Advanced Economic Theory
Models of Elections

Lecture 1

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Models of Elections

. Elections are modelled as non-cooperative games.

. There may be 2 or more office motivated candidates, possibly with different ideology or valence.

. Candidates' strategic decisions may include whether and when to run in the election, policy platform, campaign spending amount, ...

. Voters are ideologically differentiated.

. Their decisions may include whether and who to vote, and whether to support a candidate through activism or lobbyism.

. Different electoral rules may be considered.

. Repetition and private information may play a role.

Course Syllabus

Lecture 1: Median voter theorem

Readings

. P. Ordeshook 1986. *Game Theory and Political Theory: An Introduction*, Cambridge University Press, Chapter 4.

Lecture 2: Citizen candidates and probabilistic voting

Readings

. M. Osborne and A. Slivinski 1996. "A model of political competition with citizen-candidates," *Quarterly Journal of Economics*, 111(1): 65-96.

. T. Besley and S. Coate 1997. "An economic model of representative democracy," *Quarterly Journal of Economics*, 112: 85-114.

. A. Lyndbeck and J. Weibull 1993. "A model of political equilibrium in a representative democracy," *Journal of Public Economics*, 51(2): 195-209.

Lecture 3: Policy motivations

Readings

. D. Bernhardt, J. Duggan and F. Squintani 2009. "The case for responsible parties," *American Political Science Review,* 103(4): 570-587.

. S. Callander 2008. "Political motivations," *Review of Economic Studies,* 75(3): 671-697.

Lecture 4: Elections with incomplete information

Readings

. T. Feddersen and W. Pesendorfer 1996. “The swing voter’s curse,” *American Economic Review,* 86(3): 408-424.

. N. Kartik, F. Squintani and K. Tinn 2012. “Information revelation and pandering in elections,” mimeo, *Columbia University.*

Lecture 5: Agency models of elections

Readings

. J. Banks and R. Sundaram 1998. “Optimal retention in agency problems,” *Journal of Economic Theory*, 82(2): 293-323.

. J. Duggan 2000. “Repeated elections with asymmetric information,” *Economics and Politics*, 12(2): 109-135.

. D. Bernhardt, L. Campuzano, F. Squintani and O. Camara 2009. “On the benefits of party competition,” *Games and Economic Behavior*, 66(2): 685-70.

Lecture 6: Candidate valence advantage

Readings

. E. Aragones and T. Palfrey 2002. "Mixed equilibrium in a Downsian model with a favored candidate," *Journal of Economic Theory*, 103(1): 131-161.

. T. Groseclose 2001. "A model of candidate location when one candidate has a valence advantage," *American Journal of Political Science*, 45(4): 862-886.

. D. Bernhardt, O. Camara and F. Squintani 2011. "Competence and ideology," *Review of Economic Studies*, 78(2): 487-522.

Lecture 7: Lobbying and activism

Readings

. G. Grossman and E. Helpman 1996. “Electoral competition and special interest politics,” *Review of Economic Studies*, 63(2): 265-286.

. R. Srinivasan 2017. “A model of election activism, mobilization and polarization,” mimeo, *University of Warwick.*

Lecture 8: Voter turnout

Readings

. R. Shachar and B. Nalebuff 1999. "Follow the leader: theory and evidence on political participation," *American Economic Review*, 89(3): 525-547.

. T. Feddersen and A. Sandroni 2006. "A theory of participation in elections," *American Economic Review*, 96(4): 1271-1282.

Lecture 9: Legislative bargaining

Readings

. D. Baron and J. Ferejohn 1989. "Bargaining in legislatures," *American Political Science Review*, 83(4): 1181-1206.

. T. Romer and H. Rosenthal 1978. "Political resource allocation, controlled agendas, and the status quo," *Public Choice*, 33(4): 27-43.

. D. Baron 1996. "A dynamic theory of collective goods programs," *American Political Science Review*, 90(2), 316-330.

Downsian elections

. Two candidates $i = A, B$ care only about winning the election.

. Candidates i simultaneously commit to policies $x_i \in \mathbb{R}$ if elected.

. There is a continuum of voters.

. The payoff of a voter with ideology b if policy x is implemented is $u(x, b) = L(|x - b|)$, with $L' < 0$.

. Ideologies are distributed according to (continuous and strictly increasing) empirical cumulative distribution F, of median m.

. After candidates choose platforms, each citizen votes, and the candidate with the most votes wins.

. If $x_A = x_B$, then the election is tied.

Theorem (Median Voter Theorem) The unique Nash Equilibrium of the Downsian election is such that candidates $i = A, B$ choose $x_i = m$, and tie the election.
Office motivated politicians converge on median positions.

Proof. We calculate candidate payoffs as function of (x_A, x_B).

. Fix any (x_A, x_B) such that $x_A \neq x_B$.

. Because $L' < 0$, each voter with ideology b votes for the candidate i that minimizes $|x_i - b|$.

