The Eurozone game: conflicting policy preferences in an integrated capital market *

Marcus Miller†, Neil Rankin‡ and Lei Zhang§

February 2012

Abstract

In an OLG framework, capital market integration leads to interest rate equalisation as the sovereign liabilities of highly-indebted countries are transferred to partner countries with high savings and low interest rates. Indeed we find that, in equilibrium, the bulk of the existing debt of highly-indebted sovereigns will be transferred to others in this fashion. When account is taken of incentives to create more debt, however, what can emerge is a "discoordination game" with a mixed-strategy equilibrium including a crisis. Balanced-budget rules

*We are grateful to financial support from the ESRC Research Centre of Competitive Advantage in the Global Economy.

†University of Warwick.

‡University of York.

§University of Warwick.

to limit sovereign debt may be designed to resolve such discord and prevent crisis.

Key words: debt sustainability, overlapping generations, sovereign default, Eurozone debt crisis

JEL: F21, F34, F36, F41

1 Introduction

In the financial crisis of 2008/9, counter-party risk was triggered by doubts about the value of mortgage-backed assets held by highly-leveraged banks. But now markets are concerned as to the extent and sustainability of European sovereign debt, where heterogeneous countries share a unified capital market.

To study the positive effects of capital market integration, we work with two countries for simplicity, one so heavily indebted that the amount of debt in issue equals the maximum sustainable in autarky; the other debt-free. In equilibrium, after debt has spread from one to the other, interest rates and GDP converge to common values — but national incomes will differ as interest payments follow debt across national frontiers. Relative to a totally debt-free alternative, the presence of debt will lead to some crowding out in the region as a whole: nevertheless, both countries may be better off. The high-saving country may, in isolation, face the problem of excess capital formation analysed by Diamond (1965) with the danger of 'dynamic inefficiency' where the rate of return on capital falls below the rate of depreciation. The heavily-indebted country may face the opposite problem of deficient savings and capital decumulation for this reason, as analysed by Rankin & Roffia (2003). In such circumstances, capital market integration offers a clear gain to both countries.

A fascinating result we obtain is that, after capital market integration, the bulk of the debt of the highly-indebted partner must be transferred to its neighbour. (That the initially debt-free neighbour winds up holding more debt is because its national income and wealth rises thanks to the transfer payments of interest it receives as integration proceeds: and conversely for its high-borrowing neighbour.) In short, the OLG model predicts the sort of situation which exists at present, where debts of the Southern Euro-member countries are widely spread among their Northern neighbours.

What if the differing debt endowments between North and South reflect differing policy preferences, with high debt indicating myopic behaviour along the lines discussed in Rochet (2006)? In that case, giving highly indebted Southern member countries access to international capital markets by joining the Euro is a double-edged sword. On the one hand, it increases the maximum sustainable government debt in those countries by moving them well clear of the risk of a debt-induced collapse. On the other hand, it opens up the possibility of further debt issuance on their part.

When the liberalisation of global capital markets led to a succession of crisis in Emerging Markets — in Mexico and East Asia for example — Europe was seen as a haven of stability, partly because the adoption of a common money eliminated currency risk. But the build up of cross-border holdings of non-contingent sovereign debt raises the prospect of crisis — from the temptation to over-borrow in a context where the crowding out effects are widely dissipated and from ensuing creditor panic.

We treat the policy conflict between heterogeneous members of an integrated capital market as a non-corporative game where the South wishes to fiscally "free-ride" but the North may choose to protect itself by reversing capital market integration and shifting back to autarky — which we model as imposition of taxes on capital inflows from the South. There is no pure strategy equilibrium to this "discoordination game". But there exists a mixed strategy equilibrium which includes the possibility of a debt crisis where big debt expansion by the South leads closure of capital markets. The rules of the Eurozone do, of course, explicitly exclude such capital market intervention. Nevertheless, the emergence of sovereign spreads can, we argue, have much the same effect. Markets are free to charge borrowing spreads that policy-makers in the North are not allowed to impose. A country like Greece, currently facing spreads of over 20% is affectively unable to access European capital markets for years to come.

Our formal analysis looks only at steady states: and the dynamics considered in Diamond's original paper — and in a recent two country study by Farmer & Zotti (2010) — assume full employment, as with flexible wages and prices extra savings leads to more investment. But in a context where wages and prices are less flexible — as they are in Europe relative to the US, for example — there is the risk that plans to reduce debt in order to increase saving in the long run will cause a recession in the short run (as consumption falls before investment expands). So in practice credible plans for deleveraging will need to take the dynamics of adjustment into account. Another great simplification we make is to assume both economies produce the same good: but Farmer & Zotti (2010) discuss how this assumption may be relaxed.

The paper proceeds as follows. The positive effects of capital market integration are developed in Section 2 and illustrated in Section 3 with numerical examples using parameter values based on those in Rankin & Roffia (2003). This is followed in Section 4 by a comparison autarchy and integration using a graphical analysis based on the Diamond model. In Section 5 we present a "discoordination game" to highlight the possibility of crisis in the European context. In Section 6 we note that our analysis is limited to steady state and suggest short-run dynamics should be further investigated. In conclusion, we consider briefly what this analysis might imply for checking the crisis-prone nature of European sovereign bond markets.

2 A two-country overlapping generations model

Our model is a two-country version of Diamond's (1965) well-known overlapping generations model. Consider an economy with two countries: country S (representing Southern European countries, for example) which is heavily indebted, issuing a substantial amount b of sovereign debt; and country N (representing Northern European countries) which for simplicity and contrast is debt-free, i.e., issues no debt. In each country, at any given time t, these is a continuum of measure 1 of young consumers who live for two periods. A young consumer is endowed with 1 unit of labour and supplies it inelastically when he/she is young and nothing when he/she is old. Each country has equal proportional young and old generations and the size of the population (consisting of both young and old generations) in each country at any time is assumed constant (of measure 2).

For simplicity, we assume that, in both countries, consumers share identical preferences and firms share the same constant return to scale technology. In addition, both countries are assumed to produce the same traded good.

Let the capital markets for this two-country economy be integrated, so capital and bonds can be traded internationally. Strictly speaking, consumers are indifferent about the composition of their assets in this world of perfect certainty. However, it would take only an infinitesimal transaction cost of acquiring foreign assets to make them prefer to hold domestic ones, and this is also quite a realistic assumption. Hence we shall assume a form of home bias: young consumers in country N would allocate their savings first to domestic capital and then to foreign bonds, and finally in foreign capital. The discussion of the incentive for country N to prefer holding foreign debt is deferred to later sections.

