When Optimists Need Credit: Asymmetric Filtering of Optimism and Implications for Asset Prices

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Belief heterogeneity: a contributing factor to the crisis?

- Belief heterogeneity is a potential cause for increase in asset prices in the housing and complex security markets.
- Optimists need to **leverage** their investments by borrowing from moderate lenders using loans collateralized by the asset itself (e.g., mortgages, REPOs).
- Moderate lenders value the collateral (the asset) less, and thus they might be reluctant to lend.
  Endogenous constraint on optimists' ability to leverage.

This research characterizes the **types of optimism** that are disciplined by this constraint.

Main result: optimism is asymmetrically filtered

Example: a single risky asset, three future states: G, N, B.

- Moderate lenders believe each state realized with equal probability.
- Optimists borrow using collateralized debt contracts.
- Baseline setting: debt contracts simple (non-contingent) and short selling not allowed.

Main result: optimism is asymmetrically filtered

Example: a single risky asset, three future states: G, N, B.

- Moderate lenders believe each state realized with equal probability.
- Optimists borrow using collateralized debt contracts.
- Baseline setting: debt contracts simple (non-contingent) and short selling not allowed.
- **Two types of optimism:**

  Case (i): Optimists believe probability of B is less than $1/3$. $\Longrightarrow$ Price closer to moderate valuation.

  Case (ii): Optimists believe probability of B is $1/3$. They believe probability of G is more than probability of N. $\Longrightarrow$ Price closer to optimistic valuation.

Optimism is asymmetrically filtered by financial constraints. What investors disagree about matters.

Asymmetric filtering is due to asymmetry of debt contract payoffs

- Debt contract payoffs are asymmetric: default and losses in bad states.
- Disagreement about bad states $\Longrightarrow$ disagreement about how to value debt contracts $\Longrightarrow$ tighter constraints.

More specifically:

- Loans trade at an interest rate **spread** that compensates lenders for expected losses according to their **moderate beliefs**.

  Case (i): Disagreement about B. Spread appears too high to optimists. Discouraged from leveraging. Low demand and low price.

  Case (ii): Agreement about B. Spread appears normal to optimists. Enticed to leverage. High demand and high price.

Asymmetric filtering result is robust to allowing for contingent contracts and short selling

Extension with contingent contracts:

- Optimal contingent contract takes a threshold form. Zero payment in states above threshold.
  $\Longrightarrow$ A version of asymmetric filtering applies.
- **Price may exceed the valuation of the most optimistic investor.**

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Extension with short selling: Short selling reduces overvaluation in general, but less so when belief heterogeneity is about good states.

- Short selling is collateralized. Short contract payoffs are asymmetric: default and fixed payment in states above a threshold.
  $\Longrightarrow$ **Asymmetric filtering of pessimism:** Pessimism about good states reduces asset prices less than pessimism about bad states. Complements asymmetric filtering of optimism.

Speculative bubbles are also asymmetrically filtered

- **Harrison and Kreps (1978):** Belief heterogeneity and no short selling. Identity of optimists changes over time.

  $\Longrightarrow$ Price exceeds the pdv of the most optimistic investor: **Speculative bubble (Scheinkman and Xiong, 2003)**.

- **This research** introduces financial constraints.

  Case (i): Speculative bubble filtered by constraints.

  Case (ii): Speculative bubble survives constraints. A large bubble forms even if optimists have limited wealth.

Environments with uncertainty and disagreement about upside returns are conducive to bubbles financed by credit.

Closely related literature: closest work is by Geanakoplos

- **Endogenous leverage with belief heterogeneity: Geanakoplos** (2003, **2009**), Fostel and Geanakoplos (2008)...
- **Overvaluation hypothesis:** Miller (1977), Chen, Hong and Stein (2002), Diether, Malloy and Scherbina (2002), Ofek and Richardson (2003)...
- **Speculative bubbles:** Harrison and Kreps (1978), Morris (1996), Scheinkman and Xiong (2002)...

