Warwick University
Department of Economics

Prof. Francesco Squintani

EC9D3 Advanced Microeconomics
Additional Questions - Set 3

1. In a two-persons (Ms. A and Mr. B), two-commodities (apples and oranges), pure exchange economy, Ms. A likes only apples and does not care how many oranges she has. On the other hand, Mr. B likes only oranges and does not care how many apples he has. Both people behave as price-takers.

(i) Suppose that A owns all the apples and B all the oranges. Is there a Walrasian equilibrium? If so, what is (are) the equilibrium price and allocation(s)?

(ii) What are the Pareto optimal allocations in such an economy?

(iii) Suppose now that the initial endowments are such that Ms. A owns *some* oranges and Mr. B *some* apples. Is there a Walrasian equilibrium? If so, describe any equilibrium price and allocation.

2. Draw an Edgeworth box diagram for a pure exchange economy in which *no* Walrasian Equilibrium exists but in which there are many (as many as you can find) Pareto Optima.

3. In a two-consumer (Ms. A and Mr. B), two-commodity, X_1 and X_2, pure exchange economy, A's preferences are represented by the utility function:

$$U^A(x_1^A, x_2^A) = 2\,x_1^A + x_2^A.$$

Consumer B's preferences are represented by the following utility function:

$$U^B(x_1^B, x_2^B) = \alpha\,x_1^B + x_2^B$$

where x_j^i denotes the quantity of commodity $j \in \{1, 2\}$ consumed by consumer $i \in \{A, B\}$.

Assume, first, that the parameter α is such that

$$\alpha = \frac{1}{2} \tag{1}$$

The total quantities available in the economy of commodity x_1, denoted by $\bar{x}_1$, and of commodity x_2, denoted by $\bar{x}_2$, are identical. Further, assume that the endowment allocation is such that A owns half of the entire quantity available of commodity 1, $\bar{x}_1/2$, and half of the quantity available of commodity 2, $\bar{x}_2/2$.

(i) Find the *offer curves* of both Ms. A and Mr. B.

(ii) Find the set of Walrasian equilibrium prices and allocations in this economy.

(iii) Are the Walrasian equilibrium allocation(s) you find in (ii) above Pareto efficient? Explain your answer.

Now consider the following modification of this economy. Assume that A's preferences are unchanged while B's preferences are characterized by the following value of the parameter α

$$\alpha = 4 \tag{2}$$

(iv) Find the new *offer curve* of Mr. B.

(v) Find the set of Walrasian equilibrium prices and allocations in this modified economy.

(vi) Are the Walrasian equilibrium allocation(s) you find in (v) above Pareto efficient? Explain your answer.

Finally, consider the following third modification of this economy. Assume that A's preferences are unchanged while B's preferences are characterized by the following value of the parameter α

$$\alpha = 4/3 \tag{3}$$

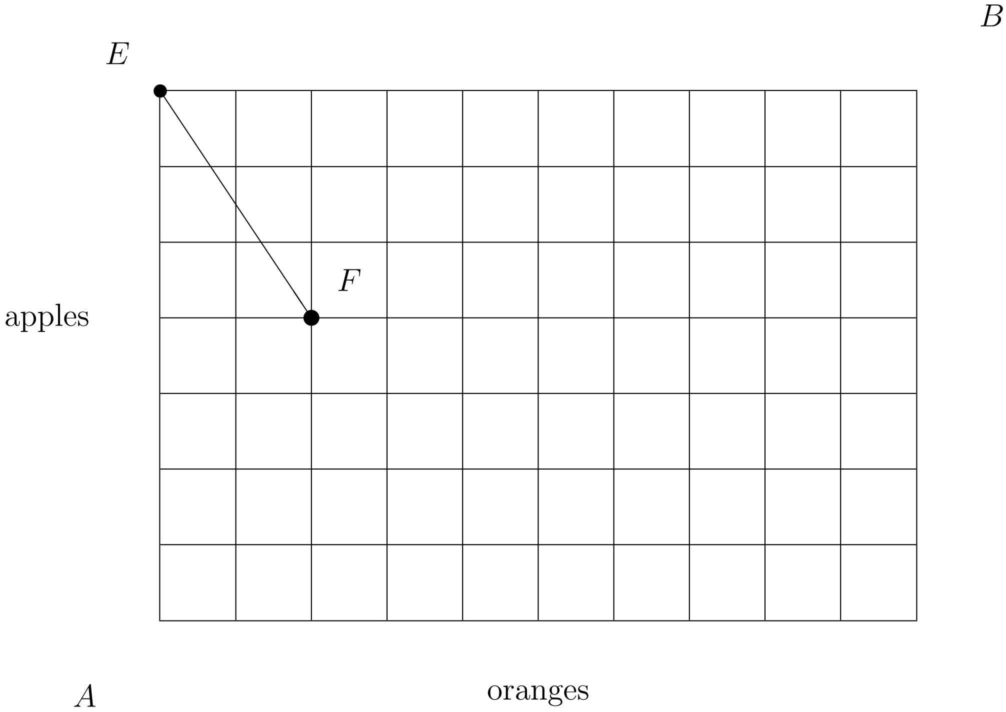
(vii) Find the new *offer curve* of Mr. B.

