Choosing Policy-Makers: Learning from Past Decisions in a Changing Environment*

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Abstract

A politician sets policy in a changing environment, while a voter learns about the ability of the politician in order to decide whether to reelect him or not. When the voter cannot observe which policies are optimal, policies are unresponsive to relevant information, and uninformative about the ability of the politician. When the voter can observe optimal policies with arbitrarily large noise and enough decisions by the politician, (i) the voter appoints good policy-makers and tosses out bad policy-makers almost always, (ii) the politician chooses policy as with no career concerns, and (iii) consistent policy records are indicative of ability when past information depreciates fast.

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Constant development is the law of life, and a man who always tries to maintain his dogmas in order to appear consistent drives himself into a false position. - Mohandas K. Gandhi.

1 Introduction

Presidents, Governors, company CEOs and central bank chairmen are elected and appointed to make decisions on behalf of others. While their tenets, principles and ideologies are typically well understood before they get to office, their ability as decision-makers is often unknown. The decisions they make while in office then stick with them as a label, defining voters' and shareholders' perceptions about their ability, and influencing their reelection chances and career prospects.

While these policy-makers typically start their administration with a clear idea of the initial policies they want to implement, the bulk of their job is to revise their policy choices in response to changes in the environment. Do we maintain a restrained fiscal approach, or is it time to actively pump the economy? Do we maintain a large military presence in a region of conflict, or scale back the operations and resort to diplomacy?

At the core of these problems is that the optimal policy in any given period evolves over time, but it is typically not independent of the optimal policy in previous periods. Because of this, past information will generally be useful to evaluate current policy alternatives, but might also mislead the policy-maker when change is required. Depending on characteristics of the environment and of the decision maker, the information accumulated in past periods can then either completely swamp current information – making executives unresponsive to new events – or depreciate with time and end up being too uninformative to influence current decisions.

The problem is made all that much harder when policy-makers have career concerns, as for example in the case of a President who wants to be reelected, or a Governor that is considering running for President after her term in office. In this case, the actions taken in office can affect the policy-maker's career prospects, and the policy-maker has to consider how his actions are going to be interpreted by the principal who can decide on his political future.

In this paper we tackle the joint problem of a career minded policy-maker, who is learning the optimal policy in a changing environment, and a principal, who is learning about the ability of the policy-maker in order to decide whether to keep/promote him or dismiss him altogether. Can voters make convincing inferences about the ability of the President based on his decisions on when to be active or passive in military interventions abroad, or on when to stimulate the economy or be fiscally prudent? Can senators evaluate the ability of the Federal Reserve chairman based on his decisions on when to pursue an expansionary monetary policy and when to be passive, or of judges based on their conviction rates?

To capture the tradeoff between experience and adaptability to a changing environment, we consider the following model. A policy-maker (the politician) makes T up-or-down decisions in an office A. The optimal t-period policy is unobservable, and evolves according to a Markov process. Before making each policy decision d_t, the politician observes an imperfect private signal about the optimal t-period policy, with better politicians having a more precise signal. A principal (the voter) observes the politician's decisions in office A, and possibly also an imperfect signal about the optimal policy in each period, and then decides whether to appoint the politician for a second office B, or instead appoint an untested politician. The politician cares about making "good" policy decisions in office A (according to his own perceptions of the different types of errors), but also about getting appointed to office B (has career/electoral concerns). The voter cares about appointing a good politician, and possibly about policy outcomes in office A, but cannot pre-commit to any appointment strategy.[1]

In this environment, all past information is useful for current decisions, but the politician's knowledge does not increase with experience in office as it would if the optimal policy was given, and time invariant. The transition probabilities of the optimal policy process can be asymmetric, so that one alternative is highly persistent, but the other is short lived. Thus for example, we can capture a situation in which a passive monetary policy is optimal most of the time, but short spells of expansionary monetary policy are optimal some of the time.

We consider two alternative informational environments for the voter. In the first scenario, the voter can observe the choices of the politician, but cannot observe the process of optimal policies. For example, the voter might be able to observe the unemployment rate, but in the absence of a deeper understanding of the economy, be ignorant about whether increasing or decreasing the deficit would reduce or increase unemployment further. We say that this voter is *uninformed*. We show that when the voter is uninformed, and the politician puts enough weight on career concerns, in equilibrium policy decisions are independent of – and uninformative about – the

[1] The voter cannot influence the politician while in office, and is forward looking when voting for reelection.

ability of the politician. As a result, elections become ineffective instruments to select good politicians. Furthermore, posturing completely dominates policy-making, in the sense that both high and low ability politicians choose the same given sequence of policies, disregarding all private information about the optimal policy process.[2]

While assuming the voter is uninformed is a useful premise in various settings, in other applications it is natural to assume that the voter can at least imperfectly observe the optimal policy in each period. This could be due for example to the presence of the media. To capture this alternative environment, we endow the voter with an imperfect signal about the realization of the optimal policy in each period. We say that in this case there is partial *transparency* about the optimal policy, and parametrize the degree of transparency in the policy-making environment by the precision of the voter's signal.

In contrast to the case in which the voter is uninformed, we show that for an arbitrarily small level of transparency, if the voter can observe a sufficiently large number of policy decisions, (i) she appoints high ability politicians and dismisses low ability politicians with probability close to one, and (ii) the politician sets policy as if he had no career concerns (implying efficiency when he has the same preferences as the voter). Thus, while a very noisy observation of the optimal policy process would not be enough for the voter to choose the right action in the absence of delegation, this limited information is enough to sort out the agency relation and possibly lead to large gains in efficiency. This suggests that even a small level of transparency can fundamentally alter the relative effectiveness of representative democracy (delegation) and direct democracy, as even an unsophisticated media can greatly improve the quality of political choices taken in office, and help voters to select and promote only high ability politicians.[3]

While in the uninformed case policy decisions are independent of the ability of the politician, with transparency policy-makers of different ability levels generate different policy outcomes. A natural question in this setting is then whether observing frequent policy reversals tells us something about the ability of the decision-maker. Does flip-flopping on policy choices reflect poor decision-making, or consistent track records virtue? We show that under some conditions, consistent records are indicative of

[2]The set of equilibria therefore includes, but is not limited to, equilibria in which the politician implements the voter's ex ante preferred policy. Thus pandering has no special stature in this context.

[3]A similar logic holds for an "almost incompetent" bureaucracy or independent monitoring agency in other principal-agent relationships (ministers/President, investors/fund managers, shareholders/CEOs.

ability. This happens for example when past information depreciates fast enough. In this case, politicians with consistent records would get reappointed (or reelected) and politicians who flip-flop thrown out of office. This, however, is not true in general, as a biased, low ability politician can be more consistent than a biased politician of high ability.

The rest of the paper is organized as follows. We review the related literature in Section 2, and present the model in Section 3. We present the results in Section 4. We begin in Section 4.1, by characterizing the optimal strategy of a politician with no career or electoral concerns. We then consider career concerns with an uninformed voter (Section 4.2) and with transparency (Section 4.3). We conclude in Section 5. All proofs are in the Appendix.

2 Related Literature

Our paper builds on the literature studying how well elections serve the purpose of selecting high ability candidates to office.[4] Canes-Wrone, Herron, and Shotts (2001) consider a model in which an incumbent attempts to signal ability. With some probability, the voter can observe the optimal policy, and otherwise, she is uninformed. They find that when the challenger and incumbent are on relatively equal footing, and the probability that the voter is informed is small, in equilibrium the low ability incumbent panders to the voter (choose the popular, ex ante preferred action). In a similar model, Canes-Wrone and Shotts (2007) show that elected officials will be more inclined to pander when there is uncertainty regarding their congruence with the electorate. Ashworth and Shotts (2010) show that introducing an independent media in the baseline model does not always have the effect of reducing pandering. As before, pandering equilibria exist only if voter is likely to be uninformed, but for different values of the asymmetry in the reputation of the incumbent and the challenger the existence of media can both eliminate and create pandering equilibria in comparison to the baseline of no media.

Pandering also arises in Maskin and Tirole (2004), who compares "judges" and politicians. They assume that the official values office per se, and also has a legacy

[4] A different although related literature studies elections as disciplining device. Here the focus is not on adverse selection but on how elections can either induce effort by the elected politicians (moral hazard) or induce them to choose the policies preferred by the voter (see for example Barro (1973), Ferejohn (1986), and Alesina and Tabellini (2007)). A classic reference is Banks and Sundaram (1998), which studies the optimal retention rule for voters in a model that incorporates both moral hazard and adverse selection.

motivation. When the office-holding motive is strong, politicians want to pander. The distortions are larger when the public is poorly informed about what the optimal action is, and when feedback about the quality of the decision is limited (and in this case non-elected officials are preferred). In our setting this conclusion holds in the extreme case in which the voter is completely uninformed, but breaks down with even limited transparency about optimal policies if the evaluation horizon is long enough.[5]

A related paper outside of the realm of politics is Prendergast and Stole (1996). Prendergast and Stole consider the problem of a manager who chooses investments on a project in each of T periods. The project has unknown profitability, and the manager receives a private signal of its return in each period, with more talented managers receiving more precise signals. The manager cares about current profits and the current market's perception of ability. The main result of the paper is that managers will initially overreact to new information but eventually become unwilling to respond to new information suggesting that their previous behavior was wrong. The model has some obvious similarities with our setting, but also important differences. The key distinction is that in their work, Prendergast and Stole consider an unchanging environment. Because the optimal t-period optimal investment policy is unchanging in time, the manager gradually learns the truth as he accumulates signals drawn from the same true state.[6]

Finally, our results are also related to the findings of a recent literature that focuses on transparency in decision making (Prat (2005) and Fox and Van-Weelden (2011)). We return to this in Section 4.3.

[5] A related line of research considers advisors with career concerns. These models build on the seminal contribution of Crawford and Sobel (1982). The expert's ability is assumed to be private information, and he has has career concerns. The overwhelming message is that the expert will generally not report truthfully (Ottaviani and Sorensen (2006a), Ottaviani and Sorensen (2006b), Morris (2001)). The closest paper to ours is Li (2007). In this model, an agent delivers an initial report and a final report about the state of the world based on two private signals of increasing quality, after which the true state becomes publicly observable. Agent's types differ in the precision of their signal and possibly also in the slope of signal quality improvement. In this setting, inconsistent reports signal high ability in equilibrium when a smart agent's signals improve faster than those of a mediocre one.

[6] There are other important differences. While in PS the manager cares (myopically) about his reputation at the end of the current period, in our setting the politician cares about how the entire decision record affects the voter's perception of his ability. Moreover, while in PS the manager's payoff is strictly increasing in her reputation (as it would be the case in a competitive market for talent), in our model the politician's payoff is discontinuous in reputation, as he is either appointed for office B or not. The distinction is similar to what Alesina and Tabellini (2007) call bureaucrats and politicians (PS consider the bureaucrats case, while we consider politicians).

3 The Model

There are two agents, the politician, and the voter, and two stages. In the first stage, the politician makes a sequence of T policy decisions in office A, $d_t \in \{0,1\}$, $t = 1,\ldots,T$. We refer to a complete sequence of policy decisions as a policy, and write this as $h_T(d)$, where for any x and $t = 1,2,\ldots$, $h_t(x) \equiv (x_1, x_2, \ldots, x_{t-1})$. In the second stage, the voter observes the policy $h_T(d)$ and possibly also some additional information Z, and decides whether to appoint the politician for an office B or to instead elect an untested politician.

In each period t, there is an optimal t-period policy $\omega_t \in \{0,1\}$, which we refer to as a *passive* or *active* policy. The realization of ω_t is unobservable, but it is common knowledge that $\Pr(\omega_1 = 1) = p_1$, and that $\Pr(\omega_{t+1} = j|\omega_t = j) = \gamma_j \geq 1/2$. Given $b \in (0,1)$, the politician suffers a loss of b if he chooses an active policy when a passive policy is optimal ($d_t = 1$ when $\omega_t = 0$), suffers a loss of $(1-b)$ if he chooses a passive policy when an active policy is optimal ($d_t = 0$ when $\omega_t = 1$), and has a payoff of zero if $d_t = \omega_t = 0$ or $d_t = \omega_t = 1$. Thus given information $\mathcal{I}$, a politician with no career concerns (whose sole motivation is given by policy considerations) chooses $d_t = 1$ if and only if $\Pr(\omega_t = 1|\mathcal{I}) \geq b$.

While the politician cannot observe the optimal t-period policy, we assume that she can observe a signal s_t, such that $\Pr(s_t = j|\omega_t = j) = \theta$, $j \in \{0,1\}$. The parameter θ represents the *ability* of the politician, and is assumed to be known by the politician but not by the voter. There are two types of politician: experts ($\theta = e$) and amateurs ($\theta = a$), where $1/2 < a < e < 1$. The probability that the politician is an amateur is $\pi \in (0,1)$.

The politician cares about both the consequences of his decisions in office A (with weight $1-\delta$) and about the expected payoff of being appointed to office B by the voter (this has a weight δ). The payoff of a politician of type θ at time t with a history of decisions $h_t(d)$ and signals $h_t(s)$ when choosing a continuation $c_t(d) \equiv (d_t, \ldots, d_T)$ is

$$V_t^\theta = \delta T y(c_t(d); h_t(d)) + (1-\delta) \sum_{m=t}^{T} E\left[u(d_m, \omega_m)|h_{t+1}(s), \theta\right]$$

where $\delta \in [0,1]$ is the career or reelection concern, and $y = 1$ if politician is appointed to office B and $y = 0$ otherwise.[7] The voter has a payoff of one if she appoints an

[7]Note that we have multiplied the career concerns term by T. This is equivalent to assuming that the politician cares about the proportion of correct decisions, and avoids changing the weight of career concerns because of the number of decisions taken in office A.

expert politician and a payoff of zero if she appoints an amateur politician. She can also possibly care about policy outcomes in office A. However, we assume that she cannot pre-commit to any appointment strategy.[8]

Let $H_t(d)$ and $H_t(s)$ denote the set of possible decision and signal histories at date $t = 1, \ldots, T+1$, and let $\Theta \equiv \{a, e\}$ be the set of types. For any $t = 1, \ldots, T$, $\theta \in \Theta$, $h_t(d) \in H_t(d)$ and $h_{t+1}(s) \in H_{t+1}(s)$, let $\sigma_{\theta,t}(h_t(d), h_{t+1}(s)) = \Pr(d_t = 1|\theta, h_t(d), h_t(s), s_t)$, and let $\sigma_\theta = \sigma_{\theta,1} \times \ldots, \times \sigma_{\theta,T}$. We denote the strategy of the agent as the mapping $\sigma \equiv (\sigma_a, \sigma_e)$. We refer to σ_θ as type θ's choice strategy (the restriction of the politician's strategy σ to its type θ).

Conditional on information $\mathcal{I}$ at time T, each type θ's choice strategy σ_θ induces a distribution over policies $h_T(d)$, call this $f(\cdot|\sigma_\theta; \mathcal{I})$. In the paper we consider two alternative informational environments. In the first case, $\mathcal{I}_T = \emptyset$, the voter has no information beyond the decision record itself. We say that in this case the voter is *uninformed*. In the second case, the voter receives a sequence of signals z_t, $t = 1, \ldots, T$, where $\Pr(z_t = j|\omega_t = j) = q > 1/2$, for $j \in \{0, 1\}$. We say that in this case the voter is *q-informed* or that the decision making environment is transparent (or q-transparent).

