Advanced Economic Theory
Models of Elections
Lecture 9

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Legislative bargaining (Baron and Ferejohn 1989)

. Bargaining within legislatures often concerns allocation of fixed surplus through bills, budget agreements, or regulations.

. Baron and Ferejohn (1989) consider repeated bargaining over fixed resources with random proposer nomination.

. There is a unique symmetric stationary equilibrium.

. Agreement is reached after the first proposal.

. The proposer obtains the largest share, but her advantage is smaller with an open amendment rule.

. Under closed amendment rule, the proposer's advantage increases in number of legislators.

The model

. Consider a legislature $N = \{1, 2, ..., n\}$ with n odd.

. The legislature has to decide how to allocate a "pie" of size 1.

. The set of possible divisions is $X = \{(x_1, ..., x_n) : x_j \geq 0$ for all $j \in N$ and $\sum_{j=1}^{n} x_j \leq 1\}$.

. There is an infinite number of periods $t = 1, 2, ..., \infty$.

. In each period t, a legislator is randomly selected as proposer, each is drawn with probability $1/n$, independently over time.

. If selected, legislator i makes a division proposal $x^i \in X$.

. A simple majority of votes is needed to pass the proposal.

. If the proposal is rejected, the game moves on to the next period.

. Subgame perfect equilibria with stage-undominated strategies. (I.e., legislators vote as if they were pivotal.)

. Consider 2 different bargaining protocols, closed and open rule.

. Under closed rule, each period-t proposal x^i is voted without modifications.

. Under open rule, each proposal x^i can be amended before vote.

. In the same period t, an amender $j \neq i$ is randomly selected.

. j may put x^i to vote, or make an alternative proposal x^j.

. If j made a proposal x^j, then legislators vote between x^i and x^j.

. Amendment process re-starts at next period $t+1$ with selection of amender $k \neq j$.

. The amendment process continues until a proposal is put to vote.

. Every legislator discount factor across periods is δ.

Proposition Any pie division $x \in X$ can be supported as a subgame perfect equilibrium if $n \geq 5$ and $\delta > \frac{n+2}{2(n-1)}$.

. The equilibrium is sustained by these strategies:

. at time $t = 0$, the drawn proposer chooses x, and all legislators accept it;

. at any time $t > 0$, if a majority rejected x at $t - 1$, the proposer chooses x, and all accept it;

. at any time t, if the proposer i chose $x^i = x' \neq x$, then i is punished as follows:

a majority $M(x')$ rejects x', and the proposer j drawn at $t + 1$ chooses $x^j = x''$ such that $x_i^j = 0$;

. if j were to deviate, the above punishment is applied on j.

- We refine the set of equilibria and focus on stationary strategies.
- A stationary strategy σ_i for any player $i \in N$ consists of
  - a mixed proposal $\pi_i \in \Delta X$ used at the period in which i is selected as proposer,
  - a voting strategy $v_i : X \to [0, 1]$, where $v_i(x)$ is the probability i accepts proposal x.

Proposition In a symmetric stationary equilibrium, any proposer i selected at any time-t chooses x^i such that $x_i^i = 1 - \frac{\delta(n-1)}{2n}$ and $x_j^i = \frac{\delta}{n}$ for some other randomly chosen $\frac{n-1}{2}$ legislators j; each legislator $j \neq i$ accepts any proposal x^i such that $x_j^i \geq \frac{\delta}{n}$.

- The first period proposal is then accepted, and game ends.

Sketch of Proof. Invoking symmetry, let v be any player's stationary equilibrium payoff at the beginning of any period.

. Because the pie is of size 1, it must be that $v \leq 1/n$.

. Consider a player j who is tendered a proposal x_j^i at any period: in equilibrium, she votes for x^i if and only if $x_j^i \geq \delta v$.

. To get proposal x^i accepted, proposer i needs $(n-1)/2$ votes.

