# Advanced Economic Theory
# Models of Elections
# Lecture 4

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## Swing voter's curse (Feddersen and Pesendorfer, 1996)

. Elections aggregate individual preferences and information.

. Voter information is often of common value, but some voters are not well informed.

. Here, uninformed voters abstain, to avoid swinging the election against common interest.

. In fact, many voters do not vote, although the cost of voting is often negligible.

. Here, strategic abstention delivers first best.

. The winning candidate is the same as if all voters knew all voters' information.

## The model

. There are 2 states $\omega = 0, 1$, with $r = \Pr(\omega = 0) \geq 1/2$, and 2 party candidates $j = 0, 1$, with platforms $x_j = 0, 1$.

. There are $N + 1$ possible voters, each votes with prob. $1 - p_A$.

. With prob. $p_0$ (prob. $p_1$), a voter is partisan for party 0 (party 1).

. With probability $p_n = 1 - p_0 - p_1$ the voter is independent: her utility is $u_n(x, \omega) = -|x - \omega|$.

. Each voter receives a signal $s \in S = \{0, a, 1\}$.

. With probability $1 - q$, $s$ is uninformative and equal to $a$.

. When signal $s$ is informative, $\Pr(s = \omega|\omega) = p > 1/2$.

. Each voter chooses $v \in \{0, A, 1\}$, where $A$ is abstention.

. I focus on symmetric Nash equilibria: voters with same type and signal vote the same candidate.

. In equilibrium, type-0 (type-1) voters vote $v_0 = 0$ ($v_1 = 1$).

. All informed independents vote according to their signal: $v_n(s) = s$ if $s = 0, 1$.

. The mixed strategy of uninformed independent agents (UIAs) is $\sigma = (\sigma_0, \sigma_1, \sigma_A) \in \Delta^3$.

## Equilibrium

. Given the strategy $\sigma$, let $\rho_{\omega,j}(\sigma)$ be the probability of a vote for $j$ if the state is $\omega$ is as follows

$$\rho_{\omega,j}(\sigma) = p_j + p_n(1-q)\sigma_j + p_n q(1-p) \quad \text{if } \omega \neq x_j,$$
$$\rho_{\omega,j}(\sigma) = p_j + p_n(1-q)\sigma_j + p_n qp \quad \text{if } \omega = x_j.$$

. Let $\rho_{\omega,A}(\sigma)$ be the probability of an abstention if the state is $\omega$:

$$\rho_{0,A}(\sigma) = \rho_{1,A}(\sigma) = \rho_A(\sigma) = p_n(1-q)\sigma_A + p_A.$$

. For any voter, the probability of a tie among the other voters is:

$$\pi_T^{\omega,\sigma} = \sum_{\ell=0}^{N/2} \frac{N!}{\ell!\ell!(N-2\ell)!}\rho_{\omega,A}(\sigma)^{N-2\ell}\rho_{\omega,0}(\sigma)^{\ell}\rho_{\omega,1}(\sigma)^{\ell}.$$

. The probability that candidate $j$ is down by 1 vote is:

$$\pi_j^{\omega,\sigma} = \sum_{\ell=0}^{(N/2)-1} \frac{N!\rho_{\omega,A}(\sigma)^{N-2\ell-1}\rho_{\omega,1-j}(\sigma)^{\ell+1}\rho_{\omega,j}(\sigma)^{\ell}}{(\ell+1)!\ell!(N-2\ell-1)!}.$$

. Let $Eu_n(v, \sigma)$ be an UIA expected payoff of voting $v$, when the other voters use $\sigma$:

$$Eu_n(1,\sigma) - Eu_n(A,\sigma) = \frac{1}{2}[(1-r)(\pi_T^{1,\sigma} + \pi_1^{1,\sigma}) - r(\pi_T^{0,\sigma} + \pi_1^{0,\sigma})]$$

$$Eu_n(0,\sigma) - Eu_n(A,\sigma) = \frac{1}{2}[r[\pi_T^{0,\sigma} + \pi_0^{0,\sigma}] - (1-r)[\pi_T^{1,\sigma} + \pi_0^{1,\sigma}]].$$

$$\begin{aligned} Eu_n(1,\sigma) - Eu(0,\sigma) = (1-r)[\pi_T^{1,\sigma} + \tfrac{1}{2}(\pi_1^{1,\sigma} + \pi_0^{1,\sigma})] \\ -r[\pi_T^{0,\sigma} + \tfrac{1}{2}(\pi_1^{0,\sigma} + \pi_0^{1,\sigma})]. \end{aligned}$$