We assume that the government in country S uses lump-sum taxes to finance the interest payment on the debt b and default is excluded. In the model below, we assume b to be exogenously given and constant over time. The government in country N charges no taxes.

In what follows, we first outline the structure of the economy for countries S and N respectively. We then analyze the properties of the steady-state equilibrium in this two-country economy.

2.1 The Heavily-indebted South

A representative young consumer in country S, born in period t, maximizes the following time-separable utility function

$$U(c_t^Y, c_{t+1}^O) \equiv u(c_t^Y) + \beta u(c_{t+1}^O), \tag{1}$$

where $U(c_t^Y, c_{t+1}^O)$ represents lifetime utility, c_t^Y and c_{t+1}^O the consumption when young and old respectively, β the discount factor, and $u(\cdot)$ the period utility, satisfying $u'(\cdot) > 0$ and $u''(\cdot) < 0$. His budget constraints are

$$c_t^Y = w_t - \gamma\tau_t - s_t, \quad c_t^Y \geq 0, \tag{2}$$

$$c_{t+1}^O = R_{t+1}s_t - (1-\gamma)\tau_{t+1}, \quad c_{t+1}^O \geq 0, \tag{3}$$

where w_t and s_t represent wages and savings when he is young, γ the fraction of taxes paid by the young generation, τ_t and τ_{t+1} the total taxes in period t and $t+1$, and R_{t+1} the gross real interest rate between periods t and $t+1$. So the allocation problem faced by the young is to find some $\{c_t^Y, c_{t+1}^O\}$ to maximise (1) subject to (2) and (3).

The aggregate taxes at t are used to finance the interest payment on the government debt b[1], i.e.,

$$1 \cdot \gamma\tau_t + 1 \cdot (1-\gamma)\tau_t = \tau_t = (R_t - 1)b, \tag{4}$$

where R_t is the real gross interest rates between period $t-1$ and t.

The first order condition for the optimal consumption allocation is given by the following Euler equation

$$u'(c_t^Y) = R_{t+1}\beta u'(c_{t+1}^O). \tag{5}$$

Given $\{w_t, b, R_t, R_{t+1}\}$, (2)–(5) determine $\{c_t^Y, c_{t+1}^O, s_t, \tau_t\}$. The consumption of the old in period t, c_t^O, is determined in a similar fashion by the young generation born at period $t-1$.

Assume perfectly competitive firms that can access a constant returns production function

$$Y = F(K, L)$$

where K, L and Y are the aggregate capital, labour and output respectively. Let $F(\cdot,\cdot)$ be of homogeneous of degree 1 such that $F(K,L) = LF(K/L, 1) \equiv Lf(k)$ with f satisfying Inada conditions.[2] For the size of the young generation assumed above, $L = 1$, the aggregate capital K is the same as the per labour capita k (scaled by the size of the young generation). So K and k will

[1]Note that in our setup, since the young generation has a measure of 1, the aggregate debt is the same as debt per capita (worker).

[2]Here we assume there is no productivity growth.

be used interchangeably below.

A representative firm maximizes its per period profits

$$F(K, L) - \hat{R}K - wL, \tag{6}$$

by choosing some $\{K, L\}$ subject to given rental cost of capital $\hat{R}$ and the wage rate w.

Given the property of F, the two first order conditions for (6) are

$$f'(k) = \hat{R}, \tag{7}$$

$$f(k) - kf'(k) = w. \tag{8}$$

Note that (7) and (8) specify aggregate demand for capital and labour. Given inelastic labour supply of $L = 1$, (8) specifies the wage rates.

Let the depreciation rate of capital be δ. No arbitrage between renting capital and buying bonds (from t to $t + 1$) implies

$$R_{t+1} = 1 + \hat{R}_{t+1} - \delta, \tag{9}$$

for given R_{t+1}, (7) and (9) determine the aggregate capital K_{t+1}, and (8) determines the wage, w_t.

We now turn to capital accumulation and the division of the output. Note that the gross aggregate investment in period t is $K_{t+1} - K_t + \delta K_t$, and the aggregate consumption, given the size of either generation is of measure 1, is

$C_t \equiv C_t^Y + C_t^O \equiv 1 \cdot c_t^Y + 1 \cdot c_t^O$. The division of the aggregate output is as follows

$$Y_t = F(K_t, 1) = C_t + K_{t+1} - K_t + \delta K_t + (R_t - 1)b_t(*) - b_{t+1}(*) + b_t(*),$$

i.e., output is divided between aggregate consumption, investment, the interest payment on the fraction of the debt held by country N, $(R_t - 1)b_t(*)Y_t$, and the reduction of debt holding (i.e., debt buyback), $-[b_{t+1}(*) - b_t(*)]Y_t$. Writing in per capita form yields

$$f(k_t) = c_t^Y + c_t^O + k_{t+1} - k_t + \delta k_t + (R_t - 1)b_t(*) - b_{t+1}(*) + b_t(*). \quad (10)$$

Finally, the clearing condition for domestic capital markets is given by

$$s_t = k_{t+1} + b - b_{t+1}(*), \quad (11)$$

where $b_{t+1}(*)$ is the per capita foreign holding of the debt issued by country S. Since the real interest rates are determined by the integrated capital markets, (11) determines the allocation of the debt between the two countries.

2.2 The Debt-free North

The young generation in country N faces a similar problem as specified in (1)–(3), except that no lump-sum taxes are levied. By setting $\tau = 0$, one can obtain respective consumption and savings $\{c_t^Y(*), c_{t+1}^O(*), s_t(*)\}$, where $(*)$

indicates variables in country N.

A representative firm in country N faces the same problem as that in country S. So demand for capital and the wages are the same as in (7) and (8).

The allocation of domestically produced output in country N is as follows

$$f(k_t(*)) = c_t^Y(*)+c_t^O(*)+k_{t+1}(*)-k_t(*)+\delta k_t(*)-(R_t(*)-1)b_t(*)+b_{t+1}(*)-b_t(*). \tag{12}$$

Note that $(R_t(*) - 1)b_t(*)$ represents the transfer from country S.

The equilibrium condition for the domestic capital markets in country N is given by

$$s_t(*) = k_{t+1}(*) + b_{t+1}(*). \tag{13}$$

2.3 The equilibrium with integrated capital markets

Here we first outline the procedure for obtaining full employment dynamics in such a neoclassical model. We then move on to study the properties of steady state equilibrium.

