Sincere Voting in Large Elections*

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Abstract

How often does a private information voting game with a large number of voters have an equilibrium in which all voters vote sincerely? Findings from the previous literature would suggest that the answer is: almost never. For example, Austen-Smith and Banks (1996) show that in the canonical Condorcet model, sincere voting is a limit equilibrium only in knife edge cases where the signal qualities are equal across the states of the world. In this paper, we study a general class of Condorcet models that enrich existing models by incorporating uncertainty about the informational environment and distribution of voter preferences. We show that in a natural class of environments in which many existing models can be embedded, there is an open and dense set of environments for which sincere voting constitutes an equilibrium in large elections. In stark contrast to the findings of the previous literature, this implies that sincere voting is generic in the space of informational environments.

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Preliminary and incomplete. Comments welcome.

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1 Introduction

In the seminal paper on voting games with private information, Austen-Smith and Banks (hereafter A-SB, 1996) revisited the setting first studied by Condorcet (1785) in which a bench of jurors, each with private information about the guilt or innocence of a particular suspect, must vote either to convict or acquit. A-SB laid out the provocative finding that with a large electorate, there is almost always a behavioral discrepancy between "sincere" voting and "strategic" (i.e., "equilibrium") voting. In other words, A-SB showed that in the Condorcet environment, rational voters must sometimes vote against their private information.

A-SB's finding launched a research agenda aimed at determining when information is aggregated by equilibrium behavior despite the fact that not everyone is voting sincerely (see Bhattacharya, 2013, for a recent characterization and survey). Very little attention has been paid, however, to the robustness and descriptive validity of the original behavioral finding of these models—that that there is a discrepancy between sincere and strategic voting. This is surprising, however, given the fact that few empirical scholars of elections are willing to embrace the idea that real-life voters vote against their private information in large elections. In fact, given the perceived implausibility of non-informative voting in large real-life elections, many scholars have proposed that voting is expressive (Tullock, 1971, Brennan and Hamlin, 1988, Hamlin and Jennings, 2011, Kamenica and Egan Brad, 2012) or that voters have non-instrumental motivations in deciding who to vote for, and whether to vote at all (Green and Shapiro, 1996; Feddersen and Sandroni, 2013). While many of these scholars find it plausible that strategic calculations can lead voters to vote against their private information in small electorates like juries, committees, and clubs, the result has been harder to digest for large electorates, where intuition suggests that real-life voters who show up at the ballots will vote informatively.

Driving the A-SB result is the fact that in large elections, conditioning on pivotality has large effects on beliefs about the payoff relevant state and almost no effect on beliefs about the behavior of other voters. If voters may actually be better informed about a single payoff relevant state as opposed to the motivations, rationality, beliefs or attentiveness to information of a large number of other "small" players, this imbalance in the nature of learning seems odd. In this paper, we allow for the possibility that voters also face some uncertainty about the preferences and intentions of other voters by extending the space of informational environments. In particular, we are interested in understanding the equilibrium incentives of purely instrumental voters who lack certainty about the motivations of others. With uncertainty about others, beliefs that condition on being pivotal involve updating about the payoff relevant state as well as on how closely related other voters' ballots and the underlying state must be. The first effect is present in existing models but the second is absent.

Our results show that for any level of uncertainty about others, the consequence of having a large enough electorate is that updating about the relationship between the behavior of others and the payoff relevant state trumps updating about the state based on pivotality. With a large enough electorate, sincere voting is a strict best response for an instrumentally rational voter and thus sincere voting equilibria exist for large electorates. We use this logic to show that in a natural set of informational environments in which the canonical A-SB/Condorcet model can be embedded, the subset of environments for which sincere voting occurs in an equilibrium for a large enough electorate is generic. We conclude that the absence of any uncertainty about what drives the voting choices of others is essential to the tension between conditioning on being pivotal and using one's private information in a large electorate.

2 Model

Consider a majoritarian election in which $2n+1$ voters must simultaneously vote for one of two alternatives $a \in \{0,1\}$. The state of the world is denoted $s = (\omega, \alpha, \varepsilon)$, and the state space is $S = \{0,1\} \times A \times [0,1]$, where $A \subset \mathbb{R}^m$ (m finite). Conditional on the state s, each voter's type is drawn independently from the set $T = \{\emptyset, 0, 1\}$. In particular, assume that conditional $s = (\omega, \alpha, \varepsilon)$, the probability that a voter is of type $t \in T$ is given by

$$\Pr\big(t \mid s = (\omega, \alpha, \varepsilon)\big) = \begin{cases} \varepsilon & \text{if } t = \emptyset \\ (1-\varepsilon) \cdot \theta^t(\omega, \alpha) & \text{if } t \neq \emptyset \end{cases} \tag{1}$$

where $\theta^0(\omega, \alpha), \theta^1(\omega, \alpha) \in [0,1]$ are numbers that satisfy $\theta^0(\omega, \alpha) + \theta^1(\omega, \alpha) = 1$ for all $(\omega, \alpha) \in \{0,1\} \times A$, and for each $t = 0, 1$ the mapping $\theta^t(\cdot, \cdot)$ is measurable on $\{0,1\} \times A$. Assume that only the first coordinate of the state, ω, is payoff-relevant for types $t \neq \emptyset$, and determines which alternative is superior. In particular, the payoff

