History as a Widespread Externality in Some Arrow-Debreu Market Games

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Abstract

Two Arrow–Debreu market games are formulated whose straightforward Nash equilibria are Walrasian. Both have an auctioneer setting prices to maximize net sales value. In the second an additional redistributive agency maximizes welfare through optimal lump-sum transfers. In intertemporal economies, however, subgame imperfections can arise because agents understand how current decisions such as those determining investment influence either future prices (with finitely many agents), or future redistribution (even in continuum economies). The latter observation undermines the second efficiency theorem of welfare economics. Indeed, when the state of the economy affects future policy, it functions like a "widespread externality."

History as a Widespread Externality

1. Introduction

Among Kenneth Arrow's many highly significant and widely cited contributions to economic science are his path-breaking papers on the two fundamental efficiency theorems of welfare economics (Arrow, 1951), on the role of securities in the allocation of risk-bearing (Arrow, 1953, 1964), and the joint article with Gérard Debreu (Arrow and Debreu, 1954) on the existence of general competitive or Walrasian equilibrium. There is also the joint monograph by Arrow and Hahn (1971).[1] Of these, the first article used what has since become the standard definition of Walrasian equilibrium, possibly modified by lump-sum redistribution of wealth, and related equilibrium to Pareto efficient allocations. The second article served to define what it means to have a complete set of securities markets in a sequence economy with uncertainty. The third paper proved existence of equilibrium under what have since become almost standard assumptions. And the monograph with Frank Hahn explores not only the main ideas in general competitive analysis, but also much of the progress that had occurred in making it applicable to real economic phenomena.

An important feature of Arrow and Debreu (1954) is its conscious use of explicitly game-theoretic ideas, previously found in the related paper by Debreu (1952) on his own.[2] However, a generalization of the usual notion of a game is involved. For there is an auctioneer whose strategy choice determines the price vector. Given this choice, agents are then constrained to choose net trades within their budget sets. Thus the strategic choice of the auctioneer limits the strategies that the other players are allowed to choose.

[1] According to Intriligator (1987), the first three articles mentioned above had been cited respectively 59, 264 (including references only to the French version that was published first), and 117 times during the period 1966–83; when ranked by their frequency of citation, that made them, respectively, the twenty-sixth, eighth, and fifteenth items in Arrow's list of publications. So, apart from the second article, these are far from being the most frequently cited of Arrow's works during that period: the first ranked item, *Social Choice and Individual Values*, was cited no fewer than 1203 times, while the second ranked, Arrow and Hahn (1971), was cited 561 times during the same period. This last figure may reflect the significance of Arrow's work in general equilibrium theory more accurately than the less frequent citations of the original articles. Indeed, these may have become classics to the extent that they are not frequently cited, just as few modern physicists or mathematicians bother to cite the works of Newton, even though many rely on ideas derived from his work. Finally it should be added that *Essays in the Theory of Risk-Bearing*, which includes a reprint of Arrow (1953, 1964), was also frequently cited.

[2] It should be noted, however, that the (then) forthcoming paper by Arrow and Debreu was cited as motivating the work reported in Debreu (1952).

One minor task accomplished in the present paper is an easy modification of this generalized game so that it becomes a game in the usually accepted sense, with agents free to choose strategies within a specified fixed strategy set that is independent of what other players may choose. The important feature of the Arrow–Debreu market game presented in Section 2 below is that agents are free to make demands that violate their budget constraints. But if they do so, then they will be reduced to autarky. This is evidently a strong enough sanction to make budget constraints self-enforcing. So the game has a set of slightly restricted "straightforward" Nash equilibria in which net trades satisfying the budget constraints are announced by all agents, including even those who have zero net trade in equilibrium and so would have nothing to lose by violating their budget constraint. It is not surprising that the set of all such equilibria corresponds exactly to the set of Walrasian equilibria, just as did the Nash equilibria of the generalized game which Arrow and Debreu themselves considered.

In order to discuss the efficiency theorems of welfare economics, one should consider a redistributive agency as an extra player, whose objective is to maximize some Paretian Bergson social welfare function. Then it is evidently necessary to consider Walrasian equilibria relative to different possible systems of lump-sum wealth redistribution. For those expected to pay lump-sum taxes, the threat of autarky may not suffice to make their budget constraints self-enforcing. Instead of a zero net trade, however, any arbitrary net trade within the budget set will suffice to deter violations of the budget constraint. This is explained in Section 3.

The main purpose of this paper, however, is not to present these trivial and not very exciting results on true games whose straightforward Nash equilibria implement desirable Walrasian equilibria. Rather, it is to point out some problems in applying the standard Arrow–Debreu methodology to intertemporal economic models. The reason is that the games formulated above raise the issue of what happens if markets cannot be prevented from re-opening in later periods. And of what happens if a benevolent welfare maximizing government cannot commit itself in advance not to make transfers at later dates. It will turn out that the Nash equilibria which correspond to Walrasian equilibria can easily be subgame imperfect, in the sense that they require incredible responses by the auctioneer or redistributive agency to trading agents' deviations from the equilibrium path (cf. Selten, 1965, 1973, 1975). Nor do these subgame imperfections result from an artifice of the particular game formulations set out in Sections 2 and 3: exactly the same imperfections would arise in any other game whose Nash equilibrium outcomes were Walrasian equilibrium allocations.

To substantiate these claims, the heart of the paper presents two simple models of an intertemporal economy lasting for two periods. The first of these involves only two agents, one of whom is a mine owner with the power to influence second period prices by choosing how much of a single exhaustible resource stock to extract in the first period, and how much to leave in the ground for later use as an input to production in the second period. The two period market game in which re-opening markets is impossible has the usual Walrasian equilibrium of an Arrow–Debreu economy. But if the mine owner believes that markets cannot be prevented from re-opening in the second period, the only subgame perfect equilibrium will involve higher extraction in the first period, but lower in the second period, since that is the way for the mine owner to exercise monopoly power. Perversely, this manipulation of the market economy makes the mine owner worse off. All this is demonstrated in Section 4 below.

Evidently an economy with a continuum of negligibly small agents will not be subject to such manipulation, even in subgame perfect equilibrium. When a redistributive agency enters the economy, however, individual agents have the power to exploit its benevolence in a dramatic fashion. Indeed, Section 5 considers a counterpart of the model in Section 4, but with a continuum of agents and also a benevolent government. The essentially unique subgame perfect equilibrium requires almost every mining firm to use up its entire resource stock in the first period, leaving nothing to produce output in the second period. The point is that optimal redistribution in the second period implies complete equality so that the mean output from using all resource stocks in production is shared equally by all. Of course, this destroys any incentive for mine owners to save by keeping some of their resource stock for the second period. In fact, redistribution in the second period makes holding back resource stock for the future an activity equivalent to the private provision of a public good, or creation of a "widespread externality" in the sense of Hammond, Kaneko and Wooders (1989), Kaneko and Wooders (1989). Such an externality affects everybody, and depends on the distribution of individual decisions in the population, even though each individual has negligible influence upon its creation. As Section 5 also argues, the only policy remedies which work are those recognizing that there really is a public good problem.

Of course, it could be argued that welfare maximizing lump-sum redistribution is impractical anyway. Indeed, I have elsewhere claimed that it is generally incentive incompatible (Hammond, 1979, 1987), especially when small coalitions can manipulate the economic system by exchanging goods on the side. But as Tesfatsion (1986) was careful to point out, previous analyses of the "time inconsistency" or incredibility of public policy were usually conducted in models where first best optimal policy was infeasible for one reason or another. In her paper and the example of Section 5 below, this is emphatically not the case.

Nevertheless, the main lesson of Section 5 is that, even when it is instruments of more realistic government policy which get affected by the current state or history of the economy, as they surely should be, and if this state is influenced by individuals' decisions, then history becomes like a privately supplied public good or widespread externality. Of course, the typical presumption is that any such externality or public good will be inefficiently supplied. So the fact that government intervention in the economy will be desirable later on provides a rationale for intervention now, in order to treat this public good problem. This is the lesson that appears to remain valid in a wide range of circumstances.

One issue this discussion raises is whether general equilibrium theory has anything interesting to say about public policy in intertemporal economies. And whether anything valid remains from the many apparently interesting analyses offered in the past, especially by macroeconomists. Many macroeconomic models, however, have contained only a single "representative" agent, for whom there can never be any problems with public goods. Indeed, one welcome feature of some recent work on endogenous growth is precisely that, even though it often retains a continuum of identical representative agents, it does nevertheless allow public good problems to arise — though through technological externalities rather than through policy reactions to economic states that are influenced by earlier private decisions.

As for past work by microeconomists on intertemporal general equilibrium theory, surprisingly little of the literature has sought to explain the scope for and influence of public policy. There seem to have been problems enough anyway, including finding a role for money, and understanding why markets are incomplete. Of course, many special models have been used to consider particular issues of intertemporal economic policy — pension and social security systems, the appropriate social rate of discount on public investment projects, the role of the national debt and its significance, the relative merits of income versus expenditure taxes, etc. What remains lacking is any general framework as broad as those that appear in Arrow (1951) and in Arrow and Debreu (1954), as well as in the static analysis of the public sector due to Diamond and Mirrlees (1971). In other words, we lack what is needed for a systematic general treatment of all relevant aspects of intertemporal economic policy, and especially of how different policy instruments interact with each other. Needless to say, this is a serious undertaking that has to be left for later work. Nevertheless, the concluding Section 6 ventures a brief sketch of what the most important ingredients for such a treatment are likely to be. It is claimed that the emphasis should turn away from frameworks such as the famous complete contingent commodity model of Debreu (1959, ch. 7), also discussed in Arrow and Hahn (1971, Section 5.6), which aims to determine the entire future course of economic history all at one go. Instead the framework should be closer to the well known Arrow (1953, 1964) model of a securities market, with the equilibrium allocation being determined sequentially by looking one period ahead within each successive period, or to the Hicksian theory of temporary equilibrium. An early modern discussion of the latter can be found towards the end of the much less well known article Arrow (1971), as well as in Arrow and Hahn (1971, Chapter 6).

