# Entrepreneurial Taxation, Occupational Choice and Credit Market Frictions 

Florian Scheuer

## MIT

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$\rightarrow$ Wages rise, benefits low to medium income workers
$\rightarrow$ Subsidize entrepreneurial effort?


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Key ingredients to model these tradeoffs:
- Heterogeneity in skill and occupational preference
- Endogenous occupational choice
- Entrepreneurs hire workers, endogenous wages


## Preview of Main Results

Baseline model without credit market frictions

- Uniform taxation, treating profits and labor income the same
$\rightarrow$ Manipulate wages to achieve more redistribution (trickle down)
$\rightarrow$ Production distortions


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$\rightarrow$ Direct redistribution (tagging), production efficiency
$\rightarrow$ Compare optimal profit and income tax schedules


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Add investment, borrowing and credit markets
- Adverse selection from private heterogeneity among entrepreneurs
- Credit market equilibrium affects entry into entrepreneurship


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Add investment, borrowing and credit markets
- Adverse selection from private heterogeneity among entrepreneurs
- Credit market equilibrium affects entry into entrepreneurship
$\rightarrow$ Cross-subsidization from high to low-quality borrowers
$\rightarrow$ Excessive (insufficient) entry of low-skill (high-skill) entrepreneurs
$\rightarrow$ Regressive entrepreneurial taxation restores efficient occupational choice


## Related Literature

- Simulating tax reforms in quantitative models with entrepreneurs Meh (2005) (flat tax reform), Cagetti/DeNardi (2009) (estate tax), Kitao (2008), Panousi (2008) (capital tax)
- Optimal savings distortions with entrepreneurial investment Albanesi (2006), (2008), Chari et al. (2002)
- Credit market interventions with adverse selection Stiglitz/Weiss (1976), DeMeza/Webb (1987), Innes (1992), Parker (2003)
- Optimal taxation with endogenous wages Feldstein (1973), Zeckhauser (1977), Allen (1982), Boadway et al. (1991), Parker (1999), Stiglitz (1982), Moresi (1998), Naito (1999)
- Nonlinear taxation with multidimensional private information Kleven et al. (2009)


## Baseline Model

Measure one of heterogeneous individuals with two-dimensional private type

$$
(\theta, \phi) \in[\underline{\theta}, \bar{\theta}] \times\left[0, \bar{\phi}_{\theta}\right], \quad \operatorname{cdf} F(\theta) \text { and } G_{\theta}(\phi)
$$

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Individuals can choose between two occupations:

- Workers supply effective labor I at (endogenous) wage w Quasi-linear preferences

$$
U(c, I, \theta)=c-\psi(I / \theta), \quad \psi(.) \text { increasing, convex }
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- Entrepreneurs hire effective labor $L$ and provide effective effort $E$ Profits

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\pi=Y(L, E)-w L, \quad Y(L, E) \text { is CRS, concave }
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Utility

$$
U(\pi, E, \theta)=\pi-\psi(E / \theta)-\phi
$$

where $\phi$ is a (heterogeneous) cost of becoming an entrepreneur

## Equilibrium without Taxes I

Given $w$, conditional on becoming a worker, type $\theta$ solves max/ $w I-\psi(I / \theta)$ $\rightarrow I^{*}(\theta, w)$ and indirect utility $v_{w}(\theta, w)$

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Critical cost value for occupational choice:

$$
\tilde{\phi}(\theta, w) \equiv v_{E}(\theta, w)-v_{w}(\theta, w)
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All $(\theta, \phi)$ with $\phi \leq \tilde{\phi}(\theta, w)$ become entrepreneurs, the others workers

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## Definition

An equilibrium without taxes is a wage $w^{*}$ and an allocation $\left\{\iota^{*}\left(\theta, w^{*}\right)\right.$, $\left.L^{*}\left(\theta, w^{*}\right), E^{*}\left(\theta, w^{*}\right)\right\}$ such that the labor market clears:

$$
\int_{\Theta} G_{\theta}\left(\tilde{\phi}\left(\theta, w^{*}\right)\right) L^{*}\left(\theta, w^{*}\right) d F(\theta)=\int_{\Theta}\left(1-G_{\theta}\left(\tilde{\phi}\left(\theta, w^{*}\right)\right)\right) I^{*}\left(\theta, w^{*}\right) d F(\theta)
$$

