Lotteries

Determinants of ticket sales and the optimal payout rate

SUMMARY

This paper considers policy issues arising in the design, regulation and taxation of lotteries, focusing on the market for an on-line lottery game. Demand determines who buys lottery tickets and in what quantities. The design of lotteries affects the terms on which tickets are supplied.

UK data suggest that its lottery may be priced too high to maximize lottery revenue – more revenue might be raised if the proportion of sales allocated to tax and other levies were smaller.

Having established the positive economics of lotteries, the paper then assesses their welfare implications. Pari-mutuel lotteries enjoy scale economies and, as natural monopolies, are invariably run either by government agencies or a regulated licensee. I estimate consumer surplus and identify the excess burden that arises from existing (over)taxation of lotteries. The large price elasticity of demand implies that revenue raised from the lottery is raised very inefficiently. Moreover, the demand for lottery tickets is inferior (and there is some evidence that such games are contagious and addictive). So using lotteries as a vehicle for raising revenue is extremely regressive.

Finally, I consider other policy implications: induced effects on charitable giving and on other forms of gambling; the impact on the government budget; perceptions of risk; and distributional considerations.

— *Ian Walker*

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The economic analysis of lotteries

Ian Walker

Keele University

1. INTRODUCTION

Many countries have lotteries, usually making considerable surpluses. Often, these are state-owned monopolies whose surpluses accrue as tax revenue; in other cases, they are privately operated but regulated, with tax (and other deductions) being a contractually specified proportion of revenue.[1] Either way, lotteries often reflect government difficulties in raising revenue through more conventional means.

Some of the largest lotteries are those in Spain, Ireland, Canada, the UK and several Australian and US states. Early US lotteries contributed to such causes as the

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[1] Munting (1996) offers a historical overview of the development of gambling in the UK and USA; Cook and Clotfelter (1993) describe lotteries in the contemporary USA. Ziemba (1986) described Canadian data relating to a 6/49 design (6 balls drawn from 49 without replacement). Fitzherbert (1996, 1997) and Creigh-Tyte and Farrell (1997) describe the UK National Lottery, further details of which may be found on the Web site http://www.national-lottery.co.uk/ of the licensed operator Camelot.

defence of (Quaker) Pennsylvania, and to Harvard and Yale Universities. Lotteries in Australia helped to fund the construction of the Sydney Opera House and the Sydney Harbour Bridge. The first UK lottery in 1566 raised funds to fight the French; until 1820 funds raised through UK lotteries supported public works projects such as the British Museum (1753) and Westminster Bridge (1694–1768); after 1826, lotteries were outlawed in the UK as a result of malpractice and corruption. In the USA phantom lotteries in Boston and bribes paid to secure the franchise in Louisiana also led to the banning of lotteries in the late nineteenth century. However, lottery games in mainland Europe continued to operate: the Italian National Lottery has operated continuously since 1530; El Gordo is an annual draw, of enormous proportions, in Spain's Loteria de Navidad, which has operated since 1763; and Germany has had city lotteries for more than 500 years and a national game for over 40 years.

During 1930–4 medical care in the Republic of Ireland was funded in part by the Irish Sweepstake (where the winning ticket was selected by UK horse race results), which raised most of its revenue from sales in the UK. More recently, the UK has operated an 'interest-lottery', where stakes are deposited in a fund, the interest from which is returned to players (who hold 'Premium Bonds') via a monthly electronic random draw.[2] Small lotteries have also been operated by charities to raise funds, and by local governments to fund local public goods, although these dwindled during the 1970s. Pressure for a more conventional national lottery arose from the potential revenue drain from foreign lotteries that could potentially have been available in the UK. Claims that EU membership left the UK vulnerable to non-UK lotteries were prominent in the arguments for the introduction of the UK National Lottery in 1994.

Since the mid-1960s there has also been a dramatic resurgence in lotteries in North America. The first modern state lottery in 1964 was in New Hampshire, a state without the power to raise a state income tax; and now 38 US states have lotteries, most with more than one game type. The early games were essentially numbers games, with players receiving a numbered ticket. Draws were infrequent and tickets highly priced, available only at restricted outlets. The first game in which players chose their own numbers was in New Jersey in 1975. This game featured cheap tickets and weekly draws. Lotto (where players chose the numbers) was introduced in New York in 1978. Computerization greatly boosted lotto, since it works best when number choices are recorded and the winners can be quickly identified. Australia and Canada have several games in each state/province. Most European countries also have some form of game; lotto is increasingly popular. Cross-border sales are often illegal, although this is impossible to enforce. This has

[2] Former UK Prime Minister Harold Macmillan denied it was a lottery, since the capital was never lost; Harold Wilson, a subsequent Prime Minister, called it a 'squalid raffle'. Nonetheless, a similar financial product has recently been launched in Spain, partly to help meet Maastricht's budgetary criteria.

led to separate games in each country (except Luxembourg, which allows two German lotteries to be sold). For example, France introduced a national draw in 1978, Ireland in 1987 and the UK in 1994.

Early US games were legal replicas of the numbers rackets: numbered tickets were bought and a single winner was drawn. Game design for illegal games was invariably in the form of a fixed cash prize, since there was no credible way to verify the level of sales that would have allowed prizes to be proportionate to sales. Raffles, in contrast, typically have a fixed non-cash prize. Both numbers games and raffles are risky for their organizers, since the prize is independent of the level of sales. Thus, participants are playing against 'the house' (the operator). The organizers need to be sure that the sales revenue will cover both their costs and the promised fixed prizes; hence, the 'take-out' rate (the proportion of revenue retained by the operator and not distributed as prizes) needs to embody a risk premium (in the absence of an insurance market). The required risk premium and take-out rate may be so high that few would wish to play. The equilibrium may be that no games exist.[3] Although lotteries with repeated draws should be able to spread risks across time, the risk of operating at a loss can be eliminated or considerably reduced in games of 'lotto' and 'scratchcards'. These two products now dominate the market.[4]

Lotto is a pari-mutuel lottery such that prizes are a *share* of sales revenue and players have the possibility of choosing their own numbers, a feature thought to promote sales, without any risk that prizes will exceed sales revenue. In lotto, participants play against each other for the prize pool and the house simply takes its risk-free commission off-the-top. However, players now face a risky prize structure, since the level of the prizes depends on overall sales and on the number of players who have chosen the same winning numbers. When the 'jackpot' prize pool has no winners, it is added to the jackpot pool of the following draw (a 'rollover' or 'carry-over'). A sequence of rollovers may generate extremely large jackpot pools that promote sales.

Scratchcards are tickets that conceal symbols revealed by scratching the surface; prizes are awarded for tickets with specific combinations of symbols. Scratchcards are not pari-mutuel, but still involve little risk for the operator: they offer a small number of relatively modest top prizes and many small prizes, and the operator continues to sell cards until the last top prize is sold or until few remain. Thus, the risk of selling insufficient cards to cover the prizes is much smaller than when there is one large prize.

[3] The operator in British Columbia runs a lottery where individuals can choose the numbers on their tickets *and* the prizes are fixed amounts, which is very risky for the operator because there could be multiple payouts of the fixed prize. This leads lottery operators, even public sector operators, to want to 'insure the tail' of the expected payout distribution. Specialized insurance is now available.

[4] Newer developments are 'Keno' and 'video lottery terminals'. Keno is a bingo-style lottery with high-frequency draws, where players match 15 from 20 drawn from 80. VLTs are slot machines offering fixed-prize lottery draws on demand. Both are 'hard' gambling lotteries, in the sense that one can play repeatedly within a short space of time.

Why are lotteries proliferating, not only in countries such as the USA, where other gambling had been illegal in most places, but also in countries such as the UK with a long history of legalized gambling markets? The pressure to operate or license lotteries reflects a desire to raise government revenue in new ways because of increased demands for state expenditure, taxpayer revolts over conventional sources of revenue, and pressure to balance budgets. These pressures are exacerbated by competition from neighbouring states: introduction of a lottery in a neighbouring state can induce a fall in tax revenue, so states may compete to introduce lotteries.[5]

As shown below, pari-mutuel lotteries exhibit scale economies, providing a motive for small states/provinces/countries to collaborate in a single cross-state game. Thus, there may be too many state-organized games in the USA, each of which may be too small. Indeed a lotto game called 'Powerball' is available across Australia; 'Tri-State' is a lotto game sold in three of the small north-eastern US states; and a single lotto game is licensed across all Canadian provinces.

The revenue from lottery games is either used as general revenue or 'hypothecated' for specific purposes: education and health spending, cultural activities, repayment of debts (e.g. Montreal's Expo and Olympics), workers' compensation (Illinois), transport infrastructure (Arizona), pensions (Pennsylvania) and even terrorist victim compensation (Spain).

My subsequent empirical work uses UK data from the National Lottery (NL), launched in 1994 and operated by a consortium of firms known as Camelot plc with a seven-year licence. The UK game is a conventional lotto game (supplemented by scratchcards). The NL rapidly became the largest game in the world in total sales, one of the largest in revenue per capita (about £100 per capita a year).[6] Annual NL sales have been approximately £5 billion per annum,[7] raising £600 million per annum in direct government tax revenue (ignoring reductions in revenue from other sources induced by changes in expenditure patterns); in addition, an annual £1.4 billion in revenue is hypothecated for 'charitable' purposes.[8] The UK game is a common design and it seems likely that inferences from the UK game apply to similar game designs in other places.

The UK is unusual in being so explicit about how funds are allocated, in ring-fencing the funds and in attempting to distance allocation of funds from state control. Nevertheless, the extent of 'additionality' is still unclear: government can strategically reduce its own spending (for example, on the arts) in response to (or in

[5] Alm *et al.* (1993) model the adoption of lotteries across US states and find that tax revenue considerations are important for early adoption.

[6] For details of games around the world, see LaFleur and LaFleur (1996) and http://www.lafleurs.com/.

[7] This is about 1% of annual UK retail sales expenditure. Approximately 80% of this comes from the on-line lotto game, 20% from scratchcard sales.

[8] The Arts Councils, Sports Council, National Heritage Memorial Fund, Millennium Fund (for infrastructure investments of lasting social value) and Charities Board each receive 5.6% of sales revenue. Tax accounts for 12%, the retailer receives 5% and the operator's costs and profit accounts for 5%. The remaining 50% is returned as prizes. A 'New Opportunities' fund has recently been added.

anticipation of) lottery funds being awarded. Moreover, the UK government has recently used the unexpectedly large levels of the 'good causes' funds as an excuse to be more interventionist, directing funds to teacher training, health spending and even venture capital funding.

This paper is largely concerned with policy issues surrounding lottery products, such as how to design, regulate and tax such products. We concentrate, though not exclusively, on the 'lotto' pari-mutuel design that dominates most markets, and which is intrinsically more interesting because of the variation in jackpot size that it offers.

To understand policy issues requires an analytical framework. Demand determines who buys lottery tickets and in what quantities. I argue that players buy tickets, despite their risk aversion and the low payout rate, because they derive pleasure from participating, doing so regularly and in small amounts. Since players typically play in repeated draws, purchasing few tickets in any one draw, we can assume that they behave as if they were risk neutral. The main determinant of how many tickets to purchase is then the expected value of a ticket. This in turn is determined by game design.

By analysing demand, and the implicit pleasure entailed, we can compute the 'benefit' of introducing such a game by working out how much players value the opportunity to play, and hence compute the lost value from taxing this opportunity. It is also possible to keep track of distributional concerns that arise if lottery tickets are bought disproportionately by those with low incomes. In such circumstances, high tax rates have implications for equity as well as efficiency.[9] Social costs, and implications for policy, may also arise from concerns about addiction or about play by those below some minimum age. Another concern is that the size distribution of prizes is too highly skewed, somehow proving too tempting.

More generally, how do we design a 'good' lottery with large social benefits and small social costs? Lottery regulation is usually motivated by the maximization of tax revenue, which is equivalent to wanting to maximize sales revenue, since taxes are a proportion of sales revenue. The prime determinant of revenue is the pricing of lottery tickets, where 'price' can be viewed as the face price minus the expected value of the prizes on offer. Revenue maximization occurs when further changes in price are just offset by changes in the sales quantity (a price elasticity of -1). To see if the revenue-maximizing design has been chosen, we need to examine the price elasticity of demand. Estimating the dependence of sales on 'price' would be impossible if there were no price variation in the observed data.[10] However, rollovers alter the effective price by changing the expected return per ticket, thereby enabling the estimation of the determinants of demand.

[9] Regressivity may arise from two distinct sources: the unequal distribution of lottery funds (e.g. subsidizing opera) and the unequal distribution of expenditures.
[10] Game design changes, being infrequent, offer a poor basis for estimating demand behaviour.

Precise estimation of the price effect requires a sufficiently large number of rollovers. The probability of a rollover can be computed from knowledge of the game design and the number of tickets sold. With truly random behaviour, the design of most lotteries implies that rollovers should be rare; but, in practice, they are commonplace. This discrepancy arises because players choose their numbers non-randomly; particular patterns of numbers are then popular choices. The existence of unpopular selections increases the chance of a rollover. This 'conscious selection' generates a higher variance in the number of jackpot winners, more rollovers and more 'price' variation than we would otherwise observe. [11]

Much of the concern about lotteries reflects a fear that players squander the little money they have in the *irrational* hope of a large win; Cook and Clotfelter (1993) refer to lotteries as 'selling hope'. The competing view is that players are rational, but play lotteries for entertainment, in the same way that they would go to the cinema, rather than solely as a financial transaction in pursuit of gain.