Outline of the talk

Baseline static model:

- Characterization of collateral equilibrium and asset prices. **Optimism is asymmetrically filtered**.
- Comparative statics of type and level of belief heterogeneity. **What investors disagree about matters for asset prices, to a greater extent than how much they disagree.**

Extension with contingent contracts:

- **Asymmetric filtering is robust to allowing for more general contracts.**

Basic environment: belief heterogeneity about asset returns

- Simple asset pricing model: one consumption good, two periods.
- Risk neutral traders have endowments in first period but consume in second.
- Resources invested in two ways:
  - Bond B supplied elastically at normalized price 1. Each unit pays $1 + r$ in second period.
  - **Asset** A in fixed supply (of one unit), yields dividends in period two, trades at price p.
- Future state $s \in \mathcal{S} = \left[s^{\min}, s^{\max}\right]$. Asset pays $v(s)$ units of consumption good in state s.
- Two types of traders: **optimists** (subscript 1) and **moderates** (subscript 0), respectively with belief distributions F_1 and F_0 over $\mathcal{S}$.

Heterogeneous priors: investors **agree to disagree**.

Optimism notion: The better the event, the greater the optimism

- **Notion of optimism** related to upper-threshold events $[\bar{s}, s^{\max}]$.

Definition

Distribution $\tilde{H}$ is **more optimistic** than H, denoted by $\tilde{H} \succ_O H$, iff $\frac{1-\tilde{H}(\bar{s})}{1-H(\bar{s})}$ is strictly increasing over $\bar{s} \in \left(s^{\min}, s^{\max}\right)$, equivalently iff

$$\frac{\tilde{h}\left(\bar{s}\right)}{1-\tilde{H}\left(\bar{s}\right)} < \frac{h\left(\bar{s}\right)}{1-H\left(\bar{s}\right)} \text{ for } \bar{s} \in \left(s^{\min}, s^{\max}\right).$$

- **Assumption (O):** $F_1 \succ_O F_0$.

Implies: $E_1\left[v\left(s\right)\right] > E_0\left[v\left(s\right)\right]$.

- **Assumption (S):** Asset A cannot be short sold.

Asset price depends on optimists' financial conditions

Asset price will satisfy

$$p \in \left[\frac{E_0 \left[v\left(s\right) \right]}{1+r}, \frac{E_1 \left[v\left(s\right) \right]}{1+r} \right].$$

Exact location depends on optimists' endowments and financial constraints.

- Endowments: (w_1, w_0) of the consumption good, $(\alpha_1, \alpha_0 = 1 - \alpha_1)$ units of the asset.
- **Financial constraints are microfounded through a collateralized loan market.**

Loans are collateralized and non-contingent

- Loans are **collateralized:** debt contract is **[promise, collateral]** pair.
- Loans are **no recourse:** Payment enforced by collateral.
- Loans are **non-contingent** in the baseline model**:** same promise in all future states.
    - Allow for different levels of promise per collateral (loan to value ratio is endogenous).

Collateralized loan market is analyzed with a competitive equilibrium notion

- A **unit debt contract** $\varphi \in \mathbb{R}_+$ is a promise of φ units by the borrower, collateralized by 1 unit of the asset.
- Contract φ defaults iff $v(s) < \varphi$. Thus, it pays

$$\min(v(s), \varphi).$$

- Contract φ traded in an anonymous market at a competitive price $q(\varphi)$ (Geanakoplos and Zame 1997, 2009).

Detour, mapping debt contracts to loans:

- Contract $\varphi = v(\bar{s})$ for some $\bar{s} \in \mathcal{S}$ is a **loan with riskiness** $\bar{s}$.
- **Loan size:** $q(v(\bar{s}))$. **Interest rate on the loan:** $\frac{v(\bar{s}) - q(v(\bar{s}))}{q(v(\bar{s}))}$.
- A menu of loans with different size (and riskiness) are traded at competitive interest rates.

