Training and Search during Unemployment*

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Abstract

This paper incorporates the enforced participation to training programs in the design of optimal unemployment policies. While training counters the loss of human capital, it also affects the willingness of the unemployed to search. The analysis provides three sets of results: (1) the introduction of training reverses previous results on the optimal consumption path during unemployment; (2) the optimal contract never stops encouraging the long-term unemployed to leave unemployment; (3) the practice of targeting training programs towards long-term unemployed is optimal only if the fall in human capital upon displacement is small relative to the depreciation of human capital during unemployment. The added value of training programs is, however, largest if the opposite is true.

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1 Introduction

Optimal unemployment insurance trades off the provision of incentives to search for work and the insurance against the consequences of unemployment. A first natural consequence of unemployment is the foregone wage while unemployed. However, after returning to work, many workers still have substantially lower wages than before displacement. In the US, one fourth of the re-employed have wages that were at least 25% lower than in their previous job (Kling 2006). It has been argued that these future income losses for the unemployed are due to the loss of human capital; displaced workers lose human capital at the moment they lose their job and their human capital continues to depreciate during unemployment. Unemployment insurance should therefore insure the unemployed against both the loss of current earnings and the expected loss of future earnings. At the same time, inducing search, and thus deviating from full insurance, is more important if finding a job avoids further depreciation of human capital.

Effective training programs counter the loss of human capital. Many countries are increasing the emphasis on training to re-integrate the unemployed in the workforce. Spending on labor market programs, active and passive, averages about 3 percent of GDP in the OECD countries. The proportion of spending on active labor market programs rather than on unemployment benefits has increased to 40-50 percent in most European countries, of which on average 40 percent is spent on training programs. The impact of training programs has been estimated in the empirical literature. While there is a lot of heterogeneity in impact across different programs (Heckman et al. 1999), a meta-analysis of recent work by Card, Kluve and Weber (2009) supports the positive long-run effect of training programs.

In this paper, I introduce training programs in the design of unemployment insurance. Unemployment benefits and taxes are conditional on the participation to training programs during unemployment. Both the monetary transfers and the intensity of the training programs may vary with the length of the unemployment spell. The paper makes two important contributions. First, the paper shows how the introduction of training programs changes the trade-off between the provision of insurance and incentives for search during the unemployment spell. This reverses previous optimality results regarding the slope of the consumption path during unemployment. Second, the paper sheds light on the value of training programs for the design of unemployment insurance and relates the optimal timing of training programs to two different sources of human capital loss, upon job loss and during unemployment.

I consider an infinite-horizon model in which training programs are an effective tool to counter the decrease in an unemployed's worker human capital. I assume that training efforts are imposed by the social planner, while the search efforts to find a job are chosen by the unemployed worker. The unemployed worker bears the cost of both the search and training efforts, which are allowed to interact as in the multi-task setting by Holmström and Milgrom (1991). If training efforts increase the marginal cost of search, the required participation to training programs implies a negative *lock-in* effect with low exit rates when programs are intensive. I assume that the same training technology is not available on the job. One justification is that employers are not willing to provide training that is not specific to their firm.

In order to focus on the fundamental trade-off between insurance and incentives for search, I ignore the use of burdensome training programs to screen between different types of job seekers or to target unemployment benefits (Akerlof 1978, Besley and Coate 1992). In the optimal design, the unemployed worker is in one of three states depending on his human capital:

- In the *training state*, the level of human capital is so low that no search is induced. Training efforts are imposed to increase the level of human capital. Since no incentives are needed, the social planner can fully smooth the unemployed's consumption.

- In the *training-and-search state*, human capital is sufficiently high so that it is optimal to induce search efforts by providing incomplete insurance. The depreciation of human capital increases both the value of insurance and value of search. By mitigating the depreciation, training relaxes the trade-off between providing insurance and inducing search.

- In the *stationary state*, the social planner makes the unemployed participate in training programs to maintain the same level of human capital. At the same time, they are given incentives to search for a job.

The distinction between the states indicates that introduction of training reverses two important results in the literature on optimal unemployment insurance. First, Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) show that consumption should be strictly decreasing with the length of the unemployment spell. However, in the training state, no search is induced and the social planner only focuses on consumption smoothing. The optimal consumption path is constant. Even in the training-and-search state, training reduces the need to induce search by mitigating the depreciation. Again, the social planner can focus more on consumption smoothing. Second, Pavoni (2009) and Pavoni and Violante (2007) show that after a finite time in unemployment, the social planner puts the unemployed on social assistance, an absorbing policy characterized by constant welfare payments and no active participation. However, if training is effective, the human capital of the long-term unemployed converges to a positive, stationary level. Hence, the social planner never stops inducing the long-term unemployed to search, whatever the length of the unemployment spell. The optimal consumption path for the long-term unemployed is always decreasing.

I perform numerical simulations for CARA preferences with monetary costs of efforts, for which the state space of the recursive problem becomes one-dimensional. The numerical simulations suggest global and monotone convergence of human capital to a unique, stationary level. The immediate policy implication is that the difference between this stationary level and the level of human capital at the start of the unemployment spell determines the optimal timing of training. If the initial level of human capital is lower, training is more intensive towards the start of unemployment. If the initial level of human capital is higher, training becomes more intensive throughout unemployment. At the same time, the simulations suggest that the welfare gain from introducing training programs in the unemployment policy design is substantial when the initial level of human capital is low, while it is negligible when the initial level of human capital is high.

The human capital level at the start is linked to the fall in human capital upon displacement, while the stationary level is linked to the depreciation rate of human capital during unemployment. Upon displacement, the unemployed may lose firm-specific or industry-specific human if they are re-employed in a different firm or industry (Neal 1995, Ljungqvist and Sargent 1998). These losses are more likely to matter in declining industries or industries shifting production abroad. During unemployment, human capital may decrease due to the explicit depreciation of skills or as a process of "unlearning by not doing" (Coles and Masters 2000). If unemployment spells persist for a long time, unemployed workers can get detached from the labor market, lose work habits and confidence in the own skills (Falk et al. 2006). The empirical evidence for the depreciation of human capital during unemployment is mixed.[1] However, the decrease in human capital, upon displacement or during unemployment, has been central in explaining the persistence of unemployment and the European unemployment dilemma (Pissarides 1992, Ljungqvist and Sargent 1998, Machin and Manning 1999), as well as the negative duration dependence of exit rates (Blanchard and Diamond 1994, Acemoglu 1995). In practice, training programs are often targeted to the long-term unemployed. The analysis, however, suggests that this is optimal only if the depreciation in human capital is sufficiently important relative to the fall upon displacement, in which case the added value of training programs may be negligible. Training programs are particularly valuable when the fall in human capital upon displacement is significant, in which case partcipation to training programs should be required from the start of the unemployment spell.

This paper builds on a recent literature on optimal unemployment insurance that departs from stationary models, like Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) by introducing depreciating human capital. Shimer and Werning (2006)

[1] For instance, Frijters and van der Klaauw (2006) find that the re-employment wage distribution deteriorates significantly, in particular during the first six months of unemployment. On the other hand, Card et al. (2007) and Van Ours and Vodopivec (2006) find no significant effect of an increase in unemployment duration on either the wage or the duration of employment in the new job.

analyze the optimal timing of benefits in a McCall search model, assuming that savings are not observable. Human capital depreciation reduces the arrival rate of job offers or deteriorates the distribution of the wages being paid on the job. Pavoni (2009) analyzes the optimal unemployment insurance contract when the unemployed agent has the binary choice to exert costly search effort or not. The depreciation of human capital reduces the output upon re-employment and the probability to become employed if searching. In this paper, I assume that human capital only determines the output, but search effort is a continuous variable. The probability to become employed thus endogenously decreases during the unemployment spell if no training facility is available. Pavoni and Violante (2007) introduce costly job monitoring as an alternative to the provision of incentives and analyze the optimal sequencing of different unemployment policies. Wunsch (2009) also focuses on the optimal sequencing of policies, but introduces costly job search assistance as well.[2]

The paper is organized as follows. In Section 2, the model is presented. In Section 3 and 4, I set up the social planner's problem and I characterize the optimal unemployment insurance contract. In Section 5, I show how the recursive problem simplifies for CARA preferences with monetary costs of efforts. In section 6, I calibrate the model to perform some numerical simulations, focusing on the optimal path of training and consumption. I also calculate the welfare gain from introducing training programs in the design of unemployment insurance. The last section concludes.

2 Model

I consider an agent at the start of an unemployment spell. The agent has human capital θ, which determines his production upon re-employment $y(\theta)$, with $y' > 0 \geq y''$ and $y(0) = 0$. During each period of unemployment, the agent exerts efforts in two dimensions, search and training. Search effort s increases the probability $\pi(s)$ to find employment with $\pi' > 0 \geq \pi''$. Once the unemployed agent has found a job, he remains employed forever.[3] Training effort t increases the unemployed's human capital θ and thus output upon re-employment. Efforts are costly and the marginal cost of search may increase with the level of training. I assume a convex cost function $\psi(s,t)$ with $\psi_{s,t} \geq 0$.[4] In practice, organizing training programs may be costly for the government or profitable when trainees are temporarily employed in public jobs. I assume that

[2] Pavoni and Violante (2005) and Wunsch (2008) had a training technology in the numerical simulations of earlier versions of their papers. They assumed that training efforts cannot be imposed, but are induced by rewarding the unemployed for high values of observable human capital with higher unemployment benefits. They also assumed that training and search efforts are extreme rivals and cannot be both exerted in the same period.

[3] I only model incentive problems during unemployment. Wang and Williamson (1996), Zhao (2000) and Hopenhayn and Nicolini (2009) also analyze incentive problems on the job and assume multiple unemployment spells.

[4] I ignore efforts during employment. With monetary costs of efforts, this is only a rescaling of the net-output produced during employment.

all costs are borne by the unemployed and captured by the cost function. Of course, the social planner can compensate the unemployed agent for these costs. I denote the agent's consumption during unemployment and upon re-employment by c and c^e respectively.

