Rethinking Distribution: Introducing Market Segmentation as a Policy Instrument*

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September 11, 2022

Abstract

Inequality and skewed distribution of 'essential' goods are pertinent problems in the 21st-century world. We consider a general equilibrium framework with a division between essential and non-essentials, and only essential goods are relevant for distributional concerns. We then compare the effects of four policies on social welfare: subsidies, direct transfers, quantity rationing, and a fourth policy that we introduce and call *Market Segmentation (MS)*. In MS, the market for essentials is segmented from non-essentials, i.e., they are not freely tradeable with each other. We find that if the relative number of low-income individuals in the economy is large and "essentials" are consumed in-elastically, MS outperforms direct transfers and subsidies. MS also weakly dominates quantity rationing. We discuss how market segmentation can help policymakers deal with issues such as automation and the superstar phenomenon (Scheuer and Werning, 2017).

Key words: Public Economics, Public Finance, Revenue, Subsidies, Taxation, Transfers, Rationing, Distribution and Welfare.

JEL classification: D3, D6, H2.

*We are grateful to Motty Perry and Herakles Polemarchakis for their supervision. We also want to thank Robert Akerlof, Costas Cavounidis, Daniele Condorelli, Ben Lockwood, Sharun Mukand, Carlo Perroni, as well as seminar participants at the CRETA (Warwick), 1st Annual Southern PhD Economics Conference (ASPEC) 2021, and International Public Economic Theory conference 2022 (PET, 2022) for helpful comments. We especially want to thank Florian Scheuer for his detailed and thoughtful comments.

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1 Introduction

The late 20th and early 21st centuries have seen a dramatic increase in wealth and income inequality (Piketty, 2014). Strikingly, many can no longer consume what most would agree to be "essential." For example, deaths in the US related to lack of healthcare are increasing at an alarming pace.[1] An increase in the prices of essentials like housing, and healthcare, that crowd out the poor, may well be exacerbating the issue.[2] In this paper, we argue that policymakers must confront this reality by expanding the set of redistributive policy instruments traditionally available to the state.

We consider a stylized general equilibrium exchange economy where some goods are considered essential, whereas others are not. We assume that only essential goods are relevant for distributional concerns. We find that under certain conditions, conventional redistributive instruments cease to achieve social welfare goals effectively. We then introduce a new policy instrument called Market Segmentation (MS henceforth) and demonstrate its superior performance to conventional instruments under these conditions.

Under MS, not every good *type* is freely tradeable for others. As we concern ourselves with distributional issues, we segment the market for essentials from non-essentials, i.e. they cannot be freely traded for each other. We formalize this by not allowing agents to spend the income generated by selling non-essential goods on essential goods above a certain threshold level. Thus, we have one budget constraint with essential goods and one for non-essentials. In such a setup, we compare the welfare effects of MS with three other policies, namely 1) Subsidies, 2) Direct transfers, and 3) Quantity rationing.

We find that if the relative number of low-income individuals in the economy is high and "essentials" are consumed in-elastically, MS outperforms transfers and subsidies.[3] The main intuition for this result is the following. By disallowing trade between essential and non-essential goods, MS lowers the price of essential goods for the poor. Importantly, it does so without creating inefficiency or dead-weight loss in the trade of non-essential goods (because there is no tax on trade between non-essential goods). To juxtapose, taxation is not efficient when the economy has a relatively large number of poor because the tax revenue generated is low and raising taxes creates dead-weight losses in an economy. Moreover, when the demand for essentials is inelastic, taxation fails to decrease the demand from the rich by much, instead leading to an increase in equilibrium price.

We then demonstrate that MS outperforms quantity rationing, when agents' preferences are heterogeneous. MS makes the agents consume the essentials they prefer. On the other hand, the rich consume the maximum of all essential goods under quantity

[1] A 2019 poll by Gallup found that the number of people putting off care due to costs has increased from 12% to 25% (Saad, 2019).

[2] For instance, see the trend of house prices in New York city at https://furmancenter.org/files/Trends_in_NYC_Housing_Price_Appreciation.pdf. For healthcare, See the following paper: Nunn et al. (2020).

[3] Inelasticity means that the price elasticity of demand for essential goods is always less than 1.

rationing, thus hampering equal distribution. MS thus exploits Walrasian equilibria's "self-selection" properties when individuals have heterogeneous preferences over essential goods to increase welfare for the poor. We defer to Weitzman (1977), "Other things being equal, the price system has greater comparative effectiveness in sorting out the deficit commodity and in getting it to those who need it most when wants are more widely dispersed."

We also show that MS and direct transfers complement each other, showing that MS and direct transfers should (almost always) be used together. This expands the set of policy instruments available to the state. Fixing the segmented categories, we characterise optimal segmentary tax in our framework.[4] Further, we consider various applications of segmentary taxes and how they can help deal with issues emerging due to rapidly changing technological and labour market conditions such as Automation (Costinot and Werning, 2020) and the superstar phenomenon (Scheuer and Werning, 2017).

Importantly to implement MS, the planner need not know individual endowments or preferences but only their distributions in society. However, to enforce segmentation, the planner must stop the rich from providing side payments to owners of essential goods. Thus, the additional information that the planner would need to implement MS is to be able to link and monitor an individual's expenditure on essential goods. This can be done by linking social security or unique identification numbers to expenditure on essentials.[5] We discuss further implementation issues in the main body of the paper.

We introduce segmentary taxes as a policy instrument and provide sufficient conditions under which MS dominates transfers and subsidies. We note that this is just a small step in finding the general solution to the problem. However, we hope that the reader has the following takeaways. MS does better than transfers and subsidies when, i) Essential goods are inelastic with respect to price, ii) There are a relatively large number of poor in the population, iii) Taxes decrease trade within the non-essential goods. Further, if needs are heterogeneous, MS dominates quantity rationing. We also demonstrate that different policies might be effective while dealing with different effects of certain technological and labour market shocks.

Our results depend on the price inelasticity of essential goods in both demand and supply. However, we think this is a natural condition when considering the demand for essential goods.[6] On the supply side, this may represent institutional constraints or fixed

[4]The kind of taxes which we call segmentary do exist in real-world settings, though they have not yet been theoretically explored as a policy instrument. One example of such a tax would be the capital gains tax in India. See (https://cleartax.in/s/capital-gains-income) and many other countries including the US; (See, Matthew Frankel (2017-12-22)). "Your Guide to Capital Gains Taxes in 2018"

[5]Such linkage of individual to consumption is already done in many social security schemes throughout the world, for instance, the public distribution system in India. See https://nfsa.gov.in/portal/PDS_page.

[6]For our results to hold, the price inelasticity of demand and supply of "essentials" is an important condition. Research estimating the price elasticity of demand on goods like housing and healthcare finds these to be inelastic. Hanushek and Quigley (1980) estimate price inelasticity of demand housing to be -0.64 in Pittsburgh and -0.45 in Phoenix. Recent papers also substantiate their claims. Albouy, Ehrlich,

factors for producing essential goods. For example, many studies point out that it is difficult to train more doctors because of technological constraints.[7] Similarly, shortages in land in the case of urban housing make supply constraints real even in the long run.[8] Hence, we think that market segmentation can be a very useful tool in the repertoire of a policy maker in many instances.

The rest of the paper is structured as follows. We conclude this section with a discussion of the related literature. Then in section 2, we present a simple example that brings forth the main argument of this paper. Section 3 presents the main model and results. Section 4 discusses applications of the taxes, including implementation in greater detail. Finally, Section 5 concludes.

1.1 Related Literature

Our paper contributes to the extensive literature on redistributive approaches in public finance. Traditionally, economics has dealt with problems of redistribution using two broad approaches. The first is taxation, pioneered by Ramsey (1927), Diamond and Mirlees (1971), where the planner collects tax revenue to facilitate direct transfers or subsidise goods. The second approach is rationing essential goods through direct quantity controls (Tobin, 1970), price controls(Weitzman, 1977)., or non-linear taxes (Gadenne, 2020). Segmentary taxes combine the two approaches in public finance by using a tax between different types of goods (in particular essential and non-essential goods) to ration essential goods by lowering their prices. Thus, it is a 'tax instrument' of price control that can avoid dead-weight loss and hence, achieves rationing via price controls, as in Weitzman (1977), Bulow and Klemperer (2012), Condorelli (2013) and Dworczak et al. (2020).

Our main result also holds in some particular non-linear labour taxation setups (pioneered by Mirrlees (1971)). In appendix B, we build on the framework of Saez (2002) and combine it with some features of Saez (2004).[9] We demonstrate that MS dominates non-linear labour taxation in such a setup when the demand and supply of essential goods are price inelastic. For ease of exposition, we keep the main body of our paper in the

and Liu (2016) state, *"Since 1970, housing's relative price, share of expenditure, and unaffordability have all grown"*. For healthcare, Ellis, Martins, and Zhu (2017) find elasticity estimates to be between -0.17 and -0.35, even lower than those for housing demand. Also, the elasticity of supply of these goods is low in the short run and is estimated to be falling even in the long run (See: Aastveit, Albuquerque, and Anundsen (2016)).

[7] https://www.washingtonian.com/2020/04/13/were-short-on-healthcare-workers-why-doesnt-the-u-s-just-make-more-doctors describes the legislative problems in expanding doctor supply. Levit et al. (2009) summarizes the institutional constraints to expanding physician supply

[8] For example: Consider the problems discussed by Indian policymakers. See https://niti.gov.in/indias-housing-conundrum

[9] Though Atkinson and Stiglitz (1976) shows that if utility is separable between labour and consumption, only labour taxes suffice to improve welfare, Naito (1999) notes that this result does not hold if relative prices in the economy can change.

framework of Diamond and Mirlees (1971). More generally, there is a vast literature on how non-linear taxes on labour can affect distribution (see Atkinson and Stiglitz (1976); Stantcheva (2014); Piketty et al. (2014); Saez and Stantcheva (2016); Stantcheva (2020)). We think combining non-linear taxes with segmentation would be promising for future work. However, in this paper, we limit ourselves to discussing linear commodity taxation, particularly because nonlinear taxation can usually be replicated by linear taxation (Scheuer and Werning, 2016).

Our paper is also related to the literature which considers how to design markets for specific commodities (for a detailed review of the literature, see Roth (2015); Vulkan et al. (2013)). Many of these 'commodities' (like organs or spots in educational institutions) are effectually segmented from the rest of the economy for moral or egalitarian reasons.[10] We give an economic rationale for this segemntation. However, if the planner is egalitarian, we show that these markets may benefit from limited interaction with others. Moreover, much of this literature treats supply as exogenous and only looks at the allocation problem (Dworczak et al., 2020; Akbarpour et al., 2020, 2021). In our paper, we endogenize supply and show how combining certain types of commodities *(essentials)* while segmenting them from others *(non-essentials)* can improve welfare.

When discussing applications, we interface with the work on **taxation and redistribution with automation** (see Guerreiro et al. (2017); Costinot and Werning (2020); Thummel (2020)). In the framework of Guerreiro et al. (2017), technical progress in automation and endogenous skill choice can lead to a massive rise in income inequality. Costinot and Werning (2020) use a sufficient statistic approach to characterise optimal tax on robots. Thummel (2020) is the closest to our work because it discusses the general equilibrium effects of taxing robots, which in turn change the relative prices, and thus distribution. Our paper departs from the above work in a few crucial aspects. First, we distinguish between essential and inessential goods, highlighting the possibility of segmenting these markets to enhance welfare. Second, we show that different policies are more efficient at increasing welfare under the different effects of automation we consider.

Dealing with yet other applications, we find that the precise nature of labour markets that produce either 'superstar effects' (Scheuer and Werning, 2017) or 'winner-take-all' (Rothchild and Scheuer, 2016) scenarios makes MS more relevant. MS allows the planner to change the distributions of essential goods without causing excess distortions in production, which is the major issue in the above-mentioned labor markets.[11]

To the best of our knowledge, our paper is the first to introduce segmentary taxes. Thus, although our results are similar in scope and goal to the papers above, our approach is entirely novel. Importantly, we expand the set of policy instruments available to the

[10] See Roth (2018)

[11] For a review on how changing labour market conditions affect optimal taxation policies, see Scheuer and Slemrod (2019).

state and demonstrate conditions under which our proposal performs better than existing instruments. The following section provides a simple example that distils our main result's central intuition.

2 Example

A stylized example is now constructed for a clear exposition of the forces behind the main results of this paper. Consider a pure exchange economy with three goods x_1 (medicine - essential good), x_2 (time/manual labour - universal good), and x_3 (gold - luxury good) and populated by 3 types of individuals (A, B and C). The proportion of each type is give by π_A, π_B and π_C respectively. We assume all individuals have identical log-linear preferences represented by $U = klogx_1 + x_2 + x_3$.

The utility function captures the intuition that a complete absence of medicines/health services is hugely detrimental to an individual's welfare. Now assume the planner is utilitarian and wishes to maximise the sum of individual utilities in this economy. We represent his objective function as,

$$W = \pi_A U_A + \pi_B U_B + \pi_C U_c$$

For simplicity, assume $\pi_A = \pi_C = \frac{1}{n+2}$ and $\pi_B = \frac{n}{n+2}$. The initial endowments are captured in the table below.

Agent Type	x_1	x_2	x_3
A (Pharma)	1	e	0
B (Poor)	0	e	0
C (Rich)	0	e	W

Type A are the pharmas - the pharmaceutical companies endowed with the medicines (and time of course), type B are the poor people endowed with *only* time, and type C are the rich who are endowed with gold and time. Consider a scenario where the planner can pick between Market Segmentation (MS) and Laissez-faire (LF) policies to maximise the social welfare function. MS is defined as the following: an agent can only trade between universal good (time) and essential (medicines/health services) and hence cannot use non-essentials to buy the essential good. Table 1 shows the different budget constraints of an individual under the above 2 regimes.

In this setup, if the endowments of the universally endowed good is small and the relative number of poor in the economy is large, social welfare under MS is higher than LF.[12] First, note that due to quasi-linearity of the utility function, only the distribution

[12]To be precise, small endowments of the universal good means that $e << k$, where e is the endowment of the universal good in the economy. For formal setup, see Appendix: Section A.1

Table 1: Different Regimes

Regime	Budget Constraint
MS	$\sum_{1,2} p_i(x_i - e_i) \leq 0$ and $p_3(x_3 - e_3) \leq 0$
LF	$\sum_{1,2,3} p_i(x_i - e_i) \leq 0$

of x_1 i.e., essential good is welfare relevant. MS works by lowering the equilibrium price of x_1, making its allocation more egalitarian by allowing the poor to consume more and increasing the social welfare. Figure 1 below demonstrates that for a low level of endowment of the universal good, welfare under MS is always greater than welfare under LF.

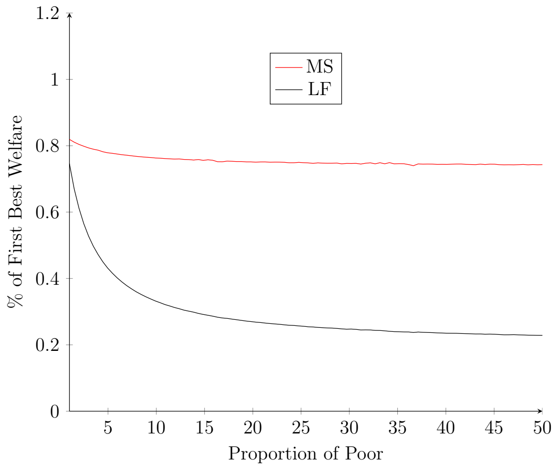


Figure 1: MS vs LF

However, it is worth noting that we can replicate the MS allocation in the above economy by using two pre-existing policy instruments (Quantity Rationing (QR) and Taxation). First, using QR, which fixes an amount which can be bought for each essential commodity, we could ration its consumption at the level arising in the MS equilibrium. However, in an economy with more than one essential good and heterogeneous preferences, MS becomes relevant. MS disallows luxury goods to be exchanged for essentials and rations 'jointly', not good by good, and allows poorer individuals greater freedom in allocating their budgets. At the same time, MS prevents the rich from buying all the available essential goods at the maximum possible levels allowed under quantity rationing. We will see study this in more detail going forward.

Second, again in an economy where there is only one luxury good, taxation weakly dominates both MS and QR. In such a setting, we could just impose a high tax on the luxury good to reduce its trade with the essential goods. At a very high level of tax, no luxury good will be traded with the essential good, thus replicating the MS equilibrium.

However, when there are more than one luxury goods, and there is heterogeneity in preferences for luxuries, high taxes reduce the trade of these goods not only for the essential goods, but also for each other. Thus, taxation can be potentially highly distortionary. Hence, MS again becomes relevant, as it creates equitable distribution of the essential good, while avoiding dead weight losses and distortions in the market for luxuries.

Hence, we now move to a setting where there are two essential goods, two luxury goods and one universal good. In this economy, we demonstrate that under certain conditions, MS achieves the objective of the planner more effectively than QR or taxation.

Five Goods and Preference Heterogeneity

In our 5-good economy, we create an environment where there are gains from trade (i.e. dead-weight losses if luxuries are taxed) and preference heterogeneity for essentials. In this setup, we first show that MS weakly dominates QR when considering the welfare of the poor in the economy. Therefore, when the number of poor in the economy is large, MS is preferable to QR. Second, we show that as the number of the poor(n) increases, MS eventually dominates taxation with transfers. In general, we want to highlight thatthe levels of taxation required to enforce the allocation of resources the planner desires are too high and create large dead-weight losses when the relative number of poor in the economy is large. In this scenario, segmentation is a middle ground.

We have 5 goods in the economy - 2 essential goods (x_1, x_2), 1 universal good x_3 and 2 luxury goods(x_4, x_5). There are still 3 kinds of individuals- A (pharmas), B (poor) and C (rich), but in C, we have two sub-kinds, i.e. they are endowed with different luxury goods. We assume that total utility for any agent is additively separable across all goods. The endowments are as follows.

Table 2: Endowments

Agents	x_1	x_2	x_3	lx_4	x_5
A	1	1	e	0	0
B	0	0	e	0	0
C_1	0	0	e	W	0
C_2	0	0	e	0	W

We assume heterogeneous preferences to make trade in luxury goods welfare improving.[13] So,

$$U^{\omega}(X) = \sum_{i=1}^{5} U_i^{\omega}(x_i)$$

[13]This is mainly for the case of linearity in non-essential goods. If convex preferences replace linearity, this should not be needed.

Table 3: Utilities

Agents	$U_1(x)$	$U_2(x)$	$U_3(x)$	$U_4(x)$	$U_5(x)$
A	$k_1^\omega logx$	$k_2^\omega logx$	x	$\frac{x}{2}$	$\frac{x}{2}$
B	$k_1^\omega logx$	$k_2^\omega logx$	x	$\frac{x}{2}$	$\frac{x}{2}$
C_1	$k_1^\omega logx$	$k_2^\omega logx$	x	$\frac{x}{2}$	x
C_2	$k_1^\omega logx$	$k_2^\omega logx$	x	x	$\frac{x}{2}$

where U_i^ω are given in table 3. Preferences for essentials are uniformly and I.I.D. as $(k_1, k_2)^\omega = \{(0, 2k), (k, k), (2k, 0)\}$ each for a third of the population. We assume that $e < k < \frac{W}{2}$, this implies that the poor only consume essential goods (are on the boundary), whereas the rich consume both essential goods and luxuries.