. Hence, when $x_i < x_j$, candidate i's vote share is $F(\frac{x_A+x_B}{2})$, and candidate j's is $1 - F(\frac{x_A+x_B}{2})$.

. Now, consider any profile (x_A, x_B) such that $x_i \neq m$ for at least one candidate $i = 1, 2$.

. j's best response is $BR_j = \{x_j : |x_j - m| < |x_i - m|\}$, by playing a best response, candidate j wins the election.

. But if j plays x_j such that $|x_j - m| < |x_i - m|$, i's best response cannot be x_i, as i can at least tie the election by playing m.

. Hence, there cannot be any Nash equilibrium where either candidate i plays $x_i \neq m$.

. Suppose now that both candidates play $x_A = x_B = m$.

. All voters are indifferent between x_A and x_B: the election is tied.

. If either candidate i deviates and plays $x_i \neq m$, then she loses the election.

. Hence, there is a unique Nash equilibrium: $x_A = x_B = m$.

. Median voter theorem corresponds to equilibrium of the "Hotelling" model of monopolistic competition.

. Producers choose to make identical products, in a model of monopolistic competition with horizontal differentiation.

. But lack of product differentiation hurts aggregate consumer welfare in Hotelling model, whereas convergence to the median benefits voters in Downsian model.

. E.g., if F is uniform on $[0, 1]$, then consumer welfare is maximal in the Hotelling model with $x_A^* = 1/4$, and $x_B^* = 3/4$.

. And for general F, the optimal products x_A^* and x_B^* are similarly differentiated.

. Matters are very different in the Downsian model.

Proposition If voters are risk averse, then the median platforms $x_A = x_B = m$ are preferred by a majority to any pair x'_A, x'_B. If x'_A, x'_B is 'competitive', i.e. $|x'_A - m| = |x'_B - m|$, then x_A and x_B are unanimously preferred to x'_A, x'_B.

Proof. Each platform x'_i in any competitive pair x'_A, x'_B, is voted by $1/2$ of voters.

. The pair x'_A, x'_B is a 'bet' with expected value equal to m.

. If voters are risk averse, $L'' < 0$, then they all prefer the sure outcome $x_A = x_B = m$.

. Consider now any distribution F and platform x'_A, x'_B: the election selects the platform x'_i closest to m.

. Thus, a majority of voters prefers $x_A = x_B = m$ to x'_A, x'_B.

Proposition If the ideology distribution F is symmetric, $F(b) = 1 - F(2m - b)$ for all b, and the loss function L is a power function, $L(|x - b|) = |x - b|^n$ for some integer n, then convergence to the median, $x_A = x_B = m$, maximizes "utilitarian" voter welfare $W(x) = -\int_{-\infty}^{+\infty} L(|b - x|)dF(b)$.

Proof. If F is symmetric around m, $F(b) = 1 - F(2m - b)$ for all b, and L is a power function, then all central moments of F coincide with the median m (the zero-th moment).

. Solving $x^* = \arg\max_x \{W(x) = -\int_{-\infty}^{+\infty} |x - b|^n dF(b)\}$, we obtain that $x^* = m$.

. When F is symmetric, there are also fairness considerations that make median convergence appealing.

. But when F is not symmetric, median convergence does not maximize utilitarian welfare W unless L is a linear function.

Ordinal preferences

- Consider a compact policy space X and a set of voters $N = \{1, ..., n\}$, with n odd.
- Preferences are single-peaked on space X with linear order $>$, if for each voter j there is a policy b_j such that for all $x, y \in X$,
  - if $b_j \geq y > x$, then $y \succ_j x$,
  - if $x > y \geq b_j$, then $x \succ_j y$.
- Preferences are single-crossing on space X with linear order $>$, for voter index permutation $p : N \to N$, whenever

  if $x > y$ and $p(j) > p(i)$, or if $x < y$ and $p(j) < p(i)$, then $x \succ_{p(i)} y$ implies $x \succ_{p(j)} y$.
- A policy x that defeats any other policy y is a Condorcet winner.

Theorem Say that an odd number of voters vote among two candidates. If policy x is the Condorcet winner, then both candidates choose x in equilibrium.

Theorem (Downs, 1975; Gans and Smart, 1996) If an odd number of voters have single-peaked or single-crossing preferences, then the Condorcet winner is the ideal point of the median voter m.

. There are preference profiles with no Condorcet winners.

1: $x \succ y \succ z$
2: $y \succ z \succ x$
3: $z \succ x \succ y$

. The two results are independent: single-crossing condition does not imply single-peakedness, nor vice-versa.

. Preferences may be single crossing but not single peaked.

$1 : x \succ y \succ z$
$2 : x \succ z \succ y$
$3 : z \succ y \succ x$

are single crossing on order $x < y < z$ but not single peaked:
$z \succ_2 y \Rightarrow z \succ_3 y$, $x \succ_2 z \Rightarrow x \succ_1 z$, $x \succ_2 y \Rightarrow x \succ_1 y$.
(Not single peaked for any $>$ as each x, y, z is the worst for a voter.)

