8. COST BENEFIT ANALYSIS AS A PLANNING PROCEDURE

P.J. Hammond

INTRODUCTION[1]

One of the most widely used tools of applied welfare economics is cost-benefit analysis. Yet the precise way in which this tool works seems not to be very well understood, even by economic theorists. In what sense, and in what circumstances, does cost-benefit analysis provide necessary or sufficient conditions for a welfare improvement? In cost-benefit analysis, should one use consumers' demand prices. or producers' supply prices, in order to value the inputs and outputs of a project? In particular, is the appropriate rate of discount the social opportunity cost of investment, or is it the social rate of time preference?

These questions seem rather basic. Yet I have found it hard to glean consistent and satisfactory answers to all of them from the extensive literature on cost-benefit analysis. Moreover, what may turn out to be the best answers so far provided are based on two crucial assumptions. First, it is often assumed that the project is sufficiently small to have a negligible impact on producer prices[2]. Second, it is assumed that the economy's production plan is optimal, at least within certain constraints. This second assumption seems especially unfortunate. After all, the reason for submitting projects to cost-benefit tests in the first place must be that the optimal production plan for the economy is not fully known. Otherwise, what would be the point of any cost-benefit calculations?

With this in mind, I venture to suggest that we should think of how cost-benefit analysis might be used in helping to locate an economy's production optimum. If cost-benefit analysis is used seriously to evaluate projects, we expect any project which passes a cost-benefit test to be undertaken, and any project which fails a cost-benefit test to be turned down. In fact, we can imagine the economy considering a series of different projects one after another, and adjusting its production plan as projects pass a cost-benefit test and become adopted. For a fixed list of projects, we therefore have an iterative planning procedure of the kind studied extensively by Malinvaud (1967) and Heal (1973), amongst others. In this sense, we can regard cost-benefit analysis as a planning procedure in its own right. In particular, a cost-benefit test can be judged by its efficacy as a planning procedure. In fact, we can go further, and deliberately tailor the cost-benefit test we shall use, including the rules for finding shadow prices, in order to generate a good planning procedure. This is precisely the aim of the present paper.

Now Malinvaud (1967) has given us particular criteria for judging the merits of an iterative planning procedure. The procedure should eventually converge to an optimum, of course, otherwise it is obviously inadequate. But since the location of the precise optimum is unlikely, and because the procedure is bound to be cut short after a finite - possibly rather small - number of steps, Malinvaud went on to advocate other important criteria. First, since the procedure may be terminated short at some stage, it is highly desirable to ensure that where it does terminate, we are left with a feasible plan, without any need for additional calculations. Second, since each successive stage of the planning procedure may turn out to be the last, it seems sensible to be sure that it leads to a better production plan. Both of these last two criteria seem especially important in project appraisal, where there can be no presumption that we are going to be looking at a whole sequence of projects in a way which will guarantee convergence to a production optimum.

Of these two criteria the first, feasibility, is easy to maintain at least in principle. Presumably the economy has some kind of feasible production plan without the project; provided that the project itself is feasible, so is the production plan which results if it is decided to adopt the project. The second criterion is clear but less easy to meet; quite simply it suggests that we should design our cost-benefit test so that any project which passes is guaranteed to be an improvement. Of course, this is a laudable aim in itself, and the appeal to properties of planning procedures may be thought unnecessary.

So my task has become one of finding what kind of cost-benefit test will succeed in identifying improvements in the economy's production plan. This is a natural step to take in parallel with, for example, the work of Dixit (1975) and Guesnerie (1977) on tax reform and the welfare effects of tax and price changes. Here I shall be examining the welfare effects of changes in the economy's production. Indeed, as is widely recognized, cost-benefit tests are particular index number tests for welfare improvements. They are also a particular kind of welfare criterion. Hence, much of what I shall have to say here applies equally to welfare criteria in general.

2. *A SIMPLE EXAMPLE*

Suppose that there are just two goods, consumption and labour, and that there is a single consumer whose welfare ordering is fixed. The government is to choose a net supply vector y for the economy from a production set Y in order to maximize the consumer's welfare ordering R. This simple example will serve to show what kinds of cost-benefit test give necessary or sufficient conditions for a welfare improvement, and so to settle what shadow prices are appropriate. Of course, the argument can be illustrated graphically.

In figure 1, the boundary of the production set Y is drawn; Y is strictly convex and its boundary is a smooth curve. The consumer's welfare contours (indifference curves) are also smooth and strictly quasi-concave. y^* is the welfare optimum. At y^* the consumer has a demand price vector q which is equal to the vector of supply prices p. Clearly, for all $y \varepsilon Y$, $qy^* > qy$ and $py^* > py$.

Now let z be any project which takes the economy to the welfare optimum y^*. Then $z = y^* - y$, where y is some vector in the production set Y. So $qz > 0$ and $pz > 0$. Thus z passes a cost-benefit test based on either the demand price vector q or the supply price vector p; in fact, the two cost-benefit tests are completely equivalent, since the price vectors must be the same.

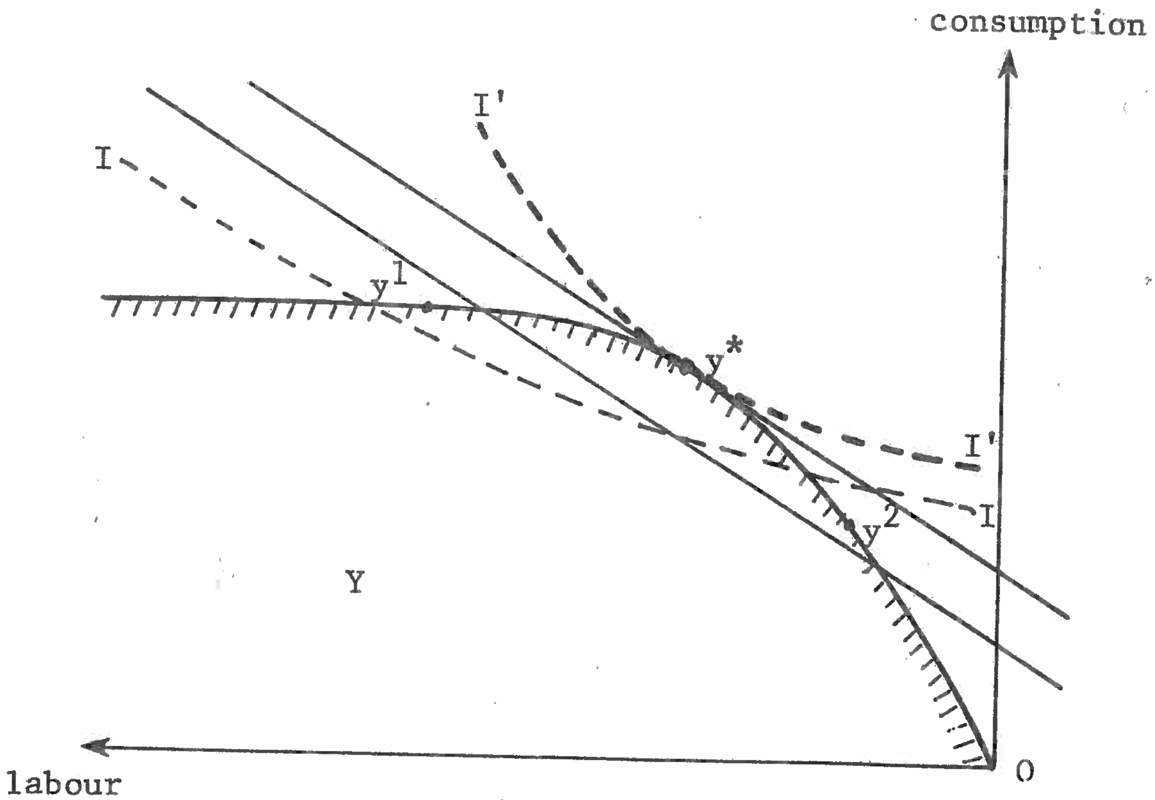

FIGURE 1.