If the true odds are misperceived, lotteries may indeed give rise to a social cost; in contrast, if rational players perceive the true odds, lotteries may confer a social gain in the same way that the existence of cinemas is beneficial. In this latter case, the gain can be computed in a conventional manner from a consumer surplus calculation based on an estimated demand function.

Section 2 outlines a theory for calculating the expected value of a lottery ticket. Section 3 examines how well this simple theory explains observed behaviour in the UK lottery, and estimates the implied price elasticity of demand. Section 4 looks at complications: conscious selection, addiction and intertemporal substitution. Section 5 uses microeconomic data to examine how demand varies with income, age and other demographic characteristics. Section 6 draws together implications for policy.

2. LOTTERY DEMAND

2.1. Rationality and participation

Rationalizing gambling, especially lottery gambles that are actuarially very 'unfair', is an old problem in economics, much of which is based on the idea that individuals are risk averse and so decline gambles in which the odds are not in the player's favour. Many observed phenomena, notably insurance, are consistent with risk aversion. Why do people play lotteries at all?

Friedman and Savage (1948) argue that people are risk averse over low and high ranges of wealth, but risk loving within intermediate wealth ranges. This might be

[11] An item from the *Irish Times* (3/12/96) illustrates the point. In the draw for the Irish National Lottery that week, there had been two winners of a large jackpot who had used the same method for number selection, based on the dates of birth, death and ordination of a particular priest (who had long been a candidate for canonization, but had lacked any miraculous associations – at least until then!).

explained by incomplete markets. For example, Ng (1975) argues that, in developing countries, credit market failures may imply that lotteries are a mechanism which allow individuals to purchase large indivisible durable goods that otherwise would be impossible to purchase. However, such discrete changes in what is affordable – technically called non-convexities – cannot explain why people play repeatedly and why play is not concentrated on the part of the wealth distribution where such non-convexities are most commonly observed.

Similar criticisms apply to the approach of Kahnemann and Tversky (1979), which allows us to reconcile insurance with small-stake/long-odds gambles, but not with small, fair bets, and to theories which assume that, instead of basing decisions on subjective probabilities, individuals are overly optimistic about long odds, but overly pessimistic about short odds (consistent with individuals knowing the true odds, but reacting to them non-linearly). Such theories can induce simultaneous gambling and insurance, but again only over specific wealth ranges. While helping explain why people participate in long shots, they remain an unconvincing story about lotteries, which are both long shots and very poor bets.

A more convincing generalization is that individuals are risk averse, and hence insure, and so would normally reject unfair bets, but that participating in gambles generates some additional non-pecuniary pleasure. Conlisk (1993) shows that even small pleasure from gambling can explain simultaneously insuring and engaging in both long- and short-odds gambles, providing the stakes are small, so that risk aversion is then unimportant. Provided the pleasure (thrill, entertainment, dreams) of the gamble offsets the monetary cost implied by the unfair odds, such gambling can co-exist with insurance against large risks, where risk aversion becomes important.

This view seems relevant to lotteries, where the stakes are invariably small and tickets are widely available. The pleasure derived from lottery gambling may extend beyond entertainment to the knowledge that revenues are used, in part, to support charitable causes or public works, especially those which otherwise are under-provided because of free riding.[12] The marketing of lottery products may stress this aspect of game design. Explaining lottery participation by the non-pecuniary pleasure derived is also compatible with empirical evidence that participation occurs throughout the income distribution. Its concentration among the poor is less than is often supposed.[13]

However, marketing and game design may also encourage misperceptions of the

[12] Morgan (1997) sets out a theory of how lotteries can be used to provide public goods. The argument relies on bundling a donation (which suffers from a negative externality because of free riding; givers would prefer others to give more) with a lottery (which exhibits a positive externality; players would prefer others to play less, thereby increasing the chance of winning the prize). However, the Morgan theory applies only to fixed-prize games and does not readily generalize to pari-mutuel designs. Indeed, fixed-prize games often feature in-kind prizes, such as a car, and are used for specific, and often local, infrastructure projects.

[13] Nevertheless, the share of income spent on lottery tickets is highest among the poor.

odds. For example, many lotteries feature small prizes for short-odds events, such as matching three out of six numbers drawn (a probability of 1 in 57 in a 6/49 design). Most players quickly learn of someone who won such a prize, and may win themselves after even a relatively short period of modest participation. Not everyone may realize that the probability of matching three is vastly higher than the probability of matching six.[14]

If individuals are risk neutral for small bets, the 'take-out' (proportion of stakes *not* returned in prizes) is the 'price' of the 'fun' from lottery play and is the *only* way in which game design affects play. In this simple approach, only the average prize affects demand.[15]

Lotto games allow players to choose their own numbers, increasing demand by giving players the 'illusion of control', a perceived ability to improve the odds in their favour by appropriate choice of numbers. Since the winning numbers are chosen randomly, this seems to imply remarkable irrationality by players. I show later that those players who make systematic choices usually reduce the expected value of their tickets below that of one whose numbers were randomly selected. Again, this suggests that the pecuniary return is not the whole story. The conscious selection of numbers may increase fun in several ways: for example, by allowing players to brag if they win.

For the above reasons, henceforth I assume that players derive fun from lottery play and view the expected financial loss on a ticket as the effective price of this fun. Players face downward-sloping demand curves for lottery fun, and the number of tickets purchased depends on the effective price of a ticket. Rollovers induce variations in expected financial return and allow us to estimate the demand for lottery fun from the correlation between variations in tickets sales and changes in the effective price. This approach is extremely powerful and will allow estimates of consumer surplus derived from lottery fun, but it comes at a price: namely, it denies the possibility that rollovers, which change the effective price of fun, also affect the amount of fun, and hence the demand for it, by altering the size of the jackpot.

2.2. A simple analytical model

In the typical pari-mutuel on-line game, players choose a given number of selections, and winners share prize pools that are fixed percentages of total sales revenue. In the basic 6/49 game, 6 balls are drawn (without replacement) from 49, but there are many variations: drawing with replacement; additional 'bonus' balls

[14] Lottery promotion invariably emphasizes that someone wins the jackpot. Hence the slogan 'It could be you'. Illinois once presented its game on billboards in poor neighbourhoods as 'Your ticket out of here'.

[15] This assumes that the fun also depends only on the average prize. If thrills and dreams depend on the size of the jackpot, higher moments of the prize distribution will be important, a point to which I return later.

that yield a wider variety of prize possibilities; requiring additional numbers to be matched from draws from a second ‘urn’; and prizes for subsets of balls drawn (e.g. matching only three of the six balls drawn). The fraction of revenue paid out is usually allocated to separate prize pools: a jackpot pool for the lowest-probability event (matching all six balls drawn), and smaller pools for higher-probability events (e.g. matching any three of the six balls drawn). Thus, within any given game design, there may be differences in the prize distribution because of differences in allocating revenue across prize pools. Many games have a single weekly draw, but twice-weekly draws are also common, and daily draws exist in some places. While games differ in the skewness of the prize distribution, most have a take-out rate of around 50%.

The UK game, for which I have data, is 6/49 (with a seventh bonus ball). Tickets cost £1 and draws were initially weekly (twice weekly since 5/2/97). Approximately 45% of the revenue from the on-line game (which is cross-subsidized by scratchcard games also operated by Camelot) is returned as prizes: all three-ball matches pay £10; 52% of the remainder is reserved for the jackpot, and the rest is split into prize pools offering smaller prizes for matching four balls, five balls, and five balls plus the seventh ‘bonus’ ball. The chance of matching all six balls (without using the bonus ball) is about 1 in 14 million.

The UK, Canada and Israel (and many other places) currently operate 6/49 games, despite vastly different populations (58 million in the UK, 29 million in Canada, only 5 million in Israel). The Irish game is currently 6/42, despite a population of under 4 million, implying a probability of jackpot winning of 1 in 2.8 million.

The balance between game design and likely sales matters: on some occasions, it has actually paid to ‘buy the pot’. After a sequence of rollovers, the jackpot can become so large that the expected (pecuniary) value of buying a ticket can exceed its nominal cost.[16] US games differ widely because of the different populations of states. The California game started in 1986 as a 6/49, but was changed (twice) to make it harder to win (and hence to generate more rollovers). In Rhode Island, with its small population, the game is 5/40.

The variation of the demand for tickets with the expected pecuniary value V of a ticket, first analysed by Sprowls (1970), has subsequently been used by Scoggins (1995), Cook and Clotfelter (1993), Gulley and Scott (1993), Lim (1995), Farrell *et al.* (1997) and Farrell and Walker (1996).

[16] On one occasion, a Canadian operator offered a special promotion – six tickets for the price of one – making the expected value of a $1 ticket well over $2. This free lunch was enthusiastically received! Two notable examples occurred in Ireland, where a syndicate bought millions of tickets when the jackpot pool had become sufficiently large. On both occasions the syndicate had to share the jackpot with non-members, but still made a profit. The game was subsequently redesigned to preclude such possibilities. In the German lottery (a variant on 6/49), Norman Faber has been very successful in exploiting knowledge of players’ favourite numbers in order to select less popular combinations.

Table 1. The simple arithmetic of lotto

Variable	Formula	Implications
$\mathcal{J}$ Jackpot size	$\mathcal{J} = (1 - \tau)S + R$	Rises with sales S and inherited rollover R Falls with take-out rate τ
P Probability of rollover	$P = (1 - \pi)^S$	Falls with sales S and easier game design π
V Expected ticket value	$V = (1 - P)\mathcal{J}/S$	**No inherited rollover ($R = 0$)** Increases with sales at a diminishing rate, towards a maximum of $(1 - \tau)$ **With rollover ($R > 0$)** Above $(1 - \tau)$ except at small S, may rise with sales, but falls towards $(1 - \tau)$ as sales rise.

Table 1 describes the simple arithmetic of lotto. Suppose individuals choose numbers randomly and there is a single prize pool.[17] This jackpot is sales revenue (net of the take-out rate) plus any rollover from the previous draw. If S is sales revenue (and tickets sold, if each costs 1 unit of local currency), R is the rollover from the previous draw (zero if the previous jackpot was won), and τ the take-out rate (akin to a tax), Table 1 shows the formula for the potential jackpot, which increases with ticket sales and the size of any inherited rollover.

The probability of the jackpot being won depends on the game design. Suppose n balls are drawn without replacement, a win requiring matching m of these. The probability π of any one ticket matching the balls chosen, which measures how hard it is to win the game, is given by $n!/m!(n-m)!$ Where $n = 49$ and $m = 6$, π is 1 in 13.984 million.[18] The probability that the jackpot will roll over (not be won) falls with ticket sales, since the more tickets are sold, the more chances there are of a matching ticket having been sold to claim the jackpot, and falls with the ease of the game.

The expected financial value of a ticket is the probability that the jackpot will be won, divided by the number of tickets sold. This formula, which can be thought of as a supply curve, relates the value of a ticket to the level of sales, a statistical relationship determined by the choice of game design.

This expected ticket value rises with the size of the rollover, but falls with the size of the take-out rate; and, in a regular draw with no rollover, rises with the level of sales, but at a successively smaller rate. The first two results are obvious: the bigger the rollover, the larger the pot to be won and hence the bigger is the return; and the larger the 'tax' slice, the smaller will be the prize pool and the smaller the return.

[17] The latter assumption is harmless provided smaller prize pools do not roll over (unlikely in theory, unheard of in practice). Farrell *et al.* (1996) relax the former assumption, showing that conscious selection of numbers leaves many theoretical properties of the game unaffected.
[18] Note that ! indicates the factorial operator. For example, $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

Explaining the shape of the relation between the expected value of a ticket and the level of sales is more complicated. Except for rollovers, the expected value of a lottery ticket would always be $1 - \tau$. Suppose rollovers are possible, but there was no rollover in the previous draw. There may be no winner in this new draw (i.e. a rollover), which is more likely the smaller are sales. The expected ticket value rises with sales because adding a further player increases the size of the prize (a fixed percentage of sales revenue) by more than the diluting effect of an additional potential winner – Cook and Clotfelter (1993) call this lotto's 'peculiar economies of scale'. However, at very high levels of sales, it is unlikely that a rollover occurs. So for large enough sales, the effect of selling an additional ticket is to raise the prize pool by the same proportion as the odds of winning fall, so that expected winnings remain constant.

The probability of rollovers introduces an externality. A player in any draw cannot appropriate the benefits that their stake confers on other players in a subsequent draw that may benefit from a rollover from the current draw.

Where a rollover has been carried forward ($R > 0$), the story is more complicated. On the one hand, adding another player increases expected ticket value (but at a decreasing rate), as in the $R = 0$ case. On the other hand, the contribution of the inherited rollover to the ticket's expected value gets unambiguously smaller as sales increase: the rollover amount is spread more thinly over more players. The overall effect of sales on expected value is then the sum of these two counteracting forces. A ticket's expected value will increase with sales when sales are small, but decrease with sales when sales are high. The inherited rollover acts as a subsidy to players in the current draw. The subsidy per ticket depends on sales, so although it shifts the 'supply curve' upwards, it does so by less the larger are current sales. Whatever the value of the rollover, the expected value of a ticket will tend towards $(1 - \tau)$ as sales tend to infinity, because the probability of a rollover (or, in the case of $R > 0$, a further rollover) tends to zero as sales tend to infinity.

For a regular draw, the expected value per ticket increases with sales, getting ever closer to $(1 - \tau)$ as sales increase. For a draw inheriting a rollover of size R, a ticket's expected value may increase with sales to some maximum, but further increases in sales then reduce the expected ticket value steadily until it converges to $(1 - \tau)$ from above.