Preferences The per-period utility during unemployment and employment are denoted by $u(c, \psi(s,t))$ and $u^e(c^e)$ respectively. The expected life-time utility for an agent starting in unemployment equals

$$u(c_0, \psi(s_0, t_0)) + \sum_{\tau=1}^{\infty} \beta^\tau [\pi_\tau^e u^e(c_\tau^e) + (1 - \pi_\tau^e) u(c_\tau, \psi(s_\tau, t_\tau))],$$

where the probability to be employed in period τ equals $\pi_\tau^e = \pi_{\tau-1}^e + \pi(s_{\tau-1})(1 - \pi_{\tau-1}^e)$. I focus on two standard types of preference specifications.

Specification 1 (Additive preferences)

$$u(c, \psi(s,t)) = u(c) - \psi(s,t) \textit{ and } u^e(c^e) = u(c^e)$$

Specification 2 (CARA preferences with monetary effort cost)

$$u(c, \psi(s,t)) = -\exp(-\sigma(c - \psi(s,t)) \textit{ and } u^e(c^e) = -\exp(-\sigma c^e)$$

Human Capital Human capital θ decreases during unemployment. First, human capital falls immediately when the agent loses his job. This fall may capture the loss of firm- or industry-specific human capital upon displacement. Second, human capital continuously depreciates during unemployment. This depreciation may capture the loss of job-skills, self-confidence or work habits, discriminatory preferences of employers or even the foregone learning-by-doing. I only model the depreciation in human capital explicitly, but characterize the optimal contract as a function of the level of human capital at the start of the unemployment spell.

Training Technology Training increases human capital. An unemployed agent with human capital θ_τ, exerting training effort t_τ in period τ, will have human capital at $\tau+1$ equal to $\theta_{\tau+1} = m(\theta_\tau, t_\tau)$ with $m_\theta(\theta_\tau, t_\tau) > 0$, $m_t(\theta_\tau, t_\tau) > 0$ and $m(\theta_\tau, 0) \leq \theta_\tau$. I focus on exponential depreciation with a linear training technology,

$$m(\theta, t) = \theta(1 - \delta) + t.$$

Both the foregone income and the decrease in expected future income due to unemployment are increasing in the level of human capital. Without training, the human capital of long-term unemployed converges to 0 for which there is no added-value of being employed. I assume that no training technology is available during employment such that the level of human capital remains constant once re-employed.

3 Social Planner's Problem

I assume that the social planner has three instruments at his disposal: unemployment consumption $\{c_\tau\}$, employment consumption $\{c^e_\tau\}$ and training $\{t_\tau\}$. The social planner can make these consumption and training levels dependent on the level of human capital.[5] If savings can be restricted, the consumption profile can be implemented by setting unemployment benefits equal to c_τ and taxes upon re-employment equal to $y(\theta_\tau) - c^e_\tau$ (Werning 2002, Shimer and Werning 2008). Search efforts are not observable, but have to be induced through the contract scheme. When deciding how intensively to search, the unemployed job seekers care about the consumption levels and not about their potential productivity $y(\theta)$ or the taxes and subsidies separately.

I follow the dual approach and minimize the expected cost of the insurance scheme providing a given level of expected life-time utility V to the agent, as in Spear and Srivastava (1987). The optimal contract can be written recursively with two state variables: the level of human capital θ and the expected discounted utility promised last period to the unemployed agent V. These two state variables summarize all relevant aspects of the agent's unemployment history. I denote the optimal contract that assigns expected life-time utility level V to the unemployed individual with human capital θ by $\{c(V,\theta), V^e(V,\theta), V^u(V,\theta), s(V,\theta), t(V,\theta)\}$. The contract solves

$$C(V,\theta) = \min_{c,V^e,V^u,s,t} c + \beta\left[\pi(s)C^e(V^e, m(\theta,t)) + (1-\pi(s))C(V^u, m(\theta,t))\right]$$

such that

$$u(c,\psi(s,t)) + \beta[\pi(s)V^e + (1-\pi(s))V^u] \geq V \qquad (PC)$$

$$s \in \arg\max u(c,\psi(s,t)) + \beta\left[\pi(s)V^e + (1-\pi(s))V^u\right]. \qquad (IC)$$

The expected total cost for the social planner consists of the cost this period and the expected cost from the next period on. The cost this period is equal to the unemployment consumption level c. The expected cost from tomorrow on depends on whether the agent finds work today, the respective promised utilities V^e and V^u and the level of human capital. The social planner is constrained to offer a contract for which the agent's expected utility is higher than V. This is captured by the promise-keeping constraint (PC). The search efforts of the unemployed agent cannot be observed. The

[5] I assume that the depreciation rate only depends on the level of human capital. If the duration of the unemployment spell is contractible as well, it suffices that the level of human capital at the beginning of the unemployment spell is contractible and the depreciation function $m(\theta,t)$ is known. The wage before becoming unemployed may be a good proxy for the level of human capital at the start of unemployment.

agent chooses the search level that maximizes his expected utility given the contract, which is captured by the incentive compatibility constraint (IC). The social planner refrains from providing full insurance and creates a wedge between V^e and V^u to give incentives for search. To guarantee the incentive compatibility of the contract, the first order condition of the agent's optimization problem is sufficient if $u(c, \psi(s,t))$ is concave in s.

The expected cost for the social planner to assign V^e to the agent after he has found employment, equals

$$C^e(V^e, m(\theta,t)) = \min_{c^e, \hat{V}^e} c^e - y(m(\theta,t)) + \beta C^e(\hat{V}^e, m(\theta,t))$$

such that

$$\frac{u^e(c^e)}{1-\beta} \geq V^e.$$

Since there is no asymmetric information once the agent is re-employed, it is optimal to keep promised utility constant and give the same level of consumption in every future period. The social planner's problem during unemployment simplifies to

$$C(V,\theta) = \min_{c, V^u, V^e, s, t} c + \beta \left[\pi(s) \tfrac{(u^e)^{-1}((1-\beta)V^e) - y(m(\theta,t))}{1-\beta} + (1-\pi(s)) C(V^u, m(\theta,t)) \right]$$

such that

$$\begin{aligned} V - u(c, \psi(s,t)) - \beta[\pi(s)V^e + (1-\pi(s))V^u] &\leq 0 & (\lambda) \\ u_\psi(c, \psi(s,t))\psi_s(s,t) + \beta\pi'(s)\left[V^e - V^u\right] &\leq 0. & (\mu) \end{aligned}$$

I proceed under the assumption that $C(V,\theta)$ is convex for the relevant pairs (V,θ).[6] The first order conditions and the two envelope conditions are in appendix.

4 Optimal Insurance Contract

In this section, I characterize how training and consumption evolve during unemployment. I distinguish between three different states depending on the level of human capital: the training state, the training-and-search state, and the stationary state. The proofs are in appendix.

4.1 Training State

If the level of human capital at the start of the unemployment spell is too low, the unemployed are required to participate to training programs to increase their human capital before being induced to search. The output on re-employment would be very

[6] In the numerical simulations, I find that for $y(\theta)$ sufficiently concave the value function is indeed convex.

low, making the transition to employment undesirable when an employer is not willing or not able to provide similar training. Since the optimal level of search is zero, no incentives for search are provided. The Lagrange multiplier on the incentive compatibility constraint μ equals zero. The first order conditions for the consumption levels coincide with those in the first best. The Lagrange multiplier on the promise-keeping constraint λ equals the shadow cost of the promised utility V and remains constant during the unemployment spell as long as no search is induced.[7]

Proposition 1 *In the training state ($s_\tau = 0$), $\Delta_{\lambda,\tau} = \lambda_\tau - \lambda_{\tau+1} = 0$.*

With $\mu = 0$, λ equals the inverse of the marginal utility of consumption. The marginal utility of consumption thus remains constant during unemployment in the training state.

Corollary 1 *In the training state, unemployment consumption c remains constant for additive preferences.*

Corollary 2 *In the training state, unemployment net-consumption $c - \psi$ is constant for preferences with monetary costs of efforts.*

Since no incentives for search are given, the social planner can completely insure the unemployed agent, as in the first best. The optimal path of consumption is thus constant during the training state. This is in contrast with the result by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) that unemployment consumption and consumption upon re-employment should be strictly decreasing with the length of the unemployment spell.

Optimal Training If the agent remains unemployed with certainty, the Euler equation characterizing training simplifies for both additive and CARA preferences with monetary costs to

$$\psi_t(0, t_{\tau-1}) = \beta\,(1-\delta)\,\psi_t(0, t_\tau).$$

When increasing training at $\tau - 1$ by one unit, training at τ can be decreased by $1 - \delta$ units without changing the level of human capital at $\tau + 1$. With no prospects for employment, deferring training is desirable because the effect of training depreciates over time and the cost of future training is discounted.

Proposition 2 *In the training state, the optimal level of training is increasing over time ($t_{\tau-1} < t_\tau$) for additive and CARA preferences and exponential decay.*

[7] From the first-order conditions, it also immediately follows that $C_V^e\,(V^e, m) = C_V\,(V^u, m)$. This implies that the (net) consumption level is also the same upon re-employment. This is only relevant if a job seeker can become employed without searching ($\pi\,(0) > 0$).

4.2 Training-and-Search State

When the level of human capital is sufficiently high, the social planner induces the unemployed to search for a job by providing incomplete insurance. The social planner changes both consumption during unemployment and upon re-employment to provide incentives.

The shadow price of the expected utility of the unemployed V decreases during unemployment in the training-and-search state. This follows immediately from the Euler equation. The intuition is that higher promised utility tomorrow relaxes the promise-keeping constraint today, but also decreases the incentives to search for a job today. At the optimum, the shadow price of promised utility tomorrow equals the shadow price of promised utility today minus its impact on incentives for search today,

$$\lambda_{\tau+1} = \lambda_\tau - \mu_\tau \frac{\pi'(s_\tau)}{(1 - \pi(s_\tau))}. \tag{1}$$

Proposition 3 follows since the Lagrange multiplier on the IC constraint is positive when search is induced, regardless of the presence of training.

Proposition 3 *In the training-and-search state ($s_\tau > 0$), $\Delta_{\lambda,\tau} = \lambda_\tau - \lambda_{\tau+1} > 0$.*

The shadow price λ_τ is equal to $C_V(V_\tau, \theta_\tau)$ and thus depends on both V and θ. For both additive preferences and CARA preferences, the shadow price takes a simple form though.