Definition of collateral equilibrium

- Type i traders choose asset and bond holdings $x_i = \left(x_i^A, x_i^B\right) \geq 0$ and debt positions $\left(z_i\left(\varphi\right)\right)_\varphi$ to maximize **expected payoffs** subject to:
  Budget constraint:
  $$px_i^A + x_i^B + \int_{\mathbb{R}_+} q\left(\varphi\right) z_i\left(\varphi\right) d\varphi \leq w_i + p\alpha_i.$$
  Collateral constraint:
  $$\int_{\mathbb{R}_+} \max\left(0, -z_i\left(\varphi\right)\right) d\varphi \leq x_i^A.$$

Collateral Equilibrium is a collection of prices $\left(p, \left[q\left(\varphi\right)\right]_{\varphi\in\mathbb{R}_+}\right)$ and allocations $\left(x_i^A, x_i^B,\ z_i\left(\cdot\right)\right)_{i\in\{1,0\}}$ such that traders choose allocations optimally, and asset and debt markets clear, that is, $\sum_{i\in\{1,0\}} x_i^A = 1$ and $\sum_{i\in\{1,0\}} z_i\left(\varphi\right) = 0$ for each $\varphi \in \mathbb{R}_+$.

Equilibrium is characterized in three steps

- There exists an (essentially unique) equilibrium in which optimists borrow and moderates lend.
- Construct an equilibrium in three steps:

1. **Lender side:** (as long as $p > \frac{E_0[v(s)]}{1+r}$) Consider contract prices

$$q(\varphi) = \frac{E_0 [\min (v(s), \varphi)]}{1+r} \text{ for each } \varphi \in \mathbb{R}_+.$$

2. **Borrower side:** (as long as $p < \frac{E_1[v(s)]}{1+r}$) optimists' collateral constraint binds. They invest all of their leveraged wealth in the asset.

   Next: Characterize optimists' optimal contract choice given p.

3. **Asset market clearing**: Solve for equilibrium price p.

Optimists' loan choice gives the asymmetric filtering result

Theorem (Asymmetric Filtering)

Suppose asset price is given by $p \in \left(\frac{E_0[v(s)]}{1+r}, \frac{E_1[v(s)]}{1+r} \right)$.

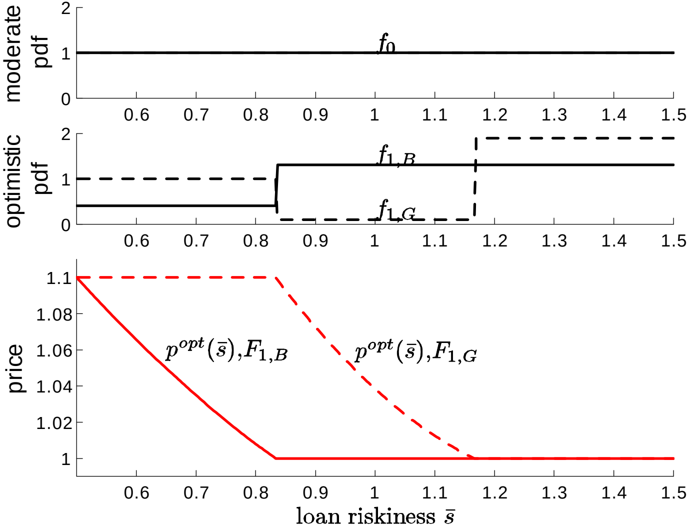
(i) There exists $\bar{s} \in \mathcal{S}$ *such that optimists only sell the debt contract* $\varphi = v(\bar{s})$*, i.e., they borrow according to a single loan with riskiness* $\bar{s}$.

(ii) The riskiness $\bar{s}$ *of the optimal loan is the unique solution to:*

$$\begin{aligned} p &= p^{opt}(\bar{s}) \\ &\equiv \frac{1}{1+r} \left(F_0(\bar{s}) E_0 [v(s) \mid s < \bar{s}] + (1 - F_0(\bar{s})) E_1 [v(s) \mid s \geq \bar{s}] \right). \end{aligned}$$

- $p^{opt}(\bar{s})$ is like an inverse demand function: decreasing in $\bar{s}$.
- **Asymmetric filtering result:** $p^{opt}(\bar{s})$ characterizes the asset price conditional on equilibrium loan riskiness $\bar{s}$.

