Country Cartels*

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April 12, 2014

Abstract

We analyze collusion under demand uncertainty by cartels such as OPEC that care about the utility derived from profits by citizens. When citizens are sufficiently risk averse and fixed operating costs are non-trivial, it becomes difficult for cartels to collusively restrict output both when demand is low and marginal dollars are highly-valued, and when demand is high and potential defection profits are high: output relative to monopoly levels becomes a U-shaped function of demand. Greater risk aversion or higher fixed operating costs make collusion more difficult to support in recessions, but easier in booms.

*We thank Odilon Camara for helpful comments.

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1 Introduction

In their pioneering analysis of collusion under demand uncertainty, Rotemberg and Saloner (1986) show that when demand is independently and identically distributed over time, and firms observe demand before taking actions, collusion is harder to support when demand is higher. The intuition is compelling: the period incentive to cheat on the cartel rises with demand, but expected continuation payoffs are unchanged. While this setting is stark, Kandori (1991) establishes that the essence of the Rotemberg and Saloner result extends to serially-correlated demand—effectively, the period incentive to cheat on the cartel is more sensitive to current market conditions than are continuation payoffs. Haltiwanger and Harrington (1991) establish related results for deterministic cyclical demand—collusion is hardest to support at the peak of a cycle.[1] Bagwell (1997) generalize these results melding a Markov demand growth process on top of the i.i.d. transitory shocks.

In contrast to this robust theoretical prediction, we have Scherer (1980)'s summary of his empirical work: *"Yet it is precisely when business conditions turn sour that price cutting runs most rampant among oligopolistic firms with high fixed costs,"* Staiger and Wolak (1992)'s assertion that "the conventional empirical wisdom [is] that tacit collusion tends to break down when business conditions turn sour," and Aiginger et al. (1998)'s survey of 113 experts, which found that most believed that price wars are more likely when demand is low. Empirical studies providing support for the premise that collusion is harder to support in downturns include Porter (1985), Scherer and Ross (1990), Suslow (2005), and Ellison (1994). However, Domowitz et al. (1987), Chevalier and Scharfstein (1996) and Borenstein and Shepard (1996) provide empirical evidence consistent with cartels being more likely to breakdown in booms. This empirical research primarily focuses on prices and price-cost margins, and with output competition, procyclical price-cost margins remain consistent with collusion being more difficult to support when demand is high. Still, even among commodity cartels, there is evidence that collusion is difficult to support in downturns. For example, the two largest production wars in OPEC occurred in 1986 and 1997 when demand was extremely low.

One approach to trying to reconcile these empirical findings is to argue that firms can only imperfectly monitor collusion, for example, seeing equilibrium prices, but not demand

[1] This prediction need not hold if firms face capacity constraints (see Fabra (2006) which features exogenous constraints or Knittel and Lepore (2010), which features endogenous constraints.)

realizations. Green and Porter (1984) and the vast imperfect monitoring literature take this approach. In this literature, low prices trigger price wars because cartel members cannot distinguish whether they are due to low demand or to cheating by a cartel member. However, given the premised lack of observability of demand, the literature delivers a very limited link between output and demand. Another approach is to introduce capacity constraints for firms, which limit defection gains when demand is high, reducing incentives to cheat, but also weaken the ability to punish in high demand states. Staiger and Wolak (1992) add a capacity-building stage to the Rotemberg and Saloner model and predict (seemingly inconsistent with the empirical evidence), that price wars are more likely in intermediate demand states than in either low demand states or high demand states (see Fabra (2006) for a related result in the Haltiwanger and Harrington (1991) cyclical demand model).

We take a different approach. We return to the insights implicit in Scherer (1980), and explore collusion by risk-averse cartel members that face positive fixed operating costs. We show that this can reconcile the mixed empirical findings in that collusively restricting output toward monopoly levels can be more difficult **both** when demand is unusually low **and** when it is unusually high. Our premise that cartel members are risk averse with CRRA preferences captures the observation that many commodity cartels consist of "country cartels" that do not care about profits per se, but rather about the utility their citizens derive from the profits. In addition to OPEC, country cartels have existed in many manufacturing and commodity cartels (natural resources such as minerals, chemicals, raw materials, metals, etc.; see Suslow (2005) for a list).[2] Because most commodity cartels choose output levels rather than prices, we model output competition when defections from cartel quotas are deterred by threats to revert to static Nash equilibrium output levels. We otherwise focus on the classical i.i.d. demand, constant marginal cost setting studied by Rotemberg and Saloner. We measure the extent of collusion by the ratio of output relative to monopoly levels that can be supported in different demand states—a higher ratio indicates that collusion is more difficult to support.

Risk averse cartel members value a marginal dollar of profit by more when profits are lower. This might lead one to conjecture that with power utility, cartels could find collusion

[2] Among non-country cartels, cartel members may inherit the risk aversion of managers, and financially-constrained firms with large debt levels may also value a marginal dollar in bad times by more because it may stave off liquidation. Consistent with this, many large commodity providers in the middle 20^{th} century were family-owned big companies/oligarchists (e.g., Brazilian coffee farmers); and Busse (2002) provides evidence that price wars in airline industry wee unilaterally initiated by financially troubled firms.

more difficult to support in low demand states. This conjecture is false: to overturn the Rotemberg-Saloner result that higher demand always makes it more difficult for firms to collusively restrain output, cartel members must not only be risk averse, but fixed operating costs of production must be positive. Absent fixed costs, the extent of risk aversion just scales the period incentive to cheat on the cartel, preserving monotonicity of the incentive in demand. Fixed operating costs magnify the marginal utility derived from an additional dollar of profit in bad times, sharply raising the incentive to cheat when demand is especially low.

More provocatively, we establish that collusion is easiest to support when demand is intermediate, neither too low, nor too high. That is, risk aversion together with fixed operating costs give rise to the opposite predictions of those in Staiger and Wolak (1992). The fixed costs of production mean that preferences effectively exhibit decreasing relative risk-aversion. Thus, when demand is especially low, the very high marginal valuation of an additional dollar of profit induced by the fixed operating costs make the incentive to cheat on the cartel very high; and when demand is much higher, the decreasing risk aversion implies that the classical effect dominates—as demand increases, there are more dollars to be gained from cheating on the cartel. We further establish that although the incentive to cheat on the cartel is a U-shaped function of the level of demand, the incentive rises more sharply in low demand states (as demand gets worse) than in higher demand states (as demand gets better).

We then show that greater fixed operating costs or risk aversion make it *harder* to support collusion when demand is low, but *easier* to support collusion when demand is high. Greater fixed costs or risk aversion raise the net continuation payoff from collusion by enhancing the threatened Nash reversion punishment for cheating on the cartel. However, greater fixed costs or risk aversion also raise the potential period utility gains from cheating on the cartel. The impact of higher operating costs on period incentives dominates when demand is especially low, making collusion more difficult to support; but the higher net continuation payoffs dominate when demand is especially high, making collusion easier to support.

In our model, the fixed operating costs together with the CRRA preferences induce cartel members to behave as if they have decreasing absolute risk averse preferences. The literature has long advocated decreasing absolute-risk-aversion utility functions as a more realistic way to describe a firm's behavior. Sandmo (1971), and Appelbaum and Katz (1986) are the first to study firm behavior under uncertainty with absolute/relative risk averse preferences.

Greenwald and Stiglitz (1990) propose micro-foundations for such preferences, arguing that firm behavior resembles a risk-averse individual who maximizes the utility of profitability, and that this utility function is likely to feature decreasing absolute risk aversion. Spagnolo (1999) argues that real world imperfections such as non-linearity of the corporate tax schedule, convexity of external financing costs, and managers' capped monetary incentives together with their fear of loss of job and reputation make a firms static objective function strictly concave, and Asplund (2002) highlights the impact of costly financial distress, liquidity constraints, and non-diversified owners. Using the intuition that "firms give relatively greater weight to realizations with low profits, Aspland looks at how the degree of risk aversion affects competition intensity. He shows that when a firms objective function features decreasing absolute risk-aversion, "fixed costs will influence best-response strategies by increasing firms risk aversion. Spagnolo (1999, 2005) shows that it is easier for firms to support collusive outcomes when they are risk averse.

We next present the model and analysis. A conclusion follows. Proofs are in an appendix.