Figure 1.

The case when z takes the economy directly to the welfare optimum is of little interest, however. Suppose the project does not lead us to y^* then. What now are appropriate prices?

Suppose one continues to use the shadow price vector $p = q$ appropriate to the optimum y^*. Then, in figure 1, $py^2 > py^1$, and so the project $z = y^2 - y^1$ is undertaken. Yet y^2 is inferior to y^1, according to the consumer's welfare ordering. So consistently using shadow prices appropriate to the optimum can lead one to accept unfavourable projects. Of course, if the society continues from y^2, and accepts new projects which increase the value of py, then ultimately it must converge to the optimum y^*. But such projects may not be identified in time, or even ever, and the economy may get stuck at y^2. Then the project z is clearly undesirable.

Nor is it any help to use the supply prices associated with either y^1 or y^2. Let p^1, p^2 denote the corresponding supply price vectors. Then $p^1y^1 > p^1y^2$, and $p^2y^2 > p^2y^1$. So the projects z and $-z$ then both get accepted as desirable projects. There is an even more fundamental problem than this, however. Why should y^1 and y^2 lie on the production frontier at all, given that cost-benefit analysis is being used to compare suboptimal, and so quite possibly inefficient, net supply vectors? And if one or both of y^1 and y^2 are inefficient, there is no corresponding supply price vector to use anyway.

The most likely remaining candidates for suitable shadow prices are what many people would regard as the obvious ones. These are the consumer's own demand price vectors at y^1 and y^2, which I shall call q^1 and q^2 respectively. In fact, it is really very well known that these will work. My detour was hardly necessary except on the grounds that, in cost-benefit analysis, there still seems to be much confusion over what exactly are the correct shadow prices.

3. *SHADOW PRICES FOR A SINGLE CONSUMER*

To make the analysis easier, I shall consistently ignore the output of private producers. And, in this section, I shall continue to assume that there is a single consumer.

Let there be a fixed set of n goods indexed by g ($g = 1$ to n); good g will be indicated by a subscript. x_g will denote the consumer's net demand for good g, and x will denote the consumer's net demand vector $x = (x_g)_{g=1}^n$. Notice that I have netted out endowments, as well as the net output of any domestic production activities.

The consumer has a feasible set of X of possible net demand vectors x, and a welfare preference ordering R defined on X. R is a weak preference relation - I will denote the associated indifference relation and P the associated strict preference relation. It will be assumed that:

(A.1) X is convex and allows extra net demand (in the sense that, if $x \in X$ and $x' \geqq x$, then $x' \in X$).

(A.2) R is convex, continuous, and monotone (in the sense that, if $x \in X$ and $x' \geqq x$, then $x' \, R \, x$, and if $x' > x$, then $x' \, P \, x$).

It is now easy to explain why the consumer's demand prices are the right ones. Given a net demand vector x, a feasible set X, and a preference ordering R, say that q is a *demand price vector at* x provided that:

(1) For all $x' \in X$, $x' \, P \, x$ implies $qx' > qx$

(2) For all $x' \in X$, $x' \, R \, x$ implies $qx' \geq qx$

Notice how the two parts of this definition correspond to two notions of competitive equilibrium for a single consumer. (1) will be true if and only if x maximizes the preference ordering R over the budget set $B = \{x' \in X \mid qx' \leqq qx\}$, so (1) corresponds to ordinary competitive equilibrium. On the other hand, (2) is true if and only if x minimizes net expenditure qx' over the preference set $R(x) = \{x' \in X \mid x' \, R \, x\}$, so (2) corresponds to compensated competitive equilibrium.

Theorem Suppose that:-

(1) X is convex, and allows extra consumption.

(2) R is convex and monotone.

Then, for any $x \in \text{int } X$, there exists a demand price vector $q \geq 0$.

And the following result is an elementary consequence of the definition of a demand price vector:

Theorem Let y^1, y^2 be two net supply vectors, and let $z = y^2 - y^1$. Let q^1, q^2 be demand price vectors at y^1 and y^2 respectively. Then:

(1) A necessary condition for z to be beneficial is that $q^1 z \geqq 0$

(2) A sufficient condition for z to be beneficial is that $q^2 z > 0$

Proof

(1) If $y^2 \; R \; y^1$, then $q^1 y^2 \geqq q^1 y^1$, so $q^1 z \geqq 0$.

(2) If $y^1 \; R \; y^2$, then $q^2 y^1 \geq q^2 y^2$. So, if $q^2 z > 0$, then, since $q^2 y^2 > q^2 y^1$, it follows that $y^2 \; P \; y^1$.

Of course, this theorem is no more than a trivial restatement of the conditions for index number tests to signify welfare improvements. Yet, trivial though it is, it is also the result which underlies all cost benefit tests which apply to discrete projects. It is the specific result given in Negishi (1972).

4. *THE MANY CONSUMER ECONOMY*

Suppose now that there is a fixed finite set of consumers I. Consumer i will be indicated by a superscript. Thus x^i denotes i's net demand vector, whose typical component x^i_g is i's net demand for good g. X^i denotes i's preference ordering. I shall assume that, for each consumer i, both X^i and R^i satisfy assumptions (A.1) and (A.2) of the previous section.

A distribution of goods between consumers is a list of net demand vectors $(x^i)_{i \in I}$ such that, for each $i \in I$, $x^i \in X^i$. I assume that there is a Bergson social welfare function $W((x^i)_{i \in I})$ defined on the space of such distributions of goods. The Bergson social welfare function will be assumed to be Pareto inclusive, in the sense that:

(a) If, for all $i \in I$, $x^i \; R^i \; \tilde{x}^i$ then $W((x^i)_{i \in I}) \geq W((\tilde{x}^i)_{i \in I})$.

(b) If (a) is true and also, for some $j \in I$, $x^j \; P^j \; \tilde{x}^j$, then $W((x^i)_{i \in I}) > W((\tilde{x}^i)_{i \in I})$.

The government is assumed to have a production set Y. A typical production or aggregate net supply vector in Y will be denoted by y. A project is a change in y, from say y^1 to y^2. What matters is the difference $z = y^2 - y^1$, which gives the net output vector of the project.

A cost-benefit test involves valuing the net outputs of a project using a vector of shadow prices $s = (s_g)_{g=1}^n$. Thus, the net benefit of the project $z = (z_g)_{g=1}^n$ is measured by $sz = \Sigma_g \; s_g z_g$. Benefits correspond to positive net outputs, and are valued positively; costs correspond to negative net outputs, and are valued negatively; sz is the difference between benefits and costs.

Where benefits and costs accrue at varying times, goods are distinguished by time, and the shadow prices will be present value prices. Where there is uncertainty, then, as in Debreu (1959, chapter 7), goods are distinguished by states of the world, and shadow prices reflect probabilities. Thus sz is the expected present discounted net benefit of the project.

5. *AGGREGATE WELFARE CRITERIA*

This is not the place to give more than the briefest discussion of the vexed topic of welfare criteria. Yet, since a cost-benefit test is a special kind of welfare criterion, nor is it possible to ignore the problems with which this topic confronts us.