Section 7 contains some brief concluding remarks.

2. A First Simple Arrow-Debreu Market Game

2.1. *Defining the Game*

Consider an economy with a finite set of private agents I and a finite set of commodities G, so that $\Re^G$ is the commodity space. Suppose that each agent $i \in I$ has a set $X^i \subset \Re^G$ of feasible net trades, and a utility function $u^i : X^i \to \Re$ representing i's preferences over net trade vectors. Assume that the no trade or autarky vector $0 \in X^i$ for all $i \in I$, and that i's preferences are locally non-satiated — i.e., the utility function u^i has no local maximum. Let $\mathbf{X}^I$ denote the Cartesian product $\prod_{i \in I} X^i$ of the trading agents' feasible sets.

Suppose too that there is an auctioneer, denoted by superscript A, whose role is to clear markets by choosing a normalized commodity price vector p in the simplex

$$\Delta := \{\, p \in \Re^G \mid p_g \geq 0 \ (g = 1 \text{ to } G); \sum\nolimits_{g=1}^{G} p_g = 1 \,\}.$$

The price vector p announced by the auctioneer will help to determine the net trade vector $t^i \in X^i$ that each agent $i \in I$ actually obtains from the economy as a function $t^i(s^i, p)$ of i's strategy choice $s^i \in X^i$, to be thought of as a net trade demand, as well as of p. Specifically, it is assumed that

$$t^i(s^i, p) = \begin{cases} s^i & \text{if } p\, s^i \leq 0; \\ 0 & \text{otherwise.} \end{cases}$$

Of course, this definition ensures that $p\, t^i(s^i, p) \leq 0$ for all $p \in \Delta$ and all $s^i \in X^i$.

Agent i's payoff as a function of the strategy profile $(\mathbf{s}^I, p)$ is accordingly given by $v^i(\mathbf{s}^I, p) := u^i(t^i(s^i, p))$. Thus agents are allowed to have the net trade vectors they each demand as long as these lie within their budget sets, but agents whose demands violate their budget constraints are reduced to autarky.

The specification of the game will now be completed by defining the auctioneer's payoff function as $v^A(\mathbf{s}^I, p) := p \sum_{i \in I} s^i$, which is just the value at prices p of the aggregate net demand vector for all agents $i \in I$. Note that $v^A(\mathbf{s}^I, p) \geq p \sum_{i \in I} t^i(s^i, p)$; a sufficient condition for equality is that all agents $i \in I$ satisfy their budget constraints $p\, s^i \leq 0$.

Finally, therefore, the market game is the collection

$$\langle I \cup \{A\}, \mathbf{X}^I \times \Delta, \langle v^h \rangle_{h \in I \cup \{A\}} \rangle$$

consisting of the set of players, their allowable strategies, and their payoff functions.

2.2. Equivalence of Straightforward Nash and Walrasian Equilibria

Let us now consider some obvious properties of the best response $\hat{s}^i$ in the market game by an agent $i \in I$ to any price vector p chosen by the auctioneer, and of the resulting net trade vector $\hat{t}^i := t^i(\hat{s}^i, p)$. One has $p\, t^i(s^i, p) \leq 0$ and also $u^i(\hat{t}^i) \geq u^i(t^i(s^i, p))$ for all $s^i \in X^i$. This implies that $p\, \hat{t}^i \leq 0$. Also, for all $s^i \in X^i$ that satisfy $p\, s^i \leq 0$ one has $t^i(s^i, p) = s^i$ and so $u^i(\hat{t}^i) \geq u^i(s^i)$. Thus each agent's best response $\hat{s}^i$ yields a net trade vector $\hat{t}^i = t^i(\hat{s}^i, p)$ equal to a Walrasian net trade vector at the price vector p. Because of local non-satiation it must also be true that $p\, \hat{t}^i = 0$.

In fact $\hat{s}^i$ will equal $\hat{t}^i$ except in the special case when $\hat{t}^i = 0$ because autarky happens to be an optimal net trade for agent i at prices p; then $\hat{s}^i$ could be zero or any net demand vector satisfying $p\, \hat{s}^i > 0$. I shall limit attention to "straightforward" best responses, and corresponding Nash equilibria, for which $\hat{s}^i = 0$ in this case.

It is now fairly easy to show that any straightforward Nash equilibrium of this game gives rise to a Walrasian equilibrium of the exchange economy, and *vice versa*. For suppose first that $(\hat{\mathbf{x}}^I, \hat{p})$ is a Walrasian equilibrium of the exchange economy. Since each agent's net demand vector $\hat{x}^i$ must maximize $u^i(x^i)$ subject to $x^i \in X^i$ and $\hat{p}\, x^i \leq 0$, it must be a best response to $\hat{p}$ in the market game. Since $\sum_{i \in I} \hat{x}^i \leqq 0$ while $\hat{p} \sum_{i \in I} \hat{x}^i = 0$, it is also true that $\hat{p} \sum_{i \in I} \hat{x}^i \geq p \sum_{i \in I} \hat{x}^i$ for all $p \in \Delta$. Thus $\hat{p}$ is a best response by the auctioneer to the profile of net demands $\hat{\mathbf{x}}^I$ in the market game.

Conversely, suppose that $(\hat{\mathbf{s}}^I, \hat{p})$ is a straightforward Nash equilibrium of the market game. Then we have already seen that each agent i's strategy $\hat{s}^i$ yields a Walrasian net trade vector $\hat{t}^i := t^i(\hat{s}^i, \hat{p}) = \hat{s}^i$ at the price vector $\hat{p}$, and also that $\hat{p}\, \hat{t}^i = 0$ for all $i \in I$. So, since $\hat{p}$ is the auctioneer's best response to the net demand profile $\hat{\mathbf{s}}^I$, it must be true that $p \sum_{i \in I} \hat{t}^i \leq \hat{p} \sum_{i \in I} \hat{t}^i = 0$ for all $p \in \Delta$. This implies that $\sum_{i \in I} \hat{t}^i \leqq 0$, and also implies the "rule of free goods" according to which any good which is in excess supply in equilibrium must have a zero price. In particular, $(\hat{\mathbf{t}}^I, \hat{p})$ must be a Walrasian equilibrium of the exchange economy.

3. A Market Game with Redistribution

In order that Arrow's (1951) second fundamental efficiency theorem of welfare economics should be valid for general Pareto efficient allocations in economies where not all agents are identical, lump-sum redistribution of wealth must be allowed. Moreover, this second theorem acquires most of its interest when applied to the maximum of any Paretian Bergson social welfare function. Accordingly, a new market game will now be considered, with an additional *redistributive player* R whose strategy is to choose any wealth distribution function $\mathbf{m}^I(\cdot) : \Delta \to \Re^I$ with levels $\langle m^i(p) \rangle_{i \in I}$ large enough to ensure that every agent $i \in I$ has a non-empty budget set $B^i(p) := \{ x^i \in X^i \mid p\, x^i \leq m^i(p) \}$ for all $p \in \Delta$, while also satisfying the obvious restriction that $\sum_{i \in I} m^i(p) = 0$. Note that it is generally impossible to ensure non-emptiness of each budget set $B^i(p)$ unless each m^i is allowed to depend on p in this way.

The auctioneer and the trading agents will function in much the same way as they did in the market game of Section 2. The only difference is that each net trade function $t^i(s^i, p)$ must now be re-defined to include the function $m^i(\cdot)$ as an extra argument, and also to make the new budget constraint $p\, x^i \leq m^i(p)$ become self-enforcing. To this end, for each $i \in I$ and each $p \in \Delta$ let $\bar{x}^i(p)$ be any fixed net trade vector in the non-empty budget set $B^i(p)$, and then let the value of the new function be

$$t^i(s^i, p, m^i(\cdot)) = \begin{cases} s^i & \text{if } p\, s^i \leq m^i(p); \\ \bar{x}^i(p) & \text{otherwise.} \end{cases}$$

The only remaining feature before the new market game

$$\langle I \cup \{ A, R \}, \mathbf{X}^I \times \Delta \times S^R, \langle v^h \rangle_{h \in I \cup \{ A, R \}} \rangle$$

becomes fully specified is the redistributor's payoff function v^R. This is taken to be $v^R := W(\langle u^i(t^i) \rangle_{i \in I})$ for some Paretian Bergson social welfare function W.

Arguing as in Section 2, given any equilibrium wealth distribution rule $\hat{\mathbf{m}}^I(\cdot)$, define a straightforward Nash equilibrium as one in which any agent $i \in I$ who enjoys a net trade vector no better than $\bar{x}^i(p)$, and so has nothing to lose from violating the budget constraint $p\, s^i \leq \hat{m}^i(p)$, in fact chooses to respect this constraint. Then the set of straightforward Nash equilibria of this market game must give allocations that coincide with the set of Walrasian equilibria relative to this rule. Furthermore, suppose that the allocation $\hat{\mathbf{x}}^I$ maximizes the welfare function $W(\langle u^i(x^i)\rangle_{i\in I})$ subject to the feasibility constraints $x^i \in X^i$ (all $i \in I$) and $\sum_{i\in I} x^i \leqq 0$, and that this optimal allocation can be achieved by facing each trader with the budget constraint $\hat{p}\, x^i \leq \hat{p}\, \hat{x}^i$ for a suitable price vector $\hat{p} \in \Delta$ because Arrow's (1951) second efficiency theorem applies. Then at least one straightforward Nash equilibrium of this new market game must yield an outcome of the form $(\hat{\mathbf{x}}^I, \hat{p}, \hat{\mathbf{m}}^I(\cdot))$, where the wealth distribution functions satisfy $\hat{m}^i(p) \equiv p\, \hat{x}^i$ for all $i \in I$ and all $p \in \Delta$. The reason is that trading agents and the auctioneer are all choosing straightforward best responses as in Section 2, while the redistributor cannot possibly do better than reach the equilibrium first-best allocation $\hat{\mathbf{x}}^I$ by adjusting the wealth distribution rule to achieve some other straightforward equilibrium and so some other feasible allocation. On the other hand, in case there are multiple Walrasian equilibria relative to the equilibrium wealth distribution rule $\hat{\mathbf{m}}^I(\cdot)$, not every straightforward Nash equilibrium need be a welfare optimum which maximizes W. This is because, as Samuelson (1974) and Bryant (1994) point out, the trading agents and the auctioneer could together steer the economy to a suboptimal equilibrium even after an optimal wealth distribution rule has been set up. Of course, this alternative equilibrium must be Pareto efficient, but the distribution of real wealth could still be suboptimal.