With additive preferences, the shadow price equals the inverse of the marginal utility of unemployment consumption, $\frac{1}{u'(c_\tau)}$. Hence, if search is positive, the result by Shavell and Weiss (1979) that unemployment consumption is decreasing still holds.

Corollary 3 *In the training-and-search state, unemployment consumption c decreases for additive preferences.*

The presence of the training technology does not change the rationale for a decreasing consumption path, as long as search is induced. If unemployment consumption were constant, the social planner could increase consumption this period and decrease consumption next period such that the social planner's expected cost remains the same for a given level of search. Since the consumption levels and therefore the marginal utilities are initially the same, the change in the consumption pattern has only a second order effect on the expected utility of the unemployed agent. The reduction in tomorrow's unemployment consumption will induce a higher search level though and thus decreases the expected payments to be made by the social planner. Interestingly, although the threat of future training requirements induces job seekers to search harder today, the social planner does not stop using the threat of lower future consumption levels for long-term unemployed job seekers.

With CARA preferences with monetary costs, the shadow price equals an inverse function of the promised utility V, independent of the level of human capital.[8]

Corollary 4 *In the training-and-search state, the promised utility during unemployment V decreases for CARA preferences with monetary costs of efforts.*

Spreading the incentives for current search efforts over all future periods of unemployment allows providing more insurance ex ante. The presence of training does not change this. However, since it mitigates the depreciation in human capital, the social planner may prefer to give less incentives for search and focus more on insurance. If the introduction of training decreases the value of $\mu_\tau \frac{\pi'(s_\tau)}{1-\pi(s_\tau)}$ in condition (1), the social planner will actually smooth consumption more. The numerical simulations in Section 6 suggest that the decrease in net-consumption during unemployment is less pronounced in the optimal scheme with training compared to the optimal scheme without training, in particular when the level of human capital is low and training is thus intensive.

The social planner also adjusts consumption upon re-employment to improve the trade-off between incentives and insurance. From the first order conditions with respect to V^e and V^u, I find that

$$C_V(V,\theta) = \pi(s) C_V^e(V^e, m(\theta,t)) + (1-\pi(s))\, C_V(V^u, m(\theta,t)). \tag{2}$$

With additive preferences, this simplifies to the Rogerson condition (or inverse Euler equation),

$$\frac{1}{u'\left(c_\tau\right)} = \pi(s_\tau)\frac{1}{u'\left(c_{\tau+1}^e\right)} + \left(1-\pi\left(s_\tau\right)\right)\frac{1}{u'\left(c_{\tau+1}\right)},$$

which implies that consumption upon re-employment must exceed unemployment consumption. The successful job seeker is immediately rewarded with higher consumption.[9] This reward upon re-employment, however, is made dependent on the duration of the unemployment spell, spreading again incentives over all future states, also during employment. Hopenhayn and Nicolini (1997) find conditions for additive preferences, without human capital decay and training, under which consumption upon re-employment is decreasing for long-term unemployed. Corollary 5 provides a generalization of their result

[8] Notice that if preferences are not additive, the Shavell and Weiss argument for decreasing consumption may not hold for two reasons. First, if efforts are changing over time, equality of unemployment consumption may not imply equality of marginal utility with respect to consumption. Second, the decrease in consumption next period may increase the marginal cost of search next period. The implied decrease in search next period may outweigh the increase in search this period. With monetary costs of efforts, equality of net-consumption does imply equality of marginal utilities. However, a decrease in net-consumption still increases the marginal cost of search.

[9] This again does not generalize for CARA preferences with monetary cost of efforts, since high unemployment consumption reduces the marginal effort cost of search.

Corollary 5 *Consumption upon re-employment cannot remain constant or be always increasing with the length of the unemployment spell, both for additive preferences and CARA preferences with monetary costs.*

The presence of training affects the degree of consumption smoothing, but does not change the nature of the conditions characterizing the optimal consumption levels in the training-and-search state. The resulting changes in human capital, however, do affect the implementation of the optimal consumption path. The social planner wants to provide insurance against the loss of human capital, disconnecting the consumption levels during unemployment and upon re-employment from the (potential) productivity of the agent. Taxes upon re-employment that increase with the length of the unemployment spell do not necessarily imply decreasing consumption levels. If human capital decreases during the unemployment spell, taxes may be lower after long unemployment spells to smooth consumption. If human capital increases with the length of the unemployment, taxes may be higher after long unemployment spells to preserve incentives for search. In general, optimal taxes upon re-employment can thus be positive or negative and depend non-monotonically on the length of the unemployment spell. Similarly, the optimal unemployment replacement rate with respect to the potential wage does not necessarily evolve monotonically during unemployment.

Optimal Training Human capital depreciation increases both the value of insurance and the value of inducing search. However, the consumption schedule cannot provide more incentives for search without reducing insurance and vice versa. By countering the depreciation, training is a valuable alternative. The value of training depends on how much the unemployed decide to search, which depends on how much they are required to train. Training and search are complements with respect to the expected value of finding employment, but they are substitutes with respect to the costs, as training increases the marginal cost of search. When the social planner requires more intensive training efforts, it can still fully control the private gains from finding employment by adjusting the consumption levels, but it cannot avoid that additional search is more costly. The value of training and search efforts depends on the level of human capital as well. The return to training is likely to be higher for low levels of human capital (if $y''(\theta) \leq 0$), whereas the return to search is higher for high levels of human capital (since $y'(\theta) > 0$). Compared to the training state, the impact of training on the incentives for search and on potential output in the next period becomes relevant. For additive preferences, the Euler equation equals

$$\frac{\psi_t(s_{\tau-1}, t_{\tau-1})}{u'(c_{\tau-1})} + \mu_{\tau-1}\psi_{s,t}(s_{\tau-1}, t_{\tau-1}) =$$
$$\beta\pi(s_{\tau-1})\frac{y'(\theta_\tau)}{1-\beta} + \beta(1-\pi(s_{\tau-1}))[\frac{\psi_t(s_\tau, t_\tau)}{u'(c_\tau)} + \mu_\tau\psi_{s,t}(s_\tau, t_\tau)]\,(1-\delta)\,.$$

For CARA preferences with monetary costs, the Euler equation equals

$$\psi_t(s_{\tau-1}, t_{\tau-1}) + \mu_{\tau-1} u'(c_{\tau-1} - \psi_{\tau-1}) \psi_{s,t}(s_{\tau-1}, t_{\tau-1}) =$$
$$\beta\pi(s_{\tau-1}) \frac{y'(\theta_\tau)}{1-\beta} + \beta(1 - \pi(s_{\tau-1})) \left[\psi_t(s_\tau, t_\tau) + \mu_\tau u'(c_\tau - \psi_\tau) \psi_{s,t}(s_\tau, t_\tau)\right] (1-\delta) .$$

For both types of preferences, increasing $t_{\tau-1}$ increases the cost of training efforts, but changes also the incentives by increasing the marginal cost of search if $\psi_{s,t} > 0$. On the other hand, the increase in training at $\tau - 1$ increases human capital at τ. This increases the output when a job is found. When no job is found, this increase allows to decrease training at τ to bring human capital back to the same level at $\tau + 1$ if $t_{\tau-1}$ had not been increased. In section 6, I use numerical simulations to get more insights in the value of training and its timing during the unemployment spell.

4.3 Stationary State

If training programs are effective, the long-term unemployed may converge to a stationary state with positive human capital θ^*. In such a stationary state, the same level of human capital is maintained with constant training effort $t^* = \delta\theta^*$. The unemployed exit unemployment with positive probability, otherwise the social planner would not impose costly training efforts. Hence, under convergence to such a state, the social planner never gives up on the unemployed and continues to induce them to find employment. This is in stark contrast with the optimality of *social assistance* when no training facility is available (Pavoni and Violante 2007). Individuals on social assistance receive welfare payments and are not encouraged to find employment. Without training, the depreciation of human capital causes the potential production to be too small compared to the cost of inducing search after a finite time of unemployment.[10] Hence, the unemployed enter social assistance within finite time and once they have entered social assistance, they never leave again.

The introduction of training thus changes the optimal consumption path for the long-term unemployed as well. Without training facility, the unemployment consumption path becomes constant in finite time, as the unemployed enter social assistance. With training facility, search continues to be induced as human capital converges to a positive level. Hence, Proposition 3 still applies.

Corollary 6 *In a stationary state with $\theta^* > 0$, unemployment net-consumption and consumption upon re-employment are decreasing with the length of the unemployment spell for CARA preferences with monetary cost of effort.*

The corollary does not prove convergence. The numerical simulations in section 6, however, show that human capital indeed converges to a positive stationary level

[10] This requires that without training technology, human capital converges to a sufficiently small level. Pavoni and Violante (2007) assume human capital converges to 0.

if training is sufficiently effective. Moreover, the convergence is monotone and global. Regardless of the level of human capital at the start θ_0, human capital converges to this unique level.

If the unemployed can train to maintain their human capital, there is no reason to stop providing incentives for search, even after very long unemployment spells. This intuition naturally generalizes to other preference specifications. That this results in ever decreasing expected utility for the long-term unemployed, and thus ever decreasing consumption for CARA preferences, of course assumes that there is no lower bound on the expected utility (see Pavoni 2007), for instance coming from limited liability or political constraints.

5 CARA Preferences with Monetary Costs

In this section, I show that for CARA preferences with monetary costs the value function is additive in V and θ efforts, $C\left(V,\theta\right)=h\left(V\right)-g\left(\theta\right)$. I guess and verify $h(V)$, as in Werning (2002) and Shimer and Werning (2008), which only leaves $g(\theta)$ to be approximated numerically.