Illustration of optimal loan and asymmetric filtering



Optimists' trade-off is between more leverage and greater borrowing costs

- Optimists choose $\bar{s}$ that maximizes the **leveraged investment return**:

$$\frac{E_1 [v(s)] - E_1 [\min(v(s), v(\bar{s}))]}{p - \frac{1}{1+r} E_0 [\min(v(s), v(\bar{s}))]}.$$

- The condition $p = p^{opt}(\bar{s})$ is the first order condition for this problem.

Optimists' trade-off features two forces:

1. Greater $\bar{s}$ allows to leverage the unleveraged return:

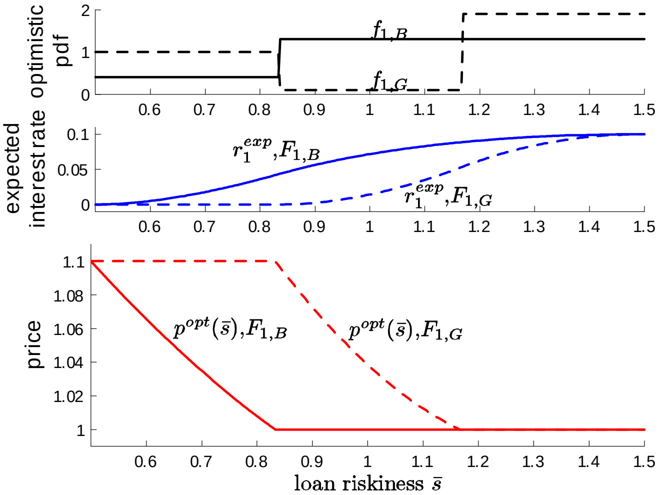
$$R^U \equiv \frac{E_1 [v(s)]}{p} > 1 + r.$$

2. Greater $\bar{s}$ comes at a greater cost. Optimists' **expected interest rate**

$$1 + r_1^{exp}(\bar{s}) \equiv \frac{E_1 [\min(v(s), v(\bar{s}))]}{\frac{1}{1+r} E_0 [\min(v(s), v(\bar{s}))]}$$

is greater than $1 + r$ and strictly increasing in $\bar{s}$.

Intuition for the asymmetric filtering result



Equilibrium price is determined by asset market clearing

- Suppose optimists borrow with a loan with riskiness $\bar{s}$. The **maximum first period consumption good** optimists can obtain is: $w_1^{\max}(\bar{s}) = w_1 + \frac{1}{1+r} E_0\left[\min\left(v(s), v(\bar{s})\right)\right]$.
- Market clearing price depends on comparison of $w_1^{\max}(\bar{s})$ and $\alpha_0 p$:

$$p = p^{mc}(\bar{s}) \equiv \begin{cases} \frac{E_1[v(s)]}{1+r} & \text{if } \frac{w_1^{\max}(\bar{s})}{\alpha_0} > \frac{E_1[v(s)]}{1+r} \\ \frac{w_1^{\max}(\bar{s})}{\alpha_0} & \text{if } \frac{w_1^{\max}(\bar{s})}{\alpha_0} \in \left(\frac{E_0[v(s)]}{1+r}, \frac{E_1[v(s)]}{1+r}\right] \\ \frac{E_0[v(s)]}{1+r} & \text{if } \frac{w_1^{\max}(\bar{s})}{\alpha_0} \leq \frac{E_0[v(s)]}{1+r} \end{cases}.$$

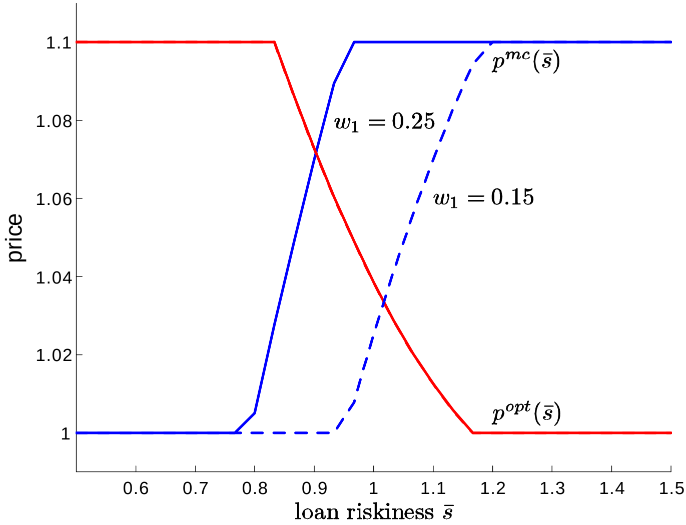
Theorem (Existence and Essential Uniqueness)