4. A Simple Economy with an Exhaustible Resource

4.1. The Model

Consider a simple economy which lasts for two periods ($t = 1, 2$) and in which there are two agents. There are also three goods — one consumption good, the stock of a single exhaustible resource, and labour. The first agent is a working miner who consumes c_t units of the consumption good and has an inelastic supply of n_t units of labour in each period t. This worker's preferences for two period consumption streams are assumed to be represented by the utility function

$$u(c_1, c_2) \equiv \ln c_1 + \ln c_2.$$

The second agent is the owner and manager of the mine, who uses the worker's labour in order to mine the resource whose initial stock is S_0 at the beginning of period 1. In each of the two periods, the quantity e_t of this resource is combined with ℓ_t units of labour in order to produce y_t units of output of the single consumption good according to the constant returns to scale Cobb–Douglas production function

$$y_t = e_t^{\gamma}\, \ell_t^{1-\gamma},$$

where $0 < \gamma < 1$. The mine owner's consumption or dividend in period t is denoted by d_t, and his preferences for two period consumption streams are assumed to be represented by the same utility function

$$v(d_1, d_2) \equiv \ln d_1 + \ln d_2.$$

4.2. Intertemporal Walrasian Equilibrium

The usual two period intertemporal Walrasian equilibrium in this economy occurs when an "auctioneer" at the beginning of the first period sets prices for both periods in order to clear all markets, knowing what the mine worker and mine owner will later supply and demand for each possible price system which the auctioneer might choose. This Walrasian equilibrium can be found by choosing a Pareto efficient allocation which maximizes the welfare weighted sum

$$\alpha\, u(c_1, c_2) + (1-\alpha)\, v(d_1, d_2)$$

for some suitable value of α satisfying $0 \leq \alpha \leq 1$. The relevant feasibility constraints are

$$c_t + d_t \leq y_t = e_t^{\gamma}\, \ell_t^{1-\gamma} \quad \text{and} \quad \ell_t \leq n_t \quad (t = 1, 2); \quad e_1 + e_2 \leq S_0.$$

Introducing non-negative Lagrange multipliers p_t for the shadow price of the consumption good and w_t for that of labour in period t ($t = 1,\, 2$), as well as ρ for the shadow price of the resource stock, the relevant Lagrangean becomes

$$\mathcal{L} \equiv \sum\nolimits_{t=1}^{2} \left[\alpha \,\ln c_t + (1-\alpha)\, \ln d_t - p_t \left(c_t + d_t - e_t^{\gamma}\, \ell_t^{1-\gamma}\right) - w_t\, (\ell_t - n_t)\right]$$
$$- \rho\, (e_1 + e_2 - S_0).$$

So the first order conditions for an optimum are the six equations

$$\frac{\alpha}{c_t} = \frac{1-\alpha}{d_t} = p_t; \quad p_t\, (1-\gamma)\, e_t^{\gamma}\, \ell_t^{-\gamma} = w_t; \quad p_t\, \gamma\, e_t^{\gamma-1}\, \ell_t^{1-\gamma} = \rho \quad (t = 1, 2)$$

together with equality versions of the five inequality constraints that define a feasible allocation. These first order conditions imply that

$$p_t\, y_t = p_t\, (c_t + d_t) = \alpha + (1 - \alpha) = 1$$

and also that

$$w_t\, n_t = w_t\, \ell_t = (1 - \gamma)\, p_t\, y_t = (1 - \gamma); \quad \rho\, e_t = \gamma\, p_t\, y_t = \gamma,$$

for each of the two periods $t = 1, 2$. The last equation implies that $e_1 = e_2$. The optimal allocation is therefore

$$e_t = \tfrac{1}{2}\, S_0; \quad y_t = (\tfrac{1}{2}\, S_0)^\gamma n_t^{1-\gamma}; \quad c_t = \alpha\, y_t; \quad d_t = (1 - \alpha)\, y_t$$

for each of the two periods $t = 1, 2$. The associated shadow prices are

$$p_t = 1/y_t; \quad w_t = (1 - \gamma)/n_t; \quad \rho = 2\,\gamma/S_0.$$

In order to achieve a Walrasian equilibrium, the welfare weight α should be chosen so that both agents just satisfy their respective budget constraints. By Walras' Law, it is enough to look at either agent on his own. In fact it is easier to consider the mine worker, whose Walrasian budget constraint

$$p_1\, c_1 + p_2\, c_2 = w_1\, n_1 + w_2\, n_2$$

is exactly satisfied in the case when $\alpha = 1 - \gamma$. The intertemporal Walrasian equilibrium therefore consists of the production plan and the price system given above, together with the consumption allocation

$$c_t = (1 - \gamma)\, y_t; \quad d_t = \gamma\, y_t$$

for each of the two periods $t = 1, 2$. Note that $p_t\, c_t = w_t\, n_t$ in each of the two periods separately. This implies that the mine worker neither saves nor borrows during the first period. Because of aggregate budget balance, the mine owner does not save or borrow either, so that there is no need for an asset market of any kind. The only form of saving in the economy comes from the mine owner deciding not to exhaust his stock of the resource all in one period. Finally, note for future reference that the mine owner's utility in this Walrasian equilibrium is given by

$$v^W := 2\, \ln [\gamma(\tfrac{1}{2}\, S_0)^\gamma] + (1 - \gamma)\, \ln(n_1\, n_2).$$

4.3. Subgame Perfect Equilibrium

The above intertemporal Walrasian equilibrium was calculated on the presumption that the "auctioneer" in the first period makes irreversible decisions regarding prices for both periods. It is as though his plans were being submitted to the "umpire" whom von Neumann and Morgenstern (1953) exploited as a device to help understand the normal rather than the extensive form of a game. Their idea was that, in the normal form of a game, the umpire carries out the players' strategies on their behalf. Then there is no opportunity for changing plans part way through the extensive form of a game, so subgame perfection is never an issue. In fact von Neumann and Morgenstern claimed that considering only the normal form would never lose any generality. Yet the work by Selten (1965, 1973, 1975) and many successors, on subgame perfect equilibria in extensive form games that are not two-person zero-sum, has made this claim completely untenable — at least for Nash equilibria, which almost everybody now agrees need to be refined.[3]

So, in the Walrasian equilibrium model of an intertemporal economy, we must ask whether the auctioneer's initially planned prices will actually emerge later on, if such prices are not irreversible commitments. In fact, for the model just considered, it turns out that they will not, in case the mine owner chooses to leave a stock of the resource which differs from the intertemporal equilibrium level $S_1 = \frac{1}{2}S_0$ at the end of the first period. And if the mine owner foresees this, he will want to exploit his power to affect market prices.

Indeed, consider a slightly different version of the above intertemporal economy in which there are:

(a) first period markets for consumption, labour, and an Arrow security for delivering consumption in the second period;

(b) second period markets for consumption and labour.

Suppose that trade takes place in an extensive game of complete information in which the players are the auctioneer, the mine owner, and the mine worker, and the order of moves is as follows:

(i) at the beginning of the first period the auctioneer sets prices for consumption and labour in the first period, and for the Arrow security, in order to clear markets in full

[3] Of course Arrow and Debreu wrote when Nash equilibrium was still a novel concept, long before Selten's fundamental work, and even longer before that work had its deserved impact.

knowledge of what the mine owner and mine worker will supply and demand at each possible price vector which he might set;

(ii) the mine owner and mine worker exchange contracts for first period labour supply, first period consumption, and the Arrow security, taking the auctioneer's price system as given, and then execute the first period part of those contracts — that is, the mine owner extracts as much as he wishes of his stock of the resource, hires what he wants of the mine worker's labour up to the limit of the worker's supply, and then produces output of which some is sold to the mine worker and the rest is consumed by the mine owner — in addition, Arrow securities may be traded;

(iii) at the beginning of the second period, and fully aware of what has happened in the first period, the auctioneer sets prices for consumption and labour in the second period in order to clear markets after the Arrow security contracts have been honoured, in full knowledge of what the mine owner and mine worker will supply and demand at each possible price vector which he might set;

(iv) the mine owner and mine worker exchange contracts for second period labour supply and consumption, taking both the auctioneer's price system and the need to honour the Arrow security contracts as given, and then execute those contracts.

Now, at the beginning of the second period of a two period economy, suppose that the mine owner's resource stock is S_1. Suppose too that any Arrow security transactions during the first period have the effect of adding a units net of conumption to the worker, and b to the mine owner. Of course, it must be true that $a + b = 0$ in order to have the Arrow security market clear in the first period. Then any Walrasian equilibrium in the second period involves the auctioneer setting the real wage w_2/p_2 equal to $(1-\gamma)\, S_1^{\gamma}\, n_2^{-\gamma}$, which is the marginal product of labour. From this and the worker's budget constraint $p_2\, c_2 = w_2\, n_2 + p_2\, a$, it follows that

$$p_2\,(c_2 - a) = w_2\, n_2 = p_2\,(1-\gamma)\, S_1^{\gamma}\, n_2^{1-\gamma} = (1-\gamma)\, p_2\, y_2.$$

So the unique Walrasian equilibrium for the second period alone consists of the allocation

$$c_2 = (1-\gamma)\, S_1^{\gamma}\, n_2^{1-\gamma} + a = (1-\gamma)\, y_2 + a; \quad d_2 = \gamma\, S_1^{\gamma}\, n_2^{1-\gamma} + b = \gamma\, y_2 + b$$

and the price system defined by the real wage given above. Note how the equilibrium allocation is a function of a, b, and S_1.