. To respect symmetry, i must offer $x_j^i = \delta v$ to $(n-1)/2$ legislators $j \neq i$, chosen at random with equal probabilities.

. Then, the selected proposer's payoff is: $v_i = 1 - \frac{n-1}{2}\delta v$.

. Indeed, because $v \leq 1/n$, we obtain that $v_i > \delta v$.

. In the symmetric stationary equilibrium, at every period t, the selected proposer i makes a proposal x^i that is accepted.

. Let us now calculate the stationary payoffs v and v_i.

. In any period, each legislator i has probability $1/n$ of becoming the proposer and getting payoff v_i.

. Likewise, i's probability of being a responder is $(n-1)/n$.

. In any symmetric equilibrium, if i a responder, then she is offered δv with probability $1/2$, and else she receives nothing.

. Thus, we can express the stationary payoff v as $v = \frac{v_i}{n} + \frac{n-1}{2n}\delta v$.

. Solving out, we obtain $v = 1/n$ and $v_i = 1 - \frac{n-1}{2n}\delta$.

. The proposer payoff v_i decreases in δ. More patient responders must be given larger shares to pass proposals.

. Payoff v_i increases in n. There are more voters to "buy off" but less must be given to each one of them to pass proposals.

. Now let's look at open-rule bargaining.

. Symmetric stationary equilibrium depends on discount factor δ.

. If the proposer is patient, she pays off $(n-1)/2$ other legislators, hoping that one of them is chosen as the amender.

. If the proposer is impatient, she makes a proposal that "pays off" all other $n-1$ legislators and is surely accepted.

. For simplicity, we are going to assume that $N = 3$.

. Consider the case of high δ first.

. Proposer offers $1-s$ to a legislator j at random, to keep s.

. If j is selected as the amender, she puts x^i to vote.

. If the other legislator ℓ is selected, she will amend x^i, and offer $1-s$ to legislator j to keep s for herself.

. Thus, proposer's equilibrium payoff is $v(s) = s/2 + \delta v(0)/2$.

. Likewise, the payoff of "excluded" legislator ℓ is $v(0) = \delta v(s)/2$.

. Finally, legislator j puts x^i to vote iff $1 - s \geq \delta v(s)$

. Hence, the proposer sets $1 - s = \delta v(s)$.

. This system of equations can be solved, obtaining

$$s = \frac{4-\delta^2}{4+2\delta-\delta^2} \qquad v(s) = \frac{2}{4+2\delta-\delta^2} \qquad v(0) = \frac{\delta}{4+2\delta-\delta^2}.$$

. Immediate to verify that no legislator has incentive to deviate.

. The excluded legislator equilibrium response minimizes the proposers' equilibrium payoff.

. The proposer is better off with the closed-rule. The possibility of proposal amendment reduces her bargaining power.

- Consider low δ.
- The proposer keeps s, and offers $\frac{1-s}{2}$ to the other 2 players.
- Let $v(s)$ be the equilibrium payoff if amending a proposal.
- The amending player will second the proposal if $\frac{1-s}{2} \geq \delta v(s)$.
- Thus, the proposer sets $\frac{1-s}{2} = \delta v(s)$.
- Each amender also chooses to offer $\delta v(s)$ if making a proposal.
- Hence, the amender keeps s and it must be that $v(s) = s$.
- In symmetric stationary equilibrium, $\frac{1-s}{2} = \delta s$, or $s = \frac{1}{1+2\delta}$.
- Again, the proposer is better off with the closed-rule.
- The threshold $\bar{\delta}$ that discriminates the 2 open-rule equilibria has: $\frac{2}{4+2\bar{\delta}-\bar{\delta}^2} = \frac{1}{1+2\bar{\delta}}$, and hence $\bar{\delta} = \sqrt{3} - 1$.

Voting over Public Policies (Romer and Rosenthal 1978)

. When changing a policy, all individuals' payoffs are affected.

