Political Economy
Theory and Experiments

Lecture 2

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Ideological candidates

. Suppose candidates are not only motivated by winning elections.

. Like voters, politicians have policy preferences.

. Although ideological, candidates who credibly commit to policy platforms "converge" to median, if voters preferences are known.

. Instead, I will show that platforms "diverge" if there is aggregate uncertainty on voters' preferences.

. Platforms also diverge if candidates who cannot commit to political platforms in conflict with their preferences.

Downsian elections with ideological candidates

. Suppose there are two candidates $i = L, R$ with ideologies b_i such that $b_L < m < b_R$, and $m - b_L < b_R - m$.

. The utility of candidate i if policy x is implemented is $u_i(x, b_i) = L(|x - b_i|)$, with $L' < 0$.

Theorem The unique Nash Equilibrium is such that candidates i choose $x_i = m$, and tie (although candidates are ideological).

Proof. For any $x_L \neq x_R$, if $x_i < x_j$, candidate i's vote share is $F(\frac{x_L + x_R}{2})$, and candidate j's is $1 - F(\frac{x_L + x_R}{2})$.

. Suppose that $x_L < m$, then candidate R wins and implements x_R by choosing x_R in $(x_L, 2m - x_L)$.

. Hence, if $x_L < 2m - b_R$, R's best response $BR_R(x_L) = \{b_R\}$, and if $2m - b_R < x_L < m$, then $BR_R(x_L)$ is empty.

. But if $x_R = b_R$, then $BR_L(x_R)$ is empty.

. If $m < x_L < b_R$, then $BR_R(x_L) = [x_L, +\infty)$.

. If $x_L > b_R$, then $BR_R(x_L) = \{b_R\}$.

. But if $x_R > x_L > m$ or $x_R = b_R$, then $x_L \notin BR_L(x_R)$.

. Hence, there is no Nash Equilibrium with $(x_L, x_R) \neq (m, m)$.

. Suppose that candidate i chooses $x_i = m$.

. Then, implemented policy is m regardless of x_j, and $BR_j(x_i) = (-\infty, +\infty)$.

. We conclude that the unique Nash Equilibrium is $x_L = x_R = m$, and the election is tied.

Aggregate uncertainty and policy-motivated candidates

. I consider a probabilistic voting model with aggregate uncertainty and policy motivated candidates.

. In unique symmetric equilibrium, candidates' platforms diverge.

. If voters update their preferences during campaigns, they are all ex ante better off when parties diverge to some extent.

. Voters are better off with moderate policy-motivated candidates than with office-motivated candidates.

. This is in contrast with models where voters preferences are fixed.

Value of platform divergence

- Each voter j with bliss point $b_j \in \mathbb{R}$ has utility $L(|b_j - x|)$, with $L' < 0$, $L'' < 0$, and $\lim_{z \downarrow 0} L'(z) = 0$, $\lim_{z \uparrow \infty} L'(z) = -\infty$.
- The ideal point b_j is decomposed as: $b_j = m + \delta_j + \varepsilon_j$:
  - δ_j is the fixed j's bias relative to the median platform m, the distribution of δ_j has compact support and zero median,
  - ε_j is i.i.d. with $E[\varepsilon_j] = 0$, symm. density on compact support.
  - m is the random median platform, with c.d.f. F and median μ.
- Assume that F is symmetric and $\mu = 0$.
- Consider divergent platforms $x_L = -x$ and $x_R = x$, with $x \geq 0$.
- Platform x_L wins if and only if $m < \frac{x_L + x_R}{2} = 0$.

. The expected welfare of voter j is:

$$W_j(x) = \int_{-\infty}^{0} L(|m+\delta_j+\varepsilon_j - x_L|)f(m)dm + \int_0^{\infty} L(|m+\delta_j+\varepsilon_j - x_R|)f(m)dm$$
$$= \int_0^{\infty}[L(|-m-\delta_j-\varepsilon_j+x|) + L(|m+\delta_j+\varepsilon_j - x|)]f(m)dm.$$

$W_j(x)$ is concave as it is the sum of integrals of concave functions.

Proposition There exists a welfare-improving threshold $\overline{x} > 0$ such that $W_j(x) > W_j(0)$ for all voters j whenever $0 < x < \overline{x}$.