Suppose that the worker and mine owner both understand this dependence, and use their power to manipulate prices in the second period by means of suitable actions in the first period. Then the worker will choose the variables c_1 and a in order to maximize his anticipated utility, which is

$$\ln c_1 + \ln \left[(1-\gamma)\,(S_0 - e_1)^\gamma \, n_2^{1-\gamma} + a\right] \equiv \ln c_1 + \ln \left[(1-\gamma)\, y_2 + a\right],$$

subject to the budget constraint

$$p_1\, c_1 + r\, a = w_1\, n_1.$$

Here r denotes the price of the Arrow security.

Using λ to denote the shadow price associated with the worker's budget constraint, the first order conditions for utility maximization become

$$\frac{1}{c_1} = \lambda\, p_1; \qquad \frac{1}{(1-\gamma)\,(S_0 - e_1)^\gamma \, n_2^{1-\gamma} + a} = \frac{1}{(1-\gamma)\, y_2 + a} = \lambda\, r.$$

After multiplying each side of the budget constraint by λ, while recognizing that it must hold with equality, and then substituting from the above first order conditions, we obtain

$$\lambda\, w_1\, n_1 = \lambda\,(p_1\, c_1 + r\, a) = 2 - \lambda\,(1-\gamma)\, r\, y_2.$$

This implies that $1/\lambda = \frac{1}{2}\,[w_1\, n_1 + (1-\gamma)\, r\, y_2]$. So substituting for $1/\lambda$ back in the first order conditions gives the worker's expenditures

$$p_1\, c_1 = \tfrac{1}{2}\,[w_1\, n_1 + (1-\gamma)\, r\, y_2]\,; \quad r\, a = \tfrac{1}{2}\,[w_1\, n_1 - (1-\gamma)\, r\, y_2]$$

on first period consumption and the Arrow security. These are functions of the first period price vector (p_1, w_1, r), and also of future output y_2 which depends on the mine owner's choice of e_1. Notice that expenditure on present and on future consumption is determined as in the linear expenditure system, but with $(1-\gamma)\, r\, y_2$ already committed to expenditure on second period consumption, in effect, and with $w_1\, n_1$ as exogenous wage income.

On the other hand, the mine owner will choose the variables d_1, e_1, ℓ_1 and b in order to maximize his anticipated utility, which is

$$\ln d_1 + \ln\left[\gamma\,(S_0 - e_1)^\gamma\, n_2^{1-\gamma} + b\right] \equiv \ln d_1 + \ln(\gamma\, y_2 + b),$$

subject to the budget constraint

$$p_1\, d_1 + r\, b = p_1\, e_1^\gamma\, \ell_1^{1-\gamma} - w_1\, \ell_1 = p_1\, y_1 - w_1\, \ell_1.$$

Using μ to denote the shadow price associated with the mine owner's budget constraint, the first order conditions for utility maximization become

$$\frac{1}{d_1} = \mu\, p_1; \quad \frac{1}{\gamma\,(S_0 - e_1)^\gamma\, n_2^{1-\gamma} + b} = \frac{1}{\gamma\, y_2 + b} = \mu\, r;$$

and

$$w_1 = p_1\,(1-\gamma)\, e_1^\gamma\, \ell_1^{-\gamma} = (1-\gamma)\, p_1\, y_1/\ell_1;$$
$$\frac{\gamma^2\,(S_0 - e_1)^{\gamma-1}\, n_2^{1-\gamma}}{\gamma\,(S_0 - e_1)^\gamma\, n_2^{1-\gamma} + b} = \mu\, p_1\, \gamma\, e_1^{\gamma-1}\, \ell_1^{1-\gamma}.$$

After multiplying each side of the budget constraint by μ, while recognizing that it must hold with equality, and then substituting from the above first order conditions, we obtain

$$\mu\,(p_1\, y_1 - w_1\, \ell_1) = \mu\,(p_1\, d_1 + r\, b) = 2 - \mu\, \gamma\, r\, y_2.$$

This implies that $1/\mu = \frac{1}{2}(p_1\, y_1 - w_1\, \ell_1 + \gamma\, r\, y_2)$. So substituting for $1/\mu$ back in the first order conditions gives the mine owner's conditional expenditures

$$p_1\, d_1 = \tfrac{1}{2}(p_1\, y_1 - w_1\, \ell_1 + \gamma\, r\, y_2); \quad r\, b = \tfrac{1}{2}(p_1\, y_1 - w_1\, \ell_1 - \gamma\, r\, y_2)$$

on consumption and the Arrow security. These are functions of the first period price vector (p_1, w_1, r), of the labour input ℓ_1, and also of the output stream y_1, y_2 which depends on the mine owner's choice of e_1. As with the mine worker, these functions represent an instance of the linear expenditure system.

Now we impose the Arrow security market clearing condition $a + b = 0$. It implies that

$$0 = 2\, r\,(a + b) = w_1\, n_1 - (1-\gamma)\, r\, y_2 + p_1\, y_1 - w_1\, \ell_1 - \gamma\, r\, y_2 = p_1\, y_1 - r\, y_2$$

where the last equality comes from using the market clearing condition $\ell_1 = n_1$ in order to simplify. Because $w_1 = p_1 (1 - \gamma) y_1 / n_1$, it follows that equilibrium price ratios are determined by the two equations

$$w_1 n_1 = (1 - \gamma) p_1 y_1 = (1 - \gamma) r y_2.$$

This, however, implies that in any equilibrium the conditional demands are

$$c_1 = (1 - \gamma) y_1, \quad a = 0; \quad \text{and} \quad d_1 = \gamma y_1, \quad b = 0.$$

Thus asset markets play no rôle whatsoever in the equilibrium allocation.

Note too that

$$1/\mu = \tfrac{1}{2}(p_1 y_1 - w_1 \ell_1 + \gamma r y_2) = \tfrac{1}{2}[p_1 y_1 - (1 - \gamma) p_1 y_1 + \gamma p_1 y_1] = \gamma p_1 y_1.$$

So, because $b = 0$, the first order condition for the mine owner's optimal choice of e_1 simplifies to

$$\frac{\gamma}{S_0 - e_1} = \mu p_1 \gamma e_1^{\gamma - 1} \ell_1^{1 - \gamma} = \frac{\mu p_1 \gamma y_1}{e_1} = \frac{1}{e_1},$$

which implies that $e_1 = S_0/(1 + \gamma)$. This contrasts with the rate of extraction $e_1 = \frac{1}{2} S_0$ which, as was shown previously, is required for any intertemporally Pareto efficient allocation. In fact, since $0 < \gamma < 1$, the mine owner who tries to exploit his monopoly power always uses an amount of the resource stock which, from the point of view of intertemporal Pareto efficiency, is too much in the first period, and too little in the second.

To complete the description of this new subgame perfect equilibrium in which the mine owner exploits his market power, notice that

$$d_1 = \gamma y_1 = \gamma e_1^{\gamma} \ell_1^{1 - \gamma} = \gamma \left(\frac{S_0}{1 + \gamma} \right)^{\gamma} n_1^{1 - \gamma}$$

and

$$c_1 = (1 - \gamma) y_1 = (1 - \gamma) \left(\frac{S_0}{1 + \gamma} \right)^{\gamma} n_1^{1 - \gamma}.$$

The corresponding equilibrium real wage is then

$$\frac{w_1}{p_1} = \frac{c_1}{n_1} = (1 - \gamma) \left(\frac{S_0}{1 + \gamma} \right)^{\gamma} n_1^{-\gamma}.$$

So the remaining stock of the exhaustible resource at the beginning of the second period is

$$S_1 = S_0 - e_1 = S_0 - \frac{S_0}{1+\gamma} = \frac{\gamma\, S_0}{1+\gamma}.$$

The resulting equilibrium allocation in the second period is therefore

$$c_2 = (1-\gamma)\, y_2 = (1-\gamma)\, S_1^\gamma\, \ell_2^{1-\gamma} = (1-\gamma) \left(\frac{\gamma\, S_0}{1+\gamma}\right)^\gamma n_2^{1-\gamma};$$
$$d_2 = \gamma\, y_2 = \gamma \left(\frac{\gamma\, S_0}{1+\gamma}\right)^\gamma n_2^{1-\gamma}.$$

Finally, the mine owner's utility from this subgame perfect equilibrium is

$$\begin{aligned} v^P := \ln d_1 + \ln d_2 &= \ln\left[\gamma \left(\frac{S_0}{1+\gamma}\right)^\gamma n_1^{1-\gamma}\right] + \ln\left[\gamma \left(\frac{\gamma\, S_0}{1+\gamma}\right)^\gamma n_2^{1-\gamma}\right] \\ &= 2\,\ln\left[\gamma \left(\frac{S_0}{1+\gamma}\right)^\gamma\right] + \ln\left(n_1^{1-\gamma}\, \gamma^\gamma\, n_2^{1-\gamma}\right). \end{aligned}$$

Notice therefore that the mine owner's utility net gain from this subgame perfect equilibium with monopoly power, $v^P - v^W$, is given by

$$\begin{aligned} &2\,\ln\left[\gamma \left(\frac{S_0}{1+\gamma}\right)^\gamma\right] + \ln\left(n_1^{1-\gamma}\, \gamma^\gamma\, n_2^{1-\gamma}\right) - 2\,\ln\left[\gamma\,(\tfrac{1}{2}\, S_0)^\gamma\right] - (1-\gamma)\,\ln(n_1\, n_2) \\ &= \; -2\,\gamma\,\ln(1+\gamma) + \gamma\,\ln\gamma + 2\,\gamma\,\ln 2 = \gamma\,\ln\left[\frac{4\,\gamma}{(1+\gamma)^2}\right] \\ &= \gamma\,\ln\left[1 - \frac{(1-\gamma)^2}{(1+\gamma)^2}\right] < 0. \end{aligned}$$

So the mine owner always loses in the end by being manipulative. At first this seems surprising, because one might think that the mine owner can always revert to the non-monopolistic intertemporal Walrasian equilibrium simply by selecting the rate of exploitation of the natural resource $e_1 = \frac{1}{2}\, S_0$ which is appropriate for that equilibrium. This is false, however, for the interesting reason that, in the first period of this subgame perfect monopolistic equilibrium, the auctioneer foresees the mine owner's manipulation and so sets prices in a way which denies him the intertemporal Walrasian equilibrium stream of resource rents and so of consumption.