With CARA preferences, the optimal response to an increase in promised utility V is to increase all consumption levels by the same amount, regardless of the level of human capital. Increasing the consumption levels equally, today and in the future, while employed and unemployed, leaves the margins for search and training unchanged. For search, this is clear from the incentive compatibility constraint and the properties of CARA preferences. Since $u(x+y)=-u(x)u(y)$ and $u(x)=-\frac{u'(x)}{\sigma}$, the promised utilities V^e and V^u and marginal utility $u'(c-\psi)$ are all rescaled by $-u(\varepsilon)$ after an ε-increase in all consumption levels. Hence, the incentive compatibility constraint,

$$\beta\pi'(s)[V^e-V^u]=u'(c-\psi)\psi_s(s,t),$$

remains binding after an equal increase in all consumption levels. The fact that an equal increase in all consumption levels is an optimal response to an increase in V implies that the optimal promised utilities V^e and V^u and current-period utility $u(c-\psi)$ are proportional to life-time utility V, for a given level of human capital θ. I can rewrite the optimal contract as $\{\alpha_u(\theta),\alpha_{V^e}(\theta),\alpha_{V^u}(\theta),s(V,\theta),t(V,\theta)\}$ with

$$\begin{aligned} V^e(V,\theta) &= \alpha_{V^e}(\theta)V && (3)\\ V^u(V,\theta) &= \alpha_{V^u}(\theta)V && (4)\\ u(c-\psi) &= \alpha_u(\theta)V. && (5)\end{aligned}$$

Given exponential utility, the optimal increase in the consumption levels in response to an increase in V is thus independent of the level of human capital and does not interact with search or training either. The value function is additive in θ and V.

Proposition 4 *For CARA preferences with monetary cost of efforts, we have*

$$C^e(V^e,\theta) = -\frac{\ln(-V^e(1-\beta))}{\sigma(1-\beta)} - \frac{y(\theta)}{1-\beta}$$

and

$$C(V,\theta) = -\frac{\ln(-V(1-\beta))}{\sigma(1-\beta)} - \frac{g(\theta)}{1-\beta}$$

for some unknown function $g(\theta)$.

I rewrite and simplify the Bellman equation for $C(V,\theta)$ in terms of $g(\theta)$ in appendix, using the expressions in Proposition 4. In the following section, I solve numerically for $g(\theta)$ and the optimal policy variables as a function of human capital θ. In a stationary state, however, human capital θ^* and thus training t^* are constant. From equations (3), (4) and (5), it follows that the relative change in per-period utility is constant during the stationary state

$$\frac{u(c_\tau - \psi(s_\tau, t^*))}{u(c_{\tau+1} - \psi(s_{\tau+1}, t^*))} = \frac{u(c^e_\tau)}{u\left(c^e_{\tau+1}\right)} = cst.$$

Like in a model without human capital and training (see Werning 2002 and Spinnewijn 2009), this implies that consumption during unemployment and upon re-employment is constantly decreasing in the length of the unemployment spell and the level of search effort exerted is constant as well. The optimal stationary level of search and training (and thus human capital) can be characterized without knowing $g(\theta)$. In appendix, I derive that the stationary state must satisfy

$$\begin{aligned} \beta\pi'(s^*)[\frac{y(\theta^*) - \psi(s^*,\delta\theta^*) + \kappa/(1-\beta)}{1-\beta(1-\pi(s^*))}] + \frac{\partial\kappa}{\partial s}/(1-\beta) &= \psi_s(s^*,\delta\theta^*) \\ \beta\pi(s^*)\frac{y'(\theta)/(1-\beta)}{1-\beta(1-\pi(s^*))(1-\delta)} + \frac{\partial\kappa}{\partial t}/(1-\beta) &= \psi_t(s^*,\delta\theta^*). \end{aligned}$$

The expected gain of search is the increased probability to produce output $y(\theta)$ rather than to search and train at cost $\psi(s,\delta\theta)$. The expected gain from training is the increase in production upon re-employment $y'(\theta)$. The difference in discounting between search and training comes from the fact that training efforts add to a depreciating stock of human capital, whereas search efforts vanish every period. The influence of the incentive compatibility is completely reflected in the function $\kappa(s,\delta\theta,\frac{\alpha_{V^u}}{\alpha_u})$, which equals 0 in the first best and is described in appendix.

6 Numerical Simulations

In this section, I calibrate the dynamic model to calculate the optimal unemployment insurance contract numerically. I analyze the optimal timing of training and search during the unemployment spell and the interaction with the optimal path of consumption. The numerical methodology is based on value function iteration with discretization of the state space. I restrict attention to CARA preferences with monetary effort costs which essentially makes the state space one-dimensional as shown in Proposition 4. The calibration exercise closely follows the previous literature when possible. In contrast with the previous literature, I explicitly model the returns and costs of continuous search and training efforts. I calibrate the search parameters to match the empirical estimates of the exit rate and the elasticity with respect to the unemployment benefit level, evaluated at the current US unemployment insurance scheme. I take the same parameters for the training effort costs, but I vary the parameter values capturing the effectiveness of training efforts and the cost rivalry with search efforts.

6.1 Calibration

CARA Preferences The unit of time is set to be one month and the monthly discount factor $\beta = 0.996$ to match an annual discount factor of 0.95. The unemployed individual has CARA preferences $u(c, \psi(s,t)) = -\exp(-\sigma(c - \psi(s,t)))$ with CARA coefficient $\sigma = 2$.

Human Capital Depreciation Human capital depreciates exponentially at a monthly depreciation rate $\delta = 0.0135$, following Pavoni and Violante (2007). Human capital determines output upon re-employment, $y(\theta) = \theta^{\omega}$ with $\omega < 1$. The stationary level of human capital in the optimal scheme is normalized to 1.

Search Costs and Returns The probability to find a job as a function of search effort s equals $\pi(s) = 1 - \exp(-\rho s)$, following Hopenhayn and Nicolini (1997). I assume a convex monetary cost of search $\psi_0 s^{\psi_1}$. The parameter values for ψ_0, ψ_1 and ρ are chosen to match the following moments evaluated for the current US unemployment insurance system; the simulated exit rate equals 0.2, matching the sample exit rate in Spinnewijn (2009), the simulated elasticity in the exit rate with respect to a change in benefits equals 0.5, matching the empirical evidence summarized in Krueger and Meyer (2002), and the monetary cost of search effort when unemployed equals one third of output when employed.

Training Costs and Returns I assume the same monetary cost of training $\psi_0 t^{\psi_1}$ and introduce a linear interaction term $\psi_{s,t} st$ in the cost function,

$$\psi(s,t) = \psi_0 s^{\psi_1} + \psi_0 t^{\psi_1} + \psi_{s,t} st.$$

I assume a linear training technology such that the next month's level of human capital equals $m(\theta, t) = (1-\delta)\theta + zt$. I show results for different values of z and $\psi_{s,t}$, capturing the effectiveness of training and the complementarity with search respectively.

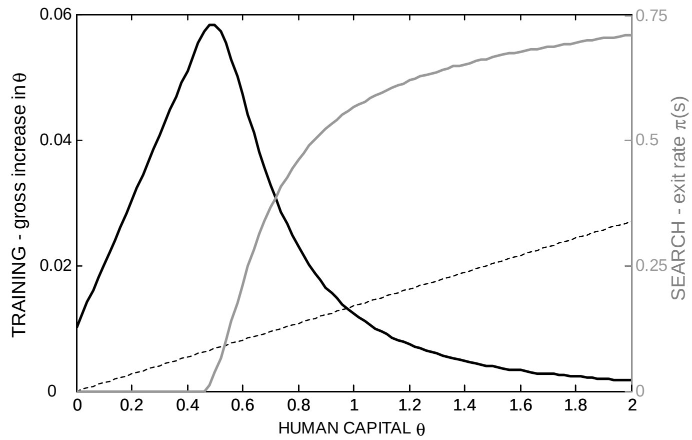

Figure 1: Policy functions: training and search efforts as a function of human capital θ. Training is expressed as the gross increase in θ ($z \times t$). Search is expressed as the exit rate ($\pi(s)$). The depreciation in human capital is presented by the dotted line.

Given the optimal contract and the standard parameter specification ($\psi_{s,t} = 0.02; z = .003; \omega = .8$), the long-term unemployed spend half as much effort on training relative to search.

6.2 Optimal Policy Functions

Figure 1 presents the optimal level of training and search efforts as a function of the level of human capital. The training and search efforts are represented by respectively the resulting gross increase in human capital $z \times t$ and the exit rate $\pi(s)$ respectively. The training efforts are imposed by the social planner, while the search efforts are chosen by the job seeker. The depreciation in human capital $\delta\theta$ is presented by the dotted line.

The unemployed agent is in the training or the training-and-search state, depending on whether his human capital is lower or higher than the cut-off level $\underline{\theta} = .46$. For levels below $\underline{\theta}$, the level of human capital is too low to induce the unemployed to search for work. The unemployed agent only exerts effort to increase the level of human capital. The training intensity is increasing in human capital, in line with Proposition 2. For levels above $\underline{\theta}$, the level of human capital is sufficiently high for the social planner to induce the unemployed to search for work at the expense of complete insurance. The induced search efforts are increasing in the level of human capital. The exponential depreciation of human capital makes that the foregone output and the expected loss in future output are increasing with the level of human capital. The social cost of remaining unemployed is thus higher, the higher the level of human capital. In the training-and-search state, the unemployed is not only induced to search for work, but at the same time obliged to participate in training programs.

The training and search policy function have opposite slopes. This is first of all

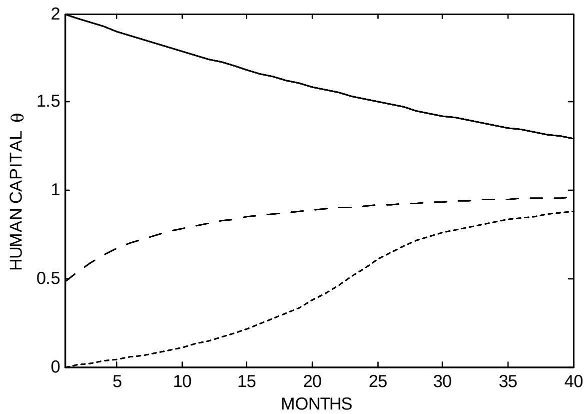

Figure 2: Human capital θ during the unemployment spell, starting with high human capital ($\theta_0 > \theta^*$ - solid), low human capital ($\theta_0 < \theta^*$ - dashed) and no human capital ($\theta_0 = 0$ - dotted).

driven by the rivalry in the cost of effort function. Time spent on training cannot be spent on search. Hence, the marginal cost of search is likely to be higher when the unemployed is required to participate in training programs (i.e. $\psi_{s,t} > 0$). Second, the value of training and search efforts interact with the level of human capital. Search is more valuable when human capital is high, whereas training is more valuable when human capital is low. These two effects dominate the complementarity between training and search efforts coming from their interaction in determining the next period's expected output $\pi(s)\, y(m(\theta, t))$.

