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Assignment 4
EC9D3 Advanced Microeconomics

1. There are three parts to the question.

(a) Show that the social welfare function that coincides with individual i's preferences satisfies U, WP, and IIA. Call such a social welfare function an individual i dictatorship.

(b) Suppose that society ranks any two social states x and y according to individual 1's preferences unless he is indifferent in which case x and y are ranked according to 2's preferences unless he is indifferent, etc. Call the resulting social welfare function a lexicographic dictatorship. Show that a lexicographic dictatorship satisfies U, WP and IIA and that it is distinct from an individual i dictatorship.

(c) Describe a social welfare function distinct from an individual i dictatorship and a lexicographic dictatorship that satisfies U, WP and IIA.

2. Let $x = (x^1, ..., x^N)$ be an allocation of n goods to agents, such $x \in \mathbb{R}_+^{nN}$, and let the set of feasible allocations be the compact and convex set T. Suppose x^* maximises the utilitarian social welfare function, $W = \sum_{i=1}^{N} u^i(x^i)$, subject to $x \in T$. For simplicity, restrict attention to continuously differentiable utility indexes u^i.

(a) Let ψ^i for $i = 1, ..., N$ be an arbitrary set of increasing continuously differentiable functions of one variable. Does x^* maximise $\sum_{i=1}^{N} \psi^i(u^i(x^i))$ over $x \in T$? Why or why not?

(b) If in part (a), $\psi^i = \psi$ for all i, what would your answer be?

(c) If $\psi^i = a^i + b^i u^i(x^i)$ for arbitrary a^i and $b^i > 0$, what would your answer be?

(d) If $\psi^i = a^i + b u^i(x^i)$ for arbitrary a^i and $b > 0$, what would your answer be?

3. Call a social choice function f strongly monotonic if $f(R) = x$ implies $f(\tilde{R}) = x$ whenever for every individual i and every $y \in X$, xR_iy implies that $x\tilde{R}_iy$.

Suppose there are two individuals, 1 and 2, and three social states, x, y, and z. Define the social choice function f to choose individual 1's top-ranked social state unless it is not unique, in which case the social choice is individual 2's top-ranked social state among those that are top-ranked for individual 1, unless this too is not unique, in which case, among those that are top-ranked for both individuals, choose x if it is among them, otherwise choose y.

(a) Prove that f is strategy-proof.

(b) Show by example that f is not strongly monotonic. (Hence, strategy-proofness does not imply strong monotonicity, even though it implies monotonicity.)

4. Suppose that there are just two alternatives x and y, and at least 3 agents. Consider the majority rule social choice function f that which chooses the outcome that is the top ranked choice for the majority of individuals if it is unique, and else picks $f(R) = x$.

(a) Show that f is Pareto efficient

(b) Show that f is strategy-proof.

(c) Show that f is non-dictatorial.