$u(t, a, s)$ for a voter of type $t \in T$ from electing $a \in \{0, 1\}$ when the state is $s \in S$ is

$$u\big(t, a, s = (\omega, \alpha, \varepsilon)\big) = \begin{cases} 1 & \text{if } a = \omega \text{ or } t = \emptyset \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Thus, voters of type $t = \emptyset$ are always indifferent between the two alternatives, while voters of type $t \neq \emptyset$ strictly prefer to elect alternative a when the state is $\omega = a$. Finally, let φ denote the prior distribution over the set of states S.

Throughout the paper, we restrict our attention to distributions φ for which ω, α and ε are pairwise-independent.

Assumption 1: φ distributes ω, α and ε independently.

This assumption enables us to define for any distribution φ the marginals in a convenient way. Let $F(\cdot) = \varphi(\cdot|\omega, \alpha)$ at any value of (ω, α) to be the conditional distribution of ε. Similarly, let $G(\cdot) = \varphi(\cdot|\varepsilon, \omega)$ at any value of (ε, ω) be the conditional distribution of α. Without loss of generality, we assume that A is the support of G. Finally, let η denote the unconditional prior probability that $\omega = 1$.

The voting game can be decomposed into the model's primitives, its informational environment, and the parameter n, which determines the number of voters $2n + 1$. The primitives of the model are $\mathcal{M} = (S, T, \{0, 1\}, u(\cdot))$, which consists of the sets of states S, types T, and alternatives $\{0, 1\}$, as well as the payoff function $u(\cdot)$. The informational environment is summarized by a pair (φ, ϑ) consisting of the distribution φ over states, and a measurable function $\vartheta : \{0, 1\} \times A \to [0, 1]^2$ that maps pairs (ω, α) to points $\big(\theta^0(\omega, \alpha), \theta^1(\omega, \alpha)\big)$ on the unit simplex. For every (ω, α) the vector $\big(\theta^0(\omega, \alpha), \theta^1(\omega, \alpha)\big)$ along with ε determines the distribution of types in the population, according to (1). We denote the game by $\Gamma = (\mathcal{M}, (\varphi, \vartheta), n)$.

Voting Strategies and Equilibrium Concept. Given a game Γ, a symmetric strategy is a function $\sigma : T \to [0, 1]$, where $\sigma(t)$ the probability that a voter votes for alternative 1 when his type is t. We will identify symmetric strategy profiles with the symmetric strategy that is used by all voters. The solution concept throughout this paper is Bayes Nash equilibrium in weakly undominated symmetric strategies, which we will refer to simply as "equilibrium."

We now introduce the familiar concept of sincere voting. Since ω, α and ε are pairwise-independent by Assumption 1, a voter's posterior probability that $\omega = 1$

given that the voter's type is $t \neq \emptyset$, is given by

$$\eta^t = \frac{\eta E_G[\theta^t(1,\alpha)]}{(1-\eta)E_G[\theta^t(0,\alpha)] + \eta E_G[\theta^t(1,\alpha)]} \tag{3}$$

where $E_G[\theta^t(\omega,\alpha)] := \int_A \theta^t(\omega,\alpha)dG(\alpha)$ denotes the expectation of $\theta^t(\omega,\alpha)$ given the prior distribution G. For type $t = \emptyset$, the posterior probability $\eta^\emptyset$ that $\omega = 1$, is simply equal to the prior η. Sincere voting is defined as follows.

Definition 1: A symmetric strategy profile σ for a game $\Gamma = (\mathcal{M}, (\varphi, \vartheta), n)$ is said to be "sincere" if for $t \neq \emptyset$

$$\sigma(t) = \begin{cases} 1 & \text{when } \eta^t > 1/2 \\ 0 & \text{when } \eta^t < 1/2 \end{cases} \tag{4}$$

A symmetric strategy profile is called a "sincere voting equilibrium" (SVE) if it is both an equilibrium of the game Γ and it is sincere.