Table 1 presents budget sets of individuals under different regimes. Note that goods 1 and 2 are the essentials, good 3 is the universal good, and good 4 and 5 are the luxuries. MS gives the individual two simultaneous budget constraints to satisfy, one for the essential and universal goods corresponding to x_1, x_2, x_3 and the other for luxuries. Under a regime with taxation and direct transfers (DT), individuals receive a lump-sum transfer of R and have to pay a tax if they are net sellers of luxuries, corresponding to $x_i - e_i < 0$. QR gives individuals one budget constraint but disallows them to buy more than a fixed amount of essentials, given by limit q_1^*, q_2^*.

When considering equilibrium prices of essential goods, this economy mimics the setup where individuals have identical preferences for the two essential goods with coefficient k. This is because of the symmetric nature of the heterogeneity introduced as a uniform I.I.D. distribution. Let us call this price p.

Regime	Budget Set
MS	$\sum_{1,2,3} p_i(x_i - e_i) \leq 0$ $\sum_{4,5} p_i(x_i - e_i) \leq 0$
DT	$\sum_{1,2,3} p_i(x_i - e_i) + \sum_{4,5} p_i(1 - \tau \mathbf{I}_{[e_i - x_i]})(x_i - e_i) \leq R$ $\mathbf{I}_{[e_i - x_i]} = 1$ if $e_i - x_i > 0$, $T =$ transfer, and $\tau =$ tax rate.
QR	$\sum_1^5 p_i(x_i - e_i) = 0$ For $i = 2, 3 \quad (x_i - e_1) \leq q_i^*$

Table 4: Budget Sets

Laissez Faire

We solve for the equilibrium allocation without any intervention as a baseline.[14]

Remark. Remember that only the distribution of the essential goods is welfare relevant. The price (with respect to the universal good - the numeraire) is $p^* = \frac{1}{2}ne + 3k$, where n is the relative number of poor in the economy. By construction, $k_1 + k_2 = 2k$ Moreover, by definition, there is no dead-weight loss in the economy. Each essential good (x_1 and x_2) is distributed as follows,

	A	B	C
x_i	$\frac{k_i^\omega}{2k}\frac{e}{\frac{1}{2}ne+3k}$	$\frac{k_i^\omega}{\frac{1}{2}ne+3k}$	$\frac{k_i^\omega}{\frac{1}{2}ne+3k}$

Market Segmentation (MS regime)

Under MS, an agent can only trade between the universal good and the essential goods, and hence cannot use non-essentials to buy essentials. However, free trade is allowed between non-essential goods. The maximization exercise is similar to the LF case, but since the rich cannot use luxuries to buy health services, their endowments are effectively identical to the poor. However, the pharma still consumes the essential good at equilibrium 'satiation level.'

Now again, as before let $e < k < \frac{W}{2} + 1$, then we know, as before, that poor people (B) will demand $x_i^{B,\omega} = \frac{k_i^\omega}{2k}\frac{e}{p_i}$ but now the rich (C) will also demand $x_1^{C,\omega} = \frac{k_i^\omega}{2k}\frac{e}{p_i}$. Moreover, assuming $m_A = p_1 + 1 > k$ in equilibrium, individual A will demand $x_i^{A,\omega} = \frac{k_i^\omega}{p_1}$

Now each type ω has a different demand. To clear markets, we take expectations over all types $\omega \in \Omega$.

$$\mathbb{E}_{\omega\in\Omega}[x_i^A] + n \times \mathbb{E}_{\omega\in\Omega}[x_i^B] + 2\mathbb{E}_{\omega\in\Omega}[x_i^C] = 1$$

$$\implies p_{MS} = \frac{1}{2}e(n+2) + k = p^* - \overbrace{2(k - \frac{e}{2})}^{\text{Price Effect}}$$

The price goes down when $e << k$. This "price reduction" increases the welfare of the poor. There is no distortion to equilibrium allocation of luxuries, i.e. goods 3 and 4 under MS. Each essential good (x_1 and x_2) is distributed as follows,

	A	B	C
x_i	$\frac{k_i^\omega}{2k}\frac{e}{p_{MS}}$	$\frac{k_i^\omega}{p_{MS}}$	$\frac{k_i^\omega}{2k}\frac{e}{p_{MS}}$

[14] To look into the detail See Appendix : Section A.3

Quantity Rationing(QR)

The welfare of the poor under MS is at least as much as the best possible QR policy. Under QR, the planner sets an upper limit on the quantity of essentials which can be purchased. If this limit is higher than the consumption of the poor under MS, aggregate demand is always greater than the aggregate supply.

To see this, suppose the planner sets limit $q^* = max_\omega\{x_{MS}\} = \frac{e}{p_{MS}} =$ and, $p_{QR} = p_{MS}$, trying replicate the MS allocation. $(k_1, k_2)^\omega = \{(0, 2k), (k, k), (2k, 0)\}$ each for a third of the population. [15] Now, the rich with $(k_1, k_2)^\omega = (0, 2k), (2k, 0)$ consume only one good and will be at the rationing limit. However, the rich with $(k_1, k_2)^\omega = (k, k)$ consume $\frac{e}{p_{MS}}$, which is greater than what they consumed under MS - $\frac{1}{2}\frac{e}{p_{MS}}$. This higher demand by one-third of the rich leads to excess aggregate demand at this price, and the market does not clear. Hence, q^* must be reduced, and the poor have their welfare reduced.

The best possible quantity rationing equilibrium under this setting has price $p_{QR} = k + \frac{ne}{2} + \frac{4}{3}e > p_{MS}$ and quantities below. Clearly, the poor are worse off under this policy than MS.

	A	B	C
x_i	$\frac{k_i^\omega}{2k}\frac{e}{p_{QR}}$	$\frac{k_i^\omega}{p_{QR}}$	$\min\{\frac{k_i^\omega}{p_{QR}}, \frac{e}{p_{QR}}\}$

Thus, MS achieves equitable distribution more effectively than QR by addressing the heterogeneity of needs in the economy. With a large number of poor in the economy, this translates into overall increase in social welfare. Section 3 provides a general proof of this phenomenon.

Taxation and Direct Transfers (DT regime)

We now discuss a regime where the policymaker can raise revenue by taxing luxuries and providing direct transfers to all individuals in the economy. We show when taxes are low, transfers are ineffective because revenues are insufficient. Then, we analyse high taxation, which generates both transfers and a reduction in price by restricting demand, but at the same time creates a dead-weight loss. We show that the possible gains of high taxes are eventually outweighed by the dead-weight loss they generate, making MS a better policy to maximize welfare. MS achieves this price reduction more efficiently as it avoids dead-weight losses within the luxury goods market.

We now consider only the "average" essential good with utility $\mathbb{E}[k_1 + k_2] \log x = k \log x$.[16] We first discuss the case of non-distortionary taxation to focus on the decline of

[15] (and for each of the sub-type, i.e., the rich, the poor and the pharmas)

[16] This is a simplification which leads to same results as the heterogeneity assumed under uniform I.I.D. distribution. We deal with a much general set of preferences in the main model of our paper. See section 3.

transfers. Then, we allow distortionary taxation and show how the welfare gain allowing distortion is bounded (for any tax rate) as n goes to infinity. However, the deadweight loss is independent of n.

A (relative) tax rate of τ implies that if the consumer pays p, the producer receives $p(1-\tau)$. The tax is imposed on luxuries and distributed equally amongst all agents in the economy.

Distortion Free Taxes

Lemma 0.1. *The maximum distortion-free tax rate is $\tau = \frac{1}{2}$, and the amount raised by taxes is W/2 in terms of the numeraire.*[17] *Observe that this is independent of n.*

Proof. As luxury goods have linear utilities and their weights in the utility of the rich are ($\frac{1}{2}$ and $\frac{1}{4}$), they will exchange all their endowment only if taxes are less than or equal to $\frac{1}{2}$. Both luxury goods are brought to the market generating a revenue of $\frac{W}{2}$. □

The revenue (R) is transferred anonymously to all agents. Under this regime, the demand for the essential good for the poor is $\frac{e+\frac{R}{(n+3)}}{p_{DT}}$ and for the pharma and the rich is $\frac{k}{p_{DT}}$. Thus, market clearing condition implies,

$$1 = 3\frac{k}{p_{DT}} + n\frac{1+\frac{R}{(n+3)}}{p_{DT}}$$

$$\implies p_{DT} = p^* + \overbrace{R - R\frac{3}{n+3}}^{\text{Price Effect}}$$

The consumption of the poor is

$$\frac{e+\frac{R}{(n+3)}}{p_{DT}} = x^* + \overbrace{\frac{R}{n}\frac{3k}{p^* + R - R/(n+3)}\frac{1}{p^*}}^{\text{Consumption Effect}}$$

Welfare comparison

Now we compare the welfare in the two regimes. We only need to analyse the welfare generated by consuming essential goods as both MS Segmentation and non-distortionary taxation allocate the non-essential goods efficiently. The consumption gain of the poor in MS is $\frac{2e(k-e)}{(n+2)e+k}\frac{1}{p^*} = \frac{2e(k-e)}{[p^*-2(k-e)]}\frac{1}{p^*}$, while the gain to the poor under transfers is $\frac{3kR}{(n+3)(p^*+R-R/(n+3))}\frac{1}{p^*}$.

[17] We say a tax rate is distortion free if there is no distortion to equilibrium allocations of luxury goods

For transfers to outperform MS, It must be that

$$\begin{aligned}\frac{3kR}{(n+3)(p^* + R - R/(n+3))}\frac{1}{p^*} &> \frac{2e(k-e)}{[p^* - 2(k-e)]}\frac{1}{p^*} \\ \implies \frac{1}{n+3}\frac{[p^* - 2(k-e)]}{[p^* + R - R/(n+3)]} &> \frac{2}{3}\frac{e}{R}\frac{(k-e)}{k}\end{aligned}$$

To hold, $\frac{e}{R}$ must be small, meaning that R must be large compared to e. When n increases and e and R are fixed, the inequality reverses, and segmentation is better than the alternative, as is shown for sample values in Figure 2.

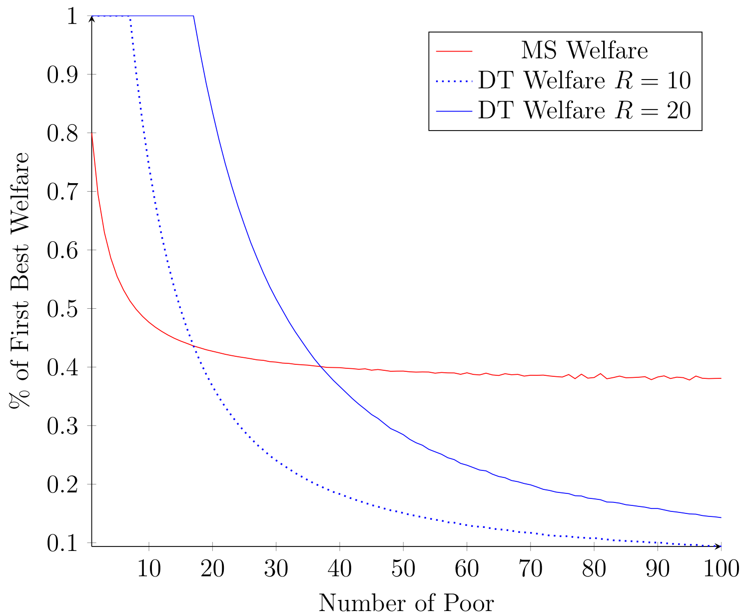


Figure 2: MS vs DT, with $e = 1, k = 2$

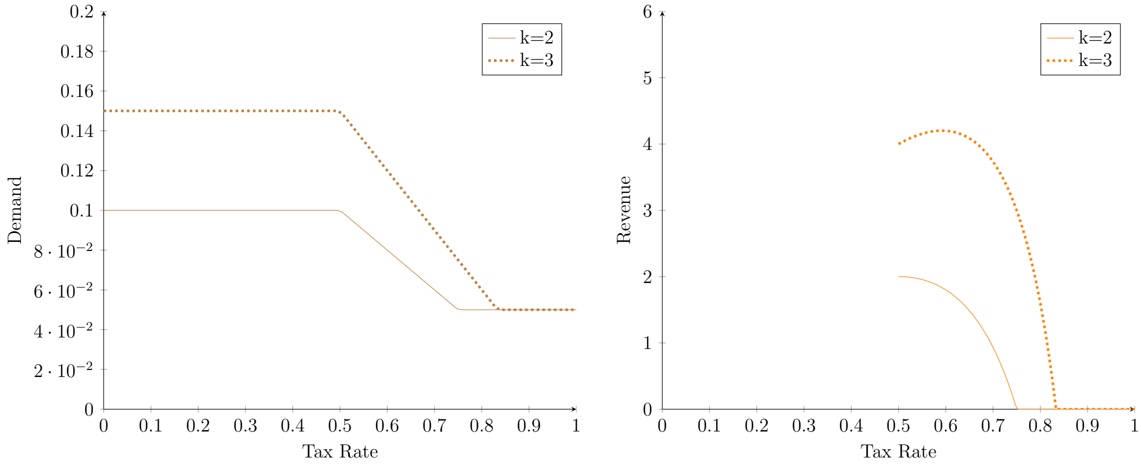
When n is large, the welfare of the poor translates to overall welfare. We show in the appendix section A.3 that as $n \to \infty$, $W_{MS} > W_{DT}$, for all $k > 1$.

Distortionary Taxes

If the tax rate is above half, the demand function of the rich changes. They will either only use universal goods to buy essential goods or sell their luxuries to buy essential goods at a higher price. The kinked demand function can be seen in figure 3a. The first thing the reader should observe in this setup is that there is a large amount of distortion, $\frac{W}{2}$, because no luxuries are traded for each other.

We know that in equilibrium, the price of luxury goods must be 1/2. If they use luxuries to buy essential goods, then the relative price faced by them is $\frac{p}{2(1-\tau)}$. Thus the demand can be given by

$$x_1 = max\{2(1-\tau)\frac{k}{p}, \frac{1}{p}\} \tag{1}$$



(a) Rich Demand
(b) Planner Revenue

To compute the total tax revenue collected by the government, we first determine the income from luxuries that the rich must earn to fulfil their demand for essentials. Since their total demand is given by equation 1, they must earn $2(1-k)\tau - 1$. Let A denote the amount he needs to sell to receive this much income. As the price of the luxury is $\frac{1}{2}$, the following must hold,

$$A \times \frac{1}{2}(1-\tau) = 2(1-k)\tau - 1$$

Let the revenue raised by the planner be R

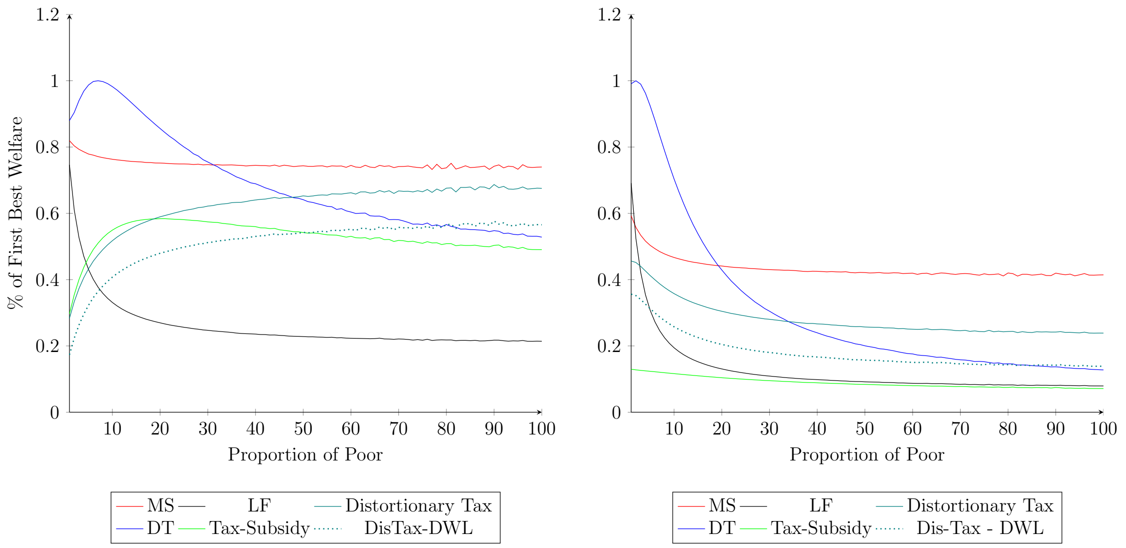
$$R = 2 \times A \times \frac{1}{2}t = 4k\tau - \frac{2\tau}{1-\tau} \tag{2}$$

Note that the above equation demonstrates that government revenue is independent of the endowment of the luxury good. Taxes are bounded from above by 1; thus, tax revenue is bounded. Thus we can see that the gain from distortion is small. On the other hand, the dead-weight loss can be arbitrary. Also, any price effect from distortionary taxes is lower than complete segmentation. Putting the above facts together explains our result. For formal proof, see the appendix section A.3.

We now give graphical representations of some examples of numerical computations of social welfare under different regimes namely MS, taxation with a subsidy on essential good (TS), and commodity taxation with direct transfers (DT) and compare how they perform.

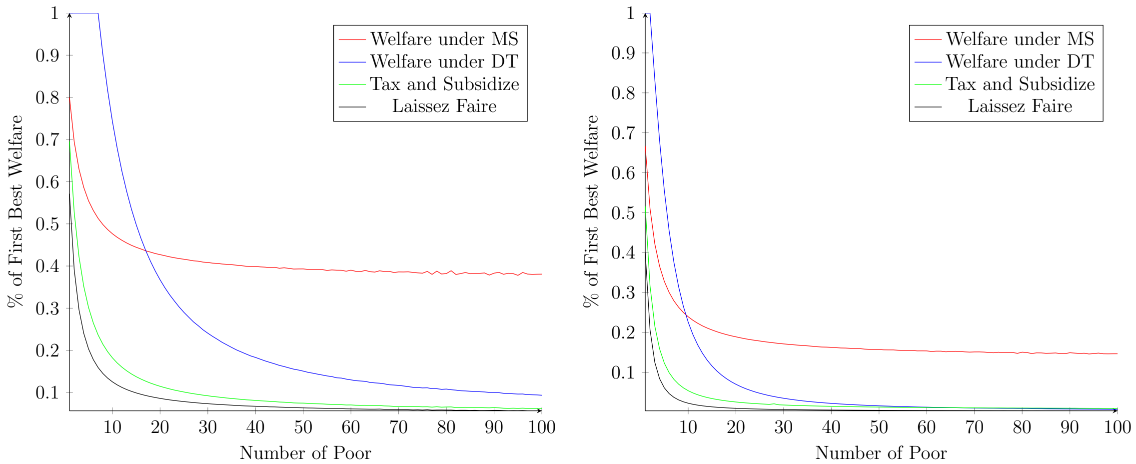
Graphs

We now plot the total welfare achieved by different regimes in our setup with the luxury good endowment (I) to be $W = 20$ and $k = 2, 3$. See, figure 4 and 5.