A pure strategy of the voter is a rule $y(\cdot)$ mapping the set of possible decision records $h_T(d)$ and (if the voter is q-informed) signals $h_T(z)$ to a decision of whether to appoint the politician ($y = 1$) or not ($y = 0$). Thus $y(h_T(d), h_T(z)) = 1$ indicates that the voter appoints the politician at office B after decision record $h_T(d)$ and signals $h_T(z)$. A mixed strategy $\varphi(h_T(d), h_T(z))$ is a probability of appointing the politician conditional on $(h_T(d), h_T(z))$.

Given the politician's strategy σ, and conditional on $\mathcal{I}_T$, after observing the policy $h_T(d)$ the voter has a belief over types $\beta^v_\sigma(h_T(d); \mathcal{I}_T) \equiv \Pr(\theta = a|h_T(d); \sigma, \mathcal{I}_T)$, and appoints the politician if and only if

$$\beta^v_\sigma(h_T(d); \mathcal{I}_T) \leq \pi \Leftrightarrow \frac{f(h_T(d)|\sigma_e; \mathcal{I}_T)}{f(h_T(d)|\sigma_a; \mathcal{I}_T)} \geq 1.$$

Thus in equilibrium $\varphi(h_T(d), h_T(z)) > 0$ only if $\beta^v_\sigma(h_T(d); \mathcal{I}_T) \leq \pi$, and similarly $\varphi(h_T(d), h_T(z)) < 1$ only if $\beta^v_\sigma(h_T(d); \mathcal{I}_T) \geq \pi$.

[8] Without assuming commitment from the voter, the voter will reelect the politician or not solely based on her beliefs about his ability at reelection time. As a result, the voter cannot credibly threaten the politician to reelect or not following particular policy choices. Since the voter doesn't have any additional instruments to affect the incumbent politician *during* his time in office, the preferences of the voter regarding policy outcomes in office A are only relevant for welfare evaluations, and not in terms of behavior.

4 Results

4.1 Policy Choices with No Career Concerns

We begin by characterizing the optimal strategy of a politician with no career or electoral concerns. In this setting, the politician's optimal policy strategy is independent of the voter's own beliefs and strategy. As a result, each decision is independent of all previous and future decisions (the decision problems are related, through information, but the decisions themselves are not). The politician therefore chooses an active policy in period t if and only if the probability that this policy is optimal is above the threshold b, that is

$$\Pr(\omega_t = 1 | h_{t+1}(s); \theta) = \sum_{\omega_{t-1}} \Pr(\omega_t = 1 | \omega_{t-1}, s_t; \theta) \Pr(\omega_{t-1} | h_t(s); \theta) > b$$

Note that the t-period posterior belief is a function of the entire history of signals, including s_t and the t-period history $h_t(s)$. However, the history $h_t(s)$ enters only through its effect on $\Pr(\omega_{t-1}|h_t(s);\theta)$. Hence, for a type θ-politician, the t-period *prior* belief $p_t^\theta(h_t(s)) \equiv \Pr(\omega_t = 1|h_t(s);\theta)$ is a sufficient statistic for $h_t(s)$. We then let $\beta_\theta(s_t, p_t^\theta(h_t(s)) \equiv \Pr(\omega_t = 1|h_t(s), s_t; \theta)$ denote the politician's t-period posterior beliefs as a function of the current period signal s_t and the prior p_t^θ. With this notation, a type θ politician chooses $d_t = 1$ in period t if and only if

$$\beta_\theta(s_t, p_t^\theta) = \frac{\Pr(s_t|\omega_t = 1, \theta) p_t^\theta}{\Pr(s_t|\omega_t = 1, \theta) p_t^\theta + \Pr(s_t|\omega_t = 0, \theta)(1 - p_t^\theta)} > b \tag{1}$$

The fundamental tradeoff between experience and adaptability follows from the fact that the information in the t-period prior can overwhelm t-period information.[9] This happens if either $\beta_\theta(1, p_t^\theta) \leq b$ or $\beta_\theta(0, p_t) \geq b$, or equivalently, if

$$p_t^\theta \leq \frac{(1-\theta)b}{(1-\theta)b + \theta(1-b)} \equiv \underline{p}(\theta, b) \quad \text{or} \quad p_t^\theta \geq \frac{\theta b}{\theta b + (1-\theta)(1-b)} \equiv \overline{p}(\theta, b)$$

[9] This is akin to what happens in models of informational cascades (Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992)). Here individuals condition their decisions on their own private information *and* the history of predecessors' decisions. In equilibrium, the information in the publicly observed action history can overwhelm one individual's private information, causing all private information from this point on to be ignored. Moscarini, Ottaviani, and Smith (1998) study this model when the state of the world changes stochastically over time, as in our setting. They show that because of the depreciation of past information, only temporary informational cascades can arise in this setting. Moreover, when the persistence of the state process is sufficiently low (past information depreciates fast) no cascade ever arises.

Therefore, with no career concerns, the politician's dominant strategy $\hat{\sigma} = (\hat{\sigma}_a, \hat{\sigma}_e)$ boils down to choosing an active policy independently of t-period information whenever the t-period prior is sufficiently favorable for the active policy, i.e., $p_t^\theta \geq \overline{p}(\theta, b)$, to be passive independently of the t-period information whenever his t-period prior is sufficiently unfavorable to taking action, i.e., $p_t^\theta \leq \overline{p}(\theta, b)$, and to follow the t-period information whenever p_t^θ is in the interval $P(\theta, b) \equiv (\underline{p}(\theta, b), \overline{p}(\theta, b))$:

$$\hat{\sigma}_{\theta,t}(h_t(d), h_t(s), s_t) = \begin{cases} 1 & \text{if } p_t^\theta \geq \overline{p}(\theta, b) \text{ or } p_t^\theta \in P(\theta, b) \text{ and } s_t = 1 \\ 0 & \text{if } p_t^\theta \leq \underline{p}(\theta, b) \text{ or } p_t^\theta \in P(\theta, b) \text{ and } s_t = 0. \end{cases} \tag{2}$$

Two simple facts will be useful in our analysis. First, note that while in general the politician's t-period decision does not match his t-period signal, when he chooses policy according to $\hat{\sigma}$, then $d_t = s_t$ whenever the politician switches from a passive to an active policy or vice-versa.

Remark 1 *Suppose the politician sets policy according to the no-career-concern strategy $\hat{\sigma}$. Then $d_t \neq d_{t-1}$ implies $d_t = s_t$.*

On the flip side, if the politician is sufficiently good relative to the persistence of the state, then "old" information depreciates too fast, and becomes essentially useless for (t-period) decision-making.

Remark 2 *There exists $\hat{\theta}(\gamma_0, \gamma_1) \in (0, 1)$ such that $p_t^\theta \in P(\theta, b)$ for all $t = 1, \ldots, T$ whenever $\theta > \hat{\theta}(\gamma_0, \gamma_1)$. Furthermore, $\hat{\theta}(\gamma_0, \gamma_1)$ is increasing in γ_0 and γ_1.*

This result is relevant because, as we show in Section 4.3, better politicians have more volatile priors, but $P(\theta, b)$ also increases with the type of the politician (as $1 - \underline{p}(\theta, b)$ and $\overline{p}(\theta, b)$ are increasing in θ). The lemma shows that the interval $P(\theta, b)$ increases faster with θ than the support of the politician's t-period prior, and as a result, high ability politicians always have beliefs contained within $P(\theta, b)$. Thus if the politician is sufficiently good relative to the persistence of the optimal policy process, all of his decisions are based only on current period information. Moreover, the speed at which the support of p_t^θ grows with θ relative to $P(\theta, b)$ is decreasing in the memory of the optimal policy. As a result, for relatively low ability levels, the support of p_t^θ is contained in $P(\theta, b)$ if past information depreciates fast, but not if past information depreciates at a slower rate. The threshold ability for which the prior is always in $P(\theta, b)$ is therefore increasing in γ_0 and γ_1.

A valid concern regarding the characterization in (2) as a positive statement is that if decision-makers care about their reputation, posturing can taint the informative content of their policy decisions. In the next sections we reconsider the problem of a decision maker with career concerns under two alternative assumptions about the transparency of the decision-making environment. First, we focus on the case in which the voter is completely uninformed about the optimal policy process. With this, we aim to capture a situation in which the voter is unable to disentangle the effect of the politician's actions from other factors affecting observable outcomes. We then move on to the case in which the environment is at least somewhat transparent, in that the voter observes the optimal policy with noise.

4.2 Career Concerns with an Uninformed Voter

We begin with the case in which the voter is uninformed. We show that when career/electoral concerns are sufficiently important, in equilibrium the politician's decisions are completely uninformative about his ability to process information. Moreover, posturing completely dominates policy-making, in the sense that both types of politician choose a single sequence of policies ignoring all of their private information about the optimal policy process. We establish the result in two steps. First, we show that if the weight that the politicians put on political prospects is sufficiently large relative to policy considerations, their decisions in office are uninformative about their ability.

Lemma 1 *Suppose the voter is uninformed. There exists a* $\underline{\delta} \in (0, 1/2)$ *such that if* $\delta > \underline{\delta}$, *then in any equilibrium the politician's decisions are uninformative; i.e.,* $\beta^P_\sigma(h_T(d); \emptyset) \equiv \Pr(\theta = a | h_T(d); \sigma, \emptyset) = \pi$ *for all policies* $h_T(d)$ *that have positive probability in equilibrium.*

When the voter is uninformed about the optimal policy in each period, low ability politicians can mimic the statistical properties of the experts' behavior, with the voter being unable to call their bluff. Since in equilibrium the voter cannot commit to appointment decisions that are not sequentially rational, her retention decisions can only be contingent on the observed policy choices. But then if observed policy choices were to be informative, there would be some policies after which the voter would appoint the politician, and others for which she would not appoint the politician. If career or reelection concerns are sufficiently important, the payoff of getting hired dominates the additional flexibility of being free of posturing.[10]

In principle, Lemma 1 allows equilibrium strategies in which the expert politician conditions policy choices on the information he observes, $h_t(s)$, and the amateur politician merely "mimics" the statistical properties of the expert's strategy. However we show in Lemma 2 that this cannot be the case: if policy decisions are uninformative in equilibrium, politicians must *pool* on a policy that is unresponsive to the politician's information. It follows that the inability of the voter to determine which policy is optimal induces even the best politicians to disregard their information completely.

Lemma 2 *Suppose the voter is uninformed. If policy choices are uninformative in equilibrium, the politician chooses a policy $h_T(d)$ that is not contingent on his information $\{h_t(s)\}_t$ or ability type: for all t and all $h_t(s)$, $\sigma_j((h_t(s), s_t = 1), h_t(d)) = \sigma_j((h_t(s), s_t = 0), h_t(d)) = \sigma(h_t(d))$ for $j = e, a$.*

The proof builds on the observation that if the expert politician's policy choices are responsive to his private information, then at some point t mimicking by the amateur politician must entail mixing over $d_t = 1$ and $d_t = 0$. If we assume that the voter appoints the politician whenever she is indifferent, then his payoffs are determined solely by policy concerns, and will therefore prefer to deviate and play his no career concern action, as dictated by $\hat{\sigma}_a(h_{t+1}(s))$. This still leaves open the possibility that there might be a non-pooling equilibrium with mimicking if the voter uses different appointment probabilities after different policies of the politician. But because the politician's payoffs are a function of the (private) belief with which he evaluates the t^{th} decision, there is no way to induce him to mix in the first place.

Lemmas 1 and 2 directly imply Theorem 1.

Theorem 1 *Suppose the voter is uninformed. Then there exists $\underline{\delta} \in (0, 1/2)$ such that if $\delta > \underline{\delta}$, in equilibrium both the expert and amateur politician implement a policy $h_T(\tilde{d})$ that is independent of the politician's private information.*

Note then that when the voter is uninformed, posturing entails a large welfare loss. In fact, in equilibrium both types of politician disregard private information

[10]Key to this result is that the voter only decides whether to reelect/appoint the politician to office or not, and cannot commit to punishing the politician for having chosen certain policies come election time. This coarseness of the set of instruments at the disposition of the voter is indeed one of the main characteristics of a political setting.

in policy-making. It follows that if the voter cares about decision-making in office A but is uninformed about optimal policy, she would do strictly better if she could commit not to use politician's policy choices to decide whether or not to appoint the politician to office B. This result echoes the insights regarding the "wrong type" of transparency identified by Prat (2005): while transparency on consequences is beneficial, transparency on action can have detrimental effects.[11]

The welfare loss incurred in the pooling equilibrium is especially severe when the politician is highly able. On the other hand, the loss cannot be too big, for the politician can always deviate and play the no career concerns strategy $\hat{\sigma}_\theta$. This is most clearly illustrated in the case of an expert politician, who can observe the state perfectly; i.e., $\theta = 1$. First, note that because the informed expert doesn't make mistakes, his payoff from following the no career concerns strategy at any point t is simply $\hat{W}_{\theta=1}(t) = 0$. Thus, he will be willing to keep playing the pooling strategy in period t as long as the per period benefit from pooling (the career concern δ) is larger than the per period expected cost of pooling, given beliefs $\Pr(\omega_t = 1 | h_{t+1}(s); \theta) = \beta_\theta \in \{0, 1\}$. Note that the best pooling equilibrium for politician is to pool on $\tilde{d}^{PL} = (1, 1, \ldots, 1)$ if $\overline{\mu} > b$ and to pool in $\tilde{d}^{PL} = (0, 0, \ldots, 0)$ if $\overline{\mu} < b$. Suppose for concreteness that $\overline{\mu} > b$, so that $\tilde{d}^{PL} = (1, 1, \ldots, 1)$. Then the per period expected cost of pooling, given beliefs β_θ is (see Lemma 3 in the Appendix)

$$(1-\delta)b \left\{ \left(\frac{T-t}{T} \right) (1-\overline{\mu}) + \frac{1}{T}(\overline{\mu} - \beta_\theta) \left[\frac{1-\kappa^{T-t+1}}{1+\kappa} \right] \right\},$$

where $\kappa \equiv \gamma_0 + \gamma_1 - 1$, and as before, $\overline{\mu} = \frac{1-\gamma_0}{2-\gamma_0-\gamma_1}$. Note that the cost of pooling is larger for $\beta_\theta = 0$ and $t = 1$. Thus, there exists an equilibrium with pooling if and only if

$$\delta \geq \frac{\left(\frac{T-1}{T}\right)(1-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^T}{1+\kappa}\right]}{1/b + \left(\frac{T-1}{T}\right)(1-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^T}{1+\kappa}\right]} \equiv \underline{\delta}(1),$$

[11] In Prat (2005)'s model, however, the agent doesn't directly care about policy outcomes. Instead, his payoff is the principal's belief that the agent is good, while the principal is better off if the agent's action matches the state, and the more certain she is about the ability of the agent (a reduced form of a two-period career concerns model). Moreover, the agent is also uninformed about whether his ability is high or low. In this setting, if the principal can observe the agent's action, the agent has an incentive to disregard useful information and act according to how an able agent is expected to act a priori. Fox and Van-Weelden (2011) show that if the prior on the state of the world is sufficiently strong, and the costs are sufficiently asymmetric, the principal is made better off observing the action but not the consequences of the action. See also Levy (2005) for the analysis of transparency in committees.