6.3 Optimal Path of Training and Human Capital

The change of human capital during the unemployment spell depends on difference between the gain in human capital due to the training efforts and the loss in human capital due to depreciation. Figure 1 shows that the training program exactly offsets the deprecation at the stationary level $\theta^* = 1$. Below this level ($\theta < \theta^*$), human capital increases. Above this level ($\theta > \theta^*$), human capital decreases. Figure 2 shows how the unemployed's human capital evolves with the length of the unemployment spell for three different levels of human capital θ_0 at the start; high ($\theta_0 > \theta^*$), low ($\theta_0 < \theta^*$) and no human capital ($\theta_0 = 0$). The convergence is global. Regardless of this level, human capital converges to the stationary level θ^*. Once the stationary level is reached, the social planner imposes training efforts such that human capital remains constant. This implies that the social planner never gives up on the unemployed. Even the long-term unemployed are trained to remain employable. This policy conclusion is in stark contrast with the optimality of social assistance when no training facility is available (Pavoni and Violante 2007).

The convergence of human capital is also monotone and the stationary level θ^* is

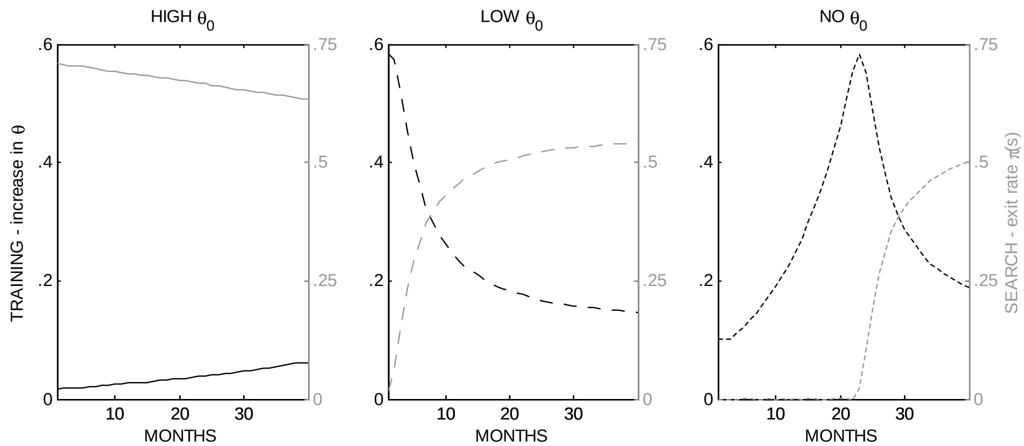

Figure 3: Training and search during the unemployment spell, starting with high human capital ($\theta_0 > \theta^*$ - solid), low human capital ($\theta_0 < \theta^*$ - dashed) and no human capital ($\theta_0 = 0$ - dotted).

unique. Two unemployed individuals who are identical except for the level of human capital at the start of the unemployment spell converge to the same level of human capital. The implications for training policies are straightforward. First, training efforts are lower the higher the human capital level at the start of the unemployment, except when the unemployed starts in the training state. Second, the difference between the human capital level at the start of the unemployment spell θ_0 and in the stationary state θ^* determines the optimal timing of training throughout the unemployment spell. Figure 3 shows the optimal training path during unemployment for different starting levels of human capital θ_0. If human capital is high at the start ($\theta_0 > \theta^*$), training is less intensive at the beginning of the unemployment spell and becomes more intensive during the unemployment spell. If human capital is low at the start ($\underline{\theta} < \theta_0 < \theta^*$), training is very intensive at the beginning of the unemployment spell and becomes less intensive during the unemployment spell. Finally, for very low human capital at the start ($\theta_0 < \underline{\theta}$), the unemployed agent starts in the training state. Training will be increasing at the beginning of the unemployment spell, but starts decreasing once θ passes $\underline{\theta}$. When the unemployed starts without any valued human capital, this training stage takes more than twenty months, as shown in the right panel of Figure 3. The panels also show the duration-dependence of the exit rates. As discussed before, the optimal levels of training and search follow opposite trends during the unemployment spell.

The difference between θ_0 and θ^* can be linked to the two sources for the loss of human capital after job loss. Human capital falls upon job loss and depreciates during unemployment. The fall in human capital when losing a job is reflected in the θ_0. Two identical agents will have different levels of human capital at the start of the unemployment spell, if the firm-specific or industry-specific capital they lose when losing their job is different. The depreciation in human capital is reflected in

θ^*. Increasing the rate of depreciation decreases the stationary level of human capital. Therefore, the more important the fall in human capital upon displacement, relative to the depreciation of human capital during unemployment, the more important training becomes towards the beginning of the unemployment spell.

In practice, training requirements are mostly imposed on the long-term unemployed. Only after some time in unemployment, one needs to enroll in particular training programs to remain eligible for unemployment benefits. Recent examples are the 'New Deal' in the United Kingdom and the 'Activation of Search Behavior' in Belgium. Such programs are more desirable if the long-term unemployed are particularly unemployable because of the depreciation of job skills during unemployment, alienation from the job market or the lack of on-the-job training. Not all programs have this particular focus on the long-term unemployed. In some countries, training is subsidized from the start of the unemployment spell and the unemployed are allowed to refuse jobs offered by the Public Employment Service if they enroll in these training programs. Some programs are aimed at young people or focus on large groups of workers who have been displaced as a result of industrial restructuring (e.g. public training programs in Germany after the unification). In general, targeting training programs to the short-term unemployed becomes more important if firm-specific and industry-specific human capital are significant and job displacement causes a big drop in human capital.

6.4 Optimal Path of Consumption

The use of training programs affects the optimal path of consumption during unemployment and upon re-employment. The introduction of training generally changes the trade-off between the provision of insurance and incentives for search. It even reverses previous results regarding the optimal consumption path by changing whether it is optimal to induce search at different times during the unemployment spell.

The introduction of (effective) training programs makes that the social planner never stops inducing the long-term unemployed to search for work. The optimal consumption path necessarily becomes downward sloping for long unemployment spells, as shown in Corollary 4. This is different when no training facility is available, as considered by Pavoni and Violante (2007). The long-term unemployed end up with social assistance in which they are not induced to search. Without training, the optimal consumption path necessarily becomes constant for long unemployment spells (right panel of Figure 4). The opposite pattern may occur at the start of the unemployment spell. If human capital is sufficiently low, the social planner only imposes training efforts and induces no search. Since no incentives for search are needed, there is no need to give up perfect consumption smoothing. Hence, consumption is constant during the training state of the unemployment spell (right panel of Figure 4). However, with no training facility, the optimal contract induces search by having strictly decreasing net-consumption levels as long as human capital is not too low at the start (Shavell

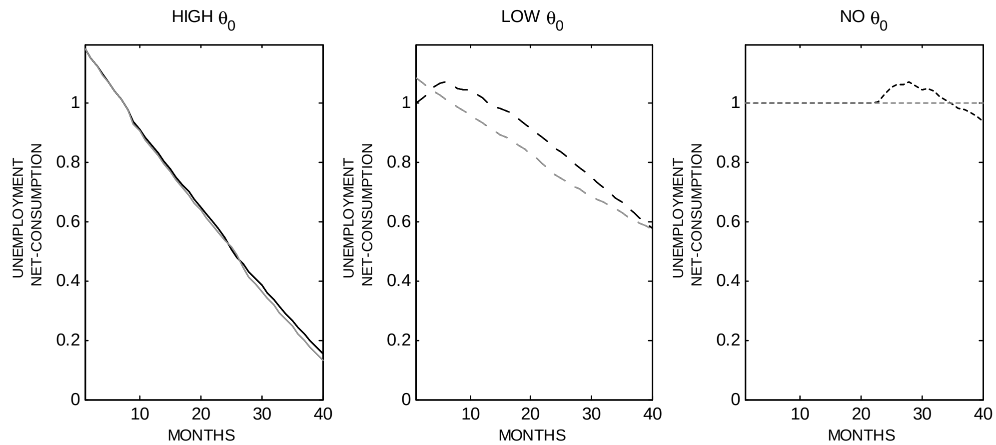

Figure 4: Net-consumption during the unemployment spell with training technology (black) and without training technology (grey), starting with high human capital ($\theta_0 > \theta^*$ - solid), low human capital ($\theta_0 < \theta^*$ - dashed) and no human capital ($\theta_0 = 0$ - dotted).

and Weiss 1979, Hopenhayn and Nicolini 1997). The consumption scheme is the social planner's only instrument to insure the unemployed and provide incentives for search. Since training mitigates the effect of human capital depreciation, it reduces the need for search for a given level of human capital. This generally allows the social planner to focus more on insurance and smooth the marginal utility of consumption. Hence, net-consumption will not be as rapidly decreasing in the beginning of the unemployment spell, particularly when it is optimal to impose intensive training as shown in the center panel of Figure 4. Due to the interactions between consumption and the search and training efforts, the path may be even upward sloping . This suggests that training as an active labor market policy is more complementary to a continental European unemployment insurance scheme with low incentives for search (high and slowly decreasing benefits) than to the US unemployment insurance scheme with high incentives for search (low and rapidly decreasing benefits).

6.5 Robustness

I illustrate how the policy functions change when changing the effort substitutability, the effectiveness of the training programs and the returns to search.

An increase in the cross-derivative $\psi_{s,t}$ increases the rivalry between training and search efforts. When subject to a given training program, the job seeker will search less when $\psi_{s,t}$ is higher. Figure 5 shows the policy functions for a higher and lower value of $\psi_{s,t}$. The higher $\psi_{s,t}$, the more likely it is that the social planner either imposes intensive training programs or induce intensive search. The level of human capital is lower in the stationary state. The social planner requires less training, but induces more search.

The optimal training intensity depends on the effectiveness of the training programs.