As capital markets in these two countries are integrated, the extra clearing condition required is the equalisation of real interest rates in both countries,

$$R_t = R_t(*), \quad \forall t. \tag{14}$$

Given (14), (7) for country S and its equivalent for country N imply

$$k_t = k_t(*), \quad \forall t. \tag{15}$$

Both (15) and (8) imply that wages are equalised across the two countries and they only depend on k_t,

$$w_t = w_t(*) \equiv w_t(k_t). \tag{16}$$

Consumption and savings for country S can be obtained using (2)–(5) which imply

$$c_t^Y = c_t^Y(k_t, R_t, R_{t+1}), \tag{17}$$

$$c_t^O = c_t^Y(k_{t-1}, R_{t-1}, R_t), \tag{18}$$

$$s_t = s_t(k_t, R_t, R_{t+1}), \tag{19}$$

and similarly for country N.

Summing (10) and (12), and (11) and (13) respectively yield

$$\begin{aligned} f(k_t) = [&c_t^Y(k_t, R_t, R_{t+1}) + c_t^O(k_{t-1}, R_{t-1}, R_t) + c_t^Y(k_t, R_t, R_{t+1}; *) \\ &+ c_t^O(k_{t-1}, R_{t-1}, R_t; *)]/2 + k_{t+1} - k_t + \delta k_t, \end{aligned} \tag{20}$$

$$2k_{t+1} + b = s_t(k_t, R_t, R_{t+1}) + s_t(k_t, R_t, R_{t+1}; *). \tag{21}$$

The above equations (together with (7) and (9) which link k_t to R_t) jointly determine the dynamics of $\{k_t, R_t\}_0^\infty$ for some given initial conditions.

Equation (17)–(19) are then used to back out consumption and savings for country S (and their counterparts for country N). Finally, (11) or (13) can be used to trace the sequence of foreign holding of the debt, $b_t(*)$.

We now turn to analysing the steady state. To make things simple, we assume $\delta = 0$, $u(c) = \ln(c)$ and $f(k) = k^\alpha$ where $0 < \alpha < 1$. The steady state equilibrium for k and R can be obtained by imposing $k_{t+1} = k_t, \forall t$ and $R_{t+1} = R_t, \forall t$, and finding the fixed point in (20) and (21). Then, other quantities can be backed out using the procedure outlined above. As pointed out in Rankin & Roffia (2003), there is an upper limit of b above which capital stock converges to zero, so we assume b is always below that limit. The results are summarized in the following proposition. Later, we will turn to the case with capital depreciation and the possibility of dynamic inefficiency.

Proposition 1 *With no capital depreciation, log utility and a Cobb-Douglas production function, the steady state capital stock for both countries is the larger root of*

$$\frac{2[\beta(1-\alpha)k^{\alpha-1} - (1+\beta)k]}{1+\beta+\beta\gamma\alpha k^{\alpha-1} - (1-\gamma)\alpha k^{\alpha-1}/(1+\alpha k^{\alpha-1})} = b. \tag{22}$$

The gross real interest rates and the wages for both countries are given by

$$R = 1 + \alpha k^{\alpha-1}, \tag{23}$$

$$w = (1-\alpha)k^{\alpha}. \tag{24}$$

Consumption and savings in country S are given by

$$c^Y = \frac{1}{1+\beta}\left[(1-\alpha)k^{\alpha} - \left(\gamma + \frac{1-\gamma}{1+\alpha k^{\alpha-1}}\right)\alpha b k^{\alpha-1}\right], \tag{25}$$

$$c^O = \frac{\beta(1+\alpha k^{\alpha-1})}{1+\beta}\left[(1-\alpha)k^{\alpha} - \left(\gamma + \frac{1-\gamma}{1+\alpha k^{\alpha-1}}\right)\alpha b k^{\alpha-1}\right], \tag{26}$$

$$s = \frac{\beta}{1+\beta}\left[\frac{(1-\gamma)b\alpha k^{\alpha-1}}{1+\alpha k^{\alpha-1}} + \beta k^{\alpha}(1-\alpha-\alpha\gamma b k^{-1})\right]. \tag{27}$$

Consumption and savings in country N are given by

$$c^Y(*) = \frac{(1-\alpha)k^{\alpha}}{1+\beta}, \tag{28}$$

$$c^O(*) = \frac{\beta(1+\alpha k^{\alpha-1})(1-\alpha)k^{\alpha}}{1+\beta}, \tag{29}$$

$$s(*) = \frac{\beta(1-\alpha)k^{\alpha}}{1+\beta}. \tag{30}$$

The fraction of the debt held by country N is

$$b(*) = \frac{\beta(1-\alpha)k^{\alpha}}{1+\beta} - k. \tag{31}$$

PROOF: See Appendix A.

The outcomes outlined in the proposition above are highly non-linear in k. The detailed comparative static analysis is done numerically in the next section when country S chooses its maximum autarky level of debt. Here we present some selected analytical comparative static results: first we give some properties of the equilibrium capital in the steady state and then the properties of the fraction of the debt held by the non-debt-issuing country.

Proposition 2 *In the steady state equilibrium:*

(i) Increasing b reduces the steady state capital stock k.

(ii) Increasing the discount factor β raises steady state capital.

(iii) Increasing, γ, the fraction of taxes levied on the young generation in country S reduces the common steady state level of capital.

PROOF: See Appendix B.

Note that properties in Proposition 2 resemble those in a closed economy and they are quite intuitive. Take, for example, Proposition 2(ii). An increase in β makes young generations in both countries value more consumption when they are old and so raises their savings. This depresses the real interest rate and results in an increase in the equilibrium capital stock.

Proposition 3 *The fraction of the debt held by country N is strictly greater than a half when γ is large and strictly less than half when γ is small.*

PROOF: See Appendix C.

A detailed discussion of the property in Proposition 3 is deferred to Section 4.

It is worth noting that capital markets integration can be welfare improving for both countries. For the Southern country, interest rates fall, both capital and consumption increase. The Northern country may, on the other hand, escape dynamic inefficiency under autarky. To see how dynamic inefficiency can arise, consider country N under autarky. As it does not issue any debt, its capital market equilibrium condition in the steady state becomes[3]

$$k = \beta(1+\beta)^{-1}(1-\alpha)k^{\alpha}$$

The gross real interest rate is given by (9), namely,

$$R = 1 + \alpha k^{\alpha-1} - \delta.$$

So if the rate of capital depreciation, δ, is large enough, $R < 1$, causing dynamic inefficiency.

With capital market integration, residents in country N can now invest in high yielding government bonds offered by country S, and so reducing their holding of domestic capital. For an appropriate level of b, the increase of the real interest rate would be high enough to move country N out of dynamic inefficiency. If δ and b are chosen suitably, it could, in principle, implement

[3]This is obtained by substituting (6)–(8) into (13) and assuming $b_{t+1}(*) = 0$.