As far as the usual kinds of compensation test are concerned, it suffices to notice that they provide neither necessary nor sufficient conditions for a welfare improvement. Even worse, as Gorman (1955) and many others have pointed out, it is all too easy to construct examples in which the usual welfare criteria lead to strict preference cycles. In particular, this is possible with the index number tests which underlie cost-benefit analysis.

There is one special case, however, where the usual welfare criteria do behave quite properly. This is the case noticed by Gorman (1953), in which all consumers have very similar tastes. In fact, their vector demand functions must take the form:-

$$h(q, m^i) = a^i(q) + b(q)\ (m^i - qa^i(q))$$

where $b(q)$ is independent of i. Thus, all consumers have income consumption curves which are parallel straight lines. In this case, redistributing income through lump-sum transfers makes no difference to competitive prices. There are well-defined community indifference curves, and also non-intersecting utility possibility curves.

Now, even in this special case, where the usual criteria do behave well, they do not provide either necessary or sufficient conditions for a welfare improvement. The criteria tell us when there has been an increase in "efficiency" but say nothing about "equity". To decide, for example, on the appropriate level of income tax, one must be prepared to trade off efficiency against equity. Then, an increase in efficiency can be combined with such a large increase in inequity that, overall, one's judgement is that welfare has declined. An a decrease in efficiency can bring about such a large gain in equity, that, overall, welfare is increased. Thus, efficiency criteria cannot, in themselves, provide either necessary or sufficient conditions for a welfare improvement in an economy with many consumers.

It may be possible to supplement the efficiency criteria with distributional criteria, as suggested originally by Little (1950), (1957). Little arrives at a sufficient condition for a welfare improvement: there must be a gain in either efficiency or equity, and no loss of either. There are some logical difficulties with such criteria, in that it seems all too easy to have cycles of changes all passing the Little test. Nonetheless, even in cases where there is no such logical difficulty, we still have no way of trading off equity against efficiency.

To make such a trade-off requires a complete social welfare ordering. This can be an ordering on the space of real income distributions. But then it may just as well be a Pareto inclusive social welfare ordering on the space of possible allocations

Given such a complete social ordering on the space of allocations, how does one step to, say, community indifference curves, defined on aggregate commodity space? As with most aggregation problems in economics, the key is to postulate some kind of optimization in the distribution of aggregates. One simply assumes that the aggregate net supply vector y gets distributed optimally between the various consumers in the economy. The optimal distribution may be affected by a number of constraints, of course. A "perfect" economy is one where there are no such constraints. This case has already been considered, in effect, by Samuelson (1950). In "imperfect" economies, all kinds of constraints are possible. One special kind of constraint is that goods are to be distributed through competitive markets, in which all consumers face an identical budget constraint because lump-sum transfers are ruled out. Then it may be possible to improve the distribution of goods by letting consumer prices diverge from producer prices, so this is the Diamond-Mirrlees (1971) problem of optimal commodity taxation. That community indifference curves can still be drawn in this case has been realised, in effect, by Mirrlees (1969).

There is no need to stop the imperfections at this point, however, and more possibilities are examined in Section 9.

Given well-defined community indifference curves, one can look for social demand prices, which will define a hyperplane supporting the community indifference curve passing through a given point. There are, however, two obstacles to finding such prices and such a supporting hyperplane. One, the less serious one, is that there may now be satiation in aggregate commodity space. Again, this has been recognized by Diamond-Mirrlees (1971) and by Mirrlees (1972), who have examples in which optimal public production is inefficient. In these examples, it is possible, but not desirable, to produce more of each good: if more is produced, the imperfections are such that to try to distribute the extra quantities leads to a worse allocation than the original one. Yet, provided that it is possible to pay each consumer some common poll subsidy, this obstacle will disappear, nearly always.

The second obstacle to the existence of social demand prices seems much more serious. It is all too easy for the community indifference curves in an imperfect economy, to be non-convex, even though each consumer has preferences which are convex, or even strictly convex. Of course, one can convexify the indifference curves by allowing randomized aggregate net supply vectors, but I am not going to take this possibility seriously.

The next three sections of this paper show the existence of community indifference curves, and suggest when social demand prices exist and how they would possibly be estimated, at least in the easiest cases. Section 8 discusses the use of relative world prices as shadow prices of traded goods.

6. *SHADOW PRICES IN A PERFECT ECONOMY*

A "perfect" economy is one in which it is possible to distribute goods between consumers in such a way as to reach a full welfare optimum. The only constraints which the economy faces are those which arise because resources are scarce and the opportunities for converting primary factors into final demands are limited by the extent of technical knowledge.

Let $X(y) = \{(x^i)_{i\varepsilon I} \mid \Sigma_{i\varepsilon I}\, x^i \leqq y;\ \ \forall i\varepsilon I{:}x^i \varepsilon X^i\}$ be the set of feasible distributions of the aggregate net supply vector y. Let $W((x^i)_{i\varepsilon I})$ be the Bergson social welfare function, which depends on $(x^i)_{i\varepsilon I}$, which determines the distribution of goods between consumers. In this case, the community indifference curves are the contours of the following "Bergson function" (cf. Graaff's (1957) definition of the "Bergson" frontier"):

$$B(y) = \max_{(x_i)_{i\varepsilon I}} \{W((x^i)_{i\varepsilon I} \mid (x^i)_{i\varepsilon I}\ \varepsilon\ X(y)\}$$

This Bergson function $B(y)$ indicates the level of social welfare which an economy will achieve, as a function of the aggregate net supply vector y, on the assumption that the distribution of y between consumers is chosen to maximize social welfare subject to the distributional constraints, as expressed in $X(y)$ the set of feasible distribution of y.

The definition of $X(y)$ above was for a totally planned economy. The behaviour of individual agents placed no restrictions at all on the distribution of goods between consumers. In a market economy, goods can only be distributed to consumers if they choose to buy them at the prevailing prices and incomes. Assuming that arbitrary lump-sum redistribution of income is allowed, and that all consumers do behave competitively, even if not all producers dò, then the set of feasible distributions of a given aggregate net supply vector y is:-

$$X^M(y) = \{(x^i)_{i\epsilon I} \mid \exists\ q \geq 0; \quad (m^i)_{i\epsilon I};\ \forall\ i\ \epsilon\ I: x^i \epsilon \xi^i(q, m^i)$$

$$\text{and } \Sigma_{i\epsilon I}\ x^i \leqq y\}$$

where $\xi^i(q, m^i)$ denotes the value of consumer i's net demand correspondence given the budget constraint $qx^i \leq m^i$. As before, define:

$$B^M(y) = \max_{(x^i)_{i\epsilon I}} \{W((x^i)_{i\epsilon I}) \mid (x^i)_{i\epsilon I}\ \epsilon\ X^M(y)\}$$

which indicates the maximum level of social welfare which is attainable in the market economy by distributing the aggregate net supply vector y.

Now, a "perfect market economy" is one with the following properties. The social welfare function $W((x^i)_{i\epsilon I})$ agrees with the consumers' own preference orderings. Consumers have monotone convex, closed consumption sets, and monotone, convex, continuous preference orderings. Finally, any optimal distribution of a given aggregate net supply vector takes each consumer off the lower boundary of his survival set. In these circumstances, we know, by the second efficiency theorem of welfare economics, that any Pareto efficient distribution of goods is competitive at some price vector $q > 0$ and some distribution of income $(m^i)_{i\epsilon I}$. So the distribuwhich maximizes $W((x^i)_{i\epsilon I})$ over the set $X(y)$ is also in the smaller set $X^*(y)$. Thus, in a perfect market economy,

$B^M(y) = B(y)$ everywhere, and a perfect market economy is effectively a perfect economy. Indeed, what I have called a "perfect market economy" is what Mirrlees (1969) described as a "perfect economy".