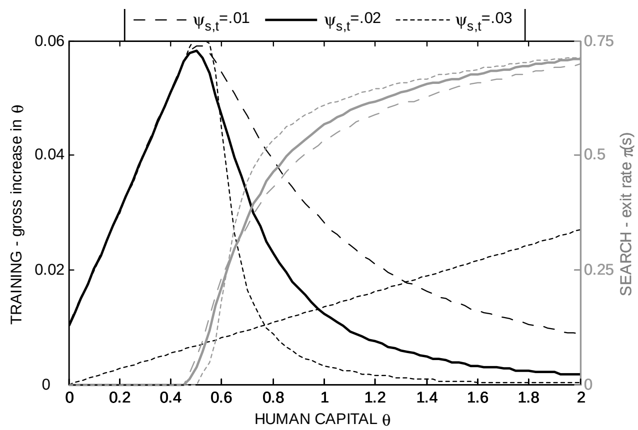

Figure 5: Effort substitutability: training and search policy functions for different parameter values for the cross-derivative of the effort cost function $\psi_{s,t}$.

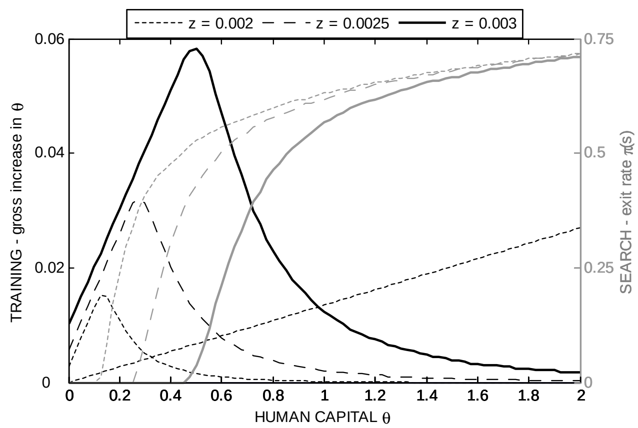

Figure 6: Training effectiveness: training and search policy functions for different parameter values for the impact of training z.

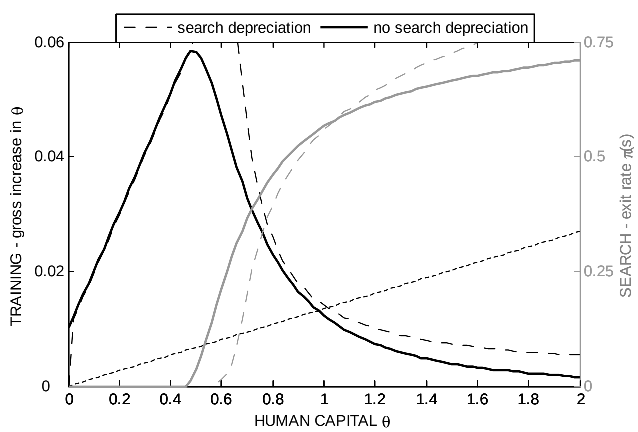

Figure 7: Search skill depreciation: training and search policy functions with and without search skill depreciation.

Empirical studies suggests that the impact of training programs is very heterogeneous (Heckman et al. 1999). Figure 6 shows the optimal policy functions for different values of z in the human capital accumulation process,

$$m(\theta, t) = \theta(1 - \delta) + zt.$$

If the impact of training z decreases, the desired level of training is lower. If training has no impact on output, the social planner will not impose or threaten to impose any training. Training plays no deterring role here. The social planner can discourage the unemployed from remaining unemployed by lowering future unemployment benefits instead and actually increase its revenues. The impact of training effectiveness on the expected unemployment duration is ambiguous. For a given level of human capital, the optimal level of search is higher, since leaving unemployment is the only way to avoid the depreciation of human capital. However, since human capital decreases more rapidly with less effective training, the long-term unemployed search less. Notice also that the cut-off level $\underline{\theta}$ below which the unemployed start in the training state is lower when the effectiveness of training is smaller. The search policy function not only shifts up, but also to the left, as shown in Figure 6.

The depreciation of human capital only affects the output upon re-employment. This naturally implies that the exit rate depends negatively on the duration of the unemployment spell. I now introduce this negative dependence exogenously as well through *search depreciation*, following Shimer and Werning (2006). I assume

$$\pi(s, \theta) = 1 - \exp(-\rho\theta s)$$

such that the marginal return to search depends directly on the level of human capital ($\pi_{s,\theta} > 0$). Training does not only increase the output upon re-employment, but also the transition rates. Figure 7 compares the policy functions. The optimal exit rate is more responsive to human capital. The optimal level of training is higher overall, particularly for low levels and high levels of human capital in the training-and-search state.

6.6 Welfare Gains

To evaluate the advantages of training programs for unemployment policies, I compare the welfare gains from the optimal schemes with and without training programs. I calculate the welfare gain as a function of the level of human capital at the start of the unemployment spell. I set the expected cost for the two schemes equal to the calibrated expected cost of the current US unemployment insurance scheme, which depends on the starting level of human capital. The current scheme provides a monthly unemployment benefit for a maximum duration of six months of about 50 percent of the pre-unemployment earnings. I set the monthly pre-unemployment earnings equal to 1,

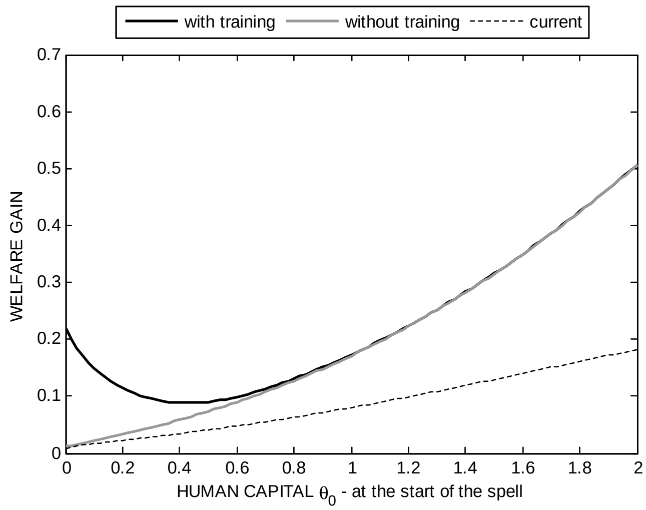

Figure 8: Welfare Comparison: welfare gain in terms of per-period consumption for the optimal policy with training, the optimal policy without training and the current policy, as a function of the level of human capital at the start of the unemployment spell.

which is the output level to which the long-term unemployed converge given the optimal contract. The welfare gains are expressed in terms of the per-period consumption an unemployed job seeker without insurance is willing to pay for the unemployment insurance scheme.

Figure 8 shows the welfare gain as a function of the level of human capital at the start of unemployment for three different schemes: the optimal scheme with training, the optimal scheme without training and the current scheme. By construction, the optimal scheme which allows for the use of training dominates the optimal scheme without training, and both schemes dominate the current scheme. However, the relative welfare gains are very different depending on the initial level of human capital. Although the effectiveness of training remains unchanged, the additional welfare gain from introducing training is negligible for high starting levels of human capital, while it is very large for low starting levels of human capital. The gain increases exponentially up to 0.2 of monthly consumption as θ_0 decreases. The pattern is opposite for the welfare gain from financial insurance. Both the value of the current scheme and the additional value of the optimal insurance scheme without training are increasing in the starting level of human capital. As human capital is higher, the output upon re-employment $y(\theta)$ is higher. This increases both the willingness to pay for unemployment benefits and the scope for consumption smoothing.

The welfare comparison suggests that incorporating training programs in the design of unemployment insurance is more valuable in declining industries where the loss of human capital upon displacement is relatively more important. Training has higher value when used to rebuild human capital than when used to mitigate depreciation. Although training is still rather intensively used at high levels of human capital, the advantage of the optimal scheme comes completely from the consumption smoothing of the high income between employment and unemployment. Training seems complementary as an unemployment policy, creating high value when financial insurance falls short. This explains the U-shape of the optimal policy's welfare gain as a function of human capital.

7 Conclusion

The secular trend of increasing production mobility, technological innovations and shifts in consumer demand forces workers to switch jobs (and industries) more frequently. Job mobility does not only involve the risk of unemployment, but also the risk of wage loss (Low, Meghir and Pistaferri 2010). Displaced workers are often reemployed at lower wages and the persistent nature of this shock makes insurance against wage risk imperative. Kling (2006) proposes to incorporate wage-loss insurance for the reemployed workers in unemployment policy. I approach training programs as a complementary unemployment policy to deal with the loss in wages. If the skill set of a displaced worker becomes redundant, incorporating training programs in unemployment policies is particularly valuable and these programs should be targeted towards the recently displaced. Only if the depreciation of human capital during the unemployment spell is sufficiently important, training should be targeted towards the long-term unemployed, as we often observe in practice.

The model has focused on the provision of insurance under moral hazard, ignoring political constraints and unobservable heterogeneity, two potential explanations for the focus on long-term unemployed. Requiring training programs only for the long-term unemployed helps *deterring* job seekers from remaining unemployed (Besley and Coate 1992) and allows the job seekers with high human capital to leave unemployment before the start of the costly programs. Notice that without political constraints, the social planner would always prefer to deter the unemployed by threatening to lower the monetary transfers rather than by imposing ineffective training programs. Moreover, if a menu of schedules could be offered, requiring intensive training from the start in the schedule designed for the job seekers with low human capital is not likely to encourage job seekers with high human capital to pretend they have low human capital.

The introduction of training programs also changes the design of the consumption scheme and may reverse previous optimality results. I have suggested the use of unemployment benefits and taxes upon re-employment under the assumption that training efforts can be imposed. The introduction of taxes and subsidies upon re-employment, however, will distort the incentives of the unemployed agent to follow effective training programs. Otherwise, the agent's incentives are aligned with the social planner's preferences, conditional on effort. Finally, I have not discussed the role of firms. The loss of skills increases the importance of inducing firms to internalize the costs of displacing workers. Firms could also be subsidized by governments to hire and train low-skilled workers who are not employable otherwise. The analysis sheds light on the subsidy governments should be willing to pay.