There exists a collateral equilibrium with asset price p and loan riskiness $\bar{s}^*$ characterized as the solution to

$$p = p^{mc}(\bar{s}) = p^{opt}(\bar{s}).$$

In any collateral equilibrium, asset price p is uniquely determined.

Illustration of collateral equilibrium



Skewness of optimism is formalized by single crossing of hazard rates

Consider the comparative statics of $p, \bar{s}^*$ and the leverage ratio

$$L \equiv \frac{p}{p - \frac{1}{1+r} E_0 \left[\min \left(v\left(s\right), v\left(\bar{s}^*\right)\right)\right]}$$

with respect to the **type** and the **level** of belief heterogeneity.

Definition

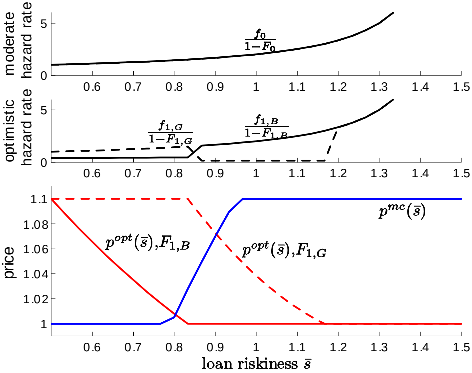
The optimism of $\tilde{F}_1$ is *weakly more right-skewed* than F_1, denoted by $\tilde{F}_1 \succeq_R F_1$, if $E\left[v\left(s\right) \;;\; \tilde{F}_1\right] = E\left[v\left(s\right) \;;\; F_1\right]$ and there exists $s^R \in \mathcal{S}$ such that:

$$\begin{cases} \frac{\tilde{f}_1(s)}{1-\tilde{F}_1(s)} \geq \frac{f_1(s)}{1-F_1(s)} & \text{if } s < s^R, \\ \frac{\tilde{f}(s)}{1-\tilde{F}_1(s)} \leq \frac{f_1(s)}{1-F_1(s)} & \text{if } s > s^R. \end{cases}$$

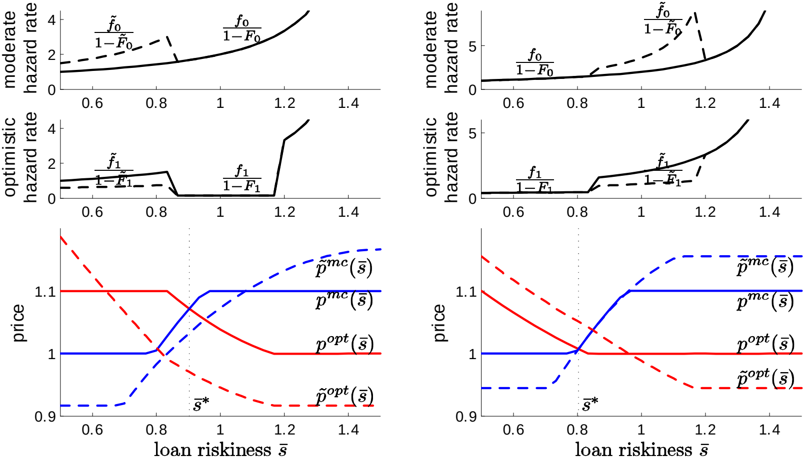
What investors disagree about matters for asset prices

Theorem

If optimists' prior is changed to $\tilde{F}_1$ that satisfies $\tilde{F}_1 \succeq_R F_1$, then: the asset price p, the loan riskiness $\bar{s}^*$, and the leverage ratio L weakly increase.