The functions $B(y)$ and $B^M(y)$ can be used to construct the community indifference curves which international trade theorists have often used, and which underlie cost benefit tests. The community indifference curves are what Graaff refers to as the "Bergson frontier". Samuelson (1950), (1956) pointed out how they could be used to overcome the difficulties brought about by the more usual "new" welfare criteria. Graaff (1957, p. 163) has criticized Samuelson's approach on the grounds that the Bergson function B may not be quasi-concave. In fact, Negishi (1963), following Gorman (1959), has given the following sufficient conditions for B to be quasi-concave, when W respects consumers' preferences : each consumer must have a *concave* utility function, and social welfare must be a quasi-concave function of consumers' utility levels. It is also quite obvious that B is a monotone function when each consumer has monotone preferences.

In a perfect market economy, then, there is unlikely to be any real problem. There will exist social demand prices everywhere, at least provided that consumers' convex preferences can be represented by a *concave* utility function. This is an extra restriction, as pointed out by Enthoven and Arrow (1961), but it hardly seems a serious one. What is more, it is obvious that the social demand prices must be equal to market prices q.

It is an unfortunate fact of life that there can never be a perfect market economy. There are, of course, many obstacles to competitive markets. But there is an even more fundamental obstacle to optimal lump-sum redistribution of income. Quite simply, this relies on information about consumers which no government is ever likely to have. If one tries to transfer income from the advantaged to the disadvantaged, or the skilled to the unskilled, then all but the most naive consumers will try to conceal their advantages or skills. More precisely and formally, as I have shown elsewhere (Hammond, 1979), introducing lump-sum transfers to try to improve equity often causes incentive-incompatibility. Thus, we are led to consider cost benefit analysis in imperfect economies, where lump-sum transfers are ruled out.

7. THE DIAMOND-MIRRLEES ECONOMY

Suppose that, to avoid the problems which arise when consumers want to mis-state their characteristics, we face each of them with an identical budget constraint. And suppose too that this is going to involve well-defined prices q. There are two cases possible. One occurs when consumers may pay a uniform poll tax or receive a uniform poll subsidy, whose net value will be denoted by m. Then the budget constraint faced by consumer i is $qx^i \leqq m$. The second case is when even such uniform poll taxes or subsidies are excluded, in which case i's budget constraint becomes $qx^i \leqq 0$. In these two cases, the set $X(y)$ of feasible distributions $(x^i)_{i\varepsilon I}$ of a given total net supply vector y is:-

$$X^1(y) = \{(x^i)_{i\varepsilon I} \mid \exists q \geq 0;\ \exists m;\ \forall i\varepsilon I\colon x^i \varepsilon\ \xi^i(q,m) \text{ and } \Sigma_i\, x^i \leqq y\}$$

and

$$X^2(y) = \{(x^i)_{i\varepsilon I} \mid \exists q \geq 0;\ \forall i\varepsilon I\colon x^i \varepsilon\ \xi^i(q, 0) \text{ and } \Sigma_i\, x^i \leqq y\}$$

respectively.

The assumption is that *any* consumer price vector q is possible, provided that markets clear. In an economy with private producers, this means that arbitrary commodity taxes can be imposed, so that consumers face different prices from those faced by producers. As remarked by Diamond and Mirrlees (1971), for some goods such as fuels, which are both intermediate goods in production and also consumer goods, this implies having different prices charged for the same good, an arrangement which may be difficult to police adequately.

Mirrlees (1969) recognized how welfare criteria can s still be used in such an imperfect economy. There are some new difficulties, however, which make the existence of a social demand price vector somewhat problematic. Such a price vector will support a contour of one of the following two functions:-

$$B^1(y) = \max_{(x^i)_{i\epsilon I}} \{W((x^i)_{i\epsilon I}) \mid (x^i)_{i\epsilon I} \epsilon X^1(y)\}$$

or $B^2(y)$, which is defined analogously from $X^2(y)$. Yet there are cases in which $B^2(y)$ may display satiation, or may not be quasi-concave. Two examples where $B^2(y)$ has a point of satiation have been provided by Diamond and Mirrlees (1971, p.18) and by Mirrlees (1972). A rather simpler version of the original Diamond Mirrlees example is example 1 of the appendix.

Example 2 of the appendix shows how neither $B^1(y)$ nor $B^2(y)$ may be quasi concave.

As remarked by Diamond and Mirrlees (1971), however, when the budget constraint takes the form $qx^i \leq m$ for each $i\epsilon I$, productive efficiency is always desirable. This, of course, is precisely because $B^1(y)$ is a monotone function, and the Diamond-Mirrlees argument establishes this. For, if $y' > y$, and if each consumer has a continuous preference ordering, then replacing the budget constraint $qx^i \leq m$ by $qx^i \leq m'$, where $m' > m$ and $m' - m$ is small, ensures that each consumer chooses a new net demand vector which is preferred to the old, and also guarantees feasibility.

8. MEADE'S FORMULA FOR SHADOW PRICES

In this kind of imperfect economy, social demand prices are no longer the same as the prices which consumers face in the market. A simple example illustrates this. Suppose that, for each $i\epsilon I$, X^i is the positive orthant, and i has the logarithmically transformed Cobb-Douglas utility function $U^i(x^i) = \Sigma_g \alpha^i_g \log x^i_g$. (where $\alpha^i_g > 0$ (each good g), $\Sigma_g \alpha^i_g = 1$.) Suppose too that each consumer i faces a budget constraint $qx^i \leq \theta^i m$, where θ^i is unalterable; then his net demand x^i_g is $\alpha^i_g \theta^i m. q_g^{-1}$ from which it follows that q and m must satisfy $q_g = m(\Sigma_i \theta^i \alpha^i_g)/y_g$ (each g) in order to clear all markets.

So $x^i_g = \theta^i \alpha^i_g y_g / \Sigma_j \theta^j \alpha^j_g$ (each i, g).

Thus, if social welfare $W = \Sigma_i U^i$, the function $B^1(y)$ takes the form:

$$B^1(y) = \text{constant} + \Sigma_i \Sigma_g \alpha^i_g \log y_g$$

So the social demand price, or shadow price, of good g is

$$p_g = \Sigma_i \alpha^i_g / y_g$$

Notice the contrast with market prices, for which the income shares θ^i weight the propensities to consume α^i_g. For shadow prices, however, it is the unweighted sum of propensities to consume which matter. Thus market prices give too much weight to goods which form a larger part of the budget of the rich, as compared to shadow prices.

More generally, consider the social demand prices p when the feasible distributions are $X^2(y)$. Then, assuming full differentiability, and letting $v^i(q)$ denote i's indirect utility function as a function of consumer prices q, and letting $F((U^i)_{i \in I})$ denote social welfare as a function of the consumers' utility levels, we have:

$$p_g = \frac{\partial B^2(y)}{\partial y_g} = \Sigma_i \frac{\partial F}{\partial U^i} \Sigma_h \frac{\partial v^i}{\partial q_h} \cdot \frac{\partial q_h}{\partial y_g}$$

where q is a vector of consumer prices which clears all markets. Write $\beta^i = \frac{\partial F}{\partial U^i} \cdot \frac{\partial U^i}{\partial m}$, the marginal social significance of consumer i's income. Then:

$$p_g = - \Sigma_i \beta^i \Sigma_h x_h^i \frac{\partial q_h}{\partial y_g} = - \Sigma_h \tilde{x}_h \frac{\partial q_h}{\partial y_g}$$

where $\tilde{x}_h = \Sigma_i \beta^i x_h^i$ is the weighted aggregate net demand for good h, the weights being the marginal social significance of each consumer's income. Alternatively, we have:

$$p_g = \frac{\partial B^2(y)}{\partial y_g} = \Sigma_i \frac{\partial F}{\partial U^i} \Sigma_h \frac{\partial U^i}{\partial x_h^i} \frac{\partial x_h^i}{\partial y_g}$$

$$= \Sigma_i \frac{\partial F}{\partial U^i} \Sigma_h \frac{\partial U^i}{\partial m} q_h \frac{\partial x_h^i}{\partial y_g}$$

(because, in the market economy, $\frac{\partial U^i}{\partial x_h^i} = \frac{\partial U^i}{\partial m} q_h$).