## Equilibrium without Taxes II

Given $E, w$, entrepreneurs of all types $\theta$ solve

$$
\max _{L} Y(L, E)-w L \quad \Rightarrow \quad Y_{L}\left(L^{c}(E, w), E\right)=w
$$

With CRS,

$$
Y\left(L^{c}(E, w), E\right)=Y_{L}\left(L^{c}(E, w), E\right) L^{c}(E, w)+Y_{E}\left(L^{c}(E, w), E\right) E,
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and thus $\pi=Y_{E}\left(L^{c}(E, w), E\right) E$

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$\rightarrow$ Entrepreneurs can be thought of just receiving a different wage $\tilde{w}=Y_{E}$
$\rightarrow$ One-to-one relationship $\tilde{w}(w)$, decreasing

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## Lemma

(i) Any no tax equilibrium involves $\tilde{w}\left(w^{*}\right)>w^{*}$ and $E^{*}\left(\theta, \tilde{w}^{*}\right)>I^{*}\left(\theta, w^{*}\right) \forall \theta$.
(ii) The critical cost value $\tilde{\phi}\left(\theta, w^{*}\right)$ is increasing in $\theta$
(iii) The share of entrepreneurs $G\left(\tilde{\phi}\left(\theta, w^{*}\right)\right)$ is increasing in $\theta$ if, for instance, $G_{\theta^{\prime}}(\phi) \succeq_{\text {FOSD }} G_{\theta}(\phi)$ for $\theta^{\prime} \leq \theta$

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$G_{\theta^{\prime}}(\phi) \succeq_{\text {FOSD }} G_{\theta}(\phi)$ for $\theta^{\prime} \leq \theta$
$G_{\theta}(\phi)$ can produce more general relationships between skill and entrepreneurship

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$\rightarrow$ Entrepreneur of type $\theta$ and worker of type $\theta^{\prime}$ s.t. $\tilde{w} \theta=w \theta^{\prime}$ choose

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\tilde{w} E(\theta)=w l\left(\frac{\tilde{w}}{w} \theta\right) \Rightarrow v_{E}(\theta)=v_{w}\left(\frac{\tilde{w}}{w} \theta\right)
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$$

$\rightarrow$ Cannot discriminate entrepreneurs of skill $\theta$ and workers of skill $(\tilde{w} / w) \theta$

## Constrained Pareto Problem

With Pareto-weights $\tilde{F}(\theta), \tilde{G}_{\theta}(\phi)$, the constrained Pareto problem is
Uniform Taxation

$$
\max _{\substack{E(\theta),,(\theta), 1(\theta)) \\ v_{E}(\theta), v_{w}(\theta), w, \tilde{w}}} \int_{\Theta}\left[\tilde{G}_{\theta}(\tilde{\phi}(\theta)) v_{E}(\theta)+\left(1-\tilde{G}_{\theta}(\tilde{\phi}(\theta))\right) v_{w}(\theta)\right] d \tilde{F}(\theta)-\int_{\theta} \int_{\underline{\phi}}^{\tilde{\phi}(\theta)} \phi d \tilde{G}_{\theta}(\phi) d \tilde{F}(\theta)
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s.t. $\tilde{\phi}(\theta)=v_{E}(\theta)-v_{W}(\theta) \forall \theta \in \Theta$

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\begin{gathered}
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$v_{E}(\theta) \geq v_{E}\left(\theta^{\prime}\right)+\psi\left(E\left(\theta^{\prime}\right) / \theta^{\prime}\right)-\psi\left(E\left(\theta^{\prime}\right) / \theta\right), v_{W}(\theta) \geq v_{W}\left(\theta^{\prime}\right)+\psi\left(/\left(\theta^{\prime}\right) / \theta^{\prime}\right)-\psi\left(/\left(\theta^{\prime}\right) / \theta\right) \forall \theta, \theta^{\prime} \in \Theta$

