# Competition for Order Flow and Smart Order Routing Systems

# Thierry Foucault

HEC School of Management Paris, GREGHEC and CEPR
1 rue de la Liberation
78351 Jouy en Josas, France
foucault@hec.fr

#### Albert J. Menkveld

Vrije Universiteit Amsterdam De Boelelaan 1105 1081 HV Amsterdam, Netherlands ajmenkveld@feweb.vu.nl

# January, 2006<sup>1</sup>

<sup>1</sup>We thank Robert Battalio, Jean-François Gajewski, Oliver Hansch (the EFA discussant), Joel Hasbrouck, Terry Hendershott, Charles Jones, Stephen Mayhew, Christine Parlour, Ailsa Roëll, Gideon Saar, Chester Spatt, Patrik Sandås, Duane Seppi (the AFA discussant), Ilias Tsiakas, Dan Weaver and seminar participants at Universitat Pompeu Fabra, the SEC, the NYSE, the University of Virginia, Paris XII, the International Conference on New Financial Market Structures in Montreal, the Workshop on Market Microstructure in Aix-en-Provence, the International Conference on Finance in Copenhagen, the 2005 European Finance Association (EFA) Meetings and the 2006 American Finance Association Meetings (AFA) for useful comments. We also thank Patricia van Dam and Friso van Huijstee for excellent research assistance and Arjen Siegmann for sharing his Perl expertise. For this project, Menkveld is grateful to NWO for a VENI grant and to Joel Hasbrouck for a visiting scholarship at NYU-Stern in 2004-2005. Last, we thank Euronext, the London Stock Exchange and the online broker Alex for their sponsorship. Of course, we bear the entire responsibility of the paper and all errors are ours.

#### Abstract

#### Competition for Order Flow and Smart Order Routing Systems

We study changes in liquidity following the introduction of a new electronic limit order market when, prior to its introduction, trading is centralized in a single limit order market. We also study how automation of routing decisions and trading fees affect the relative liquidity of rival markets. The theoretical analysis yields three main predictions: (i) consolidated depth is larger in the multiple limit order markets environment, (ii) consolidated bid-ask spread is smaller in the multiple limit order markets environment and (iii) the liquidity of the entrant market relative to that of the incumbent market increases with the level of automation for routing decisions (the proportion of "smart routers"). We test these predictions by studying the rivalry between the London Stock Exchange (entrant) and Euronext (incumbent) in the Dutch stock market. The main predictions of the model are supported.

Keywords: market fragmentation, centralized limit order book, smart routers, trading fees, trade-throughs.

"When I was CEO of Tradepoint (now virt-X), my team and I spent a considerable amount of effort 'selling' the exchange to traders. However, although they all signed up as member, they did not use the market. One major reason was that access to the market was not connected to their trading systems. Even when better bids and offers appeared on our order book, the (momentarily) inferior prices available on the LSE were hit and lifted. Potential users simply could not see, nor easily access the market. If the Tradepoint terminal was at the end of the desk, it was not accessible." (in "Is Exchange Liquidity Contestable?", by Nic Stuchfield, The Handbook of World Stock, Derivative and Commodity Exchange.)

# 1 Introduction

Automation has decreased the cost of developing trading systems, fostering the emergence of new electronic marketplaces (e.g. ECNs, International Securities Exchange, Tradepoint, etc.) and ever more fragmented financial markets, both in the US and in Europe. Whether this evolution is desirable is highly controversial and is at the heart of major regulatory overhauls in equity markets.<sup>1</sup>

In the US, for instance, it fuels a debate over the benefits and costs of a centralized limit order book (CLOB) with strict price-time priority. CLOB proponents argue that it enhances liquidity, as it greatly reduces search efforts for best execution.<sup>2</sup> Opponents argue that multiple limit order markets, linked by smart order routing systems, promote intermarket competition while retaining the benefits of order flow concentration, as smart routing systems effectively consolidate the different markets (see Stoll (2001) for a discussion).

We contribute to this debate by comparing measures of market liquidity in two situations: (i) trading in a security is centralized in a single limit order book and (ii) trading in this security is fragmented across two competing limit order books. We do this comparison by extending the model of intermarket competition developed by Parlour and Seppi (2003). We also test the predictions of the model, taking advantage of a recent reorganization of the Dutch stock market. For long, trading in this market has largely been centralized in a single limit order market, NSC, operated by Euronext. This situation changed in May 2004 when the London Stock Exchange introduced another electronic limit order market,

<sup>&</sup>lt;sup>1</sup>See for instance "Request for Comment on Issues Relating to Market Fragmentation" SEC Release  $n^{\circ}34 - 42459$  and "Regulation NMS", SEC Release  $n^{\circ}34 - 51808$  (available at http://www.sec.gov).

<sup>&</sup>lt;sup>2</sup>In the US, institutions such as Morgan Stanley, Goldman Sachs and Merril Lynch have been prominent advocates of a CLOB. In contrast, ECNs have argued against it. See "Don't CLOBber ECNs", Wall Street Journal, March 27, 2000 and "Sweeping Changes in Markets Sought", Wall Street Journal, February 29, 2000.

EuroSETS, for stocks traded in NSC. This introduction offers a unique opportunity to study empirically whether fragmentation of liquidity among competing limit order books enhances or impairs liquidity.

The nature of our empirical analysis determines the directions in which we extend Parlour and Seppi (2003), who pit a pure limit order market against a hybrid trading system (like the NYSE). First, we focus our theoretical analysis on competition between two pure limit order markets: an "incumbent" and an "entrant" market (like NSC and EuroSETS). Second, we allow competing markets to charge different trading fees. In particular, consistent with the pricing strategy of EuroSETS (and many ECNs), we allow the entrant market to charge lower fees on "passive" orders, i.e. (non-marketable) limit orders and higher fees on "aggressive" orders, i.e. market orders.<sup>3</sup> Third, we analyze how routing decisions affect intermarket competition.

When a security trades in multiple markets, brokers' routing decision is determined by a trade-off between the benefit of improved execution with the cost of searching for improved execution (which includes the cost of checking offers in various trading venues and the cost of splitting orders across these venues). The opening quotation suggests that even seemingly small search costs can result in violations of price priority ("trade-throughs"). This problem stifles intermarket competition as it reduces liquidity providers' incentives to post competitive prices. It is likely to be particularly acute for entrant markets, like EuroSETS, because, by force of habit, brokers are naturally inclined to check offers only in the incumbent market. Consequently, we assume that there are two types of brokers: (i) smart routers, who systematically search for best execution and (ii) non-smart routers, who exclusively trade in the incumbent market. We interpret smart routers as brokers equipped with a smart order routing system, a technology that consolidates quotes in different markets and routes orders accordingly, so as to minimize trading costs. More generally, they can be viewed as brokers with relatively small search costs.

The model yields three testable predictions. First, consolidated depth at a certain price (i.e. the total number of shares offered at that price or better prices in *both* markets) is always larger in the multiple limit order markets environment. The absence of time priority across markets is key for this finding.<sup>5</sup> Intuitively, it allows traders to jump in front of the

<sup>&</sup>lt;sup>3</sup>For ease of exposition, we henceforth refer to non-marketable limit orders as limit orders and to marketable limit orders as market orders.

<sup>&</sup>lt;sup>4</sup>Trade-throughs are well documented in US markets. For instance, Battalio et al.(2004) find a substantial proportion of trade-throughs in option markets. Trade-throughs also occur in equity markets (see Bessembinder (2003)), even though they are prohibited for ITS participants. See also Hendershott and Jones (2005a).

<sup>&</sup>lt;sup>5</sup>Parlour and Seppi (2003) find that competition between an hybrid market and a pure limit order market can increase or decrease consolidated depth. We obtain a different result because we focus on two

queue of limit orders placed at one price in, say, the incumbent market by submitting a limit order at the same price in the entrant market. Queue-jumping competes away the profits earned on infra-marginal limit orders (i.e. orders *strictly* ahead of the marginal order in the queue) in either market. The reduction in liquidity providers' rents translates in smaller trading costs (larger consolidated depth) at the benefit of smart routers. In addition, the queue-jumping effect implies that cumulative depth in the incumbent market (i.e. the total number of shares offered up to a given price in this market) should be smaller after introduction of the competing market. Actually, queue-jumping reduces the execution probability of limit orders in the incumbent market.

Second, the model predicts that intermarket competition can drive down the consolidated bid-ask spread (the difference between the best ask price and the best bid price in the consolidated limit order book). This occurs if intermarket competition leads competing markets to reduce fees on limit orders by a sufficiently large amount. In this case, the lowest (highest) price at which it is profitable to submit limit sells (buys) is smaller (larger) and the bid-ask spread is, therefore, smaller. Moreover, a reduction in fees on limit orders in one market always triggers (i) an increase in the cumulative depth of this market and (ii) an increase in consolidated depth.

Third, the model predicts a positive relationship between the competitiveness of the entrant market and the fraction of trades conducted by smart routers (for short, "the proportion of smart routers"). Intuitively, an increase in the proportion of smart routers raises the execution probability of limit orders submitted to the entrant market. Traders are, therefore, more willing to post competitive quotes with larger depth in this market.

We test these predictions using a sample of 22 stocks traded both in EuroSETS and NSC. We consider three different sample periods: one period in April-May 2004, just before entry of EuroSETS and two post-entry periods (August 2004 and January 2005). We group stocks in quartiles based on trading activity and test our predictions for each quartile separately.

In order to analyze the effect of EuroSETS entry on consolidated depth, we build snapshots of the *consolidated limit order book* (observed every five minutes) for each stock and each sample period. We then compare consolidated depth before and after entry of

pure limit order markets.

<sup>&</sup>lt;sup>6</sup>The absence of time priority is often viewed as deterring, instead of encouraging, liquidity provision (see Harris (1990)). In a sense the deterrence effect is present in our model: traders submit less limit orders in the incumbent market because queue-jumping reduces the likelihood of execution in this market. However, queue-jumping facilitates "entry" of relatively slow bidders (those who submit orders at the end of the queue). This second effect encourages competition between limit order traders and more than compensates the deterrence effect.

EuroSETS. After controlling for changes in market conditions in a multivariate setting, we find that consolidated depth, throughout the book, is *larger* after entry. This increase is significant for almost all quartiles and its magnitude is greater in the second post-entry period. For instance, for the most actively traded stocks, we find that consolidated depth through the fourth tick behind the best quote increased by a significant 46.3% and 100.8% in the first and second post-entry period, respectively. We find that this increase is partially due to increased depth in NSC after the entry of EuroSETS. We explain this evolution by the reduction in fees charged by NSC on limit orders, around the time of EuroSETS entry. Consistent with this interpretation, we also find that consolidated bid-ask spreads are smaller or unchanged relative to their pre-entry levels. Overall, these findings show that competition between multiple limit order market enhance liquidity and suggest that the level of fees on limit orders is an important determinant of market liquidity.

Next, we study the relationship between the proportion of smart routers and the liquidity of EuroSETS relative to that of NSC. In our theoretical analysis, the likelihood of observing a buy (sell) market order executed in the entrant market, when this market posts a strictly better ask (bid) price, is equal to the proportion of smart routers. We exploit this observation to build a proxy for the proportion of smart routers. We find that, on average, the proportion of smart routers is small (27% and 18% in the first and second post-entry period, respectively). As predicted by the model, we find a positive cross-sectional relationship between EuroSETS competitiveness and the proportion of smart routers. In particular, EuroSETS bid-ask spread relative to that of NSC is significantly smaller in stocks for which the proportion of smart routers is large. Moreover, there is a positive relationship between EuroSETS contribution to quoted depth and the proportion of smart routers (statistically significant only in the second post-entry period). This finding implies that automation of routing decisions or, equivalently in the model, trade-throughs prohibition encourage the display of limit orders.

We also show that EuroSETS liquidity is not negligible, especially for the most active stocks. Yet, EuroSETS market share is small. We show that this wedge between EuroSETS market share and its "liquidity share" results from the combination of two factors: (i) the low proportion of smart routers and (ii) the fact that brokers give priority to NSC when NSC and EuroSETS are tied at the best quotes. Again, this finding points to the importance of smart routers for vibrant intermarket competition.