The cost of pooling and the threshold $\underline{\delta}(1)$ are larger (i) the longer is politician's tenure in office A, (ii) the larger is politician's bias b to follow an active policy $d_t = 1$ in any given period, (iii) the smaller is the long-run average probability that an active policy is optimal $\overline{\mu}$, which is increasing in γ_1 and decreasing in γ_0, and (iv) the fastest past information depreciates (the smaller is $\gamma_0 + \gamma_1$).

Notice that in our framework pandering would lead the politician to choose exactly $\tilde{d}^{PL} = (1, 1, \ldots, 1)$. Theorem 1 implies that while pandering is an equilibrium in this context, it is only one out of many possibilities. The common feature of equilibria is that sorting is ineffective, and that policy choices are unresponsive to information (and hence very inefficient). The existence of an extreme asymmetry of information between the voter and the elected official leads to "obfuscation" and inefficiency, no matter whether there is pandering or not.

4.3 Career Concerns with (Partial) Transparency

We have shown that when the voter is completely uninformed, posturing dominates policy-making, in the sense that low and high ability politicians choose a single sequence of policies ignoring all information about the optimal policy process. In contrast to this negative result, we show that endowing the voter with a noisy observation of the optimal policy can be enough to break posturing. In particular, we show that even with an arbitrarily large noise about the outcomes of the politician's decisions in office, when the voter can observe enough decisions she can discriminate between good and bad decision makers with high probability. This in turn allows the existence of an equilibrium with no posturing, in which each type of politician follows the no career concern strategy $\hat{\sigma}_\theta$.

The proof of this result builds on the fact that when the expert politician chooses policy with the no career concerns strategy $\hat{\sigma}_e$, the voter can use her private information to identify almost surely decision records that are generated by an expert politician.[12] Because the inference of the voter relies on information that is not known to the politician, and the amateur is in fact of lower ability than the expert, the amateur cannot mimic the expert independently of how he chooses policy. And because in equilibrium politicians will play according to $\hat{\sigma}$, the voter will be almost sure about the politician's type almost always; i.e., $\Pr\left(\Pr(\theta = e|h_T(d))|e, \hat{\sigma}_e\right) \to_p 1$

[12] Note that since the politician's type is given by his ability and the history of signals he observed, the type space grows with T. Thus results such as Theorem 1 in Kalai and Lehrer (1993) do not apply in this setting.

and $\Pr\left(\Pr(\theta = a|h_T(d))|a, \hat{\sigma}_a\right) \to_p 1$. These conditions do not imply that the voter will always be able to appoint the expert and dismiss the amateur politician. There are some histories for which the voter will make mistakes. However for long enough policy records, these histories have a very small probability.

In the proof we make use of the following additional assumption, which simplifies the inferences of the voter following policy histories which have positive but arbitrary low probability in equilibrium.

Assumption (A1). With probability $1-\xi$ (for $\xi > 0$, small), the politician is fully rational. Conditional on being rational, he is an amateur with probability π, and an expert with probability $1-\pi$. With probability ξ the politician is a behavioral type. A behavioral type $k \in \{1, \dots, K\}$ plays $d_1 = 1$ with probability $1/2$, and then $\Pr(d_t = 1|d_{t-1} = 1) = \Pr(d_t = 0|d_{t-1} = 0) = 1/2 + k/2K$. Conditional on the politician being behavioral, he is of type k with probability r_k.

We are now ready to state formally our result.

Theorem 2 *For any precision $q > 1/2$ of the voter's information, any career concern $\delta \in (0,1)$ of the politician, and any $\varepsilon \in (0,1)$, there is a $\overline{T}(\delta, q, \varepsilon)$ such that if $T > \overline{T}(\delta, q, \varepsilon)$, then there is a PBE in which (i) both types of politicians "ignore" career concerns and choose policies in office A according to $\hat{\sigma}$, and (ii) the voter appoints an "expert" politician but not an "amateur" politician at task B with a probability of at least $1-\varepsilon$.*

Theorem 2 highlights the primary importance of transparency on equilibrium outcomes. With no transparency about the outcomes of politician's decisions in office, low ability politicians disregard useful information in order to mimic the behavior of expert politicians, and decision records become uninformative, jeopardizing selection efforts by the voter. In contrast, with even an arbitrarily small amount of transparency about the outcomes of politician's decisions in office, when politician's tenure is long enough (i) the voter appoints good types and fires bad types with probability close to one, and (ii) politician chooses policy as if he had no career concerns (implying efficiency when he has the same preferences as the voter). Furthermore, the more transparent is the environment (i.e., the larger is q), the shorter is the evaluation period $\overline{T}(\delta, q)$ needed to break posturing. (This is a straightforward corollary of Theorem 2, and follows intuitively from considering the limiting case of q approaching 1.)

The role of transparency here is indirect. While a very noisy observation of the optimal policy process would not be enough for the voter to choose the right policy in the absence of delegation, it is enough to sort out the agency relation when the evaluation period is long enough. This suggests that in a changing environment, even a small level of transparency can fundamentally alter the relative effectiveness of representative democracy (delegation) and direct democracy, as even an unsophisticated media can significantly improve the quality of political choices taken in office and help voters to select and promote only high ability politicians.

As we point out below (Section 4.4), this conclusion does not extend to the polar case of an unchanging environment, which requires the voter to be well informed about outcomes to sustain gains in both sorting and policy efficiency.

4.3.1 Ability and Policy Outcomes

Theorem 2 shows that if tenure is long enough, in equilibrium the politician chooses policy according to $\hat{\sigma}$, even after taking into account career concerns. This allows us to understand more deeply the connection between policy outcomes and the ability of the decision maker. In particular, a natural question in this setting is whether observing frequent policy reversals tells us something about the ability of the decision-maker. Does flip-flopping on policy choices reflect poor decision-making, and consistent track records virtue?

The characterization of $\hat{\sigma}$ in (2) pins down the policy chosen by the politician as a *contingent plan*. To say more about the expected *outcomes*, we need to investigate further the properties of the t-period prior process. To do this it is useful to write p_t^θ recursively, exploiting the Markovian structure of the state process. Note that $p_t^\theta = \sum_{\omega_{t-1}} \Pr(\omega_t = 1|\omega_{t-1}) \Pr(\omega_{t-1}|h_t(s);\theta)$, which can be written as

$$p_t^\theta = \beta_\theta(s_{t-1}, p_{t-1}^\theta)(\gamma_1 + \gamma_0 - 1) + (1 - \gamma_0).$$

Substituting $\beta_\theta(s_{t-1}, p_{t-1}^\theta)$ from (1), and letting $w(p_t^\theta, \theta) \equiv \theta p_t^\theta + (1-\theta)(1-p_t^\theta)$, we obtain:

$$p_{t+1}^\theta = \begin{cases} \frac{\theta p_t^\theta}{w(p_t^\theta,\theta)}(\gamma_1 + \gamma_0 - 1) + (1 - \gamma_0) & \text{if } s_t = 1 \\ \frac{(1-\theta) p_t^\theta}{1-w(p_t^\theta,\theta)}(\gamma_1 + \gamma_0 - 1) + (1 - \gamma_0) & \text{if } s_t = 0. \end{cases} \tag{3}$$

It follows that the politician's beliefs are given by the stochastic process

$$p_{t+1}^{\theta} = \begin{cases} \frac{\theta p_t^{\theta}}{w(p_t^{\theta},\theta)}(\gamma_1+\gamma_0-1)+(1-\gamma_0) & \text{w.p. } w(p_t^{\theta},\theta) \\ \frac{(1-\theta)p_t^{\theta}}{1-w(p_t^{\theta},\theta)}(\gamma_1+\gamma_0-1)+(1-\gamma_0) & \text{w.p. } 1-w(p_t^{\theta},\theta). \end{cases} \tag{4}$$

Hence, both the size of the jump from p_t^{θ} to p_{t+1}^{θ} and the transition probabilities $w(p_t^{\theta},\theta) = \theta p_t^{\theta} + (1-\theta)(1-p_t^{\theta})$ and $1-w(p_t^{\theta},\theta)$ are functions of the state of the process in period t. Note moreover that while the probability of moving away from the mean is higher than the probability of moving towards the center (by the memory of the process), the size of the jump after updating against the current belief is larger than one resulting from reinforcing the current belief (by the concavity of learning). However, expression (4) implies that

$$E[p_{t+1}^{\theta}|p_t^{\theta}] = (1-\gamma_0) + (\gamma_1+\gamma_0-1)p_t^{\theta}. \tag{5}$$

Hence $E[p_{t+1}^{\theta}|p_t^{\theta}] < p_t^{\theta}$ if and only if $p_t^{\theta} > (1-\gamma_0)/(2-(\gamma_0+\gamma_1)) \equiv \overline{\mu} \equiv \lim_{t\to\infty} \Pr(\omega_t = 1)$. Thus, politician's prior beliefs fluctuate around the long term probability $\overline{\mu}$ that $\omega_t = 1$. Figure 1 illustrates one possible realization of this process.

[Figure 1 about here]

As equation (5) shows, the expected value of p_{t+1}^{θ} conditional on p_t^{θ} does not depend on the politician's ability, θ. This property does not extend to other moments of the distribution. In particular, using (4) we can compute the conditional variance,

$$V(p_{t+1}^{\theta}|p_t^{\theta}) = \frac{(\gamma_1+\gamma_0-1)^2}{x(\theta)\,(1-x(\theta))}[p_t^{\theta}(1-p_t^{\theta})]^2,$$

where $x(\theta) \equiv \theta p_t^{\theta} + (1-\theta)(1-p_t^{\theta})$. From this it follows that

$$\frac{\partial V(p_{t+1}^{\theta}|p_t^{\theta})}{\partial\theta} = \frac{(\gamma_1+\gamma_0-1)^2}{[x(\theta)(1-x(\theta))]^2}[p_t^{\theta}(1-p_t^{\theta})(2p_t^{\theta}-1)]^2 > 0.$$

Thus, better politicians have more volatile beliefs (and more so the slowest past information depreciates; i.e., the larger are γ_0, γ_1). Now, in order to characterize behavior, we are interested in the variation in p_t^{θ} *relative to* $P(\theta, b) \equiv (\underline{p}(\theta, b), \overline{p}(\theta, b))$. And as we have shown before (see Remark 2) $P(\theta, b)$ increases faster with θ than the support of the politician's t-period prior. As a result, sufficiently high ability politicians always have beliefs contained within $P(\theta, b)$. Thus if the politician is sufficiently good relative to the persistence of the optimal policy process, all of his decisions are based only on current period information. In particular, there exists a threshold $\hat{\theta}(\gamma_0, \gamma_1)$, increasing in γ_0 and γ_1, such that $p_t^\theta \in P(\theta, b)$ for all $t = 1, \ldots, T$ whenever $\theta > \hat{\theta}(\gamma_0, \gamma_1)$.

[Figure 2 about here]

Whenever the politicians' abilities are sufficiently high, experts' policy records are more *consistent* than those of amateur politicians. We establish this under two alternative notions of consistency. First, we define the (long-run) flip-flop rate out of a $j = \{0, 1\}$ decision of a type θ politician given strategy σ_θ as $\phi_j(\theta, \sigma_\theta) \equiv \lim_{t\to\infty} \Pr(d_{t+1} \neq j | d_t = j; \sigma_\theta, \theta)$. This is the long-run probability that politician changes policy in any given period. We say that a type θ politician is more likely to flip-flop from a j decision than a type θ' politician given strategies $\sigma_\theta, \sigma_{\theta'}$ if $\phi_j(\theta, \sigma_\theta) > \phi_j(\theta', \sigma_{\theta'})$. Second, we say that a type θ politician is more consistent on active (passive) decisions than a type θ' politician given $\sigma_\theta, \sigma_{\theta'}$ if for $j = 1$ ($j = 0$) and for all $\ell' > \ell > 1$

$$\frac{\Gamma^j_{\ell'}(\theta, \sigma_\theta)}{\Gamma^j_{\ell'}(\theta', \sigma_{\theta'})} > \frac{\Gamma^j_{\ell}(\theta, \sigma_\theta)}{\Gamma^j_{\ell}(\theta', \sigma_{\theta'})} > 1,$$

where $\Gamma^j_{\ell'}(\theta, \sigma_\theta) \equiv \lim_{t\to\infty} \Pr(d_{t+\ell'} = \ldots = d_{t+1} = d_t = j; \theta, \sigma_\theta)$. Thus, a type θ politician is more consistent on active decisions than a type θ' politician if for any $\ell' > 1$, θ is more likely to generate a chain of active decisions of length ℓ' than θ', and if longer chains are increasingly likely to have been generated by type θ vis a vis θ'. Consistency is therefore a more expansive concept than flip-flopping.

In Proposition 1 we show that when all politicians' abilities are sufficiently high, so that past information depreciates fast enough (Lemma 2), the flip-flop rate out of the most likely optimal policy of both experts and amateurs is decreasing in the persistence of that policy, and increasing in the persistence of the less likely optimal policy. Furthermore, we show that experts are less likely than amateurs to flip-flop out of the long-run most likely optimal policy.

Proposition 1 *Suppose that $e > a > \hat{\theta}(\gamma_0, \gamma_1)$. If politicians have no-career concerns, (i) experts are less likely to flip-flop from a j decision than amateurs whenever*

$\gamma_j > \gamma_{-j}$; (ii) the flip-flop rate out of j decisions is decreasing in γ_j and increasing in γ_{-j} whenever $\gamma_j > \gamma_{-j}$.

Using our more expansive notion of consistency, we can also show that under the same conditions of Proposition 1, experts are also more *consistent* than amateurs. However, this is not true in general, as a biased, low ability politician can be more consistent than a biased politician of high ability.

[Figure 3 about here]

Proposition 2 *Fix $\gamma_1 \geq \gamma_0$.[13]. Suppose that in equilibrium, politicians choose policy according to the no career concerns strategy $\hat{\sigma}$. Then (i) if $e > a > \hat{\theta}(\gamma_0, \gamma_1)$, experts are more consistent than amateurs on active decisions. On the other hand, (ii) if $b < \overline{\mu}$, there exist $\overline{\theta}(\gamma_0, \gamma_1) < 1$ and $\underline{\theta}(\gamma_0, \gamma_1) \in (1/2, \overline{\theta})$ both increasing in γ_1 and decreasing in γ_0, such that if $a < \underline{\theta} < \overline{\theta} < e$, the amateur is more consistent than then expert on active decisions.*

When policy choices are informative, if all politicians are sufficiently competent and/or information depreciates at a fast rate, flip-flopping on policy choices reflects poor decision-making skills. In this case we would expect politicians with consistent policy records to get reelected, and/or climb the ladder of political offices. On the other hand, in general it is not possible to rank experts and amateurs solely on consistency.