2 The Model

Our framework features an infinitely repeated output game played by two agents 1 and 2 (e.g., country members of OPEC) that sell a homogeneous good in a market where demand evolves stochastically according to an i.i.d. process. Date t demand is given by

$$P_t = \theta_t - Q_t, \tag{1}$$

where θ_t is identically and independently distributed, with associated distribution function $F(\theta)$ on its positive support $[a, b]$, with $a > 0$, and $Q_t = q_t^1 + q_t^2$ is aggregate output.[3] Without loss of generality, we normalize the constant marginal costs of production to zero. The agents also incur fixed operating costs each period of $c \geq 0$, where $c \leq \frac{a^2}{9}$.

In the classical Rotemberg and Saloner (1986) framework, the agents are risk-neutral firms whose period payoffs equal their period profits, $U^i(\pi_t^i)) = \pi_t^i$. Thus, the marginal value that a firm derives from a dollar of profit does not vary with the level of profit, and fixed operating costs are irrelevant for a firm's decision making (assuming that exit is not a strategic consideration). We depart from this setting to investigate collusion by risk-averse agents that face

[3]Extensions to $N > 2$ agents are routine.

positive fixed operating costs. Agent i derives period utility from profit π_t^i of $U^i(\pi_t^i) = (\pi_t^i)^\alpha$, where $0 < \alpha \leq 1$. For simplicity, we assume that each period, a cartel member consumes its period profits, i.e., there is no saving and borrowing. As a result, a cartel member values an extra dollar of profit by more when profits are lower. Moreover, fixed operating costs enter decision making non-trivially, as they especially magnify the marginal value of an additional dollar in bad times. Firms use a common discount factor $\beta \in (0, 1)$ to discount future payoffs.

The fixed operating costs induce a preference ordering over period income that is a subset of the class of Hyperbolic Absolute Risk Averse utility functions, which has functional form $U(W) = \frac{1-\gamma}{\gamma}\left(\frac{a}{1-\gamma}W + b\right)^\gamma$ for which the risk tolerance (the reciprocal of absolute risk aversion) is linear in wealth[4]. HARA utility nests constant absolute-risk-aversion (exponential), constant relative-risk-aversion (power), quadratic and logarithmic utility functions.

We focus on the maximal period collusion profits that can be supported by threats to revert to the non-cooperative static Nash equilibrium outputs forever if a cartel member ever deviates from their collusive agreement. We do this because we want agents to be able to provide their citizens positive consumption in all states of the world. Our focus on output competition rather than price competition together with the assumption that $c < \frac{a^2}{9}$ ensures that the profits from Nash outputs always cover the period fixed operating costs, providing citizens subsistence consumption. Harsher threats are unlikely to be credible (for instance, failing to provide a minimal subsistence level might result in the state's overthrow). More fundamentally, output competition captures our real world motivating example of a country commodity cartel, in which cartel members choose outputs rather than prices.

After observing period demand, cartel members simultaneously choose outputs. Define $q^C(\theta)$ to be the collusive firm output supported along the equilibrium path when the demand shock is θ. Given that deviations from collusive outputs result in static Nash outputs in the future, an agent that cheats on the cartel agreement will produce the $q^F(\theta)$ that maximizes period profit, and hence period utility, solving

$$\max_{q(\theta)} (\theta - q^C(\theta) - q(\theta))q(\theta) - c \Rightarrow q^F(\theta) = (\theta - q^C(\theta))/2.$$

Let $\pi^C(\theta) = (\theta - 2q^C(\theta))q^C(\theta) - c$ and $\pi^F(\theta) = (\theta - q^C(\theta))^2/4 - c$ denote the respective period profits from cooperating and cheating on the cartel, and let $q^P(\theta) = \theta/3$ be

[4]Substituting $\gamma = \alpha$, $a = (1-\alpha)(\frac{\alpha}{1-\alpha})^{\frac{1}{\alpha}}$, and $b = \frac{-ca}{1-\alpha} = -c(\frac{\alpha}{1-\alpha})^{\frac{1}{\alpha}}$ into the HARA utility function yields $U(W) = (W - c)^\alpha$.

the Nash output and $\pi^P(\theta) = \theta^2/9 - c$ be the associated Nash period profit. Finally, let $U^C \equiv E[U(\pi^C(\theta))]$ be the expected period utility from cooperation along the equilibrium path, and let $U^P \equiv E[U(\pi^P(\theta))]$ be the expected period utility along the punishment path. Then, for each given demand shock θ, incentive compatibility requires

$$U(\pi^C(\theta_t)) + \left(\frac{\beta}{1-\beta}\right) U^C \geq U(\pi^F(\theta_t)) + \left(\frac{\beta}{1-\beta}\right) U^P. \tag{2}$$

Equation (2) can be re-arranged in terms of the "incentive to cheat":

$$U(\pi^F(\theta_t)) - U(\pi^C(\theta_t)) \leq \left(\frac{\beta}{1-\beta}\right)(U^C - U^P) \equiv v. \tag{3}$$

That is, for a cartel production schedule to be incentive compatible, the net period utility payoff from cheating when demand is θ, $U(\pi^F(\theta_t)) - U(\pi^C(\theta_t))$, cannot exceed the net expected payoff from future cooperation rather than punishment, v.

Cartel's Problem. The cartel's objective is to find the incentive compatible production schedule that maximizes their joint utility on the equilibrium path, $\sum_{t=1}^{\infty} \beta^{t-1} E[U(\pi_i^C(\theta_t)) + U(\pi_j^C(\theta_t))]$. With power utility, we can write the cartel's problem as

$$\max_{q(\theta)} 2 \int_a^b (q(\theta)(\theta - 2q(\theta)) - c)^\alpha \, \mathrm{d}F(\theta) \tag{4}$$

$$s.t. \quad (\frac{(\theta - q(\theta))^2}{4} - c)^\alpha - ((\theta - 2q(\theta))q(\theta) - c)^\alpha \leq \left(\frac{\beta}{1-\beta}\right)(U^C - U^P) \equiv v, \quad \forall\, \theta \in [a,b].$$

We measure a cartel's ability to support collusion in demand state θ by the ratio $q^C(\theta)/q^m(\theta) \geq 1$, i.e., by the ratio of output relative to monopoly levels. A higher ratio indicates that collusion is more difficult to support. Most empirical researchers measure collusion in price-cost margins (which, with constant marginal costs, is akin to measuring collusion in prices). Obviously, if price-cost margins fall with θ, then $q^C(\theta)/q^m(\theta)$ also rises with θ. However, with output competition, $q^C(\theta)/q^m(\theta)$ can rise with θ, indicating a reduced ability of the cartel to support collusion in higher demand states, even though price-cost margins rise uniformly with θ. Phrased differently, with output competition, the procyclical price-cost margins found empirically do not imply that collusion is easier to support in high demand states.

For the special case of linear utility, $U(\pi_i(\theta_t)) = \pi_i(\theta_t)$, the cartel's objective reduces to the output-competition variant of Rotemberg and Saloner (1986). In that setting, it immediately follows that the incentive to cheat increases in θ, as with i.i.d. demand, expected continuation payoffs do not vary with θ, but the current payoffs from cheating on the cartel rise when the stakes are higher. As a result, $q^C(\theta)/q^m(\theta)$ is constant when demand is low enough that monopoly profits can be supported, and it is strictly increasing once demand is high enough that threats to deviate to Nash outputs cannot support monopoly profits.

One might conjecture that risk-aversion alone, i.e., $\alpha < 1$, would be enough to reverse the result that increases in demand make it more difficult for the cartel to collusively restrict output toward monopoly levels, i.e., to reverse the result that $q^C(\theta)/q^m(\theta)$ is non-decreasing in θ. That is, one might conjecture that since the marginal utility derived from another dollar of profit is higher when profits are lower, collusion might be more difficult to support when demand is low and cartel members are sufficiently risk averse. This conjecture is false. The following proposition establishes necessary conditions for it to be harder to support collusion when demand is low than when it is high: not only must cartel members be risk averse, $\alpha < 1$, but they must also have positive fixed costs of operation, $c > 0$.