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Appendix A: First Order and Envelope Conditions

The social planner minimizes the expected costs of the insurance scheme given an individual rationality constraint and incentive compatibility constraint with Lagrange multipliers λ and μ respectively

$$C(V,\theta) = \min_{c,V^u,V^e,s,t} c + \beta\left[\pi(s)\frac{(u^e)^{-1}((1-\beta)V^e) - y(m(\theta,t))}{1-\beta} + (1-\pi(s))C(V^u, m(\theta,t))\right]$$

such that

$$\begin{aligned} V - u(c,\psi(s,t)) - \beta[\pi(s)V^e + (1-\pi(s))V^u] &\leq 0 && (\lambda) \\ u_\psi(c,\psi(s,t))\psi_s(s,t) + \beta\pi'(s)\left[V^e - V^u\right] &\leq 0. && (\mu) \end{aligned}$$

For an interior solution, the first order conditions are

$$\begin{aligned} 0 &= 1 - \lambda u_c - \mu u_{c,\psi}\psi_s && (FOC_c) \\ 0 &= C_V^e(V^e, m) - \lambda - \mu\frac{\pi'(s)}{\pi(s)} && (FOC_{V^e}) \\ 0 &= C_V(V^u, m) - \lambda + \mu\frac{\pi'(s)}{(1-\pi(s))} && (FOC_{V^u}) \\ 0 &= \beta\pi'(s)\left[C_V^e(V^e,m) - C_V(V^u,m)\right] - \mu(u_{\psi,\psi}\left(\psi_s\right)^2 && (FOC_s) \\ &\quad + u_\psi\psi_{s,s} + \beta\pi''(s)\left[V^e - V^u\right]) \\ 0 &= \beta\left[\pi(s)C_\theta^e(V^e,m) + (1-\pi(s))C_\theta(V^u,m)\right]m_t && (FOC_t) \\ &\quad -\lambda u_\psi\psi_t - \mu\left[u_{\psi,\psi}\psi_s\psi_t + u_\psi\psi_{s,t}\right] \end{aligned}$$

The envelope conditions are

$$\begin{aligned} C_V(V,\theta) &= \lambda && (EC_V) \\ C_\theta(V,\theta) &= \beta\left[\pi(s)C_\theta^e(V^e,m) + (1-\pi(s))C_\theta(V^u,m)\right]m_\theta && (EC_\theta) \end{aligned}$$

Appendix B: Proofs

Proof of Proposition 1

By the EC_V at $\tau+1$, we have

$$C_V(V_{\tau+1}, m(\theta_\tau, t_\tau)) = \lambda_{\tau+1}.$$

Since $\mu_\tau = 0$ if $s_\tau = 0$, I find from FOC_{V^u} at τ

$$C_V(V_{\tau+1}, m(\theta_\tau, t_\tau)) = \lambda_\tau.$$

Hence, $\lambda_\tau = \lambda_{\tau+1}$.$\square$

Proof of Proposition 2

With $s_\tau = 0$, EV_θ at τ simplifies to

$$C_\theta(V_\tau, \theta_\tau) = \beta C_\theta(V_{\tau+1}, \theta_{\tau+1})\,(1-\delta)\,.$$

Substituting for $C_\theta(V_\tau, \theta_\tau)$ and $C_\theta(V_{\tau+1}, \theta_{\tau+1})$ from the FOC_t at $\tau-1$ and τ respectively, I find

$$\lambda_{\tau-1} u_\psi(c_{\tau-1}, \psi_{\tau-1}) \psi_t(s_{\tau-1}, t_{\tau-1}) = \beta\,(1-\delta)\,\lambda_\tau u_\psi(c_\tau, \psi_\tau)\psi_t(s_\tau, t_\tau).$$

Since $\lambda_{\tau-1} = \lambda_\tau$ by Proposition 1 and $u_\psi(c_{\tau-1}, \psi_{\tau-1}) = u_\psi(c_\tau, \psi_\tau)$ for the respective preferences, I find

$$\psi_t(s_{\tau-1}, t_{\tau-1}) = \beta\,(1-\delta)\,\psi_t(s_\tau, t_\tau).$$

Since ψ is convex, $s_{\tau-1} = s_\tau = 0$ and $\beta\,(1-\delta) < 1$ imply $t_\tau > t_{\tau-1}$.$\square$

Proof of Proposition 3

From the FOC_{V^u} and EC_V, I find that

$$\Delta_{\lambda,\tau} \equiv \lambda_\tau - \lambda_{\tau+1} = \mu_\tau \frac{\pi'(s_\tau)}{(1-\pi(s_\tau))}.$$

From FOC_s, I find

$$\mu = \frac{\beta\pi'(s)\,[C^e(V^e, m(\theta,t)) - C(V^u, m(\theta,t))]}{\left(u_{\psi,\psi}\,(\psi_s)^2 + u_\psi \psi_{s,s} + \beta\pi''(s)\,[V^e - V^u]\right)}.$$

First, $s_\tau > 0$ only if $C^e(V^e, m(\theta,t)) - C(V^u, m(\theta,t)) < 0$. Hence, the numerator is negative. Second, the denominator equals the second derivative of the agent's expected utility with respect to search. This derivative is negative. Therefore, we have that $\mu > 0$ and with $\pi'(s_\tau) > 0$ this implies $\Delta_{\lambda,\tau} > 0$.$\square$

Proof of Proposition 4

Since an equal increase in all consumption levels only rescales the utility levels, I expect only the utility ratio's to be dependent on human capital. Here, I rescale all promised utilities with this period's utility $u(c-\psi)$ which allows me explicitly solving the two side-constraints for $\bar{V}^e \equiv V^e/u$ and $\bar{V}^u \equiv V^u/u$. The expected cost of insuring an unemployed agent becomes

$$\begin{aligned} C(V,\theta) = \min c + \beta[&\pi(s)C^e(\bar{V}^e u, \theta(1-\delta)+t) \\ &+ (1-\pi(s))C(\bar{V}^u u(c-\psi(s,t)), \theta(1-\delta)+t)] \end{aligned}$$

such that

$$1+\beta[\pi(s)\bar{V}^{e}+(1-\pi(s))\bar{V}^{u}] \leq \frac{V}{u} \qquad (\lambda)$$

$$\pi'(s)\beta[\bar{V}^{e}-\bar{V}^{u}] = \psi'_{s}(s,t)\frac{u'}{u}. \qquad (\mu)$$

Using the CARA properties, the IC constraint simplifies to

$$\pi'(s)\beta[\bar{V}^{e}-\bar{V}^{u}]=-\sigma\psi'_{s}(s,t).$$

The IC constraint does not depend on the level of consumption and the promise-keeping constraint can remain satisfied after an ε-increase in V by increasing $u(c)$ by ε (and therefore V^{e} and V^{u} with ε). The first order condition with respect to c equals

$$1+\beta\left[\pi(s)C_{V}^{e}u'\bar{V}^{e}+(1-\pi(s))C_{V}u'\bar{V}^{u}\right]-\lambda\frac{V}{u^{2}}u'=0$$

$\Leftrightarrow$

$$1+\beta\left[\pi(s)C_{V}^{e}u'\bar{V}^{e}+(1-\pi(s))C_{V}u'\bar{V}^{u}\right]=-\lambda\frac{\sigma V}{u}.$$

Notice also that the envelope condition with respect to V states that

$$C_{V}(V,\theta)=\frac{\lambda}{u}$$

or, using the first order condition,

$$C_{V}(V,\theta)=-\frac{1+\beta\left[\pi(s)C_{V}^{e}u'\bar{V}^{e}+(1-\pi(s))C_{V}u'\bar{V}^{u}\right]}{\sigma V}.$$

With CARA preferences,

$$\begin{aligned}C^{e}(V^{e},\theta') &= \frac{-\ln(-V^{e}(1-\beta))}{\sigma(1-\beta)}-\frac{\theta'}{1-\beta}\\ C_{V}^{e}(V^{e},\theta') &= \frac{-1}{\sigma(1-\beta)V^{e}}.\end{aligned}$$

Substituting, I find

$$C_{V}(V,\theta)=-\frac{1+\beta\left[\frac{\pi(s)}{1-\beta}+(1-\pi(s))C_{V}(-\sigma u)\bar{V}^{u}\right]}{\sigma V}. \qquad (6)$$

I guess and verify whether $C(V,\theta)=-\frac{\ln(-V(1-\beta))}{\sigma(1-\beta)}-\frac{g(\theta)}{\sigma(1-\beta)}$ for some function $g(\theta)$. For our guess, we have $C_{V}(V,\theta)=-\frac{1}{\sigma(1-\beta)V}$. When I plug this into (6), I get

$$\begin{aligned}C_{V}(V,\theta) &= -\frac{1+\beta\left[\frac{\pi(s)}{1-\beta}+\frac{1-\pi(s)}{1-\beta}\right]}{\sigma V}\\ &= -\frac{1}{\sigma(1-\beta)V}.\end{aligned}$$

Since this holds for any pair (V, θ), our guess and verify method has pinned down the first term of the cost function. That is,

$$C(V,\theta) = \frac{\left(-\frac{1}{\sigma}\right)\ln(-V\,(1-\beta))}{1-\beta} - \frac{g(\theta)}{1-\beta}.$$

The effect on the cost for social planner of an increase in V does not depend on the level of human capital.$\square$

Proof of Corollary 1

From FOC_c, $\frac{1}{u'(c)} = \lambda$. The result follows by Proposition 1.$\square$

Proof of Corollary 2

With $\mu = 0$, I find from FOC_c that $\frac{1}{u'(c-\psi)} = \lambda$. The result follows by Proposition 1.$\square$

Proof of Corollary 3

From FOC_c, $\frac{1}{u'(c)} = \lambda$. The result follows by Proposition 3.$\square$

Proof of Corollary 4

From EC_V, $C_V(V,\theta) = \lambda$. In Proposition 4, I will show that $C_V(V,\theta) = \frac{-1}{\sigma(1-\beta)V}$ for CARA preferences. The result follows by Proposition 3.$\square$

Proof of Corollary 6

As long as $s_\tau > 0$, Proposition 3 applies. In a stationary state with $\theta^* > 0$ and $t^* = \delta\theta^* > 0$, it cannot be optimal to have $s_\tau = 0$ and thus $\pi(s_\tau) = 0$. The social planner would not impose costly training efforts if the unemployed will never leave unemployment again. For CARA preferences, I find that

$$\begin{aligned} c^e &= (u^e)^{-1}\left((1-\beta)\alpha_{V^e(\theta)} V\right) \\ c - \psi &= u^{-1}\left(\alpha_u\left(\theta\right) V\right) \end{aligned}$$