Note that this definition places no restrictions on the behavior of the type $t = \emptyset$ or of types $t \neq \emptyset$ for whom $\eta^t = 1/2$. Such types are indifferent between the two alternatives. The definition says that a voting strategy for type $t \neq \emptyset$ is sincere if that type always votes for the alternative that is more likely, and hence has a higher expected payoff, given only the type's private information.

Environments. Our analysis will focus on "generalized Condorcet environments," which we describe below. We use the notation, $\theta^t(\omega, A)$, to denote the image of the mapping $\theta^t(\omega,\alpha)$ over all $\alpha \in A$. First, consider the following assumption.

Assumption 2: ϑ is such that $\theta^0(0,A) \subseteq \left[\frac{1}{2}, 1\right]$ and $\theta^0(1,A) \subseteq \left[0, \frac{1}{2}\right]$.

This assumption states that $t = 0$ is always at least as likely as type $t = 1$ when $\omega = 0$; similarly, type $t = 1$ is always at least as likely as type $t = 0$ when $\omega = 1$. The assumption implies that $E_G[\theta^1(1,\alpha)] \geq E_G[\theta^0(1,\alpha)]$ and $E_G[\theta^1(0,\alpha)] \leq E_G[\theta^0(0,\alpha)]$. With these inequalities, it is straightforward to show that $\eta^0 \leq \eta \leq \eta^1$, which follows by applying the definition of η^t in (3). We make the following additional assumption.

Assumption 3: $\eta^0 \leq \frac{1}{2} \leq \eta^1$.

This is an assumption on the informational environment (φ, ϑ), and it requires that $\eta \in (0,1)$. This is a standard assumption that rules out cases where sincere voting requires both types $t = 0, 1$ to vote for the same alternative. In particular, it implies that the symmetric strategy profile in which type $t = 0$ votes for alternative $a = 0$ and type $t = 1$ votes for alternative $a = 1$, is a sincere voting strategy profile.

Let Θ denote the set of all measurable function ϑ that map pairs (ω, α) to points on the unit simplex, and let $\Delta(S)$ denote the set of probability distributions on S. We consider the following set of environments:

$$\mathcal{E} = \{(\varphi, \vartheta) \in \Delta(S) \times \Theta : (\varphi, \vartheta) \text{ satisfy Assumptions 1, 2 and 3}\} \tag{5}$$

Thus, by imposing Assumptions 2 and 3, we consider only informational environments that are "generalized Condorcet environments." An environment in $\mathcal{E}$ is "Condorcet" in the sense that type $t = 0$ is always more likely in states where $\omega = 0$, type $t = 1$ is always more likely in states where $\omega = 1$, and it is not the case that sincere voting requires both types $t = 0, 1$ to vote for the same alternative. It is "generalized" Condorcet because the quality of informative signals can depend on an underlying state variable α, and we allow the possibility of the indifferent type $t = \emptyset$.

We endow each $\Delta(S)$ and Θ with the sup metric $d_\infty(\cdot, \cdot)$ and consider the usual Euclidean product metric $d_\infty((\cdot, \cdot), (\cdot, \cdot))$ on the space of environments $\mathcal{E}$.[1]

Remark 1: The canonical voting games with private information have informational environments that fall in $\mathcal{E}$. For example, A-SB's model is one in which A is a singleton; $\theta^t(\omega = t, \alpha) = q_t$ for all α and $t = 0, 1$; $\eta = 1/2$; and, F puts unit mass on $\varepsilon = 0$. In fact, for nearly all existing models, F will put unit mass on $\varepsilon = 0$ so that the probability that a voter is always indifferent across the two alternatives is zero. Our perspective, by extending the state and type spaces to include the state variable ε and type $t = \emptyset$, is that existing models should be viewed as abstractions that acknowledge the possibility of such indifferent types in real life, but for simplicity they do not include these types in the model. By allowing for the possibility of such types, we are enriching these existing models to capture a greater set of real-life possibilities.

[1] Alternatively, we could endow $\Delta(S)$ with the Lévy-Prokhorov metric, which metrizes the topology of weak convergence. Instead, we endow it with the sup metric, which provides us an even stronger notion of convergence and allows both spaces, $\Delta(S)$ and Θ, to be treated similarly.

3 The Prevalence of Sincere Voting

In this section, we fix the primitives of the model $\mathcal{M}$ and associate the concept of SVE with informational environments (φ, ϑ) for large values of n. We do this as follows.

DEFINITION 2: Fix $\mathcal{M}$, and say that the informational environment (φ, ϑ) "eventually has an SVE" if game $\Gamma = (\mathcal{M}, (\varphi, \vartheta), n)$ has an SVE when n is large enough.