**Proposition** Suppose $p_A > 0$, $q > 0$, $N \geq 2$ and $N$ even. For any symmetric $\sigma$ s.t no voter plays a strictly dominated strategy, $Eu_n(1,\sigma) = Eu_n(0,\sigma)$ implies $Eu_n(1,\sigma) < Eu_n(A,\sigma)$.

. An UIA strictly prefers to abstain whenever indifferent between voting for 1 or 0, and no voter uses a strictly dominated strategy.

. This is the swing voter's curse.

. To consider large elections, define a sequence of games with $N+1$ voters and associated strategy profiles $\{\sigma^N\}_{N=0}^{\infty}$.

**Proposition** Suppose $q > 0$, $p_n(1-q) < |p_0 - p_1|$ and $p_A > 0$. Let $\{\sigma^N\}_{N=0}^{\infty}$ be a sequence of equilibria.

. If $p_n(1-q) < p_0 - p_1$ then $\lim_{N\to\infty} \sigma_1^N = 1$, i.e., all UIAs vote for candidate 1.

. If $p_n(1-q) < p_1 - p_0$ then $\lim_{N\to\infty} \sigma_0^N = 1$, i.e., all UIAs vote for candidate 0.

. The swing voter's curse can lead to large scale abstention by the UIAs in large elections.

. This happens when the expected fraction of UIAs is too small to compensate for a candidate partisan advantage.

. Instead, when the fraction of UIAs is large enough to offset partisan bias, there are no pure strategy equilibria.

. UIAs mix between abstention and voting against the difference in partisan support to compensate exactly.

. The equilibrium winning candidate is approximately the same as the candidate that would win if voters had perfect information.

**Proposition** Suppose $q > 0$, $p_n(1-q) \geq |p_0 - p_1|$ and $p_A > 0$. Let $\{\sigma^N\}_{N=0}^{\infty}$ be a sequence of equilibria.

. If $p_n(1-q) \geq p_0 - p_1 > 0$ then UIAs mix between voting for candidate 1 and abstaining, with $\lim \sigma_1^N = \frac{p_0 - p_1}{p_n(1-q)}$.

. If $p_n(1-q) \geq p_1 - p_0 > 0$ then UIAs mix between voting for candidate 0 and abstaining, with $\lim \sigma_1^N = \frac{p_1 - p_0}{p_n(1-q)}$.

. If $p_0 - p_1 = 0$ then UIAs abstain: $\lim \sigma_A^N = 1$.

. For every $\epsilon$ there exists an $N$ such that for $\bar{N} > N$ the probability that equilibrium fully aggregates information is greater than $1 - \epsilon$.

## Information revelation and pandering (Kartik et al. 2017)

. Politicians are generally much better informed than voters, various empirical studies of voter ignorance on policy issues.

. Does electoral competition lead to informational efficiency? Or does it generates incentives to pander to electorate's prior?

. In Downsian election with informed politicians, we show platforms may overreact to information instead of pandering.

. Electoral competition induces candidates to convey some information to voters, but fails to achieve informational efficiency.

. The electorate welfare loss is as severe as if only one candidate's information were efficiently revealed.

## The model

. Given policy $x \in \mathbb{R}$ and unknown state $\omega \sim \mathcal{N}(0, 1/\alpha)$, the median voter's payoff is $L(x, \omega) = -(x - \omega)^2$.

. Two office-motivated candidates $i = A, B$ receive private signals $s_i = \omega + \varepsilon_i$, where $\varepsilon_i \sim \mathcal{N}(0, 1/\beta)$.

. They simultaneously commit to platforms $x_A$ and $x_B$.

. Then the median voter elects a candidate $i$, who implements $x_i$.

. We study perfect Bayesian equilibria where the median voter's strategy $\pi(x_A, x_B) \equiv \Pr(\textit{elect } A | x_A, x_B) = 1/2$ when indifferent.