(a) $k = 2, W = 20$ (b) $k = 3, W = 20$

Figure 4: Welfare of the Society



(a) $k = 3,\ W = 20$ (b) $k = 3,\ W = 20$

Figure 5: Welfare of the Poor

3 Model

Goods and Agents

A bundle of goods is denoted by a k-dimensional vector $X = \{x_1, ...x_k\}$ which is picked from a compact subset $\mathcal{G}$ of the k dimensional reals, $\mathbb{R}^k$, $X \in \mathcal{G} \subset \mathbb{R}^k$. These are then divided into 2 distinct sub-classes which we term **essential** and **non-essential**. The economy has finitely many agents. We refer to each agent by ω and the (finite) set of agents by Ω where $|\Omega| = N$. These agents are heterogeneous with respect to both endowments and preferences. Preferences are separable over goods, such that for individual ω,

$$U^\omega(X) = \sum_{k=1}^{K} b_k^\omega u_k(x_k)$$

Where b_i is an idiosyncratic component and $u_k(x_k)$ varies good by good but is constant across individuals. Every individual is born with an endowment $e^\omega \in \mathcal{G}$ Essential goods and non-essential goods are defined as follows.

Essential goods: A good x_k is called essential if $u'(0) = \infty$.
Non-essential goods: A good x_k is called non-essential if $u'(t) = c \quad \forall t$.

With respect to endowments e_i, individuals are divided into (at least) 3 groups.

Poor: An individual is called poor if he is *only* endowed with the universal good. We think of this good as manual labour or time. They are given by Ω_p where $|\Omega_p| = n$.
Rich: An individual is called rich if he is endowed with a positive amount of any non-essential good (along with the universal good). They are given by Ω_r where $|\Omega_r| = m$.
Don: An individual is called a don if he is endowed with a positive amount of any essential good (along with the universal good). They are given by Ω_d where $|\Omega_d| = d$.

We assume that Dons and Rich are disjoint groups. Thus, $N = n + m + d$. This lets us completely define our environment as an economy $\mathcal{E}$.

$$\mathcal{E} = (\mathcal{G}, \Omega, \bigcup_{\omega\in\Omega}\{U^\omega\}, \bigcup_{\omega\in\Omega}\{e^\omega\})$$

which is a tuple of goods, individuals, utilities and endowments.

The Social Planner

The social planner is utilitarian and wants to maximise the sum of utilities of the gents in the economy. They can levy taxes on goods traded in the market. As such, we rule out taxing endowments directly and allow taxes on only traded endowments. However, the planner is constrained to budget balance, i.e. they can only commit to revenue-neutral policies. Hence, they face a trade-off between creating equality in the consumption of essential goods on the one hand and causing excess distortions by taxing non-essential goods on the other.

Policies available to the social planner

The planner can define a function which maps each **traded endowment** to a **budget set**, and the policy space is simply a family of such functions. Thus a planner chooses suitable policy parameters (per the policies defined below), mapping each endowment to a final budget set. We analyze four policies available to the planner. The definition of each policy is given below.

Taxation and Direct Transfers (DT or transfers henceforth): The planner can tax trades on non-essentials and use the tax to provide direct transfers to agents. Formally, the DT policy is defined as for each endowment level (e), picking τ_k and R, such that

$$\mathcal{B}_{DT}(x-e) = \sum_{k=1}^{K} p_k(1+\tau_k\zeta_k)(x_k-e_k) + R \leq 0$$

where ζ_k is a variable which takes the value of 1 whenever $x_k - e_k \geq 0$, R is the anonymous transfer received by each individual in the economy, and $\tau_k \geq 0$ are taxes on each good k.

Taxation and Subsidy (TS or subsidies henceforth): The planner can tax trades on non-essentials and use the tax to subsidize the essential goods to agents in the economy. Formally, the TS policy is defined as for each endowment level (e),

$$\mathcal{B}_{TS}(x-e) = \sum_{k=1}^{K} p_k(1+(\tau_k-\sigma_k)\zeta_k)(x_k-e_k) \leq 0$$

where ζ_k is a variable which takes the value of 1 whenever $x_k - e_k \geq 0$, $\tau_k \geq 0$, and $\sigma_k \geq 0$ are the taxes subsidies on each good k respectively.

Notice that we assume that all taxes are paid by net sellers in DT and TS.

Quantity Rationing (QR or rationing henceforth): For any essential good x_k, the planner can decide the quantity level q_k, such that each individual can buy the given quantity of a good at a rationed (lower) price $\hat{p}_k$. Formally, the QR policy is defined as

$$\mathcal{B}_{QR}(x-e) = \left\{\sum_{k=1}^{K} p_k(x_k-e_k) \leq 0\right\} \bigcap \{x_k - e_k \leq q_k\}_{k=1}^{K}$$

Market Segmentation (MS or segmentation henceforth): Finally, we introduce a new policy- Market segmentation, where an individual incurs a tax if he trades one type of goods for another. In our setup, the policy entails segmentation between a set including the universal good and essential goods, and another set including all other non-essential goods.[18] Formally, the MS policy is defined as picking τ_{MS} and R_{MS} for each endowment level (e), such that

[18]We take universal good to be non-essential good, keeping in line with the example discussed in section 2.

$$\mathcal{B}_{MS}(x-e) = \left\{ \sum_{\text{essentials}} p_k(x_k - e_k) \leq y + R_{MS} \right\} \bigcap \left\{ \sum_{\text{luxuries}} p_k(x_k - e_k) \leq -(1 + \zeta_k \tau_{MS})y \right\}$$

where $\zeta_k = 1$ when $y > 0$, R_{MS} is the anonymous transfer received by each individual, and $\tau_{MS} \geq 0$ is the segmentary tax level. We refer to $\tau_{MS} = \infty$ as complete MS.

Utility Maximisation

We assume that individual agents are price takers and maximise utility subject to a budget constraint and actions of the social planner. They take prices (possibly nonlinear), endowments and social planner's policy as given.

$$\max_{X \in \mathcal{B}(x^\omega - e^\omega)} U^\omega(X)$$

Where $\mathcal{B}(x^\omega - e^\omega)$ represents the budget set corresponding to the endowment of the agent and the planner's policy.

We will always remain in an environment where the poor are on the boundary, i.e. they only consume essential goods in a laissez-faire economy. Formally that would imply $u'(\frac{m}{p}) > b_k^\omega \quad \forall \omega, k$ where m is the income of the poor and p is the price of essential goods in *laissez-faire* setup.

We now state assumptions on the economy under which the relative benefits of different policy choices of the planner are independent of the number of goods. This allows us to prove general results by only analysing the case with one (or two) essential good(s).

Aggregate Symmetry (Definition): Consider an economy with K essential goods. The preferences for each individual are defined by a vector of parameters $b = (b_1, ...b_k)$. We say that an economy satisfies aggregate symmetry if the following is true: Suppose the preferences are distributed according to $\mu \in \Delta b$ and the CDF corresponding to μ is$F(b_1, ...b_n)$, then for any $i, j \leq n$ it must be that $F(..., b_i, .., b_j) = F(..., b_j, .., b_i...)$.

Lemma 0.2. *Under aggregate symmetry, the social planner treats essential goods symmetrically.*

Proof Idea: The planner wants to maximise social welfare by using policy parameters defined above. First, notice that the frontier of allocations encloses a convex set because demand (excess supply) functions are convex in revenue spent (as demand is price inelastic). Now suppose the planner chooses an asymmetric final allocation. However, a symmetric permutation of the allocation remains feasible because of aggregate symmetry. Due to the convexity of individual utilities, the intermediate allocation has a strictly higher value to the planner[19], which is a contradiction. Thus, goods must have symmetric allocations. For formal proof, see A.4.

Lemma 0.3. *As all essential goods are treated symmetrically by the planner, it is sufficient to consider the case where we have one (or two) essential goods.*

Proof Idea: As all goods are treated symmetrically by the planner, we can simply constrain the planner to consider policies where all goods are treated identically. This reduces to a problem where the planner only chooses a policy for only one good, and the rest of the goods follow the same policy due to symmetry. For formal proof, see A.4.

Given that it is sufficient to consider one essential good let us call it x with price p. There is one unit of essential good owned by one don. We now state our main assumption.

Assumption 1. Essential Goods are price inelastic.

$$\frac{\partial}{\partial p_e}\left[p_e x_e(p_e, ..)\right] \gneq 0$$

Where p_e is the price of the essential good, and x_e is its demand.

Remark. we use n to refer to the number of poor agents in the economy.

Our first theorem compares Market Segmentation (MS) with Direct Transfers.

Theorem 1. *Under assumption 1 and significant gains from trade within luxury goods, $\exists N$ such that for all $n \geq N$. MS dominates DT.*[20]

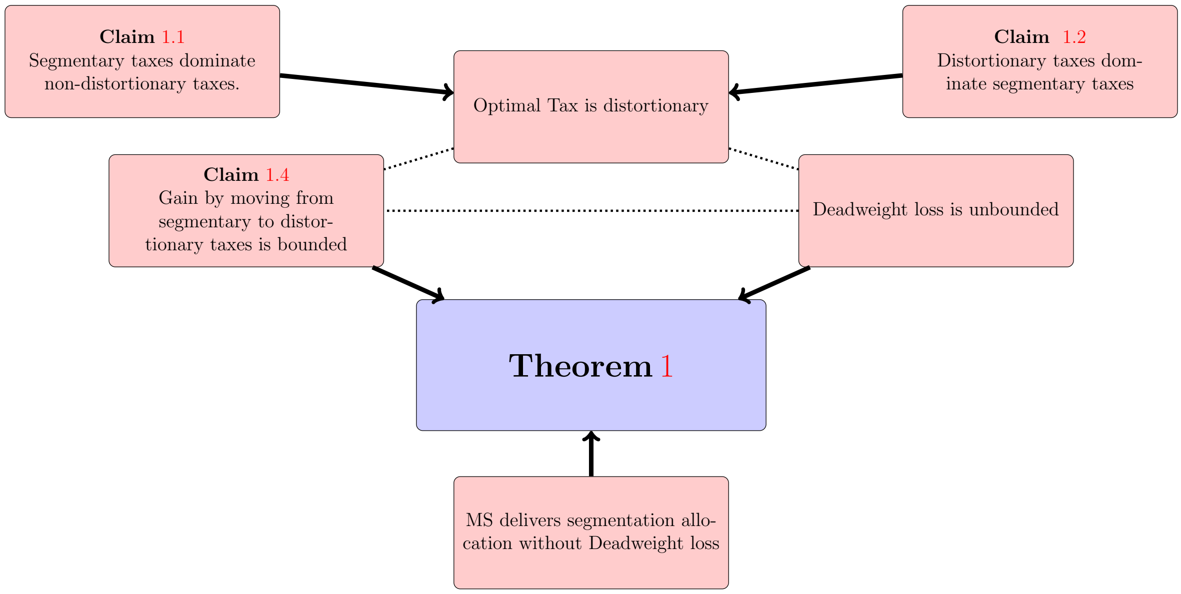
We prove this theorem through a series of claims. We first prove that if the essential good is price inelastic and the number of poor in the economy is high, commodity taxation restricting all trade in luxuries [21] gives higher social welfare than any level of non-distortionary taxation. The reason for this is three-fold. First, if n is high, the effective transfer to the poor is low. Secondly, with the inelasticity of essential goods, non-distortionary taxation does not decrease the demand of the rich by a lot and consequently just increases prices. Third, the gain from non-distortion is bounded and independent of n. We now formally state the claim.

Claim 1.1. *If assumption 1 holds, $\exists N \in \mathbb{N}$, such that, for $n \geq N$ social welfare in a regime where commodity taxation is high enough to restrict all trade in luxury goods dominates welfare for any non-distortionary tax regime.*

[19]because sums of convex functions remain convex.

[20]Significant gains from trade means that the dead weight loss in the economy at the tax level of no trade between luxury goods is greater than G. Where $G = n(U(\frac{e+\frac{R}{n+3}}{p*}) - U(\frac{e}{p_{DTS}})) + m(U(x(p*, \tau)) - U(p_{DTS}, \tau)) +$
$U(x(p*)) - U(p_{DTS}) - DWL_{DT*} + DWL_{DTS}$, we show this to be bounded

[21]we now refer to such high tax as *segmentary*.



Proof. For formal proof, see appendix A.4. ☐

The second step is to show that a taxation level that leads to no trade in luxuries (we call this the segmentation level (τ_s)) is dominated by some distortionary level of taxation which permits some trade. This is again because of inelasticity. Due to the price inelasticity of the essential good, the tax revenue increases in tax rate but becomes suddenly zero at the segmentation level. We show that the gain from going to the segmentation level is always dominated by the revenue that can be generated by taxing a little less. We now formally state this claim as well.

Claim 1.2. *Under assumption 1, there exists a distortionary level of taxation τ^*_{DT} that achieves higher welfare than τ_s.*

Proof Idea. The key observation is that while demand behaviour varies continuously at the cutoff tax τ_s, the revenue from said tax falls to zero discontinuously. For formal proof, again see the appendix A.4.

Let $\tau_{DT}*$ be the optimal tax, and let the revenue it generates be $R_{DT}*$. We now show that the welfare gain from moving from τ_s to $\tau_{DT}*$ is bounded and independent of n.

Claim 1.3. *Let the welfare gain generated by revenue of R where the relative number of poor is n be $G_n(R)$, then*

$$\exists \quad G^*(R) \quad \textit{such that} \quad \forall n \quad G_n(R) \leq G^*(R)$$

Proof Idea. Note that taxes can only be levied on non-essential goods. Thus the revenue generated is independent of n. Although some revenue (R_{DT}) is generated and distributed to all agents at τ^*_{DT}, it does not help the poor much as per capita transfers decrease with

n. Moreover, under price inelasticity, the transfers translate into price increases, and the welfare gain is limited. Again, for the formal proof, see the appendix A.4.

Claim 1.4. *If the dead-weight loss due to optimal distortionary taxation is more than the above gain (G^*), then MS outperforms segmentary taxation.*

Proof. Welfare under Market Segmentation (W_{MS}) is given by

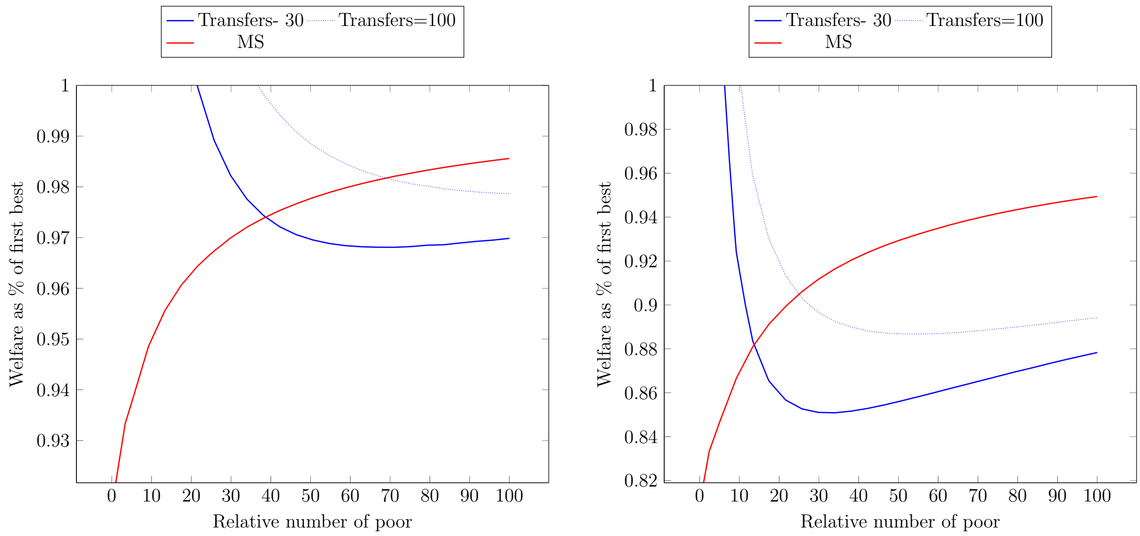
$$W_{MS} = W_{DTS} + DWL$$
$$W_{DT}^* = W_{DTS} + G^*$$

Hence,

$$W_{ms} - W_{DT^*} > 0 \iff DLW > G^*$$

□

Now we attach some computational experiments to show that under price in-elasticity of essential goods, welfare under MS overtakes welfare under DT quite fast.



(a) Price Elasticity $|\frac{1}{1-\sigma}| = 0.8$

(b) Price Elasticity $|\frac{1}{1-\sigma}| = 0.6$

Figure 6: MS vs DT

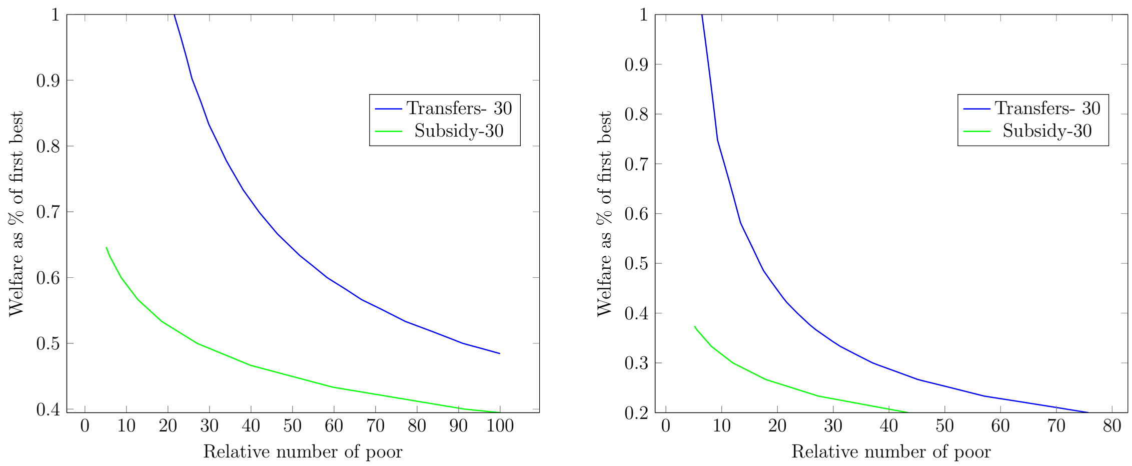
After proving that MS dominates DT when n is large and essential goods are price inelastic, we now show that DT dominates a subsidy regime under the same conditions. Together, this lets us conclude that MS also dominates the subsidy regime.