So $p_g = \Sigma_i \beta^i \Sigma_h q_h \frac{\partial x_h^i}{\partial y_g}$

which corresponds to the formula Meade (1955) gave for the welfare effect of a small change in the economy. This is equivalent to the previous formula, of course, when
$m^i = 0$ (all i), because then

$$\Sigma_h q_h x_h^i = 0 \quad (\text{all } i \varepsilon I)$$

and so

$$\Sigma_h q_h \frac{\partial x_h^i}{\partial y_g} + \Sigma_h x_h^i \frac{\partial q_h}{\partial y_g} = 0$$

Both these formulae should be compared with the following, which is obtained by differentiating the identity $\Sigma_h q_h y_h = 0$ partially with respect to y_g:

$$q_g = -\Sigma_h y_h \frac{\partial q_h}{\partial y_g} = -\Sigma_i \Sigma_h x_h^i \frac{\partial q_h}{\partial y_g} = \Sigma_i \Sigma_h q_h \frac{\partial x_h^i}{\partial y_g}$$

The difference is that, compared with market prices, shadow prices give greater relative weight to the consumption of the poor (whose marginal income is more significant) and less relative weight to the consumption of the rich.

9. *GENERAL IMPERFECT ECONOMIES*

The Diamond-Mirrlees economy introduced just one special imperfection into an otherwise perfect market economy. All consumers had to face the same linear budget constraint. There is no need, however, to stop at such imperfections. We can add to the list of imperfections, and still come up with a yet more restricted feasible set of distributions $X(y)$, and another valuation function $V(y)$, say. We can always define:

$$V(y) = \max_{(x^i)_{i \varepsilon I}} \{W(x^i)_{i \varepsilon I}) \mid (x^i)_{i \varepsilon I} \varepsilon X(y)\}$$

provided that the function $W(.)$ is continuous and the set $X(y)$ is compact, so that the maximum is well-defined.

In particular, we can allow for restrictions on the taxation system. These may mean that some goods must escape taxation altogether. Or they may mean that a broad class of goods must all bear taxes at the same *ad valorem* rate. Such restrictions were discussed in a number of papers published in the June 1970 issue of the *American Economic Review*. We can allow too, though much less easily, for the limited control which a government has over the private producers in an economy (Mirrlees (1972)). In principle, it is also possible to allow for imperfect competition, quantity constraints, involuntary unemployment and non-Walrasian equilibrium. We can even make some allowances for transactions costs, transport costs, and tax administration costs (Heller and Shell (1974)). Finally, we can also allow for what might be termed political constraints - the need to placate pressure groups, the desires of politicians to see themselves maintained in power, even for their obstinacy or lack of imagination. In the end, an aggregate net supply vector is to be valued according to how it will be distributed between the consumers in the economy. The shadow price vector gives us information about the directions in which welfare improvements lie, and have to be estimated with this end in mind.

Of course, the extra constraints make estimation of shadow prices far more difficult. They may also bring about satiation and nonconvexities. Nonetheless, it is worth noting that the earlier Meade formula:-

$$pg = - \Sigma_i \beta^i \left[\Sigma_h x_h^i \frac{\partial q_h}{\partial y_g} - \frac{\partial m^i}{\partial y_g} \right] = \Sigma_i \Sigma_h \beta^i q_h \frac{\partial x_h^i}{\partial y_g}$$

is still valid as long as consumers are, in effect, behaving competitively.

10. ON THE USE OF WORLD PRICES

In project analysis it is becoming almost customary to recommend that goods which are freely traded on competitive world markets should be valued at their world market prices. This is the approach recommended by Little and Mirrlees (1968), (1974). It is also recommended, in certain cases, by Dasgupta, Marglin and Sen (1972), although they use the domestic currency as a numeraire (unlike Little and Mirrlees, whose numeraire is foreign currency) and suggest that world prices should be converted into domestic prices at a shadow foreign exchange rate. For a more detailed discussion of when to use world prices, see Dasgupta and Stiglitz (1974).

Now this approach is in direct contrast to the social demand prices whose use I have been recommending. In fact, world prices are like the supply prices whose use was criticized in section 2 above. As Dasgupta and Stiglitz, among others, have been careful to show, world prices are the appropriate prices when the economy pursues an optimal trading policy, or even a suboptimal trading policy. Since this argument for world prices can be made quite simple, it is worth giving it here.

Let w be the world price vector for goods traded competitively on world markets: by definition, w is fixed up to a scalar multiple. Write $y = (y_T, y_N)$, where y_T is the aggregate net supply vector for goods which are traded competitively and y_N is the aggregate net supply vector for all other goods. Then, if the constrained set of possible domestic net output vectors y^D is $\tilde{Y}$ an optimal trading policy in the economy must involve a choice of a net trade vector z_T so that $V(y^D + z_T)$ is maximized subject to $y^D \in \tilde{y}$ and to an international budget constraint $wz_T \leq b$. From this, it follows that y^D should maximize $wy_T^D + s_N y_N^D$ over Y, for suitable shadow prices s_N for other goods. Thus, the shadow prices for traded goods should be proportional to w.

A similar argument has been used to establish that, *provided* taxes are optimal, then producer (or supply) prices at the optimum are appropriate shadow prices. This is already familiar from Diamond and Mirrlees (1971), Dasgupta and Stiglitz (1974), among many others. What has not been generally recognized, however, is just how much the validity of producer prices does depend on the optimality of the tax system. This dependence has recently been visibly demonstrated in unpublished work by Kevin Roberts (1978).

Despite arguments such as those given in section 2, however, it seems that the belief that world prices are appropriate retains a firm hold. The reasons for this are difficult to ascertain, let alone comprehend. While I shall try to suggest at the end of this section how a rather different justification of world prices could be attempted, let me now repeat that, as long as suboptimal policies remain in force, using world prices does *not* lead to projects being adopted which guarantee welfare improvements. This is already clear from figure 1 in section 2.

So far, it must be admitted, I have been assuming that the project is a simple take-it-or-leave-it decision. Another possibility is that, when we have a major project such as the construction of a deep-water port, the detailed design of the project should it be adopted is not entirely fixed, and the economist has a role to play in designing the project as well in deciding whether it should be adopted. Thus, we can imagine an economy faced with a choice between an initial net supply vector y^1 when the project is not adopted, and a whole set of possible net supply vectors Y^2 should the project be adopted. This is shown in figure 2 below.