Before stating the main result of this paper, we define the set

$$\mathcal{E}^s = \{(\varphi, \vartheta) \in \mathcal{E} : (\varphi, \vartheta) \text{ eventually has an SVE}\} \tag{6}$$

which is the set of informational environments that support sincere voting in large elections. The main result of this paper, stated as follows, concerns the prevalence of sincere voting in large elections—in other words, the relative size of $\mathcal{E}^s$ in $\mathcal{E}$.

THEOREM 1: $\mathcal{E}^s$ has a subset that is open and dense in $\mathcal{E}$.

Proof: See Section 4. □

The theorem states that sincere voting is generic in the space of all environments under consideration, $\mathcal{E}$. This implies that arbitrarily close to any informational environment in the set $\mathcal{E}$ is one that eventually has an SVE. Informally, this means that every game that does not eventually have an SVE, such as the Condorcet game analyzed by A-SB, can be perturbed in the space of informational environments $\mathcal{E}$ so that the perturbed game does eventually have an SVE. Moreover, the existence of an SVE is robust to further perturbations: any further perturbations cannot undo the eventual existence of SVE.

The proof of the theorem, which appears in the next section, proceeds as follows. First, we construct a set $\mathcal{E}^*$ that is (i) a subset of $\mathcal{E}^s$ and (ii) both open and dense in $\mathcal{E}$. The set $\mathcal{E}^*$ is the set of informational environments (φ, ϑ) such that the distribution F has a jump at some ε^* larger than $1/2$, and $\theta^0(0, \alpha)$ and $\theta^0(1, \alpha)$ are both bounded away from $1/2$. Clearly $\mathcal{E}^*$ is large in the space of environments $\mathcal{E}$. Therefore, the critical step is in proving that $\mathcal{E}^* \subseteq \mathcal{E}^s$. For this step, we show that the assumptions that define $\mathcal{E}^*$ enable the indifferent type $t = \emptyset$ to mix in such a way so as make the private information of both types $t = 0, 1$ relevant to their voting decision. In order to do this, it must be that conditional on either of the other two types $t = 0, 1$

being pivotal, type $t = \emptyset$ appears very likely in the population (i.e., the conditional distribution of ε places a large amount of mass on ε^* and above) and the probability with which type $t = \emptyset$ voters mixes is close enough to a coin-flip. We demonstrate in our "Calibration Lemma" of the next section that for *any* environment (φ, ϑ) in $\mathcal{E}^*$ it is possible to find a mixing strategy for type $t = \emptyset$ that satisfies these conditions. It is then the case that every informational environment $(\varphi, \vartheta) \in \mathcal{E}$ can be perturbed so that the perturbed environment belongs to $\mathcal{E}^*$. Furthermore, any further perturbations that are small enough result in environments that continue to belong to $\mathcal{E}^*$. Thus, $\mathcal{E}^*$ is open and dense in $\mathcal{E}$.

4 Intermediate Results & Proofs

In this section, we prove Theorem 1 by constructing a subset of $\mathcal{E}^s$ that is open and dense in $\mathcal{E}$. To do this, consider the following assumption for small values of $\gamma > 0$.

Assumption 2^γ: ϑ is s.t. $\theta^0(0, A) \subseteq \left[\frac{1}{2} + \gamma, 1\right]$ and $\theta^0(1, A) \subseteq \left[0, \frac{1}{2} - \gamma\right]$.

In addition to Assumptions 1 and 2^γ above, consider the following assumption.

Assumption 4: φ is s.t. F has a jump at some $\varepsilon^* > \frac{1}{2}$.

For all $\gamma > 0$, Assumption 2^γ implies Assumption 2 and Assumption 4 is an additional assumption to Assumptions 1, 2^γ and 3. Therefore, for any small γ the set

$$\mathcal{E}^\gamma = \{(\varphi, \vartheta) \in \Delta(S) \times \Theta : (\varphi, \vartheta) \text{ satisfies Assumptions 1, } 2^\gamma\text{, 3 and 4}\} \tag{7}$$

is a subset of the set of environments $\mathcal{E}$ defined in (5). Then, define the set

$$\mathcal{E}^* = \bigcup_{0<\gamma<\frac{1}{2}} \mathcal{E}^\gamma. \tag{8}$$

Clearly, $\mathcal{E}^* \subseteq \mathcal{E}$ as well. To prove Theorem 1, we will first show in Section 4.1 that $\mathcal{E}^* \subseteq \mathcal{E}^s$. Then, in Section 4.2 we will show that $\mathcal{E}^*$ is open and dense in $\mathcal{E}$.