Proof: Compare the difference one m at a time:

$$L(|\delta_j+\varepsilon_j-(m-x)|) + L(|\delta_j+\varepsilon_j-(-m+x)|)$$
$$\text{vs. } L(|\delta_j+\varepsilon_j-m|) + L(|\delta_j+\varepsilon_j-(-m)|)$$

. This is equivalent to comparing two lotteries with fixed $\delta_j+\varepsilon_j$:

even chance on $-m+x$, $m-x$ and even chance on $-m$, m.

. Clearly, when $x < m$, policy convergence is a mean-preserving spread of divergence at $-x$ and x... and voter j is better off.

. For all δ_j, ε_j in the (compact) supports, $\frac{\partial W_j}{\partial x}(x)|_{x=0} > 0$.

. By strict concavity, there is unique $x(\delta, \varepsilon) > 0$ such that $W_j(0) = W_j(x)$ and by continuity $\overline{x} = \min_{\delta,\varepsilon}\{x(\delta, \varepsilon)\} > 0$.

. The aggregate voter welfare W^* is strictly concave:

$$W^*(x) = \int_{\delta,\varepsilon} \int_0^\infty [L(|-m-\delta_j-\varepsilon_j+x|) + L(|m+\delta_j+\varepsilon_j-x|)] dF(m) dH(\delta,\varepsilon).$$

Proposition A first-order stochastic increase in $f(\cdot|m > 0)$ induces an increase in the welfare-maximizing platform x^*.

Sketch of proof: For a greater spread in f, welfare is maximized by reducing payoff of moderate m and increasing payoff of extreme m.

Quadratic-normal case

. Assume L is quadratic, i.e., $L(z) = -z^2$.

. Say m is distributed normally with mean zero and variance σ^2.

. For each voter δ, ε, simplification yields:

$W_{\delta,\varepsilon}(x) = -2\int_0^\infty (x-m)^2 dF(m) - (\delta+\varepsilon)^2 = W_{0,0}(x) - (\delta+\varepsilon)^2$.

. By mean-variance analysis, $W^*(x)$ is a quadratic function:

$$\begin{aligned} W^*(x) &= -\int_{\delta,\varepsilon}[2\int_0^\infty (x-m)^2 dF(m) + (\delta+\varepsilon)^2] dH(\delta,\varepsilon) \\ &= -2E[(x-m)^2|m>0] - E[(\delta+\varepsilon)^2] \\ &= -2(x - E[m|m>0])^2 - V[m|m>0] - V[\delta] - V[\varepsilon]. \end{aligned}$$

. The social optimum is then $x^* = E[m|m>0] = \sigma\sqrt{2/\pi}$.

. As W^* is symmetric around x^*, $\overline{x} = 2E[m|m>0] = 2\sigma\sqrt{\frac{2}{\pi}}$.

Model and equilibrium

- Candidates L and R have ideal points $-b$ and $b > 0$.
- Office benefit $w \in \mathbb{R}_+ \cup \{\infty\}$.
- Pure policy motivation is $w = 0$, pure office is $w = \infty$.
- Candidate R's payoff from (x_L, x_R) is
$$\Pr(L \text{ wins})L(|b - x_L|) + \Pr(R \text{ wins})(L(|b - x_R| + w).$$
- We focus on symmetric, pure strategy equilibria.
- We assume the hazard rate $\frac{f(m)}{1-F(m)}$ is weakly decreasing.
- Let $\overline{b}$ be the unique solution to $L'(b) = -wf(0)$.

Proposition There is a unique symmetric equilibrium, $(-x^e, x^e)$, and this equilibrium satisfies $0 \leq x^e < b$. If $b \leq \overline{b}$, then $x^e = 0$; and if $b > \overline{b}$, then x^e is the unique solution of the f.o.c.:

$$-L'(b-x) = [L(b-x) + w - L(x+b)]f(0).$$

Proof: Suppose $x_L = -x$. Candidate R's payoff for $x_R \geq 0$ is:

$$F(\tfrac{x_R - x}{2})L(b+x) + [1 - F(\tfrac{x_R - x}{2})](L(b - x_R) + w).$$

. Differentiating w.r.t. x_R and setting $x_R = x$ we obtain the f.o.c.

. The s.o.c. is satisfied as $\frac{f(m)}{1-F(m)}$ is weakly decreasing.

. Rearranging the f.o.c., I obtain: $\frac{L'(b-x)}{L(b+x)-L(b-x)-w} = f(0)$.

. LHS is strictly decreasing in $x \in [0, b)$ by strict concavity of L: by intermediate value theorem, the solution $x^e \in (0, b)$.