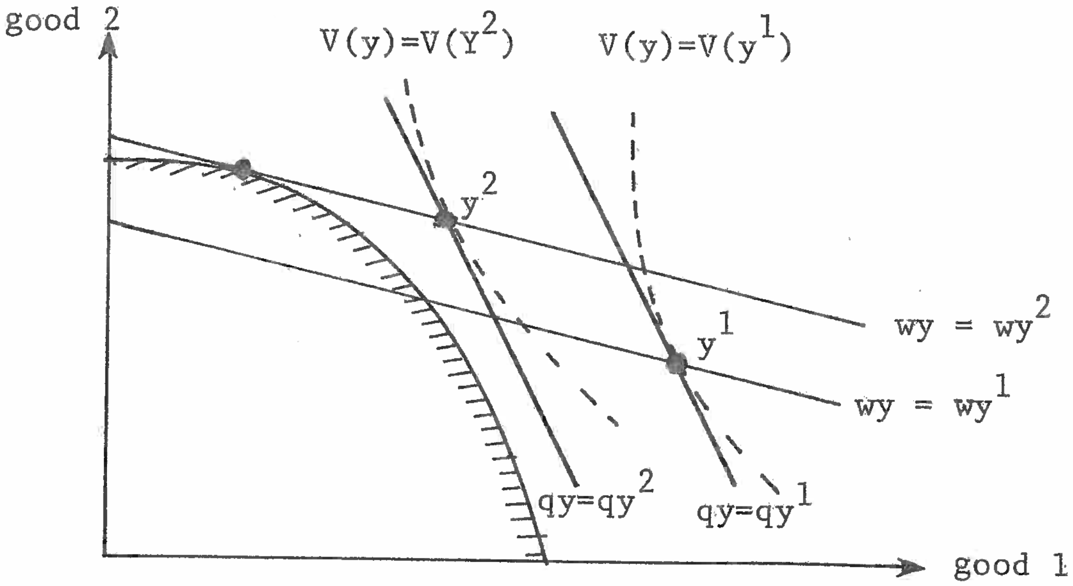

Figure 2

There are two goods, each traded competitively on world markets at the fixed world price vector $w = (w_1, w_2)$. Initially, the economy can produce at y^1. To save drawing, and also to make the trade policy initially seem a particularly bad one, I shall suppose that the suboptimal tariff structure is exactly the one which induces autarchy. After the project is introduced, there is a feasible domestic production set Y^2. The economy should use the world price vector w to decide what point of Y^2 to choose, and in this sense w is a vector of shadow prices. Once the optimal project is chosen, however, the economy's suboptimal trading policy leaves the economy at y^2 (where the tariff is the same as it was at y^1, assuming that V represents a single consumer's indifference curves and so its tangent has a slope corresponding to the consumer price vector q). As drawn, y^2 is worse than y^1, even though its value at world prices is greater. Thus, the use of world prices w leads to a welfare deterioration, whereas the use of the demand prices q would not - in fact, these recommend returning to y^1 from y^2.

Thus, even in the presence of a *fixed* suboptimal tariff, world prices may not be appropriate shadow prices. Where suboptimal tariffs are not fixed, it is certainly no harder to construct similar examples.

This brings us back once again to "social demand prices", calculated according to the Meade formula of section 8. As already argued, these are the appropriate shadow prices. If the economy's trading policy happens to be optimal, then relative world prices must be equal to relative social demand prices - indeed, this is precisely the first order condition for constrained optimality.

Thus the Meade formula is always appropriate, whereas world prices are only appropriate if trade policies are optimal - in particular, consumer prices for traded goods must be chosen optimally. Moreover, though the Meade formula may be hard to calculate, one has to go through similar calculations in any case to establish what is the optimal trade pattern in the economy. All in all, then, it seems that the Meade formula is preferable.

So why use world prices? A good reason is to avoid adopting a project which involves building up an infant industry under a protective tariff, when it is better to abolish the tariff and to do without the industry. The danger of assuming that the tariff will be maintained anyway is that it helps to reinforce suboptimal trading policies - a kind of self-fulfilling prophecy once again. A similar argument also suggests that some kinds of "political constraint" should also be ignored, and that there is a case for assuming rather more optimality in the distribution of goods than in fact there is. In this kind of situation, it is simply not possible for the economist to stay completely aloof from the political process, because what he does is bound to affect it. To analyse the proper course of action, however, if it is possible at all, would take us way beyond the scope of this paper, and of welfare economics as it is usually understood.

11. *SOME TENTATIVE CONCLUSIONS*

The student who searches the textbooks for a justification of cost benefit tests is likely to come up with the impression that there is no such justification, except in special cases. I hope to have shown what many economists must know, which is that this impression is too pessimistic. There are very many circumstances in which cost-benefit tests can give necessary or sufficient conditions for a welfare improvement.

The key to the problem is to use *optimizing* welfare criteria, based on intertemporal comparisons which are embodied into a social welfare ordering. In a "perfect market economy", where optimal lump-sum transfers are always made, cost benefit tests based on market prices will give the right answers. When optimal lump-sum transfers are not made, one is in an "imperfect" economy. Now market prices are no longer the right shadow prices, but, in many situations, valid cost-benefit tests are still possible. Assuming complete differentiability, the appropriate shadow price of good g is:-

$$-\Sigma_i \beta^i \left[\Sigma_h x_h^i \frac{\partial q_h}{\partial y_g} - \frac{\partial m^i}{\partial y_g} \right] = p_g = \Sigma_i \Sigma_h \beta^i q_h \frac{\partial x_h^i}{\partial y_g}$$

where β^i is the marginal social significance of i's income
x^i_h is i's net demand for good h
q_h is the market clearing price of good h
y_g is the aggregate net supply of good g
and m^i is i's net expenditure - the value of his net demand vector.

This formula remains correct for all types of imperfections, provided only that shadow prices are meaningful and that consumers behave competitively.

I have also tried to shed some light on the use of demand prices as against supply prices or opportunity costs. To guarantee step-by-step improvements whenever a project is accepted on the basis of a cost-benefit test, it is demand prices which are the appropriate ones. The only supply prices which can be helpful are those at the full optimum and these are unlikely to be known in situations where cost-benefit analysis has any role to play. Even then, supply prices do not guarantee improvements at each step, but only eventual convergence to an optimum.

In this connection, mention was also made of the practice of using world prices as shadow prices for traded goods. These are particular supply prices. Their use is justified provided the pattern of international trade is alway optimized, given what else is going on in the economy. But their use is not strictly justified when a government insists on maintaining suboptimal trade policies. Then, the justification is that to use true social demand prices may help to perpetuate such suboptimal trade policies, and that it may be better to adopt projects which will only be fully beneficial when more suitable trade policies come into force.

There are a number of important issues which have been considered elsewhere but which I have left aside for now, hoping to discuss them in later work. These include:

1) Measures of social surplus.
2) Externalities.
3) Imperfect labour markets and the shadow wage.
4) Imperfect capital markets and the social rate of discount.

5) The shadow price of foreign exchange (See, however, Blitzer, Dasgupta and Stiglitz, 1978)
6) Private producers.

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FOOTNOTES

1. I owe the idea of viewing cost-benefit analysis as a planning procedure to P.Warr and a seminar he presented at the Australian National University in 1975. An earlier version of this paper, entitled "Cost-Benefit Analysis in Perfect and Imperfect Economies", was presented to the Franco-Swedish Conference on Public Economics in Sarlat, April 1976. I am grateful to the participants for their encouraging comments, and to Jagdish Bhagwati, Partha Dasgupta, Kevin Roberts, T.N.Srinivasan, Alastair Smith, Jack Mintz and John Weymark for their constructive criticisms, some of which will have to be met in later work.