4.1 $\mathcal{E}^* \subseteq \mathcal{E}^s$

In this section, we prove that $\mathcal{E}^* \subseteq \mathcal{E}^s$ by showing that $\mathcal{E}^\gamma \subseteq \mathcal{E}^s$ for all $0 < \gamma < \frac{1}{4}$. Fix any $0 < \gamma < \frac{1}{2}$, and consider a game $\Gamma = (\mathcal{M}, (\varphi, \vartheta), n)$ such that $(\varphi, \vartheta) \in \mathcal{E}^\gamma$. The following claim implies that $\mathcal{E}^\gamma \subseteq \mathcal{E}^s$ for all $0 < \gamma < \frac{1}{2}$.

Claim 1: Γ has an SVE if n is large enough.

Proof. (φ, ϑ) satisfies Assumption 3, so the game has a sincere voting strategy σ^s in which $\sigma^s(1) = 1$ and $\sigma^s(0) = 0$. We denote $\sigma^s(\emptyset) = x$. Under σ^s, the probability of drawing a vote for alternative $a = 1$ when the state is $(\omega, \alpha, \varepsilon)$ is

$$\pi(x, \omega, \alpha, \varepsilon) = \varepsilon x + (1 - \varepsilon)\theta^1(\omega, \alpha) \tag{9}$$

Since voters are strategic, they condition their vote on the event that they are pivotal. The probability that a voter is pivotal can be viewed as a function of the state $(\omega, \alpha, \varepsilon)$ and the number x for a fixed value of n. Define

$$\beta(x, \omega, \alpha, \varepsilon) = (\pi(x, \omega, \alpha, \varepsilon))(1 - \pi(x, \omega, \alpha, \varepsilon)) \tag{10}$$

Then, the probability that a voter is pivotal is simply $\binom{2n}{n}[\beta(x, \omega, \alpha, \varepsilon)]^n$. This implies that voting for alternative $a = 1$ is a best response for a voter of type $t \neq \emptyset$ to the conjectured strategy profile σ^s if and only if

$$\lambda(x, t, n) := \left(\frac{\eta}{1 - \eta}\right) \left[\frac{\int_0^1 \int_A [\beta(x, 1, \alpha, \varepsilon)]^n \theta^t(1, \alpha)(1 - \varepsilon) dG(\alpha) dF(\varepsilon)}{\int_0^1 \int_A [\beta(x, 0, \alpha, \varepsilon)]^n \theta^t(0, \alpha)(1 - \varepsilon) dG(\alpha) dF(\varepsilon)}\right] \geq 1 \tag{11}$$

while voting for alternative $a = 0$ is a best response if the reverse inequality holds. Equation (11) says that conditional on the voter's private information and the event that she is pivotal, the state is more likely to be $\omega = 1$ than $\omega = 0$.

We now prove a useful intermediate result. To state the result, first define

$$\mu(x, n) := \frac{\int_0^1 \int_A [\beta(x, 1, \alpha, \varepsilon)]^n (1 - \varepsilon) dG(\alpha) dF(\varepsilon)}{\int_0^1 \int_A [\beta(x, 0, \alpha, \varepsilon)]^n (1 - \varepsilon) dG(\alpha) dF(\varepsilon)} \tag{12}$$

which is similar to the function $\lambda(x, t, n)$ in (11) except that it is not weighted by $\frac{\eta}{1-\eta}$ and $\theta^t(1, \alpha)$ and $\theta^t(0, \alpha)$ do not appear in the numerator and denominator, respectively. The result is as follows.

Calibration Lemma: Let $0 < \gamma < \frac{1}{2}$ and $(\varphi, \vartheta) \in \mathcal{E}^\gamma$, and consider the strategy profile σ^s described above. Then for all $\kappa > 0$ and n large enough, there exists a number $x_n(\kappa) \in [0, 1]$ such that $\mu(x_n(\kappa), n) = \kappa$.

Proof. Pick any $\alpha^* \in A$, and let $\underline{x} := \frac{1}{\varepsilon^*}\left(\frac{1}{2} - (1 - \varepsilon^*)\theta^1(1, \alpha^*)\right)$ and $\overline{x} := \frac{1}{\varepsilon^*}\left(\frac{1}{2} - (1 - \varepsilon^*)\theta^1(0, \alpha^*)\right)$. Here, $\underline{x}$ is defined so that $\pi(\underline{x}, 1, \alpha^*, \varepsilon^*) = 1/2$. In addition, because $\gamma < 1/2$ and $\theta^1(1, \alpha^*) > \frac{1}{2} + \gamma$ by the assumption that $(\varphi, \vartheta) \in \mathcal{E}^\gamma$, we have $0 \leq \underline{x} < 1/2$. Because $\theta^1(0,\alpha) < 1/2$ for all $\alpha \in A$, we have $\pi(\underline{x},0,\alpha,\varepsilon) < 1/2$ for all $\alpha \in A$ and $\varepsilon \in [0,1]$. Analogously, $\overline{x}$ is defined so that $\pi(\overline{x},0,\alpha^*,\varepsilon^*) = 1/2$. In this case, $1/2 < \overline{x} \leq 1$ and $\pi(\overline{x},1,\alpha,\varepsilon) > 1/2$ for all $\alpha \in A$ and $\varepsilon \in [0,1]$.