4.4 An Unchanging World

In the paper, we focused on the case of a changing environment, with partial information depreciation. Since $\Pr(\omega_{t+1} = \omega_t) \in (1/2, 1)$, knowledge of the optimal policy in the current period is an informative but imperfect signal about what would be the optimal policy in the future. In this section, we discuss briefly the polar case of an

[13] A similar statement expressed in terms of passive decisions holds when $\gamma_0 > \gamma_1$.

unchanging world (i.e. $\gamma_1 = \gamma_0 = 1$), in which the optimal decision is the same in all periods.[14]

Consider first the no-career concerns strategy. With full persistence there is an "unchanging truth" that the politician gradually learns about, becoming more and more informed about it as time goes by. Formally, in this case the politician receives independent signals from the *same* Bernoulli distribution, with success probability θ when $\omega = 1$ and $1-\theta$ when $\omega = 0$. Thus, letting $m(k,t)$ denote the event in which exactly k out of t signals are equal to one, the politician's belief that $\omega = 1$ after observing $m(k,t)$ is

$$\beta_\theta(m(k,t)) = \left[1 + \frac{1-p_1}{p_1}\left(\frac{\theta}{1-\theta}\right)^{t\left(1-\frac{2k}{t}\right)}\right]^{-1}.$$

Now suppose that $\omega = 1$. Then $k/t = \left(\sum_{\ell=1}^{t} s_\ell\right)/t \to_p \theta > 1/2$. And with $k/t > 1/2$, $\beta_\theta(m(k,t)) \to 1$ for large t. Thus $\beta_\theta(m(\cdot,t)) \to_p 1$. Similarly, $\beta_\theta(m(\cdot,t)) \to_p 0$ when $\omega = 0$. That is, with full persistence the politician gradually learns the state of the world, and after sufficiently many observations knows the state almost perfectly almost always. The same of course is true of p_t^θ, which in this case is exactly equal to β_{t-1}^θ. If follows that if $b \in (0,1)$, there is a $\tilde{t}$ such that if $t > \tilde{t}$, then $d_t = 1$ almost surely when $\omega = 1$, and $d_t = 0$ almost surely when $\omega = 0$.

Consider next strategic interactions, and the ability of the voter to distinguish between high ability and low ability politicians with partial transparency.[15] Suppose for example that both types of politician are of relatively high ability, and play the no career concerns strategy $\hat{\sigma}$. Then with high probability both expert and amateur would converge within a few periods to a decision to which they will stick for the duration of their tenure in office. This example illustrates the basic difference with the benchmark case of partial information depreciation. When the optimal policy is static, the voter cannot rely on long-run tests to discriminate among types, and instead must rely entirely on how fast the politician converges to a decision in the

[14] The case of full information depreciation (i.e. $\gamma_1 = \gamma_0 = 1/2$) falls entirely within the analysis in the paper. Theorems 1 and 2 apply unchanged, and so does the characterization of the no-career concerns strategy $\hat{\sigma}$ in Section 4.1, although in a simplified form. Note that in this case by definition $p_t^\theta = \Pr(\omega_t = 1 | s_1, \ldots, s_{t-1}) = \Pr(\omega_t = 1)$, which approaches 1/2 as t gets large. Thus, for large t, a politician with no career concerns and bias equal to b follows the t period information if $\underline{p}(\theta,b) < 1/2 < \overline{p}(\theta,b)$, and he always takes active decisions ($d_t = 1$) when $\theta/(1-\theta) < (1-b)/b$, and always takes passive decisions ($d_t = 0$) when $\theta/(1-\theta) < b/(1-b)$. Clearly, an unbiased politician ($b = 1/2$) always bases his t-period decision on t-period information.

[15] The case in which the voter is uninformed is mostly unchanged.

initial "learning" period. It follows that in this setting there is no tradeoff between transparency (q) and tenure (T) as in the benchmark model: for any T, effective sorting requires transparency to be high.

5 Conclusion

Constant development is the law of life. This is true in the policy-making realm as well, where the optimal policy in any given period evolves in response to a changing environment. In this paper we tackled the joint problem of a career minded politician, who is learning about the changing environment in order to choose policy, and a voter, who is learning about the ability of the politician in order to decide whether to keep/promote him or dismiss him altogether. A key notion in this setting is how the actions of the politician are interpreted by the voter. Here we considered two alternative informational environments. In the first case, the voter can observe the choices of the politician, but cannot observe the process of optimal policies (the voter is uninformed). In the second one, the voter can observe the optimal policy in each period with noise. The level of transparency is parametrized by the precision of the voter's signal about the optimal policy.

Our results point to a relevant strategic effect of transparency and the media. When the voter is uninformed, and the politician puts enough weight on career concerns, in equilibrium policy decisions are independent of – and uninformative about – the ability of the politician. Moreover, posturing completely dominates policy-making, as both high and low ability politicians choose the same given sequence of policies, disregarding all private information about the optimal policy process. When instead the voter can at least partially attribute policy outcomes to the actions of the politician, it is possible for representative democracy to work on both policy and sorting grounds. In fact, as long as the voter can observe a sufficiently large number of policy decisions, there is an equilibrium in which (i) the politician sets policy as if he had no career concerns, and (ii) the voter appoints high ability politicians and dismisses low ability politicians almost always. Thus, while a very noisy observation of the optimal policy process would not be enough for the voter to choose the right action in the absence of delegation, this limited information is enough to sort out the agency relation and lead to large gains in efficiency.

A key assumption that we maintained throughout the paper is that the voter only decides whether to reelect/appoint the politician to office or not, and cannot commit to punishing the politician for having chosen certain policies come election time. We believe that this coarseness of the set of instruments at the disposition of the voter is indeed one of the main characteristics of a political setting. In other environments, however, the principal might have a wider array of instruments at her disposal. In these cases the optimal contract will balance incentive and selection considerations, trading off policy loses against speed of learning.

Another interesting extension of the model is to consider the case in which the politician's bias is private information. In the paper we focused on a situation in which the voter is uninformed about the ability of the politician, but not about his ideology. In this setting we obtained the result that consistent behavior can be indicative of high ability. When this is the case, politicians with consistent records would get reappointed (or reelected) and politicians who flip-flop thrown out of office. In some applications, however, it is reasonable to assume that not only the ability but also the bias of the politician is imperfectly observed by the voter. This would be the case for example in the case of judges or CEOs. In these cases, a consistent behavior in office may signal extremism. Understanding when the voter will be able to disentangle bias and ability in a changing environment is a natural next step in this research agenda.

6 Appendix A: Proofs

Proof of Remark 1. We show that $d_{t-1} = 1$ and $d_t = 0$ imply $s_t = 0$ (the other case is symmetric). Note that $d_{t-1} = 1$ implies either $p^\theta_{t-1} \geq \overline{p}(\theta, b)$ or $p^\theta_{t-1} \in (\underline{p}(\theta, b), \overline{p}(\theta, b))$ and $s_{t-1} = 1$. Similarly, $d_t = 0$ implies that either $p^\theta_t \leq \underline{p}(\theta, b)$ or $p^\theta_t \in (\underline{p}(\theta, b), \overline{p}(\theta, b))$ and $s_t = 0$. We then have four possibilities: (i) $p^\theta_{t-1} \geq \overline{p}(\theta, b)$ and $p^\theta_t \leq \underline{p}(\theta, b)$, (ii) $p^\theta_{t-1} \geq \overline{p}(\theta, b)$, $p^\theta_t \in (\underline{p}(\theta, b), \overline{p}(\theta, b))$ and $s_t = 0$, (iii) $p^\theta_{t-1} \in (\underline{p}(\theta, b), \overline{p}(\theta, b))$, $s_{t-1} = 1$ and $p^\theta_t \geq \underline{p}(\theta, b)$, or (iv) $p^\theta_{t-1} \in (\underline{p}(\theta, b), \overline{p}(\theta, b))$, $s_{t-1} = 1$, $p^\theta_t \in (\underline{p}(\theta, b), \overline{p}(\theta, b))$, and $s_t = 0$. If (ii) or (iv), we are done. Case (iii) is impossible, because $p^\theta_t > p^\theta_{t-1}$ when $s_{t-1} = 1$ (see equation (3)). Furthermore, for case (i) to be true, it must be that $s_t = 0$ and therefore $d_t = s_t$. ■

Proof of Remark 2. First, note that for all $\theta \in (1/2, 1)$, $p^\theta_t < \Pr(\omega_t = 1|\omega_{t-1} = 1) = \gamma_1$, and $1 - p^\theta_t < \Pr(\omega_t = 0|\omega_{t-1} = 0) = \gamma_0$, so that $p^\theta_t > 1 - \gamma_0$. Note that as long as $b \in (0, 1)$, $\overline{p}(\theta, b)$ is a strictly increasing continuous function of θ with $\overline{p}(1, b) = 1$, and $\underline{p}(\theta, b)$ is a strictly decreasing continuous function of θ with $\underline{p}(1, b) = 0$. Then there exists a $\theta_H < 1$ such that $\overline{p}(\theta, b) > \gamma_1$ whenever $\theta > \theta_H$, and $\theta_L < 1$ such that $\underline{p}(\theta, b) < 1 - \gamma_0$ whenever $\theta > \theta_L$. Thus $p^\theta_t \in (\underline{p}(\theta, b), \overline{p}(\theta, b))$ for all $\theta > \max\{\theta_L, \theta_H\}$. In fact, it is easy to compute $\theta_H = \gamma_1(1 - b)/(\gamma_1(1 - b) + b(1 - \gamma_1))$, increasing in γ_1 and decreasing in b, and $\theta_L = \gamma_0 b/(\gamma_0 b + (1 - \gamma_0)(1 - b))$, increasing in γ_0 and increasing in b. ■

Proof of Lemma 1. Let $D(\sigma)$ denote the set of records that have positive probability given σ; i.e., $D(\sigma) \equiv \{d : f(h_T(d)|\sigma_\theta) > 0 \text{ for some } \theta\}$. First we show that there exists a $\underline{\delta} \in (0, 1)$ such that if $\delta > \underline{\delta}_1$, then in any equilibrium politician's decisions are uninformative; i.e., $\beta^P_\sigma(h_T(d); \emptyset) \equiv \Pr(\theta = a|h_T(d); \sigma, \emptyset) = \pi$ for all $h_T(d) \in D(\sigma)$, and the voter appoints the politician to office B with positive probability.

Take $\delta \in (0, 1)$ given, and suppose that it is not true that $\beta^P_\sigma(h_T(d); \emptyset) = \pi$ for all $h_T(d) \in D(\sigma)$ in any equilibrium. Then there exist records $h^+_T(d) \in D(\sigma)$ and $h^-_T(d) \in D(\sigma)$ such that $\beta^P_\sigma(h^-_T(d); \emptyset) > \pi$ and $\beta^P_\sigma(h^+_T(d); \sigma) < \pi$ in equilibrium. Hence, the set of records $D^-(\sigma) \equiv \{h_T(d) \in D(\sigma) : y(h_T(d)) = 0\}$ and $D^+(\sigma) \equiv \{h_T(d) \in D(\sigma) : y(h_T(d)) = 1\}$ are nonempty. Furthermore, since $\beta^P_\sigma(h^-_T(d); \emptyset) > \pi$, it must be that $\Pr(h^-_T(d)|e; \sigma, \emptyset) < \Pr(h^-_T(d)|a; \sigma, \emptyset)$. Thus in particular the amateur plays $h^-_T(d)$ with positive probability.[16]

[16] It follows immediately then that if $\delta \geq 1/2$, this cannot be possible, for in this case getting hired

Say that $h_t(d)$ is *compatible* with the set $D^+(\sigma)$ at time t (alternatively, with $D^-(\sigma)$) if $\exists\ d^0 \in D^+(\sigma)$ (in $D^-(\sigma)$) such that $h_t(d^0) = h_t(d)$. Define the function $m(\cdot;\sigma)$ mapping decision histories to $\{0,1\}$ by $m(h_t(d);\sigma) = 1$ if $h_t(d)$ is compatible with the set $D^+(\sigma)$, and $m(h_t(d);\sigma) = 0$ if $h_t(d)$ is compatible with the set $D^-(\sigma)$. Now, recall that the politician's payoff is

$$V_t(c_t(d); h_t(d), h_{t+1}(s), \theta) = \delta T y(c_t(d); h_t(d)) + (1-\delta)\sum_{m=t}^{T} E\left[u(d_m, \omega_m)|h_{t+1}(s), \theta\right]$$

and, for $t = 1, \ldots, T$, define the value $W_t^\theta(m, \beta)$ as follows

$$W_T^\theta(1, \beta) = \max_{d_T}\{\delta T m(h_T(d), d_T) + (1-\delta)E[u(d_T, \omega_T)|\beta_T = \beta]\}$$

and

$$W_T^\theta(0, \beta) = (1-\delta)\max_{d_T} E[u(d_T, \omega_T)|\beta_T = \beta],$$

where we write $\beta_t(h_{t+1}(s))$ simply as β_t. Then, for any $t = 0, \ldots, T-1$, let

$$W_t^\theta(0, \beta) = \beta) W_{t+1}^\theta(0, \beta'(\beta, s_{t+1})) = (1-\delta)E\left[\sum_{n=t}^{T}\max_{d_n} E[u(d_n, \omega_n)|\beta_n;\theta]|\beta_t = \beta;\theta\right]$$

and

$$W_t^\theta(1, \beta) = \max_{d_t}\left\{\begin{array}{c} (1-\delta)E[u(d_t, \omega_t)|\beta_t = \beta;\theta] + \\ m(h_t(d), d_t;\sigma)\sum_{s_{t+1}} W_{t+1}^\theta(1, \beta'(\beta, s_{t+1}))\Pr(s_{t+1}|\beta_t = \beta;\theta) + \\ (1 - m(h_t(d), d_t);\sigma)\sum_{s_{t+1}} W_{t+1}^\theta(0, \beta'(\beta, s_{t+1}))\Pr(s_{t+1}|\beta_t = \beta;\theta) \end{array}\right\}$$

Since $D^-(\sigma)$ is nonempty by hypothesis, and d^- is played with positive probability by the amateur, for equilibrium it must be that there exists some β that is consistent with a possible realization of signals such that

$$\left\{\begin{array}{c} (1-\delta)\max_{d_t} E[u(d_t, \omega_t)|\beta_t = \beta;\theta = a] \\ + \\ \sum_{s_{t+1}} W_{t+1}^\theta(0, \beta'(\beta, s_{t+1}))\Pr(s_{t+1}|\beta_t = \beta;\theta = a) \end{array}\right\} > \left\{\begin{array}{c} (1-\delta)\min_{d_t} E[u(d_t, \omega_t)|\beta_t = \beta;\theta = a] \\ + \\ \sum_{s_{t+1}} W_{t+1}^\theta(1, \beta'(\beta, s_{t+1}))\Pr(s_{t+1}|\beta_t = \beta;\theta = a) \end{array}\right\}$$

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dominates any possible payoff gain from better decision-making for all agent types, and thus no type of agent can play $h_T^-(d)$ in equilibrium. This is a contradiction, since $h_T^-(d) \in D(\sigma)$ by hypothesis.

Note that the difference between the LHS and the RHS is maximized for $\beta = \overline{\beta}$ if $b \geq 1/2$ and for $\beta = \underline{\beta}$ if $b \leq 1/2$. Fix $b \geq 1/2$ without loss of generality. Then this cannot be an equilibrium if

$$b + E\left[\sum_{n=t}^{T} \max_{d_n} E[u(d_n, \omega_n)|\beta_n; a]|\beta_t = \overline{\beta}; a\right] < \frac{1}{(1-\delta)} E[W_{t+1}^{\theta}(1, \beta'(\overline{\beta}, s_{t+1}))|\overline{\beta}; a]$$

While the LHS is constant in δ, the RHS is a continuous increasing function of δ. It is strictly smaller than LHS for $\delta \to 0$ and strictly larger than LHS for $\delta \to 1$. Then for any σ there exists a unique value of δ, say $\underline{\delta}(\delta) \in (0,1)$, such that if $\delta > \underline{\delta}(\sigma)$, then in any equilibrium σ, $\beta(d;\sigma) \equiv \Pr(\theta = a|d, \sigma) = \pi$ for all $d \in D(\sigma)$. Then define $\underline{\delta} = \max_{\sigma} \underline{\delta}(\sigma)$.