Theorem 2. *Under assumption 1, given any revenue R, $\exists N \in \mathbb{N}$, such that, $\forall \quad n \geq N$, direct transfers are more efficient than a subsidy on essential goods.*

Proof Idea. Fixing the amount of tax revenue makes the deadweight loss across regimes equal. Thus, we only need to focus on the distribution of essential goods in the economy.

Notice that subsidies work by driving a price wedge between sellers and buyers. This, in turn, increases commodity supply by increasing the price sellers receive. On the other hand, transfers increase the price for **both** the rich and the sellers, thus decreasing their consumption and increasing the net supply for the poor.

This means that even though the price increases may be small in the direct transfers regime relative to the subsidy case, the inelasticity of demand implies that *a large change in price for a few will not be as effective as a small change in price for many.* The number of people affected by direct transfers is always larger than subsidies, so they are more efficient. For formal proof, see appendix A.4.



(a) Price Elasticity $|\frac{1}{1-\sigma}| = 0.8$ (b) Price Elasticity $|\frac{1}{1-\sigma}| = 0.6$

Figure 7: Transfers vs Subsidies

Theorem 1 and Theorem 2 together present the primary insight of our paper. If the essential good is inelastic and society is highly unequal in terms of endowment distribution, MS outperforms taxation-based instruments, specifically transfers and subsidies.

However, one may argue that outcomes generated by MS can also be implemented using direct quantity rationing(QR). We now show that while QR mimics MS if there is one good,[22] the poor are better off under MS if there is more than one essential good and heterogeneity in preferences over essential goods.

Theorem 3. *Consider an economy with two essential goods and idiosyncratic preference heterogeneity over these. In this setup, MS improves the welfare of the poor more than the quantity rationing of essential goods.*

Proof Idea. Given the heterogeneity in preferences over essential goods, MS takes advantage of this heterogeneity which QR cannot do. Under MS, the rich consume essential goods like the poor. Under QR, an upper limit is set on the consumption of **each** essential good. Thus, the rich can consume as much as they can of each of them. This

[22]Note that quantity rationing can be implemented by non-linear taxation of the essential good. See Gadenne (2020)

implies that less is left for the poor under QR, particularly those who vastly prefer one essential good to the other. As Weitzman (1977) notes, price systems can better address heterogeneity of needs within a population. MS dominating QR is a perfect example of this astute observation. Again, for formal proof, see the appendix A.4.

We have shown that complete MS, i.e. not allowing **any** trade between essential and luxury goods, can be a more effective policy than direct transfers, subsidies and quantity rationing under certain conditions. However, do we necessarily want to 'completely' essential and luxury goods markets? In particular, a planner might allow what we call a segmentary tax. This tax is levied if the income generated by selling luxury goods is used to buy essential goods but not if luxury goods are traded amongst themselves. We call this policy - Partial MS.

The next theorem demonstrates that partial MS is always better than complete MS in our setup. At least some trade should be allowed between luxury goods and essential goods. This welfare gain occurs as the revenue earned by lowering the tax a little from the segmentary level improves welfare more than the decrease in welfare due to an increase in the price of essential goods.

Theorem 4. *Under assumption 1, partial MS dominates complete MS.*

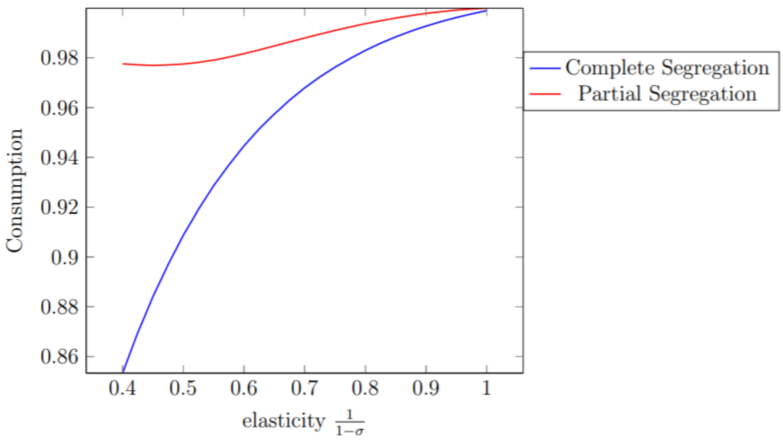
Proof Idea. The intuition for this theorem is essentially the same as that of claim 1.2 of Theorem 1. Given the price inelasticity of essential goods, the tax revenue increases in tax rate but becomes suddenly zero at the segmentation level. The gain from going to complete segmentation is always dominated by the revenue generated by taxing a little less because the demand behaviour varies continuously at the cutoff tax τ_{cms}. In contrast, the revenue from said tax falls to zero discontinuously. Please refer to appendix A.4 for the main proof.

As elasticity plays a key role in the above argument, we now show some graphs comparing consumption in the partial segmentation regime to the complete segmentation regime with varying levels of elasticity. As the price elasticity of essential goods increases, the difference between the two regimes decreases because the revenue loss due to tax decreases with the increase in elasticity. Nevertheless, for all levels of price elasticity less than 1, it is clear that partial segmentation dominates complete segmentation.

As complete MS is never optimal in our setup, we now characterize the optimal segmentary tax. Moreover, notice under the conditions where Theorem 2 holds, i.e. large n and price inelasticity of essential goods, we know that the revenue these taxes generate should be used as direct transfers rather than subsidies.

Theorem 5. *At the Optimal segmentary tax*

$$w_r\lambda_r\left[\frac{\partial t}{\partial\tau}(1+\tau)-\Big(p_1(x_1-e_1)+p_2(x_2-e_2)-t\Big)\right]+w_d.\lambda_d\frac{\partial t}{\partial\tau}+w_p\lambda_p\frac{\partial t}{\partial\tau}=\sum\mu_k\sum\frac{dx(p,\tau)}{d\tau}$$



where w_r, w_d, w_p are the weights of the rich, the don and the poor, respectively. λ is the multiplier associated with each group, and μ is the multiplier associated with the resource constraint.

Proof Idea. We must consider the cost and benefits of using a segmentary tax to understand this condition. When we increase the segmentary tax, it has two major effects. First, it increases the 'segmentary tax' revenue, which can be used as transfers. Second, it also decreases the price of essential goods by making them more expensive for the rich, thus effectively 'freeing' up the resource. In the equation above, the left-hand side represents the weighted utility to society of the transfer that an additional tax unit allows the planner to provide. It also captures the loss of utility to the rich due to an increase in taxes. The right hand side represents the increase in utility to the society by 'freeing up' the essential good via a reduction in demand of the rich. Notice that if the rich have a low weight w_r in the equation above, then the optimal segmentary taxes will be very high and close to the complete segmentation level because tax revenue is increasing in the tax rate for inelastic goods. For formal proof, see the appendix A.4.

In this section, we presented a formal general equilibrium model to demonstrate when the policy of Market Segmentation dominates Direct Transfers, Subsidies and Direct Quantity Rationing implemented via non-linear taxation. We also discussed how Partial MS is always better than Complete MS when the essential good is price inelastic and then went on to characterize the optimal segmentary tax. In the next section, we will discuss some applications of these taxes and how they can be implemented.

4 Applications and Implementation of segmentary taxes

This section discusses how segmentation and segmentary taxes can have important applications in dealing with issues facing today's world. We also discuss how they can be implemented and how they are related to other policy instruments.

4.1 Applications

4.1.1 Automation

An issue that will starkly affect the economy and distribution in the near future is automation. Scott Santens, in a Boston Globe 2016 article, summarises the problem succinctly in the following words: "nothing humans do as a job is uniquely safe anymore. From hamburgers to health-care, machines can be created to successfully perform such tasks with no need or less need for humans, and at lower costs than humans". He is not the only one who has raised this as a matter of concern. Recent work by prominent economists highlights the need to deal with automation driven inequality (see (Aghion et al., 2017) ; (Acemoglu and Restrepo, 2019); (Mookherjee and Ray, 2020)).

In this section, we study the application of various policy instruments discussed in the previous section to deal with the problems that may arise due to automation. We argue that automation can have two different types of impacts on the endowment/skill distribution in the economy. First, it can lead to a situation where machines start performing many tasks, consigning erstwhile 'skilled' labour performing these to unskilled work. We call this the 'displacement effect' of automation. It increases the relative number of unskilled (poor) workers in society. On the other hand, automation can also have a different impact on the endowment distribution if robots erode the value of unskilled workers in the labour market without having a considerable impact on skilled jobs. We call this the 'erosion effect'.

We find that this distinction is crucial because it leads to different policy prescriptions. Since the displacement effect of automation renders skilled workers unskilled, this is akin to n (i.e. the relative number of poor) increasing in our model. As discussed in Theorem 1 and Theorem 2 of this paper, if essential goods are price inelastic and there are sufficient gains from trade within the luxury good market, as n increases, MS dominates DT and subsidy regime. Hence, we argue that if automation leads to high displacement in the economy, MS can potentially be more effective than other policies at dealing with its distributional impact.

However, if automation produces more of the erosion effect, direct transfers are more effective at dealing with the issue than segmentation. Realize that when the value of the universal good in the economy decreases, the purchasing power of the poor decreases rapidly. Thus increasing the purchasing power of the poor through direct transfers becomes very important, causing the DT regime to dominate the MS and TS regimes, which only affect welfare by lowering prices of essential goods.

To study the impact of the 'erosion effect' more formally, we go back to our basic model of section 3. We model this effect as a fall in the value of parameter b associated with the universal good (e) in the utility function of all consumers in the economy. We next state a theorem formally proving that as the value of the universal endowment falls in the utility functions of individuals in an economy, DT dominates MS.

Theorem 6. *As the value of the universal endowment falls in the utility functions of individuals in an economy ($b \to 0$), direct transfers are more effective in improving social welfare than Market Segmentation.*

We again prove this theorem in a series of claims. The first claim establishes a strong intuitive and mathematical link between the weight of the universal good (b) and the actual endowment of the universal good with each individual in an economy.

Claim 6.1. *Let b be the weight associated with the universal good in all individuals' utility function, and let e be the endowment of the universal good with each individual in the economy. As $b \to 0$, the equilibrium bundle in this economy produces same the welfare as $e \to 0$.*

Proof Idea. The intuition behind this claim is simple. If the marginal value of the good goes to zero, nobody is willing to pay for it in the market. Consequently, the income it generates is zero. This is akin to an individual losing his endowment itself as it ceases to be valuable. The formal proof is given in the appendix A.4.

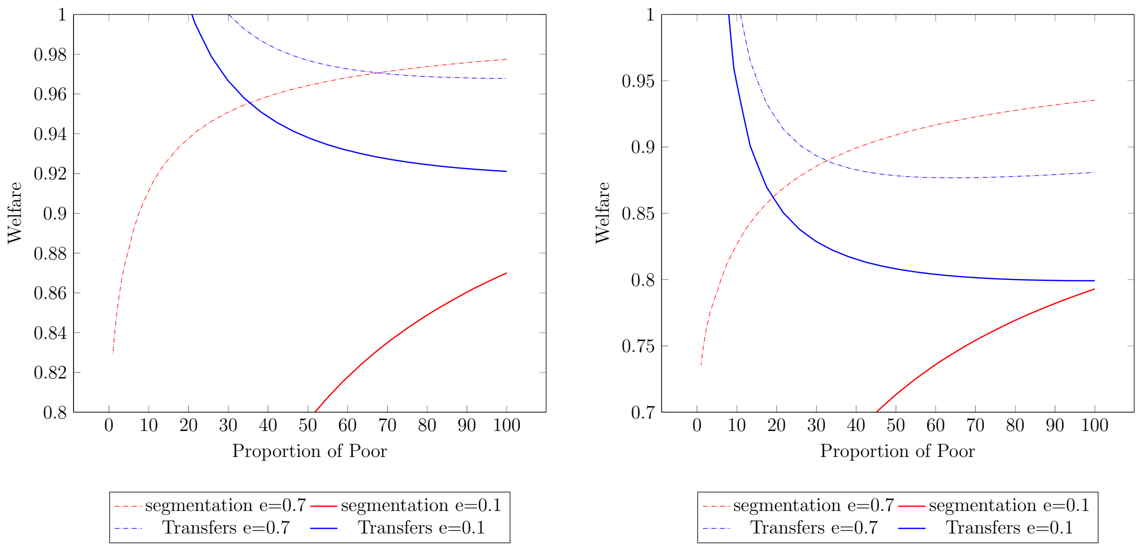
This claim implies that we can study a decrease in the marginal value of the endowment as a decrease in the amount of endowment holding the marginal value constant. Thus, we now discuss various groups' relative marginal utilities as the universal good's endowment goes to zero.

Claim 6.2. *As the endowment of the universal good goes to zero, the ratio of the poor's marginal utility to the rich's marginal utility goes to infinity. Thus, only the utility of the poor remains welfare relevant.*

Proof Idea. Again the intuition for this claim is straightforward. We know that the poor are just endowed with universal good, e. If that is falling in the economy, they cannot even consume a little of the essential good. We have assumed that as consumption of essential good goes to zero, $u'(x) \to \infty$. Given that the rich (and the dons) have positive incomes from selling other goods they are endowed with, their consumption does not fall below a certain positive level. Thus, the marginal utility is strictly bounded. Together, this proves the claim.

Finally, the claim below completes the proof.

Claim 6.3. *As the endowment of the universal good in the economy goes to zero, the direct transfer dominates complete Market Segmentation.*



(a) Price Elasticity $|\frac{1}{1-\sigma}| = 0.8$

Figure 8: Price Elasticity $|\frac{1}{1-\sigma}| = 0.6$

Proof Idea. We know that the MS regime improves social welfare by lowering the demand of the rich and thus, lowering the prices of essential goods, making it easier for the poor people to buy more. However, if poor people have no income because they have no endowment, even lower prices cannot improve welfare. Thus increasing the purchasing power of the poor through direct transfers becomes very important if automation is eroding even minimal purchasing power of even a few people in the economy. For formal proof, please refer to the appendix A.4.

Theorem 7. *As the value of the universal endowment falls in the utility functions of individuals in an economy* ($b \to 0$)*, direct transfers are more effective in improving social welfare than a subsidy on the essential good.*

Proof Idea. This theorem follows using similar claims used above.[23] As the weight associated with the universal good goes to 0 ($b \to 0$), it is as if the endowment of universal good (e) is going to zero. If e goes to 0, then the ratio of marginal utility of the poor to the rich (or the don) goes to infinity. Moreover, as in the case of MS, subsidies also work by lowering prices of the essential goods. However, as $e \to 0$, the poor have little to no purchasing power. Hence, increasing the purchasing power of the poor through direct transfers becomes very important, giving us the result. Again, the formal proof is in the appendix A.4.

[23]In particular, claim 6.1 and 6.2, remain exactly the same. Claim 6.3 needs to be modified to now discuss DT vs TS (rather than DT vs MS). However, the basic idea remains the same. For a former proof see the appendix

4.1.2 Dealing with trends like the 'Superstar phenomenon.'

Recent work shows that emergent trends in the 21st century like the 'superstar phenomenon' (Scheuer and Werning, 2017) and markets with winner-take-all characteristics (Rothchild and Scheuer, 2016) render conventional policies like non-linear labour taxation only mildly effective. Scheuer and Slemrod (2019) notes and discusses various phenomenons as to how taxation of the super-rich is becoming increasingly challenging in recent years. In such labour markets, superstars or winners cannot optimally face a high marginal income tax on labour without creating a lot of dead-weight loss in the economy or because there are convex returns to the effort exerted. We argue that Market Segmentation can help policymakers deal with these challenges.

In appendix section B, we introduce production and allow non-linear labour taxation. We show that our fundamental insight – MS can protect the prices of essential goods from rising to unaffordable levels without very large deadweight losses – holds in an economy with production, heterogeneous labour productivity, and non-linear taxation (Saez, 2004). The intuition for the results essentially remains similar. If the essential goods are inelastic, dead-weight losses in the production due to the high non-linear labour taxation required for equity concerns is large. Again, it is better to segment consumption into essential and luxury goods under this scenario. The planner can then cap expenditure on essential goods, leading to a more equitable distribution of essential goods while avoiding dead-weight loss in production.

In sub-section B.4.2, we show that in our setup if there is significant mass on the upper tails of the skill distribution (which can be interpreted as the presence of superstars in the labour market), the dead-weight loss increases under non-linear taxation with transfers, but is unaffected under MS. Hence, the gains from using MS over non-linear taxation increase in the presence of superstars in the labour market.[24]

When the planner uses MS, agents still have incentives to work and produce because they can use their incomes to consume luxuries. However, they cannot spend their 'excess superstar income' on essentials which keeps their prices low. Thus our policy allows the planner to de-link prices of essential goods from incentives to produce. To put in terms of an example, even if healthcare and affordable housing are rationed through Market Segmentation, people still have incentives to work hard to consume more i-phones, Nike sneakers and trips to Europe. Thus, when labour markets have dense right-tailed skill distributions, segmentation can avoid inefficient levels of taxation while maintaining a more equitable distribution of essential goods.

In general, it should be noted that any phenomenon that increases inequality in endowment distribution and increases dead-weight loss in taxation renders MS a more effective policy than transfers or subsidies in our setup. We think this general insight of our paper

[24]Note that we demonstrate our results in a discrete economy, thus making no assumptions at all about the return to ability being convex or concave. Indeed if these are convex our results would be starker.

might also be applicable in other important socio-economic spheres.

4.1.3 When should there be a price to cut the queue?

As a final discussion on applications of our results, we note that our discussion comparing the effectiveness of Market Segmentation (MS) to commodity taxation (DT or TS) can also be thought of as comparing the effectiveness of quantity rationing (QR) to (DT or TS). QR is a traditional policy followed in many places; for example, the National Health Service (NHS) of the UK.[25]

Our model can be used to have a more informed discussion on whether the NHS should continue to operate the way it does or whether cash transfers with a privatised health care system would fare better. Our model suggests that if healthcare provision is inelastic in demand and supply and there is significant income inequality within the population, providing health services through rationing would be better.

Perhaps, an application of Theorem 4 of our model suggests specific changes that can be made to the rationing system. Theorem 4 proves that if essential goods are price inelastic, it is better to segment the essential good market partially, implying that under our assumptions, even in NHS-style systems that ration essential goods, **there should always be a price to cut the queue.** The revenue raised should be used as transfers to improve the distribution of health services in the economy. This result resembles Dworczak et al. (2020), which shows that a similar two-price system is optimal in a specialised mechanism design setup. Dworczak et al. (2020) considers an economy where, by assumption, the marginal utility of money differs across groups. We do not assume that marginal utility differs across groups, but we can endogenize this by considering a varied endowment distribution that leads to different consumption levels. Hence, we find a result similar to Dworczak et al. (2020) in spirit, if not form.

However, given that MS dominates QR in our setup, our results suggest that integrating certain markets (essentials) while segmenting it from others (non-essentials) can improve welfare. Hence, we think that the discussion in the market design literature might be enriched if it considers the welfare effects of integrating markets in general.[26] We thus think that endogenizing the marginal utility of money and integrating markets can be a promising avenue for future research while considering market design.