. Preferences may be single peaked but not single crossing.

$1 : w \succ x \succ y \succ z$
$2 : x \succ y \succ z \succ w$
$3 : y \succ x \succ w \succ z$

are single peaked on $w < x < y < z$, but not single crossing:
for $2 < 3$, $z \succ_2 w$ but $z \not\succ_3 w$; for $3 < 2$, $y \succ_3 x$ but $y \not\succ_2 x$.

Multi-dimensional policy spaces

. Policy platforms are usually multi-dimensional.

. But often multidimensional policy can be projected on a left-right unidimensional space on which voters can be ordered.

. Consider a compact policy space $X \subset \mathbb{R}^d$ and set of voters N.

. The voters in $j \in N$ have "intermediate preferences" if every j's payoff can be written as $L_j(x) = J(x) + K(p_j)H(x)$ for some voter index permutation p, where K is monotonic, whereas $H(x)$ and $J(x)$ are common to all voters.

Proposition Say that an odd number of voters with intermediate preferences vote among two candidates. Then both candidates choose policy $x(p_m)$, the ideal point of the voter i with median p_m.

. Suppose agents preferences can be represented by $L(||x - b_i||)$, where b_i is vector describing i's bliss point in this policy space.

. L decreasing and concave in the Euclidean distance $||x - b_i||$.

Theorem (Plott, 1967) A Condorcet winner policy in a multidimensional policy space exists if and only if there is a policy $m \in \mathbb{R}^d$ median in all directions.

. The existence of a median in all direction requires strong symmetry assumptions on the distribution of individual ideal points.

. The 'top cycle' of X is the set of all alternatives $x \in X$ such that for each $y \neq x$, there are $c_1, ..., c_K$ such that $x = c_1 \succ c_2 \succ ... \succ c_K = y$, where $\succ$ represents a preference by a majority.

Theorem (McKelvey 1976) In a multi-dimensional policy space, if there is no Condorcet winner, then the top cycle is the whole set of alternatives.

Example Consider the divide the dollar game with 3 voters.

- Set of alternatives is $X = \{(x_1, x_2, x_3) \geq 0 : x_1 + x_2 + x_3 = 1\}$.
- Each voter i's payoff is increasing in x_i.
- The top cycle is $TC = X \setminus \{(1,0,0),(0,1,0),(0,0,1)\}$.
- In fact, every $x \in X$ is defeated by at least one among $(1/2, 0, 1/2)$, $(1/2, 1/2, 0)$ and $(0, 1/2, 1/2)$.
- If $x > 0$, then $x \succ (0, \varepsilon, 1-\varepsilon) \succ (1/2, 0, 1/2)$ for some small $\varepsilon > 0$ and similarly for $(1/2, 1/2, 0)$ and $(0, 1/2, 1/2)$.
- If exactly two entries of x are positive, then x beats some $x' > 0$, which then indirectly beats all other alternatives.

Agenda setting

. Suppose there are no candidates.

. Voters choose among a finite set of fixed alternatives X.

. The choice is made by sequential pairwise elimination.
E.g., voters choose x vs. y, winner is matched to z, and so on.

. The 'agenda' is the sequence in which alternatives are voted.

. If there is a Condorcet winner, it is selected for all agenda.

. If voters vote sincerely on each alternative, then for every policy x in the top cycle set, there exist agenda that select x.

. By Mc Kelvey theorem, the top cycle is X: the agenda-setter can determine the outcome.

. If voters are strategic and know the agenda, the game is solved by backward induction.

. The Banks set includes all alternatives in X that survive successive elimination by strategic voters for some agenda.

. If there is a "status quo" $\bar{x}$ in X, it is voted last against the penultimate surviving alternative in the agenda.

. The inclusion of status quo further restricts the set of alternatives "available" to the agenda setter.

Summary

. We have reviewed the Downsian model of elections.

. There are two office-motivates candidates.

. First each credibly commits to an electoral platform.

. Then, voters vote for the preferred platform candidate.

. If policies are uni-dimensional, candidates' platforms "converge" to the policy preferred by the median voter.

. If the policy space is multi-dimensional, anything goes.

. If there are no candidates and alternatives are voted sequentially, agenda setter is a dictator unless voters are strategic.

Next lecture

. I present the main alternative spatial models of elections.

. Suppose candidates have policy preferences and cannot credibly commit to platforms.

. Then there exist equilibria in which platforms “diverge” from the median policy.

. If office motivated candidates are uncertain about the voters' preferences, then platforms converge to the expected median.

. Equilibrium exist in multi-dimensional policy spaces, if candidates maximize vote shares and voters' preferences are uncertain.

. This equilibrium is Pareto efficient for the electorate.