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\text { s.t. } \tilde{\phi}(\theta)=v_{E}(\theta)-v_{W}(\theta) \forall \theta \in \Theta \\
v_{E}(\theta) \geq v_{E}\left(\theta^{\prime}\right)+\psi\left(E\left(\theta^{\prime}\right) / \theta^{\prime}\right)-\psi\left(E\left(\theta^{\prime}\right) / \theta\right), v_{W}(\theta) \geq v_{W}\left(\theta^{\prime}\right)+\psi\left(l\left(\theta^{\prime}\right) / \theta^{\prime}\right)-\psi\left(l\left(\theta^{\prime}\right) / \theta\right) \forall \theta, \theta^{\prime} \in \Theta \\
 \tag{IC}\\
\int_{\Theta} G_{\theta}(\tilde{\phi}(\theta)) L(\theta) d F(\theta) \leq \int_{\Theta}\left(1-G_{\theta}(\tilde{\phi}(\theta))\right) /(\theta) d F(\theta)
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\int_{\Theta} G_{\theta}(\tilde{\phi}(\theta))\left[Y(L(\theta), E(\theta))-v_{E}(\theta)-\psi(E(\theta) / \theta)\right] d F(\theta) \\
-\int_{\Theta}(1-G(\tilde{\phi}(\theta)))\left[v_{w}(\theta)+\psi(I(\theta) / \theta)\right] d F(\theta) \geq \tag{RC}
\end{gather*}
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\quad-\int_{\Theta}(1-G(\tilde{\phi}(\theta)))\left[v_{w}(\theta)+\psi(I(\theta) / \theta)\right] d F(\theta) \geq \quad \text { (LM) }  \tag{RC}\\
v_{E}(\theta)=v_{W}((\tilde{w} / w) \theta), \quad E(\theta)=(w / \tilde{w}) I((\tilde{w} / w) \theta) \quad \forall \theta \in[\underline{\theta},(w / \tilde{w}) \bar{\theta}] \tag{ND}
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w=Y_{L}(L(\theta), E(\theta)), \quad \tilde{w}=Y_{E}(L(\theta), E(\theta)) \forall \theta \in \Theta \tag{MP}
\end{gather*}
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\text { s.t. } \quad \tilde{\phi}(\theta)=v_{E}(\theta)-v_{W}(\theta) \forall \theta \in \Theta \\
v_{E}^{\prime}(\theta)=E(\theta) \psi^{\prime}(E(\theta) / \theta) / \theta^{2}, \quad v_{W}^{\prime}(\theta)=I(\theta) \psi^{\prime}(I(\theta) / \theta) / \theta^{2}, \quad E(\theta), I(\theta) \text { increasing } \forall \theta \in \Theta \\
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-\int_{\Theta}(1-G(\tilde{\phi}(\theta)))\left[v_{w}(\theta)+\psi(I(\theta) / \theta)\right] d F(\theta) \geq  \tag{RC}\\
v_{E}(\theta)=v_{W}((\tilde{w} / w) \theta), \quad E(\theta)=(w / \tilde{w}) I((\tilde{w} / w) \theta) \quad \forall \theta \in[\underline{\theta},(w / \tilde{w}) \bar{\theta}] \\
w=Y_{L}(L(\theta), E(\theta)), \quad \tilde{w}=Y_{E}(L(\theta), E(\theta)) \forall \theta \in \Theta \quad \text { (ND) } \tag{ND}
\end{gather*}
$$

## Optimal Taxation Results

Pecuniary externality from prices $w$ and $\tilde{w}$ entering program through (ND)

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(iv) E.g. if $\tilde{G} \succeq_{\text {FOSD }} G$ and $\tilde{F}=F$, then $T^{\prime}(w /(\underline{\theta}))>0$ and $T^{\prime}(\tilde{w} E(\bar{\theta}))<0$

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Affect wages to relax (ND) and achieve more redistribution (trickle down)

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Production distortions to relax (ND)
Production efficiency (Diamond/Mirrlees, 1971) not generally optimal