Since a ticket's expected value is always higher in rollover draws than in regular draws, irrespective of the level of sales, it is impossible to arbitrage away differences in expected values in the two draws, whatever the level of sales. Crucially, therefore, rollovers create variation in expected ticket values and thus in the effective price of fun.

Although it is theoretically possible for a lottery to offer odds that are actually favourable, it is unlikely that the expected value of a ticket would ever exceed its cost: it would require a low level of sales in a draw with a large jackpot accumulated from previous rollovers.[19] However, when a large jackpot has accumulated we expect demand to be high, so the circumstances required to observe $V > 1$ are unlikely to occur.[20]

Game design, by determining the rollover probability, affects the shape of the 'supply curve' relationship between price and quantity supplied (i.e. expected ticket value and sales). The other aspect of game design is the take-out rate τ. Surprisingly, this is close to 0.55 for most pari-mutuel games in operation. Most US states have a take-out rate between 0.55 and 0.65, but prizes are taxed as income and games pay out over twenty years rather than as an immediate lump sum.[21] In the UK game winnings are not taxed, but the 12% sales tax is half of the UK standard rate of income tax. Thus, allowing for discounting and tax, US games appear to be less attractive than the UK lottery, whose take-out rate is about 55%.

Figure 1 shows how the relation between sales and expected ticket value is affected by the difficulty of the game, determined by n, the number of possible balls, and m, the number of successful matches required. At any level of sales, the discrepancy between $(1-\tau)$, the fraction of revenue paid out conditional on a win, and the expected value of a ticket varies inversely with the rollover probability. Thus, in small states or countries we expect to see games with low n and/or high m. In Ireland, for example, the game design changed as it became more popular over time to prevent a reduction in the number of rollovers (although the game still features a higher rollover probability than the UK game). In the UK, sales for a typical Saturday draw were close to 70 million (before the introduction of the mid-week draw, which did reduce Saturday demand slightly), so the expected number of jackpot winners was close to five (i.e. 70 million $\times$ (1/14 million)), with an implied standard deviation of around 2. Thus, the UK game should generate few rollovers: the theoretical probability is about 1% when S is 70 million per draw.

In fact, as I explore below, the proportion of rollovers has been close to 20%, and on two occasions the number of jackpot winners has exceeded 50 (both remarkably unlikely if the mean is 5 and the standard deviation 2).

2.3. Demand and equilibrium

In principle, my analysis is consistent with demand depending on subjective probabilities (that determine V via π). However, aggregation conditions require that

[19] Despite attempts to organize large syndicates when the jackpot gets large, it is difficult to stop others playing, so a winning syndicate may well have to share the jackpot. Camelot, the UK operator, has indicated that it would prevent syndicates attempting to buy the pot, believing that regular players become alienated if the game can be manipulated by a coalition of players.

[20] Conscious selection, however, might allow anyone knowing which numbers are unpopular to increase the expected value of a ticket by choosing unpopular numbers.

[21] Note that this reduces the present value of the prizes by approximately 50% using a 7% discount rate. Some states offer the choice of a discounted lump-sum prize or a twenty-year annuity.

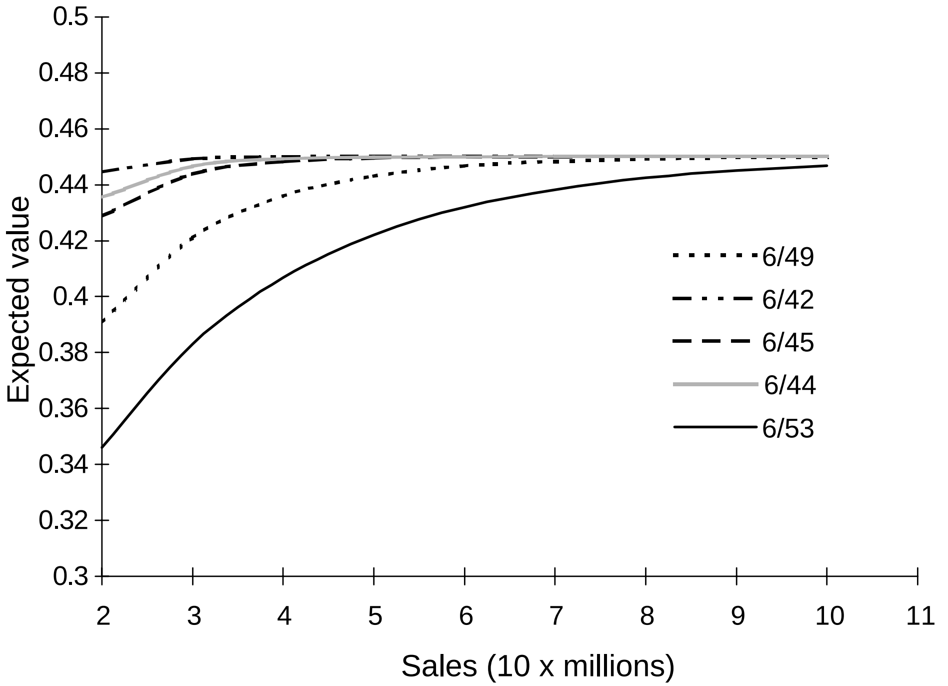

Figure 1. Game design, sales and expected ticket values

all individuals face the same price, and hence require that subjective probabilities equal actual probabilities. This should be borne in mind when considering models based on aggregate data.

In addition, one should note that individual demand depends on the expected value, which depends on actual sales for that draw, which are only revealed when the draw has taken place. Thus the analysis appeals to 'rational expectations', assuming that individuals do not make systematic mistakes in forecasting the total sales for the draw.[22]

Aggregating individual valuations, we obtain the total demand curve for lotteries, *DD* in Figure 2. Thus, a rollover, which we have seen shifts supply upwards, induces a move along the demand curve, increasing consumer surplus in the usual way.

Notice the importance of the non-linearity of the supply curve. In a regular draw, a ticket's expected value increases monotonically with sales, so at some equilibrium level of sales, S^*, each individual player would be better off if there were more players (because the rollover probability would be reduced). This positive externality to sales exists unless the game is designed so that expected ticket value converges

[22] In practice, the operator makes a forecast of the likely jackpot, and hence implicitly the sales for the draw, some time before the draw takes place. In the UK, this occurs the day before the draw. Most sales are on the day of the draw.

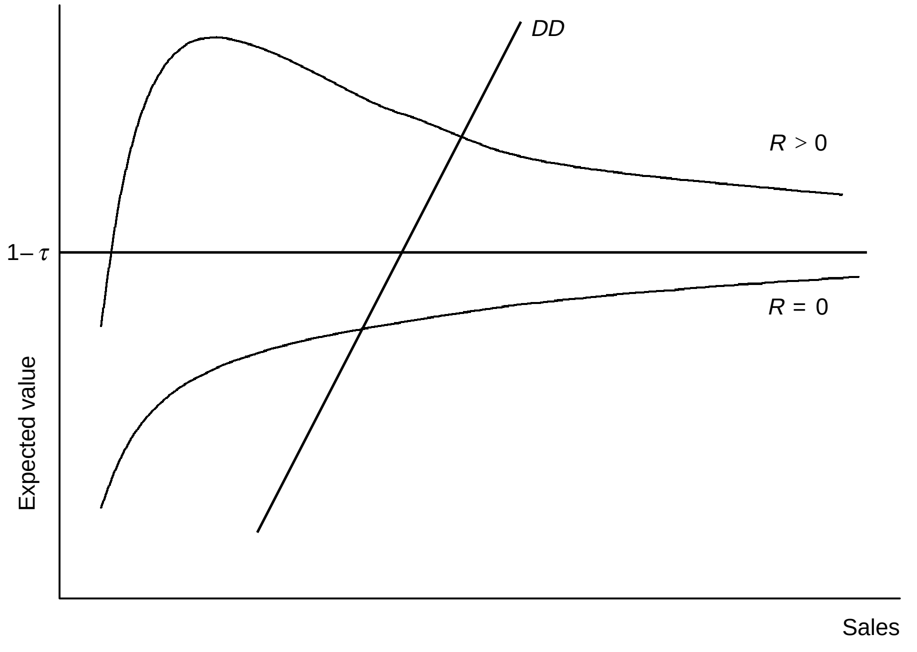

Figure 2. Demand and expected value

quickly towards $(1 - \tau)$, in which case the supply is near-horizontal at the equilibrium. In contrast, in draws where the jackpot has been inflated by a rollover, equilibrium will typically occur at a point where the supply curve is downward sloping (because the inherited rollover is being spread across more and more tickets). In this case, each participant would be better off if there were fewer other players, since the rollover bonus would be less diluted. The rollover is like a common property resource 'overfished' by free entry. In this case, players create negative externalities; for a given game design, these are larger the greater the rollover. If, however, the game is designed to minimize the positive externality that occurs in a regular draw, so that equilibrium occurs at high levels of sales, it will also imply that there are very few rollovers.

The operator can also influence the number of rollovers by encouraging conscious selection. In the UK, the facility to pick *random* numbers ('Lucky Dip' in the UK, 'Quick Pick' in many other countries) was introduced only after eighteen months, by which time many regular players had settled on their own favourite combinations of numbers. This had two beneficial effects on sales. First, since the reasons for conscious selection are correlated across individuals (e.g. using dates of famous events), this depresses the 'coverage rate' and raises the probability of a rollover, which then promotes average sales. Second, it encourages players to play in every draw to avoid the disappointment of having not bought a favourite combination on the occasion when it was actually drawn. This is important, since rollovers present players with an incentive to engage in intertemporal substitution in response to relative price variation, buying tickets only when there is a rollover. This would endanger the dynamic stability of the game: since sales would be depressed in regular draws, such draws would be more likely to generate rollovers, further diminishing sales in regular draws, and so on until the game disappeared.

Some games do indeed have a low base level of sales in regular draws, a high rollover probability and very high sales in multiple-rollover draws; the Californian state lottery is one example (Lim, 1995). In the USA, the problem is exacerbated by the possibility of atemporal substitution: players can easily substitute a lottery from a different state that is carrying forward a rollover. This tendency can be only partly countered by trying to differentiate the product from those elsewhere, and is a strong argument for having just a single game (of any type) across a market.

When the UK operator introduced a second weekly game, it replicated the existing one exactly. There are two reasons why this might be sensible. First, if players are attached to favourite combinations, a second identical game may achieve larger sales than a completely new game. Second, if, in the first game, sales are already very high, so that a ticket's expected value is large and the supply curve is flat, a reduction in sales, because of the introduction of a substitute game, will induce more rollovers by splitting sales across the two draws; when a game is close to saturation, it makes sense for the operator to introduce a second game.[23]

What is the optimal rollover probability for the operator? If the game design were such that the probability were close to unity, intertemporal substitution would completely destabilize the game; if the probability were close to zero, the game would be failing to exploit the fact that the supply curve's positive slope is flatter in a regular draw than its negative slope is in a rollover draw, whence the operator could reduce the 'price' with hardly any effect on the rollover probability. The 'optimal' (in terms of profits) rollover probability equates the marginal benefit of more rollovers (induced additional sales in draws subsequent to a rollover) and the marginal cost (lower sales in regular draw),[24] and is likely to be a good deal higher in markets where there is a single game.

The final aspect of game design is the take-out rate τ. It is common for this to be set at a high level to generate a surplus for use in promoting the games in other ways, such as occasional additions to the prize pool to create 'superdraws' (effectively a small rollover), or to ensure some minimum jackpot independently of sales, as in Ireland and Israel.

[23] It is also easier to use existing technology if the game is unchanged. Since most tickets are sold on the day of the draw, there is considerable spare capacity mid-week.

[24] Since changing the rollover probability changes the ratio of regular draws to rollovers, marginal costs and benefits apply not per draw, but to the effect on the expected profit stream when this ratio is treated as endogenous.

3. UK EVIDENCE ON SALES AND ROLLOVERS

Having developed a theory of lottery behaviour, I now turn to evidence from the UK, whose on-line draw sales (until the introduction of a second weekly draw) are shown in Figure 3. Sales rose quickly from around 40 million in early draws to a steady 65–70 million within just a few draws. In theory, sales reflect the expected value of holding a ticket, so rollovers generate additional sales. Figure 3 shows sixteen rollover draws, two of which became double rollovers, and three super-draws, each causing a spike in sales. Many more rollovers occurred during these 116 draws than might have been expected from purely random behaviour.

Figure 4 plots sales and the expected value of a ticket in each draw. Regular draws are the mass of observations at or just below 0.45. Rollovers have typically implied that the expected value rises to 0.58, reducing the effective ticket price from £0.55 to £0.42 and boosting sales from approximately 69 to 82 million. In double rollover draws, expected value rises to £0.65, price falls to £0.35 and sales rise to approximately 110 million. The superdraws in Figure 4 have effects commensurate with their interpretation as small rollovers.