. Suppose a committee with n members with quadratic utilities.

. Uni-dimensional policy and majority rule.

. Fixed agenda setter who has the monopoly over the agenda (no counter proposal).

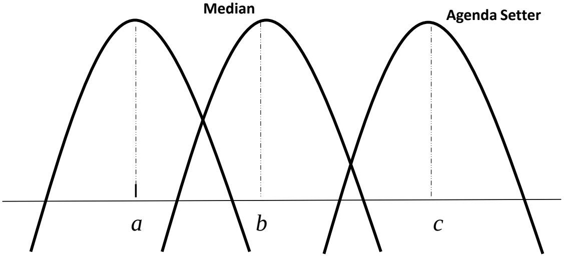
. The model predicts inertia: there is a range of policy where the status quo is not changed.

. Agenda setter is powerful, but not a dictator.

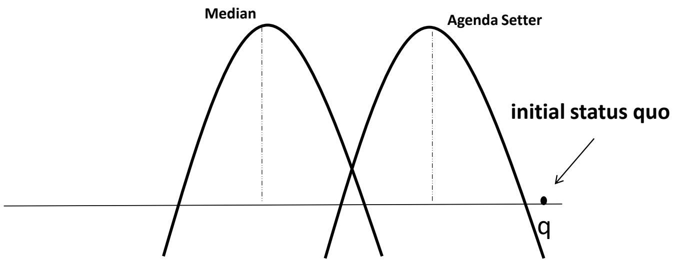
. Policy changes are asymmetric.

. If agenda setter is to the right (left) of the median, policy moves right relative to the status quo more (less) than it moves left.

Agenda Setting Model (Romer and Rosenthal, 1978)



Agenda Setting Model



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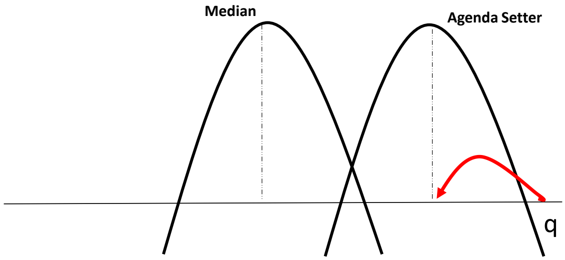

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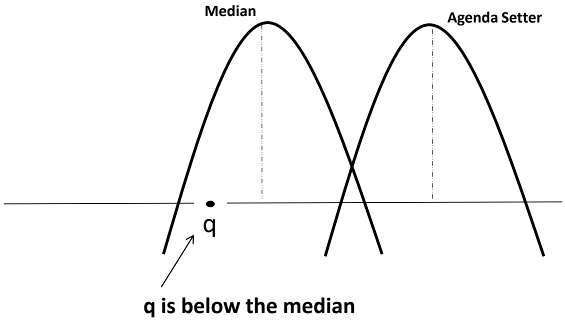

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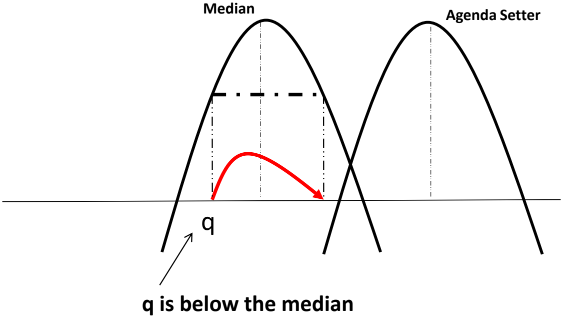

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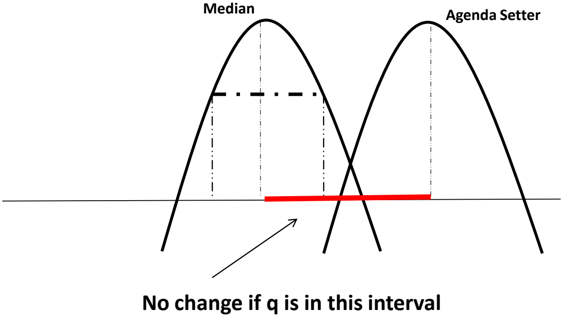