## Differential Income and Profit Taxation

Allow different tax schedules $T_{y}($.$) for labor income y \equiv w /$ and $T_{\pi}($.$) for profits \pi$

## Differential Income and Profit Taxation

Allow different tax schedules $T_{y}($.$) for labor income y \equiv w /$ and $T_{\pi}($.$) for profits \pi$ Relaxed constrained Pareto problem:

## Differential Taxation

$$
\begin{gather*}
\max _{\substack{E(\theta), L(\theta), l(\theta), v_{E}(\theta), v_{W}(\theta)}} \int_{\Theta}\left[\tilde{G}_{\theta}(\tilde{\phi}(\theta)) v_{E}(\theta)+\left(1-\tilde{G}_{\theta}(\tilde{\phi}(\theta))\right) v_{w}(\theta)\right] d \tilde{F}(\theta)-\int_{\Theta} \int_{\underline{\phi}}^{\tilde{\phi}(\theta)} \phi d \tilde{G}_{\theta}(\phi) d \tilde{F}(\theta) \\
\text { s.t. } \quad \tilde{\phi}(\theta)=v_{E}(\theta)-v_{W}(\theta) \forall \theta \in \Theta \\
v_{E}^{\prime}(\theta)=E(\theta) \psi^{\prime}(E(\theta) / \theta) / \theta^{2}, \quad v_{W}^{\prime}(\theta)=I(\theta) \psi^{\prime}(I(\theta) / \theta) / \theta^{2}, E(\theta), I(\theta) \text { increasing } \forall \theta \in \Theta  \tag{IC}\\
\quad \int_{\Theta} G_{\theta}(\tilde{\phi}(\theta)) L(\theta) d F(\theta) \leq \int_{\Theta}\left(1-G_{\theta}(\tilde{\phi}(\theta))\right) I(\theta) d F(\theta)  \tag{LM}\\
\int_{\Theta} G_{\theta}(\tilde{\phi}(\theta))\left[Y(L(\theta), E(\theta))-v_{E}(\theta)-\psi(E(\theta) / \theta)\right] d F(\theta) \\
-\int_{\Theta}(1-G(\tilde{\phi}(\theta)))\left[v_{w}(\theta)+\psi(I(\theta) / \theta)\right] d F(\theta) \geq \quad 0 \quad \text { (RM) } \tag{RC}
\end{gather*}
$$

Same as with uniform taxation, but without constraints (ND) and (MP)

## Optimal Taxation Results I

## Differential taxation eliminates pecuniary externalities

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Observations:

- Production efficiency always optimal


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Observations:

- Production efficiency always optimal
- Optimal tax formulas no longer depend on whether $Y(L, E)$ is linear or not
- In particular, $T_{\pi}^{\prime}(\pi(\underline{\theta}))=T_{\pi}^{\prime}(\pi(\bar{\theta}))=T_{y}^{\prime}(y(\underline{\theta}))=T_{y}^{\prime}(y(\bar{\theta}))=0$ in any case


## Optimal Taxation Results II

## Corollary

With a constant elasticity $\varepsilon$, the average marginal tax across occupations satisfies

$$
G_{\theta}(\tilde{\phi}(\theta)) \frac{T_{\pi}^{\prime}(\pi(\theta))}{1-T_{\pi}^{\prime}(\pi(\theta))}+\left(1-G_{\theta}(\tilde{\phi})\right) \frac{T_{y}^{\prime}(y(\theta))}{1-T_{y}^{\prime}(y(\theta))}=\frac{1+1 / \varepsilon}{\theta f(\theta)}(\tilde{F}(\theta)-F(\theta))
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Average tax in closed form and determined by redistribution across skills only

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## Corollary

The schedules $T_{\pi}, T_{y}$ inducing an allocation $(\pi(\theta), y(\theta), \tilde{\phi}(\theta))$ are Pareto optimal iff