We can estimate the relationship between sales and the expected value using the data in Figure 4. Expected value depends on sales, so the effective price is endogenous to the demand function. Since expected value depends (non-linearly, through the formulae in Table 1) on rollovers via a specific formula, we can either impose this and estimate the relationship by non-linear least squares, or use rollovers as an instrument for expected ticket value: once we control for the direct effect of the rollover on V, rollovers have no other effect on sales (provided the entertainment value of a ticket depends only on its expected value). This use of instrumental variables is essentially equivalent to fitting a (straight) line through the mean of the

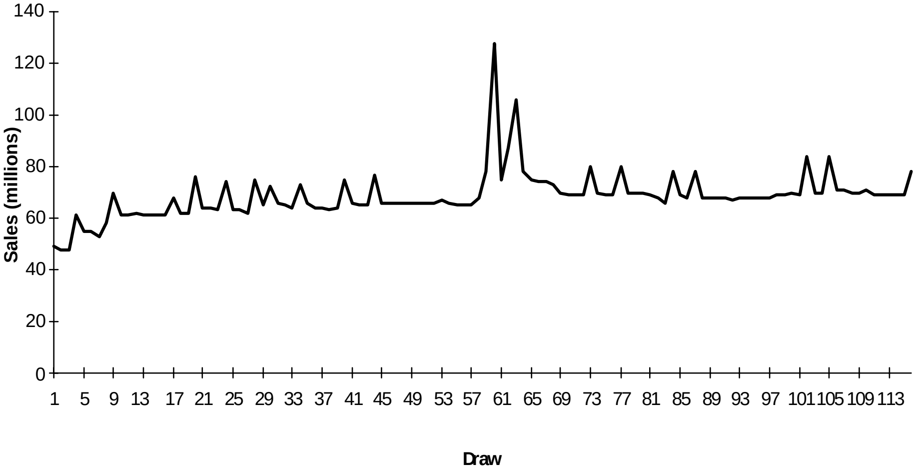

Figure 3. UK lottery sales

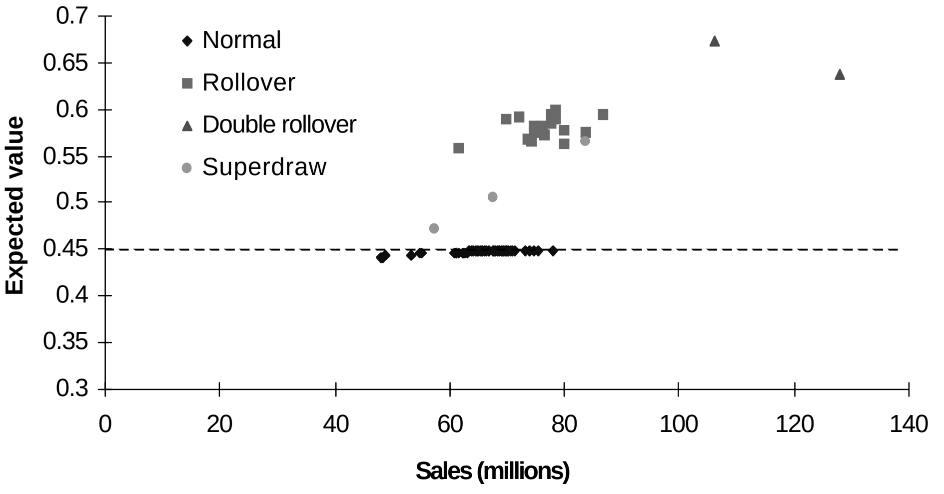

Figure 4. Sales and expected value

data for regular draws and the means for the single rollover observations and the double rollover observations. Nevertheless, without experimental variation in expected ticket values induced by random design changes, analysing how sales vary with rollovers is the only avenue available.[25]

The estimated demand curve (t-values in parenthesis) is

$$\text{Sales} = \underset{(0.7)}{4.7} + \underset{(102)}{135}^{*}\ \text{Expected ticket value} \tag{1}$$

This equation explains 44% of the variance in sales and shows that the rollovers are a statistically important determinant of sales. One could control for other factors, such as the number of previous small prize winners, the availability of a Lucky Dip facility and the number of ticket outlets. More outlets and a Lucky Dip facility both reduce transaction costs and may boost demand. Yet, omitting these variables, and any trend, makes little difference to the estimated coefficient on ticket value, since these variables are uncorrelated with variations in V arising from rollovers, which are random not systematic events!

Is the game design correct? The government's aim was to generate funds for good causes and general taxes; since these are fixed proportions of sales, the aim is to

[25] Purfield and Waldron (1997) and Beenstock and Haitovsky (1997) have longer runs of sales data, for Ireland and Israel respectively, both of which feature changes in game design as well as rollovers. The latter, in particular, look at the effect of skewness in the prize distribution as well as its expected value. There is some evidence that skewness matters. Demand rises in rollovers partly because skewness of the prize distribution rises, only partly because of a higher expected value. However, these papers can distinguish between skewness and the mean only because of design changes during their time series. It is unclear that these design changes are not themselves policy responses to the evolution of sales, and hence endogenous.

maximize sales, which occurs when marginal revenue from additional sales is zero, and the price elasticity is -1. Equation (1) implies an elasticity of -1.07 at the sample means[26] (with a standard error of 0.13). If revenue maximization is the aim, the correct design seems to have been chosen[27] and the game has been priced correctly.

The estimated elasticity also facilitates an evaluation of consumer surplus and the implication of the take-out rate not for revenue, but for consumer welfare. Equation (1) implies that the 'price' that would completely choke off demand is around £1.03.[28] The gain in consumer surplus from being able to buy tickets at a price of £0.55 (the UK take-out rate on a £1 ticket), rather than not being able to buy tickets at all, would be £0.81 billion a year (close to the additional net income that people would enjoy if there were a 0.5% cut in the income tax rate).[29]

Free-rider considerations suggest that demand should be relatively insensitive to how the money raised is allocated between good causes and general taxes. If we ignore this completely, we can ask how much higher the consumer surplus would have been were it not for the 28% of ticket sales allocated to spending on good causes. The answer is £2.04 billion a year. Thus, the deadweight loss of raising the good causes money (which, prior to the mid-week draw, was about £0.95 billion a year from the on-line draw) is a significant £0.28 billion.[30] Because of its high take-out rate (to maximize revenue), the lottery imposes a heavy tax on an activity highly valued by consumers and to which their behaviour is sufficiently price-sensitive that the high tax rate generates substantial distortions. The lottery levy is an extraordinarily economically inefficient way of raising revenue for these causes: it imposes efficiency losses on society that are 30% of the cash raised.

One might argue that demand would be lower if the 28% levy were used for some purpose other than adding to the prize pool. I offer evidence below on whether bundling the gamble with a charitable donation generates a 'warm glow' that overcomes the 'stigma' associated with gambling itself. However, the game clearly has such wide appeal that any associated stigma is small. It is therefore hard to defend using the lottery to raise money for good causes; the efficiency costs are large. I deal with inequity in section 4.

Nor is it certain that money raised for good causes is truly additional. Lottery money may displace private or government donations. I consider the available (flimsy) evidence on these issues in section 6.

[26] The same figure is obtained by Forrest and Gulley (1998).
[27] All bids for the UK lottery franchise were for 6/49 games, but differed in the take-out rates proposed. The actual take-out rate for the UK game is close to the revenue-maximizing level according to my estimates.
[28] Similar to the estimate obtained by other means by Blow and Crawford (1996).
[29] In addition, there would be additional surplus generated by the rollover weeks that would occur over a year.
[30] £0.28 = 2.04 − 0.81 − 0.95.

4. OTHER FEATURES OF LOTTO BEHAVIOUR

The short-run elasticity of sales with respect to the 'price' is close to that implied by revenue maximization. A number of important considerations, not encompassed in the simple theory, affect this conclusion and should be taken into account.

4.1. Conscious selection

The empirical finding that the number of rollovers vastly exceeds that implied by random choice of numbers by players is not confined to the UK, and is impossible to dismiss merely as a lucky sequence for the operator. Rather, it is likely to reflect conscious selection by players,[31] which typically induces a correlation across chosen ticket numbers. Unpopular combinations become sources of potential rollovers, and holders of popular combinations find their individual return on a winning ticket diluted by the number of other players with whom the payout is shared. Thus the expected value of a ticket with a popular combination of numbers will be lower than that with an unpopular combination.

Data on the distribution of numbers that people pick is seldom available.[32] The coverage rate for a draw is the proportion of possible combinations actually purchased at least once; 1 minus the coverage rate is thus the probability of a rollover in that draw. The coverage rate for the UK game is not actually published,[33] but it was initially very low and increased sharply as the game progressed. This should have led to a drop in the proportion of rollovers; in fact, the proportion was 21% in the first 58 draws and 18% in the second 58 draws, statistically surprising given the small variance in the rollover probability. It would be surprising if there were not a marked increase in the coverage rate when there is a rollover: having exhausted regular favourite combinations, extra ticket purchases are more likely to be picked randomly and hence to be less correlated with the choices of other players. Moreover, a rollover attracts people who would not otherwise play and who do not have favourite combinations, again reducing the normal correlation

[31] Thaler and Ziemba (1988) cite Langer (1975) as coining the phrase 'illusion of control'. Langer's experiments found that individuals required a larger bribe to give up a lottery ticket chosen by themselves than one they did not choose. The failure of the early New Jersey game is attributed by Thaler and Ziemba to the inability of players to select their own numbers. Hill and Williamson (1998) discuss the psychology of conscious selection.

[32] Haigh (1996) reports evidence from a Swiss lottery where the distribution of numbers chosen is available for researchers. The most popular combination was that which won the French lottery the week before. Combinations that formed patterns on the ticket, or included 'lucky' numbers such as 7, were also popular. Camelot's Web site for the UK lottery lists the top ten ways of picking numbers as: making a pattern on the playslip; Lucky Dip; lucky numbers (birthdays and anniversaries); pets; multiples of any one number; home-made draw machine (ping pong balls in washing machine!); choosing the first six numbers in any one column of a playslip; 1, 2, 3, 4, 5 and 6 or other sequences of numbers; picking the most frequently drawn numbers; and blindfolded. How Camelot compiled this ranking is unexplained, as is the means by which pets communicate their selections!

[33] The operators regard this as commercially confidential information.

of choices. Increased coverage rate when rollovers occur ought to make double rollovers less likely to occur than would otherwise be the case. With a single rollover probability of about 15%, we would expect double rollovers with a probability of 2.25% if the extent of conscious selection did not change, and this is actually close to what happened. So it appears that Camelot has been lucky both in the number of rollovers that occurred and in the number of those that became double rollovers.[34]

While significant conscious selection does occur, we need to know the implications for the expected value of a ticket. If coverage data were available, one could amend the rollover probability to $(1-\pi)^{cS}$, where c is the coverage rate, and use this in Table 1. Although this would be correct on average, the coverage rate is not actually a very complete explanation of the frequency of rollovers because luck seems to have played a large part.

In fact, Camelot does publish sufficient information to allow us to be more precise: it publishes, at each draw, the winning numbers and the number of winners of each category of prize. The frequency of winners of each class of prize has a much higher variance than one would expect, which provides indirect evidence of conscious selection. The winning numbers that are associated with draws when there are large numbers of winners are popular numbers. Although the details are complex,[35] we can formalize this intuition and estimate the probabilities that each number is drawn using the published data on the winning numbers and the number of winners of each prize pool.

Figure 5 plots the estimated distribution of numbers and a 5% confidence interval. The horizontal line indicates that we expect each number to be chosen with a probability of 1 in 49, and the line, estimated by pseudo-maximum likelihood, shows a clear bias away from numbers in excess of 31, consistent with the popular view that birthdays influence player selection.

Such knowledge can be exploited (see Ziemba, 1986) to improve one's expected winnings (or, rather, reduce one's expected losses). For example, one could choose the six least popular numbers $(36, 41, 46, 47, 48, 49)$ and avoid the six most popular $(17, 18, 19, 23, 27, 28)$. The former has an expected value of 1.11, the latter an expected value of just 0.22. A weekly return of 11% may seem very attractive, but it is also a very high-risk investment because of the innate variance. There are thousands of combinations with positive rates of return, even when there is no rollover, so playing all of these combinations at each draw would considerably reduce the risk.[36] However, others may come to learn the attraction of

[34] The Lucky Dip facility was introduced from draw 70, allowing players to choose their number selections randomly. The use of Lucky Dip has been quite small, still below 15% of players, yet we would expect to see two effects on the coverage rate. First, it should diminish correlation across players, increase coverage and reduce the rollover probability; second, the effect of rollovers on the coverage rate should become more pronounced.

[35] Farrell *et al.* (1996).

[36] There was, in fact, one occasion when a UK jackpot winner shared the prize with himself – by mistake, he bought two tickets with the same numbers on. In general this is not recommended, but in this case, he was one of several winners in that draw and so ended up getting twice the share he would otherwise have done.

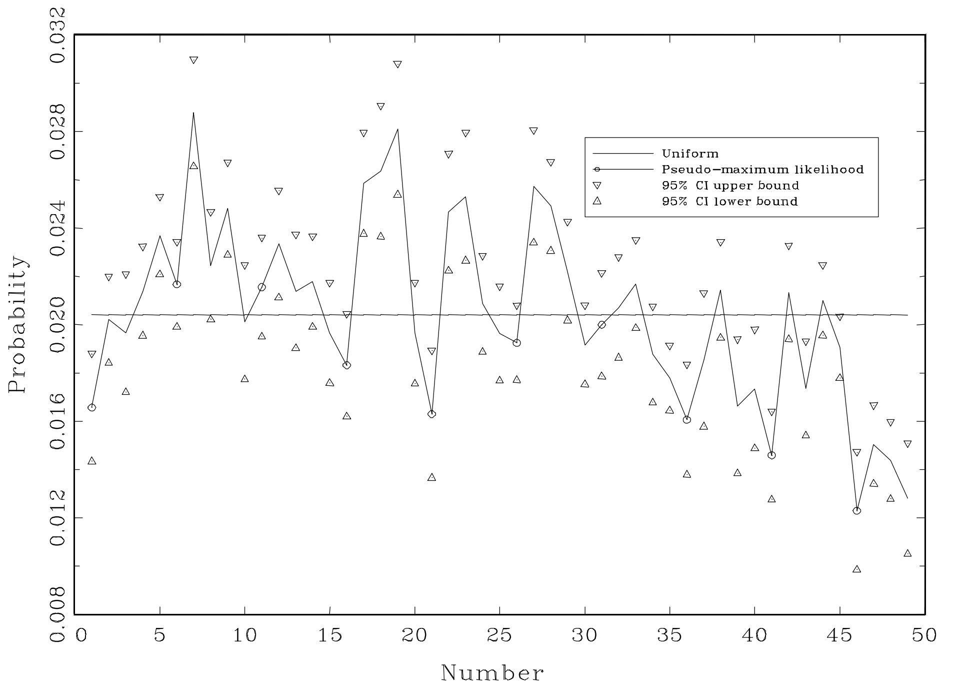

Figure 5. Estimated choice distribution

backing unpopular combinations, which would reduce the 'edge' that Figure 6 may offer.