"Golden rule" consumption for country N.

3 Comparative statics of the steady state equilibrium: numerical results

Table 1: Debt holdings, prices, consumption and asset allocations in a two-country model: the baseline.[a]

	Allocation of GNP				Asset holding		
	C^Y	C^O	GNP	$\pm(R-1)b_F$	s	k	Debt
Southern	0.157	0.342	0.500	-0.032	0.157	0.150	0.007
Northern	0.177	0.386	0.563	0.032	0.177	0.150	0.027
Prices		w=0.354	R =2.180				
Total debt		0.034					
Northern Fraction		0.796					

[a] We use $\beta = 1$, $\alpha = 1/3$, $\gamma = 1$ and b the maximum level of debt in a closed economy.

To gauge the quantitative significance of changes of parameters on the outcomes in the steady state equilibrium, we use numerical simulations. For simplicity, we ignore depreciation. As a baseline case, we set parameters to be $\beta = 1$, $\alpha = 1/3$, $\gamma = 1$ and b the maximum level of debt in a closed economy. We report the fraction of debt held by the Northern European country, the equilibrium prices, the allocation of GNP for both countries and asset holding for both countries. We then vary the parameter values of β to

Table 2: Debt holdings, prices, consumption and asset allocations in a two-country model: changing β.[a]

	Allocation of GNP				Asset holding		
	C^Y	C^O	GNP	$\pm(R-1)b_F$	s	k	Debt
Southern	0.171	0.237	0.408	-0.026	0.086	0.082	0.004
Northern	0.193	0.267	0.460	0.026	0.096	0.082	0.015
Prices		w=0.266	R =3.095				
Total debt		0.019					
Northern Fraction		0.795					

[a] We use $\beta = 1/2$, $\alpha = 1/3$, $\gamma = 1$ and b the maximum level of debt in a closed economy.

1/2 and 2, α to 0.4, γ to 1/2 and 0, to see the robustness of the results.

Table 1 reports results on the allocation of output and the holdings of assets in both countries. We also report prices, total debt and the share of the holding from country N for comparison. Tables 2 and 3 present the results when β changes. It is evident that varying the discount factor β has a negligible effect on the Northern fraction of the debt holding.

Table 4 reports results when the capital share of output, α, is increased from 1/3 to 0.4. Comparing with the baseline case, increasing the capital share raises the fraction of the debt held by country N significantly.

Tables 5 and 6 present results when, γ, the share of taxes levied on the young generation in country S is progressively reduced. Notice that the fraction of the debt held by the Northern country is larger than half in

Table 3: Debt holdings, prices, consumption and asset allocations in a two-country model: changing β.[a]

	Allocation of GNP				Asset holding		
	C^Y	C^O	GNP	$\pm(R-1)b_F$	s	k	Debt
Southern	0.121	0.456	0.577	-0.037	0.242	0.231	0.011
Northern	0.136	0.514	0.651	0.037	0.273	0.231	0.042
Prices		w=0.409	R =1.885				
Total debt		0.052					
Northern Fraction		0.795					

[a] We use $\beta = 2$, $\alpha = 1/3$, $\gamma = 1$ and b the maximum level of debt in a closed economy.

Tables 1–5. It only drops below half when the old generation in country S is heavily taxed. This confirms our results in Proposition 3.

4 Capital market integration — a diagrammatic analysis

Here we consider the impact of capital market integration in the case where the two economies are identical but differ in one respect only — the amount of sovereign debt in issue. Let N denote the Northern economy free of all sovereign debt; and S the heavily-indebted Southern economy where the extent of debt — financed entirely by lump sum taxes on the young — is at its autarky maximum. Note that, for simplicity, we again assume no capital

Table 4: Debt holdings, prices, consumption and asset allocations in a two-country model: changing α.[a]

	Allocation of GNP				Asset holding		
	C^Y	C^O	GNP	$\pm(R-1)b_F$	s	k	Debt
Southern	0.108	0.274	0.383	-0.025	0.108	0.106	0.002
Northern	0.122	0.310	0.432	0.025	0.122	0.106	0.016
Prices		w=0.245	R =2.538				
Total debt		0.018					
Northern Fraction		0.884					

[a] We use $\beta = 1$, $\alpha = 0.4$, $\gamma = 1$ and b the maximum level of debt in a closed economy.

depreciation.

Before looking at the effects of integration, consider the autarchy equilibria. Without any debt, equilibrium will be as in Figure 1, where the Young consume a fixed fraction of the wage bill, as indicated by the label c^Y. The rest of the wage bill — savings — is consumed by the Old along with the accumulated interest, components shown separately in the figure as k_N and $(R-1)k_N$ respectively, where the rate of interest corresponds to the marginal product of capital and, in the absence of debt, the accumulated interest corresponds to the profit share of GDP.

In the economy with sovereign debt, the Young are subject to a tax on their wages, $\tau = (R-1)b$, so their consumption as a fraction of GDP will fall, as indicated in Figure 2. The share of output going to the Old increases as —

Table 5: Debt holdings, prices, consumption and asset allocations in a two-country model: changing γ.[a]

	Allocation of GNP				Asset holding		
	C^Y	C^O	GNP	$\pm(R-1)b_F$	s	k	Debt
Southern	0.153	0.337	0.490	-0.036	0.167	0.145	0.021
Northern	0.175	0.387	0.562	0.036	0.175	0.145	0.030
Prices		w=0.351	R =2.206				
Total debt		0.051					
Northern Fraction		0.582					

[a] We use $\beta = 1$, $\alpha = 1/3$, $\gamma = 1/2$ and b the maximum level of debt in a closed economy.

along with the profits of enterprise — they now enjoy the transfer payment of interest. The wealth which they use to fund consumption has increased to include the stock of debt as well as the stock of capital, as indicated by the components shown separately in the figure as $k_S + b$ and $(R-1)(k_S + b)$ respectively, where $(R-1)b$ denotes the transfer of interest.

The negative effect of debt on the equilibrium level of capital is shown in Figure 3. With debt, the equilibrium condition changes from $s = k$ to $s = k + b^*$, but savings has been reduced, so the level of capital falls. That debt is at a maximum is shown by the tangency of the savings function with the line labelled $k + b^*$.