Proposition 1 *Suppose that either $c = 0$ or $\alpha = 1$. Then over-production relative to monopoly levels rises with the level of demand, i.e., $q^C(\theta)/q^m(\theta)$ is non-decreasing in θ.*

Thus, *both* risk-aversion *and* positive fixed operating costs are necessary for overproduction not to rise with θ. Intuition for why more than risk aversion is required can be gleaned from looking at those demand states θ where the net value of future cooperation v is high enough that the IC constraint is slack. For such θ, the cartel's optimization problem simplifies to a pointwise maximization of its objective. The associated first-order condition is

$$(q(\theta)(\theta - 2q(\theta)) - c)^{\alpha-1}(\theta - 4q(\theta)) = 0,$$

with solution $q^C(\theta) = \theta/4$. The two agents jointly produce the monopoly output, $\theta/2$, and each earns half of the monopoly profits net of operating costs, $\theta^2/8 - c$; and the associated fink output is $3\theta/8$, which delivers profits of $\pi^F(\theta) = 9\theta^2/64 - c$. To see how incentives to cheat on the cartel hinge on the level of demand, the extent of risk aversion and the fixed operating costs, define the (period) incentive to cheat on monopoly output as

$$h(\theta;\alpha,c) \;=\; U(\pi^F(\theta)) - U(\pi^C(\theta)) \;=\; \left(\frac{9\theta^2}{64} - c\right)^{\alpha} - \left(\frac{\theta^2}{8} - c\right)^{\alpha}. \tag{5}$$

When there are no fixed costs, $h(\theta, \alpha, c = 0)$ simplifies to

$$h(\theta; \alpha, c = 0) = \left(\frac{9\theta^2}{64}\right)^\alpha - \left(\frac{\theta^2}{8}\right)^\alpha = \left(\frac{\theta^2}{64}\right)^\alpha (9^\alpha - 8^\alpha).$$

Thus, without fixed costs, the extent of risk aversion scales the incentive to cheat, preserving monotonicity in θ. A similar result holds when monopoly output cannot be supported.

We now show that for the incentive to cheat on the cartel not to rise monotonically with the level of demand, the impact of risk aversion must be higher in low demand states than high, i.e., the preferences induced by the fixed operating costs must exhibit decreasing relative risk aversion. With positive fixed operating costs, preferences effectively take a subsistence utility form, and the associated Arrow-Pratt measure of relative risk-aversion, $RRA(W) = -WU''(W)/U'(W) = (1-\alpha)\frac{W}{W-c}$, decreases in W if and only if $\alpha < 1$ *and* $c > 0$. Then, when demand is low, the higher marginal valuation of an additional dollar of profit induced by the fixed operating costs cause the incentives to cheat to rise further when demand drops lower, and agents become more desperate for another marginal dollar of profit. In contrast, when demand is much higher, the decreasing risk aversion implies that risk aversion matters less, with the result that the standard effect dominates—as demand rises, there are more dollars to be gained from cheating on the cartel. Putting these two observations together suggests that the incentive to cheat on the cartel will be a U-shaped function of θ. We now formalize this intuition and begin to address the question of exactly where the separation between good and bad times occurs. The theorem shows that to deliver the U-shaped relationship, agents must have intermediate levels of risk aversion: for the incentive to cheat on the cartel not to rise monotonically with θ, for a given level of fixed costs, agents must be sufficiently risk averse; and for the incentive not to fall monotonically with θ, they must not be too risk averse.

Proposition 2 *There exist critical levels of risk aversion, $\underline{\alpha}(c)$ and $\bar{\alpha}(c)$, indexed by the fixed costs c, such that if and only if cartel members have intermediate levels of risk aversion, $\underline{\alpha}(c) < \alpha < \bar{\alpha}(c) < 1$, then h is a U-shaped function of θ, achieving a minimum at $\hat{\theta}(\alpha, c) \in (a, b)$. That is, $h'(\theta) < 0$ for $\theta < \hat{\theta}(\alpha, c)$, and $h'(\theta) > 0$ for $\theta > \hat{\theta}(\alpha, c)$. Further, $\underline{\alpha}(c)$, $\bar{\alpha}(c)$ and $\hat{\theta}(\alpha, c)$ rise with the fixed cost c, and $\hat{\theta}(\alpha, c)$ increases in risk aversion α.*

The proof shows that, as in Figure 1, there is a unique intermediate demand level $\hat{\theta}$ that minimizes the incentive to cheat. As demand falls below $\hat{\theta}$, the incentive to cheat rises due to

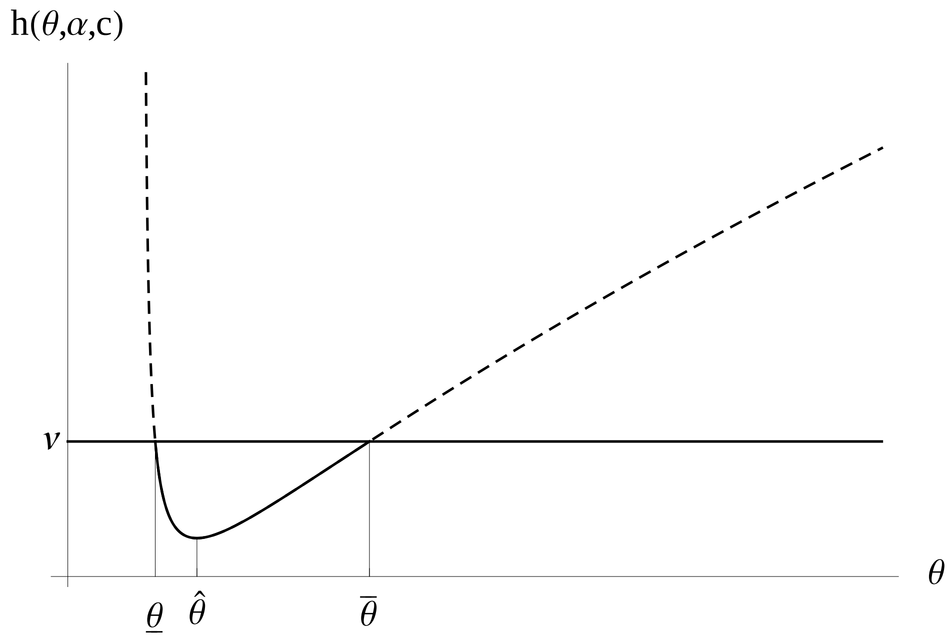

Figure 1: Incentive to cheat $h(\theta;\alpha,c)$ as a function of θ. Parameters are: $\alpha = 1/3$ and $c = 1/9$.

the high marginal valuation of another dollar of profit; and as demand rises above $\hat{\theta}$, so too does the incentive to cheat due to the greater profit that can be gained. The comparative statics reveal that when agents are more risk averse or fixed costs are greater, demand does not have to be as bad for the incentive to cheat to begin to rise as demand drops lower.

Monopoly outputs are supportable when the period benefit from cheating, h, is less than the expected net value of future cooperation, v, which is independent of θ. When h is a U-shaped function of θ, it directly follows that monopoly outputs can only be sustained for intermediate values of demand whenever cartel members are neither so patient that they can support monopoly outputs in every state, nor so impatient that they can support monopoly outputs in no state (see Figure 1). Corollary 1 formalizes the necessary conditions.

Corollary 1 *There exist $\underline{\beta}$, $\bar{\beta}$ with $\underline{\beta} < \bar{\beta}$ such that if and only if $\beta \in [\underline{\beta}, \bar{\beta}]$ the cartel can support monopoly profits only if demand is neither too low nor too high: If and only if $\beta \in [\underline{\beta}, \bar{\beta}]$, there exist $\underline{\theta}(\beta)$, $\bar{\theta}(\beta)$ with $a < \underline{\theta}(\beta) < \bar{\theta}(\beta) < b$ such that monopoly profits can be supported if and only if $\theta \in [\underline{\theta}(\beta), \bar{\theta}(\beta)]$.*

We have identified two forces that can drive the cartel away from supporting monopoly outputs: temptations rooting from the larger potential profit gain when times are good, and desperateness for an extra dollar of profits when times are bad. But, which force is stronger? In Proposition 3, we show that the ability to support collusion drops off more quickly when demand falls below $\hat{\theta}$ than when it rises past $\hat{\theta}$.

Proposition 3 *The incentive to cheat on monopoly output rises more quickly as low demand states become worse than as high demand states improve:* $h(\hat{\theta}-\delta) > h(\hat{\theta}+\delta)$ *for all* $\delta > 0$.

The intuition for Proposition 3 devolves from the increasing desperation to obtain another dollar of profit when its marginal valuation is high that is implicit in Scherer (1980)'s summary that *"Yet it is precisely when business conditions turn sour that price cutting runs most rampant among oligopolistic firms with high fixed costs."* Proposition 3 goes beyond Proposition 2. Proposition 2 showed that the incentive to cheat on monopoly output rises not only when demand is larger, but also when market conditions turn sour. Proposition 3 documents an asymmetry in the incentive to cheat function: monopoly can be supported in a narrower range of bad states than good ones. Put differently, $\hat{\theta}$ is closer to $\underline{\theta}$ than to $\bar{\theta}$.