(viii) Find the set of Walrasian equilibrium prices and allocations in this modified economy.

(ix) Are the Walrasian equilibrium allocation(s) you find in (v) above Pareto efficient? Explain your answer.

Answers

1. Consider the situation depicted in the Edgeworth box below.



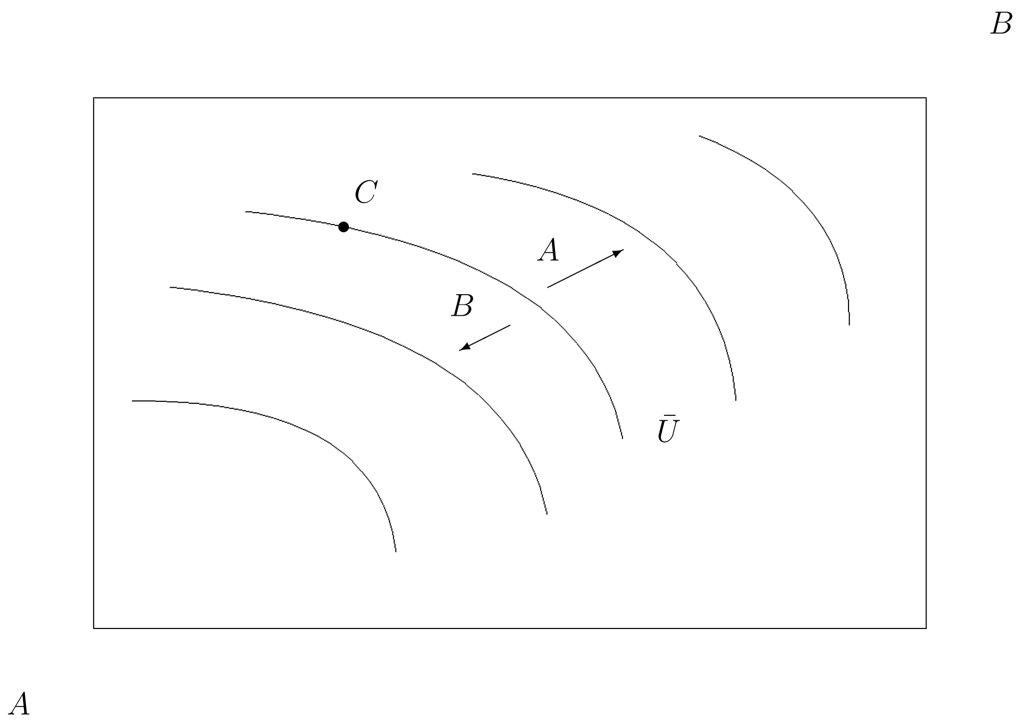
A's indifference curves are straight lines parallel to the horizontal axe while B's indifference curves are straight lines parallel to the vertical axe.

The point E denotes the unique *Walrasian Equilibrium* of this economy. Furthermore E also denotes the unique *Pareto Optimal* allocation.

If E is also the endowment point of this economy the following properties hold. No trade will occur at the competitive equilibrium and any price will sustain the equilibrium allocation E. The Walrasian equilibrium price is *indeterminate*.