Figure 6 shows how conscious selection affects the supply curve, for three cases: no rollover, a £10 million rollover and a £15 million rollover. The expected value of a ticket is computed for the case where everybody chooses randomly and compared with the expected value using the degree of conscious selection estimated in Figure 5. Although conscious selection was shown to have a big effect on the rollover probability, it has very little effect on the expected value at the levels of sales usually observed.

The extent of conscious selection that we observe seems to be high enough to generate a useful number of rollovers and yet not too high that it induces any significant welfare loss in regular draws. Of course, the extent of conscious selection is something that the operators can and do manipulate (e.g. by making a random selection available when many players have become attached to favourite combinations). Since conscious selection has little effect on expected values, it has little effect on the estimated price elasticity; so our earlier conclusions about take-out rate and consumer surplus are essentially unaffected. That conscious selection has little effect on the expected value of a ticket is unsurprising; if it did, we would expect it to be much less prevalent.

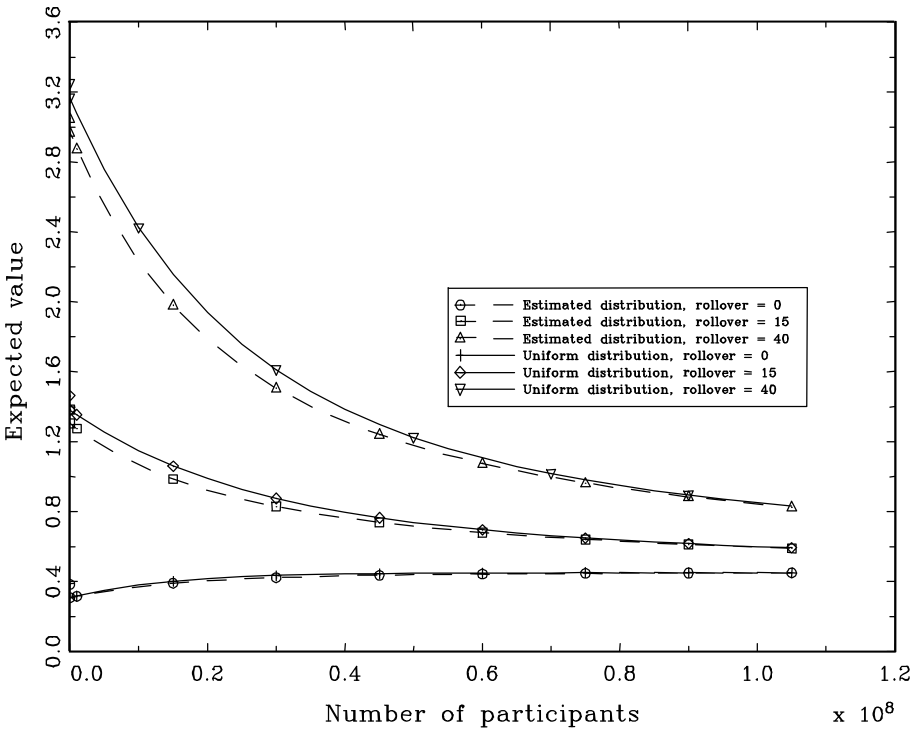

Figure 6. Conscious selection and the supply curve

4.2. Addiction

There are two reasons to expect sales in one draw to be correlated with sales in the next. First, people may become addicted, as with heroin or opera. If so, rollovers have a double dividend, boosting sales not only for the next draw, but also for subsequent draws. If a further rollover occurs before sales have returned to their pre-rollover level, successive rollovers can even cause sales to ratchet upwards over time.

However, too many rollovers may also damage sales in the long run, since players may shift out of regular draws entirely. The high and sustained sales of the UK lottery suggest that this second effect is weak. However, it would be dangerous to make the frequency of rollovers (or other injections to the prize pool) predictable in their timing.

Following Becker *et al.* (1994), I test whether last period's sales affect current sales by incorporating the lagged value of sales in equation (1). Such dynamics allow a distinction between the long-run and short-run effects of a change in the 'price' of a lottery ticket. I found a short-run elasticity of sales with respect to 'price' of approximately -1.1 as before; but the effect of lagged sales is statistically significant and quite large (a coefficient of 0.33), although smaller than those found in the cigarette and alcohol literature.[37] The implication of this lagged sales effect is that the long-run elasticity is approximately −1.5, implying that the game design could be changed to increase sales revenue (and hence tax and good causes revenue) by *reducing* the take-out rate.

The rollover effects implied by the estimates are plotted in Figure 7, which also shows how sales are predicted to evolve after rollovers, comparing this with post-rollover sales. A typical rollover raises sales by 20%, but sales do not return to their previous level immediately. Thus a rapid succession of rollovers can ratchet sales upwards.

One reservation about this analysis is that it is unwise to exploit a relationship that may not be stable *when it is exploited*.[38] If Camelot tried to manipulate the behaviour of players as suggested here, that behaviour itself might change, invalidating the relationship. For example, if players began to expect that a superdraw would be timed to follow a rollover, intertemporal substitution in ticket purchases would change. However, it might not be a problem if it were done relatively infrequently and unpredictably.

Gambling analysts regard lotto as 'soft' gambling: draws are infrequent and at fixed intervals. In contrast, scratchcards are 'hard' gambling: players can immediately generate another draw, and hence can more easily pursue compulsive behaviour such as 'chasing losses'. This interpretation of 'addiction' is quite different

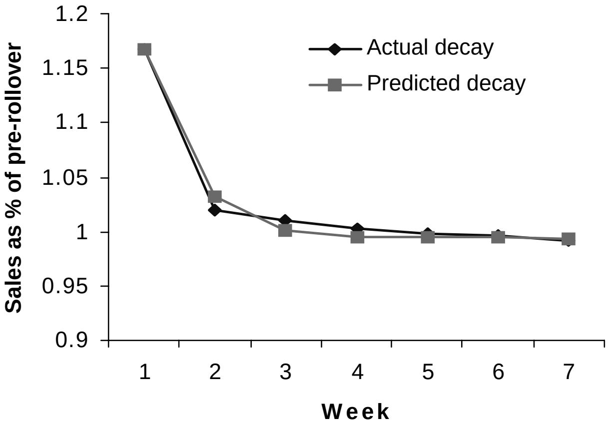

Figure 7. Post-rollover sales

[37] See Farrell *et al.* (1997). Attempts to generalize the specification to allow for more sophisticated dynamics proved not to add to the model. Note that sales increase in rollover weeks and, while they fall afterwards, they remain higher than prior to the rollover occurring. This means that rollovers heighten the potential addictiveness of the game; the more rollovers, the more potential there is for addiction to develop.

[38] This insight, the 'Lucas critique', was first applied by Nobel laureate Robert Lucas to macroeconomic stabilization policy based on empirical relationships between macroeconomic aggregates.

from that analysed above, and in practice we rarely have adequate micro data to describe such behaviour in any detail. Specialist surveys target groups that are potentially vulnerable to such temptations, notably children. Such considerations also explain why a minimum age limit is invariably set for lottery participation – in many countries, eighteen; in the UK, sixteen.

Related to addiction is contagion, in which an individual's tastes are affected not by their recent behaviour, but by the current behaviour of other individuals in close proximity. UK Family Expenditure Surveys give expenditure data for each *individual* within a household and show that the probability of one individual playing within a household is sensitive to whether others in the household are playing. Nor is this a spurious correlation; individuals start and stop playing when others in the household start and stop. Contagion may be something the operator may wish to encourage to promote sales, but lottery regulators may fear that it encourages play by children within households where parents play.

4.3. Competition from future draws

If the *anticipation* of rollovers switches demand away from regular draws, it could undermine the interpretation of my estimated price elasticities, which may have uncovered short-run and temporary responses, whereas both the operator and the government will care primarily about long-run design.[39] What is required to separate these two effects is some (unforeseen) design change. Can we make use of the introduction of the UK mid-week draw?

Camelot decided simply to replicate the Saturday draw without any design changes. Since the Wednesday draw is therefore a close substitute for the Saturday draw, changes in relative prices, induced by a rollover, should make players switch the timing of ticket purchases between draws. One could test whether the sensitivity of Saturday sales to rollover-induced changes in expected value became smaller after the availability of a Wednesday draw. This would imply that consumers engage in substitution across draws.

Figure 8 shows the relationship between expected value and sales for Wednesdays and Saturdays, before and after the launch of a mid-week draw.[40] Econometric analysis of these data confirms what is apparent from the diagram: the Wednesday demand curve is effectively parallel to the Saturday one, but lower (i.e. to the left); sales in both draws are equally sensitive to changes in expected value, and the slope of the demand curve is the same as that when only the Saturday game existed. The Saturday demand curve has shifted inwards slightly, as Figure 8 suggests, with a

[39] Similar considerations arise in labour supply analysis. See Blundell and Walker (1986).
[40] Camelot tried to boost Wednesday sales by having superdraw-guaranteed jackpots for many of the early Wednesday draws.

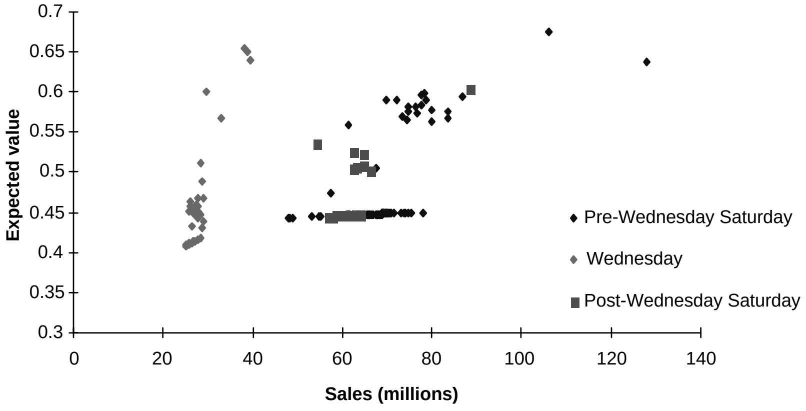

Figure 8. Expected ticket value, sales and the Wednesday draw

corresponding fall in the consumer surplus enjoyed on Saturdays, but this loss is more than compensated by the consumer surplus that can now be enjoyed on Wednesdays. Introducing the new draw increased welfare, but did not generate any significant intertemporal substitution. The implication of this analysis is that play is sufficiently habitual that intertemporal substitution is small, so we can rely on our previous estimates to assess game design.

The only public data about play in consecutive draws is in the Family Expenditure Survey, which records expenditure in great detail, including for on-line draw purchases. FES respondents record expenditures in diaries for two consecutive weeks, hence we can examine the stability of individual behaviour over time. Figure 9 shows expenditures in consecutive (non-rollover) draws from early FES data for on-line draw sales.[41] The number of people at each point is reflected in the density of each blob. While the majority of observations are on the diagonal (people spend the same amount in each draw, usually just a few pounds), a sizeable minority move into and out of lottery participation even in the absence of rollovers. This is reasonably convincing evidence that there is a strong 'fixed effect' in behaviour, but there is also sizeable random variation.

4.4. Odds misperceptions and other game characteristics

My simple framework has assumed that rollovers affect demand only through their effect on expected value. However, this belies the fact that game designs are much more complex than such a theory would suggest.

[41] The data, from April to October 1995, do not separate out scratchcards from the on-line draw. Moreover, the 1995/6 FES data do not allow us to isolate the data by week and individual. The data from April 1996 will be more informative.

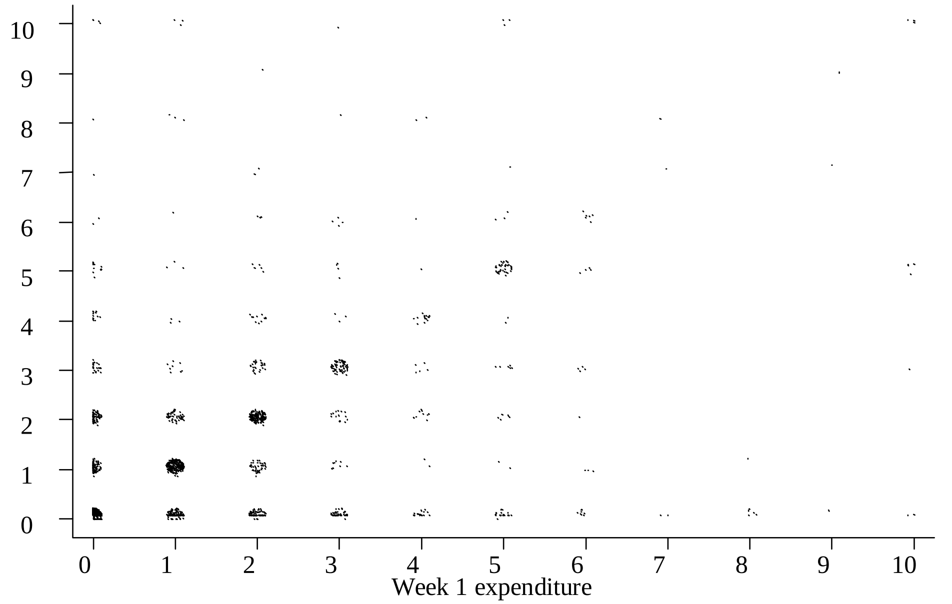

Figure 9. Sales in consecutive weeks

The UK Consumers' Association surveyed 2029 individuals in a particular draw in 1995, finding widespread evidence that respondents knew little about the odds in the game. The survey asked about the odds of winning any prize (1 in 57) and of winning the jackpot (1 in 14 million). The true probabilities were overestimated by a third of respondents, underestimated by a third and assessed broadly correctly by a third. It is hard to see a *systematic* bias in people's beliefs. Nor was misperception correlated with individual characteristics, such as class or education.