4.4. *Continuum Economies*

This fully worked out example is intended only to illustrate what is perhaps anyway obvious: once agents foresee how market clearing prices in the future depend on their current actions, they will wish to exploit their monopoly power over such future prices. This problem does not arise, however, in continuum economies with each agent having negligible power to influence future prices. So the next section of this paper will be concerned with such economies. It will be shown how redistributive policies of the kind presumed in the second efficiency theorem of welfare economics can nevertheless create new subgame perfection problems of their own.

5. Redistribution in a Simple Continuum Economy

5.1. *An Intertemporal Welfare Optimum*

So consider now an economy with a continuum of identical workers and another continuum of identical mine owners. Suppose that each worker and each mine owner are identical to the worker and to the mine owner, respectively, in the two agent economy of Section 4 above. Suppose that the proportions of workers and of mine owners in the economy are μ and $1-\mu$ respectively. Suppose too that the government of this economy chooses a symmetric allocation to all the workers and mine owners which maximizes the appropriately weighted sum

$$W := \mu\,(\ln c_1 + \ln c_2) + (1-\mu)\,(\ln d_1 + \ln d_2)$$

of the utilities of the typical worker and mine owner. The physical feasibility constraints are that

$$\mu\, c_t + (1-\mu)\, d_t \leq (1-\mu)\, y_t = (1-\mu)\, e_t^{\gamma}\, \ell_t^{1-\gamma}; \quad \ell_t \leq n_t$$

for both time periods $t = 1, 2$, and that

$$e_1 + e_2 \leq S_0.$$

With shadow prices p_t for consumption, w_t for labour, and ρ for the exhaustible resource, the appropriate Lagrangean can be written as

$$\mathcal{L} = \sum_{t=1}^{2} \{\mu\, \ln c_t + (1-\mu)\, \ln d_t - p_t\,[\mu\, c_t + (1-\mu)\, d_t - (1-\mu)\, e_t^{\gamma}\, \ell_t^{1-\gamma}] - w_t\,(\ell_t - n_t)\} - \rho\,(e_1 + e_2 - S_0).$$

So the first order conditions for an optimum include, for $t = 1, 2$, the equations

$$\frac{1}{c_t} = p_t = \frac{1}{d_t}; \quad (1-\mu)\, p_t \, \frac{(1-\gamma)\, y_t}{\ell_t} = w_t; \quad (1-\mu)\, p_t \, \frac{\gamma\, y_t}{e_t} = \rho$$

Together with the resource constraint $\mu\, c_t + (1-\mu)\, d_t = (1-\mu)\, y_t$, the first two of these equations imply that

$$(1-\mu)\, p_t\, y_t = \mu\, p_t\, c_t + (1-\mu)\, p_t\, d_t = \mu + (1-\mu) = 1.$$

So, since it must be true that $\ell_t = n_t$ for any optimum, we also have

$$w_t\, n_t = 1 - \gamma; \quad \rho\, e_t = \gamma.$$

From this and the constraints it follows that the welfare optimal allocation is given by

$$e_1 = e_2 = \tfrac{1}{2}\, S_0; \quad y_t = (\tfrac{1}{2}\, S_0)^\gamma\, n_t^{1-\gamma}; \quad c_t = d_t = (1-\mu)\, y_t.$$

This involves a perfectly egalitarian allocation of consumption. Moreover, half the stock of the resource in every mine is used up in each of the two periods, as in Section 4.1.

Finally, this optimal allocation can be decentralized by means of markets in which suitably normalized prices for consumption and labour in each of the two periods are the Lagrange multipliers $p_t = 1/(1-\mu)\, y_t$ and $w_t = (1-\gamma)/n_t$ which were found above.

5.2. Subgame Perfect Policy

This optimum effectively presumes that the government is commited to its redistribution policy in advance, and that each mine owner also commits himself to keep half his resource stock until the second period. If such commitments are impossible, however, a subgame imperfection arises. For suppose that the government bases its second period optimal redistributive policy upon what the mine owners have already chosen in the first period. In this economy, what consumption was in the first period is irrelevant to the second period decisions. Indeed, the optimal second period policy involves maximizing the second period welfare integral

$$W_2 := \mu \int \ln c_2 + (1-\mu) \int \ln d_2$$

subject to the resource constraints

$$y_2 \leq (1-\mu)\, e_2^\gamma\, \ell_2^{1-\gamma}; \quad e_2 \leq S_1$$

for each mine separately, and subject to the overall resource constraints

$$\mu\,\bar{c}_2 + (1-\mu)\,\bar{d}_2 \leq (1-\mu)\,\bar{y}_2; \quad (1-\mu)\,\bar{\ell}_2 \leq \mu\, n_2.$$

Here $\bar{c}_2$ denotes the mean level of c_2 among the population of mine workers, while $\bar{d}_2$, $\bar{y}_2$, and $\bar{\ell}_2$ denote the mean levels of d_2, y_2, and ℓ_2 respectively among the population of mine owners. Note that asymmetric allocations are now being considered, as is appropriate when issues of subgame perfection arise. Since all mine workers have identical exogenous supplies of labour, there is no need to write $\bar{n}_2$.

Obviously it is optimal for all mine workers and all mine owners to have the same consumption levels $c_2 = d_2 = \bar{c}_2 = \bar{d}_2$. Also, an optimal allocation of labour is one that maximizes mean output $\int e_2^\gamma\, \ell_2^{1-\gamma}$ per mine by equalizing labour's marginal product $(1-\gamma)\, e_2^\gamma\, \ell_2^{-\gamma}$ in each mine. Because $e_2 = S_1$ is required in (almost) every mine for mean output to be maximized, for this special case labour should therefore be supplied to each mine in proportion to its available resource stock S_1. This implies that

$$\ell_2 = \mu\, S_1\, n_2/(1-\mu)\,\bar{S}_1 = S_1\,\bar{\ell}_2/\bar{S}_1,$$

where $\bar{S}_1$ denotes the mean level of resource stock per mine, and $\bar{\ell}_2$ is the mean labour supply per mine, which must equal $\mu\, n_2/(1-\mu)$. So optimal output in each mine is

$$y_2 = S_1^\gamma\,[S_1\,\bar{\ell}_2/\bar{S}_1]^{1-\gamma} = S_1\,(\bar{\ell}_2/\bar{S}_1)^{1-\gamma},$$

which implies that optimal mean output per mine is $\bar{y}_2 = \bar{S}_1^\gamma\,\bar{\ell}_2^{1-\gamma}$.

The welfare optimal distribution of this second period mean output is then given by

$$c_2 = \bar{c}_2 = d_2 = \bar{d}_2 = (1-\mu)\,\bar{y}_2$$

for each mine worker and each mine owner respectively.

Note in particular how the allocation of consumption to each mine owner in the second period is completely independent of how much resource stock he has available that period. Not surprisingly therefore, each mine owner has every incentive to exhaust his resource stock entirely in the first period, in case markets are used in an attempt to decentralize the intertemporal optimal allocation. For with d_2 fixed, each mine owner chooses ℓ_1 and

e_1 in the first period in order to maximize current consumption d_1 subject to the budget constraint

$$p_1\, d_1 \leq p_1\, y_1 - w_1\, \ell_1 = p_1\, e_1^\gamma\, \ell_1^{1-\gamma} - w_1\, \ell_1$$

and the resource constraint $e_1 \leq S_0$. The mine owner's optimum clearly involves having $e_1 = S_0$. After all, by exhausting his stock of the resource immediately, each mine owner can maximize his first period resource rents without forfeiting any second period consumption. So, if mine owners correctly foresee the optimal second period redistributive policy of the government, then it is a dominant strategy for them all to exhaust their resource stocks in the first period. This would imply that output must fall to zero in the second period, yielding minus infinite utility for all agents.

Indeed, even if markets are not used to arrange net trade vectors in the first period, in many cases it is still a dominant strategy for each mine owner to exhaust his resource stock immediately. For instance, suppose that the prescribed net trade to some mine owner consisted of a net payment of t_1 units of consumption in exchange for receiving $\ell_1 (> 0)$ units of labour — i.e., the net trade vector is $(-t_1, \ell_1)$. Then, by using e_1 units of the resource stock in the first period, the mine owner's consumption stream would become

$$(d_1, d_2) = (e_1^\gamma\, \ell_1^{1-\gamma} - t_1, \bar{d}_2),$$

where $\bar{d}_2$ is independent of his personal decisions. The corresponding utility is

$$v(d_1, d_2) = \ln d_1 + \ln \bar{d}_2 = \ln(e_1^\gamma\, \ell_1^{1-\gamma} - t_1) + \ln \bar{d}_2$$

which is clearly maximized by choosing $e_1 = S_0$. So this is indeed the dominant strategy of each mine owner — or, to be more accurate, of each mine owner who is allowed to use a positive amount of labour in the first period.

5.3. A Market for the Resource Stock

Since resource stocks are being excessively exploited, a standard economists' remedy might be to create a market for them, just as it is often thought that excessive exploitation of fishery stocks or of a common could be overcome through making those resources become private property. Yet in fact such a policy is of no help here, and would actually create chaos even in the first period. This is because every agent would want to take an indefinitely large short position in the market for the resource stock during the first period, knowing that this would have no bearing on the allocation of consumption in the second period. Indeed, suppose that in the first period there were perfect markets for consumption, labour supply, and the resource stock, with prices p_1, w_1, and r_1 respectively. Let S_1^O and S_1^W denote the net holdings of the resource stock at the end of the first period by the typical mine owner and the typical mine worker respectively.