. A strategy $x_i(s_i)$ is unbiased if $x_i(s_i) = E[\omega | s_i] = \frac{\beta}{\alpha+\beta} s_i$,
$x_i(s_i)$ has pandering if $|x_i(s_i)| < |E[\omega | s_i]|$,
$x_i(s_i)$ has overreaction if $|x_i(s_i)| > |E[\omega | s_i]|$.

## Results

. The strategies $x_A = x_B$ are a perfect Bayesian equilibrium.

**Proposition** There is no equilibrium in which both candidates $i$ use unbiased strategies.

*Proof:* As the strategy $x_i(\cdot)$ is invertible, each candidate $i$'s platform $x_i$ reveals signal $s_i$.

. Given $x_i(s_i) = E[\omega|s_i]$, platform midpoint is $\frac{x_A+x_B}{2} = \frac{\beta(s_A+s_B)}{2\alpha+2\beta}$.

. Seeing $x_A, x_B$, median voter updates $E[\omega|s_A, s_B] = \frac{\beta(s_A+s_B)}{\alpha+2\beta}$.

. If $|s_i| > |s_j|$ then $|x_i(s_i) - E[\omega|s_A, s_B]| < |x_j(s_j) - E[\omega|s_A, s_B]|$, then the candidate with the more extreme platform wins.

. There is a profitable deviation is to overreact (not to pander).

. Despite overreaction incentives, information can be revealed.

**Proposition** There is a symmetric fully revealing equilibrium.
It has overreaction: $x_i(s_i) = E[\omega|s_i, s_{-i} = s_i] = \frac{2\beta}{\alpha+2\beta}s_i$.
Voter chooses each candidate with prob. $\frac{1}{2}$ for all platform pairs.

*Proof:* As the strategy $x_i(\cdot)$ is invertible, each candidate $i$'s platform $x_i$ reveals signal $s_i$.

. It suffices to show that for any $s_A$ and $s_B$,

$$(x_A(s_A) - E[\omega|s_A, s_B])^2 = (x_B(s_B) - E[\omega|s_A, s_B])^2.$$

. Substituting $x_i(\cdot)$ in the expression yields

$$\left(\frac{2\beta}{\alpha+2\beta}s_A - \frac{2\beta}{\alpha+2\beta}\left(\frac{s_A+s_B}{2}\right)\right)^2 = \left(\frac{2\beta}{\alpha+2\beta}s_B - \frac{2\beta}{\alpha+2\beta}\left(\frac{s_A+s_B}{2}\right)\right)^2,$$

. This is true for all $s_A, s_B$.

. The previous equilibrium is fully revealing, but platforms are distorted by overreaction.

. Is this the best equilibrium, in terms of ex-ante voter utility?

**Proposition** There is an equilibrium in which one candidate $i$ wins for all platform pairs: $x_i(s_i) = E[\omega|s_i] = \frac{\beta}{\alpha+\beta} s_i$, $x_j(s_j) = s_j$. This 'unbiased dictator' equilibrium yields higher ex-ante voter utility than the symmetric fully revealing equilibrium.

. Unbiased dictator equilibrium uses only one signal efficiently.

. Is it possible to improve voter welfare by using information from both informed candidate in equilibrium?

**Proposition** The unbiased dictator equilibrium strictly maximizes the voter ex-ante expected utility.

**Lemma** For any informative equilibrium $(x_A, x_B, \pi)$, there is $p^* \in \{0, \frac{1}{2}, 1\}$ such that for all platforms $(x_A, x_B)$ on the equilibrium path, $\pi(x_A, x_B) = p^*$.

- This lemma implies the previous Proposition because:
  - Uninformative equilibria are dominated by unbiased dictatorship.
  - Any 'dictatorial' equilibrium with $p^* \in \{0, 1\}$ is weakly worse than the unbiased dictator equilibrium for the voter.
  - If $p^* = 1/2$, then the voter welfare is the same as if either candidate were always elected.
  - At least one of them cannot be playing unbiased strategy, hence this equilibrium is dominated by unbiased dictatorship.

*Proof:* Fixing any voter strategy $p(x_A, x_B)$, induces a complete-information constant-sum game between candidates with

- strategy sets $Y_A = Y_B = \mathbb{R}$;
- payoffs: $u_A(y_A, y_B) = \pi(y_A, y_B)$, $u_B(y_A, y_B) = 1 - \pi(y_A, y_B)$.