If the endowment allocation is F then the following properties hold. E is the only competitive equilibrium allocation. The slope of the line on which the segment $\bar{EF}$ lies is the relative price at which equilibrium trade occurs. It is also the relative price that sustains the Walrasian equilibrium.

2. Consider the situation described in the following Edgeworth box.



A's indifference curves are concave, while B's indifference curves are convex and both set of indifference curves have exactly the same shape. In other words $\bar{U}$ is one of the indifference curves of A and B.

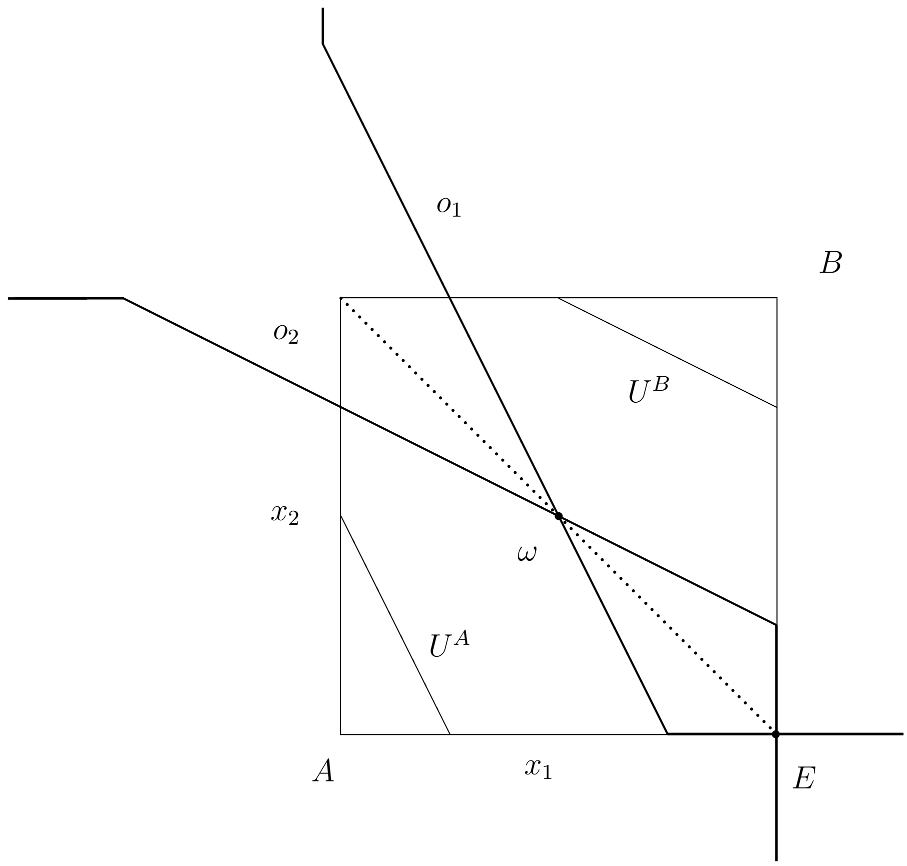
A north-east movement increases A's utility but decreases B's utility.

In this framework there does *not* exist any Walrasian equilibrium (continuity of excess demand functions or upper hemi-continuity of excess demand correspondences fails).

There might exists only one Walrasian equilibrium at the south-west corner of the box provided that the slope of the indifference curve going through the origin is neither 0 nor ∞. In this case the Walrasian equilibrium price will coincide with the slope of the indifference curve through that corner.

However, *any* point in the box is a Pareto optimal allocation. Indeed, any movement from say point C in any direction will make A or B weakly worse off or at most both indifferent. Hence C cannot be Pareto improved upon.

3. When $\alpha = 1/2$ the economy can be described in the Edgeworth box represented below.



(i) The offer curve of consumer A, denoted o_1, is the solution to A's utility maximization problem:

$$\begin{aligned} \max_{x_1^A, x_2^A} \quad & 2\,x_1^A + x_2^A \\ \text{s.t.} \quad & p\,x_1^A + x_2^A \leq p\,\frac{\bar{x}_1}{2} + \frac{\bar{x}_2}{2} \end{aligned} \tag{4}$$

for every value of p where we normalize the price of x_2 to 1. This offer curve is plotted in the Edgeworth box above.