In this first period, each mine worker will choose his planned consumption stream c_1, c_2, and his position S_1^W in the resource market in order to maximize utility $u(c_1, c_2) \equiv \ln c_1 + \ln c_2$ subject to the budget constraint

$$p_1 \, c_1 + r_1 \, S_1^W \leq w_1 \, n_1 .$$

The mine worker also faces the constraint that $c_2 = \bar{c}_2$, completely independent of what he chooses in the first period, because of the government's second period redistribution policy. Then there is no optimal policy in the first period unless $r_1 = 0$. For, since it must be true that $p_1 > 0$, the mine worker can make c_1 arbitrarily large by having $S_1^W \to -\infty$ if $r_1 > 0$, and $S_1^W \to +\infty$ if $r_1 < 0$.

The position of each mine owner is little different. Since second period consumption $d_2 = \bar{d}_2$ is not influenced at all by his decisions, he will choose d_1, e_1, ℓ_1, y_1, and his position S_1^O in the resource market in order to maximize utility $v(d_1, d_2) \equiv \ln d_1 + \ln \bar{d}_2$ subject to the production constraint $y_1 \leq e_1^\gamma \, \ell_1^{1-\gamma}$, the resource constraint $e_1 \leq S_0$, and finally the first period budget constraint

$$p_1 \, d_1 + r_1 \, S_1^O \leq y_1 \; - w_1 \, \ell_1 + r_1 \, (S_0 - e_1).$$

Once again there is no optimal policy unless $r_1 = 0$. For, since it must be true that $p_1 > 0$, the typical mine owner can make d_1 arbitrarily large by having $S_1^O \to -\infty$ if $r_1 > 0$, and

$S_1^O \to +\infty$ if $r_1 < 0$.

So only if $r_1 = 0$ can there be equilibrium in the resource market. But then, of course, the resource market has no role to play. This implies that we are back with the same equilibrium as before, with each mine owner choosing $e_1 = S_0$ and so exhausting the resource stock in just the first period.

5.4. Remedial Policy

This disastrous outcome of total immediate exhaustion of the resource stock arises because optimal redistributive policy in the second period has the effect of converting output in the second period, and so the resource stock at the end of the first period, into a kind of privately provided public good or widespread externality. No matter how much resource stock any mine owner retains, his consumption in the second period is always the same. This encourages mine owners to "ride free" by exhausting all their stocks in the first period, in order to enjoy the largest possible resource rent.

Recognizing that a public good or externality problem has arisen suggests various kinds of policy remedy. Obviously direct controls like rationing could work in principle, with each mine owner being allowed to use no more than half his resource stock in the first period. Alternatively, one could subsidize resource retention or penalize excessive resource use. This could involve a subsidy s being imposed on each unit of resource saving, with a corresponding tax rate s on each unit of excessive resource use. Specifically, each mine owner is faced with the new first period budget constraint

$$p_1\, d_1 \leq p_1\, e_1^{\gamma}\, \ell_1^{1-\gamma} - w_1\, \ell_1 + s\, (\tfrac{1}{2} S_0 - e_1)$$

as well as the earlier resource constraint $e_1 \leq S_0$. The subsidy rate s will be set so that each mine owner wants e_1 to be at its efficient level of $\frac{1}{2} S_0$ when maximizing d_1 subject to this budget constraint. Bearing in mind that $p_1 = 1/(1-\mu)\, y_1$ is the price of output and consumption in the first period which sustains an intertemporal optimum, this requires that

$$s = p_1\, \gamma\, e_1^{\gamma-1}\, n_1^{1-\gamma} = \gamma\, p_1\, y_1/e_1 = \gamma/(1-\mu)\, y_1 = 2\, \gamma/(1-\mu)\, S_0.$$

In effect, this amounts to having the government buy out the right to use each mine in the first period for the sum of $s \cdot \frac{1}{2} S_0 = \gamma\,/(1-\mu)$, which could be regarded as the value of that half of its total stock which will be depleted during the first period. Since in each period mean resource rents per head of population are $\rho\, e_t = \gamma$, this also represents each mine owner's share of these rents. Thereafter each mine owner is allowed to lease back half his mine in the first period in exchange for royalty payments at the rate s on each unit of resource depletion.

It would not do, however, to have the government pay for the whole resource stock of each mine, with a view to allowing each mine owner to save some of the proceeds in order to finance his own second period consumption. This would not work because the government's optimal second period redistribution policy always ignores completely any asset holdings held at the start of that period. So if there were some kind of perfect capital market on which mine owners could save for future consumption, each of them would like not only to spend immediately all the compensation paid by the government for using the mine, but also to borrow indefinitely large sums in the first period knowing that these would be repaid, in effect, by the government's optimal redistribution program in the second period. That is why, in the first period, the government should buy the right to use the resource stock for the first period only. Trying to set up any kind of normal capital markets of the kind which economists have become accustomed to advocating would be nothing short of disastrous in this model.

6. Sequential Economic Policy Analysis

6.1. Subgame Imperfections

While formally correct, the fundamental efficiency theorems of welfare economics do not explicitly consider the subgame perfection of a competitive market mechanism in an intertemporal setting. With a small number of agents in a static economy, one can imagine reaching competitive equilibrium through a game in which there is an omniscient auctioneer who sets prices that clear all markets, and by allowing agents to trade as they wish at these prices. In an intertemporal economy, however, unless there are many agents, or unless there is complete separation between the economies in different periods, some individuals will be able to manipulate equilibrium prices in the "subeconomies" which begin in later periods. As the example of Section 4 shows, they can do this by adjusting their previous investment plans or any other earlier economic decisions which affect preferences or feasible sets in the subeconomy. Thus, unlike for static economies, there are conceptual difficulties in considering competitive market economies unless there is a continuum of agents whose investment and other decisions have a negligible effect on later prices.

It is not really surprising that competition requires many agents. Rather, the surprise is that, in a single period economy, it may be compatible with having only a few agents. More deeply troubling, however, is the subgame imperfection that accompanies lump-sum redistribution even in continuum economies. This is similar to a phenomenon noticed by Tesfatsion (1986) in particular, though related to earlier work by Kydland and Prescott (1977, 1980), Calvo (1978), Fischer (1980), and many others on the time inconsistency of macroeconomic policy. In a static economy, and under the usual conditions ensuring the validity of Arrow's second efficiency theorem of welfare economics, a benevolent government that wishes to maximize some Paretian Bergson social welfare function can do so by instituting complete perfectly competitive markets along with an optimal redistribution of income by means of lump-sum transfers. With many periods, however, such optimal redistribution in any subeconomy will typically depend upon the past decisions of economic agents. If the benevolent welfare objective is sufficiently egalitarian, for instance, it will mandate lump-sum taxes on those who have saved a lot in the past in order to finance transfers to those who have left themselves with little or no wealth. Agents who understand this dependence will see that their incentives to save, to invest, or to create future wealth for themselves are all seriously blunted by the egalitarian redistributive policies which the government will pursue later. The example of Section 5 illustrated this.

Of course, one may well object that the redistributive policy contemplated in that example is clearly absurd precisely because it destroys all incentives to save, to invest, or to conserve resource stocks. Nevertheless, that policy, as in Tesfatsion (1986), is just the logical implication of the government's first best welfare maximization in the second period economy. The absurd policy of this example fulfils its purpose — namely, to show that subgame imperfections arise even in standard first best welfare economics, when one tries to apply it to simple intertemporal settings. This is true even for "perfect" markets, without public goods, externalities, distortionary taxes, asymmetric information, or any other form of "market failure". The point is that the prospect of what will appear in the future to be non-distortionary lump-sum redistribution actually functions now like "taxes on history".

Such taxes, of course, typically distort individual decisions to "supply history". In this simple example, "history" was just the distribution of resource stocks left in each mine at the end of the first period.

This example, and the one in the previous section, do nothing to contradict the usual Arrow–Debreu theory, even in an intertemporal setting. That theory remains valid when trading plans, prices, and lump-sum transfers can all be fixed *now*, remaining unchanged at all future times. Yet what is so special about "now", which allows trading plans, prices, and lump-sum transfers to be set at this time and then to remain fixed for ever thereafter? Why are the trading plans, prices, and lump-sum transfers for the future not those that were already set some time ago in the past? And, if changes to past plans are allowed now, what will happen when a new "now" comes around a little while later? Do trading plans, prices, and lump-sum transfers remain at the levels which are being planned now, or will they respond to changed circumstances if individuals happen to deviate from their original (Nash equilibrium) trading plans? Since changes in trading plans do induce changes in the prices needed to maintain equilibrium, as considered in Section 4, or in the transfers needed for distributive justice, as considered in Section 5, subgame imperfections cannot just be assumed away. The Arrow–Debreu theory remains logically consistent, but its plausibility as a description of an ideally functioning intertemporal economy disappears almost completely.

6.2. Sequential Allocations

The above discussion points to the conclusion that, even in a simple two period sequence economy, each period's equilibrium prices and economic policy variables will typically all depend upon the state of the economy at the beginning of the period. Here "state" is to be understood in its Markovian sense, as a sufficient statistic for all the past history of the economy, so that knowledge of the state is enough to know everything relevant for the future of the economy. Recent work in general equilibrium theory has begun to consider equilibrium Markov processes (see especially Duffie, Geanakoplos, Mas-Colell and McLennan, 1994, as well as Stokey and Lucas with Prescott, 1989), but much remains to be done. However, it seems to me that a great deal can still be learned from models with finite horizons or even just two periods. These allow the technical problems of recursion in general infinite horizon Markov processes to be avoided while an appropriate conceptual framework is still being formulated.

This dependence of policy variables and equilibrium prices on the current state of the economy suggests the need for an approach to intertemporal economics rather different from that of Irving Fisher (1907, 1930), Hicks (1946), and Debreu (1959). Their apparatus of "dated commodities" or "dated contingent commodities" is reminiscent of "open loop" policy in control theory, in that it specifies what will happen at each future date in a model with certainty, and at each future "date-event" when there is uncertainty. The proposal here is to consider instead the equivalent of "closed loop" policies, describing what single period allocation will come about as a function of the state at the start of that period. We may call these "sequential allocations". Such a description of the outcome of the economic system is more general because it allows consideration of what will happen off any equilibrium path of our dynamic model. The discussion of Section 6.1 above points to the need for a richer formulation of this kind.