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& \frac{\theta f_{E}(\theta)}{1+1 / \varepsilon} \frac{T_{\pi}^{\prime}(\pi(\theta))}{1-T_{\pi}^{\prime}(\pi(\theta))}+F_{E}(\theta)-\int_{\underline{\theta}}^{\theta} \frac{g_{\hat{\theta}}(\tilde{\phi}(\hat{\theta})) f(\hat{\theta})}{\bar{G}} \Delta T(\hat{\theta}) d \hat{\theta} \quad \text { and } \\
& \frac{\theta f_{W}(\theta)}{1+1 / \varepsilon} \frac{T_{y}^{\prime}(y(\theta))}{1-T_{y}^{\prime}(y(\theta))}+F_{W}(\theta)+\int_{\underline{\theta}}^{\theta} \frac{g_{\hat{\theta}}(\tilde{\phi}(\hat{\theta})) f(\hat{\theta})}{1-\bar{G}} \Delta T(\hat{\theta}) d \hat{\theta}
\end{aligned}
$$

are increasing in $\theta$, where $\bar{G} \equiv \int_{\Theta} G_{\theta}(\tilde{\phi}(\theta)) d F(\theta)$ and
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$$

Only identification of $F(\theta)$ required
Reintepretation: testing for Pareto optimality of $T_{\pi}, T_{y}$

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## Comparing Profit and Income Taxes

## Assumption 1

$\theta$ and $\phi$ are independent and $g(\phi)$ is non-increasing

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Redistribution across cost types

## Proposition

Suppose $\tilde{F}(\theta)=F(\theta)$ and $\tilde{g}(\phi)<g(\phi)$ for all $\phi \leq \tilde{\phi}(\bar{\theta})$. Then (i) $T_{y}^{\prime}(y(\theta))<0, T_{\pi}^{\prime}(\pi(\theta))>0$ for all $\theta \in(\underline{\theta}, \bar{\theta})$

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(iii) Compared with the no tax equilibrium, $w=Y_{L} \downarrow, \tilde{w}=Y_{E} \uparrow, \tilde{\phi}(\theta) \downarrow, L(\theta) \uparrow$

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(iii) Compared with the no tax equilibrium, $w=Y_{L} \downarrow, \tilde{w}=Y_{E} \uparrow, \tilde{\phi}(\theta) \downarrow, L(\theta) \uparrow$

Redistribution across skill types

## Proposition

Suppose that $\tilde{G}(\phi)=G(\phi)$ but $\tilde{F}(\theta) \neq F(\theta)$. If occupations are fixed, then

$$
\frac{T_{\pi}^{\prime}(\pi(\theta))}{1-T_{\pi}^{\prime}(\pi(\theta))}=\frac{T_{y}^{\prime}(y(\theta))}{1-T_{y}^{\prime}(y(\theta))}=\frac{1+1 / \varepsilon}{\theta f(\theta)}(\tilde{F}(\theta)-F(\theta)) \text { for any } w, \tilde{w}
$$

## Numerical Illustration: Calibration I

Data on profits, income and entrepreneurship from 2007 SCF

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Entrepreneur: (i) self-employed, (ii) own business, (iii) actively manage it, (iv) $\geq 2$ employees

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Descriptive Statistics

|  | Entrepreneurs |  | Workers |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Mean | St. Dev. | Mean | St. Dev. |
| Age | 48.4 | 10.2 | 42.1 | 11.6 |
| Yearly Income (in 1000\$) | 88.5 | 234.7 | 69.5 | 128.3 |
| Hours per Week | 48.3 | 14.1 | 43.4 | 10.5 |
| Weeks per Year | 50.2 | 6.0 | 50.4 | 5.7 |
| Wage per Hour (in \$) | 55.5 | 243.8 | 34.6 | 124.9 |

## Numerical Illustration: Calibration II

Constant elasticity disutility of effort $\psi(e)=e^{1+1 / \varepsilon} /(1+1 / \varepsilon)$ with $\varepsilon=.25$ Cobb-Douglas technology $Y(L, E)=L^{\alpha} E^{1-\alpha}$ with $\alpha=.63$ (workers' share of income in SCF data)

## Numerical Illustration: Calibration II

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Identify $f(\theta)$ from empirical income distributions
$\rightarrow$ Impute marginal tax using functional form (Gouveia/Strauss, 1994)