We now characterize output levels following demand realizations—both high and low—that are sufficiently extreme that the cartel cannot support monopoly outputs. To prevent agents from cheating, cartel output must be increased to a level that makes agents indifferent between cheating and cooperation. More formally, at each $\theta \in [a,\underline{\theta}]\bigcup[\bar{\theta},b]$ incentive compatible quotas, $q(\theta)$, solve

$$(\frac{(\theta-q(\theta))^2}{4}-c)^{\alpha}-((\theta-2q(\theta))q(\theta)-c)^{\alpha}=\left(\frac{\beta}{1-\beta}\right)(U^C-U^P)\equiv v. \tag{6}$$

Define the normalized production level $z \equiv q(\theta)/\theta$: z is an index for overproduction relative to monopoly output, as $4z = q(\theta)/(\theta/4) = q(\theta)/q^m(\theta)$. When monopoly output can be supported, there is no overproduction, so that $z = 1/4$; and when the cartel breaks down and agents revert to Nash outputs, then $z = 1/3$. That is, outside the monopoly support region, we have $z \in (1/4, 1/3)$, and profits decrease in z.

We rewrite the left-hand side of equation (6) in terms of z and define $H(z,\theta)$ to be this period incentive to cheat:

$$H(z,\theta)=\left(\theta^2\frac{(1-z)^2}{4}-c\right)^{\alpha}-\left(\theta^2(1-2z)z-c\right)^{\alpha}. \tag{7}$$

When $z = 1/4$, then $H(1/4, \theta; \alpha, c)$ reduces to the period incentive to cheat on monopoly output, $h(\theta; \alpha, c)$. As in Proposition 2, one can show that $H(z, \theta)$ is a U-shaped function of θ for every $z \in (1/4, 1/3)$. Proposition 4 shows that when demand realizations make it more attractive to cheat on the cartel, members must reduce this attraction by increasing output relative to the monopoly level, but that output increases become less and less effective at reducing this incentive. Further, collusion is harder to sustain both for more extreme low demand realizations and for more extreme high demand realizations, requiring greater overproduction:

Proposition 4 *Outside the monopoly support region* $[\underline{\theta}, \bar{\theta}]$*, the period incentive to cheat is a continuously decreasing, convex function of output relative to monopoly levels:* $\frac{\partial H(z,\theta)}{\partial z} < 0$*, and* $\frac{\partial^2 H(z,\theta)}{\partial z^2} > 0$*. Overproduction relative to monopoly output rises when demand is further from the monopoly support region:* $\partial \left(\frac{q(\theta)}{q^m(\theta)} \right) / \partial(\underline{\theta} - \theta) > 0$ *for* $\theta < \underline{\theta}$*, and* $\partial \left(\frac{q(\theta)}{q^m(\theta)} \right) / \partial(\theta - \bar{\theta}) > 0$ *for* $\theta > \bar{\theta}$*.*

One might conjecture that when fixed operating costs, c, are higher, or cartel members are more risk averse (lower α), it becomes more difficult to support collusion in every demand state. The intuition underlying such a conjecture is that such changes raise the period utility gain from cheating on any given level of output. However, the intuition underlying this conjecture is incomplete. The conjecture that greater fixed costs or increased risk aversion make collusion harder to support would follow directly *if* the net continuation payoffs from collusion versus punishment did not rise. However, as c is increased (or agents become more risk averse), the threat to punish cheating on the cartel by reverting to Nash equilibrium outputs becomes harsher relative to the gain from a given level of cooperation. If, as a result, v rises by enough with greater operating costs or risk aversion to offset the increased period incentive to cheat on the cartel, then greater collusion may be facilitated.

We next characterize how the extent of risk aversion or fixed costs affect the ability to support collusion in different demand states. We establish a single-crossing property characterizing which states collusion is easier to support. We show that provided that increases in operating costs c or in risk aversion (reductions in α) do not uniformly raise or lower the incentive to cheat on the cartel, then greater fixed costs and greater risk aversion make collusion harder to support when demand is low, but easier when demand is high.

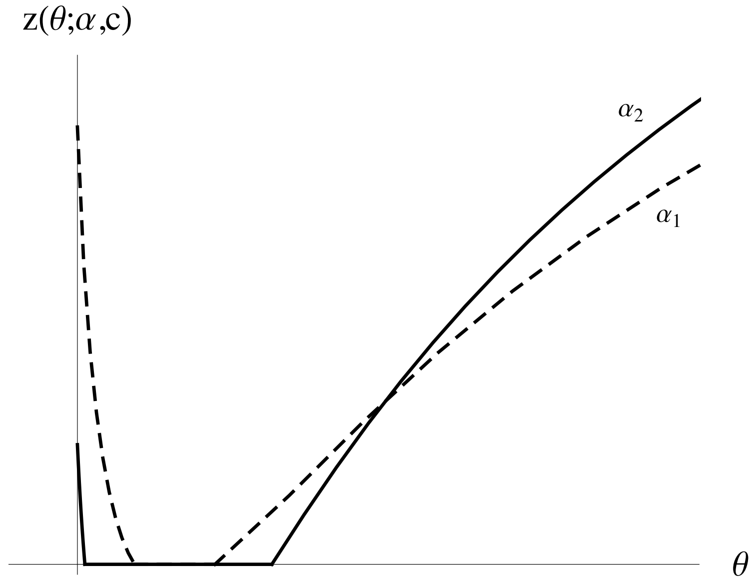

Figure 2: Dashed (solid) line presents the ratio of cartel-to-monopoly output for more (less) risk averse agents ($\alpha_2 = 2/5 < \alpha_1 = 1/2$). Other parameters: $c = \frac{1}{9}$, $\beta = 0.43$, $\theta \in [1, 5]$.

Proposition 5 *Consider $\alpha_2 < \alpha_1$. Suppose there exists a θ^* such that $z_1(\theta^*) = z_2(\theta^*) > \frac{1}{4}$. Then outside the monopoly support region, more risk averse agents find it harder to support collusion in bad times, but easier in good times: For $\alpha_2 < \alpha_1$, for all $\theta < \theta^*$, if $z_2(\theta) > 1/4$, then $z_2(\theta) > z_1(\theta)$; and for all $\theta > \theta^*$, if $z_1(\theta) > 1/4$ then $z_2(\theta) < z_1(\theta)$.*

Proposition 6 *Consider fixed operating costs, $c_2 > c_1$. Suppose there exists a θ^* such that $z_1(\theta^*) = z_2(\theta^*) > \frac{1}{4}$. Then outside the monopoly support region, greater fixed costs make it harder to support collusion in bad times, but easier in good times: For $c_2 > c_1$, for all $\theta < \theta^*$, if $z_2(\theta) > 1/4$, then $z_2(\theta) > z_1(\theta)$; and for all $\theta > \theta^*$, if $z_1(\theta) > 1/4$ then $z_2(\theta) < z_1(\theta)$.*

The key to these proofs is to show that the impact of an increase in c or in risk aversion on the period gain from cheating, $H(z, \theta)$ falls with θ for a fixed $z = q(\theta)/\theta$, i.e., that $\frac{\partial^2 H(z,\theta;c)}{\partial c \partial \theta} < 0$ and $\frac{\partial^2 H(z,\theta;\alpha)}{\partial \alpha \partial \theta} > 0$. Hence, if we ever have $z_1(\theta^*) = z_2(\theta^*)$, then holds at a unique θ^*.

