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**Solutions to Assignment 3**
**EC9D3 Advanced Microeconomics**

**1.** The equilibrium in which there is no surplus labour or capital is characterized by three sets of conditions.

The first set of conditions is the full employment conditions that solves for the total production of commodities $(x_1, x_2)$

These conditions are:

$$3x_1 + 2x_2 = 22$$
$$5x_1 + 4x_2 = 40$$

From the conditions above we obtain the following equilibrium outputs: $x_1 = 4$ and $x_2 = 5$.

The second set of conditions that helps in the characterization of the equilibrium is the zero profit conditions generated by the CRS technology. This conditions require the price of the output to equal the marginal cost of inputs which in the fixed coefficient technology at hand coincides with the average cost:

$$3r + 5w = p$$
$$2r + 4w = 1 - p$$

where $r$ denotes the unit price of capital, $w$ the unit price of labour, and we let the price of the consumption commodities 1 and 2 be $p$ and $(1 - p)$ respectively

From the zero profit conditions above we obtain:

$$r = \frac{9p - 5}{2} \tag{1}$$
$$w = \frac{3 - 5p}{2} \tag{2}$$

We can now compute the income of the whole group of capitalists:

$$r\bar{K} = 11(9p - 5)$$

and of the whole group of workers:

$$w\bar{L} = 20(3 - 5p).$$

The Marshallian demands of the whole group of capitalists are:

$$x_1^c = \frac{8(9p-5)}{p} \qquad x_2^c = \frac{3(9p-5)}{1-p}$$

while the Marshallian demands of the whole group of workers are:

$$x_1^w = \frac{8(3-5p)}{p} \qquad x_2^w = \frac{12(3-5p)}{1-p}.$$

Using Walras Law we focus on the market of commodity $x_1$. The equilibrium condition is then:

$$x_1^c = \frac{8(9p-5)}{p} + x_1^w = \frac{8(3-5p)}{p} = 4$$

which yields: $p^* = \frac{4}{7}$ and $r^* = \frac{1}{14}$, $w^* = \frac{1}{14}$.

The equilibrium allocation associated with these relative prices is then: $x_1^c = x_1^w = 2$ and $x_2^c = 1$, $x_1^w = 4$.

**2.** We proceed in sequence as follows.

(i) We start deriving the *aggregate supply:* each firm's cost function is:

$$c(w, y, \alpha) = w\,\frac{y^2}{\alpha}$$

hence each firm's supply function is:

$$y(p, w, \alpha) = \frac{\alpha\,p}{2\,w}$$

and the aggregate supply is:

$$Y(p, w) = \int_1^3 y(p, w, \alpha)\frac{1}{2}d\alpha = \int_1^3 \frac{\alpha\,p}{2\,w}\,\frac{1}{2}d\alpha = \frac{p}{w}.$$

We can now compute the *Walrasian equilibrium:* the market clearing condition is:

$$Y(p, w) = Q(p)$$

or

$$\frac{p}{w} = 10 - p$$

which gives:

$$p^* = \frac{10\ w}{w+1}.$$

Firm $\alpha$'s profits are:

$$\Pi_\alpha = \frac{25\ w\ \alpha}{(w+1)^2}$$

from which we get:

$$\frac{\partial \Pi_\alpha}{\partial w} = \frac{25\ \alpha\ (1-w)}{(w+1)^3}.$$

(ii) We conclude that:

- $\dfrac{\partial \Pi_\alpha}{\partial w} > 0$ if and only if $w < 1$,
- $\dfrac{\partial \Pi_\alpha}{\partial w} < 0$ if and only if $w > 1$, and
- $\dfrac{\partial \Pi_\alpha}{\partial w} = 0$ if and only if $w = 1$.

(iii) The intuition behind this conclusion can be stated as follows. The equilibrium price $p^*$ depends on $w$ because the slope of the aggregate supply depends on $w$ hence three cases may occur:

- if $w < 1$ an increase in the slope of aggregate supply induces a more than proportional increase in the output price with respect to input price increase, hence profits increase,
- if $w = 1$ an increase in the slope of aggregate supply induces a proportional increase in output price with respect to the input price, hence profits do not change,
- if $w > 1$ an increase in the slope of aggregate supply induces a less than proportional increase in output price with respect to the increase in input price, hence profits decrease.

**3.** There are four parts to the answer.

(i) We start from the analysis of the *no-grade regime*. The student problem is:

$$\begin{array}{ll} \max\limits_{x} & G(x) \\ \text{s.t.} & 0 \leq x \leq 24 \end{array}$$

The solution will depend on the shape of the function $G(\cdot)$. For example:

- if $G'(x) > 0$ for every $x \in [0, 24]$ then $x^* = 24$,
- if $G'(x) < 0$ for every $x \in [0, 24]$ then $x^* = 0$,
- if $G'(0) > 0$ and $G'(24) < 0$ then

$$G'(x^*) = 0$$

  and

$$G''(x^*) < 0.$$

(ii) Assume from now on $G''(x) < 0$ for all $x \in [0, 24]$. Consider now the grade regime. The student problem is now:

$$\begin{array}{ll} \max\limits_{x} & G(x) + F\left(\dfrac{x}{y}\right) \\ \text{s.t.} & 0 \leq x \leq 24 \end{array}$$

We shall focus on the case in which this problem has an interior solution. The first order condition is:

$$G'(x^{**}) + \frac{1}{y} F'\left(\frac{x^{**}}{y}\right) = 0 \tag{3}$$

and the second order condition is:

$$G''(x^{**}) + \frac{1}{y^2} F''\left(\frac{x^{**}}{y}\right) < 0. \tag{4}$$

Since all students are identical, symmetry implies:

$$y = \frac{1}{n} \sum_{i=1}^{n} x_i^{**} = x^{**}$$

hence (3) becomes:

$$G'(x^{**}) + \frac{1}{x^{**}} F'(1) = 0 \tag{5}$$

which given that by assumption

$$\frac{1}{x^{**}} F'(1) > 0$$

implies

$$G'(x^{**}) < 0$$

hence

$$G'(x^{**}) < G'(x^{*})$$

which if $G'' < 0$ implies:

$$x^{**} > x^{*}$$

that is over-investment in working hours.

Moreover given that $F(1) = 0$ we get:

$$G(x^{**}) + F(1) = G(x^{**}) < G(x^{*})$$

In fact, $x^{*}$ is the value of $x$ that maximizes $G(x)$.

(iii) We shall now derive the symmetric Pareto optimal choice of hours. This is the solution to the following problem:

$$\max_{\hat{x}} \sum_{i=1}^{n} G(x) + F(1) = \max_{\hat{x}} G(x). \tag{6}$$

We obtain:

$$\hat{x} = x^{*}.$$

(iv) A quota is easy to use to obtain the symmetric Pareto optimal choice of hours just set an upper-bound on the amount of hours worked: $h = x^{*}$.

Consider now a tax/subsidy scheme. Let $t$ be the per unit of hour tax and $g$ the lump-sum subsidy. The student problem facing such a tax is:

$$\begin{array}{ll} \max\limits_{x} & G(x) + F\left(\dfrac{x}{y}\right) - t\,x + g \\ \text{s.t.} & 0 \le x \le 24 \end{array}$$

The first order conditions are:

$$G'(x) + \frac{1}{x}\,F'(1) - t = 0$$

hence $t = \frac{F'(1)}{x^*}$, or $T = t\ x^* = F'(1)$ is the optimal tax. The optimal subsidy is $g = t\,x^*$.