Now consider $\delta > \underline{\delta}$. Our previous argument establishes that the sets $D^{-}(\sigma) \equiv \{h_T(d) \in D(\sigma) : y(h_T(d)) = 0\}$ and $D^{+}(\sigma) \equiv \{h_T(d) \in D(\sigma) : y(h_T(d)) = 1\}$ cannot both be nonempty. Then either $\varphi(h_T(d)) > 0$ for all $h_T(d) \in D(\sigma)$, or $\varphi(h_T(d)) < 1$ for all $h_T(d) \in D(\sigma)$. Moreover, in this case there must exist a set of decision histories of the politician for which the voter appoints the politician with positive probability. Otherwise $y(h_T(d)) = 0$ for all $h_T(d) \in D(\sigma)$, the politician doesn't have a career concern, and best responds by playing $\hat{\sigma}$. But this contradicts $\beta(d;\sigma) \equiv \Pr(\theta = a|d, \sigma) = \pi$ for all $h_T(d) \in D(\sigma)$. ■

Proof of Lemma 2. Suppose first that if indifferent, the voter appoints the politician with probability one. First, note that $\Pr(h_T(d)|\sigma_e) = \Pr(h_T(d)|\sigma_a)$ for all $h_T(d) \in D(\sigma)$ implies that $\Pr(h_t(d)|\sigma_e) = \Pr(h_t(d)|\sigma_a)$ for all t, for all $h_t(d)$ consistent with some $h_T(d) \in D(\sigma)$. So suppose (towards a contradiction) that there is a t and a signal realization $\tilde{h}_t(s)$ such that $\sigma_e(\tilde{h}_t(s), s_t = 1) \neq \sigma_e(\tilde{h}_t(s), s_t = 0)$. The choice strategy $\sigma_{e,\{1,2,\ldots,t\}}(\cdot)$ induces a probability distribution $f(\cdot|\sigma_e)$ over records $h_t(d)$. Then we can compute $\Pr(h_{t+1}(d)|\sigma_e) = \Pr(h_t(d)|\sigma_e)\Pr(d_t|\sigma_e, h_t(d))$. Since $\Pr(h_t(d)|\sigma_e) = \Pr(h_t(d)|\sigma_a)$ for all t and $h_t(d)$ consistent with some $h_T(d) \in D(\sigma)$, we must have that

$$\Pr(h_t(d)|\sigma_e)\Pr(d_t|\sigma_e, h_t(d)) = \Pr(h_t(d)|\sigma_a)\Pr(d_t|\sigma_a, h_t(d))$$

and $\Pr(h_t(d)|\sigma_e) = \Pr(h_t(d)|\sigma_a)$, so that we must have $\Pr(d_t|\sigma_e, h_t(d)) = \Pr(d_t|\sigma_a, h_t(d))$ as well. Now, since $\sigma_e(\tilde{h}_t(s), s_t = 1) \neq \sigma_e(\tilde{h}_t(s), s_t = 0)$, then $\Pr(d_t|\sigma_e, h_t(d)) \in (0,1)$, say $\Pr(d_t = 1|\sigma_e, h_t(d)) = \alpha \in (0,1)$. Inducing $\Pr(d_t|\sigma_a, h_t(d)) = \alpha$ will generically entail mixing by the amateur at either $s_t = 1$ or $s_t = 0$ (or both). But recall that the no career concerns strategy $\hat{\sigma}_a$ generically involves playing $d_t = 1$ or $d_t = 0$ after every history $h_{t+1}(s)$. Thus the amateur politician will generically not be indifferent and will prefer to deviate and play $\hat{d}(h_{t+1}(s))$. Therefore it must be that for all t and all $h_t(s)$, $\sigma_e(h_t(s), s_t = 1) = \sigma_e(h_t(s), s_t = 0)$. But this is perfect pooling on one record.

The same result holds if we allow the possibility that the voter mixes, using different conditional appointment probabilities for different decision histories of the politician. As before, suppose that there is a t and a signal realization $\tilde{h}_t(s)$ such that $\sigma_e(\tilde{h}_t(s), s_t = 1) \neq \sigma_e(\tilde{h}_t(s), s_t = 0)$. Then the amateur must be mixing over $d_t = 1$ and $d_t = 0$ in decision t. We established before that if the voter appoints the politician with probability one when indifferent, the amateur politician has incentives to deviate, and play the decision consistent with his no-career concerns strategy $\hat{\sigma}_a$, say $\hat{d}_t$. So it must be that there are (at least) two classes of records, say D_1 and D_2, with associated probabilities of appointment ϕ_1 and $\phi_2 \neq \phi_1$. Now suppose without loss of generality that the decision history up to period t was consistent with D_1, and that $\hat{\sigma}(h_{t+1}(s)) = 1$. If both $d_t = 1$ and $d_t = 0$ are consistent with D_1, then the amateur politician does not have incentives to mix (this is just as in the case in which the voter always appoints when indifferent). So it must be that $(h_t(d), d_t = 1)$ is consistent with D_1 and $(h_t(d), d_t = 0)$ is consistent with D_2, and that ϕ_1 and ϕ_2 are such that the amateur politician is indifferent between $d_t = 1$ and $d_t = 0$ at t. But this is not possible. Suppose in fact that ϕ_1 and ϕ_2 are such that given a history of signals $h_{t+1}(s)$, the amateur politician is indifferent between $d_t = 1$ and $d_t = 0$ at t. Then the amateur will not be willing to mix for a history of signals $h'_t(s) = h_t(s)$ for all $t \neq i$ and $s'_i = 1 - s'_i$. We conclude that it must be that for all t and all $h_t(s)$, $\sigma_e(h_t(s), s_t = 1) = \sigma_e(h_t(s), s_t = 0)$. ■

Lemma 3 *Consider an expert politician that can observe the state perfectly,* $\theta = 1$, *and suppose* $\overline{\mu} > b$. *The expected payoff from following the no career concerns strategy at any point* t *is* $\hat{W}_{\theta=1}(t) = 0$. *The (average) expected payoff from following the pooling equilibrium strategy from period* t *on, given beliefs* $\Pr(\omega_t = 1|h_{t+1}(s); \theta) = \beta_\theta$ *is*

$$W^{\theta}_{PL}(t, \beta_\theta) = \delta - (1-\delta)b\left\{\left(\frac{T-t}{T}\right)(1-\overline{\mu}) + \frac{1}{T}(\overline{\mu} - \beta_\theta)\left[\frac{1-\kappa^{T-t+1}}{1+\kappa}\right]\right\},$$

where $\kappa = \gamma_0 + \gamma_1 - 1$, *and* $\overline{\mu} = \frac{1-\gamma_0}{2-\gamma_0-\gamma_1}$. *Thus, there exists an equilibrium with pooling if and only if*

$$\delta \geq \frac{\left(\frac{T-1}{T}\right)(1-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^T}{1+\kappa}\right]}{1/b + \left(\frac{T-1}{T}\right)(1-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^T}{1+\kappa}\right]} \equiv \underline{\delta}(1),$$

Moreover, for any $\theta \in (1/2, 1]$, $\underline{\delta}(\theta) \geq \tilde{\delta}$, *where*

$$\tilde{\delta} \equiv \frac{\left(\frac{T-1}{T}\right)(b-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^T}{1+\kappa}\right]}{1/b + \left(\frac{T-1}{T}\right)(b-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^T}{1+\kappa}\right]}$$

Proof of Lemma 3. The best pooling equilibrium for the politician is to pool in

$$\tilde{d}^{PL} = \begin{cases} (1,1,\ldots,1) & \text{if } \overline{\mu} > b \\ (0,0,\ldots,0) & \text{if } \overline{\mu} < b. \end{cases}$$

So suppose for concreteness that $\overline{\mu} > b$, so that $\tilde{d}^{PL} = (1,1,\ldots,1)$. In period t, politician has a belief $\Pr(\omega_t = 1|h_{t+1}(s);\theta)$. Thus, if in period t, politician has a belief β_θ, his equilibrium payoff in the best pooling equilibrium is

$$\begin{aligned} W^{\theta}_{PL}(t,\beta_\theta) &= \delta T + (1-\delta)\sum_{m=t}^{T} E[u(1,\omega_m)|h_{t+1}(s),\theta] \\ &= \delta T - (1-\delta)\sum_{m=t}^{T}\Pr(\omega_m = 0|h_{t+1}(s),\theta)b. \end{aligned} \tag{6}$$

Now, iterating,

$$\Pr(\omega_{t+n} = 1|h_{t+1}(s),\theta) = \beta_\theta(\gamma_1+\gamma_0-1)^n + (1-\gamma_0)\left[\frac{1-(\gamma_1+\gamma_0-1)^s}{2-\gamma_1-\gamma_0}\right]$$

Then

$$W^{\theta}_{PL}(t,\beta_\theta) = \delta T - (1-\delta)\sum_{m=t}^{T}\left\{1-\beta_\theta(\gamma_1+\gamma_0-1)^{m-t} - (1-\gamma_0)\left[\frac{1-(\gamma_1+\gamma_0-1)^{m-t}}{2-\gamma_1-\gamma_0}\right]\right\} \tag{7}$$

Now consider the payoff for a type θ politician of deviating and playing $\hat{\sigma}_\theta$. This is

$$\hat{W}_\theta(t,\beta_\theta) = -(1-\delta)\sum_{m=t}^{T}\sum_{h_{m+1}(s)}\Pr(h_{m+1}(s)|h_{t+1}(s))\left[\begin{array}{c}\Pr(\omega_m = 1|h_{m+1}(s),\theta)(1-\hat{\sigma}_{\theta,m}(h_{m+1}(s)))(1-b) \\ \Pr(\omega_m = 0|h_{m+1}(s),\theta)\hat{\sigma}_{\theta,m}(h_{m+1}(s))b\end{array}\right] \tag{8}$$

Note that for $\theta = 1$, $\hat{W}_{\theta=1}(t, \beta_\theta) = 0$. Then for a $\theta = 1$ not to deviate from the pooling eq., (write $\kappa = \gamma_1 + \gamma_0 - 1$, and recall that $\overline{\mu} = \frac{1-\gamma_0}{2-\gamma_1-\gamma_0}$)

$$\begin{aligned}
\delta &\geq (1-\delta)b\frac{1}{T}\sum_{m=t}^{T}\left\{1 - \beta_\theta\kappa^{m-t} - (1-\gamma_0)\left[\frac{1-\kappa^{m-t}}{1+\kappa}\right]\right\} \\
&= (1-\delta)b\frac{1}{T}\left\{(T-t)(1-\overline{\mu}) + (\overline{\mu} - \beta_\theta)\sum_{m=t}^{T}\kappa^{m-t}\right\} \qquad (9) \\
&= (1-\delta)b\left\{\left(\frac{T-t}{T}\right)(1-\overline{\mu}) + \frac{1}{T}(\overline{\mu} - \beta_\theta)\left[\frac{1-\kappa^{T-t+1}}{1+\kappa}\right]\right\}.
\end{aligned}$$

Since this has to hold for all β_θ (for all possible histories $h_t(s)$), then

$$\delta \geq (1-\delta)b\left\{\left(\frac{T-t}{T}\right)(1-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^{T-t+1}}{1+\kappa}\right]\right\}.$$

And since this has to hold for all $t = 1, \ldots, T-1$, then in particular for $t = 1$ (where it binds), so

$$\delta \geq (1-\delta)b\left\{\left(\frac{T-1}{T}\right)(1-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^{T}}{1+\kappa}\right]\right\}.$$

Now consider an arbitrary $\theta \in (1/2, 1]$. Recall that

$$\hat{\sigma}_{\theta,m}(h_{m+1}(s)) = \begin{cases} 1 & \text{if } \Pr(\omega_m = 1|h_{m+1}(s), \theta) \geq b \\ 0 & \text{if } \Pr(\omega_m = 1|h_{m+1}(s), \theta) < b. \end{cases}$$

Call the term in brackets in (8) Z. Then when $\Pr(\omega_m = 1|h_{m+1}(s), \theta) \geq b$, $Z = \Pr(\omega_m = 0|h_{m+1}(s), \theta)b < (1-b)b$, and when $\Pr(\omega_m = 1|h_{m+1}(s), \theta) < b$, $Z = \Pr(\omega_m = 1|h_{m+1}(s), \theta)(1-b) < (1-b)b$. Thus $Z < (1-b)b$. But then

$$\hat{W}_\theta(t, \beta_\theta) > -(1-\delta)b(1-b)(T-t)$$

Then cannot have pooling for whatever expert this is if this is larger than the pooling payoff , so here we need

$$\delta > (1-\delta)b\left\{\left(\frac{T-t}{T}\right)(b-\overline{\mu}) + \frac{1}{T}(\overline{\mu} - \beta_\theta)\left[\frac{1-\kappa^{T-t+1}}{1+\kappa}\right]\right\},$$

and since this has to hold for all (feasible) β_θ (for all feasible histories $h_t(s)$) then so with $\beta_\theta = 0$ (approx), and $t = 1$, which gives

$$\delta \geq (1-\delta)b\left\{\left(\frac{T-1}{T}\right)(b-\overline{\mu}) + \frac{1}{T}\overline{\mu}\left[\frac{1-\kappa^{T}}{1+\kappa}\right]\right\},$$

■

Proof of Theorem 2. Consider first the case $e \geq \hat{\theta}(\gamma_0, \gamma_1)$. Here $p_t^e \in (\underline{p}, \overline{p})$ for all t, and thus according to $\hat{\sigma}_e$, $d_t^e = s_t$ for all t. It follows, in particular, that in equilibrium d_t^e is independent of the history of signals up to period t, $h_t(s)$.

Suppose that the Principal could observe the realization of the state, $h_{T+1}(\omega) = \{\omega_1, \ldots, \omega_T\}$. Let $\mathcal{D}_\omega \equiv \{d_t : \omega_t = \omega\}$, with $|\mathcal{D}_1| = U$, and $|\mathcal{D}_0| = B$. Then relabel these so that the first element of $\mathcal{S}_\omega$ is d_1, the second d_2, and so on. Then the observations $d_t \in \mathcal{D}_1$ are i.i.d. with $\Pr(d_t = 1 | d_t \in \mathcal{D}_1) = \theta$, and the observations $d_t \in \mathcal{D}_0$ are i.i.d. with $\Pr(d_t = 1 | d_t \in \mathcal{D}_0) = 1 - \theta$. Thus by the LLN, $\frac{1}{U} \sum_{d_t \in \mathcal{D}_1} d_t \rightarrow_p \theta$, and $\frac{1}{B} \sum_{d_t \in \mathcal{D}_0} d_t \rightarrow_p 1 - \theta$.