Proposition Say L is a power function $L(z) = -z^{\alpha}$ with $\alpha > 1$. If $b > \overline{b}$, then $\frac{\partial x^e}{\partial b} > 0$, $\frac{\partial x^e}{\partial f(0)} < 0$, $\frac{\partial x^e}{\partial w} < 0$.

. Platform divergence increases as parties are more polarized, likelihood of electoral tie decreases, office benefits decrease.

. The limiting properties of equilibria are as follows:

- . If $w = 0$, then x^e is a solution of $\frac{L'(b-x)}{L(b+x)-L(b-x)} = f(0)$.
- . If $w \geq -\frac{L'(b)}{f(0)}$, then $x^e = 0$
- . If $f(0) \to 0$, then $x^e \to$ solution of $\frac{L'(b-x)}{L(b+x)-L(b-x)-b} = 0$
- . If $f(0) \to \infty$, then $x^e \to 0$
- . If $b \to 0$, then $x^e \to 0$
- . If L is a power function, then as $b \to \infty$, we have $x^e \to \frac{1}{2f(0)}$.

. We now turn to relating voter welfare to candidates' ideologies.

. Let $\overline{\overline{b}}$ be the ideology such that the equilibrium platform $x^e = \overline{x}$

. If $0 \leq b \leq \overline{b}$, then platforms converge at zero.

. If $\overline{b} < b < \overline{\overline{b}}$, then the ex ante welfare of all voters is higher with policy-motivates candidates than with platforms convergence.

. If $b > \overline{\overline{b}}$, then ex ante welfare of some voters is strictly lower.

Proposition In the quadratic-normal model, $\overline{\overline{b}} = \infty$:

$$\lim_{b\to\infty} x^e = \frac{1}{2f(0)} = \sigma\sqrt{\frac{\pi}{2}} < 2\sigma\sqrt{\frac{2}{\pi}} = 2E[m|m > 0] = \overline{x}.$$

. All voters are always better off with policy-motivates candidates.

Citizen candidate models

. Key assumption of Downsian models is that politicians can commit to any policy platform, regardless of their preferences.

. Convergence to median obtains with office-motivated candidates, but also with policy motivations (if voters' preferences are known).

. What happens if politicians cannot commit and can only implement their preferred policy?

. Say voters vote for the candidate with platform they prefer.

. Then, there exist equilibria in which two or more candidates differentiate platforms.

. If voters coordinate not to vote for losing candidates, then exactly two candidates run in the election.

Osborne and Slivinski 1996

. Policy space is $X = \mathbb{R}$ and there is a continuum of citizens i.

. The citizens' ideal platforms b_i empirical distribution F is continuous with unique median m.

. Each citizen i chooses to run or not in the election, $e_i \in \{E, N\}$.

. If a citizen i enters, she becomes a "candidate" with platform $x_i = b_i$ (citizens cannot commit to a different platform).

. After all citizens have simultaneously chosen on entry, they vote.

. Voting is "sincere:" each voter i with bliss point b_i votes for the candidate(s) j whose platform x_j is closest to b_i.

. Votes are split equally if multiple candidates platforms coincide.

- A citizen who chooses E incurs the cost $c > 0$,
and derives benefit $w > 0$ if she wins.
- Let the platform of the election winner be x_W.
- If citizen i with ideal platform b_i chooses N then i's payoff is

  $$u_i(N, e) = -|x_W - b_i|.$$
- If citizen i with ideal platform b_i chooses E,

  then her payoff is $u_i(E, e) = w - c$ if she wins,

  and $u_i(E, e) = -|x_W - b_i| - c$ if she loses.
- If no citizen enters, then they all obtain the payoff of $-\infty$.

Results

. There exist equilibria with one, two or more candidates.

. In multi-candidate equilibria platforms may diverge.

Proposition There is a one-candidate equilibrium iff $w \leq 2c$.
If $c \leq w \leq 2c$, then the candidate's platform is $x_W = m$.
If $w < c$, then $x_W \in [m - \frac{c-w}{2}, m + \frac{c-w}{2}]$.

. If $w > 2c$, then a second candidate would enter even just to tie.

. If $x = m$, then no entrant can defeat the candidate.

. If $w < c$, and $|m - x_W| \leq \frac{c-w}{2}$, then no-one who can defeat the candidate would strictly benefit by entering.

Proposition In any 2-candidate equilibrium the platforms are $x_A = m - e$ and $x_B = m + e$ for some $e \in (0, \bar{e}(F)]$.
Any such equilibrium exists if and only if $2e \geq c - w/2$, $c \geq |m - s(e, F)|$ and either $e < \bar{e}(F)$ or $e = \bar{e}(F) \leq 3c - w$.