The offer curve of consumer B, denoted o_2, is instead the solution to B's utility maximization problem:

$$\begin{aligned} \max_{x_1^B, x_2^B} \quad & \frac{1}{2}\,x_1^B + x_2^B \\ \text{s.t.} \quad & p\,x_1^B + x_2^B \leq p\,\frac{\bar{x}_1}{2} + \frac{\bar{x}_2}{2} \end{aligned} \tag{5}$$

and it is also represented in the same Edgeworth box above.

(ii) The Walrasian equilibrium is denoted by E. The equilibrium allocation is: $x_1^A = \bar{x}_1$, $x_2^A = 0$, $x_1^B = 0$ and $x_2^B = \bar{x}_2$. The equilibrium relative price is $p^* = 1$.

(iii) The Pareto-efficient allocations are:

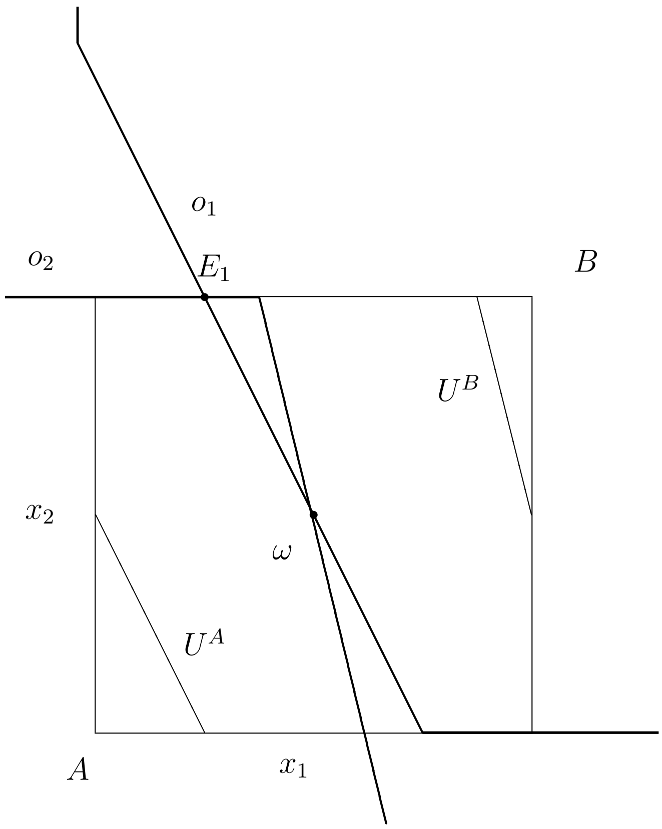
$$\{0 \leq x_1^A \leq \bar{x}_1, x_2^A = 0, x_1^B = \bar{x}_1 - x_1^A, x_2^B = \bar{x}_2\}$$

and

$$\{x_1^A = \bar{x}_1, x_2^A = \bar{x}_2 - x_2^B, x_1^B = 0, 0 \leq x_2^B \leq \bar{x}_2\}$$

Therefore the Walrasian equilibrium allocation in (ii) above is Pareto efficient.

Consider the economy under the alternative specification of B's preferences: $\alpha = 4$.



(iv) The offer curve of consumer A is obviously unchanged while the offer curve of consumer B is now the solution to the following problem

$$\begin{aligned} \max_{x_1^B, x_2^B} \quad & 4\, x_1^B + x_2^B \\ \text{s.t.} \quad & p\, x_1^B + x_2^B \leq p\, \frac{\bar{x}_1}{2} + \frac{\bar{x}_2}{2} \end{aligned} \tag{6}$$

and it is depicted in the Edegeworth box above.

(v) E_1 is a Walrasian equilibrium. This equilibrium corresponds to the allocation: $x_1^A = \bar{x}_1/4$, $x_2^A = \bar{x}_2$, $x_1^B = 3\,\bar{x}_1/4$ and $x_2^B = 0$. The equilibrium relative price is $p^* = 2$, the absolute value of the slope of A's indifference curves.