Now define the sets $\hat{\mathcal{D}}_0$ and $\hat{\mathcal{D}}_1$ by $\hat{\mathcal{D}}_\omega \equiv \{d_t : z_t = \omega\}$. Intuitively, $\hat{\mathcal{D}}_\omega$ is the set of observations in periods t that are classified as consistent with a realization $\omega_t = \omega$ given only the information in z_t. Call the set of observations $d_t \in \hat{\mathcal{D}}_\omega$ that are correctly classified $C_\omega \equiv \{d_t \in \hat{\mathcal{D}}_\omega : z_t = \omega = \omega_t\}$, and the set of observations $d_t \in \hat{\mathcal{D}}_\omega$ that are incorrectly classified $I_\omega \equiv \{d_t \in \hat{\mathcal{D}}_\omega : z_t = \omega \neq \omega_t\}$. Now, focusing on $\hat{\mathcal{D}}_1$,

$$\frac{1}{|\hat{\mathcal{D}}_1|} \sum_{d_t \in \hat{\mathcal{D}}_1} d_t = \frac{|C_1|}{|\hat{\mathcal{D}}_1|} \left[\frac{1}{|C_1|} \sum_{d_t \in C_1} d_t \right] + \frac{|I_1|}{|\hat{\mathcal{S}}_1|} \left[\frac{1}{I_1} \sum_{d_t \in I_1} d_t \right]$$

As before, here $\frac{1}{|C_1|} \sum_{d_t \in C_1} d_t \rightarrow_p \theta$, and $\frac{1}{|I_1|} \sum_{d_t \in I_1} d_t \rightarrow_p 1 - \theta$. Moreover, $\lim_{t \to \infty} \Pr(z_t = 1 | \omega_t = 1) \Pr(\omega_t = 1) = \mu q$, so that in the long-run distribution the z's are draws from almost identical distributions. Then $|C_1| / |\hat{\mathcal{D}}_1| \rightarrow_p q$. Thus $\frac{1}{|\hat{\mathcal{D}}_1|} \sum_{d_t \in \hat{\mathcal{D}}_1} d_t \rightarrow_p q\theta + (1-q)(1-\theta) \equiv \phi(\theta)$. Similarly, $\frac{1}{|\hat{\mathcal{D}}_0|} \sum_{d_t \in \hat{\mathcal{D}}_0} d_t \rightarrow_p \phi(\theta)$. In other words, if the record $h_T(d)$ is generated by an agent of type θ with choice strategy $\hat{\sigma}_\theta$, then for any $\varepsilon > 0$ and $\eta > 0$ there is a T^* such that if $T > T^*$, then $\forall \omega \in \{0, 1\}$.

$$\Pr \left(\phi(\theta) - \varepsilon \leq \frac{1}{|\hat{\mathcal{D}}_\omega|} \sum_{d_t \in \hat{\mathcal{D}}_\omega} d_t \leq \phi(\theta) + \varepsilon | (\theta, \hat{\sigma}_\theta) \right) > 1 - \eta. \tag{10}$$

Define for any $\varepsilon > 0$ the sets $A_e(\varepsilon) \equiv (\phi(e) - \varepsilon, \phi(e) + \varepsilon)$ and $A_a(\varepsilon) \equiv (\phi(a) - \varepsilon, \phi(a) + \varepsilon)$, and let $\mathcal{A}_\theta(\varepsilon) \equiv \left\{ h_T(d) : \frac{1}{|\hat{\mathcal{D}}_\omega|} \sum_{d_t \in \hat{\mathcal{D}}_\omega} d_t \in A_\theta(\varepsilon) \; \forall \omega \in \{0, 1\} \right\}$. It follows from our previous argument that for any positive ε, η there is a T^* such that if $T > T^*$, then $\Pr(\mathcal{A}_\theta(\varepsilon) | \theta, \hat{\sigma}_\theta) > 1 - \eta$. Moreover, for small enough ε, these sets are disjoint. Furthermore, for small ε, $\lim_{T \to \infty} \Pr(\mathcal{A}_\theta(\varepsilon) | B_k) = 0$.

So consider a record $h_T(d) \in \mathcal{A}_e(\varepsilon)$. Then, writing $m(d) \equiv \Pr(h_T(d) | \mathcal{A}_e(\varepsilon))$ for simplicity

$$\Pr(\theta = e|h_T(d)) > \frac{m(d)(1-\eta)(1-\pi)\lambda}{m(d)[(1-\eta)(1-\pi)+\eta\pi]\lambda + (1-\lambda)\sum_k \Pr(h_T(d)|\theta = B_k)} \equiv 1-\alpha,$$

which can be made arbitrarily close to 1 for large enough T. (The inequality comes from (10), and the following: we know the amateur is going to be in $\mathcal{A}_a(\varepsilon)$ with probability at least $1-\eta$, so he will be everywhere else with probability at most η, and in particular, will be in $\mathcal{A}_e(\varepsilon)$ with probability strictly less than η.) Now, because for $T > T^*$ we have $\Pr\left(d : \frac{1}{|\hat{\mathcal{D}}_\omega|}\sum_{d_t \in \hat{\mathcal{D}}_\omega} d_t \in A_\theta|\theta, \hat{\sigma}_\theta\right) > 1-\eta$, it follows that when the decision record is generated by an expert following choice strategy $\hat{\sigma}_e$, then for any $\alpha > 0$, $\eta > 0$ there exists $\overline{T}$ such that if $T > \overline{T}$ then

$$\Pr\left[\Pr(\theta = e|h_T(d)) > 1-\alpha|e, \hat{\sigma}_e\right] > 1-\eta,$$

or equivalently, $\Pr\left(\Pr(\theta = e|h_T(d))|e, \hat{\sigma}_e\right) \to_p 1$. By the same reasoning, if $h_T(d) \in \mathcal{A}_a(\varepsilon)$, then $\Pr\left(\Pr(\theta = a|h_T(d))|a, \hat{\sigma}_a\right) \to_p 1$. Thus in equilibrium with large T, the voter appoints the expert politician and doesn't appoint the amateur politician with probability close to one.

Next consider a deviation by a politician of type θ to a choice strategy $\sigma_\theta \neq \hat{\sigma}_\theta$. Note that any deviation from $\hat{\sigma}_\theta$ is costly at the decision-making stage. Thus, if it is to be profitable, it must induce a higher probability of getting hired. Now, by definition, $\hat{\sigma}_\theta$ is the optimal choice strategy with no career concerns, and therefore for any $\sigma_\theta \neq \hat{\sigma}_\theta$ and $\omega \in \{0,1\}$ is must be that $\Pr(d_t = \omega|\omega_t = \omega, h_t(d), h_t(s), \hat{\sigma}_\theta) = \Pr(s_t = \omega|\omega_t = \omega, \hat{\sigma}_\theta) \geq \Pr(d_t = \omega|\omega_t = \omega, h_t(s), h_t(d), \sigma_\theta)$ for all $t, h_t(s), h_t(d)$, with strict inequality for some $t, h_t(s), h_t(d)$. Therefore, for $\theta = a, e$,

$$\Pr\left(h_T(d) \notin \mathcal{A}_\theta(\varepsilon)|\theta, \sigma_\theta\right) > \Pr\left(h_T(d) \notin \mathcal{A}_\theta(\varepsilon)|\theta, \hat{\sigma}_\theta\right),$$

and (for $\theta = a$)

$$\Pr\left(h_T(d) \notin \mathcal{A}_e(\varepsilon)|a, \sigma_a\right) > \Pr\left(h_T(d) \notin \mathcal{A}_e(\varepsilon)|a, \hat{\sigma}_a\right).$$

But because for large enough T $\Pr\left(d : \frac{1}{|\hat{\mathcal{D}}_1|}\sum_{d_t \in \hat{\mathcal{D}}_1} d_t \in A_\theta|\theta, \hat{\sigma}_\theta\right) > 1-\eta$ for η arbitrarily small, it follows that for any record $h_T(d)$ such that $\frac{1}{|\hat{\mathcal{D}}_1|}\sum_{d_t \in \hat{\mathcal{D}}_1} d_t \notin A_\theta$ $\Pr\left(d|\theta, \hat{\sigma}_\theta\right) < \eta$ for η arbitrarily small, and therefore that for any such record $h_T(d)$

$$\Pr\left(\Pr(\theta \in B|h_T(d))|a, \sigma_a\right) \to_p 1,$$

where as before $B = \cup_k B_k$.

Because deviations to $\sigma_a \neq \hat{\sigma}_a$ are costly in terms of task A, and put higher probability on these decision histories $h_T(d) \notin A_e(\varepsilon) \cup A_a(\varepsilon)$ and do not increase the probability of decision histories in $A_e(\varepsilon)$, the amateur doesn't have a profitable deviation. Differently to the amateur, the expert could feasibly deviate to a σ_e generating decision histories consistent with $A_a(\varepsilon)$ with high probability, but this only decreases his payoffs.

We now drop the assumption of $e \geq \hat{\theta}(\gamma_0, \gamma_1)$, and consider the general case. When e is relatively large, $p_t^e \in (\underline{p}(e,b), \overline{p}(e,b))$ for all t, and thus according to $\hat{\sigma}_e$, $d_t^e = s_t$ for all t. As a result,

$$\frac{1}{|C_1|} \sum_{d_t \in C_1} d_t \rightarrow_p \theta, \quad \text{and} \quad \frac{1}{|C_0|} \sum_{d_t \in C_0} d_t \rightarrow_p 1 - \theta.$$

In general, however, for both the expert and the amateur, $p_t^\theta \notin (\underline{p}(e,b), \overline{p}(e,b))$, and thus according to $\hat{\sigma}_e$, it is possible that $d_t^e = 1$ when $s_t = 0$ (if $p_t^e > \overline{p}(e,b)$), or $d_t^e = 0$ when $s_t = 1$ (if $p_t^e < \overline{p}(e,b)$). This implies that the probability of $d_t = 1$ for $(\theta, \hat{\sigma}_\theta)$ now depends on the realization of p_t^θ, which in general will be different for different decisions, depending on the realization of $h_T(\omega)$ and then $h_T(s)$. As we show, however (see (i) below), it is still the case that $\mu(e) \equiv \lim_{t\to\infty} \Pr(d_t^e = 1 | \omega_t = 1, \hat{\sigma}_e) > \lim_{t\to\infty} \Pr(d_t^a = 1 | \omega_t = 1, \hat{\sigma}_a) \equiv \mu(a) > 1/2$. Furthermore, while the observations d_t are drawn from distributions that are not i.i.d., these differences average out in the long run distribution (this is conceptually equivalent to drawing a random sample from C_1), so that it is still the case that $1/|C_1| \sum_{d_t \in C_1} d_t \rightarrow_p \mu(\theta)$. We argue this more formally below, in (ii). With these amenpoliticianents, the previous proof then extends to the general case.

We begin by showing (i) that $\mu(e) \equiv \lim_{t\to\infty} \Pr(d_t^e = 1 | \omega_t = 1, \hat{\sigma}_e) > \lim_{t\to\infty} \Pr(d_t^a = 1 | \omega_t = 1, \hat{\sigma}_a) \equiv \mu(a) > 1/2$. To do this, consider any realization of the process ω_t, and define the random variable

$$X_t = \begin{cases} 1 & \text{if } d_t^e - d_t^a \geq 0 \\ 0 & \text{if } d_t^e - d_t^a < 0. \end{cases}$$

Given a particular history of states, equation (3) describes the evolution of p_t^θ for each type of agent, and the random variable X_t records the (random) events in which the decision of the expert would differ from the decision of the amateur. We want to show that $\mathbb{E}(X_t | \omega_t = 1, \hat{\sigma}_\theta) > 1/2$ for large t. If the latter inequality is true, it follows that in the long period $\Pr(d_t^e \geq d_t^a | \omega_t = 1, \hat{\sigma}_e, \hat{\sigma}_a) > 1/2$ and i) must follow, i.e., when $\omega_t = 1$, the expert choose $d_t = 1$ with higher probability than the amateur. We proceed under the assumption that t is large, so that for all s close to t, $\Pr(\omega_s = 1)$ is close to $(1-\gamma_0)/(2-(\gamma_1+\gamma_0))$. Let $m_t^\theta \equiv \{p_t^\theta : p_t^\theta \in (\underline{p}(\theta,b), \overline{p}(\theta,b))\}$, $u_t^\theta \equiv \{p_t^\theta : p_t^\theta > \overline{p}(\theta,b)\}$, and $l_t^\theta \equiv \{p_t^\theta : p_t^\theta < \underline{p}(\theta,b)\}$. Then we have that

$$E(X_t|\omega_t = 1, \hat{\sigma}_\theta) = \Pr(u_t^e|\omega_t = 1) + \Pr(m_t^e, u_t^a|\omega_t = 1)e + \Pr(m_t^e, m_t^a|\omega_t = 1)(1-(1-e)a) +$$

$$+\Pr(m_t^e, l_t^a|\omega_t = 1) + \Pr(l_t^e, m_t^a|\omega_t = 1)(1-a) + \Pr(l_t^e, l_t^a|\omega_t = 1).$$

Since $1-(1-e)a > e$, it follows that

$$\Pr(m_t^e, u_t^a|\omega_t = 1)e + \Pr(m_t^e, m_t^a|\omega_t = 1)(1-(1-e)a) + \Pr(m_t^e, l_t^a|\omega_t = 1) > \Pr(m_t^e|\omega_t = 1)e,$$

and therefore

$$\begin{aligned}
E(X_t|\omega_t = 1, \hat{\sigma}) &> \Pr(u_t^e|\omega_t = 1) + \Pr(m_t^e|\omega_t = 1)e + \Pr(l_t^e, m_t^a|\omega_t = 1)(1-a) + \Pr(l_t^e, l_t^a|\omega_t = 1) \\
&> \Pr(u_t^e|\omega_t = 1) + \Pr(m_t^e|\omega_t = 1)e + (\Pr(l_t^e|\omega_t = 1) - \Pr(l_t^e, u_t^a|\omega_t = 1))(1-a) \\
&> \Pr(u_t^e|\omega_t = 1) + \Pr(m_t^e|\omega_t = 1)e + \Pr(l_t^e|\omega_t = 1)(1 - \Pr(u_t^a|\omega_t = 1))(1-a),
\end{aligned}$$

where the second inequality follows from $\Pr(l_t^e, m_t^a|\omega_t = 1)(1-a) + \Pr(l_t^e, l_t^a|\omega_t = 1) > (\Pr(l_t^e|\omega_t = 1) - \Pr(l_t^e, u_t^a|\omega_t = 1))(1-a)$, and in the last inequality we use the fact that since $p_t^e(\cdot|\omega_t = 1)$ and $p_t^a(\cdot|\omega_t = 1)$ are affiliated, $\Pr(l_t^e, u_t^a|\omega_t = 1) < \Pr(l_t^e|\omega_t = 1)\Pr(u_t^a|\omega_t = 1)$. But now

$$\Pr(u_t^e|\omega_t = 1) + \Pr(m_t^e|\omega_t = 1)e + \Pr(l_t^e|\omega_t = 1)(1 - \Pr(u_t^a|\omega_t = 1))(1-a) >$$

$$\Pr(u_t^e|\omega_t = 1)\frac{1}{2} + \Pr(m_t^e|\omega_t = 1)e + \Pr(l_t^e|\omega_t = 1)\left(\frac{1}{2} + (1 - \Pr(u_t^a|\omega_t = 1))(1-a)\right)$$

where we used the fact that $\Pr(u_t^e|\omega_t = 1)$ is larger than $\Pr(l_t^e|\omega_t = 1)$. But since the last inequality above is a convex combination of numbers bigger than 1/2, it must be the case that $E(X_t|\omega_t = 1, \hat{\sigma}) > 1/2$.

