100·0	71·1	

Note: The first letter of the strategy denotes the kicker's choice and the second the goalkeeper's choice. "R" denotes the R.H.S. of the goalkeeper, "L" denotes the L.H.S. of the goalkeeper, and "C" denotes centre.

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Palacios-Huerta (2003)

TABLE 2

Distribution of strategies and scoring rates by kicker type

| Score difference | #Obs. | Left-footed kickers | | | | | | | | | Scoring rate |
		LL	LC	LR	CL	CC	CR	RL	RC	RR	
0	174	17·8	1·7	20·1	6·3	0	8·6	22·9	0·5	21·8	82·7
1	73	28·7	0	30·1	4·1	0	2·7	19·1	0	15·0	78·0
−1	92	29·3	1·0	26·0	1·0	0	2·0	21·7	1·0	18·4	82·6
2	29	51·7	0	13·7	3·0	0	0	10·3	0	20·6	72·4
−2	30	40·0	0	13·3	3·0	0	3·0	20·0	0	20·0	76·6
All penalties	406	29·3	1·4	20·4	4·4	0	3·9	23·8	0	16·5	
Scoring rate	81·0	62·1	100	95·1	94·4	0	81·2	93·8	0	61·2	
		Right-footed kickers									
0	406	16·4	1·2	21·4	3·4	1·2	4·4	20·4	0·7	30·5	83·2
1	162	14·8	0	14·2	4·3	0	2·4	32·1	0	32·1	77·7
−1	222	15·7	1·0	25·6	2·2	0	0	19·3	1·0	35·1	80·6
2	68	11·7	2·9	19·1	5·8	0	0	25·0	1·4	33·8	76·4
−2	84	21·4	0	29·7	3·5	0	3·5	15·4	0	26·2	78·5
All penalties	1011	15·8	0·6	22·5	3·2	0·5	3·4	20·8	0·6	32·1	
Scoring rate	79·8	50·0	100	93·8	93·9	60·0	82·8	97·6	100	73·2	

Note: The first letter of the strategy denotes the kicker's choice and the second the goalkeeper's choice. "R" denotes the R.H.S. of the goalkeeper, "L" denotes the L.H.S. of the goalkeeper, and "C" denotes centre.

Palacios-Huerta (2003)

- Penalty kickers in the data fall in one of two categories: left footed or right footed.

- A right footed "kicker's natural side" is to place the ball to the RHS of the GK; a left footed "kicker's natural side" is to place the ball to the LHS of the GK

- L is used to denote the kicker's non-natural side

- The probability of success matrix and the mixed strategy Nash equilibrium and actual frequencies (aggregate data):

	g_L	$1 - g_L$
k_L	58·30	94·97
$1 - k_L$	92·91	69·92

	g_L (%)	$1 - g_L$ (%)	k_L (%)	$1 - k_L$ (%)
Nash predicted frequencies	41·99	58·01	38·54	61·46
Actual frequencies	42·31	57·69	39·98	60·02

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Chiappori, Levitt & Groseclose *AER* (2002)

- First major contribution to test indirectly Mixed Strategy Equilibria (MSE) using penalty kicks (using a variety of econometric techniques)
- Data on penalty kicks (total in sample 459) from French and Italian leagues over a three year period from 1997-2000
- Data do not reject simultaneous move assumption for the GK and PK
- GK strategy today is conditioned on PK's past history
- PK strategy today as if all GKs are identical
- Data do reject null hypothesis that scoring probabilities are equal for PKs across *R*, *L* and *C*
  - *This is the crucial test for indifference across the pure strategies*
- Analyse dataset by looking at *natural side* of PKs, allowable given PKs treat GKs as identical
- **Coloma (2007)** uses the same dataset to test directly for MSNE and claims consistency with Nash equilibrium play

Professionals *vs* Students

- **Palacios-Huerta & Volij (PH&V) *Econometrica* (2008)**
  - Professionals placed in the laboratory and shown to conform to minimax and serial independence, including for the O'Neill game, while students apparently do not conform to minimax

- **Wooders, John *Econometrica* (2010)**
  - Re-examines PH&V data, showing that, in fact, the play of professionals is inconsistent with the minimax hypothesis in several important respects: including negatively correlated strategies. He also shows the behaviour of students conforms more closely to the minimax hypothesis

Mixed Up about Strategies?

- With apologies to **Reinhard Selten (1975)** are penalty kicks a case of a *trembling foot perfect equilibrium*!