Having discussed some applications of the MS policy, we move on to discuss how it can be implemented. We discuss its implementation alone and with the complimentary policy instrument of commodity taxation with direct transfers.

[25]For more detail on how NHS works see their stated constitution https://www.gov.uk/government/publications/the-nhs-constitution-for-england/the-nhs-constitution-for-england.

[26]See Roth (2015) for a review of the market design literature.

4.2 Implementation

In this sub-section, we discuss how a social planner can implement MS if she decides to do so. It is important to note that the planner need not know individual endowments or preferences of different people in the economy, but only their overall distribution. In this regard, the policy of Market Segmentation and commodity taxation require equivalent information to be implemented.

However, to enforce segmentation, the planner must stop the rich from providing side payments to "the dons," i.e. owners of essential goods. To enforce this, it is sufficient that the planner can link and monitor an individual's expenditure on essential goods. This can be done by linking social security or unique identification numbers to expenditure on essentials. Many social security and welfare schemes across the globe require such monitoring of individual level consumption. For instance, the 'consumption' of Covid vaccines was linked to individuals and monitored. Similarly, the Public Distribution System (PDS) in India, which provides food grains on the principle of quantity rationing (see Gadenne (2020)), links and monitors household level purchases of 'essential items.' Thus, monitoring might be costly but feasible with better technology at the disposal of the social planner.

Admittedly, implementing MS would require some cost of monitoring and the possibility of black market transactions. However, similar problems exist in implementing policy instruments like taxation and quantity rationing. Tax evasion is a topic well studied in economics and research papers note its high prevalence in many socio-economic settings.[27] Moreover, tax compliance and revenue collection are costly.[28] Similarly, quantity rationing has its problems with implementation, including high cost and corruption.[29]

We think that all these costs, the feasibility of monitoring, and the possibility of the emergence of black markets should be considered while weighing the costs and benefits of different policies considered in this paper before implementing them. However, we see no reason ex-ante to disregard MS on these grounds because other policies also deal with similar issues.

Another possible issue with implementing MS can be a lack of a universal good in the real world. In the main model discussed in section 3, MS segments the market for essential and universal goods from non-essential goods. Since everyone is endowed with universal good in our setup, this creates an equitable distribution of all essential goods. However, it might be the case that there is no universal good in the economy. After all, manual labour might also have heterogeneous productivity across individuals.

[27]see, Slemrod (2007)

[28]Pope (2002)

[29]The Public Distribution System (PDS) in India works on the principle of quantity rationing (see, Gadenne (2020)). Many have noted corruption in its implementation at various levels. See, https://www.ndtv.com/india-news/corrupt-public-distribution-system-says-supreme-court-panel-412887

In such scenarios, MS can simply be implemented by *expenditure ceilings* on goods typed as essential. By an expenditure ceiling, we mean that agents will not be taxed till their level of expenditure on essential goods is below the ceiling. However, if agents spend more than the ceiling on essential goods, they would have to pay a segmentary tax. The planner can just determine the Rawlsian income in the economy, i.e. the income earned by the least well-endowed individual in the economy and set that as an expenditure ceiling.[30] In effect, this policy will produce the same equilibrium outcomes as the model with a universal good. This is because, in our model, the price of the universal good times the endowment of it determines the expenditure of essential goods by the agents, which will be equal to the Rawlsian income described above.

The expenditure ceilings can also be determined as a function of the minimum wage in an economy. It is worth noting that using expenditure ceilings allows planners to choose the ceiling at a possibly different level than the Rawlsian income. The optimum level of the ceiling will depend on the distribution of endowments in the economy and thus can be a topic of future research.

Finally, we note in this section that Market Segmentation and commodity taxation with direct transfers are two policies that are complementary in nature. Thus, the planner does not necessarily have to choose one over the other but can implement them together. We now present a theorem that proves that unless one policy (MS or DT) in itself achieves the first-best outcome, then the policy of Market Segmentation and the policy of commodity taxation should be used together to improve social welfare in our setup.

Theorem 8. *Assume that the poor are consumption constrained even after implementing either of the two policies (i.e. segmentary taxes or commodity taxation with direct transfers). Under this condition, using the second policy along with the first improves social welfare.*

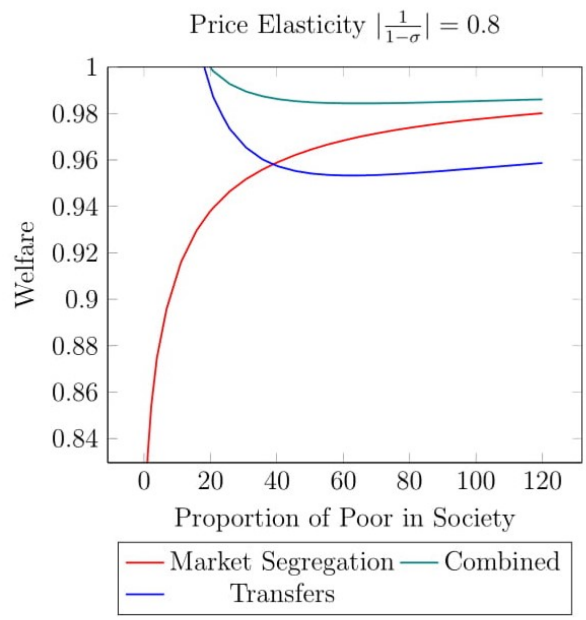
Proof Idea. This is a natural result because DT and MS work to improve welfare using completely different approaches. Transfers increase the purchasing power of the poor, thus helping to create equitable consumption of essential goods. On the other hand, segmentation drives down the prices of these essential goods. Hence, they turn out to be complementary and work well together. For formal proof, see the appendix A.4.

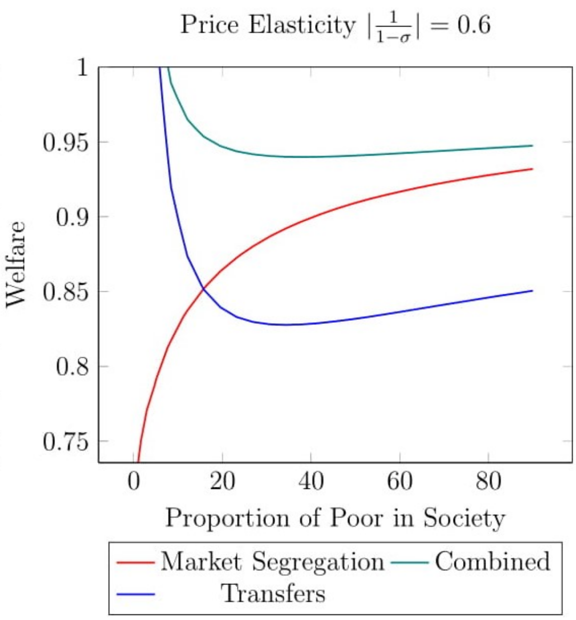
An example of direct transfers and quantity rationing working well together has been recently seen in India. Despite a mediocre macroeconomic performance, India's poverty rate has declined rapidly. Some have attributed this to the welfare programs carried out by the Indian state.[31] Indian state has combined direct transfers with quantity rationing

[30] The name Rawlsian income follows the tradition of social choice literature where the social welfare function which maximised of the least well-off individual is called Rawlsian. See https://plato.stanford.edu/entries/social-choice/#DocParDisDil

[31] See, the discussion in Bhalla et al. (2022).

to improve the welfare of the poor. We have discussed that MS is equivalent to quantity rationing in the case of only one essential good. Together, these facts suggest some evidence in line with the theorem above. We demonstrate this insight below with the help of figures plotting welfare on the y axis against the number of poor people on the x-axis. The figures illustrate that social welfare improves when transfers and MS are implemented together.





5 Conclusion

It is common for policymakers to intervene in a laissez-faire market equilibrium to improve social welfare, particularly to alter the final allocation of goods for redistributive purposes. This paper introduces a new policy instrument – Market Segmentation – and compares it to existing instruments – Direct Transfers, Subsidies and Quantity Rationing. Our model, though stylized, provides a systematic framework to think about how different policies can be more welfare-enhancing in different conditions.

Policymakers and researchers need to understand the conditions under which to use specific policies. For Market Segmentation (MS) to work at-least two types of goods must exist in the economy. In our model, there is one type of goods which are distribution relevant to the planner – essentials.[32] Second, there is another type (non-essentials), that is not relevant in terms of distribution. However, they being traded (or produced) is important for social welfare. As long as there are some goods which are distributionally

[32]Formally, under separability, this means the utility function for the essential good is concave and satisfies the Inada conditions

relevant and another type on which taxation produces dead-weight losses, MS becomes a policy of significant relevance to the policy maker.

Even when the above conditions hold, MS may not be the best policy. Perhaps the main contribution of our work is to demonstrate that under certain conditions, w.r.t. elasticity of essential goods and the number of low-income individuals, MS performs better than two commonly used policies, i.e. subsidies and direct transfers. We also show MS weakly dominates quantity rationing (QR) in our model.

Crucially, different policies' relative effectiveness depends on the price inelasticity of demand and supply of essential goods. The lack of response of demand and supply to price is what renders direct transfers and subsidies ineffective compared to MS. Thus, policymakers need to carefully study the elasticity condition of the essential goods before making a policy decision.

The price elasticity of demand being inelastic might be an innocuous condition; however, the price elasticity of supply being inelastic is not, particularly in the long run. The supply of many essential goods may significantly respond to prices in the long run, for example, food grains. However, there might be many scenarios where supply is constrained even in the long run due to technological constraints. For example, affordable housing in a city is constrained by the availability of land; similarly, increasing the supply of cardiac surgeons might be difficult even in the long run due to the nature of training needed.[33] Thus, we think that policymakers should be careful in applying our results to the real world. However, the complimentarity of MS and DT as policy instruments is a particularly nice property helping the policy makers. Since complimentarity implies MS should be considered (alongside DT) even if the conditions we lay out are not entirely verifiable.

In our framework, we also demonstrate that under the same elasticity conditions discussed above, transfers (DT) is better than subsidies (TS). Under DT, both rich and essential good providers reduce their demand for essential goods, leading to more equitable distribution. However, under TS, only essential good providers are affected. The inelasticity of demand implies that a large change in price for a few will not be as effective as a slight change in price for many. We also find that partial segmentation is better than complete segmentation under inelasticity because the segmentary tax revenue increases in tax rate but jumps to zero, in a discontinuous fashion, at the complete segmentary level. We also consider different applications of our results and discuss how our model and results might have many real-world applications.

We also think that many more sophisticated instruments can be constructed using our method of taxing transactions between types of goods. We think our paper can provide a base for promising future research to deal with questions of inequality, taxation and

[33] It takes 16 years of training post high school to train a cardiothoracic surgeon in the USA and similar time in Germany according to Tchantchaleishvili et al. (2010)

even instruments to promote (dis)saving and investment.

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A Appendix

A.1 Example 3 good

Since the utility of each type is identical, the utility maximization will lead to the following demand functions.

$$
\begin{aligned}
x_1 &= \frac{k}{p_1} & & if\ m \geq k \\
x_1 &= \frac{m}{p_1} & & if\ m < k \\
x_2 = x_3 &= 0 & & if\ m < k \\
x_2 + x_3 &= m - k & & if\ m \geq k
\end{aligned}
$$

where $m = p_1.e_1 + p_2.e_2 + p_3.e_3$

For markets to clear, we require $p_2 = p_3$. Let that be the numeraire. Hence we know $m_B = e$ and $m_C = I + 1$ and $m_A = p_1 + e$. If $1 < k < I + e$, then we know that type B will demand $x_1^B = \frac{e}{p_1}$ and individual C will demand $x_1^C = \frac{k}{p_1}$. Moreover assuming in equilibrium, $m_A = p_1 + e > k$ individual A will also demand $x_1^C = \frac{k}{p_1}$. Market clearing condition requires $x_1^A + nx_2^B + x_1^C = 1$, hence we get $p_1^* = ne + 2k$ and the following equilibrium allocations. (Note this implies $m_A > k$)

Market Segmentation

In this example, we define complete Market Segmentation as the following:-

Definition 1 (Market Segmentation)**.** There is an infinite tax if one uses capital good income to buy the essential or universal good. There is no tax on any other transaction.

Notice that this definition of complete Market Segmentation implies that capital good income cannot be used to buy essential or universal goods. Still, essential good income can be used to buy capital goods without any tax. Now let us solve for equilibrium and calculate the social welfare with this policy intervention.

The utility maximisation exercise remains identical but under new endowments. However p_2 need not be equal to p_3 in equilibrium. Importantly though, since the rich type cannot use wealth to buy health services, his endowment is identical to the poor. Thus we get the following demand functions.

Now again as before let $1 < k < 6$, then we know, as before, that type B will demand $x_1^B = \frac{e}{p_1}$ but now type C will also demand $x_1^C = \frac{e}{p_1}$. Moreover again assuming in equilibrium, $m_A = p_1 + e > k$ individual A will demand $x_1^C = \frac{k}{p_1}$

Market clearing condition requires $x_1^A + nx_2^B + x_1^C = 1$, hence now we get $p_1^* = k + ne + 1$ and the following equilibrium allocations. (Note this implies $m_A > k$)

Table 5: Consumption under Regimes

Type	x_1^{LF}	x_1^{MS}
A	$\frac{e}{2k+e\pi_B}$	$\frac{e}{k+e(\pi_B+1)}$
B	$\frac{k}{2k+e\pi_B}$	$\frac{k}{k+e(\pi_B+1)}$
C	$\frac{k}{2k+e\pi_B}$	$\frac{e}{k+e(\pi_B+1)}$

The social welfare with any such allocation would be

$$W^{MS} = (n+1)klog[\frac{1}{k+n+1}] + klog[\frac{k}{k+n+1}] + I + 3 \tag{3}$$

A.2 Welfare comparison

Given the social welfare in the two regimes, we can compare them and see when segmenting markets would be better.

$$W^{MS} > W^{LF}$$

$$\Leftrightarrow (n+1)k\log\left[\frac{e}{k+n+1}\right] + k\log\left[\frac{k}{k+n+1}\right] > nk\log\left[\frac{e}{n+2k}\right] + 2k\log\left[\frac{k}{n+2k}\right]$$

$$\Leftrightarrow log\left[\frac{ke^{n+1}}{(n+1+k)^{n+2}}\right] > log\left[\frac{k^2e^n}{(n+2k)^{n+2}}\right]$$

Since the log is an increasing function, this implies that for the above to hold, we need

$$\frac{ke^{n+1}}{(n+1+k)^{n+2}} > \frac{k^2e^n}{(n+2k)^{n+2}}$$

$$k(n+2k)^{n+2} > e(n+1+k)^{n+2}$$

Given that we had assumed that $e < k < I + 1$ we can check that the above inequality holds for a large enough n. Hence we can say that Market Segmentation improves social welfare for these values of k.

The rationale behind such a result is simple. Market Segmentation removes the wealth endowment from the economy and thus lowers the price of good 1 (the health services) in equilibrium. Thus, the poor can afford more of that good, and their welfare improves. The welfare of the rich guy falls due to his inability to use his wealth now to buy health services, but that fall is offset by the gains of others for a sufficiently high value of k, which captures the importance of the essential good to an individual's welfare.

It is clear from the above example that Market Segmentation can improve social welfare in some scenarios. But a similar result could have been achieved using some kind of commodity taxation as well. At least to us, it is unclear which policy would be more welfare-enhancing. Let us now state a Lemma which shows that in the conditions described above, a commodity tax regime is weakly preferred to complete Market Segmentation. To do that, we formalise the conditions of the example below.

Lemma 8.1. *Suppose the preferences of all individuals are identical in an n- good economy with 1 essential good, 1 universal good and n-2 non-universal goods(linear), which are valued equally. In that case, a tax regime is weakly preferred to Market Segmentation.*

Proof If we have log-linear utility functions, a utilitarian social welfare function and preferences are identical. Social welfare will be independent of trade in linear goods. Hence, the maximum social welfare under Market Segmentation can be simply achieved by a very high tax on all non-universal goods.

Hence, we can say that in the example discussed above, the Market Segmentation equilibrium can just be achieved by a very high commodity tax on good x_3.

Therefore, Market Segmentation can only be better if trade in linear goods also improves social welfare. Thus we expand our horizon a little and discuss a five good economy with preference heterogeneity,

A.3 Example 4 goods

Laissez Faire allocation

Again because of the utilities, for the market to clear $p_2 = 2p_3 = 2p_4$ take $p_2 = 1$

We assume there are n poor people, there are 2 rich people and 1 pharma. Utility maximisation gives us

$$\begin{aligned} x_1 &= \frac{k}{p_1} && if\ m \geq k \\ x_1 &= \frac{m}{p_1} && if\ m < k \end{aligned}$$

Let $1 < k < \frac{I}{2} + 1$, therefore for market clearing we need.

$$n \times \frac{1}{p_1} + 3 \times \frac{k}{p_1} = 1$$

which implies the market clearing price is n + 3k. The equilibrium allocations are given in the Appendix.

Type A (The pharma) - $(\frac{k}{n+3k}, x_2^{A*}, x_3^{A*}, x_4^{A*})$

Type B (The poor guy) - $(\frac{1}{n+3k}, 0, 0, 0)$

Type C1 - $(\frac{k}{n+3k}, x_2^{C*}, 0, x_4^{C*})$

Type C2 - $(\frac{k}{n+3k}, x_2^{C*}, x_3^{C*}, 0)$

where $x_2^{A*} + nx_2^{B*} + 2x_C^{A*} = n+3$, $x_3^{A*} + nx_3^{B*} + 2x_3^{C*} = I$ and $x_4^{A*} + nx_4^{B*} + x_4^{C*} = I$

Market Segmentation allocation

Type A (The pharma) - $(\frac{k}{n+2+k}, x_2^{A*}, x_3^{A*}, x_4^{A*})$

Type B (The poor guy) - $(\frac{1}{n+2+k}, 0, 0, 0)$

Type C1 - $(\frac{1}{n+2+k}, x_2^{C*}, 0, x_4^{C*})$

Type C2 - $(\frac{1}{n+2+k}, x_2^{C*}, x_3^{C*}, 0)$

where $x_2^{A*} + nx_2^{B*} + 2x_C^{A*} = n+3$, $x_3^{A*} + nx_3^{B*} + 2x_3^{C*} = I$ and $x_4^{A*} + nx_4^{B*} + x_4^{C*} = I$

Direct Transfer regime

Type A (The pharma) - $(\frac{k}{p_{DT}}, x_2^{A*}, x_3^{A*}, x_4^{A*})$

Type B (The poor guy) - $(\frac{1+\frac{I}{2(n+3)}}{p_{DT}}, 0, 0, 0)$

Type C1 - $(\frac{k}{p_{DT}}, x_2^{C*}, 0, x_4^{C*})$

Type C2 - $(\frac{k}{p_{DT}}, x_2^{C*}, x_3^{C*}, 0)$

where $x_2^{A*} + nx_2^{B*} + 2x_C^{A*} = n+3$, $x_3^{A*} + nx_3^{B*} + 2x_3^{C*} = I$ and $x_4^{A*} + nx_4^{B*} + x_4^{C*} = I$

Welfare comparison: Non-distortionary case

Comparing, we get,

$$klog\frac{k}{p_{MS}} + (n+2)klog\frac{1}{p_{MS}} > klog\frac{k}{p_{DT}} + 2klog\frac{k}{p_{DT}} + nklog\frac{1+\frac{I}{2(n+3)}}{p_{DT}}$$

which would imply that

$$log(\frac{k}{(p_{MS})^{n+3}}) > log\frac{k^3(1+\frac{I}{2(n+3)})^n}{p_{DT}^{n+3}}$$

which again means that

$$\left\{\frac{p_{DT}}{p_{MS}}\right\}^{n+3} > k^2(1+\frac{I}{2(n+3)})^n$$

We can compute

$$\frac{p_{DT}}{p_{MS}} = \frac{3k+n+I\frac{n}{2(n+3)}}{n+k+2} \tag{4}$$

$$= \left\{1 + \frac{2k-2+I\frac{n}{2(n+3)}}{n+k+2}\right\} \tag{5}$$

As n goes to infinity the LHS

$$\left\{\frac{p_{DT}}{p_{MS}}\right\}^{n+3} \to e^{2K-2+\frac{I}{2}} = e^{2k-2}(e)^{\frac{I}{2}}$$

the RHS goes to

$$k^2(e)^{\frac{I}{2}}$$

now as

$$e^{k-1} > k$$

which is true for all $k > 1$. Thus, there exists some n after which the LHS is bigger than the RHS.