There is a vast literature on market fragmentation (see Lee (2002) for a survey). The theoretical literature usually concludes that fragmentation is harmful for liquidity (Mendelson (1987)) and traders' welfare (Pagano (1989)). This literature, however, has not considered the effect of fragmentation on *consolidated depth*, as we do here. There also exist

many empirical studies on this topic.<sup>7</sup> Most related to our paper are the studies that compare measures of market liquidity before and after entry of new competitors in the provision of trading services (e.g. Battalio (1997), Mayhew (2003), DeFontnouvelle et al. (2003), Boehmer and Boehmer (2004), Battalio et al. (2004), Biais, Bisière and Spatt (2004)). However, to the best of our knowledge, there is no empirical evidence regarding competition between pure limit order markets.<sup>8</sup>

Glosten (1994) analyzes theoretically competition between pure limit order markets. He shows that consolidated depth is identical when order flow concentrates in a single market or fragments between several limit order markets (see Proposition 8 in Glosten (1994)). In this model, limit order traders get zero expected profits because (i) there is free-entry of limit order traders and (ii) a pro-rata allocation rule is used when several limit order traders post the same price. Introduction of a new limit order market is therefore redundant. Glosten (1998) argues that this prediction does not hold if time priority is used instead of a pro-rata allocation rule. In this case, infra-marginal limit orders in a given book get strictly positive expected profits. Introduction of a new limit order market allows traders to compete away these expected profits if time priority is not enforced across markets. As a result, consolidated depth can be larger in the multiple markets environment. Our framework is very different from Glosten (1998) (e.g. there is no asymmetric information in our setting) but, interestingly, a similar economic intuition explains why intermarket competition enhances consolidated depth.

The plan of the paper is as follows. Section 2 describes the model. Section 3 derives the main implications of the model. Section 4 describes the natural experiment conducted in this paper and reports the empirical findings. Section 5 concludes. All proofs are in Appendix A.

<sup>&</sup>lt;sup>7</sup>For instance, Hamilton (1979), Bessembinder (2003), Barclay et al. (2003), Fink et al. (2004), Hendershott and Jones (2005b).

<sup>&</sup>lt;sup>8</sup>Board and Wells (2000) study competition between SETS and Tradepoint (2 UK based limit order markets). They do not study the effect of this competition on consolidated depth, however.

<sup>&</sup>lt;sup>9</sup>Some authors study competition between a pure limit order market and a market using a different trading mechanism, e.g. a dealership market (as in Viswanathan and Wang (2002) or Sabourin (2004)) or a hybrid market (as in Parlour and Seppi (2003)). Other papers on competition between markets (e.g. Pagano (1989), Chowdry and Nanda (1991) or Hendershott and Mendelson (2000)) do not consider limit order markets.

<sup>&</sup>lt;sup>10</sup>Biais, Martimort and Rochet (2000) relax the free-entry assumption. They show that liquidity suppliers earn strictly positive expected profits when their number is finite. These profits decrease and the market becomes deeper as the number of liquidity suppliers increases.

# 2 Competition for Order Flow with Smart Routers

# 2.1 Model

As explained in the introduction, the model builds upon Parlour and Seppi (2003). There are three periods in this model. In period 1, limit order traders post offers in the competing markets. In period 2, a broker arrives. She must execute immediately an order for  $\widetilde{X}$  shares on behalf of a client. In period 3, payoffs are realized. We now explain in more detail the trading rules and the actions available to the traders active in periods 1 and 2.

Competing markets. There are two markets: (a) the *incumbent* market, denoted I and (b) the *entrant* market, denoted E. A risky security, with expected final payoff  $v_0$ , trades on these two markets. Both markets are organized as limit order markets. In each market, traders position their limit orders on a price grid. The tick size, i.e. the difference between two successive prices on the grid,  $(p_{i+1} - p_i)$ , is equal to  $\Delta$ . We set  $p_0 \stackrel{def}{=} v_0$ .

We exclusively focus on the number of shares offered at the "ask side" of the book (that is at prices  $\{p_1, p_2, ...\}$ ).<sup>11</sup> It is straightforward to derive symmetric results for the "bid side" of the book. We denote by  $S_{jk}$  the number of shares offered for sale at price  $p_k$  ( $k \ge 1$ ) in market j. The cumulative depth at price  $p_k$  in market j is denoted by  $Q_{jk}$  (with  $Q_{j0} = 0$ ). We denote by  $Q_k$  the consolidated depth at price  $p_k$ , that is the total number of shares offered up to price  $p_k$  in markets I and E. Formally:

$$Q_{jk} = Q_{jk-1} + S_{jk}, \forall k \ge 1 \tag{1}$$

and

$$Q_k = Q_{Ik} + Q_{Ek}, \forall k \ge 1. \tag{2}$$

Price and time priority are enforced within each market but not across markets. When a buy market order is routed to market j, it executes against the limit orders standing in this market at progressively higher prices until completion of the order. Moreover, limit orders standing at a given price are filled in the sequence in which they were submitted (time priority).

**Limit order traders.** Limit orders in each market are submitted by risk-neutral traders who arrive at stochastic points in times during period 1. The trader who arrives first in market  $j \in \{I, E\}$  sees an empty book and fills the book. Then a new trader arrives, observes the book, and decides to add depth to the book or not. The process stops

In this model, it is never optimal to submit a sell (buy) limit order below (above)  $v_0$  (see below).

when there is no price in each book at which submitting a limit order is profitable.<sup>12</sup> Then, the trading game proceeds to period 2.

Limit order traders active in market j bear two distinct costs. First, they bear an execution cost  $f_j \geq 0$ , per share, when their order executes in market j. This cost includes the execution fee charged by market j and other order-handling costs (back-office, clearing and settlement costs, etc.). Second, they bear an order entry cost  $c_j > 0$  per share, whether their order is executed or not. This cost includes the costs associated with the time required to submit and then monitor the order and, possibly, an entry fee charged by market j. Importantly, this cost is paid whether execution takes place or not, in contrast to the execution fee,  $f_j$ . The model encompasses the particular case in which  $c_I = c_E$ . We interpret differences in order entry costs or execution costs across markets as resulting from differences in the fees charged by each market (as order-handling costs and monitoring costs are likely to be identical across markets). For our empirical analysis, the relevant case is  $c_I > c_E$  as EuroSETS does not charge order entry fees while NSC does (see Section 4).

Market order traders and smart routers. The broker arriving in period 2 must fill a market order on behalf of a client. A client is characterized by (a) the size of his order,  $\widetilde{X}$ , (b) the direction of his order (buy/sell) and (c) his reservation price. We assume that the client is a buyer with probability  $\alpha$  and that  $\widetilde{X}$ , is random with a cumulative probability distribution F(.). The client's reservation price is fixed and equal to  $p_m > p_1$ . Thus, the broker never executes a market order at a price strictly larger than  $p_m$ . For this reason, limit orders at prices strictly larger than  $p_m$  have a zero execution probability and are never submitted.

The broker has one of two types: (i) either she is a smart router (probability  $\gamma$ ) or (ii) she is not (probability  $(1-\gamma)$ ). In the latter case, she only considers the incumbent market as a possible trading venue. In contrast, a smart router consolidates offers in both trading systems and optimally splits her order between markets I and E so as to minimize total trading cost. If the broker's order size is  $x > Q_m$  then she executes all limit orders placed at prices  $p_1, p_2, ..., p_m$  in each market. The residual portion is left unexecuted. If the broker's order size is  $x \in [Q_{s-1}, Q_s]$  with  $s \leq m$  then she executes all limit orders placed at prices  $p_1, p_2, ..., p_s$  in each market.

In general, at the stop-out price  $p_s$ , there are several ways to allocate the residual portion,  $(x - Q_{k-1})$ , of the market order between the two competing markets. We assume

 $<sup>^{12}</sup>$ As in Parlour and Seppi (1997), we assume that the number of potential liquidity suppliers is infinite.  $^{13}$ This assumption is innocuous as our results do not depend on  $p_m$ . It just guarantees that a buy order does not get executed at an infinite price because the limit order book is too thin to absorb the entire order.

that smart routers use one of two tie-breaking rules in this case: (a) trade first in market I and then execute the residual (if any) in market E or (b) trade first in E and then in I. A proportion  $\delta_I$  ( $\delta_E = 1 - \delta_I$ ) of smart routers uses the first (second) tie-breaking rule. For the moment, we assume that markets I and E charge no fees on market orders. In Section 4.2. we relax this assumption and show that  $\delta_I$  should be determined by the fees on market orders.

We interpret smart routers as traders equipped with Smart Order Routing Systems (SORSs). For these traders, the cost of searching for the best routing strategy and implementing it is nil as the search process is completely automated. Smart routers can be viewed more generally as brokers who systematically search for the smallest trading costs. In contrast, we view non-smart routers as brokers who handle orders manually. In order to economize on search costs, they trade only in one market.<sup>14</sup> In Section 4.2, we show how to extend the model when non-smart routers use a more complex routing strategy.

Strictly speaking, parameter  $\gamma$  is the proportion of trades intermediated by smart routers. For brevity, we refer to this parameter as being "the proportion of smart routers". Two polar cases are of special interest. Obviously, the case  $\gamma=0$  is similar to the situation in which the incumbent market operates alone. We can therefore develop predictions about the effects of introducing a new trading venue by comparing liquidity when (i)  $\gamma=0$  and (ii)  $\gamma>0$ . The case  $\gamma=1$  constitutes another interesting situation. In this case, brokers can costlessly use both systems and thereby orders freely flow to the cheapest market. This case occurs when all brokers are smart routers. Alternatively, it represents a situation in which trade-throughs (i.e. violations of price priority) are prohibited or avoided by adequate intermarket linkages. For instance, in the US, markets participating in the ITS (Intermarket Trading System) agreements for NYSE-listed stocks must either (a) match the intermarket best offer (best bid) or (b) reroute market buy (sell) orders to the market posting the best offer (bid price) when the order arrives.<sup>15</sup>

There are at least two factors that determine the proportion of smart routers: (a) the fraction of brokerage firms adopting SORSs and (b) smart routers' market share. One may expect this market share to increase over time as smart routers achieve smaller trading costs for their clients. In the short run however, brokerage clienteles are likely to be sticky

<sup>&</sup>lt;sup>14</sup>The quotation opening the present article suggests that the mere fact of switching terminals to check offers in the entrant market might be perceived as being "too costly" ("Potential users simply could not see, nor easily access the market. If the Tradepoint terminal was at the end of the desk, it was not accessible.").

<sup>&</sup>lt;sup>15</sup>There is no trade-through regulation for the stocks considered in our empirical analysis and we find frequent occurences of trade-throughs in our empirical analysis (see Section 5.2). Moreover, in the Dutch equity market, best execution is defined with respect to a benchmark, which is the price posted in the "main" market (NSC), rather than the best price across all trading venues. Thus, trading at the price posted in the main market is sufficient to comply with best execution in the Dutch market.

as (i) it takes time for investors to realize which brokers achieve smaller costs and (ii) switching brokers is costly. Thus, we take  $\gamma$  as given in our analysis. We discuss incentives of brokerage firms to adopt SORSs in Section 5.2.

# 2.2 The Competitive Equilibrium

We focus on *competitive equilibria*, i.e. limit order books at the end of period 1 that leave no profit opportunities. In this section, we define these equilibria formally.

Let  $P_{Ik}(Q_{Ik-1}, Q_{k-1}, S_{Ik}, S_{Ek})$  and  $P_{Ek}(Q_{k-1}, S_{Ik}, S_{Ek})$  be the probabilities that all shares offered at price  $p_k$  in markets I and E, respectively, get executed. As time priority is enforced within each market, execution of all shares offered at price  $p_k$  in market j is equivalent to execution of the last share offered at price  $p_k$  in market j ("the marginal limit order" at this price). For this reason, we refer to  $P_{jk}$  as being the execution probability of the marginal order at price  $p_k$  in market j. The next lemma relates  $P_{jk}$  to the probability distribution of the market order size. We denote the likelihood that a market order is larger than x by  $\overline{F}(x) \stackrel{def}{=} 1 - F(x)$ .