There is some empirical support for the idea that optimistic people spend more than pessimistic people: multivariate modelling of expenditure patterns reveals that the elasticity of expenditure, conditional on participating, with respect to the *perceived* odds in the data is not significantly different from -1 (but rather imprecisely estimated).

The Consumers' Association data also contain information about attitudes to other game features, such as how the revenue was used. Contrary to popular opinion, attitudes to a wide variety of aspects of game design seem to be uncorrelated with expenditure in any systematic way. Although people may say that they would prefer a different game design (more money to charity, smaller jackpots, etc.), what they say they prefer is not reflected in what they actually do.

Misperception may also be generated by prize-winning experience. Individuals who win a prize may subsequently feel that the odds are better. Are recent winners more likely to play than recent losers? The FES data yield no statistically significant relationship between winning one week and play the following week (controlling for the extent of play last week); nor do aggregate data show a significant relationship between weekly sales and the number of £10 prizewinners in the previous week. I find these results surprising. If they are true, they undermine the most obvious rationale for designing a game with smaller, but more frequently won, prizes.

5. INCOME, LOTTERY DEMAND AND TAX REGRESSIVITY

The earlier estimate of 'price' elasticity of demand could not control for income, since in time-series data there is no observable variation in income. I therefore use micro data, collected by National Opinion Polls on behalf of the Office of the National Lottery (OFLOT), the industry regulator, to examine how income differences across players in regular draws compared to rollover draws, and estimate how income affects demand, estimating price and income elasticities from these micro data. I use these estimates to decompose welfare gains from the introduction of the UK lottery to see which individuals benefited most. I contrast the commonly expressed *ex post* view about the losses associated with playing, with the consumer surplus *ex ante* view, in which play is necessarily beneficial (since playing is voluntary).

The micro data pool five sample surveys, of about 1800 individuals each.[42] One of these surveys, by chance, coincided with a 'double rollover'. Thus 20% of the sample is obtained from a survey where the expected value was exogenously higher. The final sample contained 9077 observations, some 1795 of which were drawn from the double rollover survey.[43] Only individuals above the legal minimum age for buying a lottery ticket were interviewed.

As before, the expected value of a ticket reflects the level of sales and the size of any rollover: the average V for regular draws was 0.45 compared with 0.62 for the double rollover. Some 63% of people participated in the regular draws, 73% in the double rollover draw. Participation and expenditure also changed markedly in rollover draws. The respective average levels of expenditure were £1.50 and £2.30. Mean expenditures, conditional on participating, were £2.40 and £3.10. These data (excluding the 0.12% of individuals buying over 49 tickets per draw, whom I dropped) match the aggregate sales data quite well. Scaling up the samples to reflect the aggregate UK population aged sixteen and over would imply average aggregate sales of £69 million over the four regular draws (compared with actual average sales

[42] These surveys corresponded to the draws in weeks 35, 43, 60, 65 and 78, where the first draw of the National Lottery was the second week of November 1994. More comprehensive descriptions of the data can be found in OFLOT (1995, 1996a,b).

[43] The income question was unanswered in 48% of cases and, in any case, is grouped. To estimate an income elasticity, I 'merge' FES income data by extending the 'complementary matching' technique of Arrelano and Meghir (1992). This involves estimating an income equation from FES data, and predicting the income for those in OFLOT data using FES coefficients. Here, I exploit the fact that the variable of interest is not missing (for half the sample), but reported as being within a range.

of £69 million), and £104 million in the double rollover (compared to £128 million in the actual aggregate sales for this draw).[44]

The distribution of expenditure is given in Figure 10. The typical player in regular draws spends a modest £1 or £2; in rollover draws there is a much stronger tendency to spend £5. The data also show that men spend more than women; working individuals spend more than the retired, unemployed or students; and there is a clear lifecycle effect, whereby the young and old spend little.

The theory suggested that, at high sales at which expected value asymptotes to $(1-\tau)$, the expected value is therefore insensitive to sales. The variance around this figure, excluding rollover draws, is indeed small. We really observe demand at just two points: average demand at a high price (when $V = 0.45$) and average demand at a lower price (a rollover draw, when $V = 0.63$). Since we effectively compare the means across the two samples, we need to partial out the effects of other factors that change between the two samples, especially income. Empirically, I find that richer people are more likely to play in rollover draws than regular draws.[45] The effect of

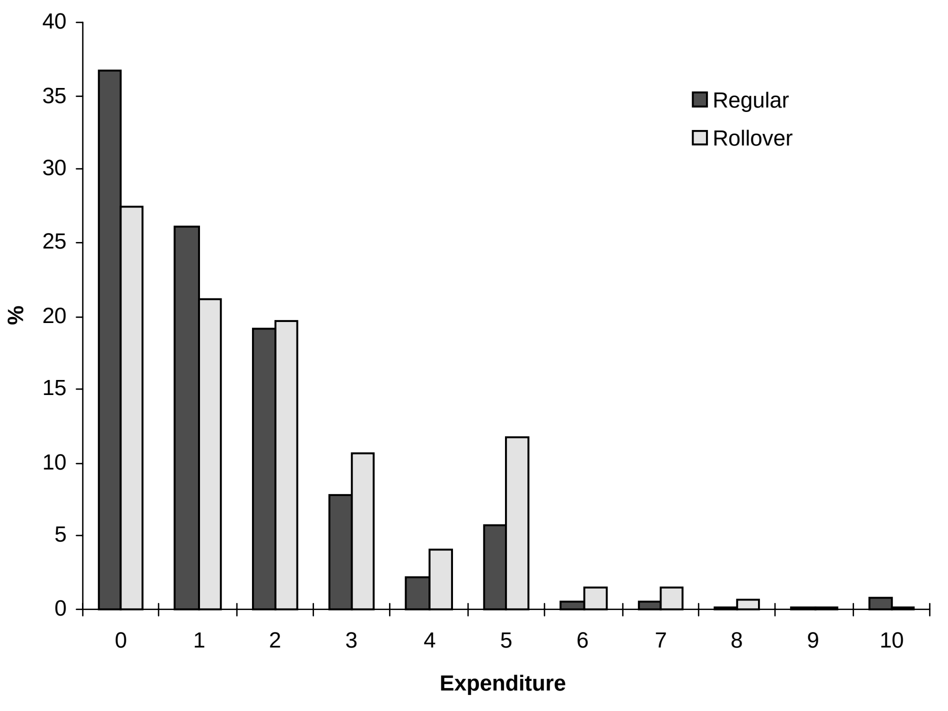

Figure 10. Expenditure distributions in rollover and regular weeks

[44] The data ask about tickets bought for oneself and may exclude syndicates, which are likely to spend more heavily in rollover weeks.

[45] Income is recorded only in discrete categories, and some income data are missing. As in Stewart (1983), I estimate an equation for income from the grouped data that allows me to predict income as a continuous variable.

income may be non-linear – the poor are more likely to participate, but the rich can afford more tickets – so I enter predicted income as a quadratic (in £ per draw). I also include dummy variables for missing income, female, married and age groups.

Ignoring the discrete nature of the dependent variable (tickets bought), but recognizing that purchases cannot be negative, I use essentially a Tobit estimation procedure to estimate individual demand for tickets (which for a significant number of people is zero demand). This Tobit specification is used by Scott and Garen (1994), a recent example from a small literature which uses techniques suitable when the distribution of tickets bought per player has a mass point at zero.[46]

The estimated effects of individual characteristics show the extent to which women play less; married individuals play more; and play rises with age, plateaus in middle age, and falls steeply in old age. The missing income indicator never achieves statistical significance. The estimated price elasticity[47] (−1.3, with a standard error of 0.25) is (insignificantly) higher than the short-run estimate from aggregate data, although my aggregate equation omitted income from the specification. The explanation for the higher elasticity may be that the micro data include a *double* rollover, and that the relationship between demand and ticket value may be non-linear; in addition, the large increase in the skewness of the prize distribution in the double rollover may be important.

The estimated income elasticity is negative. Lottery tickets are an inferior good once the regression results are purged of lifecycle and education variation in income by the inclusion of years of education and a quadratic in age. This is an important result. The income elasticity determines the incidence of the tax element in the price. The inferiority of lottery tickets makes the (high) lottery tax strongly regressive.

Leaving aside distribution for a moment, the welfare gain of introducing lottery tickets can be measured by consumer surplus as before. To do so, we need to incorporate the participation effect: although many individuals in the sample did not participate, there is some probability that they might, and we need to incorporate this probability into the analysis to capture the variance in effects across individuals. However, I ignore any distinction between compensated and uncompensated demand because the income elasticity is relatively small and the share of lottery tickets in total expenditure is very small at about 1%.

The results, averaged across participants and non-participants, imply a consumer surplus of £0.42 per ticket in a regular draw, £0.62 in a rollover and £0.71 in a

[46] In fact, I use an extension of the Tobit model, the Censored Least Absolute Deviation estimator, which uses 'symmetric trimming' to overcome the dependence of Tobit results on the assumption that residuals are normally distributed. See Farrell and Walker (1996) for further details.

[47] The price elasticity is the elasticity with respect to V multiplied by $-(1-V)/V$, since the cost of a ticket is £1, so that the 'price' is $1-V$. The elasticity here is short run because it takes no account of greater sales in the weeks subsequent to the double rollover draw.

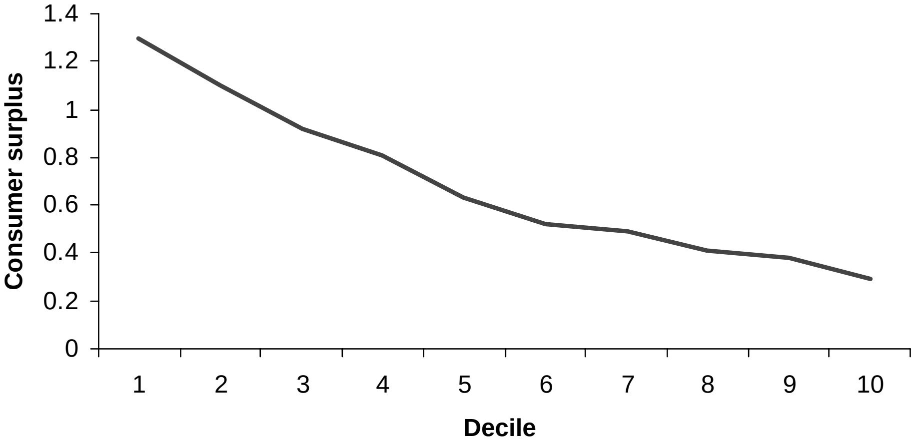

Figure 11. Distribution effects

double rollover. The annual consumer surplus is therefore about £1 billion, a little higher than the figure from aggregate data because of the higher elasticity.

Figure 11 ranks individuals by the residual from my income-determining equation (purging income of its age correlation) and shows the means of the surpluses for individuals in each decile of the age-adjusted income distribution. The largest surplus is at the bottom of this distribution, where the expected number of tickets purchased is largest: after controlling for age and education, lotteries are an inferior good. Note that, while the poor enjoy the largest *ex ante* consumer surplus, since the financial return to lottery participants is heavily negative, the poor also feature disproportionately *ex post* among financial losers from lottery play.

6. POLICY IMPLICATIONS

The analysis has several implications for policy. First, I find no significant evidence that players suffer from systematic misperceptions about the way in which lotto operates; nor much evidence that there are markedly inappropriate levels of play: the young play little (but we need better data on under-age play, where there are significant recording problems), and the unemployed and students also play little. There is some evidence of a small proportion of heavy players, but data here are, by their nature, thin. We need much larger samples before we can pass judgement.

There is a more sophisticated argument that bundling a lottery ticket with a contribution to some public good can overcome the free-rider problem: the positive externality from contributing to a public good is just balanced by the negative externality associated with buying a lottery ticket. The later arises in the fixed-prize game (raffles or numbers games) because the prize is common property. Thus the argument is a second-best one, trying to produce an equilibrium in which the correct quantity of the public good is funded. However, the argument works only with fixed-prize games and hence is irrelevant to lotto.

Thus, I find no compelling evidence to suspend the usual consumer sovereignty considerations. My analysis suggests that big welfare gains were derived from having a lottery in the UK (having two draws has increased these gains). It would be surprising if the same were not true elsewhere. Whether the UK should have more than two draws is a delicate question. The 'natural' scale economies of lotto[48] must be balanced against the desire for greater product diversity. Camelot has focused demand on the present brand by encouraging conscious selection; it is unlikely that there would be strong demand for further lotto games. If conscious selection falls further, there may be a case for yet another draw (perhaps, a second Saturday draw) to cut further sales on Saturdays, causing more rollovers.