With capital market integration debt is redistributed between the two economies so as to equalise the interest rate paid. Since this corresponds

Table 6: Debt holdings, prices, consumption and asset allocations in a two-country model: changing γ.[a]

	Allocation of GNP				Asset holding		
	C^Y	C^O	GNP	$\pm(R-1)b_F$	s	k	Debt
Southern	0.131	0.308	0.439	-0.058	0.200	0.123	0.077
Northern	0.166	0.389	0.555	0.058	0.166	0.123	0.043
Prices		w=0.331	R =2.350				
Total debt		0.120					
Northern Fraction		0.356					

[a] We use $\beta = 1$, $\alpha = 1/3$, $\gamma = 0$ and b the maximum level of debt in a closed economy.

to the marginal product of capital, the effect is to ensure a common level of capital and GDP, as indicated by the point labelled I in the figure — corresponding to equilibrium in a closed economy with a debt level of $b/2$.

Though production is equalised, national incomes will differ as some of the debt interest is transferred from Young in country S to the older generation in country N. How much? The striking result obtained reported in the previous section for economies where the young bear the taxes is that *more than half* of the debt and debt interest are transferred.

To see how this arises, consider first the condition for equilibrium in the integrated market. As it happens, this mirrors equilibrium in a closed

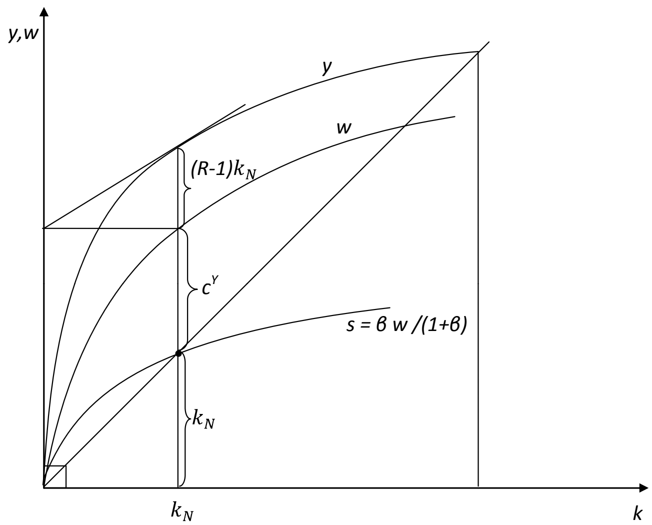

Figure 1: Division of output between young and old — the debt free case.

economy with debt of $b/2$, with the capital stock determined as:

$$\frac{1}{1+\beta}[w(k) - (R(k) - 1)b/2] = k + b/2$$

with the wage and the rate of return determined by production conditions.

Combining this with the savings and asset accumulation equation for either of the countries one can solve for σ, the share of b held outside its country of origin. For the partner country this is:

$$\frac{1}{1+\beta}w(k) = k + \sigma b$$

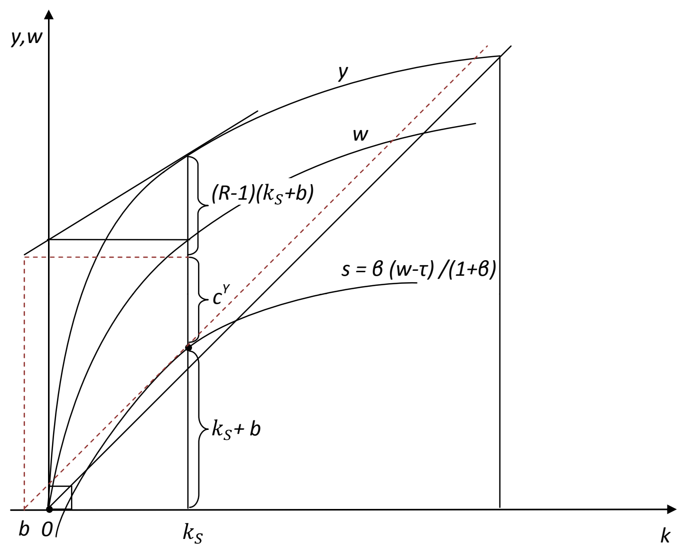

Figure 2: Division of output between young and old — the high debt case.

By substitution it can readily be found that

$$\sigma = \frac{1}{2}\left\{1 + \frac{\beta[R(k) - 1]}{1 + \beta}\right\} \geq \frac{1}{2}.$$

The implication for savings and for national income in the heretofore debt-free economy are illustrated in Figure 4, where $\hat{k}$, the equilibrium capital stock after integration, is smaller than under autarchy and the excess of savings above this level, i.e. $\beta w(\hat{k})/(1 + \beta) - \hat{k}$ indicates the holdings of foreign debt. The interest paid to the older generation on this is $[R(\hat{k}) - 1]\sigma b$, and it enables the old to consume more without reducing the consumption of the young, as is shown in the figure. With transfer, national income now exceeds national production.

It is clear that economy N will hold more of the sovereign debt issued by

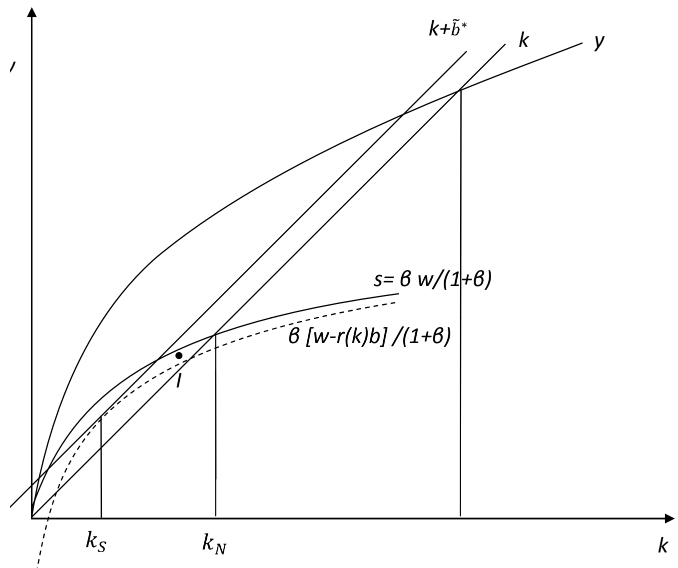

Figure 3: Autarky equilibria and the effect of capital market Integration

S than S itself when $\gamma = 1$, i.e., the young in country S bear all the (interest) tax burden. But the simulations show, see Table 6, that this is no longer true when $\gamma = 0$ and the (interest) tax burden falls entirely on the old. The effect of raising γ on the savings function in S is to shift upwards the dotted line in 3, so when $\gamma = 0$, it will lie above the solid line showing savings in N. For some threshold value of γ, the two will coincide, and the logic used above indicates that sovereign debt will be equally shared at this threshold. Our simulations suggest, however, the relevant threshold value of γ would require taxes to fall mostly on the old — Table 5 shows that for $\gamma = 1/2$, $\sigma > 1/2$. *We conclude that the result that* $\sigma > 1/2$ *is robust to all reasonable parameter variations.*