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\begin{equation*}
\frac{T(y)}{y}=b-b\left[s y^{p}+1\right]^{-1 / p} \tag{1}
\end{equation*}
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$\rightarrow w \theta$ and $\tilde{w} \theta$ from

$$
1-T_{y}^{\prime}(y)=\left(\frac{y}{w \theta}\right)^{1 / \varepsilon} \text { and } 1-T_{\pi}^{\prime}(\pi)=\left(\frac{\pi}{\tilde{w} \theta}\right)^{1 / \varepsilon},
$$

$\rightarrow w$ and $\tilde{w}$ such that $\tilde{w} / w$ equals ratio of mean wages of entrepreneurs and workers, and $\tilde{w}=(1-\alpha)(\alpha / w)^{\frac{\alpha}{1-\alpha}}$

## Numerical IIlustration: Calibration III




Kernel estimate of inferred skill density $f(\theta)$, truncated at 99 percentile

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Kernel estimate of inferred skill density $f(\theta)$, truncated at 99 percentile Iso-elastic cost distribution $G_{\theta}(\phi)=\left(\phi / \bar{\phi}_{\theta}\right)^{\eta}$ with $\eta=.5$ Adjust $\bar{\phi}_{\theta}$ to generate the pattern of the share of entrepreneurs in the right panel

## Numerical Illustration: Redistribution Across Cost Types






Pareto weights $\tilde{G}_{\theta}(\phi)=G_{\theta}(\phi)^{\rho_{\Phi}}, \rho_{\Phi}=2$
$\rightarrow$ Redistribution from low to high cost agents (entrepreneurs to workers)
$\rightarrow w$ falls by $10 \%$ as a result of tax policy

## Numerical Illustration: Redistribution Across Skill Types






Pareto weights $\tilde{F}(\theta)=F(\theta)^{1 / \rho_{\Theta}}, \rho_{\Theta}=2$
$\rightarrow$ Redistribution from high to low skill agents
$\rightarrow w$ falls by $3 \%$ as a result of tax policy

## Numerical Illustration: Redistribution Across $\theta$ and $\phi$






Pareto weights $\rho_{\Theta}=2, \rho_{\Phi}=2$
$\rightarrow$ Redistribution in both dimensions
$\rightarrow w$ falls by $12 \%$ as a result of tax policy

## Numerical Illustration: Higher $\varepsilon$



Pareto weights $\rho_{\Theta}=2, \rho_{\Phi}=2$, increased elasticity $\varepsilon=.5$ rather than $\varepsilon=.25$ $\rightarrow$ Lower optimal marginal tax rates

## Introducing Credit Markets

Suppose each entrepreneur has to invest $I$ to set up a firm
Entrepreneurs have no wealth, need to borrow funds from banks in competitive credit market

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\begin{aligned}
& \quad \pi=Y(L)-w L+\epsilon, \quad \epsilon \sim H_{\epsilon}(\epsilon \mid \theta) \\
& H_{\epsilon}(\epsilon \mid \theta) \succeq M L R P H_{\epsilon}\left(\epsilon \mid \theta^{\prime}\right) \text { for } \theta>\theta^{\prime}
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- Banks offer menus of credit contracts that supply funding $I$ in return for repayment schedule $R_{\theta}(\pi)$
$\Rightarrow$ Entrepreneurs' expected utility $\int\left(\pi-R_{\theta}(\pi)\right) d H(\pi \mid \theta)-\phi$
$\Rightarrow$ Given any $\left\{R_{\theta}(\pi)\right\}$, all entrepreneurs hire the same $L$ s.t. $Y_{L}=w$
$\Rightarrow$ Can work with $H(\pi \mid \theta)$ directly, with support $\Pi$