Welfare comparison: distortionary case

First of all, it should be clear that the goods that matters in the welfare calculation are only the essential goods and the capital goods that are not brought into the market (which produces dead weight loss). We will show that the welfare gain from using the distortionary tax regime rather than the non-distortionary tax regime is bounded as n increases. Furthermore, welfare loss is dependent on I and increases as I increases. This essentially means that for a large enough n and I, the non-distortionary tax regime is strictly worse than distortionary tax regime.

the total welfare under this policy is

$$klog\frac{k}{p} + 2klog\frac{1/2(1-t)k}{p} + nklog\frac{1+\frac{T}{n+3}}{p}$$

Further we know distrotionary price p is

$$p = (1-t)k + k + n + \frac{Tn}{n+3}$$

The welfare gain to the poor is

$$nlog(\frac{1+\frac{(((1-t)k-2)\times\frac{t}{1-t})}{n+3}}{p}) - nlog\frac{1}{p_{MS}}$$

plugging in $p_{MS} = n + 2 + k$ and simplifying, we get,

$$\begin{aligned}
&nlog(\frac{n+k+2}{n+(1-t)k+Tn/(n+3)} \times (1+\frac{((1-t)k-2)\times\frac{t}{1-t}}{n+3})) \\
&= nlog(\frac{n+k+2}{n+(1-t)k+Tn/(n+3)}) + nlog(1+\frac{((1-t)k-2)\times\frac{t}{1-t}}{n+3}) \\
&= nlog(1+\frac{(2+kt-Tn/(n+3))}{n+(1-t)k+Tn/(n+3)}) + nlog(1+\frac{((1-t)k-2)\times\frac{t}{1-t}}{n+3})
\end{aligned}$$

which is an expression of the sort

$$nlog(1+\frac{f_1}{n})+nlog(1+\frac{f_2}{n})$$

which we know is bounded above as n goes to infinity.

Thus, welfare gain from using distortionary commodity taxation is bounded above. However, the dead weight loss due to this commodity taxation can be arbitrarily high and is independent of n. Dead weight loss in our example depends on I and thus for a high enough I and as n increases, segmentation becomes better than taxation and subsidy regime.

A.4 Model: Proofs

Proof of Lemma 0.2

Proof: Set of achievable allocations is convex

Convexity of Excess Supply The convexity of the excess supply function can be shown in two steps. Firstly, notice that the assumption of aggregate symmetry means that the excess supply functions are identical across tax rates.

The excess supply function is simply

$$\sum_{dons}(e_i - x_i)(p, \tau)$$

This gives us

$$\frac{dS}{dR} = -\sum \frac{dx}{dp}\frac{dp}{dR}$$

$$\implies \frac{d^2S}{dR^2} = -\sum \left[\frac{d^2x}{dp^2}\left(\frac{dp}{dR}\right)^2 + \frac{dx}{dp}\frac{d^2p}{dR^2}\right]$$

we know that $\left(\frac{dp}{dR}\right)^2 > 0$ becuase it is a square and $\frac{d^2x}{dp^2} > 0$due to inelasticity. Now looking at the second term, $\frac{dx}{dp} < 0$ by normality and $\frac{d^2p}{dR^2} < 0$ by inealsticity/

We know that

$$u'(x) = \beta p \implies u''(x)\frac{dx}{dp} = \beta$$

or

$$\frac{dx}{dp} = \beta\frac{1}{u''} \implies \frac{d^2x}{dp^2} = -...u''' > 0$$

which gives us what we need.

Secondly notice that marginal utility of the goods is diminishing, therefore the revenue needed to get one more good supplied from those who are endowed is higher if the supply is already high, thus, the supply functions are convex. we know that

$$p = p(m+1)x(p) + R + ne \implies \frac{dp}{dR} = \frac{dp}{dR}(m+1)x(p) + (m+1)p\frac{dx}{dp}\frac{dp}{dR} + 1$$

$$\implies \frac{dp}{dR} = \left[1 - (m+1)^{-1}(x(p) + p\frac{dx}{dp})\right]^{-1} > 0$$

$$\implies \frac{d^2p}{dR^2} = -\left[1 - (m+1)^{-1}(x(p) + p\frac{dx}{dp})\right]^{-2}\left[-(m+1)^{-1}\frac{dp}{dR}\frac{d}{dp}(p \cdot x)\right] < 0$$

We know from above that the set of achievable allocations in an economy is convex. Let $(x_1, ...x_k)$ be an asymmetric allocation that is optimal i.e. maximises the social welfare function. Also given we have aggregate symmetry in the economy for any i, j pair $x_i \neq x_j$, if $(x_1, ...x_i, .., x_j, .x_k)$ is feasible so is $(x_1, ...x_j, .., x_i, .x_k)$. Now because the set of individual allocations are convex, $(x_1, ...\frac{x_i+x_j}{2}, .., \frac{x_i+x_j}{2}, .x_k)$ must also be feasible. However, because utility functions of the agents are convex, these average allocations (which are feasible) increase the social welfare. Hence, the original asymmetric allocation cannot be the maximum. Thus, we can have a maxima under aggregate symmetry only if the allocation is symmetric.

Proof of Lemma 0.3

We have established that the planner must treat all essential goods symmetrically to maximise social welfare.

Formally, Define

$$V^*_{sym} = \max_{\tau_1 = \tau_2 .... \tau_n = \tau} V(\tau_1, , , , \tau_n)$$

and

$$V^* = \max_{\tau_1, ... \tau_n} V(\tau_1, , , , \tau_n)$$

notice that $V^*_{sym} \leq V^*$ because it is a maximization on a restricted set, however by the above lemma V^* must be such that $\tau_1 = \tau_2 .... = \tau_n$

which means that $V^*_{sym} \geq V^*$ so they must be equal, and we can only conider the maximisation over the space $\tau_1 = \tau_2 = ...\tau_n$

Further notice that $V^*_{sym} = \sum_{individuals} U_i(x_i(\tau_k))$

by aggregate symmetry, prices of all goods must be equal, so we can just consider the aggregate good(average) and the aggregate(average) consumer to compute the ideal social utility.

Proof of Theorem 1

The consumption of essential goods is given by x_d, x_r, x_p for the don, the rich and the poor, respectively. We now state a lemma showing that the demand for the essential good is just a function of its price in our setup.

Lemma 8.2. *With the utility function as defined above (quasilinear) and the price of the universal good taken as the numeraire, the demand for essential goods is independent of the prices of other goods.*

Proof. Follows quite simply from quasilinearity; see (Varian, 2014, p. 104). □

Proof of Claim 1.1

Proof. We have to show that commodity taxation at the segmentation level dominates non-distortionary taxation. The welfare at the segmentary level of commodity taxation (τ_{DTS}) is

$$(n+m)u(\frac{e}{p_{DTS}}) + u(x(p_{DTS})) - DWL = W_{DTS}$$

the welfare at non-distortionary taxes is

$$nu(\frac{e}{p_{dt}}) + mu(x(p_{dt},\tau)) + u(x(p_{dt}) = W_{dt}$$

subtracting we get,

$$W_{DTS} - W_{dt} = n(u(\frac{e}{p_{DTS}}) - u(\frac{e}{p_{dt}})) + \\ m(u(\frac{e}{p_{DTS}}) - u(x(p_{dt},\tau))) + (u(x(p_{DTS})) - u(x(p_{dt}))) - DWL$$

for this to be positive

$$DWL < n(u(\frac{e}{p_{DTS}}) - u(\frac{e}{p_{dt}})) + m(u(\frac{e}{p_{DTS}}) - u(x(p_{dt}))) - (u(x(p_{DTS})) - u(x(p_{dt})))$$

RHS can be written as

$$n\left[(u(\frac{e}{p_{DTS}}) - u(\frac{e}{p_{dt}})) + \frac{m}{n}((u(\frac{e}{p_{DTS}}) - u(x(p_{dt},\tau)))) + \frac{1}{n}((u(x(p_{DTS})) - u(x(p_{dt}))))\right]$$

because all terms are of the same order of magnitude and n is increasing, eventually only the first term remains relevant. Hence we get,

$$n(u(\frac{e}{p_{DTS}}) - u(\frac{e}{p_{dt}})) \approx nu'(\frac{e}{p_{dt}})\left[\frac{e}{p_{DTS}} - \frac{e}{p_{dt}}\right]$$

which gives us

$$= nu'(\frac{e}{p_{dt}})\frac{1}{n}\left[m(x_r(p_{dt}) - \frac{e}{p_{DTS}}) + x_d(p_{dt}) - x_d(p_{DTS})\right]$$

=

$$u'(\frac{e}{p_{dt}})\left[m(x_d(p_{dt}) - \frac{e}{p_{DTS}}) + x_d(p_{dt}) - x_d(p_{DTS})\right]$$

Now for a large n and a small e, the following is true.

$$m(x_r(p_{dt}) - \frac{e}{p_{DTS}}) > \frac{e}{p_{dt}}$$

we can use this to bound the above expression by

$$u'(\frac{e}{p_{dt}})\left[\frac{e}{p_{dt}} + x_d(p_{dt}) - x_d(p_{DTS})\right]$$

This now can be written as

$$u'(\frac{e}{p_{dt}})\left[\frac{e}{p_{dt}}\right] - u'(\frac{e}{p_{dt}})\left[x_d(p_{DTS}) - x_d(p_{dt})\right]$$

The first term is unbounded, to see this, notice that

$$u'(\frac{e}{p_{dt}})\left[\frac{e}{p_{dt}}\right]$$

can be written as $p^*x(p^*)$ where p^* is the price which supports $\frac{e}{p}$ or $x^{-1}(\frac{e}{p})$ further as $\frac{e}{p}$ goes to zero its inverse goes to infinity, which means $p^*x(p^*)$ is unbounded by the elasticity condition.

The second term is bounded, to see this

$$u'(\frac{e}{p_{dt}})\left[x_d(p_{dt}) - x_d(p_{DTS})\right] \approx u'(\frac{e}{p_{dt}})\frac{dx}{dp}(p_{dt} - p_{DTS})$$

further, at the segmentation price,

$$u'(\frac{e}{p_{dt}}) = \lambda p_{dt}$$

this gives us

$$\lambda p_{dt}\frac{dx}{dp}(p_{dt} - p_{DTS})$$

by inelasticity

$$\frac{d}{dp}(px) \geq 0 \implies p\frac{dx}{dp} \leq x(p)$$

which can be plugged in to bound the above expression

$$x(p_{dt})(p_{dt} - p_{DTS}) = \frac{e}{p_{dt}}(p_{dt} - p_{DTS})$$

which is clearly bounded and less than e. □

Proof of claim 1.2

Proof. Let τ_{DTS} be the complete segmentation level of commodity tax. A slight decrease in tax rate from this level has three effects on welfare. First, decrease in tax rate raises some tax revenue for the planner which can be used as a direct transfer to improve welfare. This is given by the following expression $\frac{dW}{dT} \times \frac{\delta T}{\delta \tau}$. On the other hand, the decrease in tax rate causes the price of the essential good to rise for the poor people which decreases welfare. This is given by $\frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau}$. Third, the dead weight-loss in the economy also decreases. This is given by $\frac{dW}{dDWL} \times \frac{\delta DWL}{\delta \tau}$

Thus what we want to show is that as we go from τ_{DTS} to $\tau_{DTS} - \epsilon$, where $\epsilon > 0$ but arbitarily small

$$\frac{dW}{dT} \times \frac{\delta T}{\delta \tau} + \frac{dW}{dDWL} \times \frac{\delta DWL}{\delta \tau} > \frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau} \tag{6}$$

Now notice that close to the complete segmentation tax rate i.e $\tau_{DTS} - \epsilon$ the tax derivative of demand is close to zero. However, the income raised close to τ_{DTS} is large and positive. This is because the price derivative of demand is a continuous function of price and price is a continuous function of tax rate, thus we can say that tax derivative of demand is continuous function in price. Moreover, we know that it is exactly zero at the cutoff tax, thus it must be close to zero around it. However, if the good is inelastic, the total expenditure on the commodity is an increasing function of price (and tax rate) and thus tax revenue which is an increasing function of tax rate till τ_{DTS} where it falls discontinuously to zero. Hence, as $\tau \to \tau - \epsilon$ then $\frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau} \to 0$ but $\frac{\delta T}{\delta \tau} \to R >> 0$.

Now given that $\frac{dW}{dT} > 0$, we get that 6 holds. *Q.E.D*

□

Proof of claim 1.3

Now we claim that the gain from revenue which the planner gets when he implements distortion can be *less* than the DWL.

If the planner extracts revenue R by a tax τ the total welfare of the economy is

$$W* = n(U(\frac{e + \frac{R}{n+3}}{p*}) + mU(x(p*, \tau)) + U(x(p*)) - DWL_{DT*}$$

comparing this to the full segmentation tax rate, we get,

$$W * -W_{DTS} = n(U(\frac{e+\frac{R}{n+3}}{p*}) - U(\frac{e}{p_{DTS}})) + m(U(x(p*,\tau)) - U(p_{DTS},\tau)) +$$
$$U(x(p*)) - U(p_{DTS}) - DWL_{DT*} + DWL_{DTS}$$

It is clear that $m(U(x(p*,\tau)) - U(p_{DTS},\tau)) + U(x(p*)) - U(p_{DTS}) - DWL_{DT*} + DWL_{DTS}$ is bounded and hence we call it M. Thus, we get

$$n(U(\frac{e+\frac{R}{n+3}}{p*}) - U(\frac{e}{p_{DTS}})) + M$$

$$= nu'(\frac{e}{p_{DTS}})\frac{1}{n}\left[1 - mx(p*,\tau) - x(p*) - (1 - mx(p_{DTS},\tau) - x(p_{DTS},\tau))\right] + M$$

Given that change in utility can be approximated as the marginal utility times the change in consumption we get,

$$= u'(\frac{e}{p_{DTS}})\left[-m(x(p*,\tau) + x(p_{DTS},\tau) + x(p_{DTS}) - x(p*))\right] + M$$

$$= u'(\frac{e}{p_{DTS}})\left[-m\frac{dx}{d(p*(1-\tau))}(p*(1-\tau) - p_{DTS}) - \frac{dx}{dp}(p* - p_{DTS})\right] + M$$

Now we know that at segmentation level of commodity segmentation, marginal utility of the poor must be equal to the marginal utilty of the rich i.e. $u'(\frac{e}{p_{DTS}}) = \lambda p_{DTS}$

$$= \lambda p_{DTS}\left[-m\frac{dx}{d(p*(1-\tau))}(p*(1-\tau) - p_{DTS}) - \frac{dx}{dp}(p* - p_{DTS})\right] + M$$

Now this expression is bounded due to the in-elasticity condition *Q.E.D.*

Proof of Theorem 2

Proof. We know

$W_{DT} = nu(\frac{1+\frac{R}{n+m+1}}{p_{DT}}) - nu(\frac{1}{p_{LF}}) + mu(x_r(p_{DT})) - mu(x_r(p_{LF})) + u(x_d(p_{DT})) - u(x_d(p_{LF}))$

$$p_{LF} = \frac{n}{1-(m+1)x_{LF}}$$
$$p_{DT} = \frac{n + R\frac{n}{n+m+1}}{1 - mx_r(p_{DT},\tau) - x_d(p_{DT},\tau)}$$

In an market equilibrium with subsidy on essential good, the following equation must hold.

$$1 = x(p_s + \Delta p) + \frac{n}{p_s} + mx(p_s)$$
$$\implies p_s = \frac{n}{1 - mx(p_s) - x(p + \Delta p)}$$

where p_s is the price in the subsidy regime and Δp is the price paid by government to the don on each unit sold. Therefore, in a balanced budget

$$R = (1 - x_{ds})\Delta p$$

So we get,

$$p_s = \frac{n}{1 - mx(p_s) - x(p + \frac{R}{1-x_{ds}})} \tag{7}$$

where x_{ds} is the consumption of don in the subsidy regime

Further $W_s = nu(\frac{1}{p_s}) - nu(\frac{1}{p_{LF}}) + mu(x_r(p_s)) - mu(x_r(p_{LF})) + u(x_d(p_s)) - u(x_d(p_{LF}))$

$$\begin{aligned} W_{DT} - W_S = n\Bigg[u\bigg(\frac{1}{n}(1 - mx_r - x_d)\bigg) - u\bigg(\frac{1}{n}(1 - mx(p_s) - x(p + \frac{R}{1 - x_{ds}}))\bigg)\Bigg] \\ + m\Bigg[u(x_r(p_{DT})) - u(x_r(p_s))\Bigg] + \Bigg[u(x_d(p_{DT})) - u(x_d(p_s))\Bigg] \end{aligned} \tag{8}$$

Since all terms are of the same order of magnitude and n is increasing, eventually only the first term remains relevant. Hence, for a large n we can say the following

$$W_{DT} - W_S = n\Bigg[u'(.)\bigg(\frac{1}{n}(1 - mx_r - x_d) - \frac{1}{n}(1 - mx_r(p_s) - x_d(p_s))\bigg)\Bigg] \tag{9}$$

Given $u'(.) > 0$, to show that $W_{DT} - W_{MS} > 0$ we need to show that

$$m(x(p_s) - x(p_{DT})) + (x_r(p_s + \frac{R}{1 - x_d(p_s)}) - x_r(p_{DT})) > 0$$

$$\begin{aligned} &\approx m\frac{dx}{dp}|_{p=p_{DT}}(p_s - p_{DT}) + \frac{dx}{dp}|_{p=p_{DT}}(p_s + \frac{R}{1 - x_d} - p_{DT}) \\ &= \frac{dx}{dp}|_{p=p_{DT}}\Bigg[(m+1)(p_s - p_{DT}) - \frac{R}{1 - x_d}\Bigg] \end{aligned} \tag{10}$$

Notice that $p_s < p_{LF}$ which means that

$$p_{DT} - p_s > p_{DT} - p_{LF} = \frac{n + R\frac{n}{n+m+1}}{1 - mx_r(p_{DT}, \tau) - x_d(p_{DT}, \tau)} - \frac{n}{1 - (m+1)x_{LF}}$$

Given that $x_r(p_{DT},\tau) < x_r(p_{LF})$ and $x_d(p_{DT},\tau) < x_d(p_{LF})$ we know

$$p_{DT} - p_s > p_{DT} - p_{LF} > R\frac{n}{n+m+1}\frac{1}{(1-mx_r(p_{DT},\tau) - x_d(p_{DT},\tau))(1-(m+1)x_{LF})}$$

For a large n, the above equation implies

$$p_s - p_{DT} \geq R$$

Putting this back in equation 10, we get

$$\frac{dx}{dp}|_{p=p_{DT}}\left[(m+1)R - \frac{R}{1-x_d}\right]$$

This implies that for a large n as long as $m+1 > \frac{1}{1-x_d}$ direct tranfers are better than subsidy. Also, notice that as $n \to \infty, x_d \to 0$. Thus, the condition simply implies $m > 0$ i.e. there are positive number of rich people in the economy.