- The PK chooses a strategy aiming for a zone, there is a probability of error in executing the action (alternatively, the strategy realised may vary from the strategy planned – *the trembling foot*)

- GK does not make an error, PK has probability of error

- If the ball hits the target at any zone with equal probability, the goalkeeper may as well dive randomly or not dive at all

- Obviously, the GK seeks to signal a weak side (separating equilibrium) to try and influence the PK's choice and then defends that side

- I am alluding to *deliberate randomisation e.g.* **Reny and Robson** *GEB* **(2004)**

- Alternatively could be a *Quantal Response Equilibrium* (McKelvey and Palfrey (1995)), where an error in choosing a pure strategy is interpreted as bounded rationality – difficult to view players in penalty sub-game as consistent with bounded rationality

- Another possibility is an application of *Level-k reasoning* (**Nagel (1995); Stahl and Wilson (1995); Costa-Gomes and Crawford (2006)** While penalty kicks involve guessing, the game structure is not one where iterative dominance is apparent - but level-k reasoning could apply

- Problem of identification: can we distinguish randomisation from heterogeneous play?

- *Ambiguity aversion*? Players are uncomfortable with unknowns and behave in ways contrary to expected utility theory – randomisation is a hedge against making a poor choice

Mixed Up about Strategies?

- The empirical research using penalty kick data is based on real situations but has serious limitations – most notably a lack of *aggregation* (though this is claimed to be adequately addressed in **Chiappori *et al.* (2002)**)

- The issue of testing for mixed strategies has long been a challenge, pre-dating O'Neill's experiment

- Ideally what is required is experimental data on penalty kicks that can be controlled for *action bias,* includes financial incentives and involves players of known abilities

- Looking at recent matching penny experiments may help design appropriate penalty kick experiments to overcome *action bias* and *framing* problems

- **Goeree and Holt *AER* (2001)** look at matching pennies and show for experiments that the MSNE seems only to hold coincidentally for the case of symmetric payoffs – in other words making one cell disproportionately larger led to systematic deviation from the MSNE

  - *Penalties in shoot-out situations are highly likely to be asymmetric – in that the winner stands to gain far more than the loser loses (the outcome is not the goal exchange as typically assumed in the literature above)*

  - *Goeree and Holt discuss the validity of level-k reasoning (introspection) in the matching pennies case and add noise into a Logit based decision rule (choice probabilities positively but imperfectly related to payoffs) which makes higher levels of reasoning costly (complex)*

Mixed Up about Strategies?

- **Ariel Rubinstein (1994)** "[Matching Pennies] is classically used to motivate the notion of mixed strategy equilibrium, but randomization is a bizarre description of a player's deliberate strategy in the game. A player's action is a response to his guess about the other player's choice; guessing is a psychological operation that is very much deliberate and not random" (**Osborne and Rubinstein (1994), p.37**)

- **Eliaz and Rubinstein (2010)** Examine framing effects (labelling of players and actions, sequencing of moves, in a finitely repeated matching pennies game motivated by a game of marbles described in Edgar Allan Poe's short story "The Purloined Letter" – where one boy chooses the number of marbles (1 or 2), and the second guesses the choice – *this is much like the PK choosing where to kick and the GK guessing where he is kicking*

- **Timothy Dang (2009)** (U Arizona) looks at a game of matching pennies with guessers and players (the guessers mirror the players using *z-Tree* to see whether different players play different pure strategies and his results appear to show ambiguity aversion is the majority motivation for randomization

Conclusion

- The penalty kick in football has been viewed as a two-person one-shot simultaneous move zero sum game

- This interpretation makes it analogous to a *matching pennies* game

- Analyses of penalty kick data appear to show that professionals play mixed strategies, as the matching pennies game predicts

- Doubtful whether the PKs at the moment of a kick randomize (though some may do occasionally), though GKs may

- Preparation involves choosing a pure action in an informed fashion (using history where available) and adding randomness to avoid prior detection

- Heterogeneous players and settings makes field data potentially unreliable, as well sitting within a larger game subject to review

- A controlled setting involving real penalty kicks with real players would be very interesting and could reveal insights about choice of actions, test for ambiguity aversion, the effect of coaching (preparation) and external influences

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Some online resources

- http://www.scienceofsocceronline.com/2009/04/penalty-kicks-by-numbers.html
  Summarises some research papers

- http://www.opposingviews.com/i/understanding-the-statistics-behind-world-cup-penalty-kicks
  Looks at Bar-Eli research

- Statistics on penalties: http://www.rsssf.com/miscellaneous/penalties.html

- On the Longest Penalty Ever: http://www.buchmesse.de/en/blog/argentina/2010/05/11/the-true-story-of-the-world%E2%80%99s-longest-penalty-kick-a-look-at-osvaldo-soriano-and-his-famous-short-story-but-in-real-life-the-better-team-won/

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