Level of disagreement has ambiguous effects



Ambiguity is resolved by controlling for the type of the additional disagreement

Theorem

Let $\bar{s}^*$ denote the equilibrium loan riskiness. Suppose beliefs are changed to $\tilde{F}_1$ and $\tilde{F}_0$ that satisfy $\tilde{F}_1 \succeq_O F_1$ and $F_0 \succeq_O \tilde{F}_0$:

(i) Suppose $\frac{\tilde{f}_i(\bar{s})}{1-\tilde{F}_i(\bar{s})} = \frac{f_i(\bar{s})}{1-F_i(\bar{s})}$ over $\bar{s} \in \left(s^{\min}, \bar{s}^*\right)$. Then the asset price p, the loan riskiness $\bar{s}^*$, and the leverage ratio L weakly increase.

(ii) Suppose $\frac{\tilde{f}_i(\bar{s})}{1-\tilde{F}_i(\bar{s})} = \frac{f_i(\bar{s})}{1-F_i(\bar{s})}$ over $\bar{s} \in (\bar{s}^*, s^{\max})$. Then the asset price p weakly decreases.

What investors disagree about is a more robust predictor of the asset price than the level of the disagreement.

A version of asymmetric filtering applies with contingent loans

- A **unit contingent debt contract**, denoted by $\varphi : \mathcal{S} \to \mathbb{R}_+$, is a collection of promises $(\varphi(s))_{s\in\mathcal{S}}$, collateralized by 1 unit of the asset.
- Equilibrium defined similarly. Characterized under:

Assumption (MLRP). $\frac{f_1(s)}{f_0(s)}$ is strictly increasing over $\mathcal{S}$.

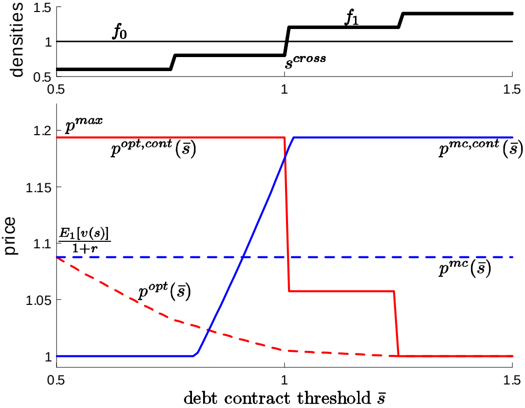
- Optimal contingent loan takes a threshold form:
  $$\varphi_{\bar{s}}(s) \equiv \begin{cases} v(s) & \text{if } s < \bar{s} \\ 0 & \text{if } s \geq \bar{s}. \end{cases}$$
- For each $p \in \left(\frac{E_0[v(s)]}{1+r}, p^{\max}\right)$, the threshold $\bar{s}$ is the solution to:

$$p = p^{opt,cont}(\bar{s}) \equiv \frac{1}{1+r}\left(\int_{s^{\min}}^{\bar{s}} v(s)\,dF_0 + \frac{f_0(\bar{s})}{f_1(\bar{s})}\int_{\bar{s}}^{s^{\max}} v(s)\,dF_1\right).$$

- Optimism about relative likelihood of states above $\bar{s}$ increases the price. Optimism about relative likelihood of states below $\bar{s}$ does not.

With contingent loans, the price can exceed the most optimistic valuation

Maximum price at which optimists demand the asset exceeds the optimistic valuation. It is given by $p^{\max} = \frac{1}{1+r} \int_{s^{\min}}^{s^{\max}} v(s) \max(f_0(s), f_1(s))\, ds$.



Conclusion

- Theoretical analysis of effect of belief heterogeneity on asset prices.
- Endogenous constraint on lending due to belief heterogeneity.
- Focus: Types of belief heterogeneity that are disciplined by this constraint.
- Result: **Asymmetric filtering of optimism.**

Economic environments with considerable uncertainty about the upside returns are conducive to asset price increases (and bubbles) financed by credit.