This comes from the fact that subsidy only decreases consumption of the 'dons' but direct transfers decreases the consumption of both the rich and the dons. Given, inelasticity and large number of poor in the economy, this makes direct transfers more efficient than subsidy. □

Proof of Theorem 3

Proof. Suppose there are 2 *essential goods* called x^1 and $x^2 2$. Different people in economy prefer x^1 and x^2 differently. The preference heterogeneity in the population is indexed by parameter $\alpha \in [0,1]$. Thus utility function can be given as

$$U_\alpha(X) = \alpha u(x^1) + (1-\alpha)u(x^2) + \sum b_i x_i$$

The preferences in the population are described by a density $f(\alpha) \in \Delta[0,1]$. For sake of simplicity, we assume the distribution is symmetric i.e. $f(\alpha) = f(1-\alpha)$. Let the endowment of the universal good be e.

Let us first consider the MS regime. Under MS, the symmetry in the distribution of α implies that prices of both x^1 and x^2 must be equal in equilibrium. Let us call that p. Equilibrium condition under Market Segmentation gives us the following equation

$$x_d^1(p) + n\mathbf{E}_{\alpha\sim f}[x_p^{1\alpha}(p)] + m\mathbf{E}_{\alpha\sim f}[x_r^{1\alpha}(p)] = 1$$

similarly for the second good,

$$x_d^2(p) + n\mathbf{E}_{\alpha\sim f}[x_p^{2\alpha}(p)] + m\mathbf{E}_{\alpha\sim f}[x_r^{2\alpha}(p)] = 1$$

Adding both equations we get,

$$x_d^1(p) + n\mathbf{E}_{\alpha\sim f}[x_p^{1\alpha}(p)] + m\mathbf{E}_{\alpha\sim f}[x_r^{1\alpha}(p)] + x_d^2(p) + n\mathbf{E}_{\alpha\sim f}[x_p^{2\alpha}(p)] + m\mathbf{E}_{\alpha\sim f}[x_r^{2\alpha}(p)] = 2$$

Now given that under Market Segmentation both the poor and the rich can only use universal good to buy essential goods, we must have $x^{1\alpha}(p) + x^{2\alpha}(p) = \frac{e}{p}$. Thus, multiplying the above equation by p, we get

$$p(x_d^1(p) + x_d^2(p)) + (n+m)e = 2p \implies p = \frac{(n+m)e}{2 - x_d^1(p) - x_d^2(p)}$$

At these prices, different individuals buy different amount of essential goods, depending on the their respective value of α.

Now, suppose the planner were to attempt to mimic the outcome of the above Market Segmentation policy with a rationing policy. This rationing policy can be implemented by a non-linear tax[34]. This would mean an infinite tax on consumption above a certain level $\bar{q}$.

If the poor have to to be given the same utility with the rationing policy, then it must be the case that $\bar{q_1} = \bar{q_2} = \frac{e}{p_{ms}} = \frac{2-x_d^1(p)+x_d^2(p)}{(n+m)}$. This is because if $\bar{q_1} < \frac{e}{p_{ms}}$, then the poor with $\alpha = 1$ will be definitely worse off than the segmentation regime. However, if $\bar{q}$ is as above then, the aggregate consumption of the rich with is not $m\mathbf{E}_{\alpha\sim f}[x_\alpha(p)]$. It is between $m\mathbf{E}_{\alpha\sim f}[x_\alpha(p)]$ and $m\bar{q}$ for each essential good, where $\bar{q} >> \mathbf{E}_{\alpha\sim f}[x_\alpha(p)]$. This is because the rich are no longer income constrained. Thus, we have a situation of excess demand in the market. Under excess demand market is not cleared and thus the the social planner has to set a new $\bar{q_n}$ which is less than $\bar{q}$ to clear the market. As discussed above, this makes the poor worse off.

□

Proof of Theorem 4

Proof. Let τ^{cms} be the complete segmentation level of tax. A slight decrease in tax rate from this level has two effects on welfare. First, decrease in tax rate raises some tax revenue for the planner which can be used as a direct transfer to improve welfare. This is given by the following expression $\frac{dW}{dT} \times \frac{\delta T}{\delta \tau}$. On the other hand, the decrease in tax rate causes the price of the essential good to rise for the poor people which decreases welfare. This is given by $\frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau}$.

Thus what we want to show is that as we go from τ^{cms} to $\tau^{cms} - \epsilon$, where $\epsilon > 0$ but arbitarily small

$$\frac{dW}{dT} \times \frac{\delta T}{\delta \tau} > \frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau} \tag{11}$$

[34]see, Gadenne (2020) for details

Now notice that close to the complete segmentation tax rate i.e $\tau^{cms} - \epsilon$ the tax derivative of demand is close to zero. However, the income raised close to τ^{cms} is large and positive. This is because the price derivative of demand is a continuous function of price and price is a continuous function of tax rate, thus we can say that tax derivative of demand is continuous function in price. Moreover, we know that it is exactly zero at the cutoff tax, thus it must be close to zero around it. However, if the good is inelastic, the total expenditure on the commodity is an increasing function of price (and tax rate) and thus tax revenue which is an increasing function of tax rate till τ^{cms} where it falls discontinuously to zero. Hence, as $\tau \to \tau - \epsilon$ then $\frac{\delta x_{ess}}{\delta p}\frac{\delta p}{\delta \tau} \to 0$ but $\frac{\delta T}{\delta \tau} \to R >> 0$.

Now given that $\frac{dW}{dT} > 0$, we get that 11 holds. $Q.E.D$

□

Proof of Theorem 5

The planners problem is to maximise the following equation with respect to segmentary tax rate τ.

$$w_p V_p(p,\tau) + w_d V_d(p,\tau) + w_r V_r(p,\tau) \tag{12}$$

subject to

$$\sum x_i(p,\tau) = \sum e_i \forall k \tag{13}$$

where V_p is the indirect utility function of the poor, V_r is the indirect utility function of the rich, V_d is the indirect utility function of the don, k are the different goods and w is the weight associated with each good.

Thus, setting up the Lagrange for the planner in order to maximise with respect to τ, we get

$$L = w_p V_p(p,\tau) + w_d V_d(p,\tau) + w_r V_r(p,\tau) + \sum \mu_k \left[\sum e_i - \sum x_i(p,\tau)\right]$$

Differentiating with respect to τ, we get

$$w_p \frac{dV_p}{d\tau} + w_d \frac{dV_d}{d\tau} + w_r \frac{dV_r}{d\tau} = \sum \mu_k \sum \frac{dx(p,\tau)}{d\tau} \tag{14}$$

Now to get the expressions of the indirect utility functions for each type of individuals, we carry on the utility maximisation exercise for each of them.

The market is segmentd i.e. essential goods and universal goods are in one sub-market (x_1, x_2) and capital goods are in another sub-market $(x_3.....x_K)$. Also, suppose there is a market for assets introduced by the planner, which we call y. This asset enables people to transfer income from the market for capital goods to the market for essential goods. And this is taxed by the planner at the rate of τ, that is to say there is a wedge between the buying price and selling price for the asset. This tax generates a tax revenue which we assume is equally distributed between all individuals. Let this be t. In these conditions, the utility maximization problem of the rich individual is the following.

Maximise

$$U_r(x)$$

subject to

$$p_1(x_1 - e_1) + p_2(x_2 - e_2) \leq y + t$$
$$\sum p_k(x_k - e_k) \leq -(1+\tau)y$$

Assuming that both equations bind

$$(1+\tau)\left[p_1(x_1 - e_1) + p_2(x_2 - e_2) - t\right] + \sum p_k(x_k - e_k) \leq 0$$

Let $V_r(p,\tau)$ be the indirect utility function which we get from maximising the above rich agent's problem. Using the envelope theorem and differentiating with respect to the tax and we get that.

$$\frac{\partial V}{\partial \tau} = \frac{\partial L}{\partial \tau} = -\lambda_r\left[(p_1(x_1 - e_1) + p_2(x_2 - e_2) - t - \frac{\partial t}{\partial \tau}(1+\tau)\right]$$

If preferences are quasi-linear, then the λ_r is 1.

Now the don doesn't own any capital endowment so his utility maximisation problem can be given as the following.

Maximise $U_d(x)$ subject to

$$p_1(x_1 - e_1) + p_2(x_2 - e_2) - t + \sum p_k(x_k - e_k) \leq 0$$

So,

$$\frac{dV_P}{d\tau} = -\lambda_d(-\frac{\partial t}{\partial \tau})$$

Again, if preferences are quasi-linear, then the λ_d is 1.

Now looking at the problem for the poor, we know that they own only poor goods Max $U_p(x)$ subject to

$$p_1(x_1 - e_1) + p_2(x_2 - e_2) - t \leq 0$$

Which gives us

$$\frac{dV_P}{d\tau} = -\lambda_p(-\frac{\partial t}{\partial \tau})$$

where $\lambda_p = \frac{\mu_p}{p_1}$

Putting the above values of the derivative of the indirect utility function back into equation 14, we get

$$w_r\lambda_r\left[\frac{\partial t}{\partial\tau}(1+\tau)-\left(p_1(x_1-e_1)+p_2(x_2-e_2)-t\right)\right]+w_d.\lambda_d\frac{\partial t}{\partial\tau}+w_p\lambda_p\frac{\partial t}{\partial\tau}=\sum\mu_k\sum\frac{dx(p,\tau)}{d\tau} \tag{15}$$

Proof of Theorem 6

Proof of claim 6.1

The utility function we defined for the economy can be given by

$$U = u(x_1) + bx_2 + ...$$

Further let each individual be endowed with e units of x_2 good. In this economy, the first order conditions from the maximization problems of the rich and the don, we get

$$\frac{u'(x_1)}{b} = \frac{p_1}{p_2}$$

$$u'(x_1) = \frac{p_1}{p_2}b$$

Let us define, $p^* = \frac{p_1}{p_2}b$.

The utility maximization problems of the poor gives us,

$$x_1 = \frac{p_2 \times e}{p_1}$$

which can be written as

$$x_1 = \frac{b \times e}{p^*}$$

Thus, the 3 equations below uniquely pin down consumption of the rich and the poor.

$$u'(x) = p^* \tag{16}$$

$$x_p = \frac{b.e}{p^*} \tag{17}$$

$$\sum x_i = 1 \tag{18}$$

Now since the $u'(x)$ is independent of b and x and is bounded above zero by assumption, thus $p*$ is also bounded above zero. Thus, we find that in equilibrium

$$b \to 0 \implies x_p \to 0 \iff e \to 0 \implies x_p \to 0$$

Proof of claim 6.2

Proof. Suppose, for the rich the marginal utility of consuming the essential good is λp where λ is the marginal utility of income, which we know to be bounded above zero given all other goods are linear. Thus, the ratio of marginal utility of the poor to the marginal utility of the rich

$$\frac{u'(\frac{e}{p})}{\lambda p}$$

Now we know that as $e \to 0$, λp is bounded. However, $e \to 0 \implies u'(0) \to \infty$ by assumption.

$\square$

Proof of claim 6.3

Proof.

$$W_{DT} = n\frac{\epsilon + \frac{T}{n+m+1}}{p_{DT}} \tag{19}$$

where,

$$p_{DT} = \frac{\epsilon n + T\frac{n}{n+m+1}}{1-(m+1)x_{DT}}$$

and x_{DT} is the consumption of don and rich people is the transfer regime.

Under Market Segmentation, we know that

$$W_{ms} = (n+m)\frac{\epsilon}{p_{ms}} \tag{20}$$

$$p_{ms} = \frac{\epsilon(n+m)}{1-x(p_{ms})}$$

where x_{ms} is the consumption of don in segmentation regime

Subtracting the 20 from 19, we get

$$W_{DT} - W_{ms} = 1-(m+1)x(p_{DT}) - (1-x(p_{DT})) \tag{21}$$

which reduces to the condition

$$x(p_{ms}) - (m+1)x(p_{DT})$$

Now as $\epsilon \to 0$, we know that consumption of the don in the Market Segmentation regime i.e. $x(p_{ms}) \to 1$. This is because he ends up consuming all the good in this regime as both the rich and the poor people are not able to demand anything when their endowment goes to zero.

On the other hand, under the direct transfer regime, the poor people do receive positive transfers and thus demand strictly positive amount of essential good even when their endowment goes to zero. Thus the consumption of the don and the rich people combined is bounded strictly below 1 i.e. as $\epsilon \to 0, (m+1)x(p_{DT}) \to z << 1$.

Hence, 21 is strictly positive. *Q.E.D*

$\square$

Proof of Theorem 7

The first two parts of the proof are the same as claim 6.1 and 6.2 as above. Below we prove another claim that concludes the proof.

Claim 7.1. *As endowment of the universal good in the economy goes to zero, the direct transfer increases social welfare more than subsidy on the essential good.*

Proof of claim 7.1

Proof. Welfare under direct transfers can be given by

$$W_{DT} = n\frac{\epsilon + \frac{T}{n+m+1}}{p_{DT}} \tag{22}$$

where,

$$p_{DT} = \frac{\epsilon n + T\frac{n}{n+m+1}}{1-(m+1)x_{DT}}$$

and x_{DT} is the consumption of don and rich people is the transfer regime.

Under subsidies, we know that

$$W_S = n\frac{\epsilon}{p_s} \tag{23}$$

where,

$$p_s = \frac{\epsilon n}{1 - m.x(p_s) - x_{ds}(p + \frac{T}{1-x_{ds}})}$$

and x_{p_s} is the consumption of rich people and x_{ds} is the consumption of the don in the subsidy regime.

Subtracting the two welfare equations above, we get we get

$$W_{DT} - W_S = 1-(m+1)x_{DT} - \left(1 - x(p_s) - mx(p_s + \frac{T}{1-x_{ds}})\right) \tag{24}$$

Which reduces to the condition

$$mx(p_s) + x(p_s + \frac{T}{1-x_{ds}}) - (m+1)x(p_{DT})$$

Now as $\epsilon \to 0$, we know that consumption of the don and the consumption of the rich people in the subsidy regime i.e. $mx(p_s) + x(p_s + \frac{T}{1-x_{ds}}) \to 1$. This is because they end up consuming all the goods in this regime as the poor people are not able to demand anything when their endowment goes to zero even in presence of the subsidy.

On the other hand, under the direct transfer regime, the poor people do receive positive transfers and thus demand strictly positive amount of essential good even when their endowment goes to zero. Thus the consumption of the don and the rich people combined is bounded strictly below 1 i.e. as $\epsilon \to 0, (m+1)x(p_{DT}) \to z << 1$.

Hence, 24 is strictly positive. *Q.E.D*

□

Proof of theorem 8

First, we show that in a regime with only commodity taxation and direct transfers, some positive value of segmentary taxes makes the poor better off. We know that because of market clearing condition the following equation must be true.

$$1 = x_d(p) + nx_p(p) + mx_r(p) \tag{25}$$

where $x_d(p)$ is the consumption of the essential good by the don, $x_p(p)$ is the consumption of the poor and $x_r(p)$ is the consumption of the rich.

Let the segmentary tax be high enough so that the price of the essential commodity increases for the rich[35]. Thus, we know at this level of segmentary tax the consumption of the rich decreases at any given price. Thus the following equation must be true after the implementation of the segmentation policy So we now get

$$1 = x_d(p + \delta p) + nx_p(p + \delta p) + m(x_r(p) - \delta S) \tag{26}$$

where δS is the change in rich consumption due to segmentation.

Subtracting 25 from 26, we get

$$\Big(x_d(p + \delta p) - x_d(p)\Big) + n\Big(x_p(p + \delta p) - x_p(p)\Big) = m\delta S \tag{27}$$

Given that the RHS of the equation is positive, the LHS must also be positive, which means δp must be negative. Thus the welfare of the poor, which is $\frac{e+\frac{T}{n+m+1}}{p}$ increases with the use of segmentary taxes. Now we show that the converse is also true. If any regime with optimal segmentary taxes does not achieve first best outcome, then if commodity taxation generates some tax income, it makes the poor better off.

The consumption of the poor in the segmentation economy can be given by [1 −

[35]We know such a tax exists because complete segmentation level of tax will always exist that will definitely increase the price for the rich

$x_d(p_{ms}) - mx_r(p_{ms})]$, where $x_d(p_{ms})$ is the consumption of the don $x_r(p_{ms})$ is the consumption of the rich. We want to show that

$$[1 - x_d(p_{ms} + \delta p) - mx_r(p_{ms} + \delta p)] > [1 - x_d(p_{ms}) - mx_r(p_{ms})] \tag{28}$$

where δp is the change in price after commodity taxation and direct transfers. So we want to show that for any given price p and endowment i, introduction direct transfers increases the price i.e. δp is positive which concludes the above proof.

So, let us suppose that in the segmentary equilibrium, poor have income i[36], and the price of the commodity is p, from the market clearing condition we get,

$$p = px_d + ni + mpx_r$$

Now suppose we provide direct transfers of T to the population, we get,

$$p_{DT} = p_{DT}x_d(p_{DT}) + p_{DT}mx_r(p_{DT}) + ni + T\frac{n}{n+m+1}$$

Implicitly differentiating this equation with respect to T, we get

$$\frac{dp}{dT} = \frac{n}{n+m+1} + [mx_r + x_d]\frac{dp}{dT} + p[\frac{mdx_r}{dp} + \frac{dx_d}{dp}]\frac{dp}{dT} \tag{29}$$

which means that

$$\frac{dp}{dT} = \frac{1}{(1 - [mx_r + x_d] - p[\frac{mdx_r}{dp} + \frac{dx_d}{dp}])}\frac{n}{n+m+1} \tag{30}$$

which is positive because $mx_r + x_d < 1$ and $[\frac{dX_r}{dp} + \frac{dx_p}{dP}] < 0$. Thus, increasing T always increases price.