**Lemma 1**: The execution probabilities of the marginal order at price  $p_k \leq p_m$  in the incumbent and the entrant market are, respectively:

$$P_{Ik}(Q_{Ik-1}, Q_{k-1}, S_{Ik}, S_{Ek}) = \alpha[(1-\gamma)\overline{F}(Q_{Ik-1} + S_{Ik}) + \gamma(\delta_I \overline{F}(Q_{k-1} + S_{Ik}) + \delta_E \overline{F}(Q_{k-1} + S_{Ik} + S_{Ek}))],$$
(3)

and

$$P_{Ek}(Q_{k-1}, S_{Ik}, S_{Ek}) = \alpha \gamma [\delta_E \overline{F}(Q_{k-1} + S_{Ek}) + \delta_I \overline{F}(Q_{k-1} + S_{Ik} + S_{Ek})]. \tag{4}$$

For a given state of the limit order book in each market, consider a trader who adds a limit order for an infinitesimal number of shares at price  $p_k$  in market j. As the order is at the end of the queue of limit orders submitted at price  $p_k$  in market j, its execution probability is  $P_{jk}$ . The trader's expected profit is equal to his revenue in case of execution times the execution probability minus the order entry cost. Thus, if the trader operates in market I, his expected profit (per share) is:

$$\Pi_{Ik}^{m}(Q_{Ik-1}, Q_{k-1}, S_{Ik}, S_{Ek}) = P_{Ik}(Q_{Ik-1}, Q_{k-1}, S_{Ik}, S_{Ek})(p_k - v_0 - f_I) - c_I.$$
 (5)

Similarly, if the trader operates in market E, his expected profit is:

$$\Pi_{Ek}^{m}(Q_{k-1}, S_{Ik}, S_{Ek}) = P_{Ek}(Q_{k-1}, S_{Ik}, S_{Ek})(p_k - v_0 - f_E) - c_E.$$
(6)

Other things equal, the execution probability of the marginal order at a given price is decreasing with the number of shares  $(S_{jk})$  offered at this price (see equations (3) and (4)). Hence, as the number of shares offered at price  $p_k$  in market j increases, the expected profit on the marginal order at this price decreases. When it is equal to zero, no trader finds optimal to add depth at price  $p_k$  in market j. If this zero profit condition holds at each price in each book, there are no profit opportunities left for liquidity suppliers and, in this sense, a competitive equilibrium is reached (see Seppi (1997), Sandås (2001), and Parlour and Seppi (2003)). Let  $Q_k^*(\gamma)$ ,  $Q_{jk}^*(\gamma)$ ,  $S_{jk}^*(\gamma)$  denote the equilibrium values at price  $p_k$ . Formally, a competitive equilibrium is defined as follows (see Definition 2 in Parlour and Seppi (2003)).

**Definition 1**: A competitive equilibrium is a set of depths  $\{S_{Ik}^*, S_{Ek}^*\}_{k=1}^m$  such that the expected profit of the marginal limit order at each price  $p_k$  in each limit order book is non-positive if  $S_{jk}^* = 0$  and nil if  $S_{jk}^* > 0$ . Formally, for  $k \in [1, m]$ :

$$\Pi_{Ik}^{m}(Q_{Ik-1}^{*},Q_{k-1}^{*},S_{Ik}^{*},S_{Ek}^{*}) = 0 \text{ if } S_{Ik}^{*} > 0 \text{ and } \Pi_{Ik}^{m}(Q_{Ik-1}^{*},Q_{k-1}^{*},S_{Ik}^{*},S_{Ek}^{*}) \leq 0 \text{ if } S_{Ik}^{*} = 0,$$

$$\Pi_{Ek}^{m}(Q_{k-1}^{*},S_{Ik}^{*},S_{Ek}^{*}) = 0 \text{ if } S_{Ek}^{*} > 0 \text{ and } \Pi_{Ek}^{m}(Q_{k-1}^{*},S_{Ik}^{*},S_{Ek}^{*}) \leq 0 \text{ if } S_{Ek}^{*} = 0.$$

It is worth emphasizing two features of competitive equilibria, as they play an important role in the rest of the analysis.

- 1. Limit orders at the end of the queue at a given price in each book just break-even. But infra-marginal orders get strictly positive expected profits (e.g.  $\Pi_{Ik}^m(Q_{Ik-1}^*, Q_{k-1}^*, S_{Ik}, S_{Ek}^*) > 0$  if  $0 < S_{Ik} < S_{Ik}^*$ ). This explains why, in this model, the incumbent market is not "competition-proof".
- 2. A limit order book can feature prices at which no limit orders are submitted  $(S_{jk}^* = 0)$ . This happens when the execution probability at a given price price is small so that a limit order at this price is not profitable. If, for a given market, there are no prices at which submitting a limit order is profitable then the book in this market is empty and the market is not viable.

The second feature implies that there are two possible equilibrium outcomes: either (i) each market attracts some limit orders or (ii) one market does not attract any limit order. In the first case, the execution probability of the limit orders standing in each book must be strictly positive, which means that trades will occur in both markets. Hence, we say that

the two markets *coexist* when the books in each market are non-empty in equilibrium. In the second case, obviously, trading concentrates in the market where limit orders cluster. We say that this market is the *dominant market*. To fix notation, let index j refer to one market (I or E) and index "-j" to the competing market (i.e. if j = I then -j = E).

**Definition 2**: Market j is dominant if the book of the competing market is empty for all possible reservation prices, that is:  $S_{-jk}^*(\gamma) = 0, \forall k, \forall m$ . Otherwise, markets E and I coexist.

Let  $\widehat{c}_j \stackrel{def}{=} \frac{c_j}{\alpha}$  and  $R_{jk} \stackrel{def}{=} \frac{\widehat{c}_j}{(p_k - v_0 - f_j)}$ . Using equations (5), (6) and the equilibrium definition, it is immediate that a competitive equilibrium is reached when the  $S_{jk}^*(\gamma)$ s solve the following system of 2m equations:

$$P_{Ik}(Q_{Ik-1}^*, Q_{k-1}^*, S_{Ik}^*, S_{Ek}^*) = R_{Ik} \text{ if } S_{Ik}^* > 0, k \in \{1, ..., m\},$$

$$P_{Ik}(Q_{Ik-1}^*, Q_{k-1}^*, S_{Ik}^*, S_{Ek}^*) \le R_{Ik} \text{ or } R_{Ik} \le 0 \text{ if } S_{Ik}^* = 0, k \in \{1, ..., m\}.$$

$$(7)$$

$$P_{Ek}(Q_{k-1}^*, S_{Ik}^*, S_{Ek}^*) = R_{Ek} \text{ if } S_{Ek}^* > 0, k \in \{1, ..., m\},$$

$$P_{Ek}(Q_{k-1}^*, S_{Ik}^*, S_{Ek}^*) \le R_{Ek} \text{ or } R_{Ik} \le 0 \text{ if } S_{Ek}^* = 0, k \in \{1, ..., m\}.$$
(8)

For fixed values of parameters  $\gamma$  and  $\delta_I$ , the equilibrium  $S_{jk}^*$ s are completely determined by the  $R_{jk}$ s. An increase in the order entry cost  $(c_j)$  or in the order execution cost  $(f_j)$ triggers an increase in  $R_{jk}$ . Thus, qualitatively, an increase in the order execution cost in market j has the same effect on the equilibrium as an increase in the order entry cost in this market. In order to simplify the exposition, we normalize  $f_I$  and  $f_E$  to zero. We have checked that all our results are robust when  $f_j > 0$ . In particular, qualitatively, our statements regarding the effect of order entry costs also apply to order execution costs.

For some values of the parameters and an adequate specification of  $\overline{F}(.)$ , it is possible to solve the previous system of equations and obtain closed-form solutions. However, our implications (derived in the next section) do not rely on a specific parametrization of the model. They follow from the equilibrium conditions (7) and (8).

**Benchmark:** When market I operates alone ( $\gamma = 0$ ), the expected profit on the marginal limit order at price  $p_k$  in market I is:

$$\Pi_{Ik}^m = \alpha [\overline{F}(Q_{Ik-1} + S_{Ik})(p_k - v_0) - \widehat{c}_I] = \alpha [\overline{F}(Q_{Ik})(p_k - v_0) - \widehat{c}_I] \text{ for } k \le m.$$

In a competitive equilibrium, the cumulative depth at price  $p_k$  (that we denote by  $Q_{Ik}^*(0)$  when  $\gamma = 0$ ) adjusts in such a way that the marginal limit order at this price just breaks

even. Thus,  $Q_{Ik}^*(0)$  solves:

$$\overline{F}(Q_{Ik}^*(0))(p_k - v_0) - \hat{c}_I = 0 \quad \text{for} \quad k \le m.$$
 (9)

Observe that  $\overline{F}(x)$  decreases with x and is bounded by 1. Thus, if  $(p_k - v_0) < \widehat{c}_I$ , there is no solution to this equation and  $Q_{Ik}^*(0) = 0$ . Hence, the first price on the grid at which some sell limit orders are submitted (the best offer price) is the smallest price  $a^*$  such that  $(a^* - v_0) - \widehat{c}_I > 0$ . To fix this price at  $p_1$ , we assume:

$$\mathbf{A.1} \qquad \widehat{c}_I < p_1 - v_0, \tag{10}$$

Under this assumption,  $Q_{Ik}^*(0) > 0$ ,  $\forall k \geq 1$ . Assumption A.1 is not crucial for the results but simplifies the exposition. However, it fixes the size of the bid-ask spread in the consolidated market. We therefore relax this assumption in Section 3.3 when we derive predictions about the effect of intermarket competition on the consolidated bid-ask spread.

# 3 The Effects of Intermarket Competition

# 3.1 Are Competing Books Harmful for Liquidity?

We first study the conditions under which markets I and E coexist or not (Propositions 1 and 2). The next proposition shows that market E is viable if and only if the proportion of smart routers exceeds a critical mass.

**Proposition 1** (critical mass): Let  $\gamma^*(\widehat{c}_E, \widehat{c}_I, \delta_I) \stackrel{def}{=} Min\{\frac{\widehat{c}_E}{(2-\delta_I)\widehat{c}_I}, \frac{\widehat{c}_E}{\delta_I\widehat{c}_I+(1-\delta_I)\Delta}\}$ . If  $\gamma \leq \gamma^*$  ( $\widehat{c}_E, \widehat{c}_I, \delta_I$ ) then market I is dominant in equilibrium. If  $\gamma^*$  ( $\widehat{c}_E, \widehat{c}_I, \delta_I$ )  $< \gamma < 1$  then both markets coexist.

This result underscores the importance of routing systems for the entrant. The latter is able to steer away some orders from the entrant market *iff* the proportion of smart routers is larger than the *critical mass*  $\gamma^*$   $(\hat{c}_E, \hat{c}_I, \delta_I)$ .<sup>16</sup> The intuition for this result is simple. Other things equal, the likelihood of execution for limit orders placed in the

<sup>&</sup>lt;sup>16</sup>This can explain why the LSE has repeatedly encouraged brokerage firms to develop Smart Order Routing Systems since the introduction of EuroSETS. For instance, it organized meetings between Dutch brokerage firms and developers of SORSs. On the day following the introduction of EuroSETS, the FT wrote: "The London Stock Exchange, which yesterday started an assault on Amsterdam stock, is drawing attention to traders' increasing need for smart order routing to take advantage of increased competition." ("LSE Tries the Smart Order Route", Financial Times, May 25, 2004).

entrant market increases with the proportion of smart routers. When this proportion is small, the expected revenue from submitting a limit order in the entrant market is too small to cover the order entry cost in this market, at any price. As a result, no limit orders are submitted in the entrant market. If  $\gamma^*$  ( $\hat{c}_E, \hat{c}_I, \delta_I$ )  $< \gamma < 1$ , the two markets coexist. The situation in which  $\gamma = 1$  deserves special treatment, because the incumbent market has no captive base of users in this case. Yet, as shown in the next proposition, the two markets can charge different fees and coexist in this case as well. Let  $\hat{c}_E^* \stackrel{def}{=} Min\{\frac{\hat{c}_I}{1+\delta_I}, \frac{\hat{c}_I - \delta_I \Delta}{1-\delta_I}\}$  and  $\hat{c}_E^{**} = Max\{(2 - \delta_I)\hat{c}_I, \delta_I\hat{c}_I + (1 - \delta_I)\Delta\}$ .

**Proposition 2**: Suppose that all traders are smart routers ( $\gamma = 1$ ). If  $\widehat{c}_E^* < \widehat{c}_E < \widehat{c}_E^*$  then both markets coexist. If  $\widehat{c}_E \leq \widehat{c}_E^*$  then market E is dominant. If  $\widehat{c}_E \geq \widehat{c}_E^*$  then market E is dominant.

Using the conditions derived in Propositions 1 and 2, Figure 1 depicts the set of values for  $\gamma$  and  $\hat{c}_E$  such that (a) market I dominates or (b) the two markets coexist (for fixed values of  $\hat{c}_I$  and  $\delta_I$ ). Notice that the coexistence conditions encompass the polar cases in which  $\delta_I = 1$  or  $\delta_E = 1$ . These polar cases are discussed in more details in Corollary 2 below.

#### [insert Figure 1]

Even when  $\gamma = 1$ , the two markets coexist for a wide range of parameter values in which the two markets charge different order entry fees. That is, trading does not necessarily concentrate in the market charging the lowest fee. This means that routing decisions for limit orders are not infinitely elastic to fees. Why?