Let $E_G\big[[\beta(\underline{x},1,\alpha,\varepsilon)]^n\big] := \int_A [\beta(\underline{x},1,\alpha,\varepsilon)]^n dG(\alpha)$. Then, we have

$$\int_0^1 E_G\big[[\beta(\underline{x},1,\alpha,\varepsilon)]^n\big](1-\varepsilon)dF(\varepsilon) = \int_{[0,1]\setminus\{\varepsilon^*\}} E_G\big[[\beta(\underline{x},1,\alpha,\varepsilon)]^n\big](1-\varepsilon)dF(\varepsilon) + E_G\big[[\beta(\underline{x},1,\alpha,\varepsilon^*)]^n\big](1-\varepsilon^*)p \quad (13)$$

where we have defined $p := F(\varepsilon^*) - F(\varepsilon^{*-}) > 0$ to be the size of the jump in F occurring at ε^*. Since by construction $\beta(\underline{x},1,\alpha^*,\varepsilon^*) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, we know that $n\binom{2n}{n}$ times the term $E_G\big[[\beta(\underline{x},1,\alpha,\varepsilon^*)]^n\big](1-\varepsilon^*)p$ on the right hand side of (13) goes to $+\infty$ as $n \to \infty$. This means that $n\binom{2n}{n}$ times the quantity on the left-hand-side of (13) goes to $+\infty$ as $n \to \infty$. This characterizes the limit behavior of $n\binom{2n}{n}$ times the numerator of $\mu(\underline{x},n)$ in (12).

Let us now study the denominator of $\mu(\underline{x},n)$ in (12). Since $E_G\big[[\beta(\underline{x},0,\alpha,\varepsilon)]^n\big]$ is bounded and continuous in ε and F is bounded and increasing, the mean value theorem of integration implies that there exists $\tilde{\varepsilon} \in [0,1]$ such that

$$\begin{aligned}\int_0^1 E_G\big[[\beta(\underline{x},0,\alpha,\varepsilon)]^n\big](1-\varepsilon)dF(\varepsilon) &= E_G\big[[\beta(\underline{x},0,\alpha,\tilde{\varepsilon})]^n\big](1-\tilde{\varepsilon})\left[F(1)-F(0)\right] \\ &\leq \sup_{\varepsilon\in[0,1],\alpha\in A} [\pi(\underline{x},0,\alpha,\varepsilon)(1-\pi(\underline{x},0,\alpha,\varepsilon))]^n \\ &\leq \left[\left(\max\left\{\underline{x}, \tfrac{1}{2}-\gamma\right\}\right)\left(\max\left\{\underline{x}, \tfrac{1}{2}-\gamma\right\}\right)\right]^n \quad (14)\end{aligned}$$

where the first inequality follows because $(1-\tilde{\varepsilon})[F(1)-F(0)] = (1-\tilde{\varepsilon}) \leq 1$, and the second follows because $\theta^1(0,\alpha) \leq \frac{1}{2} - \gamma$ for all α and $\max\left\{\underline{x}, \frac{1}{2}-\gamma\right\} < \frac{1}{2}$. Then, note that because $\max\left\{\underline{x}, \frac{1}{2}-\gamma\right\} < \frac{1}{2}$, it must be that $n\binom{2n}{n}$ times the term on the right-hand-side of the last inequality in (14) goes to 0 as $n \to \infty$. This implies that $n\binom{2n}{n}$ times the integral in (14) goes to 0 as $n \to \infty$. This characterizes the limit behavior of $n\binom{2n}{n}$ times the denominator of $\mu(\underline{x},n)$ in (12).

Gathering these observations, it follows that $\lim_{n\to\infty} \mu(\underline{x},n) = +\infty$. Analogous arguments establish that $\lim_{n\to\infty} \mu(\overline{x},n) = 0$. Since $\mu(x,n)$ is continuous in x for all n, these facts along with the intermediate value theorem imply that for all $\kappa > 0$, there exists $x_n(\kappa) \in [\underline{x},\overline{x}] \subseteq [0,1]$ such that $\mu(x_n(\kappa),n) = \kappa$ when n is large enough. □

Remark 2: As $\varepsilon^* \to 1$, the number $\underline{x}$ goes to 1/2 from below while $\overline{x}$ goes to 1/2 from above. This implies that we can make the number $x_n(\kappa)$ in the Calibration

Lemma arbitrarily close to 1/2 by perturbing any F so that there is jump close enough to $\varepsilon = 1$, and picking n large enough.