Now consider (ii). Note that according to $\hat{\sigma}_\theta$, $d_t^\theta = 1$ iff either $p_t^\theta > \overline{p}(\theta,b)$ or $p_t^\theta \in (\underline{p}(\theta,b), \overline{p}(\theta,b))$ and $s_t = 1$. Now, we can show (see (3), (4)) that

$$p_{t+1}^\theta = \begin{cases} \frac{\theta p_t^\theta}{\theta p_t^\theta + (1-\theta)(1-p_t^\theta)}(\gamma_1+\gamma_0-1) + (1-\gamma_0) & \text{w.p. } \theta p_t^\theta + (1-\theta)(1-p_t^\theta) \\ \frac{(1-\theta)p_t^\theta}{1-[\theta p_t^\theta + (1-\theta)(1-p_t^\theta)]}(\gamma_1+\gamma_0-1) + (1-\gamma_0) & \text{w.p. } 1-[\theta p_t^\theta + (1-\theta)(1-p_t^\theta)]. \end{cases}$$

Then

$$E[p_{t+1}^\theta|p_t^\theta] = (1-\gamma_0) + (\gamma_1+\gamma_0-1)p_t^\theta.$$

Hence it follows that $E[p_{t+1}^\theta | p_t^\theta] < p_t^\theta$ if and only if $p_t^\theta > \frac{1-\gamma_0}{2-(\gamma_0+\gamma_1)} \equiv \overline{\mu} \equiv \lim_{t\to\infty} \Pr(\omega_t = 1)$. Thus the politician's beliefs fluctuate around the long term probability that $\omega_t = 1$, $\overline{\mu}$. This implies that $y_t^\theta \equiv p_t^\theta - \overline{\mu}$ is a supermartingale. To see this, note that

$$y_{t+1}^\theta = \begin{cases} \left(\frac{\theta\left(y_t^\theta+\overline{\mu}\right)}{\theta\left(y_t^\theta+\overline{\mu}\right)+(1-\theta)\left(1-\left(y_t^\theta+\overline{\mu}\right)\right)} - \overline{\mu}\right)(\gamma_1+\gamma_0-1) & \text{w.p. } \theta\left(y_t^\theta+\overline{\mu}\right)+(1-\theta)\left(1-\left(y_t^\theta+\overline{\mu}\right)\right) \\ \left(\frac{(1-\theta)\left(y_t^\theta+\overline{\mu}\right)}{1-\left[\theta\left(y_t^\theta+\overline{\mu}\right)+(1-\theta)\left(1-\left(y_t^\theta+\overline{\mu}\right)\right)\right]} - \overline{\mu}\right)(\gamma_1+\gamma_0-1) & \text{otherwise.} \end{cases}$$

It follows that

$$E[y_{t+1}^\theta | y_t^\theta] = \left(y_t^\theta + \overline{\mu}\right)(\gamma_1+\gamma_0-1) - \overline{\mu}(\gamma_1+\gamma_0-1) = (\gamma_1+\gamma_0-1)y_t^\theta < y_t^\theta.$$

Since $E[|y_t^\theta|] \in [l(\theta;\gamma_1,\gamma_0) - \overline{\mu}, u(\theta;\gamma_1,\gamma_0) - \overline{\mu}]$ is always bounded, by Doob's Forward Convergence Theorem, y_t^θ converges almost surely to a random variable Z. It follows that p_t^θ converges to a random variable X with c.d.f. F. We can then apply the strong law of large numbers to X to conclude that the empirical distribution function $\hat{F}_t(a) = \frac{1}{T}\sum_{t=1}^T 1\{p_t^\theta < a\}$ converges to $F(a)$ as $T \to \infty$ almost surely, for every value of a. This then implies that

$$\frac{1}{|C_1|}\sum_{d_t \in C_1} d_t \to_p [1 - F(\theta)] + [F(\theta) - F(1-\theta)]\theta = \mu(\theta).$$

This completes the proof. ■

Proof of Proposition 1. Note that because $\theta > \hat{\theta}(\gamma_0,\gamma_1)$, $\phi_1(\theta) \equiv \phi_1(\theta, \hat{\sigma}_\theta) = \lim_{t\to\infty}\Pr(s_{t+1} = 0 | s_t = 1)$. Now,

$$\begin{aligned} \Pr(s_{t+1} = 1 | s_t = 1) &= \sum_{\omega_{t+1}=0,1} \Pr(s_{t+1} = 1 | \omega_{t+1}) \Pr(\omega_{t+1} | s_t = 1) \\ &= \sum_{\omega_{t+1}=0,1} \Pr(s_{t+1} = 1 | \omega_{t+1}) \left[\sum_{\omega_t=0,1} \Pr(\omega_{t+1}|\omega_t)\Pr(\omega_t | s_t = 1)\right] \\ &= [\theta(1-\gamma_0) + (1-\theta)\gamma_0] + \left(\frac{\theta\mu_t}{\theta\mu_t + (1-\theta)(1-\mu_t)}\right)(\gamma_0+\gamma_1-1)(2\theta-1), \end{aligned} \tag{11}$$

where $\mu_t = \Pr(\omega_t = 1)$. Recall $\lim_{t\to}\mu_t = (1-\gamma_0)/(2-(\gamma_0+\gamma_1)) \equiv \overline{\mu}$. Let $q \equiv \theta\overline{\mu}/(\theta\overline{\mu} + (1-\theta)(1-\overline{\mu})) = \theta(1-\gamma_0)/(\theta(1-\gamma_0) + (1-\theta)(1-\gamma_1)) \in (0,1)$. Then

$$\phi_1(\theta) = 1 - [\theta(1-\gamma_0) + (1-\theta)\gamma_0] - q(\gamma_0+\gamma_1-1)(2\theta-1)$$

Note that $q|_{\theta=1/2} = \overline{\mu} > 1/2$ iff $\gamma_1 > \gamma_0$ and

$$\frac{\partial q}{\partial \theta} = \frac{(1-\gamma_0)(1-\gamma_1)}{(\theta(1-\gamma_0)+(1-\theta)(1-\gamma_1))^2} = q^2 \frac{1-\gamma_1}{\theta^2(1-\gamma_0)} = \frac{q(1-q)}{\theta(1-\theta)} > 0.$$

Therefore $q > 1/2$ whenever $\gamma_1 > \gamma_0$. Furthermore,

$$\frac{\partial q}{\partial \gamma_1} = \frac{\theta(1-\theta)(1-\gamma_0)}{(\theta(1-\gamma_0)+(1-\theta)(1-\gamma_1))^2} = q^2 \frac{1-\theta}{\theta(1-\gamma_0)} = \frac{q(1-q)}{1-\gamma_1} > 0,$$

and

$$\frac{\partial q}{\partial \gamma_0} = -\frac{\theta(1-\theta)(1-\gamma_1)}{(\theta(1-\gamma_0)+(1-\theta)(1-\gamma_1))^2} = -q^2 \frac{(1-\theta)(1-\gamma_1)}{\theta(1-\gamma_0)^2} = -\frac{q(1-q)}{1-\gamma_0} < 0.$$

Using these results we can conclude that

$$\frac{\partial \phi_1(\theta)}{\partial \gamma_1} = -(2\theta-1)\left((\gamma_0+\gamma_1-1)\frac{\partial q}{\partial \gamma_1} + q\right) = -\frac{q(2\theta-1)}{1-\gamma_1}(\gamma_0(1-q)+q(1-\gamma_1)) < 0,$$

and

$$\frac{\partial \phi_1(\theta)}{\partial \gamma_0} = -(2\theta-1)\left(-(1-q)+(\gamma_0+\gamma_1-1)\frac{\partial q}{\partial \gamma_0}\right) = +\frac{(1-q)(2\theta-1)}{1-\gamma_0}((1-\gamma_0)(1-q)+q\gamma_1) > 0$$

This establishes part (i). For part (ii), note that

$$\frac{\partial \phi_1(\theta)}{\partial \theta} = (2\gamma_0-1) - (\gamma_0+\gamma_1-1)\left(2q+(2\theta-1)\frac{\partial q}{\partial \theta}\right),$$

and notice that when $\gamma_1 \geq \gamma_0$ we have that

$$\frac{\partial \phi_1(\theta)}{\partial \theta} < 2\gamma_0 - 1 - \gamma_0 - \gamma_1 + 1 = \gamma_0 - \gamma_1 \leq 0,$$

where the first inequality follows from the fact that $\gamma_1 \geq \gamma_0$ implies $q > 1/2$. ■

Proof of Proposition 2. Part (i). Suppose without loss of generality that $\gamma_1 > \gamma_0$. Then we want to show that better politicians are more consistent on active decisions. Because $e > a > \hat{\theta}(\gamma_0, \gamma_1)$, then for all types θ, $\hat{\sigma}_\theta$ implies $d_t = s_t$. Then

$$\Gamma^j_{\ell'}(\theta, \sigma_\theta) = \lim_{t\to\infty} \prod_{\ell=1}^{\ell'} \Pr(s_{t+\ell} = 1 | s_t = \ldots = s_{t+\ell-1} = 1) \Pr(s_t = 1).$$

Furthermore, $\lim_{t\to\infty} \Pr(s_t = 1) = \overline{\mu}\theta + (1-\overline{\mu})(1-\theta)$, and $C_{\ell'} \equiv \lim_{t\to\infty} \Pr(s_{t+\ell'} = 1 | s_t = \ldots = s_{t+\ell'-1} = 1)$ is

$$\begin{aligned} C_{\ell'} &= \lim_{t\to\infty} \sum_{\omega_{t+\ell'}} \Pr(s_{t+\ell'} = 1|\omega_{t+\ell'}) \Pr(\omega_{t+\ell'}|s_t = \ldots = s_{t+\ell'-1} = 1) \\ &= (1-\theta) + (2\theta - 1) \lim_{t\to\infty} \Pr(\omega_{t+\ell'} = 1|s_t = \ldots = s_{t+\ell'-1} = 1). \end{aligned} \tag{12}$$

From (3), $x_{\ell'} \equiv \lim_{t\to\infty} \Pr(\omega_{t+\ell'} = 1|s_t = \ldots = s_{t+\ell'-1} = 1)$ is given

$$x_{t+1} = \frac{\theta x_t}{\theta x_t + (1-\theta)(1-x_t)}(\gamma_1 + \gamma_0 - 1) + (1-\gamma_0), \tag{13}$$

evaluated at $t + 1 = \ell'$, with initial condition $x_t = \overline{\mu}$. Now, from (12),

$$\frac{\partial C_{\ell'}}{\partial \theta} = (2x_{\ell'} - 1) + (2\theta - 1)\frac{\partial x_{\ell'}}{\partial \theta}$$

Because $x_t = \overline{\mu} > 1/2$ if and only if $\gamma_1 > \gamma_0$ and (13) defines an increasing sequence, it follows that $x_{\ell'} > 1/2$. Therefore $\partial C_{\ell'}/\partial\theta > 0$ whenever $\partial x_{\ell'}/\partial\theta > 0$. This in turn follows from the fact that (i) $x_t = \overline{\mu}$ is increasing in θ when $\gamma_1 > \gamma_0$ and that (ii) each step in (13) is increasing in θ. Since each term $C_{\ell'}$ is increasing in θ and $\lim_{t\to\infty} \Pr(s_t = 1) = \overline{\mu}\theta + (1-\overline{\mu})(1-\theta)$ is increasing in θ when $\gamma_1 > \gamma_0$, then

$$\frac{\Gamma^j_{\ell'}(\theta, \sigma_\theta)}{\Gamma^j_{\ell'}(\theta', \sigma_{\theta'})} > \frac{\Gamma^j_{\ell}(\theta, \sigma_\theta)}{\Gamma^j_{\ell}(\theta', \sigma_{\theta'})} > 1$$

for all $\ell' > \ell > 1$.

Part (ii). First, note that p^θ_t is bounded above by a number $u(\theta) \in (1/2, 1)$, and bounded below by $l(\theta) \in (0, 1/2)$. To see this, note that the process p^θ_t must be below the deterministic sequence given by (3) with $s_t = 1$ for all t, and above the deterministic sequence given by (3) with $s_t = 0$ for all t. First consider the upper sequence. Because this upper sequence is increasing and bounded, it converges. To compute the limit, solve

$$p^\theta_{t+1} = \frac{\theta p^\theta_t}{\theta p^\theta_t + (1-\theta)(1-p^\theta_t)}(\gamma_1 + \gamma_0 - 1) + (1-\gamma_0)$$

for $p^\theta_{t+1} = p^\theta_t = u(\theta; \gamma_0, \gamma_1)$. Note that $u(\theta; \gamma_0, \gamma_1)$ solves

$$W(u) \equiv u(\theta u + (1-\theta)(1-u)) - \theta u(\gamma_1 + \gamma_0 - 1) - (1-\gamma_0)(\theta u + (1-\theta)(1-u)) = 0$$

Since $W(u)$ is convex, $W(0) = -(1-\gamma_0)(1-\theta) < 0$ and

$$W\left(\frac{1-\gamma_0}{2-(\gamma_1+\gamma_0)}\right) = -\frac{(1-\gamma_0)(1-\gamma_1)(\gamma_1+\gamma_0-1)(2\theta-1)}{(2-(\gamma_1+\gamma_0))^2} < 0,$$

$$W(\gamma_1) = (1-\theta)(1-\gamma_1)(\gamma_1+\gamma_0-1) > 0,$$

and $\gamma_1 > (1-\gamma_0)/(2-(\gamma_1+\gamma_0))$, it follows that $u(\theta;\gamma_0,\gamma_1) \in ((1-\gamma_0)/(2-(\gamma_1+\gamma_0)),\gamma_1)$ exists and it is unique. Furthermore, when $\gamma_1 \geq \gamma_0$, we have that

$$\frac{\partial u(\theta;\gamma_0,\gamma_1)}{\partial\theta} = -\frac{2u^2 - u(2+\gamma_1-\gamma_0) + (1-\gamma_0)}{2\theta u + (1-\theta)(1-2u) - \theta(\gamma_1+\gamma_0-1) - (1-\gamma_0)(2\theta-1)} > 0.$$

To sign the derivative, notice that the denominator is positive since it is the derivative of $W(u)$ evaluated at $u(\theta;\gamma_0,\gamma_1)$, i.e., where $W(u)$ is increasing. As for the numerator, when $\gamma_1 \geq \gamma_0$ it is always increasing in u, and hence $2u^2 - u(2+\gamma_1-\gamma_0) + (1-\gamma_0) < 2\gamma_1^2 - \gamma_1(2+\gamma_1-\gamma_0) + (1-\gamma_0) = -(\gamma_1+\gamma_0-1)(1-\gamma_0) < 0$. Furthermore,

$$\frac{\partial^2 u(\theta;\gamma_0,\gamma_1)}{\partial^2\theta} = -\frac{\frac{\partial^2 W(u)}{\partial^2\theta}\left(\frac{\partial W(u)}{\partial u}\right)^2 - 2\frac{\partial^2 W(u)}{\partial\theta\partial u}\frac{\partial W(u)}{\partial u}\frac{\partial W(u)}{\partial\theta} + \frac{\partial^2 W(u)}{\partial^2 u}\left(\frac{\partial W(u)}{\partial\theta}\right)^2}{\left(\frac{\partial W(u)}{\partial u}\right)^3} =$$

$$\frac{\partial^2 u(\theta;\gamma_0,\gamma_1)}{\partial^2\theta} = -2\frac{-(4u-(2+\gamma_1-\gamma_0))\frac{\partial W(u)}{\partial u} + (2\theta-1)\frac{\partial W(u)}{\partial\theta}}{(2\theta u + (1-\theta)(1-2u) - \theta(\gamma_1+\gamma_0-1) - (1-\gamma_0)(2\theta-1))^3}\frac{\partial W(u)}{\partial\theta} < 0.$$