Limit order traders' placement decision is determined by a trade-off between execution probability and order entry cost. When time priority is not enforced across markets, submitting a limit order in one market is in effect a way to jump in front of the queue of limit orders placed at the same price in the competing market.<sup>17</sup> Traders are willing to pay for this possibility as it raises their execution probability and, in this way, the two markets can coexist even if they charge different fees on limit orders. To see this point, consider the case in which the incumbent market charges larger fees ( $\hat{c}_E < \hat{c}_I$ ). Initially, when books are empty, limit order traders should choose the entrant market. But, as the queue of limit

<sup>&</sup>lt;sup>17</sup>Anedoctal evidences suggest that limit order traders do use queue-jumping strategies in presence of multiple trading venues. For instance, the August 2004 issue of the EuroSETS newsletter mentions that: "[...] some firms are using the relatively lower volumes on the spread offered by the Dutch Trading Service to "queue-jump" rather than waiting elsewhere for execution" (EuroSETS newsletter, Issue August 2004, 4).

orders in the entrant market becomes larger, the execution probability of a limit order in this market diminishes. In contrast, the execution probability of the first limit order submitted in the incumbent market remains high because it does not yield time priority to orders in the entrant market. For a sufficiently large queue in the entrant market, the benefit of a relatively high execution probability in the incumbent market counterbalances the benefit of a relatively small fee on limit orders in the entrant market, if  $\hat{c}_E > \hat{c}_E^*$ . It is then optimal to start filling the book in the incumbent market.

We now analyze the effect of intermarket competition on the cumulative depth of the incumbent market. If the cumulative depth in the incumbent market at, say, price  $p_k$  were unchanged after entry, the execution probability of the marginal limit order at this price in the incumbent market would necessarily be smaller as part of the order flow executes against the entrant limit order book. The marginal limit order would then lose money. Thus, in equilibrium, the cumulative depth in the incumbent market must diminish throughout the book after entry of market E, as shown in the next proposition.

**Proposition 3**: Other things equal, when the two markets coexist or when market E is dominant, the cumulative depth in the incumbent market is smaller than when the incumbent market operates alone, i.e.  $Q_{Ik}^*(\gamma) \leq Q_k^*(0)$ ,  $\forall k \geq 1$ , when  $\gamma \in [\gamma^*, 1]$ .

Now consider the effect of intermarket competition on consolidated depth. Suppose that the books in both markets have filled in such a way that consolidated depth at price  $p_k$  is smaller than or equal to its level when the incumbent market operates alone (i.e.  $Q_k \leq Q_k^*(0)$ ). This situation cannot be an equilibrium because profit opportunities remain, at least in the incumbent market. Actually, the marginal limit order in the incumbent market yields time priority to the limit orders offered in this market only, not to all the limit orders in the consolidated market. Thus, its execution probability is larger than if offers were consolidated in a single market with time priority enforced. Formally, this follows from equation (3), which yields

$$\alpha \overline{F}(Q_k) < P_{Ik}(Q_{Ik-1}, Q_{k-1}, S_{Ik}, S_{Ek}),$$
(11)

since  $Q_{k-1} + S_{Ik} < Q_k$  (if both markets coexist, there are some limit orders posted at price  $p_k$  in both markets). The expected profit on the marginal limit order in the incumbent market is then strictly larger than if offers were consolidated in a single market with time priority enforced. As a result, the level of consolidated depth for which no further profit opportunities remain in the centralized market (i.e.  $Q_k^*(0)$ ) leaves room for profit opportunities in the fragmented market. Thus, consolidated depth in the fragmented market must be larger than when a single market operates, as claimed in the next proposition.

**Proposition 4**: When the two markets coexist, consolidated depth is larger than when the incumbent market operates alone, i.e.  $Q_k^*(\gamma) \geq Q_k^*(0)$ ,  $\forall k \geq 1$ . Moreover, there exists  $k_0 \geq 1$  such that consolidated depth is strictly larger for  $k \geq k_0$  (i.e.  $Q_k^*(\gamma) = Q_k^*(0)$ ,  $\forall k < k_0$  and  $Q_k^*(\gamma) > Q_k^*(0)$ ,  $\forall k \geq k_0$ ).

Key to this finding is the absence of time priority across markets, not the pricing policy of the entrant market since this result obtains even if fees on limit orders are larger in the entrant market.<sup>18</sup> Absence of time priority across markets enables traders to bypass time priority on limit orders standing at a given price in one market by submitting a limit order at the same price in the competing market. We call this effect of intermarket competition, the "queue-jumping effect". It intensifies competition among limit order traders. Hence, rents on infra-marginal limit orders in either market are reduced and overall trading costs for smart routers are reduced (i.e. consolidated depth increases).

Intermarket competition is likely to prompt a reduction in the fees charged on passive orders. The next proposition analyzes the effect of a reduction in these fees on consolidated depth. We focus on a reduction in order entry costs as execution fees have been normalized to zero. However, the same result obtain if we consider variations in execution fees instead of variations in order entry costs (see the remark following equations (7) and (8)).

**Proposition 5**: If market j is viable (i.e. it coexists with its competitor or dominates) then a reduction in the order entry cost of market j results in an increase in cumulative depth in market j and an increase in consolidated depth.

To understand this result, suppose that the incumbent market reduces its fees on passive orders. Cumulative depth in the incumbent market increases as providing liquidity in this market becomes less expensive. As a result, the likelihood of execution for limit orders placed in the entrant market becomes smaller. Limit order trading in this market is less attractive and it becomes thinner. However, consolidated depth increases as the increase in cumulative depth in the incumbent market more than compensates the decline in cumulative depth of the entrant market.

This proposition implies that a reduction in order entry fees is a second channel through which intermarket competition can enhance consolidated depth. To see this, consider a scenario in which the incumbent market reduces its fee on passive orders after introduction

<sup>&</sup>lt;sup>18</sup>The result only relies on the coexistence of the two markets, not directly on their fees. Fees indirectly matter however because they determine in part whether markets coexist. For instance, for  $\delta_I = 1$ , the two markets coexist only if  $c_E < c_I$  (see Corollary 2 below). Thus, in this polar case, Proposition 4 indirectly requires a smaller fee in the entrant market. Moreover, the value of  $k_0$  depends on the order entry costs. For instance, it can be shown that  $k_0$  decreases as  $c_E$  becomes smaller.

of a competing market (this is indeed the case in our experiment). Propositions 4 and 5 imply that this introduction triggers an increase in consolidated depth for two reasons: (i) the queue-jumping effect and (ii) the reduction in its fees on passive orders by the incumbent market. The following example illustrates this point, which is important for the interpretation of our empirical findings.

**Example:** Suppose that  $\gamma = 1$ ,  $\delta_I = \delta_E = 0.5$  and that  $\widetilde{X}$  has a uniform distribution on  $[0, \overline{Q}]$ . Let  $\widehat{c}_I^b$  and  $\widehat{c}_I^a$  be the order entry costs in the incumbent market before and after entry  $(\widehat{c}_I^a \leq \widehat{c}_I^b < \Delta)$ . In this case, if  $\frac{\widehat{c}_I^a - \delta_I \Delta}{1 - \delta_I} < \widehat{c}_E < \frac{\Delta + \widehat{c}_I^a}{2}$ , the equilibrium depths at price  $p_1$  in the incumbent market and the entrant market are respectively:

$$Q_{I1}^*(1) = \frac{2\overline{Q}}{3}(1 - \frac{2\widehat{c}_I^a - \widehat{c}_E}{\Delta}),$$

and

$$Q_{E1}^*(1) = \frac{2\overline{Q}}{3} \left(1 - \frac{2\widehat{c}_E - \widehat{c}_I^a}{\Delta}\right).$$

In contrast, when the incumbent market operates alone, the quoted depth is:

$$Q_{I1}^*(0) = \overline{Q}(1 - \frac{\widehat{c}_I^b}{\Delta}),$$

Therefore, the change in consolidated depth at price  $p_1$  is:

$$Q_{I1}^{*}(1) + Q_{E1}^{*}(1) - Q_{I1}^{*}(0) = 2\overline{Q}(\frac{\Delta - \hat{c}_{I}^{b}}{3\Delta}) + 2\overline{Q}(\frac{\hat{c}_{I}^{b} - \hat{c}_{E}}{3\Delta}) + 2\overline{Q}(\frac{\hat{c}_{I}^{b} - \hat{c}_{I}^{a}}{3\Delta})$$
(12)

Suppose first that order entry costs are identical before and after entry  $(\widehat{c}_E = \widehat{c}_I^a = \widehat{c}_I^b)$ . The increase in quoted depth is then given by the first term on the RHS of the previous equation. It is due to the possibility of using queue-jumping strategies when the two markets coexist. Now, suppose that the entrant market charges a strictly smaller fee on passive orders than the pre-entry fee  $(\widehat{c}_E < \widehat{c}_I^b)$ . This triggers an additional increase in quoted depth given by the second term in the RHS of equation (12). Last, if the incumbent market reduces its fee on passive orders, consolidated depth increases by an additional amount given by the third term in the previous equation. This example illustrates (for the depth at the top of the book) the two mechanisms (queue-jumping and reduction in fees on passive orders) by which intermarket competition can trigger an increase in consolidated depth.

We have not yet discussed the evolution of consolidated depth when, after entry, the entrant market is dominant. Observe that market E is dominant if and only if  $\hat{c}_E \leq \hat{c}_E^*$  (Proposition 2). It is immediate that  $\hat{c}_E^* \leq \hat{c}_I$ , with a strict inequality when  $\delta_I > 0$ . Thus, when market E dominates, the equilibrium is identical to the equilibrium in the

benchmark case with a *smaller* order entry cost. Hence, applying Proposition 5, we obtain the following result.

Corollary 1: If the entrant market dominates then consolidated depth is larger than when the incumbent market operates alone.

In summary, we find that, other things equal, consolidated depth is larger when multiple limit order markets compete for order flow relative to a centralized limit order book. Moreover, a reduction in the fees on passive orders further increases consolidated depth.

### 3.2 Model Extensions

Raw vs. net prices and the tie-breaking rule. In the baseline model, we assume that execution costs on market orders are zero. In reality, traders also pay execution fees on market orders, which can be different from execution fees on limit orders. In this case, the routing decision should be determined by net prices (prices including execution fees) instead of raw prices, i.e. displayed quotes.<sup>19</sup> The model can easily be generalized to account for execution fees on market orders. Let  $\Phi_{jb}$  be the fee, per share, charged by market j on market orders submitted by broker b and suppose, to simplify the exposition, that  $0 \leq \Phi_{jb} < \Delta$ . We allow the execution fee to depend on the brokerage firm's identity because exchanges may choose to charge different fees for different brokerage firms. The next lemma gives the optimal routing strategy for smart routers given their execution fee on market orders.

**Lemma 2**: Consider a smart router, b, who must execute an order with size  $x \in [Q_{s-1}, Q_s]$ . If  $\Phi_{jb} < \Phi_{-jb}$  then the smart router's optimal routing strategy is as follows: (i) execute all limit orders placed at prices  $p_1, p_2, ..., p_{s-1}$  in markets E and I and (ii) at the stop-out price  $p_s$ , give priority to market j.

This result is straightforward. For instance, suppose that  $\Phi_{Ib} < \Phi_{Eb}$ . Observe that  $p_k + \Phi_{Eb} < p_{k+1}$  as  $\Phi_{Eb} < \Delta$ . Thus, the quote montage in terms of net prices for smart router b ranks the offers standing in markets E and I in the following order: (i)  $Q_{1I}$  shares at  $p_1 + \Phi_{Ib}$ , (ii)  $Q_{1E}$  shares at  $p_1 + \Phi_{Eb}$ , (iii)  $Q_{2I}$  shares at  $p_2 + \Phi_{Ib}$ , (iv)  $Q_{2E}$  shares at  $p_2 + \Phi_{Eb}$  etc. It is then immediate that the routing strategy described in the previous lemma is optimal.

<sup>&</sup>lt;sup>19</sup>In reality, it is unclear whether routing decisions are made on the basis of net or raw prices. Actually, there are practical difficulties in building quote montages based on net prices (in particular because execution fees can vary across investors). See McCleskey (2004) for a discussion.

The previous lemma implies that execution fees on market orders should determine which market is given priority in case of a tie, i.e.  $\delta_I$  and  $\delta_E$ . Indeed, if these are the only determinants of the tie-breaking rule then  $\delta_I$  is the probability that  $\{\Phi_{Ib} < \Phi_{Eb}\}$ . Thus, the results obtained in the previous sections are still valid in presence of fees on market orders since these results hold for any value of  $\delta_I$ .

Fee structure and coexistence. We can interpret variations in  $c_j$ ,  $f_j$  or  $\delta_j$  as being due to variations in the fees chosen by market j. Specifically, the effect of a decrease in fees on limit orders in market j can be analyzed in this model by studying the effect of a decrease in  $c_j$ . Moreover, the discussion in the previous paragraph implies that a decrease in fees on market orders in market j materializes in an increase in  $\delta_j$ .

The fees determine the ability of the two markets to coexist. Two observations substantiate this claim. First, the fees determine the critical mass of smart routers,  $\gamma^*$ , required for the viability of the entrant market. Using the expression for  $\gamma^*$ , it is immediate that the critical mass decreases with  $\hat{c}_I$ , increases with  $\hat{c}_E$  and increases with  $\delta_I$ . Hence, charging relatively small fees can be a way for the entrant market to reduce the critical mass required to attract some trading activity. Conversely, reducing its fees is a way for the incumbent market to predate entry of a new competitor. Second, differentiation in the fee structures of the two markets facilitates their coexistence when  $\gamma = 1$ , as shown by the next corollary.