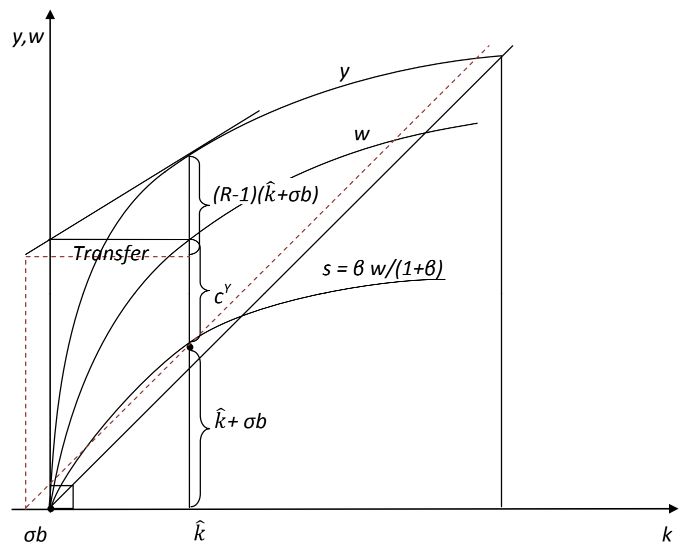

Figure 4: The income transfer to the erstwhile debt free economy.

5 The Eurozone Game

The outcome of capital market integration may be a benign shift of consumption towards the golden rule as described in the previous section; but is this incentive compatible? There is reason to believe it is not — as the current crisis might suggest. Here we explore the effects of adding political preferences which differ as between North and South; what emerges is a "discoordination game" with a mixed strategy Nash equilibrium and a positive probability of crisis. The game is as shown in Table 7.

The actions used by the North are whether to accept Southern bonds or not. One way to describe the actions available to governments in the North is that they can choose whether to introduce a tax on holdings of Southern debt by their citizens: a tax of x per unit of Southern debt rebated

Table 7: The Eurozone Game[a]

	Capital		
	Integration		Autarky
Fiscal prudence	(2,2)	←	(1,1)
	↓		↑
Fiscal Free-riding	(3,0)	→	(0,1)
			Debt Crisis

[a] Column player is North and Row player is South. Payoffs are for (South, North).

as a lump sum to the young in the North, so $r = r^* + x$. The premium, x, causes the Southern capital stock, k, to be lower in equilibrium than its Northern counterpart $k*$, although the two are still held in a unique positive relationship by the equality of the interest rate and capital's marginal product in each country. If x is sufficiently large, this can drive holdings of Southern debt by Northern agents to zero, reproducing the resource allocation under autarchy.

As for the actions of the South, following Rochet (2006) on myopic governments, it can be supposed that when governments can borrow at interest rates set in the North they will expand the volume of debt. As Rochet (2006) argues, there may be incentive issues in the debtor country that lead to over-borrowing and default risk even when costs are sufficient to rule out strategic default.[4]

[4]Inability to pay may arise from myopia on the part of a government which borrows

Table 7 shows the normal form of the policy game played between the North and the South. The top left is the payoff when the South issues a moderate level of debt which the North is happy to hold — representing the outcome of the positive analysis of the last section. But with most of its debt held externally and the low interest rate in the integrated capital markets, the South would have an incentive to issue more debt, and so moving down to the bottom left cell (where the increase in payoffs to the South from 2 to 3 reflect the myopia of the Southern government). Given such "fiscal free-rising", however, the North could impose a tax x on inflows of sovereign debt from the South, so precipitating a crisis, as represented by the payoffs at the bottom right. Given the threat of being denied access to capital markets, the South would have an incentive to limit its debt issuance, leading to the outcome in the top right cell.

As can be seen from the arrows in the table, this is what is sometimes called a 'discoordination game' (Rasmusen 1994, p.79) with no pure strategy Nash equilibrium. There is however a mixed strategy equilibrium, which for the parameters given is equiprobable randomisation for both participants. Thus the North will revert to autarky with probability half. Likewise, the South will fiscally free ride with probability half. The prediction of this game is that the Debt Crisis in the bottom right hand corner will occur with

as much as it can against a stochastic stream of tax and finds it has insufficient funds to service the debt when tax receipts fail to grow. Lenders, being aware of this, will impose a default premium on interest charges so debt will follow a 'rational bubble', characterized by periods of steady (and procyclical) capital inflows, ending in crisis periods where the country defaults and investors stop lending for a time', Rochet (2006, pp. 15-16).

probability of a quarter.

Current plans to provide financing subject to strict limits on sovereign deficits may represent efforts to attain the Pareto superior outcome at top left by constitutional change.[5]

In the above game it is assumed the North can impose capital controls at will, as a way of checking fiscal free-riding. This is however inconsistent with European rules and regulations, but the market may read the preferences of policy makers in the North and act accordingly. As in the case of Henry II, it was enough for the King to say "Will no one rid me of this turbulent priest?" for his trusty knights to make a martyr of Thomas Becket, Archbishop of Canterbury. In the current context, as we have seen, the markets can put a substantial sovereign spread on Southern debt, which effectively imposes financial autarky on the South.

This might constitute a shift of equilibrium as in from the model of Calvo (1988) who warns of the risk that market forces may create self-fulfilling debt crises, with sharp rises in borrowing costs making default incentive-compatible even when there no solvency issues in the first place. Events in Mexico in 1994-5 were interpreted in this way by Cole & Kehoe (1996); and Radelet & Sachs (2000) analysed the 1997-8 crisis in South East Asia in similar terms.

[5]But if these limits were to prevent the operation of 'automatic stabilisers', then (as with appointing an ultra-conservative Central Banker in the Barro-Gordon (1983) model of inflation) there is surely a risk that they could generate income volatility in the face of aggregate demand shocks and financial rescue operations.

As explained by Cohen and Portes (2004, p.11):

> The intuition is quite simple: perception of high risk raises the spread, which in turn raises the debt service burden, which in turn provokes the crisis. Beliefs are self-fulfilling because the fundamentals themselves are partly endogenous. If default reduces the amount that a country pays to its creditors below what it would normally pay then lenders perceptions do change how much a country will eventually pay.