Sequential allocations like this can be considered in a rather general stochastic overlapping generations model with a continuum of agents having bounded lifetimes. Each agent can be both a consumer and a producer. There can be both aggregate and idiosyncratic uncertainty. Each agent can have a personal "history" or state described in part by personal capital stocks. But the personal state can also include any other variables affecting preferences, consumption and production possibilities, etc. Then there will be a Markov process determining the same agent's personal history one period later, conditional on consumption, production, labor supply, and other economic decisions in the current period. Such personal state variables can capture the effects of age, health, family circumstances, education, etc. Each agent's idiosyncratic Markov process can also be affected by an aggregate state which can represent the physical environment, together with aspects of the economic system, public goods, etc. In fact, the "history" or state of the economy as a whole is described by this systemic state variable, together with the entire frequency distribution of individual agents' personal states in the population of all living agents. This will be called the "macro state", for obvious reasons.

Of course, there are some technical problems even in the formulation of such a model. There will be a continuum of individuals who, conditional on the evolution of the macro state, have personal states following stochastically independent Markov processes. In particular, if we consider just one period, and neglect the macro state for a moment, there will be a continuum of independent random variables describing the individuals' personal states. As Gale (1979), Feldman and Gilles (1985) and Judd (1985) in particular have pointed out, this leads to non-measurable sample functions describing the empirical distribution in the population.

In fact, this should not be surprising. An American taxpayer's social security number gives no information about that person's height, and we can think of the function from social security number to height as approximately the realization of a continuum of independent random variables. The function is extremely irregular, and not measurable in the limit as one goes to a continuum of individuals. Yet, the joint distribution of Americans' social security numbers and heights is no doubt rather close to the product of two independent distributions — one a well defined distribution of social security numbers, and another of people's heights.

This suggests a simpler escape from the technical problem of non-measurability than that presented by resorting to finitely additive charges (Feldman and Gilles, 1985), non-standard analysis (Anderson, 1991, Section 5), or alternative integrals (Bewley, 1986). Instead of a fixed population of individual labels and then random personal states for each, one can consider instead a joint distribution of both individual labels and personal states. For large enough populations the strong law of large numbers applies and tells us that frequency distributions will match probability distributions.

A similar construction based on joint distributions can be used for the continuum of individual stochastic processes used in the model described informally here. The result is a description of a sequence economy that appears to be immensely rich, capable of embracing many dynamic phenomena. It is also a step toward generalizing the Arrow–Debreu model so that it becomes a collection of interacting dynamic processes, rather than just a means of allocating resources once and for all.

6.3. Sequential Competitive Equilibria

Within the sequential model described above, it is then natural to think of a sequential competitive equilibrium as a process specifying the price vector in each period as a function of the macro state, and also specifying each agent's allocation as a function of both the macro state and the agent's own personal state. This would be for an equilibrium without lump-sum transfers. The second efficiency theorem, however, forces us to consider equilibria with transfers. They will typically specify how much each agent is allowed to spend each period on current goods and services, as a function of both the macro state and the personal state. Note that such transfers can substitute for all financial markets (including those for credit, futures, insurance, etc.) because such markets are no more than a device for reallocating claims to current expenditure between agents in different time periods and contingently upon different events. Where there are traded financial assets, their holdings should be included in the description of each agent's personal state. Then each agent will be faced with a separate budget constraint for each period, and for each macro state and personal state that could occur at the start of that period.

Sequential equilibria of this kind raise many issues. One obvious question concerns their existence. Standard techniques should be able to prove existence of an equilibrium in each subeconomy that depends only on the macro state at the beginning of the subeconomy. It is by no means obvious, however, even in the two period case, that later equilibria can be selected in a way that depends only on the macro state at the beginning of these later subeconomies. Typically, therefore, it may be necessary to allow the entire history of previous macro states to affect what equilibrium prices emerge in each subeconomy. Then each macro state would have to be expanded to include within its description a complete history of all previous macro states. Some reduction in the dimensionality of the relevant state space may well be possible, however, as indeed the results of Duffie *et al.* (1988) for infinite horizon models suggest.

The presence of transfers or of other instruments of government economic policy, and their dependence on the macro state, is an important new feature of such sequential models. In particular, the macro state includes the interpersonal distribution of personal states, which in turn is affected by the distribution of individual agents' consumption, saving, investment, production and work decisions in all previous periods. As was remarked in

Section 1, this makes the macro state into a kind of "widespread externality." This is the public good aspect of the macro state in intertemporal models mentioned in Section 6.1.

6.4. Sequential Policy Analysis

The example of Section 5 shows the need to consider more carefully how the dependence of future government policy on past actions by agents affects those agents' incentives. However, it does also show how limited may be the ability of governments to pursue redistributive programs, even in the absence of information failures and other obstacles to ideal lump-sum redistribution. Issues of "time (in)consistency" or, more exactly, of subgame (im)perfection, arise not only in macroeconomics and not only when there are "distortionary" taxes or public goods — they are inherent in intertemporal models of an economic system. More interestingly, perhaps, the above example also shows how any individual decision in an intertemporal economy has a public good aspect, because of the predicted reaction of government policy to earlier decisions by individual agents. This vital feature seems to be missing from all but a very few of the intertemporal models that have been used for policy analysis, especially those used by microeconomists. For some of the exceptions, see Rogers (1986), Staiger and Tabellini (1987), Klein (1987), Chari (1988), Stokey (1989, 1991), Karp and Newbery (1989), Maskin and Newbery (1990), Bliss (1991). Macroeconomists, on the other hand, are very familiar with the "Lucas critique" concerning the government's policy reactions (Lucas, 1976), but do not seem to have brought out all its public good aspects.

Another limitation of the literature on public finance is that much of it appears to consider only rather inflexible policies. Examples include a permanent change in income tax rates, a permanent and constant new tax on fuels which produce carbon dioxide when burnt, a permanent and irreversible move toward freer trade or market integration, etc. Yet sequential economies call out for sequential policies that depend upon the macro state. Previous sections of this paper have considered sequential lump-sum redistribution. It is natural for policy economists to consider sequential commodity and income taxes, sequential regulation of industry, sequential policies to combat pollution, etc. And for economic theorists to consider sequential budget decentralizations of incentive compatible sequential allocation mechanisms, and sequential rules for providing public goods.

Indeed, sequential policies of this kind do play an important role in macroeconomic discussion of issues like stabilization policy. It is surely time to embody them in microeconomic policy models. In real economies, they are inevitable anyway. Even a government which tries to ignore fluctuations in the macroeconomy, and maintains constant tax rates and public expenditure programs regardless of the state of the business cycle, will find itself balancing its finances and meeting its budget deficit with a countercyclical level of borrowing. In other words, unless there is no uncertainty whatsoever, its borrowing policy at least will be sequential, in the sense of depending upon the macro state.

This is a simple observation, yet it has profound implications. Following Sen (1972), in Hammond (1986, 1990) the need for a balancing policy to accompany any public (or private) sector project or tax reform was pointed out. The reason is that, unlike the private sector which is usually modelled as balancing its finances optimally, the public sector usually relies on unspecified tax or borrowing policies to meet any shortfall it may experience. Yet these balancing policies are a crucial part of a project or tax reform — indeed, in Hammond (1986) there are non-trivial cases where the only immediate effect of a project is to generate a surplus for the government, which will then only benefit individuals once it gets spent in some useful way (including the possibility of reducing existing tax revenue requirements). This is the balancing part of the policy. And, as should be clear from the argument of the previous paragraph, this balancing policy will have to depend upon the macro state, as well as on the size of the increased deficit or surplus that has to be balanced. A similar phenomenon applies in connection with the gains from increased production efficiency, from trade liberalization, or from forming a customs union. To achieve the Pareto improvements claimed in most textbooks, the gainers from such "supply-side" policies have to pay lump-sum compensation to the losers — to those who have human or physical capital invested irreversibly in industries which cease to be competitive, for instance. This lump-sum compensation depends on what the individual's net trade would have been in the absence of a reform. In a sequential economy, there has to be something equivalent to a lump-sum adjustment to each individual's budget constraint in each period, and for each macro and personal state in each period. Moreover, each such adjustment depends on what the individual would have consumed and produced in that period, for that macro and personal state. We are back with sequential lump-sum transfers.

In the more realistic case where lump-sum compensation is impossible, Dixit and Norman (1980, 1986) were still able to demonstrate the gains from trade, using commodity (and income) tax adjustments to compensate losers. In the static model of Hammond and Sempere (1995) we show how to generate Pareto improvements by instituting a total freeze on consumer prices, wages, and dividends, combined with poll subsidies and perfectly flexible tax rates. This leaves producer prices free to adjust so as to clear all markets. In a sequential setting, the corresponding policy requires freezing consumer prices, wages and dividends at levels they would have had, as functions of the macro state, in the absence of any reform. Obviously there are enormous practical difficulties in determining what these would have been, let alone in arranging the required freeze. This leads us to regard the possibility of Pareto gains as largely illusory. Instead, one should be evaluating combinations of sequential supply side policy reforms together with imperfect sequential policies that lighten the burden on the most deserving losers.

In fact, there is no reason why only balancing or compensating policies should be sequential. Public sector projects can often be improved by making them more responsive to changes in the macro state — e.g., by concentrating projects for (re)constructing a country's infrastructure during periods when the economy would otherwise be in recession, or by adapting the intensity of (re)training programs to the level of unemployment. The same is true of tax policies, of course. Economic policy as a whole is almost inevitably sequential; good economic policy will take into account the sequential nature of the economy; models for evaluating policy must then also be sequential.

7. Concluding Remarks

The paper started by formulating two Arrow–Debreu market games whose straightforward Nash equilibria are Walrasian. Both games had an auctioneer setting prices to maximize net sales value. In the second an additional redistributive agency maximizes welfare through optimal lump-sum transfers. In intertemporal economies, however, it was shown that subgame imperfections can arise because agents understand how current decisions such as those determining investment influence either future prices (with finitely many agents), or future redistribution (even in continuum economies). The latter observation undermines the second efficiency theorem of welfare economics in sequential environments. Indeed, when the state of the economy affects future policy, it functions like a "widespread externality."