Corollary 2: Consider the case in which  $\gamma = 1$ . If market j charges uniformly smaller fees on market orders (i.e. if  $\delta_j = 1$ ) then the two markets coexist iff its competitor charges a smaller fee on limit orders (i.e. iff  $c_{-j} < c_j$ ). Otherwise, (i.e. if  $c_j < c_{-j}$ ), trading concentrates in market j.

This corollary derives directly from Proposition 2 in the special case in which  $\delta_j = 1$ . It shows that even in this polar case the two markets can coexist but, for this, their fee structure must be differentiated: one market is more competitive, in term of fees, for passive orders and the other for aggressive orders. This results in a situation in which one market charge relatively small fees on one side (e.g. limit orders) and relatively large fees on the other side (e.g. market orders). Interestingly, ECNs (e.g. Inet) or the LSE for EuroSETS have adopted such asymmetric fee structures.<sup>20</sup> The previous corollary suggests that this might be a way to soften intermarket competition as entering into a price war on

<sup>&</sup>lt;sup>20</sup>For instance, Inet (a US based ECN) offers a rebate of \$0.002 per executed share on limit orders adding liquidity to its book and charges a fee of \$0.003 per executed share on orders removing liquidity from its book.

both passive and aggressive orders results in a situation in which trading concentrates in a single market (so that there is room for only one market). A full analysis of this question requires to determine the profit maximizing fee structures for the competing markets, a problem that is beyond the scope of this paper.

Varying search costs. In the baseline model, we distinguish two types of brokers: (i) smart routers and (ii) non-smart routers. For a given order size, smart routers only base their routing decisions on the quotes available in each system and split their orders so as to minimize trading costs. In contrast, non-smart routers ignore the prices available in the entrant market. In reality, non smart routers' incentives to search for best execution might vary with market conditions. In particular, these incentives are smaller in fast moving markets because (i) the quotes standing in either market change quickly compared to the time needed to split orders and (ii) the opportunity cost of time is larger. In contrast, in slow markets, brokers are likely to be more patient and to consider the quotes in both trading venues to execute their orders.

This possibility however does not change the results of the model. To see this point, assume that a broker searches for best execution if her search cost,  $C_b$ , is smaller than a threshold  $T_{bt}$ . This threshold can be a function, for instance, of market conditions (e.g. trading activity, volatility) and trade size. In this case, the likelihood that, on a given order t, a broker behaves as a smart router is:

$$\gamma_t = \gamma + (1 - \gamma) \Pr(0 < \widetilde{C}_b \le \widetilde{T}_{bt}),$$

as  $\gamma$  is the proportion of brokers who systematically search for best execution. The results of the model are unchanged (as  $\gamma_t$  simply plays the role of  $\gamma$  in all equations).

### 3.3 Predictions

In the remainder of this section, we develop refutable predictions that we test through an event analysis of the introduction of a new limit order market in the Dutch stock market. The entrant market, EuroSETS (operated by the London Stock Exchange), competes with the incumbent limit order market, NSC (operated by Euronext). In the next section, we explain that this event is an ideal laboratory as it mirrors the nature of competition in our theoretical model.

#### 3.3.1 Market Depth Before and After Entry

Proposition 4 generates the most basic prediction that, other things equal, consolidated depth is higher after the introduction of EuroSETS due to the queue-jumping effect. Moreover, as explained in the empirical section, fees on passive orders are unambiguously lower after EuroSETS introduction. We expect this fee reduction to amplify the increase due to the queue-jumping effect (see the discussion following Proposition 5).

The queue-jumping effect and the lower-fees effect have opposite predictions for depth in the incumbent market, NSC. The queue-jumping effect predicts NSC depth to be lower after EuroSETS entry (Proposition 3). Lower fees, however, increase NSC depth (Proposition 5). The net result is, therefore, ambiguous. Thus, the model does not deliver clear-cut predictions for the effect of EuroSETS introduction on NSC depth.

#### 3.3.2 Bid-Ask Spread Before and After Entry

As we focus on the sell side of the book, we define the bid-ask spread as the difference between the best ask and  $v_0$ . Assumption A.1 (equation (10)), however, sets this difference equal to 1 tick irrespective of whether the two markets coexist or not. If we relax this assumption, we obtain the following result.

**Proposition 6**: Suppose that prior to the introduction of market E, the order entry cost in market I is such that  $\hat{c}_I > \Delta$  (i.e. A.1 does not hold). If, after entry of market E, the order entry cost for limit orders is unchanged or smaller in at least one of the two markets then the inside spread in the consolidated market is unchanged or smaller compared to the pre-entry bid-ask spread.

The intuition is straightforward. The smallest offer at which providing liquidity is profitable increases with the order submission fee in this model. Thus, a fee reduction either leaves unchanged the bid-ask spread or reduces it (if the reduction is large enough). Fees on limit orders are unambiguously lower after EuroSETS introduction. Thus, we expect the bid-ask spread in the consolidated market (i.e. the difference between the best bid and offer in the consolidated market) to be unchanged or reduced after the introduction of EuroSETS.

# 3.3.3 Cross-Sectional Relation between Entrant Liquidity and Proportion of Smart Routers

We now show that the model predicts a positive relationship *cross-sectionally* between the liquidity of the entrant market relative to that of the incumbent market and the proportion of smart routers. We consider two measures for the entrant relative liquidity.

Relative entrant liquidity in terms of depth. Let  $C_{1E}(\gamma) \stackrel{def}{=} \frac{Q_{E1}^*(\gamma)}{Q_1^*(\gamma)}$  be the ratio of entrant depth at the best ask price (i.e.  $p_1$ ) to consolidated depth at this price for a given value of  $\gamma$ . This ratio measures the entrant's contribution to consolidated depth at the best ask price.

**Proposition 7**: Other things equal, when the two markets coexist, the entrant's contribution to consolidated depth at price  $p_1$  increases with the proportion of smart routers (i.e.  $\frac{\partial C_{1E}(\gamma)}{\partial \gamma} > 0$ ).

The intuition is that an increase in smart routers enlarges the execution probability of the marginal limit order submitted at the best ask in the entrant market. Thus, it raises the expected profit and, as a result, the quoted depth in the entrant market (i.e.  $Q_{E1}^*(\gamma)$ ) increases. This, at the same time, decreases the execution probability of the marginal limit order at the best ask in the incumbent market and thus reduces depth in this market  $(Q_{I1}^*(\gamma))$ . The result of these effects is that the entrant's contribution to consolidated depth increases.

Relative entrant liquidity in terms of spread. Depending on the value of  $\gamma$ , the incumbent best ask can be strictly smaller than, equal to, or strictly larger than the entrant best ask, as shown by the next proposition. Let  $\gamma^{**} \stackrel{def}{=} \frac{\widehat{c}_E}{\delta_I \widehat{c}_I + (1 - \delta_I) \widehat{\Delta}}$  and  $\gamma^{***} \stackrel{def}{=} Min\{1, \frac{\Delta - \widehat{c}_I + (1 - \delta_I)\widehat{c}_E}{(1 - \delta_I)\Delta}\}$ . It is easily shown that  $\gamma^{**} < \gamma^{***}$ .

#### Proposition 8:

- 1. When  $\gamma^* < \gamma \leq \gamma^{**}$ , the two markets coexist and the entrant best ask is strictly larger than the incumbent best ask. The interval  $(\gamma^*, \gamma^{**}]$  is non-empty if and only if  $\widehat{c}_I > \frac{\Delta}{2}$ .
- 2. When  $Max\{\gamma^*, \gamma^{**}\} < \gamma < \gamma^{***}$ , the two markets coexist and the incumbent best ask is identical to the entrant best ask.
- 3. When  $\gamma^{***} < \gamma < 1$ , the two markets coexist and the incumbent best ask is strictly larger than the entrant best ask. The interval  $[\gamma^{***}, 1]$  is non-empty if and only if  $\widehat{c}_E < \frac{\widehat{c}_I \delta_I \Delta}{1 \delta_I}$ .

When  $\gamma^* < \gamma \le \gamma^{**}$ , a sell limit order placed at price  $p_1$  in the entrant market cannot break-even because its execution probability is too small. However, it is profitable to submit sell limit orders at higher prices. Hence, the two markets coexist but the incumbent ask is strictly smaller than the entrant ask. A symmetric situation occurs when  $\gamma^{***} < \gamma < 1$ . Thus, for given order entry fees and for a fixed value of the tie-breaking parameter  $\delta_I$ , the model predicts that the ratio of the incumbent spread to the entrant spread is positively related to the proportion of smart routers.

# 4 Empirical Analysis

In this section, we study the introduction of a second limit order book in the Dutch market to test the empirical predictions derived in the previous section. We organize the section in three subsections. First, we give the details of the experiment, introduce the data, discuss the main methodology, and provide summary statistics. Second, we test the empirical predictions. Third, we do additional empirical analyses to further understand our findings.

# 4.1 Background, Data, and Methodology

#### 4.1.1 Market Structures and Fees

On October 14, 2003, the London Stock Exchange announced that it would introduce a new trading platform, EuroSETS, for the 50 stock constituents of the two major Dutch indices, the AEX and AMX. This introduction was encouraged by the Dutch brokerage community as a way to prompt the incumbent market, Euronext, to cut its trading fees.<sup>21</sup> Euronext operates the main electronic trading platform for Dutch stocks, NSC, since the merger of the Amsterdam Stock Exchange and the Paris Bourse in 2001. Trading in EuroSETS began on May 24, 2004.

The difference between EuroSETS and NSC is minimal for three reasons. First, these trading systems use very similar trading rules. They operate over the same trading hours (9:00 am to 5:30 p.m.) and they are both organized as continuous electronic limit order markets (see Biais, Hillion and Spatt (1995) for a detailed description of standard trading rules in these markets). As both systems are fully automated, they offer the same speed

<sup>&</sup>lt;sup>21</sup>See "Discontented Dutch Brokers Prompt Foreign Intervention", *Euromoney*, December 2003 or "Dutch Auction: Can the LSE Snatch Dutch Equities Trading from Euronext?", *The Economist*, March 6, 2004.

of execution. They have the same amount of transparency: (i) participants can see all limit orders standing in each book (except for hidden orders) and (ii) trading is completely anonymous in each market. Also, EuroSETS and NSC use the same tick size, €0.01, for stocks that trade below €50. Moreover, price-time priority is enforced within each market (but not across markets). Second, the members of both markets are almost identical as virtually all brokers signed up for EuroSETS.<sup>22</sup> Third, the two markets use the same clearing and settlement system, Clearnet. Thus, in deciding where to trade, brokers should mainly trade-off (i) prices in both systems, (ii) trading fees, and (iii) search costs, consistent with our model.

Fees on passive orders are smaller after the introduction of EuroSETS for two reasons. First, the fees charged on passive orders in EuroSETS are smaller than those charged in NSC before and after entry. In fact, EuroSETS charges no order entry fee and offers rebates in case of execution. Second, Euronext reduced its fees on passive orders several times since the LSE announced its plan to enter the Dutch market. The first reduction was 50% and happened on April 23, 2004, a few weeks before EuroSETS entry. Then, just a few days before entry, Euronext launched the "European Soccer Championship" summer special, running from May 24 (the day of EuroSETS introduction) through July 31, 2004. During this period, rebates were offered on passive orders submitted to NSC and aggressive orders were free of order entry fees.

EuroSETS charges lower fees than NSC on passive orders, but not necessarily on aggressive orders. As an illustration, consider a broker submitting less than 60,000 orders per month.<sup>23</sup> Based on documents released by Euronext and the LSE, we estimate the fees for this broker as follows:

		Passive Order		Aggressive Order	
(€)		Entry	Execution	Entry	Execution
Fee Structure	NSC	0.15	$0.665/\mathrm{trade}$	0.30	$0.665/\mathrm{trade}$
	EuroSETS	0.00	-0.02/€1,000	0.00	$0.08/ \in 1,000^a$

a: This fee has a lower bound of €0.40 and an upperbound of €5.00. The majority of orders, however, falls within these bounds.

Suppose that the broker submits a median size order for 1,500 shares at median price €15. If the order is passive, EuroSETS is cheaper (zero entry fee plus a rebate in case of execution). If the order is aggressive, NSC is cheaper as the order is charged €0.965 (€0.30

<sup>&</sup>lt;sup>22</sup>In its December 2004 newsletter, the LSE lists the 53 brokerage firms that use the service. It includes all major brokers e.g. ABN AMRO, RABO, Van Der Moolen, Goldman Sachs, Merrill Lynch, and Morgan Stanley.