An exhaustive numerical analysis indicates that whenever demand is uniformly distributed, and agents are sufficiently risk averse with high enough operating costs that the monopoly support region is interior, then continuation payoffs always rise with c or with risk aversion by amounts that, consistent with Figures 2 and 3 and the two propositions, give rise to asymmetric effects on the cartel's ability to support collusion. That is, with uniform de-

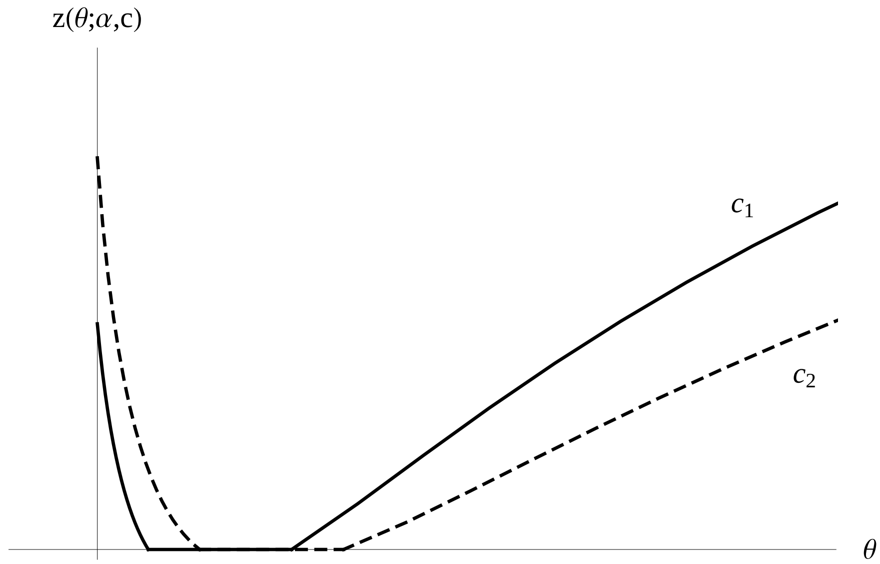

Figure 3: Dashed (solid) line presents the ratio of cartel-to-monopoly output for lower (higher) fixed costs ($c_2 = 1/9 > c_1 = 1/13$). Other parameters: $\alpha = .5$, $\beta = .4$, $\theta \in [1, 5]$.

mand shocks, the single-crossing property always holds. Indeed, numerically we find that the "crossing point" is always at a high demand state.[5] That is, the effect of an increase in c or reduction in α on the increased incentive to cheat dominates the impact on net continuation payoffs for sufficiently low demand shocks where agents are especially desperate for another dollar of profit. However, net continuation payoffs rise with increased operating costs and increased risk aversion, and this effect dominates once demand is sufficiently high, making collusion easier to sustain. These results reflect the induced decreasing relative risk aversion in preferences—the effect of an increase in operating costs or risk aversion on the period utility gain from cheating on a given level of collusion falls as demand, and hence profits, rise.

Asymmetric cartels. Although we do not analyze it formally, Propositions 5 and 6 have suggestive implications for how heterogeneous agents with different levels of fixed operating costs or risk aversion should collude. For example, in practice, OPEC countries do not rely solely on oil revenues, and higher non-oil revenues effectively imply lower fixed operating costs. From this perspective Saudi Arabia with $1,789 per capita in non-oil export revenues may effectively have lower fixed operating costs than OPEC member such as Venezuel which only has $127 per capita in non-oil export revenues.[6]. Then the propositions would suggest

[5]Not surprisingly, one can construct very asymmetric distributions such that on some parameter range, changes in parameters have uniform effects in all demand states on the incentives to cheat on the cartel.

[6]Data source: 2013 OPEC Statistical Bulletin and 2013 IMF World Economic Outlook

that Saudi Arabia should have a lower share of output in low demand states (where high operating cost cartel members find collusion more difficult to sustain), but a higher share of output when demand is high (and high operating cost cartel members mind ceding share by less, and are willing to do so in order to obtain a greater share in low demand states where they care more about their share). In particular, high operating cost countries gain relatively more utility from a marginal dollar when demand is low, and their incentive to cheat on the cartel in low demand states is higher; so that a cartel that seeks to maximize a weighted sum of the uitilities of each caretel member will allocate relatively greater shares to high operating cost/more risk-averse countries when demand is low, and relatively lower shares when demand is high. Consequently, the low operating cost or less risk averse cartel member's output should be more sensitive to the level of demand than is the output of higher operating cost or more risk-averse cartel members. Thus, Saudi Arabia should be the swing producer, with relatively lower outputs in bad times, and relatively higher outputs in good times, so that its output would appear to be the primary driver determining cartel outcomes.

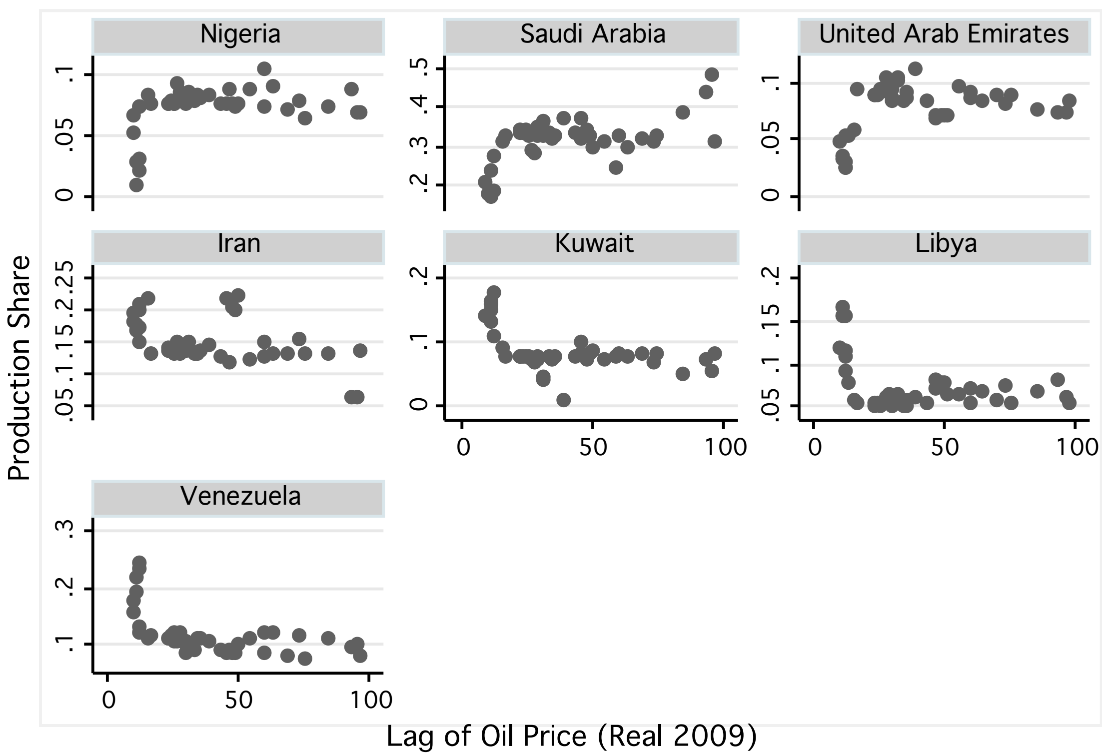

Figure 4: Production shares of selected OPEC countries as a function of price, 1965-2009, excluding Iraq's production.

	Venezuela	Iran	Kuwait	Libya	Nigeria	UAE	Saudi
Oil Price	-0.085***	-0.070***	-0.066***	-0.043**	0.025**	0.030**	0.158***
	(0.00)	(0.00)	(0.00)	(0.02)	(0.03)	(0.02)	(0.00)
Constant	0.151***	0.179***	0.113***	0.089***	0.063***	0.066***	0.250***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.29	0.24	0.23	0.13	0.11	0.11	0.36
N	44	44	44	44	44	44	44

Table 1: **Price sensitivity of production shares 1965-2009:** In each regression the dependent variable is the share of OPEC production for each country. The independent variable is the lag of 2009 real oil price. The numbers in parenthesis are p-values. ***, ** denote significance at 5%, and 10% levels.

Figure 4 offers evidence consistent with such a premise.[7] It reveals that production shares are very asymmetric, with four countries—Iran, Kuwait, Libya and Venezuela—consistently having sharply higher shares when oil prices are very low; and three countries—Saudi Arabia, United Arab Emirates and Nigeria—consistently having sharply lower production shares when oil prices are very low, and higher shares when oil prices are at their highest. [8] Table 1 summarizes this figure by measuring the slope of the price-share relationship using regression analysis. It shows that Saudi Arabia among the pro-cyclical producers and Venezuela among the counter-cylycal producers have the highest (absolute) price sensitivity. On average, for every dollar drop in real oil price, in the following year Saudi Arabia cuts back on its share by 0.158%, whereas Venezuela increases its share by 0.0805%. Obviously, conclusions about why these patterns obtain are speculative in nature. Nonetheless, the patterns are suggestive.

3 Conclusion

A robust prediction of the theoretical literature on collusion under demand uncertainty when cartel members observe demand and can monitor each other's actions is that collusion is more difficult when demand is higher. In contrast to this theoretical prediction, most empirical

[7]The pattern is robust to including year-fixed effects to control for events in this period that can impact all OPEC members such as Iraq war, Gulf war, Iran revolution, etc.

[8]We omit Algeria, Indonesia and Qatar from Figure 4 because their shares of production do not systematically vary with price. We exclude Iraq's share due to the impact of the war and the fact that Iraq did not fully participate in OPEC's production agreements in this period.

researchers have concluded that price wars are more common when demand is low.