We now return to the proof of Claim 1. By definition of the functions λ in (11) and μ in (12), we have

$$\frac{1-\eta}{\eta}\lambda(x,0,n) \leq \frac{\sup_{\alpha\in A}\theta^0(1,\alpha)}{\inf_{\alpha\in A}\theta^0(0,\alpha)}\mu(x,n) \leq \frac{\frac{1}{2}-\gamma}{\frac{1}{2}+\gamma}\mu(x,n) \tag{15}$$

$$\frac{1-\eta}{\eta}\lambda(x,1,n) \geq \frac{\inf_{\alpha\in A}\theta^1(1,\alpha)}{\sup_{\alpha\in A}\theta^1(0,\alpha)}\mu(x,n) \geq \frac{\frac{1}{2}+\gamma}{\frac{1}{2}-\gamma}\mu(x,n) \tag{16}$$

Then, by the Calibration Lemma, we know that if n is large enough, there is a number $x_n \in [0,1]$ such that $\mu(x_n, n) = \frac{1-\eta}{\eta}$. (Simply choose $\kappa = \frac{1-\eta}{\eta} > 0$.) Since $\gamma > 0$, this implies from (15) and (16) that $\lambda(x_n, 0, n) \leq \frac{\frac{1}{2}-\gamma}{\frac{1}{2}+\gamma} < 1$ and $\lambda(x_n, 1, n) \geq \frac{\frac{1}{2}+\gamma}{\frac{1}{2}-\gamma} > 1$. This implies that when $\sigma^s(\emptyset) = x_n$, the unique best response to the strategy profile σ^s is for each type $t = 0, 1$ to vote sincerely. Since type $t = \emptyset$ is always indifferent across the two alternatives, this establishes the existence of SVE for large values of n, completing the proof of Claim 1. $\square$

4.2 $\mathcal{E}^*$ is open and dense in $\mathcal{E}$

Now, we prove that $\mathcal{E}^*$ is open and dense in $\mathcal{E}$. Lemma 1 below shows that it is dense. Lemma 2 then shows that it is open.

LEMMA 1: For all $\rho > 0$ and every $(\varphi, \vartheta) \in \mathcal{E}$, there exists $(\hat{\varphi}, \hat{\vartheta}) \in \mathcal{E}^*$ such that $d_\infty((\varphi, \vartheta), (\hat{\varphi}, \hat{\vartheta})) < \rho$.

Proof. Fix $0 < \gamma < \frac{1}{2}$ and $\varepsilon^* > \frac{1}{2}$, and let δ_{ε^*} be the degenerate distribution that puts unit mass on ε^*. Construct $\hat{\varphi}$ be setting $\hat{\eta} = \eta$ and $\hat{G} = G$, but let $\hat{F} = (1-\nu)F + \nu\delta_{\varepsilon^*}$ for some small $\nu > 0$. Then $\hat{F}$ must have a jump at ε^*. Construct $\hat{\vartheta}$ by setting

$$\hat{\theta}^0(0,\alpha) = \begin{cases} \frac{1}{2}+\gamma & \text{if } \theta^0(0,\alpha) < \frac{1}{2}+\gamma \\ \theta^0(0,\alpha) & \text{otherwise} \end{cases} \tag{17}$$

$$\hat{\theta}^0(1,\alpha) = \begin{cases} \frac{1}{2}-\gamma & \text{if } \theta^0(1,\alpha) > \frac{1}{2}-\gamma \\ \theta^0(1,\alpha) & \text{otherwise} \end{cases} \tag{18}$$

Now note that by construction $(\hat{\varphi}, \hat{\vartheta}) \in \mathcal{E}^\gamma$ for all $\gamma, \nu > 0$. By choosing ν and γ close enough to 0 we can make $d_\infty((\varphi, \vartheta), (\varphi, \varphi^*)) < \rho$ for any $\rho > 0$. $\square$

Lemma 2: For every $(\varphi, \vartheta) \in \mathcal{E}^*$, there exists $\rho > 0$ such that every $(\hat{\varphi}, \hat{\vartheta})$ for which $d_\infty((\varphi, \vartheta), (\hat{\varphi}, \hat{\vartheta})) < \rho$ belongs to the set $\mathcal{E}^*$, i.e.

$$\mathcal{B}_\rho(\varphi, \vartheta) := \{(\hat{\varphi}, \hat{\vartheta}) \in \mathcal{E} : d_\infty((\varphi, \vartheta), (\hat{\varphi}, \hat{\vartheta})) < \rho\} \subseteq \mathcal{E}^*$$