<sup>&</sup>lt;sup>23</sup>An exact comparison of the fees charged by EuroSETS and NSC is difficult as charges per order can vary according to several factors specific to each broker (e.g. its monthly total trading volume within a given market).

entry plus  $\leq 0.665$  execution) on NSC and  $\leq 1.80$  on EuroSETS (zero entry plus 0.08 times 22.5).

Overall, the nature of competition between EuroSETS and NSC fits well with the assumptions of our model as (i) both markets are pure electronic limit order markets with similar rules and (ii) the routing decision should mainly be driven by fees, prices, and search costs as both systems have almost identical membership and the same clearing and settlement system. Thus, the introduction of EuroSETS creates an ideal laboratory to test the predictions of the theoretical model.

#### 4.1.2 Data

We compare measures of market liquidity before and after entry of EuroSETS. To this end, we select a pre-entry period and two post-entry periods of 21 trading days. The pre-entry period starts on April 23 and lasts until May 21, 2004 (the last day before entry). In order to identify *permanent* equilibrium effects of EuroSETS entry, we consider two distinct post-entry periods. The first post-entry period runs from August 2 through August 30, 2004, the second from January 3 through January 31, 2005.

We emphasize that our pre- and post-entry periods are chosen so that the fee structure of Euronext is, in principle, fixed over all periods. In fact, the pre-entry period starts on the day that Euronext announced and implemented its new fee structure, whereas the post-entry periods do not include the "summer special" (May 24 through July 31). Hence, we expect NSC depth to decrease if the impact of the initial fee reduction by Euronext is fully reflected in NSC depth as of April 23. If, however, NSC depth slowly adjusts, the prediction for NSC depth after EuroSETS entry is ambiguous (see predictions developed in Section 3.3).

Our sample includes all 25 stocks of the AEX index. In order to avoid confounding effects, we remove (i) stocks with a price higher than €50 (as tick size is not the same across markets), (ii) stocks that drop out of or enter the index during the sample period, and (iii) stocks that implement or cancel an ADR program during the sample period. This leads us to remove three stocks from our sample: Unilever, Gucci, and Numico. We group the remaining 22 stocks in quartiles based on trading volume, with Q1 containing the most actively traded stocks. The classification is based on 2003 volume to ensure an exogenous ranking. The composition of the quartiles is given in Appendix B. Stocks in the first quartile account for 66% of total volume and 71% of total market capitalization.

We use two types of data in our analysis: (i) five-minute order book snapshots and (ii) "continuous" time data on trades and best bid and ask quotes. The snapshot data contain

the five best bid and ask quotes and the number of shares offered at these quotes sampled every five minutes in both EuroSETS and NSC. We use these data to build snapshots of the consolidated limit order book every five minutes. The continuous time data consists of time-stamped (to the nearest second) trades for both markets, best bid and ask quotes for NSC, and best bid and ask quotes (including depth) for EuroSETS.<sup>24</sup>

Our data have two important limitations. First, we do not have data on trades occurring in venues other than NSC and EuroSETS. There are three main alternative venues: (i) an OTC market that, effectively, is an upstairs market, (ii) "Xetra star," a trading platform operated by Deutsche Börse (introduced in 2003), and (iii) foreign markets where the stocks are cross-listed. The market share of Xetra star is very small and trades (unlike EuroSETS) do not clear and settle through the clearinghouse used by NSC. Half of the stocks in our sample have ADRs in the US. We expect US brokerage firms to be more active in these stocks. This is important as these firms are used to trade in a more fragmented environment (the US equity market). Thus, they are more likely to be equipped with smart routing technology (see Hallam and Idelson (2003)) or, at least, to behave like smart routers.

Second, we do not have data on iceberg orders i.e. orders that display only a fraction of total size to the market. Both NSC and EuroSETS allow for this type of orders. Thus, we can measure the change in displayed consolidated depth following EuroSETS entry, not the change in overall (hidden and displayed) consolidated depth. The change in displayed depth can, therefore, underestimate the change in total depth if hidden depth has increased and/ or hidden depth in EuroSETS is large. Alternatively, the change in displayed depth can overestimate the change in total depth if hidden depth in NSC has decreased. We see no ways to discard the first possibility, but address the second possibility by analyzing the evolution of effective spreads in NSC (see below).

#### 4.1.3 Methodology and Summary Statistics

Most of the empirical analysis that follows consists of comparing quartile means for various market variables (e.g. quoted spread) before and after the entry of EuroSETS. We estimate these quartile means, changes in these quartile means, and test for the statistical significance of these changes using panel data techniques.

The econometric specification assumes that the variable of interest  $y_{it}$  for stock i on day t can be expressed as the sum of a stock-specific mean  $(\mu_i)$  and event effect  $(\delta_i)$ , potential

<sup>&</sup>lt;sup>24</sup>The continuous time data are regular "TAQ" data from Euronext and the LSE. The EuroSETS snapshots are made available to us by the LSE. The NSC snapshots are downloaded (with permission of Euronext and Alex) from a Dutch retail broker, Alex, who offers a live feed to the Euronext trading platform.

control variables  $(X_{it})$ , and an error term  $(\varepsilon_{it})$ :

$$y_{it} = \mu_i + \delta_i 1_{[t \text{ in post-entry period}]} + \beta' X_{it} + \varepsilon_{it}$$
 (13)

$$\varepsilon_{it} = \xi_t + \eta_{it} \tag{14}$$

where  $1_{[A]}$  is an indicator function that is 1 if A is true, zero otherwise, and  $\xi_t$  is a common factor across all stocks that, for example, captures the widely documented commonality in liquidity in case  $y_{it}$  is a liquidity measure. As we are interested in quartile changes, we define:

$$\mu_q \stackrel{def}{=} \frac{1}{N_q} \sum_{i \in I_q} \mu_i \quad \text{(quartile mean pre-entry)}$$
 (15)

$$\delta_q \stackrel{def}{=} \frac{1}{N_q} \sum_{i \in I_q} \delta_i \quad \text{(quartile change: post-entry vs. pre-entry)}$$
(16)

where q is a quartile index that runs from 1 (most active) to 4,  $N_q$  is the number of stocks in quartile q, and  $I_q$  contains the indices of the stocks contained in quartile q. We are particularly interested in  $\delta_q$ , which measures the impact of EuroSETS entry on the dependent variable for each quartile. Following Petersen (2005), our t-statistics are based on Rogers standard errors to account for heteroskedasticity and non-zero (stock-specific) autocorrelation in the error term  $(\eta_{it})$ .<sup>25</sup> A first application of the methodology is to analyze and compare the three trading periods based on summary statistics.

## [insert Table 1]

Table 1 provides summary statistics on daily volume, number of trades, average trade size, realized volatility (based on five minutes midquote returns), and average price level. It also provides EuroSETS and NSC market shares (in terms of trading volume and number of trades) for each quartile. The first column reports the average values in the pre-entry period and testifies to the heterogeneous nature of the sample. For instance, the average daily trading volume ranges from €167.33 million for Q1 to €9.4 million for Q4. Realized volatility (annualized) is 35.25% for Q4 and declines to 19.13% for Q1. We observe a significant drop in trading activity (-13% for volume) and volatility (-7%) in the first post-entry period. In contrast, trading activity in the second post-entry period is not significantly different from pre-entry levels, but volatility, however, is significantly lower (-36%). We will account for changes in volatility, volume, and price levels by including these variables as controls in equation (13).

<sup>&</sup>lt;sup>25</sup>Roger standard errors are similar in spirit to Newey-West standard errors, but are designed for panel data analysis. The main difference is in the weights of the cross-terms. See Petersen (2005) for details.

On average, EuroSETS daily market share in terms of total number of trades is 3.5% (2.0%) in the first (second) post-entry period. For Q1, EuroSETS share is 6.1% (3.6%) in the first (second) post-entry period. For the other quartiles, it is less than 1%. Market share in terms of volume gives similar results. We note that the Q1 market share is of the same order of magnitude as the market share of the most active regional exchanges in the US, as Bessembinder (2003) finds these shares to range from 2.2% (Philadelphia Stock Exchange) to 8.2% (Chicago Stock Exchange). It is worth stressing that EuroSETS low market share for stocks in Q2, Q3, and Q4 does not mean that EuroSETS is completely inactive in these stocks. First, some trades do occur in these stocks (see Table 1). Second, EuroSETS relative liquidity is not negligible in these stocks (see Section 5). In Section 5, we show that this wedge between EuroSETS market share and EuroSETS "liquidity share" can be explained by a relatively low proportion of smart routers and a tie-breaking rule very favorable to NSC.

# 4.2 Testing the Predictions

This subsection tests the predictions developed in Section 3.3 and is structured accordingly.

#### 4.2.1 Market Depth Before and After Entry

The first two predictions are on liquidity change after EuroSETS entry. We report this change in terms of (i) the relative quoted spread (i.e. the ratio of the inside spread and the midquote) and (ii) the value (number of shares times the midquote) offered up to k ticks behind the best quotes. To save space, we only report the findings for k = 0 (depth at the best quotes) and k = 4, as the conclusions are identical for the intermediate values  $(k \in \{1, 2, 3\})$ . We also only report ask side depth, as bid side depth shows similar results.

[insert Table 2]

[insert Table 3]

Table 2 reports the "univariate" results i.e. the results without control variables. It gives the average level of each liquidity measure in (i) the consolidated market and (ii) NSC, before and after entry of EuroSETS. It also reports the level of these liquidity measures for EuroSETS. Table 3 reports the "multivariate" results, i.e. the estimates of equation (13) with volume, volatility, and price level as control variables. As the conclusions that emerge from both sets of results are similar, we focus our discussion on Table 3. The coefficient of

interest is  $\delta_q$ , which measures, for a given quartile, the impact of EuroSETS introduction on liquidity, after controlling for other variables that affect liquidity.

Change in consolidated depth. Consolidated depth increases substantially after EuroSETS entry across all quartiles, although we find these increases to be statistically significant only for the most active stocks (Q1 and Q2) and for Q3 in the first post-entry period. Including control variables, we find for Q1 that quoted depth at the best ask (Depth0) in the consolidated market increased by 46.3% in the first post-entry period and by 100.8% in the second post-entry period (see Table 3). For Q2, these increases are, respectively, 48.1% and 97.8%. For Q3 and Q4, although predominantly insignificant, we also find increases in excess of 30%.

Change in NSC depth. In general, the evolution of NSC depth is similar to that of consolidated depth. First, the change is positive throughout the limit order book for all quartiles and for both post-entry periods. Second, in economic terms, the increase is substantial. For instance, for Q1 stocks, NSC depth increases by 24.8% in the first post-entry period and 74.4% in the second post entry period. Third, the change in NSC depth is statistically significant only for Q1 and Q2 and for Q3 in the first post-entry period.

At first glance, the NSC depth increase is not consistent with the model as the fee structure of NSC is unchanged across all periods in our experiment (see Section 3.3). In this case, Proposition 3 predicts a decline in depth for the incumbent market, not an increase. However, recall that our pre-entry period starts on the day that NSC announced its fee cut (April 23, 2004). The logic of the model implies that NSC depth increases on this day. But, in reality, depth might not adjust to its new equilibrium level instantaneously. If adjustment takes time, then we observe a positive drift in the evolution of NSC depth, consistent with our results.

An alternative explanation is that limit order traders in NSC have decided, under the pressure of competition, to display more of their iceberg orders after EuroSETS entry. In this scenario, total NSC depth might still have decreased after entry if reduced hidden depth outweighs increased displayed depth. In this case, price impact of market orders executed in NSC should be unchanged or larger after entry. We explore this possibility through a study of the effective spread in NSC, defined as twice the amount by which the average (volume-weighted) execution price of a buy (sell) market order exceeds (is below) the contemporaneous midquote in NSC. In the second post entry period, we find a decrease in effective spreads in NSC (see Table 2). In contrast, effective spreads in NSC are larger in the first post entry period, but the increase is significant only for stocks in Q2 and Q4. Overall, the evolution of effective spreads does not point to substantially less hidden depth in NSC after EuroSETS entry. Thus, it is unlikely that the increase in NSC depth reflects

#### 4.2.2 Bid-Ask Spread Before and After Entry

Generally, spreads seem to decline after EuroSETS entry, but the effects are small relative to the changes in depth. The inside spread in the consolidated market (i.e. the best ask minus the best bid across both systems) is significantly smaller for all quartiles in the second post-entry period, but only for Q1 in the first post-entry period. In economic terms, these reductions are small relative to depth changes and range from -8.1% to -20.8% (see Table 3). The other quartiles in the first post-entry period do not show any significant changes. The spread changes in NSC before and after entry are similar to those of the consolidated market, except for Q1. For this quartile, we find much smaller reductions for NSC spread (-1.6% and -7.8% for the first and second post-entry periods, respectively) as compared to the consolidated spread (-14.7% and -13.3%), which shows that the reduction for this quartile primarily stems from very competitive spreads in EuroSETS.