We provide a simple theory of collusion by risk averse agents that face positive fixed operating costs that can reconcile these literatures by providing conditions under which it is most difficult to collusively restrict output when demand is especially low, but that it also becomes difficult to support collusion when demand is high. The idea that cartel members are risk averse captures the observation that many effective cartels are comprised of countries that collusively restrict output of various commodities. Such cartels do not care directly about profits, per se, but rather about the utility derived by their risk-averse citizens who receive those profits. As a result, the marginal value of a dollar of profit is greater when demand, and hence profits, are lower; and this high marginal valuation is magnified by the large fixed operating costs that Scherer (1980) cites as playing a vital role in making collusion difficult.

We show that for aggregate cartel output relative to monopoly levels to be a U-shaped function of the level of demand, both ingredients are necessary—cartel members must be risk averse, and operating costs must be positive. We further establish that when cartel members are more risk averse or fixed operating costs are higher, then it becomes more difficult to support collusion in bad demand states, but easier in good ones.

Appendix

Proof of Proposition 1 : When $\alpha = 1$, it is immediate from Rotemberg and Saloner (1986) that $q(\theta)/q^m(\theta)$ is non-decreasing in θ.

If $c = 0$, and the IC constraint does not bind, then $q^C(\theta)/q^m(\theta) = 1$. Now suppose that the IC constraint binds, and let $\theta_1 < \theta_2$ be two arbitrary values of θ outside the monopoly support region. Since $q^m(\theta) = \theta/4$, to show that $q(\theta)/q^m(\theta)$ increases in θ we must show that $q(\theta_2)/\theta_2 > q(\theta_1)/\theta_1$, where $q(\theta_i)/\theta_i \equiv z_i \in (1/4, 1/3)$. To prove that $z_2 > z_1$, suppose instead that $z_1 \geq z_2$. Rewrite the IC constraint in terms of z_i when $c = 0$ as:

$$\Big(\theta_i^2 \frac{(1-z_i)^2}{4}\Big)^\alpha - \big(\theta_i^2(1-2z_i)z_i\big)^\alpha = v.$$

Since v is independent of θ,

$$\Big(\theta_1^2 \frac{(1-z_1)^2}{4}\Big)^\alpha - \Big(\theta_1^2(1-2z_1)z_1\Big)^\alpha = \Big(\theta_2^2 \frac{(1-z_2)^2}{4}\Big)^\alpha - \Big(\theta_2^2(1-2z_2)z_2\Big)^\alpha.$$

Since $\theta_1 < \theta_2$, it follows that

$$\Big(\frac{(1-z_1)^2}{4}\Big)^\alpha - \Big((1-2z_1)z_1\Big)^\alpha > \Big(\frac{(1-z_2)^2}{4}\Big)^\alpha - \Big((1-2z_2)z_2\Big)^\alpha.$$

Calling the four terms in this inequality from left to right as A, B, C and D, rewrite the inequality as: $A - B > C - D$. Under the assumption $z_1 > z_2$, and recalling that cooperation profits decrease in z, i.e., $(1-2z)z$ decreases in $z > 1/4$, we have $B/D < 1$. Therefore, $A - B > C - D$ implies that $\frac{A-B}{B} > \frac{C-D}{B} > \frac{C-D}{B}\frac{B}{D} = \frac{C-D}{D}$. Therefore, $\frac{A}{B} > \frac{C}{D}$, i.e.,

$$\Big(\frac{\frac{(1-z_1)^2}{4}}{(1-2z_1)z_1}\Big)^\alpha > \Big(\frac{\frac{(1-z_2)^2}{4}}{(1-2z_2)z_2}\Big)^\alpha,$$

or equivalently, $\frac{(1-z_1)^2}{(1-2z_1)z_1} > \frac{(1-z_2)^2}{(1-2z_2)z_2}$ for $z_2 \geq z_1$. But $g(z) = \frac{(1-z)^2}{(1-2z)z}$ is a decreasing function of z, i.e., $g'(z) < 0$, a contradiction. $\square$

Proof of Proposition 2: The first-order condition is

$$h'(\theta) = 2\alpha\theta \left(\frac{9}{64}\Big(\frac{9\theta^2}{64} - c\Big)^{\alpha-1} - \frac{1}{8}\Big(\frac{\theta^2}{8} - c\Big)^{\alpha-1} \right) = 0.$$

Solving yields

$$\frac{\frac{\theta^2}{8} - c}{\frac{9\theta^2}{64} - c} = \Big(\frac{8}{9}\Big)^{\frac{1}{1-\alpha}} \equiv k \Rightarrow \hat{\theta} = 8\sqrt{\frac{(1-k)c}{8-9k}}.$$

Notice that $k = (\frac{8}{9})^{\frac{1}{1-\alpha}} < \frac{8}{9} < 1$. Clearly, $\theta < \hat{\theta}$ implies that

$$\frac{\frac{\theta^2}{8} - c}{\frac{9\theta^2}{64} - c} < (\frac{8}{9})^{\frac{1}{1-\alpha}} \quad \Rightarrow 9(\frac{9\theta^2}{64} - c)^{\alpha-1} < 8(\frac{\theta^2}{8} - c)^{\alpha-1}.$$

Therefore, $h'(\theta) < 0$. A similar argument holds for $\theta > \hat{\theta}$.

The requirement that $a < \hat{\theta} < b$ imposes bounds on the range of α. We require

$$a < \hat{\theta} = 8\sqrt{\frac{(1-k(\alpha))c}{8-9k(\alpha)}} < b.$$

Solving yields the upper and lower bounds:

$$\bar{\alpha}(c) = 1 + \frac{\log(9/8)}{\log(\frac{8a^2-64c}{9a^2-64c})} \quad \text{and} \quad \underline{\alpha}(c) = 1 + \frac{\log(9/8)}{\log(\frac{8b^2-64c}{9b^2-64c})}.$$

Since $\alpha(x;c) = 1 + \log(9/8)/\log(\frac{8x^2-64c}{9x^2-64c})$ is a decreasing function of x, with a limit of zero as x goes to infinity, $\hat{\theta} \in (a,b)$ exists as long as $\underline{\alpha}(c) < \alpha < \bar{\alpha}(c)$.

Finally, differentiating $\hat{\alpha}(c)$ and $\hat{\theta}(\alpha, c)$ with respect to c and α delivers the comparative statics results:

$$\frac{\partial\hat{\alpha}(c)}{\partial c} = \frac{8a^2\log(9/8)}{(9a^2-64c)(a^2-8c)\left(\log(1-\frac{a^2}{9a^2-64c})\right)^2} \geq 0,$$

which is non-negative since $c < \frac{a^2}{9}$; and

$$\frac{\partial\hat{\theta}(\alpha,c)}{\partial\alpha} = -\frac{2^{\frac{2\alpha-1}{\alpha-1}}9^{\frac{1}{1-\alpha}}\log\left[\frac{9}{8}\right]c}{(\alpha-1)^2\left(2^{\frac{3\alpha}{\alpha-1}}9^{\frac{1}{1-\alpha}}-9\right)^2\sqrt{\frac{\left(\left(\frac{9}{8}\right)^{\frac{1}{1-\alpha}}-1\right)c}{2^{\frac{3\alpha}{\alpha-1}}9^{\frac{1}{1-\alpha}}-9}}} < 0; \quad \frac{\partial\hat{\theta}(\alpha,c)}{\partial c} = \frac{4}{c}\sqrt{\frac{\left(\left(\frac{8}{9}\right)^{\frac{1}{1-\alpha}}-1\right)c}{8^{\frac{1}{1-\alpha}}9^{\frac{\alpha}{\alpha-1}}-8}} > 0. \quad \square$$

Proof of Corollary 1: Let $\underline{\theta} < \bar{\theta}$ be the two roots of $h(\theta;\alpha,c) = v$ when it has two roots for $a \leq \theta \leq b$. Note that v is independent of θ. Since $h(\theta;\alpha,c)$ is a U-shaped function of θ (Proposition 2), for intermediate values of θ where $h(\theta;\alpha,c) < v$ the IC constraint (3) is slack. Therefore, monopoly profits can be supported for $\theta \in [\underline{\theta}, \bar{\theta}]$.