Free entry is unlikely to be a sensible policy, since this would simply dissipate the surplus (which the good causes now enjoy). In federal countries, like the USA and Australia, the power of each state to raise its own revenue is limited by federal statute and confined to a few methods, typically duties on cigarettes and alcohol. Licensing of gambling has been a lucrative way for states to generate independent sources of revenue,[49] and there has been a proliferation of casinos[50] and lottery games. In parts of the USA, in particular, strong competition between local games works to the detriment of players and operators alike: scale economies are sacrificed when sales are low in any one game. Regulation in the UK (and elsewhere) tries to prevent such competition developing.

Whether lotto games should be operated directly by a government agency (as in most countries and many US states) or by a licensed private operator regulated by government (as in the UK and some US states) is harder to judge. In principle, competitive tendering for the licence ought to allow the government to extract all of the surplus, and the UK operator appears to be at least as efficient as any other operator.[51]

There is a long-term threat to the viability of individual games through Internet lotto, presently unpopular because players are reluctant to trust an untried company not to abscond with the prize pool. Operators with a reputation for operating a successful game, perhaps acquired through operating a conventional lotto, may be able to engender sufficient trust to operate such a game.

[48] These are additional to any scale economies arising from the large fixed costs associated with the infrastructure.
[49] Indeed, in Australia there are now no differences in income or sales taxes across states, and gambling duty is the only vehicle that gives states the power to vary their revenue.
[50] Some states have licensed 'riverboat' casinos – although there is usually no requirement that they actually set sail. Other states have licensed casinos on the reservations of Native Americans, where they serve as a source of employment as well as revenue.
[51] The 1997 White Paper suggests that subsequent licences may be awarded to a not-for-profit operator. However, it is unclear whether a not-for-profit operator might have higher costs than a profit-maximizing operator.

However, the profitability of such an extension to conventional sales outlets might have to weigh the gains from additional (foreign) Internet sales with any loss in domestic sales from domestic customers preferring not to play in games in which foreigners might win. A game with considerable scale economies from strong domestic sales might be attractive to foreign players whose own lotteries were smaller or non-existent. Conversely, an Internet operator, should it get off the ground, might hope to drive domestic lotteries out of business, both because a larger market would convey greater scale economies and because it would be likely to face lower tax rates than those currently imposed on domestic lotteries.

I have argued that there is a large consumer surplus from being able to play a lottery. Moreover, the estimates of the long-run elasticity suggest that the UK game seems to be operating at an elastic part of the demand curve, which indicates that it could increase revenue if the take-out rate were reduced. Apart from this incorrect 'pricing', the UK game seems to be efficient and there are grounds for thinking that it is operated at lower cost than any other lottery despite (perhaps, because of) the large (absolute level of) profits that the operator earns. Since lotteries in other countries have similar take-out rates, appropriate pricing may not hold in other countries either, though one would need to examine demand in each country to verify such a conjecture. Despite too high a take-out rate having been chosen, the UK operator has maintained demand by encouraging a high level of rollovers (given to the game design) and introducing a mid-week draw.[52]

The UK lottery regulator OFLOT is charged with maximizing good causes revenue, subject to not encouraging inappropriate levels of play. There remain some worrying aspects of the game. First, there is some evidence that the game is addictive (habit forming rather than physically damaging). However, my estimates suggest that addiction is modest: post-rollover sales return to within 0.1% of the original level of sales within four weeks. Moreover, it has the side-effect that, combined with rollovers, it roughly offsets the tendency for people to tire of the game. Otherwise, the lottery operator would have to devote more resources to promotion and product development.

While there is evidence from elsewhere on under-age play (OFLOT, 1996a, b), no research yet conducted can show what the effect of increasing the age limit might be on play by those under age. What is required is data that would allow us to model the probability of retailers selling to particular individuals: information on the height and appearance of youths might be adequate. Thus far, research on under-age play has been only descriptive, without much thought as to how to inform policy.

[52] The scratchcard game has been relatively unsuccessful. It experienced a decline in sales from a peak of £40 million per week shortly after its introduction, but sales have since stabilized at around £20 million per week. Recent product development and promotion seem not to have improved sales markedly.

Survey data do suggest a very small number of people (perhaps 80,000) playing in excess of £50 each draw. As yet, we have no way of knowing whether this tail in the distribution arises from a persistent group of individuals or from the occasional syndicate ploughing small winnings back into the game. Moreover, we simply do not yet have enough data to be confident about the size of the group of heavy players. Panel data are required to assess this, but none are yet in the public domain. The regulator could easily rectify these data deficiencies at modest cost. Indeed, this information is required to show that regulation is being effective.

Neither regulator nor operator is concerned with consumer surplus *per se*. I have argued that the introduction of the lottery created considerable consumer surplus. By the same token, further large gains in consumer surplus could be obtained by reducing the take-out rate; having a high elasticity of demand, the purchase of lottery tickets is reasonably price sensitive and a high tax rate therefore creates a large distortion. The deadweight loss imposed by the good causes levy is remarkably high. In no other case do we impose such large tax rates on price-sensitive commodities that are not subject to some other motive (congestion, pollution, etc.) for taxation. If the government believes that revenue for good causes is desirable, and that some market failure, such as free riding, prevents people making adequate voluntary contributions, good causes should be centrally financed through taxation, perhaps higher VAT or income tax. The high take-out rate, which increases the effective price of a ticket, needlessly limits the huge enjoyment that the game engenders.

One can understand the arguments for taxing either sales (as is done) or winnings (as income). It is harder to rationalize why players must donate to good causes as a condition of play. Since no other commodity is supplied under similar conditions, lotto is singled out to carry this burden. Moreover, since the poor spend a higher fraction of their income on lottery tickets than do the rich, and since the good causes supported (e.g. opera) may favour the rich, game design is not merely inefficient but inequitable. Indeed, attempts to widen the distributive benefits of the good causes money (to support education and health care) seem likely to fall foul of the additionality critique – the government would be tempted to use lottery funds to cut regular expenditure of such causes.

One suggestion for using the lottery to fund good causes without imposing such a high welfare loss might be to discriminate between sales in regular draws and those following rollovers. This has *some* deadweight loss, since confiscation of part (or indeed all) of the rollover would raise the effective ticket price. Confiscating rollovers for the deserving many seems eminently fairer than donating it to a lucky few in the following draw. Against this, one needs to consider the role that rollovers currently play in bolstering long-run demand.

Finally, there are other aspects of behaviour that I have not discussed in detail. One concern is about the additionality of good causes revenue: it may simply displace an equal amount of government (or other) funding that otherwise would have occurred.[53] Connolly and Bailey (1996) argue that the requirement for good causes to raise matching funding from elsewhere reduces this problem, but the creation of the 'New Opportunities' fund in mid-1997 does not seem to have this requirement, and appears to fund activities, such as education and health care, that are traditionally central government responsibilities.

The additionality argument also arises in the context of charitable giving. Some charities have argued that lottery funding partially crowds out voluntary contributions. Evidence is sketchy, but a comparison of charitable giving in the FES data (which are less than comprehensive) before and after the introduction of the game shows no significant difference (Banks and Tanner, 1997). Since it is unclear what would have happened in the absence of the game, this 'before and after' comparison is not fully convincing. However, there is no difference in charitable giving levels in the *weekly* FES data when there is a rollover compared to regular draws; since rollovers are exogenous random events, this might imply that the effect on giving is modest.

Some other sectors of the economy also claim to have been adversely affected. However, good evidence is difficult to find because we do not know what would have happened in the absence of the lottery. In particular, before and after comparisons are unrevealing, especially in identifying effects on expenditures that were already trending (such as soccer 'pools' betting). Rollovers can rescue us from this impasse again: simply comparing the budget shares in the FES data in rollover draws with regular draws reveals no difference to the third decimal place. This suggests that the lottery shifts expenditure away from other commodities in the same expenditure category (Leisure Services). Thus, at least some of the decrease in soccer pools spending may be due to the introduction of the lottery.[54]

To sum up, well-designed lotteries should exist, since there is a large demand for them and little compelling evidence that they do any significant harm, although a badly designed game might. Moreover, the games benefit the poor disproportionately. Imposing a large take-out rate generates large efficiency losses as well as being regressive, and there is a strong temptation for governments to appropriate the take-out as a form of general revenue. While the UK lotto game seems to be intelligently run, my estimates of long-run demand elasticity suggest that the UK game is priced too high to maximize revenue (which is the objective of the regulator), and effective regulation is still required to monitor play closely. This requires data not yet available to researchers, either in the UK or elsewhere.

[53] Clotfelter and Cook (1989) note that expenditure on education in US states where lottery revenue is hypothecated to education appears to be no higher than expenditure in states that do not do this.

[54] However, Creigh-Tyte (1997) estimates, using aggregate data, that other gambling expenditure has not fallen since the introduction of the NL.

Discussion

Karl-Gustaf Löfgren

Umeå Universitet, Sweden

This paper deals with an unusual topic within our science, but a common activity in everyday life. The scant interest shown by economists in the economics of gambling perhaps reflects our difficulty in rationalizing gambles that are actuarially unfair. Most of our theory about behaviour in risky situations is based on the idea that individuals are risk averse, which means that they would not participate in an unfair game. Walker gives a short, but essentially complete survey of the main attempts to solve this puzzle. Like him, I am attracted by the ideas put forward by Conlisk (1993), who assumes that individuals are risk averse, but that participating in gambles generates some additional non-pecuniary pleasure (thrill). Hence, for small bets, the thrill of the gamble can offset the monetary cost generated by the unfair odds. However, as the stakes become larger, risk aversion becomes more important, and people find it worthwhile to insure against large risks such as house fires.

Walker's own work is empirical and policy oriented, but based on a simple analytical model, which unfortunately does not encompass a reason why individuals participate in the lotto game. This is problematic when he is conducting a welfare analysis of the lotteries. He derives the expected value of a lottery ticket in terms of ticket sales, the take-out ratio and any possible rollover. This he calls a supply curve, although it is simply the expected value of a ticket as a function of sales.

Walker estimates the demand curve by using the fact that the value of the ticket is typically higher in a rollover than in a regular draw. The price variation is used to estimate sales as a linear function of the expected value of the ticket, from which he computes a 'demand' elasticity of about -1, taken as an indication that the game is optimally designed to maximize tax revenue. He also uses the demand curve to compute the consumer surplus of the game design, and the deadweight loss from raising the government revenues by a lotto game. The details are hard to follow, but I am more worried about the principles involved.

As Walker points out, the conditions for aggregating demand curves require that all individuals face the same price, which here means the subjective probability of winning. This requires players to forecast sales (and know their dependence on the subjective estimates made by other participants). In such circumstances, misperceptions of the odds must be common.

Walker's own data refute the basis for his extreme rational expectations assumption. The survey he cites shows that, although there is no severe and systematic misperception of the odds, the respondents knew little about the odds in the game; nor did the survey ask about the odds conditional on the number of tickets sold. Suppose, for example, that instead of the 'extreme rational expectations assumption' being valid, players underestimate the value of a ticket in a regular game (a positive externality), but overestimate it in a rollover game (a negative externality). Walker's empirical procedure would then obtain an incorrect estimate of the demand elasticity, and hence draw inappropriate conclusions about consumer surplus.

The need for an appropriate theory as a basis for a welfare analysis is also indicated by the fact that the excess burden of taxation is what the consumers would be willing to pay to get rid of the tax, minus what the tax raises in terms of revenue. Why would a lotto player be willing to pay more than the increase in the expected value from a decrease in the take ratio? Under approximate risk neutrality, the answer is that the 'thrill' of the game increases with the magnitude of the prize. One way to get more information about the thrill component would be to conduct controlled contingent valuation experiments. In particular, the answer to a question about the willingness to pay to participate in games with different, and sometimes unknown, experimental designs can be much more accurate than the answer to the corresponding question, not uncommon in environmental economics, about the willingness to pay for, say, the preservation of wolves.

One of the strengths of the paper is the rich data material used. There is also a micro dataset consisting of five sample surveys, of about 1800 individuals each. One of these surveys coincided with a double rollover, so there is a variation in the expected value of a ticket, which can be used to obtain an alternative estimate of the response to changes in the expected value of a ticket. It also contains the incomes of the players, so a response in the number of tickets bought with respect to income can also be estimated. Although, as with macro data, I have a problem with using the resulting estimates to conduct welfare analysis, I do find it interesting that the elasticity with respect to the expected value does not significantly differ from the macro estimate, that the income elasticity is negative and small (lotto is an inferior good), that women play less than men, that married individuals play more and that play rises with income.

I think, however, that there may be a minor statistical problem with the estimates. Since there are many people in the sample who do not buy lottery tickets, Walker uses a Tobit estimation procedure to estimate the individual demand for tickets. This is typically a very useful approach when the distribution has a mass point of zero. However, the dependent variable, the number of tickets, is count data, which violate the normality of the residual in the Tobit-model. Therefore, it might have been more appropriate to use a count data model, such as a Poisson regression.

Despite these criticisms, the paper contains a lot of useful empirical information on the design, regulation and taxation of lotteries. The dataset is unique, and Walker handles it with considerable imagination and creativity. I am convinced that this paper will become required reading for anyone wanting to do further research on the economics of lotteries.

Urs Schweizer

University of Bonn

It is a pleasure to comment on Ian Walker's paper. I am intrigued that so many people are willing to play regularly in lotteries in which the expected value of a ticket is less than half its purchase price. I shall offer further evidence on lotteries from Germany, make some critical remarks on Walker's interpretation of demand for lottery tickets, and propose another interpretation of demand to show that consumer surplus depends crucially on the particular interpretation.