To sum up, we find that, following EuroSETS entry, consolidated depth has increased and bid-ask spreads are unchanged or smaller. These findings are consistent with our predictions. In theory, the increase in consolidated depth can be due to both (i) the queue-jumping effect and (ii) the reduction in fees on limit orders following EuroSETS entry. The strong increase in NSC cumulative depth and the reduction in bid-ask spreads in this market, however, is explained theoretically by the fee reduction in NSC. This observation suggests that the reduction in fees on passive orders plays a significant role in our findings.

Finally, could the liquidity improvement be a systematic change in liquidity for reasons other than the event? The methodology yields an estimate of the volatility of the common factor  $\xi_t$  (see equation (14)). We report this estimate in the last line of Table 3 and compare it to the changes in liquidity. For spreads, we find it to be of the same order of magnitude (for Q1 in the first post-entry period, for example, the spread drops by 1.16 bp and  $\sigma_{\xi} = 1.1$  bp). For depth, on the other hand, we find the changes to be a factor 2 to 6 larger than  $\sigma_{\xi}$ . It is, therefore, highly unlikely that systematic changes have driven the depth results, in particular since the improvement is similar across both post-entry periods. The spread results, however, are weaker in this respect.

# 4.2.3 Cross-Sectional Relation between Entrant Liquidity and Proportion of Smart Routers

We exploit considerable cross-sectional variation in relative EuroSETS liquidity to test our third prediction. For relative bid-ask spreads, for example, Table 2 shows that EuroSETS spreads for Q4 stocks are, on average, 242% higher than NSC spreads in the first post-entry period. For Q1 stocks, EuroSETS spreads are only 24% higher.

Proxies for  $\gamma$  i.e. the proportion of smart routers. Consider all cases when the entrant posts a strictly better ask (bid) than the incumbent. In these cases, conditional on the arrival of a market buy (market sell), the likelihood of observing a trade in the entrant market is equal to  $\gamma$  in the model. This observation suggests to estimate  $\gamma$  as the proportion of all market buys (market sells) executed in EuroSETS when EuroSETS posts a strictly better ask (bid) than NSC. We denote this proxy by  $\hat{\gamma}_1$ . Notice that  $(1 - \hat{\gamma}_1)$  is the trade-through rate at the expense of EuroSETS.

#### [insert Table 4]

Panel A of Table 4 reports  $\widehat{\gamma}_1$  for each stock and each quartile. On average, the proportion of smart routers is equal to 27% in the first and 18% in the second post-entry period. Moreover, in both periods, the proportion of smart routers varies across stocks. Q1 stocks stand out as they are characterized by the highest proportion, 54% in both periods, as opposed to less than 23% for the other quartiles. The cross-sectional (and time) variation in  $\gamma$  reflects the proportion of trades brokered by smart routers. In fact, even if the number of smart routers is fixed and equal across stocks (i.e. if a broker has one, she will use it for all stocks), the fraction of total order flow channeled through smart routers can vary across stocks (and time). For instance, we already argued that the proportion of smart routers is likely to be larger for stocks cross-listed in the US. In support of this conjecture, we find a high and positive correlation between  $\widehat{\gamma}_1$  and a US listing dummy ( $\rho = 0.63$  and  $\rho = 0.44$  for the first and second post-entry period, respectively).

The proxy  $\widehat{\gamma}_1$  is valid only if non-smart routers never trade in the entrant market. In reality, as discussed in Section 3.2, these traders may occasionally check quotes in EuroSETS. If so,  $\widehat{\gamma}_1$  overestimates the proportion of smart routers. Hence, for robustness, we propose a second proxy for  $\gamma$ . We use Probit to model the likelihood of observing a buy (sell) order executed in EuroSETS conditional on EuroSETS posting a strictly better ask (bid) price, as a function of variables that capture relevant market conditions (discussed below). We then use the likelihood of observing a trade in EuroSETS under very adverse conditions as a more conservative proxy for  $\gamma$ , denoted  $\widehat{\gamma}_2$ . Intuitively, trades occurring

in these conditions must come from traders who only pay attention to prices, i.e. smart routers.

We expect brokers who handle orders manually to (i) prefer executing orders in single trades (as opposed to splitting orders into multiple trades) and to (ii) suffer high opportunity cost of checking EuroSETS quotes in fast markets. The single-trade preference suggests to consider (i) EuroSETS depth at the best quote and (ii) trade size as explanatory variables in the Probit model for the routing decision. The opportunity cost argument, further discussed in Section 3.2, suggests two additional explanatory variables that capture trade intensity: the number of trades and volatility in the 10 minutes prior to submission of the market order.

The Probit estimates in Panel B of Table 4 show that all explanatory variables are statistically significant and carry the expected sign. We now use these estimates to calculate the likelihood of observing a market order routed to EuroSETS, conditional on strictly better prices in EuroSETS, under very adverse conditions for EuroSETS (i.e. low depth in EuroSETS, a large market order, high trading activity). The estimates for this proxy,  $\hat{\gamma}_2$ , are reported in Panel A of Table 4 (2nd column). By construction,  $\hat{\gamma}_2$  is a conservative estimate relative to  $\hat{\gamma}_1$  ( $\hat{\gamma}_2 < \hat{\gamma}_1$ ), but it is clear from Table 4 that the two proxies are highly correlated, cross-sectionally.

Cross-sectional relation smart routers and EuroSETS liquidity. Armed with proxies for  $\gamma$ , we now test our third prediction of the model: EuroSETS liquidity relative to NSC liquidity should increase with the proportion of smart routers. For each stock and each post-entry period, we compute (i) the average "spread ratio" i.e. the ratio of NSC spread to EuroSETS spread and (ii) the average "depth ratio" i.e. the ratio of the number of shares offered in EuroSETS at the best consolidated ask to the total number of shares offered at the best consolidated ask (the ratio is zero when EuroSETS is not active at the best ask). The model predicts that both ratios increase with the proportion of smart routers (see Propositions 7 and 8). We test these predictions by running cross-sectional regressions in each post-entry period, where the dependent variable is either the depth or spread ratio and the explanatory variables include the smart-router proxy ( $\hat{\gamma}_1$  or  $\hat{\gamma}_2$ ) with and without volume and volatility as control variables. Incidentally, we find considerable cross-sectional dispersion in the dependent variables, as, for the spread ratio, the standard deviation is 0.27 (0.37) times the mean in the first (second) post-entry period. For the depth ratio, this factor is 0.58 (0.78).

#### [insert Table 5]

The results in Table 5 are consistent with our predictions. For the spread ratio (Panel

A), we find a significantly positive coefficient for both proxy  $\hat{\gamma}_1$  and  $\hat{\gamma}_2$ . In the univariate regressions, we find that these proxies explain between 47% and 89% of the cross-sectional variation. The result is robust to adding volume and volatility as control variables. Hence, as implied by Proposition 8, EuroSETS quotes are more competitive for stocks where the proportion of smart routers is large. For the depth ratio (Panel B), we also find that it is positively related to the proportion of smart routers, as implied by Proposition 7. For this ratio, however, the relationship is statistically significant only in the second post-entry period and the explanatory power is lower (the proxies, univariately, explain between 10% and 63% of the cross-sectional variation).

**Policy implications.** The positive relationship between the entrant market liquidity and the proportion of smart routers has two interesting implications.

First, it vindicates the view that violations of price priority (here at the expense of EuroSETS) reduce limit order traders' willingness to supply liquidity.

Second, it implies that brokerage firms' decisions to use a smart routing technology are complements. That is, the reduction in trading costs achieved by a smart router increases in the fraction of brokerage firms adopting SORSs. In such conditions, two types of self-fulfilling equilibria can arise, a low intensity competition equilibrium or a high intensity competition equilibrium. In the low intensity competition equilibrium, few brokers behave as smart routers because small gains are expected from smart routing. As a result, the gains turn out to be small because too few brokers are smart routers. In the high intensity competition equilibrium, many brokers are smart routers, because they expect large reductions in trading cost and these reductions will indeed be large, as many brokers are smart routers. Market participants can be stuck in the low intensity competition equilibrium as brokerage firms make their decision to invest in SORSs by calculating savings, based on *current* market conditions, not the conditions that would prevail if they could coordinate to collectively invest in SORSs. Prohibiting trade-throughs can be a way to move away from this equilibrium and to solve the coordination problem. Actually, in this case, the outcome is as if  $\gamma = 1$  since violations of price priority are explicitly forbidden.<sup>26</sup>

<sup>&</sup>lt;sup>26</sup>Coordination problems associated with the adoption of new technologies and regulatory solutions are analyzed in Dybvig and Spatt (1983). Battalio et al. (2004) show that trade-throughs decreased considerably in the option markets between June 2000 and January 2002 (overall the trade-through rate declined from 11.09% to 3.74%). This decline occurred despite the absence of (i) formal intermarket linkages or (ii) trade-through prohibitions in US options markets. However, as pointed by Battalio et al. (2004), US options markets have been under strong pressures from the SEC to develop intermarket linkages enforcing price priority. They conclude that the *threat* of regulatory intervention might be sufficient to induce market participants to avoid trade-throughs.

# 5 Further Analysis of EuroSETS Market Share and the Benefits of Smart Routing

In this final section, we further analyze liquidity in the EuroSETS system. First, we attempt to explain why EuroSETS market share is low despite its relatively large "liquidity share". Second, we study the benefits of being a smart router given EuroSETS liquidity.

# 5.1 EuroSETS Market Share vis-à-vis Liquidity Share

We further study EuroSETS contribution to liquidity through (i) the frequency at which it is present at the best ask in the consolidated market and (ii) the depth it offers conditional on being present. Although relative bid-ask spreads and depth as reported in Table 2, give insight into EuroSETS contribution to liquidity, they do not, perhaps, answer the most relevant question to a trader who is focused on the best price. How often does EuroSETS show the best quote and, if it does, what depth does it offer? Figure 2 answers this question for the ask side of the book. For completeness, we report the same statistics for NSC.

#### [insert Figure 2]

We find that EuroSETS contribution to liquidity is non-negligible, in particular for Q1 stocks. For the most actively traded stocks, Q1, we find that the frequency that EuroSETS is present at the best ask is 76.88% (87.97%) in the first (second) post-entry period. The depth it provides conditional on being present is €27,000 (€27,000), which is non-negligible relative to the average Q1 trade size of approximately €35,000 in both post-entry periods (see Table 1). For NSC, we find that it is present at the best ask 90.34% (95.01%) of the time with considerably more depth than EuroSETS, i.e. €145,000 (€217,000). This implies that 9.64% (4.99%) of the time EuroSETS offers strictly lower asks (100% minus the NSC frequency). For the other quartiles, EuroSETS is less frequently present at the inside as its frequency ranges from 9% to 32%. Yet, this frequency is not negligible. For instance, Bessembinder (2003) finds that the fraction of time regional exchanges in the US are at the inside ranges from 17.4% (Chicago Stock Exchange) to 6.44% (Philadelphia Stock Exchange). Thus, at least in comparison to US regional exchanges, EuroSETS appears quite competitive.

These statistics indicate that EuroSETS liquidity share (its contribution to overall displayed liquidity) is substantially larger than its market share. For Q1 stocks in the first post-entry period, for example, EuroSETS is present at the best ask 77% of the time and

the proportion of smart routers is 54%. This suggests a market share (in number of trades) higher than the 6.1% that we observe, except if the tie-breaking rule used by brokers is very favorable to NSC. We now show that this is indeed the case.

Tie-breaking rule and market share. In order to estimate the tie-breaking parameter  $\delta_I$ , we proceed as follows. Consider all market buys (sells) that occur when (i) both the entrant and the incumbent are at the best (consolidated) ask and (ii) the entrant depth at the best ask is larger than the size of the trade  $(Q_{E1}^* > X)$ . In our model, the fraction of these market orders that execute (at least partially) in the incumbent market is:

$$\kappa = (1 - \gamma) + \gamma \delta_I \tag{17}$$

Thus, the estimate of  $\kappa$  combined with our estimate of  $\gamma$ , say  $\widehat{\gamma}_1$ , allows us to calculate  $\delta_I$ .

[insert Table 6]

Table 6 reports our estimates for all quartiles and both post-entry periods. In all cases, the estimate of  $\kappa$  is close to 1. Thus, when both markets are at the inside, trades almost always occur in NSC. The resulting estimates for  $\hat{\delta}_I$  are large too, in particular for the first post-entry period where they exceed 95% for all quartiles. Large values for  $\hat{\delta}_I$  are consistent with the fact that large aggressive orders are charged less in NSC (see the discussion following Lemma 2).

