Problem Set 5 - EC123

Q4 (a)

Writing $z = e^{x(t)y(t)^3}$ and using the chain rule we find that $\frac{dz}{dx} = \left(\frac{dx}{dt}y^3 + x3y^2\frac{dy}{dt}\right)e^{xy^3}$. Substituting in for $x'(t)$ and $y'(t)$ we get

$$\frac{dz}{dx} = \left(2ty^3 + x3y^2 3t^2\right)e^{xy^3} = ty^2 e^{xy^3}(2y + 9x)$$

Q4 (b)

Writing $z = x^2 h(x(t), y(t))$ again we use the chain rule to get

$$\frac{dz}{dt} = 2x\, x'(t)\, h(x(t), y(t)) + x^2 \frac{dh(x(t), y(t))}{dt}$$

$$\frac{dz}{dt} = 2x\, 2t\, h(x, y) + x^2\left(\frac{\partial h(x, y)}{\partial x}\frac{dx}{dt} + \frac{\partial h(x, y)}{\partial y}\frac{dy}{dt}\right)$$

$$\frac{dz}{dt} = 2tx\, h(x, y) + x^2\left(\frac{\partial h(x, y)}{\partial x}2t + \frac{\partial h(x, y)}{\partial y}3t^2\right)$$

Q5 (a)

We are given $z = F(x, y) = x(t)^2 + e^{y(t)}$ and then compute

$$\frac{dz}{dt} = 2x\frac{dx}{dt} + e^y\frac{dy}{dt} = 2x3t^2 + e^y 2$$

Simplifying this eventually leads to $\frac{dz}{dt} = 2\left(3t^5 + e^{2t}\right)$

Q5 (b)

Starting with $Y(K, L) = K(t)\, L(t)^2$ we get

$$\frac{dY}{dt} = \frac{\partial Y}{\partial K}\frac{dK}{dt} + \frac{\partial Y}{\partial L}\frac{dL}{dt} = L^2 K'(t) + 2KLL'(t)$$

Q5 (c)

As I think I said in the seminar, it might be easier to rewrite the function as $F(r, u(r), v(r))$ where $u(r) \equiv (1-r)$ and $v(r) \equiv (1-r)^{-1}$. Then differentiating F with respect to r we get

$$\frac{\partial F}{\partial r} + \frac{\partial F}{\partial u}\frac{du}{dr} + \frac{\partial F}{\partial v}\frac{dv}{dr} = \frac{\partial F}{\partial r} - \frac{\partial F}{\partial u} + \frac{\partial F}{\partial v}(1-r)^{-2}$$

Q5 (d)

Finding $\frac{dz}{dt}$ and $\frac{dz}{ds}$ we should see that

$$\frac{dz}{dt} = \frac{\partial F}{\partial f}\frac{df}{dt} + \frac{\partial F}{\partial g}\frac{dg}{dt} \text{ and } \frac{dz}{ds} = \frac{\partial F}{\partial g}\frac{dg}{ds}$$