(vi) The Pareto-efficient allocations are now

$$\{x_1^A = 0, 0 \leq x_2^A \leq \bar{x}_2, x_1^B = \bar{x}_1, x_2^B = \bar{x}_2 - x_2^A\}$$

and

$$\{0 \leq x_1^A \leq \bar{x}_1, x_2^A = \bar{x}_2, x_1^B = \bar{x}_1 - x_1^A, x_2^B = 0\}$$

Therefore the Walrasian equilibrium allocation in (v) above is Pareto efficient.

Consider the economy under the alternative specification of B's preferences: $\alpha = 4/3$.

(vii) The offer curve of consumer A is once again unchanged while the offer curve of consumer B is now the solution to the following problem

$$\begin{aligned} \max_{x_1^B, x_2^B} \quad & \frac{4}{3} x_1^B + x_2^B \\ \text{s.t.} \quad & p\, x_1^B + x_2^B \leq p \frac{\bar{x}_1}{2} + \frac{\bar{x}_2}{2} \end{aligned} \tag{7}$$

and it is depicted in the Edegeworth box below.

(viii) E_2 is a Walrasian equilibrium. This equilibrium corresponds to the allocation: $x_1^A = 7\,\bar{x}_1/8$, $x_2^A = 0$, $x_1^B = \bar{x}_1/8$ and $x_2^B = \bar{x}_2$. The equilibrium relative price is $p^* = 4/3$, the absolute value of the slope of B's indifference curves.

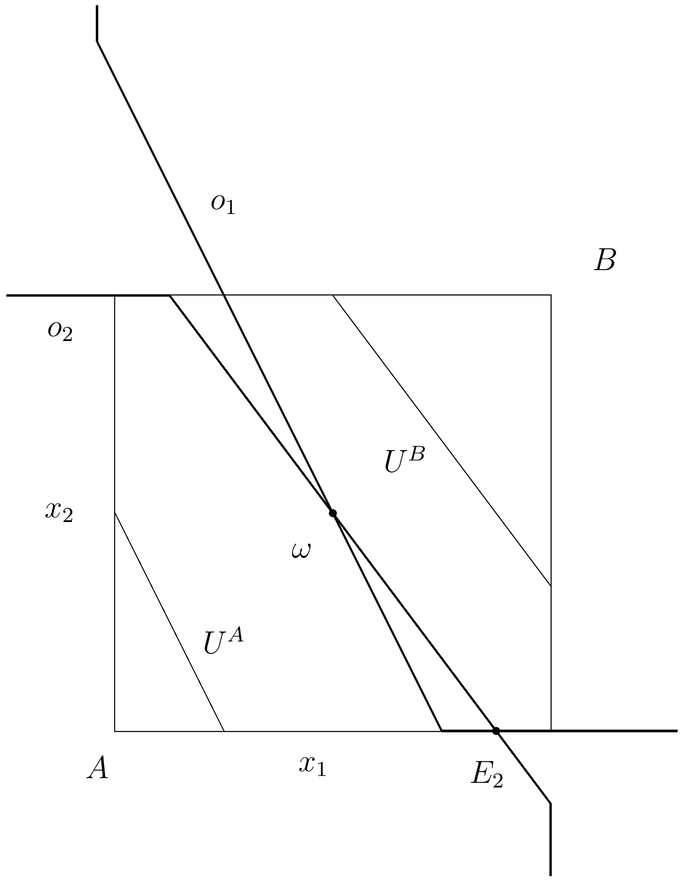
(ix) The Pareto-efficient allocations are now

$$\{0 \leq x_1^A \leq \bar{x}_1, x_2^A = 0, x_1^B = \bar{x}_1 - x_1^A, x_2^B = \bar{x}_2\}$$

and

$$\{x_1^A = \bar{x}_1, x_2^A = \bar{x}_2 - x_2^B, x_1^B = 0, 0 \leq x_2^B \leq \bar{x}_2\}$$

as in (iii) above. Therefore the Walrasian equilibrium allocation in (viii) above is Pareto efficient.


o_1
B
o_2
U^B
x_2
ω
U^A
A
x_1
E_2