Problem Set 4

Exercise 1. Whether candidate 1 or candidate 2 is elected depends on the votes of two citizens. The economy may be in one of two states, A and B. The citizens agree that candidate 1 is best if the state is A and candidate 2 is best if the state is B. Each citizen's preferences are represented by the expected value of a Bernoulli payoff function that assigns a payoff of 1 if the best candidate for the state wins (obtains more votes than the other candidate), a payoff of 0 if the other candidate wins, and payoff of 1/2 if the candidates tie. Citizen 1 is informed of the state, whereas citizen 2 believes it is A with probability 0.9 and B with probability 0.1. Each citizen may either vote for candidate 1, vote for candidate 2, or not vote.

a. Formulate this situation as a Bayesian game. (Construct the table of payoffs for each state.)

b. Show that the game has exactly two pure Nash equilibria, in one of which citizen 2 does not vote and in the other of which she votes for 1.

c. Show that one of the player's actions in the second of these equilibria is weakly dominated.

d. Why is the "swing voter's curse" an appropriate name for the determinant of citizen 2's decision in the second equilibrium?

Exercise 2. Firm A (the "acquirer") is considering taking over firm T (the "target"). It does not know firm T's value. It believes that this value, when firm T is controlled by its own management, is at least \$0 and at most \$100, and assigns equal probability to each of the 101 dollar values in this range. Firm T will be worth 50% more under firm A's management than it is under its own management. Suppose that firm A bids y to take over firm T, and firm T is worth x (under its own management). Then if T accepts A's offer, A's payoff is $\frac{3}{2}x - y$ and T's payoff is y; if T rejects A's offer, A's payoff is 0 and T's payoff is x.

a. Model this situation as a Bayesian game in which firm A chooses how much to offer and firm T decides the lowest offer to accept.

b. Find the Nash equilibria of this game.

Exercise 3. Consider a sealed-bid first-price and second-price auctions with private values, in which the players are risk averse. Specifically, suppose each of the n players' preferences are represented by the expected value of the Bernoulli payoff function $x^{1/m}$, where x is the player's monetary payoff and $m > 1$. Suppose also that each player's valuation is distributed uniformly between 0 and 1.

a. Show that the Bayesian game that models a first-price sealed-bid auction under these assumptions has a (symmetric) Nash equilibrium in which each type v_i of each player i bids $(1 - 1/[m(n-1)+1])v_i$.

b. Compare the auctioneer's revenue in this equilibrium with her revenue in the weakly dominant symmetric solution of the second-price sealed-bid auction.