Thus we can be sure that introducing taxes increases the equilibrium price of the essential good i.e. δp is positive. So the new welfare for the poor which is $1 - x_r(p + \delta P) - x_d(p + \delta p)$ is higher than the old. *Q.E.D*

B Analysing Market Segmentation in a Labour Supply Model

We augment the basic discrete job labour model of Saez (2002) with a distinction between two *types* of consumption goods. The first type is luxury goods which are produced using labour and a CRS production technology. The second type is essential goods which are endowed to a few people. Keeping up with the main model, we call the people endowed

[36]This includes the transfers recieved from tax revenue generated from segmentary tax

with the essential goods as 'dons'. Again, we assume aggregate symmetry and thus consider only one essential good.

B.1 Consumers

Consumers are identical in utility, but have a distribution of ability $\theta_i = [0, h]$. We interpret θ_i as the effort agent i has to exert to do the high productivity job. Each individual is endowed with 1 unit of labour, which they use to work at high productivity or low productivity jobs. Let $f(.)$ and $F(.)$ be the probability density function and cumulative density function for θ_i .

Individuals earn wages from labour and consume essentials and luxuries, taking prices as given. Other than the effort level required for the individuals in the high productivity jobs, individuals are identical. So we can write the utility of individual i as

$$U_i(x, e) = u(e) + x - \theta_i$$

where x is the luxury and e is the essential good. $u'(.) > 0, u"(.) << 0$ and $\lim_{x \to 0} u"(.) \to \infty$

For sake of simplicity of exposition we can assume that

$$u(e) = \frac{x^\alpha}{\alpha} \quad \text{where} \quad \alpha < 0$$

In this setup, the demand of the essential good for the rich is

$$e(p) = p^{\frac{1}{\alpha - 1}}$$

and expenditure on the good is

$$pe(p) = p^{\frac{\alpha}{\alpha - 1}}$$

which is an increasing function of price.

B.2 Production

Both the high productivity job and the low productivity job produce the luxury good. However, they both differ in the production function.

For the low skill job, we

$$f(L) = c_l L$$

For the high skill job, we have

$$f(L) = c_h L$$

We assume that 'dons' are endowed with 1 unit of essential goods and do not provide labour for production.

Due to perfect competition, each individual gets a wage equal to his marginal product. So the individual working in a high skill job gets paid $w_h = c_h$, and the individual working in a low skill job gets paid $w_l = c_l$

B.3 Laissez Fair (LF) Equilibrium

Suppose that individual i works at the high skill job we can write his (indirect) utility function as a function of wage and prices. If he takes the high skill job, we get

$$V_i(c_h, p) = V(c_h, p) - \theta_i$$

and at the low skill job,

$$V_i(c_l, p) = V(c_l, p)$$

An individual takes a high skill job if and only if the following condition holds

$$V(c_l, p) < V(c_h, p) - \theta_i \iff \theta_i < V(c_h, p) - V(c_l, p)$$

Let us define m as a measure of people who take up the high skill job under LF regime, i.e.

$$m = \int_0^{V(c_h,p)-V(c_l,p)} f(\theta)\, d\theta$$

Similarly, n is defined as a measure of people who take up the low skilled job.

$$n = \int_{V(c_h,p)-V(c_l,p)}^{h} f(\theta)\, d\theta$$

Assuming that $c_h > a$, and $c_l < a$, where is the expedityure satiation on the essential good, consumption of the essential good of the high skill job takers (rich) and the dons is $e(p)$, the consumption of the low skill job takers (poor) is $\frac{c_l}{p}$. Thus, market clearing condition for the essential good is

$$n\frac{c_l}{p_{LF}} + me(p_{LF}) + e(p_{LF}) = 1$$

which gives the following equilibrium price

$$p_{LF}\big(1 - (m+1)e(p_{LF})\big) = nc_l$$

or

$$p_{LF} = \frac{1}{\big(1 - (m+1)e(p_{LF})\big)}nc_l$$

B.4 Government Intervention

Now suppose the government wants to intervene in the economy and maximize the utilitarian social welfare function $\sum U_i$.

We prove a lemma that shows that with a utilitarian social welfare function and quasi-linear utility function, maximizing $\sum U_i$ is equivalent to (almost) maximizing the consumption of the poor when the essential good is price inelastic, and the number of poor goes to infinity. More formally, the social welfare function becomes **lexicographic** in the consumption of the poor. The planner first tries to maximize the welfare of the poor. Only after that, he cares about other criteria that affect welfare.

Lemma 8.3. *If the essential good is price inelastic and the relative number of poor in the economy is very large*[37]*, then the planner becomes lexicographic in the utility of poor people. This is because the ratio of marginal utility of the poor to the marginal utility rich goes to infinity.*

Proof. Suppose, for the rich, the marginal utility of consuming the essential good is λp where λ is the marginal utility of income and p is the price of essential good. We also know that λ is bounded above zero given luxury good is linear.
Thus, the ratio of marginal utility of the poor to the marginal utility of the rich is

$$\frac{u'(\frac{c_l}{p})}{\lambda p}$$

Suppose this does not go to infinity as n tends to infinity for an inelastic good, i.e. it is bounded.

Then, we get

$$\frac{1}{\lambda p}u'(\frac{c_l}{p}) \leq c \quad as \quad n \to \infty$$

We know that since p is increasing in n, thus, as n goes to infinity p also goes to infinity. Therefore we get

$$\frac{1}{\lambda p}u'(\frac{c_l}{p}) \leq c \quad as \quad p \to \infty$$

With a quasi-linear utility function, if this is true for $p \to \infty$, then it must be true for all p, i.e.

$$u'(\frac{c_l}{p}) \leq \lambda c.p \quad \forall p$$

Since this is true for all p, it must be true for $p = \frac{c_l}{x(p)}$ as well, where $x(p)$ is the demand for the essential good. Thus we get that

[37] Formally this means there exists $\bar{n}$, such that $\forall n \geq \bar{n}$, the result holds

$$u'(x(p)) = u'(\frac{c_l}{\frac{c_l}{x(p)}})) \leq \lambda c \frac{c_l}{x(p)} \tag{31}$$

$$\tag{32}$$

Now from the first order conditions, we also know that $p = \lambda u'(x(p))$ Thus we get that $px(p) \leq \lambda^2.c.c_l$ which means that $p(x(p))$ i.e. expenditure is bounded above.

However, given that the good is strictly inelastic, we know that the expenditure function increases in p, i.e.

$$\frac{d}{dp}(px) >> 0 \tag{33}$$

and thus expenditure on the commodity must not be bounded. Q.E.D.

□

Now that we have established that under the above conditions, the social planner has a lexicographic preference to maximise the consumption of essential goods, we now compare two interventions that increase the welfare of the poor. One is that of a non-linear labour taxation regime, and the second is that of segmenting the market for essential goods. We first discuss the taxation regime.

B.4.1 Non-Linear Taxation

Remark. In any regime in the measure of poor is n, the following equation must hold

$$1 = (n + m - m(t))\frac{c_l}{p} + m(t)e(p,t) + e(p)$$

which can be rearranged to

$$p = (n + m - m(t))c_l + p(m(t)e(p,t) + e(p))$$

this means that

$$p \geq (n + m - m(t))c_l$$

which means it goes to infinity as n goes to infinity.

Our setup, with two jobs, means the government can tax the high skilled job holders and use that money to give transfers to low-skilled jobholders. Let the government set t_h as tax for the high skilled job and t_l as a transfer for the low skilled job.

Under this setup, If a worker takes the high skill job then he gets the utility

$$V_i(c_h - t_h, p) = V(c_h - t_h, p) - \theta_i$$

and at the low skill job, he gets

$$V_i(c_l + t_l, p) = V(c_l + t_l, p)$$

An individual takes a high skill job if and only if the following condition holds

$$V(c_l + t_l, p) < V(c_h - t_h, p) - \theta_i \iff \theta_i < V(c_h - t_h, p) - V(c_l + t_l, p)$$

Let us define m_1 as a measure of people who take up the high skill job under this system of non-linear taxation, i.e.

$$m_1 = \int_0^{V(c_h - t_h, p) - V(c_l + t_l, p)} f(\theta)\, d\theta$$

Given tax rate is set at $t_h > 0$, this leads to a reduction of supply at high skill jobs, i.e. $m_1 < m$. Also due to the budget balance condition, the transfer to the poor has to be

$$t_l = \frac{t_h m_1}{n + m - m_1}$$

Also, in equilibrium the following condition must hold

$$1 = (n + m - m(t_h))(\frac{c_l + t_l}{p}) + m(t)e(p, t) + e(p)$$

$$p = (n + m - m(t_h))(c_l + t_l) + pm(t)e(p, t) + pe(p)$$

which gives the following equilibrium price.

$$p = \frac{1}{1 - m(t)e(p, t) - e(p)}(n + m - m(t_h))(c_l + t_l)$$

Clearly, the price is an increasing function of n. Now also notice that both m_1 and t_l are functions of t_h. So we can say that number of people working in the high productive job m is a function of t_h where $t_h = 0$ is the special case of laissez faire economy. Also notice that if $t_h \geq c_h - c_l$, no one works in the high productivity jobs. Thus, $m_1 = 0$. Hence, effectively t_h and m_1 are both bounded in our setup.

Moreover, in this economy, the dead-weight loss is given by

$$DWL = \int_{m_1}^{m} (c_h - c_l - \theta_i) f(\theta)\, d\theta \tag{34}$$

Having set up the non-linear taxation regime, we now discuss the segmentation regime.

B.4.2 Segmentation

Under this setup, complete segmentation is a scenario where the planner caps the expenditure on essential goods at c_l. Thus, the people who work in the high productivity jobs and the low productivity jobs consume an equal amount of essential goods in complete segmentation. However, the high productivity workers can potentially consume luxuries. We discuss the social welfare and dead-weight loss in this setup.

Given individual ability θ_i, an agent works in the high productivity jobs iff

$$V_i(c_h, p) = u(\frac{c_l}{p}) + c_h - c_l - \theta_i \geq u(\frac{c_l}{p}) = V_i(c_l, p)$$

and for production

$$\theta_i \leq c_h - c_l$$

In equilibrium, the following must hold.

$$1 = (n+m)\frac{c_l}{p} + e(p)$$

where $\frac{c_l}{p}$ is the consumption of both high and low productivity workers and $e(p)$ is the consumption of the don. Moreover, the dead-weight loss is given by

$$DWL = \int_{c_h - c_l}^{m} (c_h - c_l - \theta_i) f(\theta)\, d\theta \tag{35}$$

Now we prove our main result. We know that when $n \to \infty$, the social welfare is lexicographic in the welfare of the poor. Thus we first compare the welfare of the poor in both regimes and then compare the dead-weight loss. We show that when $n \to \infty$, consumption of the poor is always increasing in t_h. Thus, t_h is set at the maximum level possible i.e. $t_h = c_h - c_l$. Moreover, at this level $m(t_h) = 0$. At this level, the dead-weight loss in luxury good production is at the highest level, i.e. no one works in high-productivity jobs. Hence, the production of luxury goods is at the lowest possible level. On the other hand, if we use segmentation, i.e. allow the rich to spend only c_l on the essential goods, we have the same allocation of essential goods but greater production of luxury goods in the economy. Formally, we state this result as a theorem below.

Theorem 9. *If the essential good is price inelastic, as the number of poor in the economy increases, complete Market Segmentation dominates a non-linear labour supply taxation.*

First, we consider what tax level t_h maximises welfare under the non-linear taxation regime. Given that the tax rate for the planner is t_h, the measure of high productivity job workers in the economy is $m(t_h)$. Let e_{poor} be the consumption of the essential good of the poor, $e(p(t_h), t_h)$ be the consumption of the rich and the $e(p(t_h)))$ be the consumption of the 'don.' Notice that the taxes affect the rich in two ways: Directly and indirectly (through price). For the dons the effect is only indirect.

The consumption of the poor in this regime can be written by

$$e_{poor} = \frac{1}{n}\left[1 - m(t_h)e(p(t_h), t_h) - e(p(t_h))\right]$$

Differentiating w.r.t. t_h

$$\frac{de_{poor}}{dt_h} = \frac{1}{n}\left[-\frac{dm(t_h)}{dt_h}e(p(t_h), t_h) - m(t_h)\frac{\delta e(p(t_h), t_h)}{\delta t_h} - (m(t_h)+1)\frac{\delta e}{\delta p}\frac{dp}{dt_h}\right]$$

Because of inelasticity of essential good we know that

$$\frac{\delta e}{\delta p}p + e(p) \geq 0 \implies |\frac{\delta e}{\delta p}| \leq \frac{e(p)}{p}$$

Given that $\frac{\delta e(p(t_h),t_h)}{\delta t_h}$ and $\frac{dm(t_h)}{dt_h}$ are always less than equal to zero, we can lower bound the above term by

$$\frac{1}{n}\left[-\frac{dm(t_h)}{dt_h}e(p(t_h), t_h) - m(t_h)\frac{\delta e(p(t_h), t_h)}{\delta t_h} + (m(t_h)+1)\frac{e(p)}{p}\frac{dp}{dt_h}\right] \tag{36}$$

Now we show that for a large n the above term is always positive.

First notice that given

$$t_l = \frac{t_h m_1}{n + m - m_1}$$

and

$$p = (n + m - m(t_h))(c_l + t_l) + pm(t)e(p, t) + pe(p)$$

$\implies$

$$p = (n + m - m(t_h))(c_l) + m(t_h)t_h + m(t_h)e(p, t_h)p + e(p)p$$

$$\frac{dp}{dt} = \frac{dm}{dt}\left[t - c_l + pe(p, t)\right] + m(t) + m(t)\left[p(\frac{\delta e(.)}{\delta p}\frac{dp}{dt_h} + \frac{\delta e(.)}{\delta t_h}) + e(p)\frac{dp}{dt}\right] + \frac{dp}{dt}\left[e(p) + p\frac{\delta e}{\delta p}\right]$$

which gives us

$$\frac{dp}{dt} = \left[1 - (m(t)+1)\left[e(p) + p\frac{\delta e}{\delta p}\right]\right]^{-1}\left[\frac{dm}{dt}(t - c_l + pe(p(t), t)) + m(t)(1 + p\frac{\delta e(p(t), t)}{\delta t})\right]$$

Now let us compute

$$\frac{e(p)}{p}\frac{dp}{dt} = \left[1-(m(t)+1)\left[e(p)+p\frac{\delta e}{\delta p}\right]\right]^{-1}\left[e(p)\frac{dm}{dt}\left[\frac{t}{p}-\frac{c_l}{p}+e(p(t),t)\right]\right] + \left[1-(m(t)+1)\left[e(p)+p\frac{\delta e}{\delta p}\right]\right]^{-1}\left[\frac{m(t)e(p)}{p}\left[1+p\frac{\delta e(p(t),t)}{\delta t}\right]\right]$$

Now putting this term back in 36, we get

$$\frac{1}{n}\Bigg[-\frac{dm(t_h)}{dt_h}e(p(t_h),t_h)-m(t_h)\frac{\delta e(p(t_h),t_h)}{\delta t_h} + (m(t_h)+1)\left[1-(m(t)+1)\left[e(p)+p\frac{\delta e}{\delta p}\right]\right]^{-1}\left[e(p)\frac{dm}{dt}\left[\frac{t}{p}-\frac{c_l}{p}+e(p(t),t)\right]\right] + \left[1-(m(t)+1)\left[e(p)+p\frac{\delta e}{\delta p}\right]\right]^{-1}\left[\frac{m(t)e(p)}{p}\left[1+p\frac{\delta e(p(t),t)}{\delta t}\right]\right]\Bigg] \quad (37)$$

Given $\left[1-(m(t)+1)\left[e(p)+p\frac{\delta e}{\delta p}\right]\right]$ is positive and $\frac{1}{p}\geq\frac{-\delta e}{\delta t}$, we know that 1st, 2nd and 4th term are positive. We now compare 1st and 3rd term and show that their sum is positive.

Adding we get

$$-\frac{dm(t_h)}{dt_h}e(p(t_h))\left[1-\left[1-m(t)e(p(t))-\left[e(p)+p\frac{de}{dp}\right]\right]^{-1}\right]\left[\frac{t}{p}-\frac{c_l}{p}+e(p(t),t)\right]$$

Now we know as $n\to\infty$

$$\left[1-m(t)e(p(t))-\left[e(p)+p\frac{de}{dp}\right]\right]\to 1$$

and

$$\left[\frac{t}{p}-\frac{c_l}{p}+e(p(t),t)\right]\to 0$$

Thus,

$$-\frac{dm(t_h)}{dt_h}e(p(t_h))\left[1-\left[1-m(t)e(p(t))-\left[e(p)+p\frac{de}{dp}\right]\right]^{-1}\right]\left[\frac{t}{p}-\frac{c_l}{p}+e(p(t),t)\right]\geq 0$$

Thus, we know as $n\to\infty$ $\frac{de_{poor}}{dt_h}\geq 0$, hence t_h is set at the maximum level i.e $t_h=c_h-c_l$. Consumption of the low skilled job holders (in this case everyone except the dons) is given by $\frac{c_l}{p}$

Moreover, at this level of taxation $m(t_h)=0$ and thus the dead-weight loss is given by

$$DWL = \int_0^m (c_h - c_l - \theta_i) f(\theta)\, d\theta$$

Under complete segmentation, the consumption of the low skilled job holders (again everyone except the dons) is same as the above i.e. $\frac{c_l}{p}$.

However, the dead-weight loss is

$$DWL = \int_{c_h - c_l}^m (c_h - c_l - \theta_i) f(\theta)\, d\theta$$

Given $c_h - c_l >> 0$ by assumption, the dead-weight loss in the segmentary regime is much more lower. Q.E.D.

Remark. Notice that DWL under non linear taxation is

$$DWL = \int_0^m (c_h - c_l - \theta_i) f(\theta)\, d\theta$$

Hence, if the distribution of skills is very dense at the tail (close to zero), DWL is large. We believe this is the analogue of 'superstar phenomenon' in our model, because this means that the skill distribution in the economy is such that the number of very high skill workers (or superstars) is large. Hence very high taxes deter them from producing which creates a lot of dead-weight loss. However, DWL in the case of segmentation is

$$DWL = \int_{c_h - c_l}^m (c_h - c_l - \theta_i) f(\theta)\, d\theta$$

which is unaffected 'superstars' (the mass at or close to zero: the highest skilled individuals). Thus, compared to non-linear taxation MS performs better. A simple way to see this is that the efficiency gain under MS over taxation is

$$Gain = \int_0^{c_h - c_l} (c_h - c_l - \theta_i) f(\theta)\, d\theta$$

so if the mass near zero increases the efficiency gain also increases.