## Credit Market Equilibrium I

## Definition

A credit market equilibrium is a set of contracts $\left\{R_{\theta}(\pi)\right\}$ such that (i)

$$
\begin{equation*}
\int_{\Pi}\left(\pi-R_{\theta}(\pi)\right) d H(\pi \mid \theta) \geq \int_{\Pi}\left(\pi-R_{\theta^{\prime}}(\pi)\right) d H(\pi \mid \theta) \forall \theta, \theta^{\prime} \in \Theta, \tag{IC}
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$$
\begin{equation*}
\int_{\Theta} G(\tilde{\phi}(\theta))\left[\int_{\Pi} R_{\theta}(\pi) d H(\pi \mid \theta)-I\right] d F(\theta) \geq 0 \tag{NNP}
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Note: Allow for arbitrary sets of contracts, thus cross-subsidization not ruled out Restrict to contracts s.t. (i) $0 \leq R_{\theta}(\pi) \leq \pi$ (limited liability), and
(ii) $R_{\theta}(\pi)$ non-decreasing (monotonicity)

## Credit Market Equilibrium II

## Proposition

Under Assumption 1, the credit market equilibrium is such that only the single contract $R^{*}(\pi)=\min \left\{\pi, z^{*}\right\}$ is offered and $z^{*}$ solves

$$
\int_{\Theta} G\left(\tilde{\phi}_{z^{*}}(\theta)\right)\left[\int_{\Pi} \min \left\{\pi, z^{*}\right\} d H(\pi \mid \theta)-I\right] d F(\theta)=0
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- By MLRP, low-skill borrowers have more probability weight in low-profit states
- Debt contracts put the maximal repayment in low-profit states


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The equilibrium is a pooling equilibrium with a standard debt contract offered Intuition:

- By MLRP, low-skill borrowers have more probability weight in low-profit states
- Debt contracts put the maximal repayment in low-profit states
$\rightarrow$ Debt contracts are least attractive to low-skill borrowers
$\rightarrow$ Any deviation would attract a lower quality borrower pool and earn negative profits


## Efficiency of Occupational Choice

Efficiency: type $(\theta, \phi)$ should become entrepreneur if and only if

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\int_{\Pi} \pi d H(\pi \mid \theta)-I-\phi \geq v_{W}
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$\Rightarrow$ Efficient critical cost value $\tilde{\phi}_{e}(\theta)=\int_{\Pi} \pi d H(\pi \mid \theta)-I-v_{W}$

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There exists a skill-type $\tilde{\theta}$ s.t. $\int_{\Pi} \min \left\{\pi, z^{*}\right\} d H(\pi \mid \tilde{\theta})=I$ and

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Cross-subsidization in the credit market leads to occupational misallocation:

- Excessive entry of low-skilled types into entrepreneurship
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Cross-subsidization in the credit market leads to occupational misallocation:

- Excessive entry of low-skilled types into entrepreneurship
- Insufficient entry of high-skilled types
$\Rightarrow$ Too many and too few entrepreneurs simultaneously


## Corrective Tax Policy

## Lemma

If the profit $\operatorname{tax} T(\pi)$ is such that after-tax profits $\hat{\pi} \equiv \pi-T(\pi)$ are increasing, then the credit market equilibrium given $T(\pi)$ is a single debt contract $R_{z_{T}^{*}}(\hat{\pi})=\min \left\{\hat{\pi}, z_{T}^{*}\right\}$, where $z_{T}^{*}$ solves

$$
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and $\tilde{\phi}_{z_{T}^{*}, T}(\theta) \equiv \int_{\Pi}\left(\pi-T(\pi)-\min \left\{\pi-T(\pi), z_{T}^{*}\right\}\right) d H(\pi \mid \theta)-v_{W} \forall \theta \in \Theta$.

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## Proposition

If the tax policy $T(\pi)$ is introduced such that $\pi-T(\pi)$ is increasing and, for all $\theta \in \Theta$,

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(i) the resulting credit market equilibrium is s.t. $\tilde{\phi}_{z_{T}^{*}, T}(\theta)=\tilde{\phi}_{e}(\theta)$ for all $\theta \in \Theta$, (ii) the gov't budget is balanced.

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Regressive profit tax counteracts cross-subsidization and restores efficiency

## Conclusion

Uniform profit and income taxation...

- ... provides some justification for trickle down based arguments
- ... calls for additional tax distortions, e.g. on inputs

Role of differential profit and income taxation in ...

- ... removing pecuniary externalities from uniform taxation
- ... correcting inefficient sorting into occupations with credit market frictions