(see Proposition 4), with the inverse functions $(u^e)^{-1}$ and u^{-1} strictly increasing and $\alpha_{V^e}(\theta)$ and $\alpha_u(\theta)$ positive. Since θ is constant in a stationary state and V does decrease by Corollary 4, the result immediately follows.$\square$

Proof of Corollary 5

Using $\Delta_{\lambda,\tau} = \mu_\tau \frac{\pi'(s_\tau)}{(1-\pi(s_\tau))}$, I find from FOC_{V^e}

$$\frac{1}{u^{e\prime}(c^e_{\tau+1})} - \lambda_\tau = \Delta_{\lambda,\tau} \frac{1-\pi(s_\tau)}{\pi(s_\tau)}. \tag{7}$$

Evaluating (7) at τ and $\tau - 1$, I find

$$\frac{\Delta_{\lambda,\tau-1}}{\pi(s_{\tau-1})} = \left[\frac{1}{u^{e\prime}(c^e_\tau)} - \frac{1}{u^{e\prime}(c^e_{\tau+1})}\right] + (1-\pi(s_\tau))\,\frac{\Delta_{\lambda,\tau}}{\pi(s_\tau)}.$$

Notice that $\Delta_{\lambda,\tau} \to 0$ for $\tau \to \infty$. Either $\Delta_{\lambda\tau} = 0$ or $\Delta_{\lambda,\tau} > 0$ by Proposition 1 and 3. If for CARA preferences $\Delta_{\lambda,\tau} > 0$, then $V_\tau > V_{\tau+1}$, since $\Delta_{\lambda,\tau} = \frac{-1}{\sigma(1-\beta)V_\tau} - \frac{-1}{\sigma(1-\beta)V_{\tau+1}}$. Either V converges to $\bar{V}$ or V converges to $-\infty$. In both cases, $\Delta_{\lambda,\tau} \to 0$. If for additive preferences $\Delta_{\lambda,\tau} > 0$, then $c_\tau > c_{\tau+1}$ since $\Delta_{\lambda,\tau} = \frac{1}{u'(c_\tau)} - \frac{1}{u'(c_{\tau+1})}$. Either c converges to $\bar{c}$ or c converges to $-\infty$. In both cases, $\Delta_{\lambda,\tau} \to 0$ as well. By integration,

$$\frac{\Delta_{\lambda,\tau-1}}{\pi(s_{\tau-1})} = \left[\frac{1}{u^{e\prime}(c^e_\tau)} - \frac{1}{u^{e\prime}(c^e_{\tau+1})}\right] + \sum_{k=1}^{\infty}\left(\prod_{l=0}^{k-1}(1-\pi(s_{\tau+l}))\right)\left[\frac{1}{u^{e\prime}(c^e_{\tau+k})} - \frac{1}{u^{e\prime}(c^e_{\tau+k+1})}\right].$$

Since $\Delta_{\lambda,\tau-1} > 0$ for $s_{\tau-1} > 0$, we cannot have that $c^e_{\tau+k} \leq c^e_{\tau+k+1}$ for all $k > 0$. Hence, whenever search is positive during unemployment, it must be that at some later time consumption upon re-employment is strictly decreasing with the length of the unemployment spell.□

Appendix C: CARA Preferences with Monetary Cost of Efforts

Bellman Equation for $g(\theta)$ I rewrite the Bellman equation for $C(V,\theta)$ in terms of $g(\theta)$. I use the expressions in Proposition 4 to substitute for $C^e(\alpha_{V^e}V, \theta(1-\delta)+t)$ and $C(\alpha_{V^u}V, \theta(1-\delta)+t)$ with

$$\begin{aligned} C(V,\theta) = \min c + \beta &\left[\pi\frac{-\ln(-\alpha_{V^e}V(1-\beta))}{\sigma(1-\beta)} + (1-\pi)\frac{-\ln(-\alpha_{V^u}V(1-\beta))}{\sigma(1-\beta)}\right] \\ &- \beta\left[\pi(s)\frac{y(\theta(1-\delta)+t)}{1-\beta} + (1-\pi(s))\frac{g(\theta(1-\delta)+t)}{1-\beta}\right] \end{aligned}$$

such that

$$\begin{aligned} \alpha_u + \beta[\pi(s)\,\alpha_{V^e} + (1-\pi(s))\alpha_{V^u}] &= 1 && (\lambda) \\ \alpha_{V^e} - \alpha_{V^u} &\leq -\frac{\sigma\alpha_u\psi_s(s,t)}{\beta\pi'(s)}. && (\mu) \end{aligned}$$

The first two terms in the objective function can be rewritten to

$$\begin{aligned} & -\tfrac{\ln(-u)}{\sigma} + \psi(s,t) - \tfrac{\beta\ln(-V(1-\beta))}{\sigma(1-\beta)} + \beta\left[\pi\tfrac{-\ln(\alpha_{V^e})}{\sigma(1-\beta)} + (1-\pi)\tfrac{-\ln(\alpha_{V^u})}{\sigma(1-\beta)}\right] \\ = & -\tfrac{\ln u/V}{\sigma} - \tfrac{\ln(-V)}{\sigma} + \psi(s,t) + \tfrac{\ln(-V(1-\beta))}{\sigma} - \tfrac{\ln(-V(1-\beta))}{\sigma(1-\beta)} + \beta\left[\pi\tfrac{-\ln(\alpha_{V^e})}{\sigma(1-\beta)} + (1-\pi)\tfrac{-\ln(\alpha_{V^u})}{\sigma(1-\beta)}\right] \\ = & -\tfrac{\ln(-V(1-\beta))}{\sigma(1-\beta)} + \psi(s,t) - \frac{1}{\sigma(1-\beta)}\left[(1-\beta)\ln(\tfrac{\alpha_u}{1-\beta}) + \beta\pi\ln(\alpha_{V^e}) + \beta(1-\pi)\ln(\alpha_{V^u})\right]. \end{aligned}$$

Since

$$C(V,\theta) = -\frac{\ln(-V(1-\beta))}{\sigma(1-\beta)} - \frac{g(\theta)}{1-\beta},$$

I find that the Bellman equation for $g(\theta)$ equals

$$\begin{aligned} g(\theta) = \max_{\alpha,s,t} & \ \beta \left[\pi\left(s\right) y(\theta(1-\delta)+t)) + (1-\pi\left(s\right))g(\theta(1-\delta)+t) \right] - (1-\beta)\psi(s,t) \\ & + \frac{1}{\sigma}\left[(1-\beta)\ln(\frac{\alpha_u}{1-\beta}) + \beta\pi\left(s\right)\ln(\alpha_{V^e}) + \beta(1-\pi\left(s\right))\ln(\alpha_{V^u}) \right] \end{aligned}$$

such that the IR and IC constraint hold.

Stationary State for CARA Preferences Assuming an interior solution, the value function $g(\theta)$ in the second best can be characterized by an unconstrained maximization after substituting in for the IR and IC constraint. That is,

$$\begin{aligned} g(\theta) &= \max \beta\left\{ \pi(s) y\left(\theta\left(1-\delta\right)+t\right) + (1-\pi(s)) g\left(\theta\left(1-\delta\right)+t\right) \right\} \\ & \quad -(1-\beta)\psi(s,t) + \kappa(s,t,\tfrac{\alpha_{V^u}}{\alpha_u}) \end{aligned}$$

with

$$\begin{aligned} \kappa(s,t,\tfrac{\alpha_{V^u}}{\alpha_u}) = \frac{1}{\sigma}\{ & \beta\pi\left(s\right)\ln(\tilde{\kappa}(s,t)+\tfrac{\alpha_{V^u}}{\alpha_u}) + \beta(1-\pi\left(s\right))\ln(\tfrac{\alpha_{V^u}}{\alpha_u}) \\ & - \ln(1+\beta[\pi\left(s\right)\tilde{\kappa}(s,t)+\tfrac{\alpha_{V^u}}{\alpha_u}]) - (1-\beta)\ln(1-\beta) \} \end{aligned}$$

and

$$\tilde{\kappa}(s,t) = -\frac{\sigma\psi_s(s,t)}{\beta\pi'(s)}.$$

Notice that only three control variables remain. With $g(\theta)$ concave, the first order conditions with respect to s, t and $\frac{\alpha_{V^u}}{\alpha_u}(=\frac{V^u}{u})$ are

$$\begin{aligned} \beta\pi'(s)[y\left(\theta\left(1-\delta\right)+t\right) - g\left(\theta\left(1-\delta\right)+t\right)] - (1-\beta)\,\psi_s(s,t) + \frac{\partial\kappa}{\partial s} &= 0 \\ \beta\left\{ \pi(s) y'\left(\theta\left(1-\delta\right)+t\right) + (1-\pi(s)) g'\left(\theta\left(1-\delta\right)+t\right) \right\} - (1-\beta)\psi_t(s,t) + \frac{\partial\kappa}{\partial t} &= 0 \\ \frac{\partial\kappa}{\partial\frac{\alpha_{V^u}}{\alpha_u}} &= 0. \end{aligned}$$

The envelope condition equals

$$g'(\theta) = \beta\left\{ \pi(s) y'\left(\theta\left(1-\delta\right)+t\right) + (1-\pi(s)) g'\left(\theta\left(1-\delta\right)+t\right) \right\}(1-\delta).$$

In a steady state θ^*, training equals the depreciation in human capital to maintain the same level, that is $t^* = \delta\theta^*$. Hence, from the objective function and envelope condition, I get

$$
\begin{aligned}
g(\theta) &= \frac{\beta\pi(s(\theta))y(\theta) - (1-\beta)\psi(s(\theta), \delta\theta) + \kappa(s, t, \frac{\alpha_V u}{\alpha_u})}{1 - \beta(1 - \pi(s(\theta)))} \\
g'(\theta) &= \frac{\beta\pi(s(\theta))y'(\theta)\,(1-\delta)}{1 - \beta(1 - \pi(s(\theta)))(1-\delta)}.
\end{aligned}
$$

Substituting this in the first order conditions, we find the expressions in the text.