Proof. If $(\varphi, \vartheta) \in \mathcal{E}^*$ then $(\varphi, \vartheta) \in \mathcal{E}^{\gamma'}$ for some $0 < \gamma' < \frac{1}{2}$. Pick such a γ'. Also, note that F has a jump at some $\varepsilon^* > \frac{1}{2}$. Let $p := F(\varepsilon^*) - F(\varepsilon^{*-})$ be the size of this jump. Pick ρ such that $0 < \rho < \min\{\gamma', p/2\}$. Then, if $(\hat{\varphi}, \hat{\vartheta}) \in \mathcal{B}_\rho(\varphi, \vartheta)$, we have $\hat{\theta}^0(0, A) \subseteq [\frac{1}{2} + \overline{\gamma} - \rho, 1]$ and $\hat{\theta}^0(1, A) \subseteq [0, \frac{1}{2} - \overline{\gamma} + \rho]$ and $\hat{F}$ has a jump of size at least $p - 2\rho > 0$ at the point $\varepsilon^* > \frac{1}{2}$. Therefore, $(\hat{\varphi}, \hat{\vartheta})$ satisfies Assumptions 2^γ and 4 with $\gamma = \gamma' - \rho > 0$; hence, $(\hat{\varphi}, \hat{\vartheta}) \in \mathcal{E}^{\gamma' - \rho}$. Since $0 < \gamma' - \rho < 1/2$, we have $(\hat{\varphi}, \hat{\vartheta}) \in \mathcal{E}^*$. $\square$

Remark 3: The use of perturbations that involve atoms at $\varepsilon^* > \frac{1}{2}$ is convenient but it is also possible obtain SVE eventually by perturbing environments smoothly (i.e., so that the perturbed environment has smooth distributions). As long as the perturbed environment has a support that contains an interval close enough to $\varepsilon = 1$, we can obtain SVE eventually. For example, arguments similar to those employed by Good and Mayer (1975) and Chamberlain and Rothschild (1981) can be used to establish an analogue to the Calibration Lemma. (For this, however, we would have to generalize the results of these authors and apply the change of variables theorem.)

Remark 4: Assumption 2 is substantively well motivated but is also technically essential for our approach. With overlap in the images of the mappings $\theta^t(\omega, \alpha)$ (as in Mandler, 2012) a larger subset of S can support ties in large elections and the proof strategy used to establish Claim 1 breaks down. In particular, Mandler's (2012) setup corresponds to the case where $\varepsilon = 0$ with probability one and $\theta^t(\omega, A) = [0, 1]$. Our assumption that the images do not overlap captures the idea that agents know a priori which types of signals are more supportive of certain states, but we allow for uncertainty about the precisions of the signals.

Remark 5: Our perturbation approach in this paper is different from that of a recent literature on the robustness of equilibria to incomplete information. For example, Weinstein and Yildiz (2007) use large perturbations of players' higher order beliefs to show that every rationalizable action for a player type in a normal form game is the unique rationalizable action for that type in a perturbed game. Instead of perturbing higher order interim beliefs in the universal type space, we share the inclination of Kajii and Morris (1997) that " 'perturbations' should be explicitly modeled as arising from player types with different payoffs (see Kreps 1990)." To our knowledge, ours is the first paper to adopt this approach in a voting game of private information.

5 Discusssion

Contributions from Feddersen and Pesendorfer (1997) and Wit (1998) to Bhattacharya (2013) more recently, have focused on information aggregation in large voting games with private information. In these papers information aggregation is shown to typically emerge in a sequence of equilibria in which the fraction of sincere voters vanishes. Here we reconsider the possibility of supporting sincere voting in equilibrium when the electorate is large and focus on a key aspect of the canonical set up that leads to the tension between sincere voting and equilibrium–strong assumptions about what players know about the informational environment.

We find that adding even a small amount of uncertainty about the distribution of payoffs can avoid this tension. Feddersen and Pesendorfer (1997) introduce uncertainty about the distribution of preferences as a robustness check. In their extension, the behavior of the types that are voting uninformatively is fixed and thus equilibrium adjustments require mixing by voters with relevant information. They find that the fraction of informative voters need not vanish but that information aggregation fails. In contrast we allow for types that are indifferent and assume there is uncertainty about the size of this segment of the population. Indifference by these types allows equilibrium balancing to occur without distorting the behavior of types with relevant private information. The presence of types which are likely to be quite negligible but can be large with some probability turns out to have a profound impact on the incentives of others. Posterior beliefs that condition on being pivotal in a large election will put substantial probability on this group turning out to be large even though the group is likely to be arbitrarily small in the probability model. This features of the beliefs is closely related to several interesting results on voting with uncertainty about the distribution of voters (Good and Mayer 1975, Chamberlain and Rothschild 1981).

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