For $h(\theta;\alpha,c) = v$ to have two roots, v can be neither too small nor too large. Since

$v \equiv \left(\frac{\beta}{1-\beta}\right)(U^C - U^P)$, there is one-to-one mapping between v and β [9]. Thus, we must bound β appropriately: β must exceed the $\underline{\beta}$ that solves

$$h(\hat{\theta};\alpha,c) = \left(\frac{\underline{\beta}}{1-\underline{\beta}}\right)(U^C - U^P),$$

and be less than the $\bar{\beta}$ that solves

$$\text{Min}\{h(a;\alpha,c), h(b;\alpha,c)\} = \left(\frac{\bar{\beta}}{1-\bar{\beta}}\right)(U^C - U^P).$$

Hence, $h(\theta;\alpha,c) = v$ has two roots for $\theta \in [a,b]$ if and only if $\beta \in [\underline{\beta}, \bar{\beta}]$. $\square$

Proof of Proposition 3: We must show that $h(\hat{\theta}-\delta) > h(\hat{\theta}+\delta)$, i.e.,

$$\left(\frac{9}{64}(\hat{\theta}-\delta)^2 - c\right)^{\alpha} - \left(\frac{1}{8}(\hat{\theta}-\delta)^2 - c\right)^{\alpha} > \left(\frac{9}{64}(\hat{\theta}+\delta)^2 - c\right)^{\alpha} - \left(\frac{1}{8}(\hat{\theta}+\delta)^2 - c\right)^{\alpha}.$$

Let $m_1 = \frac{1}{8}(\hat{\theta}^2+\delta^2) - c$, and $m_2 = \frac{9}{64}(\hat{\theta}^2+\delta^2) - c$. Also let $n_1 = \frac{1}{8}(2\hat{\theta}\delta)$, and $n_2 = \frac{9}{64}(2\hat{\theta}\delta)$. Now rewrite the inequality to be established as

$$(m_1+n_1)^{\alpha} - (m_1-n_1)^{\alpha} > (m_2+n_2)^{\alpha} - (m_2-n_2)^{\alpha}.$$

Given that $m_2 > m_1$ and $n_2 > n_1$, to prove the above inequality it suffices to show that the cross-derivative of $L = (m+n)^{\alpha} - (m-n)^{\alpha}$ with respect to m and n is negative, and we have:

$$\frac{\partial^2 L}{\partial m \partial n} = -\alpha(1-\alpha)\left((m+n)^{\alpha-2} + (m-n)^{\alpha-2}\right) < 0. \quad \square$$

Proof of Proposition 4:

$$\frac{\partial H(z,\theta)}{\partial z} = \frac{1}{2}\alpha\theta^2\left(2(4z-1)(\theta^2(1-2z)z-c)^{\alpha-1} - (1-z)(\frac{\theta^2}{4}(1-z)^2-c)^{\alpha-1}\right).$$

[9]We ignore the endogeneity of q with respect to β and its effect on U^C. As β rises there is an indirect effect on v via changes in U^C. That is, U^C is a function of $q(\theta)$ at each state θ, but its effect is reinforcing: (i) from $v \equiv \left(\frac{\beta}{1-\beta}\right)(U^C - U^P)$, we see that fixing q state-by-state, increasing β raises v, i.e., $dv/d\beta > 0$ for fixed q's, and (ii) from the constrained optimization in (4), increasing v relaxes the IC constraint, weakly increasing period payoffs, i.e., $dU^C/dv \geq 0$. It follows from (i) and (ii) that $dU^C/d\beta \geq 0$. Since $U^P = E[U(\theta^2/9 - c)]$ is independent of q and therefore of β, the indirect effect of β on v must be reinforcing, i.e., $d(U^C - U^P)/d\beta > 0$. Therefore, there is a one-to-one relationship between β and v.

To show $\frac{\partial H(z,\theta)}{\partial z}$ is negative, equivalently we must prove:

$$\frac{1-z}{2(4z-1)} > \left(\frac{\theta^2 \frac{1}{4}(1-z)^2 - c}{\theta^2 z(1-2z) - c}\right)^{1-\alpha}.$$

Since the cheat payoff, $(1-z)^2/4$ always exceeds the cooperation payoff, $z(1-2z)$, the right-hand side exceeds one. Therefore, it suffices to show that

$$\frac{1-z}{2(4z-1)} > \frac{\theta^2 \frac{1}{4}(1-z)^2 - c}{\theta^2 z(1-2z) - c}.$$

Define $c' \equiv c/\theta^2$ and rearrange the above inequality as

$$\frac{1-z}{2(4z-1)} - \frac{\frac{1}{4}(1-z)^2 - c'}{z(1-2z) - c'} = \frac{\frac{1}{2}(3z-1)(z-1+6c')}{2(4z-1)(z(1-2z)-c')} > \frac{\frac{1}{6}(3z-1)^2}{2(4z-1)(z(1-2z)-c')} > 0,$$

for $1/4 \leq z < 1/3$. The next to the last inequality follows since the above expression decreases in c' and thus is minimized when c' equals its upper bound of $Max(c/\theta^2) = (a^2/9)/a^2 = 1/9$, implying that $3(z-1+6c') > (3z-1)$.

To prove convexity of H, we bound the second derivative of H/α strictly away from zero (we divide by α because the derivative of H goes to zero as α goes to zero). We also write H/α in terms of $c' = c/\theta^2 \in [0, \frac{1}{9}]$ to make the domain compact:

$$\frac{1}{\alpha} H(z;\alpha,c') = \frac{1}{\alpha}\left[\left(\frac{(1-z)^2}{4} - c'\right)^{\alpha} - ((1-2z)z - c')^{\alpha}\right],$$

with associated second derivative

$$\begin{aligned}\frac{1}{\alpha}\frac{d^2H}{dz^2} = {} & \frac{1}{4}(1-\alpha)\left[4(4z-1)^2\left((1-2z)z-c'\right)^{\alpha-2} - (1-z)^2\left(\frac{1}{4}(1-z)^2 - c'\right)^{\alpha-2}\right] \\ & + \frac{1}{2}\left[\left(\frac{1}{4}(1-z)^2 - c'\right)^{\alpha-1} + 16\left((1-2z)z-c'\right)^{\alpha-1}\right].\end{aligned} \tag{8}$$

The compact domain has $z \in [1/4, 1/3]$, $\alpha \in [0,1]$ and $c' \in [0, 1/9]$. Further, $\frac{1}{\alpha}\frac{d^2H}{dz^2}$ is continuous and twice differentiable on its domain, with derivatives bounded from below, so that in an ϵ ball around any point (z,α,c'), $\frac{1}{\alpha}\frac{d^2H}{dz^2}$ cannot drop too far below its value at (z,α,c'). Therefore, to establish convexity, it suffices to bound $\frac{1}{\alpha}\frac{d^2H}{dz^2}$ strictly away from zero on an appropriately fine grid. An exhaustive search on a grid with increments of 0.001 for z, α and c' reveals that it achieves a lower bound of $9/2$ when $\alpha = 1$. See Figure 5.

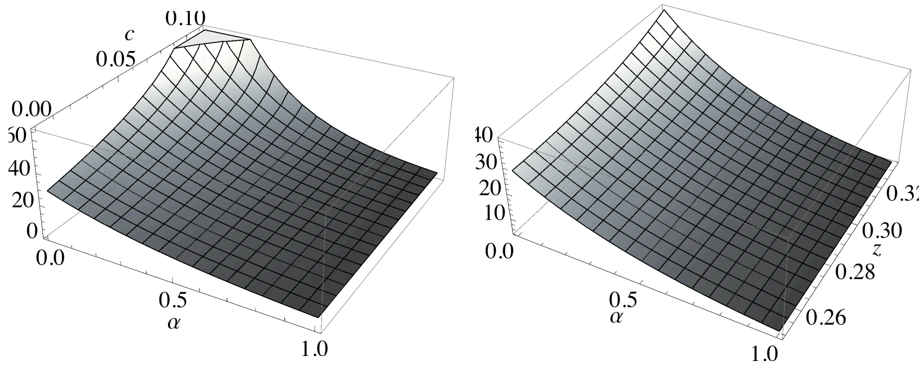

Figure 5: 3-dimensional plot of the second derivative of H/α; (a) the left panel shows H/α as a function of α and c at $z = 1/4$, and (b) the right panel shows H/α as a function of α and z at $c = 0$.