2. For a recent exception, see Harris (1978).

APPENDIX

Two Examples

Example 1

There are two consumers and two goods. The consumers' utility functions are:

$$U^1(x^1) = (x_1^1 + 1)\, x_2^1$$

$$U^2(x^2) = x_1^2(x_2^2 + 1) \text{ respectively.}$$

The social welfare function is: $W = \min\{U^1, U^2\}$

Faced with the budget constraint $qx \leqq 0$, the two consumers' net demands vectors are respectively:

$$x_1^1 = -\tfrac{1}{2},\; x_2^1 = q_1/2q_2$$

$$x_1^2 = q_2/2q_1,\; x_2^2 = -\tfrac{1}{2}$$

So the indirect utility functions are respectively:

$$v^1(q) = q_1/4q_2,\ v^2(q) = q_2/4q_1$$

The function W therefore achieves a global maximum when $q_1 = q_2$. Then aggregate net demands are $(0, 0)$. It follows that, in this economy, the function $B^2(y)$ has a point of global satiation at $(0, 0)$.

Example 2

Once again, there are two consumers and two goods. The consumers' utility functions are:

$$U^1(x^1) = (x_1^1 + 1)^{\alpha}(x_2^1)^{1-\alpha}$$

$$U^2(x^2) = (x_1^2)^{\beta}(x_2^2)^{1-\beta}$$

respectively, where $0 < \alpha < 1$ and $0 < \beta < 1$.

The social welfare function is:

$$W = \min\{\gamma^1 U^1, \gamma^2 U^2\}$$

where $\gamma^1, \gamma^2 > 0$.

Faced with the budget constraint $qx \leq m$, the two consumers' net demand vectors are respectively

$$x_1^1 = -1 + \frac{\alpha(m + q_1)}{q_1},\quad x_2^1 = \frac{(1-\alpha)(m + q_1)}{q_2}$$

$$x_1^2 = \frac{\beta m}{q_1} \qquad x_2^2 = \frac{(1-\beta)m}{q_2}$$

So the indirect functions are respectively:

$$v^1(q,m) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(m + q_1)\ q_1^{-\alpha}\ q_2^{\alpha-1}$$

$$v^2(q,m) = \beta^{\beta}(1-\beta)^{1-\beta} m q_1^{-\beta}\ q_2^{\beta-1}$$

For market clearing, we must have:

$$y_1 = \alpha - 1 + \frac{(\alpha + \beta)m}{q_1}, \quad y_2 = \frac{[2 - (\alpha + \beta)]\, m + (1 - \alpha)q_1}{q_2}$$

$$\text{So } q_1 = \frac{(\alpha+\beta)m}{y_1+1-\alpha}, \quad q_2 = \frac{[2(y_1 + 1 - \alpha) - (\alpha + \beta)y_1]m}{y_2(y_1 + 1 - \alpha)}$$

By appropriate choice of the constants γ_1, γ_2, it follows that $B(y)$ takes the form:

$$B(y) = \min \{V^1(y),\ V^2(y)\} \quad \text{where}$$

$$V^1(y) = (y_1 + 1 + \beta)\, y_2^{\,1-\alpha}\, [2(y_1 + 1 - \alpha) - (\alpha + \beta)y_1]^{\alpha-1}$$

$$V^2(y) = (y_1 + 1 - \alpha)\, y_2^{\,1-\beta}\, [2(y_1 + 1 - \alpha) - (\alpha + \beta)y_1]^{\beta-1}$$

Suppose β is close to 0, α is close to 1, and y_1 is close to 1.

Take $y_2 < 2$. Then $V^1(y)$ is close to 2, $V^2(y)$ is close to y_2, and so $V_2(y) < V_1(y)$, $B(y) = V_2(y)$.

Thus, along a B-indifference curve:

$$y_2 = \left(\frac{B}{y_1 + 1 - \alpha}\right)^{\frac{1}{1-\beta}} [2(y_1 + 1 - \alpha) - (\alpha + \beta)y_1]$$

so

$$\frac{(1-\beta)\ (y_1+1-\alpha)^{\left(1+\frac{1}{1-\beta}\right)}}{B^{\frac{1}{1-\beta}}} \quad \frac{dy_2}{dy_1}$$

$$= (\alpha+\beta) - [2\beta + (\alpha+\beta)(1-\beta)]\,(y_1 + 1-\alpha)$$

and

$$\frac{(1-\beta)^2 (y_1 + 1 - \alpha)^{(2 + \frac{1}{1-\beta})}}{B^{\frac{1}{1-\beta}}} \frac{d^2 y_2}{dy_1^2} = -(\alpha+\beta)(2-\beta)$$

$$+ \left[2\beta + (\alpha+\beta)(1-\beta)\right] (y_1 + 1 - \alpha)$$

With β close to 0, α close to 1, y_1 close to 1, it follows that $\frac{d^2 y_2}{dy_1^2} < 0$, which shows that the function B is not quasi-concave.

DISCUSSION : PARTHA DASGUPTA

A typical Peter Hammond paper is graced by clarity of thought, a judicious choice of problems, elegance in the manner in which the arguments are executed, and a deep appreciation of the literature on the subject he is writing. The first three characteristics are fully reflected in this paper. As regards the fourth, though, the paper leaves much to be desired.

But then, how does it matter? Most of us find it easier to write than read, and in any case we have better things to do. There are children to raise, friends to entertain, and theatres to go to. So perhaps one should ignore it. But I think it matters on this occasion. For Hammond is not concerned with proving new theorems. Rather, he is reviving a methodology for conducting social cost-benefit analysis, criticizing a currently much-used one, and trying to link various bits of the existing literature on some specific problems, such as the possible desirability of using border prices. It matters then that in the process of doing this he has allowed himself to overlook a voluminious literature. It matters to me especially, since he attributes to me views that I did not know I held. And I shall come back to it.

But first, to the methodology Hammond is advocating. The methodology consists first in characterizing an economy in equilibrium. The equilibrium can be a very general one, in the sense that the economy may be shot through with distortions. It may be a non-Walrasian one, as in the Harris-Todaro model. In this economy a small feasible project is envisaged. The idea is to trace out the general equilibrium effects of the project, were it to be under-taken, valuing these effects, and then checking whether the move is worth society's while. Deep problems beset such an exercize. For unless strong assumptions are made on preferences, the underlying technology and the constraint sets that agents are assumed to face, the "general equilibrium" effects of a small perturbation in production may not be amenable to simple analysis. To take only one example, one may not be able to guarantee that in under-taking the production perturbation the economy finds itself a new resting place. As Hammond remarks, James Meade, having developed the methodology, made a systematic study based on it. But uncharacteristically Meade did not squeeze the last drop of juice from this particular lemon, and there was (and still is) much work to be done. Subsequently, economists as diverse as Dorfman, Harberger and Marglin appealed to this approach, and more recently one strand of the UNIDO Guidelines for Project Evaluation has pursued it in a more elaborate context.[1]

Hammond does not note this. Nor does he note at the end of Section 8 that the use of income distributional weights (which he wants to recommend in project appraisal) were advocated strongly in the UNIDO Guidelines. Instead he complains that the recent literature on project evaluation has shied away from the Meade approach and has instead chosen to study cost-benefit rules based on shadow prices obtained from optimization exercizes. The reason why he finds the second route otiose ("after all, the reason for submitting projects to cost-benefit tests in the first place must be that the optimal production plan for the economy is not fully known. Otherwise, what would be the point of any cost-benefit calculations?") has in fact been elaborated on earlier by Rudra (1973). But it is worth asking why economists within the last decade have chosen to pursue this second route when developing rules for social cost-benefit analysis. I think the reason is that in recent years those economists who have been developing the analysis have primarily been interested in national development planning. For one thing, the total volume of public investment may be large, and may not consist simply in providing society with "public" goods. But more important, public investment would in such a context provide the government with only one among several sets of controls. It is then immediate that one wants to study the manner in which a government ought simultaneously to wield its various controls. The Diamond-Mirrlees (1971) paper is a pristine example of such an exercise.