Finally, some readers may find it odd that I should contribute a piece to this volume which points out some limitations of the Arrow–Debreu methodology. Yet my enjoyable interaction with Kenneth Arrow over many years makes me keenly aware that nobody is readier than he is to accept that there are such limitations, while also seeking to adapt the Arrow–Debreu framework in order to overcome them. Unfortunately, limitations of space and time prevent me from exploring here how an equilibrium theory of stochastic allocation processes retains much of the formalism and essential insights of Arrow's pioneering work on the role of securities markets in a simple sequence economy, while at the same time overcoming the subgame imperfections noticed in this contribution.

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REFERENCES

R.M. Anderson (1991) "Non-Standard Analysis with Applications to Economics", ch. 39, pp. 2145–2208 in W. Hildenbrand and H. Sonnenschein (ed.) *Handbook of Mathematical Economics, Vol. IV* (Amsterdam: North-Holland).

K.J. Arrow (1951) "An Extension of the Basic Theorems of Classical Welfare Economics", in J. Neyman (ed.) *Proceeedings of the Second Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley: University of California Press) pp. 507–532; reprinted in Arrow (1983), ch. 2, pp. 15–45.

K.J. Arrow (1953, 1964) "Le rôle des valeurs boursières pour la répartition la meilleure des risques", *Econométrie* (Paris: Centre National de la Recherche Scientifique) pp. 41–48; translation of English original, "The Role of Securities in the Optimal Allocation of Risk-bearing," later published in *Review of Economic Studies*, **31**: 91–96; reprinted in Arrow (1983), ch. 3, pp. 48–57.

K.J. Arrow (1971) "The Firm in General Equilibrium Theory", *The Corporate Economy: Growth, Competition, and Innovative Potential* in R. Marris and A. Wood (ed.) (London: Macmillan; and Cambridge, Mass.: Harvard University Press) pp. 68–110; reprinted in Arrow (1983), ch. 8, pp. 156–198.

K.J. Arrow (ed.) (1983) *Collected Papers of Kenneth J. Arrow, Vol. 2: General Equilibrium* (Cambridge, Mass.: Belknap Press of Harvard University Press).

K.J. Arrow and G. Debreu (1954) "Existence of an Equilibrium for a Competitive Economy", *Econometrica*, vol. 22, pp. 265–290; reprinted in Arrow (1983), ch. 4, pp. 59–91 and in Debreu (1983), ch. 4, pp. 68–97.

K.J. Arrow and F.H. Hahn (1971) *General Competitive Analysis* (San Francisco: Holden-Day).

T.F. Bewley (1986) "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers", pp. 79–102 in W. Hildenbrand and A. Mas-Colell (ed.) *Contributions to Mathematical Economics: In Honor of Gérard Debreu* (Amsterdam: North-Holland).

C.J. Bliss (1991) "Adjustment, Compensation and Factor Mobility in Integrated Markets", *Unity with Diversity in the European Economy: The Community's Southern Frontier* in C.J. Bliss and J.B. de Macedo (ed.) (Cambridge: Cambridge University Press).

G.A. Calvo (1978) "On the Time Consistency of Optimal Policy in a Monetary Economy", *Econometrica*, vol. 46, pp. 1411–1428.

V.V. Chari (1988) "Time Consistency and Optimal Policy Design", *Federal Reserve Bank of Minneapolis: Quarterly Review*, vol. 12, (4, Fall) pp. 17–31.

G. Debreu (1952) "A Social Equilibrium Existence Theorem", *Proceedings of the National Academy of Sciences*, vol. 38, pp. 886–893; reprinted in Debreu (1983), ch. 2, pp. 50–58.

G. Debreu (1959) *Theory of Value: An Axiomatic Analysis of Economic Equilibrium* (New York: John Wiley).

G. Debreu (1983) *Mathematical Economics: Twenty Papers of Gerard Debreu* (Cambridge: Cambridge University Press).

P. Diamond and J. Mirrlees (1971) "Optimal Taxation and Public Production, I and II", *American Economic Review*, vol. 61, pp. 8–27 and 261–278.

A. Dixit and V. Norman (1980) *Theory of International Trade* (Welwyn, Herts.: James Nisbet).

A. Dixit and V. Norman (1986) "Gains from Trade without Lump-Sum Compensation", *Journal of International Economics*, vol. 21, pp. 99–110.

D. Duffie, J. Geanakoplos, A. Mas-Colell and A. McLennan (1994) "Stationary Markov Equilibria", *Econometrica*, vol. 62, pp. 745–781.

M. Feldman and C. Gilles (1985) "An Expository Note on Individual Risk without Aggregate Uncertainty", *Journal of Economic Theory*, vol. 35, pp. 26–32.

S. Fischer (1980) "Dynamic Inconsistency, Cooperation, and the Benevolent Dissembling Government", *Journal of Economic Dynamics and Control*, vol. 2, pp. 93–107.

I. Fisher (1907) *The Rate of Interest: Its Nature, Determination, and Relation to Economic Phonomena* (New York: Macmillan).

I. Fisher (1930) *The Theory of Interest* (New Haven: Yale University Press).

D.M. Gale (1979) "Large Economies with Trading Uncertainty", *Review of Economic Studies*, vol. 46, pp. 319–338.

P.J. Hammond (1979) "Straightforward Individual Incentive Compatibility in a Large Economies", *Review of Economic Studies*, vol. 46, pp. 263–282.

P.J. Hammond (1986) "Project Evaluation by Potential Tax Reform", *Journal of Public Economics*, vol. 30, pp. 1–36.

P.J. Hammond (1987) "Markets as Constraints: Multilateral Incentive Compatibility in Continuum Economies", *Review of Economic Studies*, vol. 54, pp. 399–412.

P.J. Hammond (1990) "Theoretical Progress in Public Economics: A Provocative Assessment", *Oxford Economic Papers* (Special Issue on Public Economics), vol. 42, pp. 6–33.

P.J. Hammond, M. Kaneko and M.H. Wooders (1989) "Continuum Economies with Finite Coalitions: Core, Equilibrium, and Widespread Externalities", *Journal of Economic Theory*, vol. 49, pp. 113–134.

P.J. Hammond and J. Sempere (1992) "Limits to the Benefits from Market Integration and Other Supply-Side Policies", European University Institute, Working Paper ECO No. 92/79.

J.R. Hicks (1939, 1946) *Value and Capital (2nd edn.)* (Oxford: Oxford University Press).

M.D. Intriligator (1987) "The Impact of Arrow's Contribution to Economic Analysis", in G.R. Feiwel (ed.) *Arrow and the Foundations of the Theory of Economic Policy* (London: Macmillan, and New York: New York University Press) ch. 30, pp. 683–691.

K. Judd (1985) "The Law of Large Numbers with a Continuum of Random Variables", *Journal of Economic Theory*, vol. 35, pp. 19–25.

M. Kaneko and M.H. Wooders (1989) "The Core of a Continuum Economy with Widespread Externalities and Finite Coalitions: From Finite to Continuum Economies", *Journal of Economic Theory*, vol. 49, pp. 135–168.

L. Karp and D. Newbery (1989) "Intertemporal Consistency Issues in Depletable Resources", Centre for Economic Policy Research, Discussion Paper No. 346.

D. Klein (1987) "The Microfoundations of Time Inconsistency under a Benevolent Rule", GEA Working Paper 87-01, New York University; revised as "The Microfoundations of Rules vs. Discretion" (1988).

F.E. Kydland and E.C. Prescott (1977) "Rules Rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy*, vol. 85, pp. 473–491.

F.E. Kydland and E.C. Prescott (1980) "Dynamic Optimal Taxation, Rational Expectations, and Optimal Control", *Journal of Economic Dynamics and Control*, vol. 2, pp. 79–91.

R.E. Lucas (1976) "Econometric Policy Evaluation: A Critique", in K. Brunner and A.H. Meltzer (ed.) *The Phillips Curve and Labor Markets, Carnegie-Rochester Conference Series on Public Policy* vol. 1, pp. 19–46; reprinted in R.E. Lucas, *Studies in Business Cycle Theory* (Cambridge, Mass.: M.I.T. Press, 1981) (Amsterdam: North-Holland).

E. Maskin and D.M. Newbery (1990) "Disadvantageous Oil Tariffs and Dynamic Consistency", *American Economic Review*, vol. 80, pp. 143–156.

J. von Neumann and O. Morgenstern (1943; 3rd. edn. 1953) *Theory of Games and Economic Behavior* (Princeton: Princeton University Press).

C.A. Rogers (1986) "The Effect of Distributive Goals on the Time Inconsistency of Optimal Taxes", *Journal of Monetary Economics*, vol. 17, pp. 251–269.

R. Selten (1965) "Spieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit", *Zeitschrift für die gesamte Staatswissenschaft*, vol. 121, pp. 301–324 and 667–689.

R. Selten (1973) "A Simple Model of Imperfect Competition, where 4 Are Few and 6 Are Many", *International Journal of Game Theory*, vol. 2, pp. 141–201.

R. Selten (1975) "Re-examination of the Perfectness Concept for Equilibrium Points of Extensive Games", *International Journal of Game Theory*, vol. 4, pp. 25–55.

A.K. Sen (1972) "Control Areas and Accounting Prices: An Approach to Economic Evaluation", *Economic Journal*, vol. 82, pp. 486–501.

R.W. Staiger and G. Tabellini (1987) "Discretionary Trade Policy and Excessive Protection", *American Economic Review*, vol. 77, pp. 823–837.

N.L. Stokey (1989) "Reputation and Time Consistency", *American Economic Review, Papers and Proceedings*, vol. 79, pp. 134–139.

N.L. Stokey (1991) "Credible Public Policy", *Journal of Economic Dynamics and Control*, vol. 15, pp. 627–656.

N.L. Stokey and R.E. Lucas (with E.C. Prescott) (1989) *Recursive Methods in Economic Dynamics* (Cambridge, Mass.: Harvard University Press).

L. Tesfatsion (1986) "Time-Inconsistency of Benevolent Government Economies", *Journal of Public Economics*, vol. 31, pp. 25–52.