. Any (Nash) equilibrium of our Bayesian game is a correlated equilibrium $\rho$ of this complete-information game.

. Because this is a constant-sum game, for all $y_i, y_i'$ played in a correlated equilibrium $\rho$, $Eu_i[y_i|y_i;\rho] = v_i^* = Eu_i[y_i'|y_i;\rho]$.

. Back in the Bayesian game equilibrium of electoral competition, each candidate $i$'s interim expected probability of winning $E[\pi(x_i, x_j)|s_i]$ is constant in $s_i$ and for all $x_i$ played in equilibrium.

. We want to conclude that the ex-post probability of winning $\pi(x_A, x_B)$ is constant in $x_A$ and $x_B$.

. This is not obvious: one counterexample is the matching pennies game.

| | L | R |
|---|---|---|
| L | 1, 0 | 0, 1 |
| R | 0, 1 | 1, 0 |

. The unique correlated equilibrium strategy is $\{(\frac{1}{2}, \frac{1}{2})\}$.

. Interpret it as the Nash Equilibrium of a Bayesian game.

. Each player $i$ has type $s_i \in \{L, R\}$ with probability 1/2.

. In equilibrium, she plays the action $x_i = s_i$.

. Regardless of her type $s_i$, player $i$'s interim expected payoff is 1/2 for both possible actions $x_i$.

. Yet, the ex-post payoff differs across pairs of actions $(x_A, x_B)$.

. Fix an informative equilibrium of the election game.
(Suppose $B$'s strategy is informative.)

. Consider an arbitrary finite number of on-path platforms for $B$, say $(y_B^1, \dots, y_B^m)$ with $m > 1$.

. By previous result, for any platform on-path $x_A$ and signals $s_A, s'_A$,

$$v_A^* = E[\pi(x_A, x_B)|s_A] = E[\pi(x_A, x_B)|s'_A].$$

. This implies that for any $m$ signals $(s_A^1, ..., s_A^m)$,

$$\begin{pmatrix} \Pr(x_B^1|s_A^1) & \cdots & \Pr(x_B^m|s_A^1) \\ \vdots & & \vdots \\ \Pr(x_B^1|s_A^m) & \cdots & \Pr(x_B^m|s_A^m) \end{pmatrix} \begin{pmatrix} \pi(x_A, x_B^1) \\ \vdots \\ \pi(x_A, x_B^m) \end{pmatrix} = \begin{pmatrix} v_A^* \\ \vdots \\ v_A^* \end{pmatrix}.$$

. Since rows of coefficient matrix change nonlinearly in $s_A^j$, the only solution for all $(s_A^1, ..., s_A^m)$ is a constant $\pi(x_A, \cdot)$.

## The value of pandering

. Now consider a (hypothetical) ‘benevolent candidates’ game: each candidate maximizes the voter’s welfare.

**Proposition** In the benevolent candidates game, there is a symmetric fully-revealing equilibrium with pandering: $x_i(s_i) = E[\omega|s_i, |s_j| < |s_i|]$, and voter elects the more extreme candidate. This equilibrium improves upon unbiased strategies.

. With unbiased strategies, the winner has more extreme signal.

. Optimality then requires moderation of one’s platform when conditioning on winning, i.e. pandering.

## Summary

. I have considered how well elections aggregate information.

. I have presented a model in which voters have different information about candidates' valence.

. I have shown that there exists an equilibrium in which informed non-partisan voters are pivotal, and the "best" candidate is elected.

. I have presented a model in which candidates are better informed than voters.

. Electoral competition induces candidates to convey some information to voters, but fails to achieve informational efficiency.

. The electorate welfare loss is as severe as if only one candidate's information were efficiently revealed.

## Next lecture

. I present agency models of election.

. Voters do not care about electoral promises.

. They retain effective incumbents, and dismiss incumbents with poor performance to elect the challenger.

. If candidates' valence and ideologies are known, retention rules are ineffective.

. If candidates' valence or ideologies are uncertain, such retention rules encourage high effort/platform moderation.

. Party competition encourage even more moderation and improves voter welfare.