There is of course a strong relationship between the methodology Hammond is advocating and the optimization approach. If some optimization is going on in the background then some of the terms appearing in the expression describing the change in social welfare due to the marginal project will drop out, (this is an application of the envelope theorem), and the Meade expression quoted by Hammond in Section 8 will be what is usually called a first-order condition of the optimization exercise.

The idea of a single central planner is, of course, a myth. What one has in mind is a set of departments, each perhaps involved in the wielding of a few controls. In this context the fact that not all departments may share the same social goal is of importance, and one may be forced to pose the problem as a game. But even if they were to share the same goal, there is the problem of co-ordination. One may wish to suppose that each department optimizes the shared social welfare function given the actions of the other departments, and in this context it may seem reasonable to regard a Nash equilibrium as a viable outcome. Then, if the environment is a classical one, an equilibrium is an outcome which a central planner would have chosen had he (or perhaps I should say, HE) optimized by wielding all the controls simultaneously. In this event the co-ordination is perfect. This is the situation, for example, in the Diamond-Mirrlees (1971) economy. Such perfect co-ordination can in principle be attained in some non-classical environments as well, as the Marschak-Radner (1972) theory of teams has shown us. Usually though an equilibrium will lack perfect co-ordination. The beginnings of a study of such problems have recently been made in the deep explorations of Sanford Grossman (1977) in his analysis of equilibria with incomplete markets.

A simple manner of highlighting the possible lack of co-ordination among different government departments and then to derive social cost-benefit rules is to suppose the decisions of all other agencies to remain fixed and to allow only the department in charge of project evaluation to optimize. It supposes of course a particularly vigorous project evaluation department co-existing with unusually slothful government colleagues. But one must bear in mind that the analysis is directed for the benefit of the department of project evaluation. If this department is slothful as well the entire exercise is a non-starter. But as I have said, lack of co-ordination is only one problem. The fact that in practice different departments will pursue somewhat separate goals needs recognition as well. For example, the department involved in project evaluation may have reasons for believing that the tariff on X will be raised due to the influence of pressure groups if it recommends that Y be produced in the public sector. This kind of game problem is hard to come to grips with satisfactorily. A conceptually simple route is to allow the project evaluation department to anticipate the response of other departments to its choice and then to optimize. This is similar to the route that von Stackelberg followed in his analysis of imperfect competition. Rules for project evaluation with such problems as these in the foreground was the focus of attention in the analysis in Dasgupta and Stiglitz (1974) and more recently in the context of foreign exchange constraints in Blitzer, Dasgupta and Stiglitz (1977).

The aim in these theoretical explorations is not to produce a catalogue of cost-benefit rules. Rather, it is to see what arguments are involved in the choice of projects. In this connection it is useful to check the robustness of some simple rules that have recently been advocated. Perhaps the most celebrated rule advocated in recent years is the use of border prices for tradeable commodities in project evaluation (Little and Mirrlees (1969, 1974)). I need hardly point out why such a rule would be so appealing. Hammond seems to think that the UNIDO Guidelines advocates this as well. Here I am caught in a box. My teacher James Mirrlees admonishes me that I don't subscribe to the rule even though I should. Now Peter Hammond accuses me of subscribing to it when in fact I shouldn't. The issue is not one of analytical soundness. I suspect Little and Mirrlees concentrated on environments in which such a rule obtains and I rather suspect the authors of the UNIDO Guidelines concentrated on those where the rule would not be appropriate.[2] The thing to do is to identify these environments. Here I can do no more than to illustrate the nature of the problem. Details have been discussed in the references at the end of these comments. There are also several unsolved problems here.

For ease of exposition suppose the economy in question to be small. There are T tradeable commodities and N non-tradeable ones. Let $\underline{w} = (w_1, \ldots, w_T)$ be the border price vector of tradeable goods, $\underline{p} = (p_1, \ldots, p_T, p_{T+1}, \ldots, p_{T+N})$ the vector of domestic private producer prices, $\underline{q} = (q_1, \ldots, q_T, \ldots, q_N)$. The vector of prices faced by consumers and $\underline{s} = (s_1, \ldots, s_T, s_{T+1}, \ldots, s_{T+N})$ the vector shadow prices. Of these, only $\underline{w}$ is exogenously given. It is this fact which gives the border price rule its operational appeal.

Let me without further ado suppose there is no loss in generality in normalizing by setting $w_1 = p_1 = s_1 = q_1 = 1$. Now if overall production efficiency characterizes the optimum, then $\underline{p} = \underline{s}$ and $\underline{w} = (s_1, \ldots, s_T)$. This is certainly so at the full optimum. It is also so in the Diamond-Mirrlees (1971) optimum. But the "border price" rule which I am discussing here is less stringent. It consists in the recommendation that $(s_1, \ldots, s_T) = \underline{w}$. It does not claim that $p = \underline{s}$. As one would expect, this rule is appropriate for a much larger class of economies than the Diamond-Mirrlees (1971) one. For example, if the government is unable to tax domestic production but must raise all its revenue by border taxes and does so optimally, then in general the rule holds (see Dasgupta and Stiglitz (1976) and Diamond and Mirrlees (1976). A direct approach to the problem of investigating when the border price rule holds consists in looking at circumstances in which the public sector ought to be production-efficient. Since, presumably, the government can engage in trade, public sector production efficiency implies that the border price rule holds. But co-ordination between <u>all</u> project evaluators in the public sector may be too much to expect. Thus one may be forced not to amalgamate all production sets in the public sector. A given project evaluator in charge of one such production set may have reasons for believing that other project evaluators are using "wrong" sets of shadow prices. Suppose then that the public sector as a whole is not production-efficient. Presumably he can still ask whether <u>he</u> ought to aim at production efficiency. So far as I know this is still an open problem. A further interesting problem is to identify circumstances in which

$\underline{w} = (s_1, \ldots, s_T)$ and $(s_{T+1}, \ldots, s_{T+N}) = \mu(p_{T+1},\ldots,p_{T+N})$, where $\mu \neq 1$. It is under these circumstances that a project evaluator can, without error, use a single shadow foreign exchange rate. So far as I know, these circumstances have not been identified.

But even if circumstances are such that the border price rule does not hold for all tradeable commodities it may for some of them and it is convenient to have rules for identifying tradeable goods for which the border price rule holds. These matters have been discussed at length in Little and Mirrlees (1969, 1974), UNIDO Guidelines (1972), Joshi (1972), and Dasgupta and Stiglitz (1974). It is in fact often easier operationally to identify goods for which the border price rule does not hold than it is to identify those for which it does. For example, if an import quota exists on a commodity and if the quota is biting, then clearly the rule does not hold for it.

Professor Hammond catalogues several specific problems which he hopes to tackle in the future. I look forward eagerly to his analysis. But I warn him that if I find subsequently that he claims that I have claimed somewhere that the shadow wage rate of unskilled labour in LDC's is its market wage divided by the square root of two, our close friendship will undergo a mild strain.

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FOOTNOTES

1. Friends occasionally have complained to me that the UNIDO Guidelines possesses too many strands.

2. I am, of course, caricaturing somewhat. There are many telling qualifications in Little-Mirrlees (1974) cautioning the project evaluator. These matters, among others, were discussed at length, though not always illuminatingly, in a symposium on Little-Mirrlees (1969) in the Bulletin of the Oxford University Institute of Economics and Statistics (February 1972).