Dynamic extension to analyze speculative bubbles

- Infinite horizon OLG economy with single consumption good. Periods and generations $n \in \{0, 1, ...\}$.
- Generation n traders have endowments in period n, but consume in period $n + 1$.
- Resources invested in two ways: Bond B and **Asset** A, as before.
- Asset yields a_n in each period n.
- Log dividends follow a random walk:

$$a_{n+1} = a_n s_{n+1},$$

where s_{n+1} has distribution F_{true} over $\mathcal{S}$, with $E_{true}[s_{n+1}] = 1$.

Belief heterogeneity is about the next period shock

- **Optimists** and **moderates** in each generation n, respectively with belief distributions F_1 and F_0 about s_{n+1}.

Assumption ($\mathbf{O_d}$). $F_0 = F_{true}$ and $F_1 \succ_O F_0$, with $E_1[s_{n+1}] = 1 + \varepsilon$. In addition, traders' beliefs for $\{s_{n+k}\}_{k=2}^{\infty}$ are identical and given by F_{true}.

- Present discounted valuations:

$$p_0^{pdv}(a_n) \equiv \frac{a_n}{r} \text{ and } p_1^{pdv}(a_n) \equiv \frac{a_n(1+\varepsilon)}{r}.$$

Without financial constraints, speculative bubbles form

- **Lemma:** Current dividend realization $a \equiv a_n$ is a sufficient statistic. Denote the next period shock with $s \equiv s_{n+1}$.
- Recursive equation:

$$p(a) = \frac{1}{1+r}\left(a(1+\varepsilon) + \int_{\mathcal{S}} p(as)\, dF_1\right).$$

- Solution:

$$p(a) = \frac{a(1+\varepsilon)}{r-\varepsilon} > p_1^{pdv}(a) = \frac{a(1+\varepsilon)}{r}.$$

- **Speculative bubble:**

$$\lambda = \frac{p(a) - p_1^{pdv}(a)}{p(a)} = \frac{\varepsilon}{r}.$$

Financial constraints are as in the two period model

- **Loan market in each period** is same as in the static model given value function $v(a, \cdot)$ defined by:

$$v(a, s) \equiv as + p(as) \text{ for each } s.$$

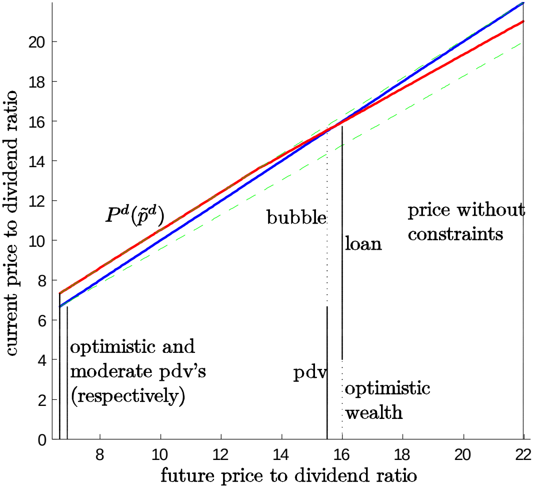
Dynamic Equilibrium: collection of prices $\left(p(a), [q(a, \varphi)]_{\varphi}\right)_a$ and allocations $\left(\left\{x_i(a), [z_i(a, \varphi)]_{\varphi}\right\}_i\right)_a$ such that, for each a, traders' allocations are optimal and markets clear.

- Equilibrium characterized by a fixed point argument.
- Assume endowments are given by $w_i = \omega_i a$.

Theorem

There exists a dynamic equilibrium with $p(a) = p_d a$ and $\bar{s}^*(a) = \bar{s}_d^*$ for each $a \in \mathbb{R}_{++}$.

Speculative bubbles may form even if optimists have a small amount of wealth



Speculative bubbles are also asymmetrically filtered

Theorem

If optimists' prior is changed to $\tilde{F}_1$ that satisfies $\tilde{F}_1 \succeq_R F_1$, then: the price to dividend ratio p_d, the loan riskiness $\bar{s}_d^*$, and the share of the speculative component λ_d weakly increase.