Evidence from Germany

Germany also uses a 6/49 design, with a seventh bonus ball (BB), and the last digit of the ticket's identification number is used as a super number (SN). A 6/49 game has been run every Saturday since October 1955, and a Wednesday game since January 1988. The latter has two draws, and the winning ticket must match only one of them. The take-out rate is 50%. There are seven prize pools. Pool 1 requires matching the 6 balls and the super number; pool 2 requires only a match of the 6 balls; pool 3, a match of 5 of the 6 balls and the bonus ball; and so on down to pool 7, where only 3 of the 6 balls need be matched. Sales revenue, net of take-out and operating costs, is assigned to the pools in fixed proportions. Table 2 captures the details. The design of Saturday draws changed in 1997.

The design allows for rollovers in all pools, pool by pool. In practice, however, rollovers occur only in pool 1. During 1996:1 to 1998:4, pool 1 was typically shared by between 0 and 3 winners; pool 2 by between 1 and 17 winners, though there were 125 winners on one occasion. Pool 1 has seen fourteen single rollovers, double rollovers five times, triple rollovers three times, four consecutive rollovers twice, and

Table 2. The winning pools in German lotteries (allocation of net revenue)

Pool	Balls matched	Saturday, pre-1997 (%)	Saturday, post-1997 (%)	Wednesday (×2) (%)
1	6 + SN	4	6	12
2	6	12	10	10
3	5 + BB	6	6	6
4	5	20	20	19
5	4	20	20	17
6	3 + BB	14	14	9
7	3	24	27	27

I thank Hans Peter Grüner and Patrick Schmitz for helpful comments on both theoretical aspects and German lottery design.

even one occasion with six consecutive rollovers! Rollover frequency far exceeds that expected from random choice; it must reflect conscious choice. Sales revenues per Saturday draw are about DM130 million in normal weeks, up to DM160 million after consecutive rollovers.

Conscious choice is nicely documented by two examples. In 1977, 205 winners shared a jackpot of DM6.3 million; in 1988, 222 winners shared a jackpot of DM18.8 million. The winning combination was the same as the week before in the Dutch and the German lottery, respectively. A statistics service lists the winning combination, the prize money per pool, and the number of winners in each pool, yielding information on the numbers that tend to be preferred by players. It may help players who want to avoid popularly chosen ticket numbers.

Norman Faber is well known among lotto players in Germany. He used to offer free tickets in exchange for personal information, probably including further information on preferred combinations. He now organizes successful pools of up to 218 persons, sharing 1000 tickets per Saturday draw, submitting combinations randomly chosen from his list of unpopular combinations. His letterhead documents his performance during August–December 1997: two single wins in pool 1, 4 in pool 2 when nobody won pool 1, and two in pool 2 when there was a single winner of pool 1. He not only wins, but avoids crowded pools. In any case, there is demand for intermediation in lotteries. Faber seems to make a good living out of it.

Revisiting the simple analysis of lottery demand

The expected value of holding a lottery ticket depends on the number of tickets sold and the amount rolled over from previous weeks. Demand in normal weeks is fairly stable. If, due to rollovers, the expected value rises, demand increases. Walker estimates a relation between sales and expected ticket value. One can in fact use this to deduce that, for the UK, tax revenue is maximized by a take-out rate of 52%. Whether or not the UK tax authorities were aware of my estimate, their rate of 55% is amazingly close to this rate.

Walker suggests we interpret the difference between unit purchase price of a ticket and its expected value V as the perceived price $P = 1 - V$. He assumes that lottery tickets yield intrinsic utility (pleasure), and that a player's total willingness-to-pay rises (but at a diminishing marginal rate) with the number of tickets bought, the cost of N tickets being perceived as $(1 - V)N$. A rational player buys tickets up to the point where the marginal utility equals the perceived cost per ticket. The area under the demand relationship is perceived consumer surplus. In a normal week (no rollovers), perceived cost is the take-out rate, which for the UK implies consumer surplus of £15.8 million and tax revenues of £36 million per week. Consumer surplus plus tax revenues are about £51.8 million per week. Although this looks like quite a free lunch, such a welfare evaluation, based on perceived prices, is questionable.

If demand depended solely on the expected value of a ticket, as Walker assumes, participants would be indifferent between different designs of lotteries yielding the same expected values. But we know that people prefer lotteries with a smaller chance of a larger prize. Nobody would play a game where everyone got back with certainty $(1-\tau)$ of their ticket price every time! Given Walker's pleasure motive, demand for lottery tickets should depend on game design, including the size of the big prize. Hence his basic estimated relationship, relating sales to expected value alone, is an unsound basis for evaluation of consumer surplus and policy conclusions based on this.

A different foundation of demand

Walker claims that participating in an unfair lottery is at odds with conventional expected utility theory, yet reports various attempts by the literature to solve the apparent puzzle. Some, I think, are dismissed prematurely. Ng (1975), for example, sees lotteries as a mechanism for allowing some individuals to purchase large indivisible and durable goods. Suppose people play to have a chance of being able to afford an expensive, indivisible good (e.g. education), which yields a higher rate of return on other investments.[55] Let x_0 be the cost of the indivisible good. Without it, the return on investment x is bx; with it, the return is $B(x-x_0)$, where B exceeds b.

If j is the expected value of a ticket showing the winning combination of balls, π the probability of matching this combination, and V the expected value of a ticket, then $\pi j = V$. Suppose nobody has wealth w sufficient to buy the indivisible good that a winning ticket then makes affordable. The usual design of lotteries is likely to fulfil these requirements (unlike my trivial example, above, of an unattractive lottery!).

A player with T tickets has a probability $T\pi$ of picking the winning combination and accessing the higher returns; with probability $(1-T\pi)$, the player is stuck with the low rate of return. Players compute the expected gain from buying tickets and choose the number of tickets to maximize consumer surplus, which occurs[56] when consumer surplus is $\pi(B-b)T^2$. The shape of the demand curve is shown in Figure 12.

[55] Hans-Werner Sinn has pointed out to me that Daniel Bernoulli, in a paper published in Latin in 1738 (English translation in *Econometrica*, 1954, was the first to think of such a technology: 'Thus, though a poor man generally obtains more utility than does a rich man from an equal gain, it is nevertheless conceivable, for example, that a rich prisoner who possesses two thousand ducats but needs two thousand ducats more to repurchase freedom, will place a higher value on a gain of two thousand ducats than does another man who has less money than he.' Obviously, to get out of prison, it may be perfectly rational to participate in a lottery among prisoners.

[56] The expected gain from buying T tickets is $T\pi B(w+j-x_0-T)+(1-T\pi)b(w-T)$. Since $\pi j = V$, the expected surplus, from buying T rather than 0 tickets, is $-\pi(B-b)T^2-[\pi B(x_0-w)+b(1+w\pi)]T+BVT$. Choosing tickets to maximize consumer surplus implies $BV=2\pi(B-b)T+\pi B(x_0-w)+b(1+w\pi)$, whence maximized consumer surplus occurs at $\pi(B-b)T^2$. This interior solution prevails if V is in the range $V_L=\{\pi(x_0-w)+[b/B](1+w\pi)\}<V<\{[(B-b)/B]\pi w+\pi x_0+b/B\}=V_H$. Note that the preceding formulae imply that maximized consumer surplus can also be written as $B\Delta$, where $\Delta = T(V-V_L)/2$

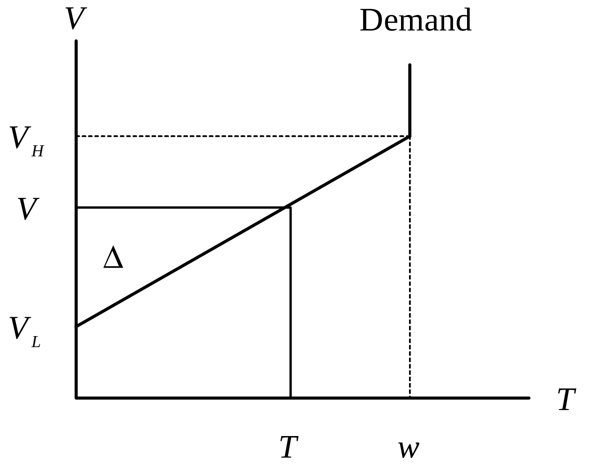

Figure 12. An alternative demand curve

The area to the left of this demand curve, denoted by $\Delta = T(V - V_L)/2$, is the consumer surplus used in Walker's welfare analysis. With my interpretation of demand, consumer surplus is not Δ but $B\Delta$. It is the high rate of return times the area to the left of the demand curve. The analysis could be developed further; at present, my aim is merely to show that the welfare conclusions drawn depend heavily on the interpretation of demand.

Concluding remarks

Conscious choice and misperception of odds are well-established aspects of human behaviour. Successful lottery intermediation, as by Norman Faber, exploits the bounded rationality of ordinary lottery players; in such activities, the theory of fully rational decisions has hit its limits. To develop a theory of bounded rationality of lottery players, the experimental approach might be of use, as it has been previously in similar areas. However, this comes at a price. By giving up individual utility functions, the main ingredients for welfare analysis are lost. Yet, from a theoretical viewpoint, lotteries are such an interesting topic of welfare analysis – as Walker's paper and, hopefully, also my note have shown – that, as a benchmark, the fully rational approach has its own merits.

General discussion

A large part of the discussion dealt with the question of how the misperception of chances affects the welfare analysis. Paul Klemperer argued that people do not really understand risk. Even though people might be on average right about the probability of winning the jackpot, they nevertheless have difficulties in understanding what these numbers mean. For a typical person buying a lotto ticket an hour before the lottery, the probability of winning is still only one-fortieth of the probability of that person dying in the hour before the draw takes place.

Urs Schweizer suggested introducing another 'lottery' scheme that produces the same expected value per ticket, but makes misperceptions of probabilities on the participants' side impossible. For instance, there could be a lottery in which each £1 ticket pays 45p for sure. Despite having the same expected value as an actual lottery ticket, this ticket (a loss of 55p with certainty) would hold few attractions! To base the welfare analysis on actual demand based only on expected values, it has to be explained how misperception occurs in the transfer from true to perceived values.

David Begg noted that the UK market for retail gambling is an excellent case for studying the misperception issue. When there are two candidates in a bet (e.g., a cup final in football), the profit margins for the competitive betting chains in the UK are about 8–9%, just enough to run the business. The profit margin quickly rises when the number of candidates in a bet increases. For the 32 horses in the Grand National, the profit margin might be over 20%: people have difficulty in calculating the chances, and the industry has a good idea of the degree of complexity at which the misperception starts. Karl-Gustaf Löfgren argued that it is difficult to estimate the consumer surplus based on subjective probabilities, as the demand function itself is not observable.

Friedrich Breyer defended the paper's approach to welfare analysis. Even if people systematically overstate their chances of winning – and there is a lot of empirical evidence for this in the experimental literature – we can rationalize their behaviour by applying subjective probabilities. The resulting demand function can be used for welfare evaluations. Consumers do not have to be correct in their absolute estimate of the probabilities. They only have to be correct about how the probabilities change with the size of the jackpot.

Paul Klemperer suggested making use of the existing literature on smoking and drinking to get a better understanding of the complicated welfare issues at stake. In this parallel literature, the issues of addiction, taxing the consumption and the regressive nature of these taxes also play an important role. If 'selling hope' is a justification for lotteries, it might be desirable to limit the number of tickets that can be bought so as to limit the addictive behaviour of some players.

Philippe Aghion wondered why people buy single tickets each week even though playing lotteries is addictive and people know that. He suggested using the hyperbolic approach to preferences to rationalize this phenomenon.

Alberto Alesina strongly advocated a *laissez-faire* approach in the lottery business. It may be true that playing the lottery is addictive, as argued in the paper. However, there are many activities that are addictive, such as smoking, drinking and watching TV. The addictive nature of lotteries alone is no reason for prohibiting this good.

Furthermore, there is no need to reserve gambling for a public monopoly. The regulation creates additional social costs by making gambling, other than lotteries, illegal and this, therefore, criminalizes almost the whole business. The policy implication could not only be to tax less, but also to liberalize gambling.

Paul Dobson wondered whether there is some evidence that only one gamble might be ideal. In the UK, a number of organizations are lobbying the government to relax gambling laws and to set up alternatives to lotto. One advantage would be a reduction in the deadweight loss because the government would no longer be able to take such a high cut once there was competition. This advantage has to be weighed against the loss in economies of scale.

Eva Gutierrez suggested including the rent-seeking activities in the social cost of lotteries. Social institutions that benefit from the proceeds of a public lottery do a significant amount of lobbying to prevent the entry of new lotteries.

Kai Konrad criticized the paper's policy conclusion that the organizer should choose the lottery with the lowest operating costs. As participation in the lottery itself creates benefits for the consumers, what really matters is how the whole game is organized.

Alberto Alesina saw a parallel with the literature on voting. In a large election, the probability that you are pivotal in voting is essentially zero; nevertheless people vote. This phenomenon does not imply that people misperceive their chance of being pivotal. It is just that they like to vote. As in the case of lotteries, it is the good 'excitement' that people are really interested in.

Friedrich Breyer raised the question of why there are intermediate classes of prizes and not only one big prize. One explanation could be that there is some contagion effect, where the willingness to participate increases if people know someone who has won (even if it was only a small prize). Paul Klemperer noted that the experience of winning a small prize, such as three out of six correct numbers, adds to the misperception of the chances for a jackpot.

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