As mechanisms to avoid such self-fulfilling debt crises, Cohen and Portes discuss debt workouts and the IMF acting as Lender of First Resort.

5.1 More on the European context

Alan Greenspan (2011) argues that, in the European Union, there is a useful distinction to be drawn between the economies of North and South in terms of sovereign spreads:

> The [debt] burden is primarily on southern Europe, where sovereign bond credit spreads (relative to the German Bund) range from 370 basis points (Italy) to 1,960 basis points (Greece). The northern eurozone countries have tight spreads against Germany — a narrow 40 to 80 basis points for the Netherlands, Austria, Finland and France. There are thus two distinctly defined eurozone areas: in the north and in the south. (FT 7Oct 2011).

While overvalued currencies may be the principal driver, the exposure of these Southern sovereigns surely involves the other two factors. Greece, for example, widely criticised for fiscal laxity, seems to correspond to the case of sovereign over-borrowing, with debts growing at an unsustainable rate as a prelude to default. Well before the current crisis, in fact, Buiter & Sibert (2005) pointed out that the discounting practices of the ECB had the effect of subsidising high risk borrowers by lending at triple A rates; and — by suppressing market signals — this may have provided incentives for loose fiscal policy. Alan Greenspan (2011), it appears, also endorses this view:

> Subsidised borrowing may have accounted for much of the acceleration in the ratio of euro-south consumption relative to that of Germany. It rose between 1995 and 1998 at a 1.26 percent annual rate. Presumably as a consequence of subsidised euro credit, that ratio accelerated to a 1.63 per cent annual rate of increase between 1998 and 2007. (FT Oct 7).

On the other hand, the risk of contagion to other Southern members — and beyond — seems more like creditor panic as discussed by Calvo (1988), the early experience of East Asia in 1997/8, where one country after another on a dollar peg was subject to speculative attack, offers salutary warning.[6] Hence the calls for the 'big bazooka' of massive short-term support to restore confidence in the survival of the Eurozone.

[6]Typically successful, except for Hong Kong which beat the speculators at their own game, as discussed by Miller & Zhang (2000) and Goodhart & Lu (2003).

6 Dynamics: the short run versus the long run

At least as important as default is the need to consider the dynamics of adjustment. Farmer & Zotti (2010) have studied issues of stability and debt limits in a two country OLG setting; but this is under the assumption of full employment, where increases in saving are assumed to lead automatically to an increase in investment.

Dropping the convenient assumption of Say's Law leads to a more Keynesian perspective, where income in the short run is determined by aggregate demand. To these capture aggregate demand effects, we need to depart from Diamond's assumption of fully flexible prices and replace it by the more Keynesian assumption of price and wage rigidities. An example of such a modification to the Diamond model is Rankin (1987).

In fact, the OLG framework, with its sharp distinction between Old and Young in terms of their marginal propensities to consume, lends itself naturally to such a treatment. Take for example the implications for consumer demand of shifting tax from the Young to the Old. Because the Old have an higher marginal propensity to consume (unity) this will increase savings and increase the capital stock in the long run full employment equilibrium. But in the short run consumer demand will fall and, in the absence of a rise in investment demand, there is a risk of recession.[7]

Reducing the level of sovereign debt itself could likewise have negative demand effects until such time as investment increases to offset the decline in consumer demand predicted by an OLG approach. How long — and how profound — these short term effects might be is an open question. The difference of focus — on long run versus short — may indeed lie at the heart of the debate on whether fiscal contraction is the right medicine for European countries at this time.

7 Conclusion

The analytical results on the redistribution of sovereign debt in an integrated market have been derived in setting that is deliberately stylised and simplified. We postulate large initial differences in debt in countries otherwise identical, for example; and there is no explicit account of incentives and dynamics in the formal model.

We show that in a context where the highly indebted economy would in isolation be saving too little and the debt free economy too much, capital market integration would offer welfare gains to both parties. Despite

[7]We may write the level of consumer demand

$$y(l_t) = \beta(wl_t - \gamma\tau) + (1 + r)(s_{t-1} - (1 - \gamma)\tau)$$

So

$$y(l_t) = \beta wl_t + (\beta - (1 + r_t - \beta)\gamma)\tau + (1 + r_t)s_{t-1}$$

So assuming w and r constant, aggregate consumer demand will fall as γ decreases, with multiplier effects if the fall in demand reduces employment income.

such gains, it has to be acknowledged that — given the initial conditions as specified — the financial exposure of the highly indebted economy remains a potential heel of Achilles for the integrated market. In Section 5 with the strategic analysis, we find that irreconcilable policy differences leads to the prospect of crisis with capital market closure and leaving the South with debt cannot be sustained in isolation.

Note, however, that the benefits of debt would remain — without the risks of cross-border exposure — if both countries were to issue roughly the same amount of debt. One is tempted to ask: Is there any way of getting to such an outcome starting from the heterogeneous initial conditions we postulate? One way would be for the heavily-indebted country to reduce its debts — possibly by default — in combination with debt-financed deficits by its erstwhile debt free neighbour. This may sound a fanciful and artificial way of changing the initial conditions: but it seems to be the direction in which Europe is heading. There is considerable pressure on Southern countries to rein in their deficits — with Greece expected to write down its sovereign debt by more than half; and Northern countries are being encouraged to spend more to avoid the recession that will otherwise be associated with such deleveraging by Southern neighbours.

To prevent a recurrence of capital market crisis, however, it will be necessary to prevent returning to the original initial conditions. This means facing up to the incentive issues flagged up by Buiter & Sibert (2005) and analysed in our policy game. As Rochet (2006) suggests the solution could well involve institutional rules imposed by supranational bodies: this is presumably what those who want to rewrite the Treaty of Europe have in mind. The rules on budget deficits and on debt limits would help to eliminate the conflict in the policy "discoordination game": whether introducing such rules is advisable while Europe is in recession is a key issue which we do not examine.

References

Barro, R. J. & Gordon, D. B. (1983). Discretion and reputation in a model of monetary policy, *Journal of Monetary Economics* **12**: 101–121.

Buiter, W. & Sibert, A. (2005). How the eurosystems treatment of collateral in its open market operations weakens fiscal discipline in the eurozone (and what to do about it), CEPR Discussion Papers, no. 5387.

Cohen, D. & Portes, R. (2006). Toward a lender of first resort, IMF Working Paper, WP/06/66.

Cole, H. L. & Kehoe, T. J. (1996). A self-fulfilling model of mexico's 1994-1995 debt crisis, *Journal of International Economics* **41(3-4)**: 309–330.