We now establish that over-production relative to monopoly increases in $\theta - \bar{\theta}$ for $\theta > \bar{\theta}$; and in $\underline{\theta} - \theta$, for $\theta < \underline{\theta}$. First consider any $\theta_2 > \theta_1 \in (\bar{\theta}, b]$. To establish that $q(\theta)/q^m(\theta)$ increases in $(\theta - \bar{\theta})$, we show that $z_2 > z_1$. Suppose instead that $z_1 > z_2$. We have:

$$H(z_i, \theta_i) = \left(\theta_i^2 \frac{(1 - z_i)^2}{4} - c\right)^\alpha - \left(\theta_i^2 (1 - 2z_i) z_i - c\right)^\alpha = v \quad \text{for} \quad i = 1, 2.$$

Consider the two functions $H(., \theta_1)$ and $H(., \theta_2)$. From Proposition 2 for $\theta > \bar{\theta}$, h increases in θ, so

$$h(\theta_2; \alpha, c) > h(\theta_1; \alpha, c) \Rightarrow H(1/4, \theta_2) > H(1/4, \theta_1).$$

Also from incentive compatibility,

$$H(z_2, \theta_2) = H(z_1, \theta_1) = v,$$

at the premised $z_1 > z_2$, and since $H(z, \theta)$ is decreasing in z for any θ, this implies that $H(z_2, \theta_1) > H(z_2, \theta_2)$. But if $H(1/4, \theta_2) > H(1/4, \theta_1)$ and $H(z_2, \theta_2) < H(z_2, \theta_1)$ then by the intermediate value theorem there exists a z' with $1/4 < z' < z_2$ such that $H(z', \theta_2) = H(z', \theta_1)$, a contradiction of $\theta_2 > \theta_1$ and $z' < 1/3$.

An identical proof by contradiction establishes that if $\theta_1 < \theta_2 < \underline{\theta}$, then $z_1 > z_2$. That is, $z_1 < z_2$ would imply $H(z_2, \theta_2) = H(z_1, \theta_1)$ at $z_1 < z_2$ (by incentive compatibility), and hence $H(z_1, \theta_2) > H(z_1, \theta_1)$, but here, $H(1/4, \theta_1) > H(1/4, \theta_2)$ yields a contradiction via the intermediate value theorem. $\square$

Proof of Proposition 5: Let $H_1(z,\theta) \equiv H(z,\theta;\alpha_1,c)$ and $H_2(z,\theta) \equiv H(z,\theta;\alpha_2,c)$ for $\alpha_1 > \alpha_2$. We prove that if there exists a θ such that $z_1(\theta) = z_2(\theta)$, then it is unique. Call these values θ^* and z^*. To establish this single-crossing result, we prove that for a fixed z, $H_1 - H_2$ increases in θ by showing that $\frac{\partial H}{\partial \theta}$ increases in α. Therefore, there exists a neighborhood of θ^* and z^*, such that for a fixed z, H_1 is a steeper function of θ than H_2.

When both IC constraints bind (i.e., $H_i = v_i$ for $i = 1, 2$) then $H_1 - H_2 = v_1 - v_2$ does not vary with θ, i.e., $\frac{\partial H}{\partial \theta} = \frac{\partial v}{\partial \theta} = 0$.

$$H(z,\theta) = \left(\theta^2 \frac{(1-z)^2}{4} - c\right)^{\alpha} - \left(\theta^2(1-2z)z - c\right)^{\alpha} = v \equiv v(\alpha, c), \tag{9}$$

$$\frac{\partial H}{\partial \theta} = \frac{1}{2}\theta\alpha \left((1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - c\right)^{\alpha-1} - 4z(1-2z)\left(\theta^2(1-2z)z - c\right)^{\alpha-1}\right) = 0.$$

Defining $\gamma_F \equiv (1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - c\right)^{\alpha-1}$ and $\gamma_C \equiv 4z(1-2z)\left(\theta^2(1-2z)z - c\right)^{\alpha-1}$, we must have $\gamma_F = \gamma_C \equiv \gamma$. We now prove that $\frac{\partial H}{\partial \theta}$ increases in α, i.e., $\frac{\partial^2 H}{\partial \theta \partial \alpha} > 0$:

$$\begin{aligned}\frac{\partial^2 H}{\partial \theta \partial \alpha} = \frac{1}{2}\theta\Big(&(1-z)^2\Big(\theta^2\frac{(1-z)^2}{4} - c\Big)^{\alpha-1} - 4z(1-2z)\Big(\theta^2(1-2z)z - c\Big)^{\alpha-1} \\ &+ \alpha\Big((1-z)^2\Big(\theta^2\frac{(1-z)^2}{4} - c\Big)^{\alpha-1}\log\Big(\theta^2\frac{(1-z)^2}{4} - c\Big) - 4z(1-2z)\Big(\theta^2(1-2z)z - c\Big)^{\alpha-1}\log\Big(\theta^2(1-2z)z - c\Big)\Big)\Big).\end{aligned}$$

Substituting γ_F and γ_C, and using $\gamma_F = \gamma_C \equiv \gamma$, rewrite this as:

$$\begin{aligned}\frac{\partial^2 H}{\partial \theta \partial \alpha} &= \frac{1}{2}\theta\Big(\gamma_F - \gamma_C + \alpha\Big(\gamma_F \log\Big(\theta^2\frac{(1-z)^2}{4} - c\Big) - \gamma_C \log\Big(\theta^2(1-2z)z - c\Big)\Big)\Big) \\ &= \frac{1}{2}\theta\alpha\gamma\Big(\log\Big(\theta^2\frac{(1-z)^2}{4} - c\Big) - \log\Big(\theta^2(1-2z)z - c\Big)\Big) > 0.\end{aligned}$$

The inequality holds since $\frac{(1-z)^2}{4} > (1-2z)z$ for $z \in [1/4, 1/3)$. When monopoly output cannot be supported in both environments, then $\frac{\partial^2 H}{\partial \theta \partial \alpha} > 0$, implies that for $\theta > \theta^*$, we need $z_1(\theta) > z_2(\theta)$ to retrieve $H_1 = v_1$ and $H_2 = v_2$; and $\theta < \theta^*$ demands $z_1(\theta) < z_2(\theta)$. $\square$

Proof of Proposition 6: Let $H_1(z,\theta) \equiv H(z,\theta;\alpha_1,c)$ and $H_2(z,\theta) \equiv H(z,\theta;\alpha_2,c)$ for $c_1 < c_2$. We prove that if there exists a θ such that $z_1(\theta) = z_2(\theta)$, then it is unique. Call these values θ^* and z^*. To establish this single-crossing result, we prove that for a fixed z, $H_1 - H_2$ increases in θ by showing that $\frac{\partial H}{\partial \theta}$ decreases in c. Therefore, there exists a neighborhood of θ^* and z^*, such that for a fixed z, H_1 is a steeper function of θ than H_2.

When both IC constraints bind then $\frac{\partial H}{\partial \theta} = \frac{\partial v}{\partial \theta} = 0$. We have

$$\frac{\partial H}{\partial \theta} = \frac{1}{2}\theta\alpha \left((1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - c\right)^{\alpha-1} - 4z(1-2z)\left(\theta^2(1-2z)z - c\right)^{\alpha-1}\right) = 0.$$

We now prove that $\frac{\partial H}{\partial \theta}$ decreases in c, i.e., $\frac{\partial^2 H}{\partial \theta \partial c} < 0$:

$$\frac{\partial^2 H}{\partial \theta \partial c} = \frac{1}{2}\alpha(1-\alpha)\theta\left((1-z)^2\left(\theta^2\frac{(1-z)^2}{4} - c\right)^{\alpha-2} - 4z(1-2z)\left(\theta^2(1-2z)z - c\right)^{\alpha-2}\right).$$

Substituting γ_F and γ_C, and using $\gamma_F = \gamma_C \equiv \gamma$, rewrite this as:

$$\begin{aligned}\frac{\partial^2 H}{\partial \theta \partial c} &= \frac{1}{2}\alpha(1-\alpha)\theta\left(\frac{\gamma_F}{\theta^2\frac{(1-z)^2}{4} - c} - \frac{\gamma_C}{\theta^2(1-2z)z - c}\right)\\ &= \frac{1}{2}\alpha(1-\alpha)\theta\gamma\left(\frac{1}{\theta^2\frac{(1-z)^2}{4} - c} - \frac{1}{\theta^2(1-2z)z - c}\right) < 0.\end{aligned}$$

The inequality holds since $\frac{(1-z)^2}{4} > (1-2z)z$ for $z \in [1/4, 1/3)$, and hence its reciprocal is smaller. When monopoly output cannot be supported in both environments, then $\frac{\partial^2 H}{\partial \theta \partial c} > 0$, implies that for $\theta > \theta^*$ we need $z_1(\theta) > z_2(\theta)$ to retrieve $H_1 = v_1$ and $H_2 = v_2$, and $\theta < \theta^*$ demands $z_1(\theta) < z_2(\theta)$. $\square$

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