. $s(e, F)$ is the platform such that A and B still tie their votes if a third candidate C enters with $x_C = s(e, F)$.

. $\bar{e}(F)$ is the value of e such that A and B lose to C iff $e > \bar{e}(F)$.

. If $e > \bar{e}(F)$, then a third candidate enters and wins.

. If $e = \bar{e}(F) > 3c - w$, then a third candidate enters and ties.

. If $e < c - w/2$, then one of the two candidates drops out.

. If $c < |m - s(e, F)|$, then an entrant may want to enter and lose.

Proposition Every 3-candidate equilibrium is such that:

. either the election is a 3-way tie, and the platforms are $x_A = t_1 - e_1$, $x_B = t_1 + e_1 = t_2 - e_2$, $x_C = t_2 + e_2$ for some $e_1, e_2 \geq 0$, where $t_1 = F^{-1}(1/3)$, $t_2 = F^{-1}(2/3)$,

. or candidates A and C tie the election and B loses for sure, and the platforms are $x_A < x_B < x_C$.

. A necessary condition for 3-way tie is $w \geq 3c + 2|e_1 - e_2|$.

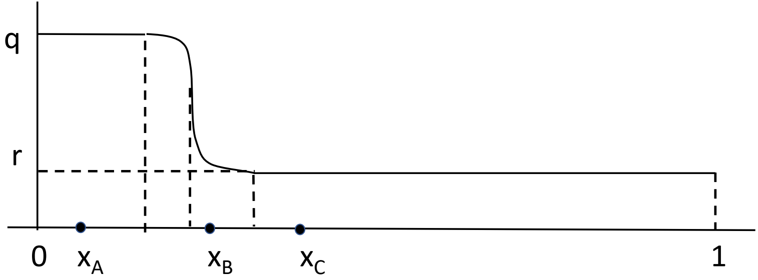
. In the 2-way tie equilibrium, candidate 2 enters to lose the election and induce a tie.

. If B did not enter, her worst candidate would win for sure.

. A necessary conditions for 2-way tie is $w \geq 4c$ and $c < t_2 - t_1$:

  . if $c > t_2 - t_1$, then B would not enter,

  . if $w < 4c$, then one of the two winning candidates drops out.



. Candidate B enters to lose the election.

. B’s entry makes A and C tie: $q(x_A + x_B)/2 = r[1-(x_B + x_C)/2]$.

. By entering B steals more votes to A than to C.

. B is closer to C than to 1: $x_C - x_B < x_B – x_A$.

Proposition A necessary condition for the existence of an equilibrium in which $k \geq 3$ candidates tie for first place is $w \geq kc$. A necessary condition for the existence of an equilibrium in which there are three or more candidates is $w \geq 3c$.

. There may be multiple candidates elections.

. These equilibria generalize the logic of the 3-way tie equilibrium in the previous proposition.

. Each pair of contiguous candidates is symmetrically located around an ideologically k-tile, t_1, t_2, ..., t_{k-1}.

Besley and Coate 1997

. Besley and Coate 1997 assume that voters vote strategically.

. Voters do not waste vote on candidates who are ideologically close to their bliss point, but have no chance to win.

. As there is a continuum of voters, no voter is pivotal.
This assumption requires coordination among voters.

. There are no equilibria in which 3 or more candidates tie election.

. There are no equilibria in which a candidate enters the election and loses for sure.

. These equilibria are upset by strategic voters who vote second best candidate, to break a tie with a candidate they dislike more.

Summary

. I have introduced policy motivation in spatial models of elections.

. Suppose candidates have policy preferences in the aggregate uncertainty probabilistic model.

. Because of uncertainty, equilibrium platforms diverge.

. If voters' preferences may change during campaigns, then platform divergence improves electorate welfare.

. Suppose candidates have policy preferences, cannot credibly commit to platforms, and choose whether to run or not.

. There exist equilibria where platforms "diverge" from the median.

. Candidate may enter elections in the expectation of losing, only to steal votes from perspective winner.

Next Lecture

. I will present agency models of election.

. Voters do not care about electoral promises.

. They retain effective incumbents, and dismiss incumbents with poor performance to elect the challenger.

. If candidates' valence and ideologies are known, retention rules are ineffective.

. If candidates' valence or ideologies are uncertain, such retention rules encourage high effort/platform moderation.

. Politicians seeking re-election may choose to pander.

. Independent bureaucracy is immune to pandering.