To sign the derivative, notice that as we show above $\partial W(u)/\partial u$ is positive, the denominator is positive, $\partial W(u)/\partial\theta$ is negative, and

$$4u - (2+\gamma_1-\gamma_0) > 4\frac{1-\gamma_0}{2-(\gamma_1+\gamma_0)} - (2+\gamma_1-\gamma_0) = \frac{\gamma_1^2-\gamma_0^2}{2-(\gamma_1+\gamma_0)} > 0 \text{ iff } \gamma_1 > \gamma_0.$$

We also have that

$$\frac{\partial u(\theta;\gamma_0,\gamma_1)}{\partial\gamma_1} = -\frac{-\theta u}{2\theta u + (1-\theta)(1-2u) - \theta(\gamma_1+\gamma_0-1) - (1-\gamma_0)(2\theta-1)} > 0,$$

and

$$\frac{\partial u(\theta;\gamma_0,\gamma_1)}{\partial\gamma_0} = -\frac{(1-\theta)(1-u)}{2\theta u + (1-\theta)(1-2u) - \theta(\gamma_1+\gamma_0-1) - (1-\gamma_0)(2\theta-1)} < 0.$$

Recall that $\overline{p}(1/2,b) = b$, $\overline{p}(1,b) = 1$, and notice that $\overline{p}(\theta,b)$ is always increasing in θ, and convex (concave) in θ if and only if $b < 1/2 (b > 1/2)$. Since $1 > u(1;\cdot) = \gamma_1 >$

$(1-\gamma_0)/(2-(\gamma_1+\gamma_0)) = u(1/2;\cdot)$, it follows that if $u(1/2;\cdot) = \overline{\mu} > b$ then $u(\theta;\gamma_0,\gamma_1)$ and $\overline{p}(\theta,b)$ must cross an odd number of times. By letting $\overline{\theta}(\gamma_0,\gamma_1)$ the largest value of θ such that $u(\theta;\gamma_0,\gamma_1) = \overline{p}(\theta,b)$, it follows that if $\theta > \overline{\theta}(\gamma_0,\gamma_1)$, $p_t^\theta < \overline{p}(\theta,b)$ for all t. Notice that $\overline{\theta}(\gamma_0,\gamma_1)$ is increasing in γ_1 and decreasing in γ_0. Furthermore, if $b \leq 1/2$, $\overline{\theta}(\gamma_0,\gamma_1)$ must be unique.

Consider now the lower sequence. Because this lower sequence is decreasing and bounded, it converges. To compute the limit, solve

$$p_{t+1}^\theta = \frac{(1-\theta)p_t^\theta}{(1-\theta)p_t^\theta + \theta(1-p_t^\theta)}(\gamma_1+\gamma_0-1) + (1-\gamma_0)$$

for $p_{t+1}^\theta = p_t^\theta = l(\theta;\gamma_0,\gamma_1)$. Note that $l(\theta;\gamma_0,\gamma_1)$ solves

$$K(l) \equiv l((1-\theta)l + \theta(1-l)) - (1-\theta)l(\gamma_1+\gamma_0-1) - (1-\gamma_0)((1-\theta)l+\theta(1-l)) = 0$$

Since $K(u)$ is concave, $K(1) = (1-\theta)(1-\gamma_1) > 0$ and

$$K(1-\gamma_0) = -(1-\theta)(1-\gamma_0)(\gamma_1+\gamma_0-1) < 0,$$

$$K\left(\frac{1-\gamma_0}{2-(\gamma_1+\gamma_0)}\right) = \frac{(1-\gamma_0)(1-\gamma_1)(\gamma_1+\gamma_0-1)(2\theta-1)}{(2-(\gamma_1+\gamma_0))^2} > 0,$$

and $(1-\gamma_0)/(2-(\gamma_1+\gamma_0)) > 1-\gamma_0$, it follows that $l(\theta;\gamma_0,\gamma_1) \in (1-\gamma_0, (1-\gamma_0)/(2-(\gamma_1+\gamma_0)))$ exists and it is unique. Furthermore, when $\gamma_1 \geq \gamma_0$, we have that

$$\frac{\partial l(\theta;\gamma_0,\gamma_1)}{\partial \theta} = -\frac{-2l^2 + l(2+\gamma_1-\gamma_0) - (1-\gamma_0)}{2(1-\theta)l + \theta(1-2l) - (1-\theta)(\gamma_1+\gamma_0-1) + (1-\gamma_0)(2\theta-1)} < 0.$$

To sign the derivative, notice that the denominator is positive since it is the derivative of $K(l)$ evaluated at $l(\theta;\gamma_0,\gamma_1)$, i.e., where $K(l)$ is increasing. As for the numerator, when $\gamma_1 \geq \gamma_0$ it is easy to check that is concave in l, positive at the boundaries, and hence always positive. Since $l(\theta;\gamma_0,\gamma_1)$ is decreasing in θ, $l(1/2;\gamma_0,\gamma_1) = u(1/2;\gamma_0,\gamma_1) = \overline{\mu}$, when $b < \overline{\mu}$, there exist a unique $\underline{\theta}(\gamma_0,\gamma_1)$ such that if $\theta < \underline{\theta}(\gamma_0,\gamma_1)$, $l(\theta;\gamma_0,\gamma_1) > \overline{p}(\theta,b)$. But this implies that whenever $a < \underline{\theta} < \overline{\theta} < e$ the amateur is more consistent than then expert on j decisions. Since

$$\frac{\partial l(\theta;\gamma_0,\gamma_1)}{\partial \gamma_1} = -\frac{-(1-\theta)l}{2(1-\theta)l + \theta(1-2l) - (1-\theta)(\gamma_1+\gamma_0-1) + (1-\gamma_0)(2\theta-1)} > 0,$$

and

$$\frac{\partial l(\theta;\gamma_0,\gamma_1)}{\partial \gamma_0} = -\frac{\theta(1-l)}{2(1-\theta)l + \theta(1-2l) - (1-\theta)(\gamma_1+\gamma_0-1) + (1-\gamma_0)(2\theta-1)} < 0,$$

it also follows that $\underline{\theta}(\gamma_0,\gamma_1)$ is increasing in γ_1 and decreasing in γ_0. ■

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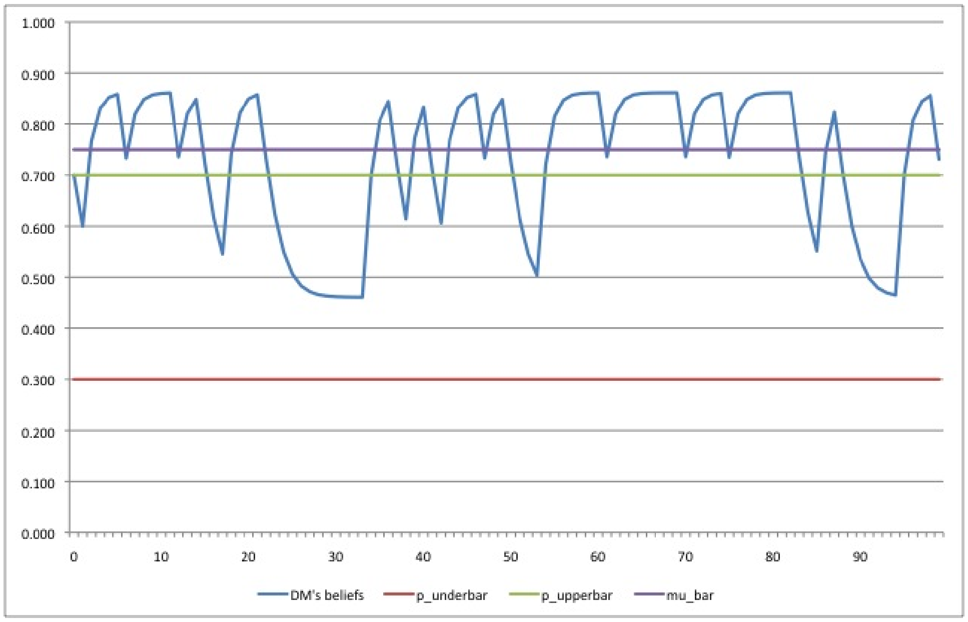

Figure 1: A possible path of the politician's t-period prior beliefs, together with $\overline{\mu}$ (upper purple line), $\overline{p}$ (mid green line) and $\underline{p}$ (lower red line). In this example, $\gamma_0 = 0.7$, $\gamma_1 = 0.9$, and $b = 1/2$.

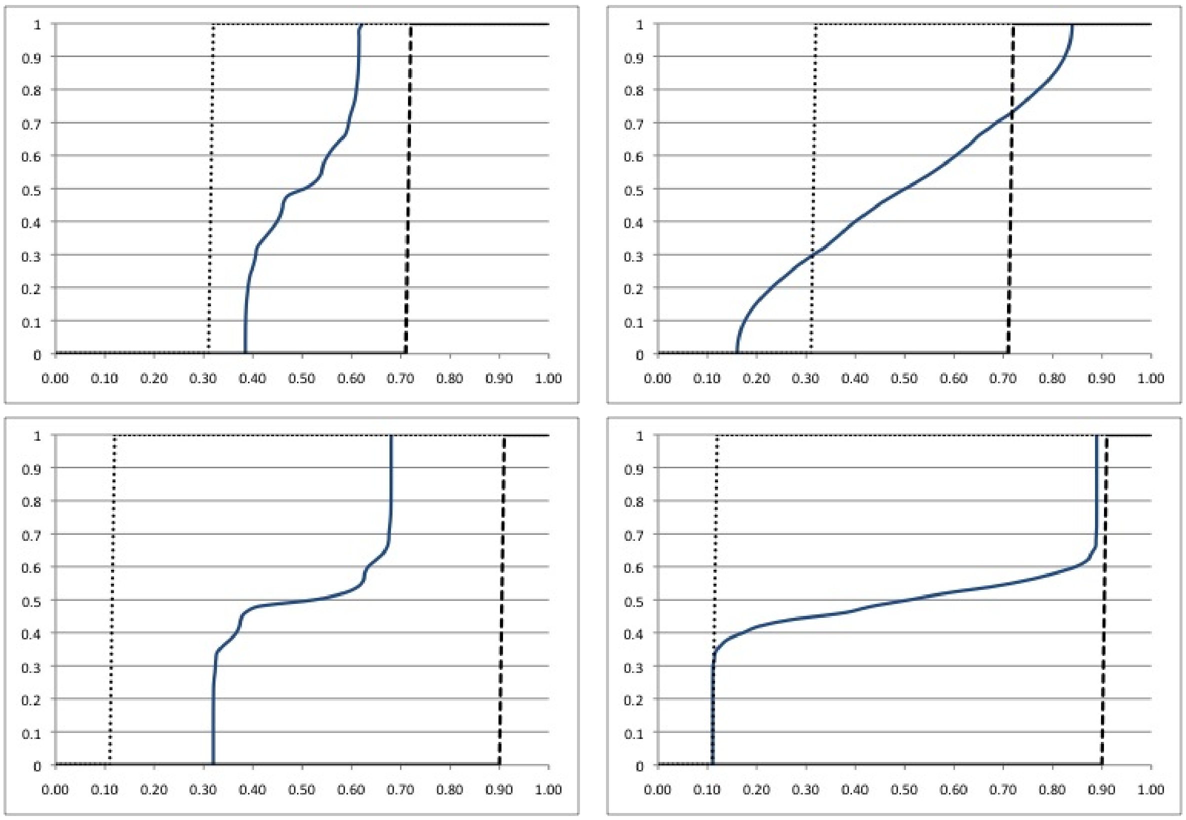

Figure 2: DM's Beliefs Long-Run CDF, and set $P(\theta;\gamma)$ for an unbiased politician ($b = 1/2$). Columns are different (γ_0, γ_1) configurations: $(\gamma_0, \gamma_1) = (0.7, 0.7)$, $(\gamma_0, \gamma_1) = (0.9, 0.9)$, Rows are different θ values: $\theta = 0.7$, $\theta = 0.9$.

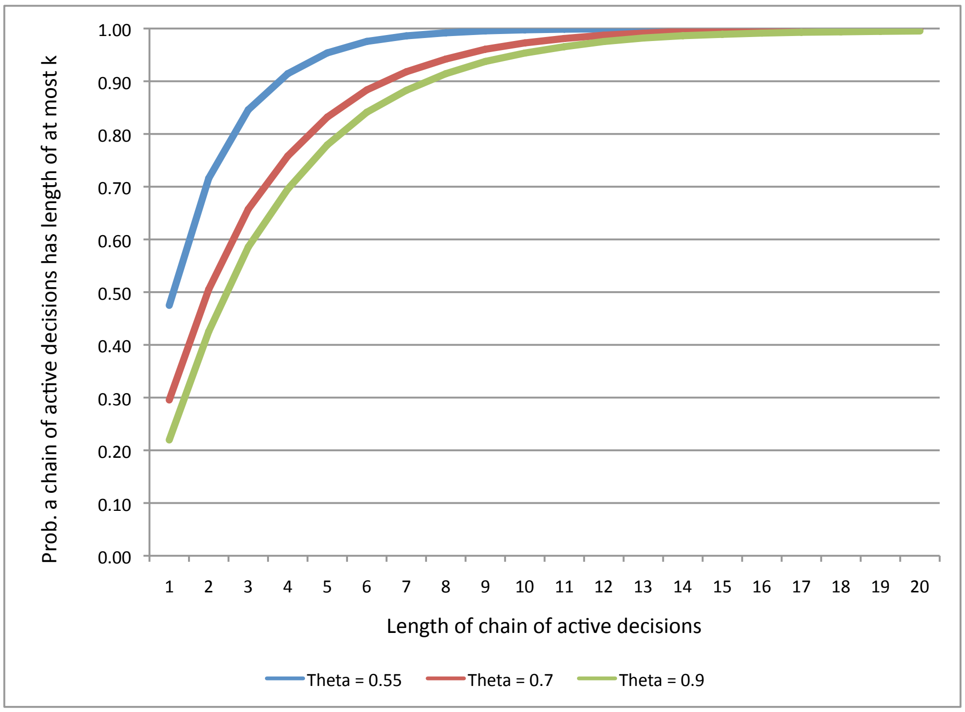

Figure 3: For various ability levels, the figure plots the long-run cumulative distribution functions of the length of chains of active decisions (In the example, $\gamma_0 = \gamma_1 = 0.7$). The distribution of chain lengths for higher ability types FOSD the distribution of chain lengths for lower ability types.