Farmer, K. & Zotti, J. (2010). Sustainable government debt in a two-good, two-country overlapping generations model, *International Review of Economics* **57 (3)**: 289–316.

Goodhart, C. & Lu, D. (2003). *Intervention to save Hong Kong: counter speculation in financial markets*, OUP.

Greenspan, A. (2011). Europes crisis is all about the north-south split, Financial Times.

Miller, M. & Zhang, L. (2000). Creditor panic, asset bubbles and sharks: Three views of the asian crisis, *in* U. Dadush, D. Dasgupta & M. Uzan (eds), *Private Capital Flows in the Age of Globalisation*, Edward Elgar: MA, USA.

Radelet, S. & Sachs, J. (2000). The onset of the East Asian currency crisis, *in* P. Krugman (ed.), *Currency Crises*, Chicago: University of Chicago Press.

Rankin, N. (1987). Disequilibrium and the welfare-maximising levels of government spending, taxation and debt, *Economic Journal* **97**: 65–85.

Rankin, N. & Roffia, B. (2003). Maximum sustainable government debt in the overlapping generations model, *Manchester School* **71 (3)**: 217–241.

Rasmusen, E. (1994). *Games and Information*, Blackwell.

Rochet, J.-C. (2006). Optimal sovereign debt: An analytical approach, Inter-American Development Bank, Research Department: RES Working Papers 4477.

A Proof of Proposition 1

From (14) to (16), real interest rates, wage rates and capital stock for both countries are identical. Given the steady state k exists, (7)–(9) imply (23) and (24) since $\delta = 0$.

In the steady state equilibrium, applying log utility to (2), (3) and (5) gives

$$c^Y = \frac{1}{1+\beta}[w - \gamma\tau - (1-\gamma)\tau/R], \tag{A.1}$$

$$c^O = \frac{\beta R}{1+\beta}[w - \gamma\tau - (1-\gamma)\tau/R], \tag{A.2}$$

$$s = \frac{\beta R(w - \gamma\tau) + (1-\gamma)\tau}{1+\beta}. \tag{A.3}$$

Substitution of (4), (23) and (24) into (A.1)–(A.3) yields (25)–(27).

Similarly, one can solve for consumption and savings for country N. This leads to

$$c^Y(*) = \frac{w}{1+\beta}, \tag{A.4}$$

$$c^O(*) = \frac{\beta R w}{1+\beta}, \tag{A.5}$$

$$s(*) = \frac{\beta w}{1+\beta}. \tag{A.6}$$

Substitution of (23) and (24) into (A.4)–(A.6) yields (28)–(30).

To obtain the fraction of the debt held by country N, we use (13) when k is at the steady state. Substitution of (30) into (13) leads to (31).

Finally, to obtain the fixed point equation for the steady state k, we first impose stationary conditions to k_t and R_t in (21). We then replace all savings functions in (21) by (27) and (30) to obtain the fixed point equation (22). The reason to choose the larger root is because that root corresponds to the stable steady state (see Rankin and Roffia, 2003). QED

B Proof of Proposition 2

Rewrite the fixed point equation (22). Imposing stationarity in (21) and replacing the two savings function by (27) and (30) yield

$$\frac{\beta R(w-\gamma\tau)+(1-\gamma\tau)}{(1+\beta R}+\frac{\beta w}{1+\beta}=2k+b. \tag{B.1}$$

Rearranging to obtain

$$g(k;\alpha,\beta,\gamma)\equiv\frac{\beta R(w-\gamma\tau)+(1-\gamma\tau)}{(1+\beta R}+\frac{\beta w}{1+\beta}-2k=b. \tag{B.2}$$

Note that $g(k;\alpha,\beta,\gamma)$ is the same as the left hand side of (22).

As has been shown in Rankin & Roffia (2003), function $g(k;\alpha,\beta,\gamma)$ is increasing when k is small and decreasing when k is large. It has a maximum at some $\tilde{k}>0$ such that $g(\tilde{k};\alpha,\beta,\gamma)>0$. As long as $b\leq g(\tilde{k};\cdot)$, the larger root to (B.2) exists. As the fixed point is the intersection between b and the decreasing part of $g(k;\cdot)$, increasing b results in an decrease in the fixed point k.

Differentiating g with respect to β, one obtains

$$\frac{\partial g(k;\alpha,\beta,\gamma)}{\partial\beta} = \frac{1}{(1+\beta)^2}\{[w-\gamma\tau-(1-\gamma)\tau/R]+w\}. \tag{B.3}$$

The first term inside the bracket of the right hand side of (B.3) is the life time wealth of the young generation in country S. So $w-\gamma\tau-(1-\gamma)\tau/R \geq 0$ and $\partial g/\partial\beta > 0$. This implies that the decreasing part of the function $g(k;\beta,\cdot)$ shifts upwards; intersecting a constant b results in an increase in fixed point solution k.

Differentiating g with respect to γ, one obtains

$$\frac{\partial g(k;\alpha,\beta,\gamma)}{\partial\gamma} = -\frac{(1+\beta R)\tau}{(1+\beta)R} < 0. \tag{B.4}$$

Using the similar argument as that for β, the fixed point solution k decreases. QED

C Proof of Proposition 3

Dividing the both sides of (B.1) by 2 and substituting in $\tau=(R-1)b$ yields

$$\frac{\beta}{1+\beta}\left(w-\frac{\gamma(R-1)b}{2}\right)+\frac{1}{2}\frac{(1-\gamma)(R-1)b}{(1+\beta)R} = k+b/2. \tag{C.1}$$

Let $b(*) = \sigma b$, substitution of (A.6) into (13) in the steady state yields

$$\frac{\beta w}{1+\beta} = k + \sigma b. \tag{C.2}$$

Subtracting (C.1) and (C.2) and rearranging to obtain

$$\sigma = \frac{1}{2}\left[1 + \frac{\beta\gamma(R-1)}{1+\beta} - \frac{(1-\gamma)(R-1)}{(1+\beta)R}\right] \equiv \frac{1}{2}h(\gamma). \tag{C.3}$$

Note that

$$h(\gamma = 0) = \frac{1+\beta R}{(1+\beta)R} < 1,$$
$$h(\gamma = 1) = 1 + \frac{\beta(R-1)}{(1+\beta)} > 1$$

So country N will hold more than half of the debt when γ is large (when the young generation in country S is taxed heavily) and less than half of the debt when γ is small (when the old generation in country S is taxed heavily).

The threshold of γ above which country N will hold more than half of the debt is given by

$$\tilde{\gamma} = 1/(\beta R(\tilde{\gamma}) - 1).$$

QED