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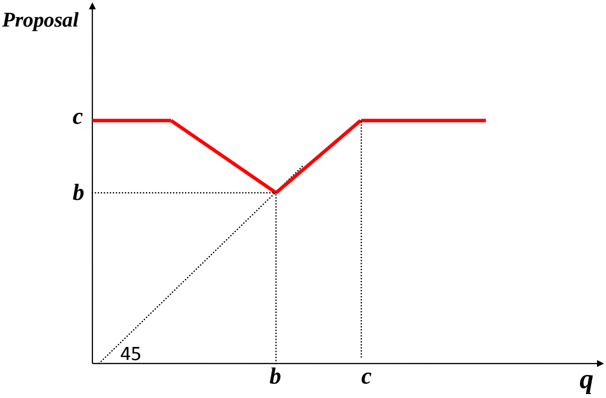

Endogenous Status quo (Baron 1996)

. Before we have assumed that once an agreement is made at t, the game ends and players get the agreed policy from t on.

. Suppose instead that reconsideration is possible in future periods.

. What is the default after an agreement?

. Suppose that when an agreement is reached, the default option in tomorrow's bargaining coincides with the policy agreed today.

. Current policy becomes default, or status quo, tomorrow.

. We say that the status quo is endogenous.

. Players recognize that when they change policy, this will change future bargaining.

The model

. Three committee members with quadratic utilities:
$u_i = -(x - b_i)^2$ where b_i is the ideal point of legislator i.

. Assume $0 < b_1 < b_2 < b_3 < 1$, equally distanced.

. Two periods.

. Suppose that member 1 is recognized in $t = 1$, and that recognition probabilities in $t = 2$ are $1/3$ for each player.

. What is decided in first period affects bargaining tomorrow via the change in default; q_t denotes status quo at t.

. The continuation value function depends on the status quo (the status quo is a state variable).

Analysis

- Solve backwards.
- In $t = 2$, the solution is as in the model by Romer and Rosenthal.
- In $t = 1$ player 1 chooses proposal:

  $z^1 \in \arg\max_{z \in [0,1]} -(z - b_1)^2 + \delta V_1(z)$

  subject to $-(z - b_j)^2 + \delta V_j(z) \geq -(q_1 - b_j)^2 + \delta V_j(q_1)$

  for at least one j, with $j \in \{2, 3\}$.
- Proposal z^1 is a function of the status quo q_1.
- q_1 determines the bargaining power of the opponents.
- Suppose that default at $t = 1$ is $q_1 = 0$.
- If $\delta = 0$, the proposal by player 1 will be b_1.

. If $\delta > 0$, player 1 (player 3) realizes that choosing a policy closer to b_2 will make the proposal by 3 (player 1) more centered.

. Extremist players propose policies more moderate than their preferred policies.

. The median player 2 can propose her preferred policy b_2 and that policy is unchanged in $t = 2$.

. To conclude, either b_2 is proposed or a policy close to it: there is dynamic convergence to the median policy.

. An endogenous status quo makes proposals more centered and provides insurance against political risk

. Zapal (2016) shows that there is dynamic convergence to the median also in the infinite horizon version of this game.

Summary

. We have considered legislative bargaining.

. Repeated bargaining over fixed resources with random proposer nomination yields a unique stationary equilibrium.

. Agreement is reached after the first proposal.

. The proposer obtains the largest share, but her advantage is smaller with an open amendment rule.

. Under closed amendment rule, the proposer's advantage increases in number of legislators.

. Bargaining over policies leads to change of policies with inertia.

. An endogenous status quo induces more moderate proposals, and provides insurance to the legislators.