These high values of  $\delta_I$  in addition to the low proportion of smart routers explain, at least partially, why EuroSETS market share (in number of trades) is small compared to its contribution to liquidity. Suppose that market orders execute at the best quote (either in NSC or in EuroSETS). This is a reasonable approximation as, in reality, a large fraction of orders do not walk up the book. Furthermore, suppose that there is a probability  $p_E^{strictly}(p_E^{jointly})$  that the best ask or the best bid in EuroSETS is strictly better (equal) than that in NSC. In this case, the likelihood of observing a market order routed to EuroSETS is:

$$p_E^{strictly} \gamma + p_E^{jointly} (1 - \delta_I) \gamma.$$

Clearly, this likelihood is small when  $\gamma$  is small and  $\delta_I$  is large, even if  $(p_E^{strictly} + p_E^{jointly})$  is large. For instance, for Q1 stocks in the first post-entry period, we have  $p_E^{strictly} \simeq 10\%$ ,  $p_E^{jointly} \simeq 67\%$  (see Figure 2),  $\gamma \simeq 54\%$ , and  $\delta_I \simeq 98\%$ . Hence, the likelihood that EuroSETS is at the inside is 77%, yet the the likelihood that it receives the market order is 6.1%, a figure very close to its actual market share (in number of trades) in Q1 stocks.

## 5.2 Are the Benefits of Smart Routing Small?

Table 4 shows that the proportion of market orders routed to NSC when EuroSETS offers strictly better prices ("trade-throughs") is large. This suggests that the two markets are locked in a low intensity competition equilibrium. In this case, we expect the benefits of smart routing to be small compared to the cost. Although the cost of developing smart routers is hard to estimate, we can calculate the opportunity cost of market orders that execute in NSC, but could have obtained better execution by being (at least in part) executed in EuroSETS.

To this end, we identify all buy (sell) orders routed to NSC that execute at an average price strictly larger (smaller) than the contemporaneous best ask (bid) in EuroSETS. We refer to these trades as generalized trade-throughs.<sup>27</sup> For each generalized trade-through, we compute the difference between (i) the actual (average) execution price of the market order and (ii) the average price that could have been achieved by executing the market order (partially) against EuroSETS best quotes. This difference times the size of the order is the opportunity cost of the generalized trade-through. For example, let us suppose our transaction data show an NSC transaction that was triggered by a market buy order for 2,000 shares. It executed 1,000 shares at the best NSC ask of  $\leq 15.00$  and 1,000 shares at the second best ask of  $\leq 15.01$ . If EuroSETS quote data show a best ask of 1,000 shares at  $\in 15.00$  at the time of the transaction, we label the transaction a generalized tradethrough with opportunity cost  $1.000* \le 0.01 = \le 10.00$ . Our data only allow us to find a lower bound for total opportunity cost as (i) in continuous time we only have the best bid and ask quote for EuroSETS including depth and (ii) we do not observe hidden depth in EuroSETS. Thus, we underestimate the total gain of consuming EuroSETS liquidity in addition to NSC liquidity. As execution fees differ in each system, we also compute the opportunity cost net of execution fees on market orders (based on the fee structure of Section 4.1.1).

We realize that since the NSC charges smaller fees on aggressive orders, our approach based on raw prices may misclassify trades. For instance, EuroSETS can have a strictly lower ask, yet reallocating the order might not be optimal as execution fees might be higher (see Section 3.2). Hence, in addition to identifying generalized trade-throughs based on raw prices, we also repeat the analysis with net prices.

<sup>&</sup>lt;sup>27</sup>Trade-throughs are usually defined as trades occuring when, say, a market buy executes in market A when a competing market, B, posts a strictly lower ask. Our definition is more general, because it also encompasses the case in which the ask in B is not better than in A and still the buy market order executes at an average price larger than the best ask in B, because the market order walks up the book in market A. This is why we refer to these trades as generalized trade-throughs.

Table 7 shows that the findings are very similar whether we use raw (Panel A) or net (Panel B) prices to classify trades. We, therefore, focus our discussion on the results based on raw prices. For each quartile, we report a monthly estimate of the opportunity cost before (third line) and after (fourth line) substracting additional trading fees. The monthly cost is much larger for Q1 stocks than for any other stocks. For instance, in the first post-entry period, the opportunity cost net of fees is equal to €271,000 for stocks in Q1 and is less than €40,000 for stocks in the other quartiles. This is primarily the result of a larger number of generalized trade-throughs in Q1 vis-à-vis the other quartiles. Per order, this cost in the first post entry-period ranges from €43.24 for Q4 stocks to €16.55 for Q1 stocks.

Presumably, the opportunity cost for non-smart routers are spread over a large number of brokerage firms. Hence, for a single firm, the potential savings in trading cost associated with a smart routing technology appears small. This supports the idea that EuroSETS and NSC are trapped in the low intensity competition equilibrium described in Section 4.2.3.

# 6 Conclusion

The proliferation of multiple trading platforms for identical securities has amplified the fragmentation of securities markets. One important question is whether liquidity is larger with (i) centralization of order flow in a single limit order book or with (ii) fragmentation of order flow among competing limit order markets, linked by smart routers. In this paper, we shed light on this issue, both theoretically and empirically. We model competition between pure limit order markets, accounting for possibly different fee structures across markets. We compare two situations: (i) a single limit order market operates (the "incumbent" market) and (ii) two limit order markets (the incumbent and the "entrant" market) compete for order flow. We also study the effect of a change in the proportion of smart routers (brokers who systematically split their orders so as to minimize trading costs) on the liquidity of the entrant market. The model generates three predictions:

- 1.a Consolidated depth is always larger in the multiple limit order market environment if intermarket competition is effective (i.e. if the entrant market is viable).
- 1.b Cumulative depth in the incumbent market is smaller in the multiple markets environment, other things equal. This result can be reversed if intermarket competition

prompts a fee reduction on passive orders (non-marketable limit orders) in the incumbent market.

- 2 Bid-ask spreads in the consolidated market are either identical or smaller in the multiple markets environment.
- 3 The liquidity of the entrant market relative to that of the incumbent market increases with the proportion of smart routers.

On May 24, 2004, the London Stock Exchange introduced a new trading platform, EuroSETS, for a subset of Dutch stocks. EuroSETS is a pure limit order market, competing with another limit order market, NSC, operated by Euronext. NSC was the main trading venue for Dutch stocks before EuroSETS entry. We take advantage of this change in the organization of the Dutch equity market to test the predictions of our model. For a sample of 22 stocks that trade in both systems, we reconstruct the consolidated limit order book before and after entry of EuroSETS. We find that consolidated depth is significantly larger after EuroSETS entry and that the inside bid-ask spread is unchanged or slightly smaller, in line with the model predictions. Moreover, we find that cumulative depth has significantly increased in NSC. This improvement in NSC liquidity is consistent with the reduction in NSC fees on passive orders just before EuroSETS entry. Last, as predicted by the model, we find a positive relationship between measures of EuroSETS relative liquidity and a proxy for the proportion of smart routers. This finding supports the view that violations of price priority (trade-throughs) undermine limit order traders' incentives to supply liquidity.<sup>28</sup>

There are several possible extensions that could be considered in future research. First, it would be interesting to analyze the welfare effects of market fragmentation. Our findings indicate that fragmentation of order flow between competing limit order books improves consolidated liquidity. But, it does not necessarily follow that it enhances welfare. In fact, in our model, only smart routers unambiguously benefit from market fragmentation. Traders who do not behave as smart routers and limit order traders can be hurt. Hence, the benefits of fragmentation must be balanced with (i) the cost of becoming a smart router and (ii) potential losses for liquidity providers. Second, it would be interesting to study the factors determining the maximal number of possible entrants when several limit order markets compete and the way consolidated depth varies with the number of markets. Last, we pointed out that externalities associated with the adoption of smart routing technologies could result in equilibria with high trade-through rates. A more in depth analysis of this problem is another interesting topic for future research.

<sup>&</sup>lt;sup>28</sup>Recent regulatory changes in US equity markets (so called "RegNMS") aim at preventing tradethroughs, precisely because they could reduce liquidity supply.

# Appendix A: Proofs

#### Proof of Lemma 1

Let  $H_0$ ,  $H_1$  and  $H_2$  be, respectively, the following events:

- 1.  $H_0$ : { The broker arriving in period 2 is not a smart router}.
- 2.  $H_1$ : {The broker arriving in period 2 is a smart router and gives priority to market I in case of tie}.
- 3.  $H_2$ : {The broker arriving in period 2 is a smart router and gives priority to market E in case of tie}.

Now consider the execution probability,  $P_{Ik}$ , of the marginal order placed at price  $p_k$  in the incumbent market. Suppose that the broker arriving in period 2 is a buyer. If she is not a smart router  $(H_0)$ , the marginal limit order at price  $p_k$  executes if and only if  $\widetilde{X} \geq Q_{Ik-1} + S_{Ik}$ . If the broker arriving in period 2 is a smart router then two cases must be distinguished. If she gives priority to the incumbent market in case of a tie  $(H_1)$  then the marginal order at price  $p_k$  in the incumbent market executes iff  $\widetilde{X} \geq Q_{k-1} + S_{Ik}$ . If instead she gives priority to the entrant market in case of a tie  $(H_2)$  then the marginal order at price price  $p_k$  in the incumbent market executes iff  $\widetilde{X} \geq Q_{k-1} + S_{Ik} + S_{Ek}$ . Thus, we deduce that:

$$P_{Ik} = \alpha[(1-\gamma)\Pr(\tilde{X} > Q_{Ik-1} + S_{Ik}) + \gamma(\delta_I\Pr(\tilde{X} > Q_{k-1} + S_{Ik}) + \delta_E\Pr(\tilde{X} > Q_{k-1} + S_{Ik} + S_{Ek}))].$$

As  $\Pr(\widetilde{X} \geq x) = \alpha \overline{F}(x)$ , this rewrites:

$$P_{Ik} = \alpha [(1 - \gamma)\overline{F}(Q_{Ik-1} + S_{Ik}) + \gamma(\delta_I \overline{F}(Q_{k-1} + S_{Ik}) + \delta_E \overline{F}(Q_{k-1} + S_{Ik} + S_{Ek}))].$$

This proves the first part of the lemma. The second part is obtained following a similar reasoning.

■

#### **Proof of Proposition 1**

**Part 1:** We first show that market I is dominant if and only if  $\gamma \leq \gamma^*$ . When I is dominant,  $S_{Ek}^* = 0$ ,  $\forall k$ . Then,  $Q_k^* = Q_{Ik}^*$ ,  $\forall k$  (i.e. the consolidated depth is equal to the cumulative depth in market I at each price level). This situation constitutes an equilibrium **iff** the following conditions are satisfied (use equations (7) and (8)):

$$\gamma(\delta_E \overline{F}(Q_{Ik-1}^*) + \delta_I \overline{F}(Q_{Ik}^*)) \le \frac{\widehat{c}_E}{(p_k - v_0)}, \forall k \ge 1.$$
 (18)

$$\overline{F}(Q_{Ik}^*) = \frac{\widehat{c}_I}{(p_k - v_0)}, \ k \ge 1.$$
(19)

Substituting this expression for  $\overline{F}(Q_{Ik}^*)$  in equation (18), we deduce that an equilibrium in which market I dominates is obtained iff,

$$\gamma\left(\frac{\delta_E \widehat{c}_I}{(p_{k-1} - v_0)} + \frac{\delta_I \widehat{c}_I}{(p_k - v_0)}\right) \le \frac{\widehat{c}_E}{(p_k - v_0)}, \, \forall k \ge 2.$$
 (20)

and

$$\gamma(\delta_E + \frac{\delta_I \widehat{c}_I}{(p_1 - v_0)}) \le \frac{\widehat{c}_E}{(p_1 - v_0)} . \tag{21}$$

Using the fact that  $\delta_E = 1 - \delta_I$ , Condition (20) is equivalent to

$$\gamma \le \frac{\widehat{c}_E}{(2 - \delta_I)\widehat{c}_I},$$

while Condition (21) is equivalent to

$$\gamma \le \frac{\widehat{c}_E}{\delta_I \widehat{c}_I + (1 - \delta_I) \Delta}.$$

Hence, we deduce that an equilibrium in which market I dominates can be sustained if and only if:

$$\gamma \leq Min\{\frac{\widehat{c}_E}{(2-\delta_I)\widehat{c}_I}, \frac{\widehat{c}_E}{\delta_I\widehat{c}_I + (1-\delta_I)\Delta}\}.$$

Part 2: We deduce (from Part 1) that if  $\gamma^* < \gamma < 1$  then market I cannot dominate. This leaves us with two possibilities: (a) either market E dominates or (b) both markets coexist. We show that Case (a) does not occur in equilibrium when  $\gamma^* < \gamma < 1$ . The proof proceeds by contradiction. Suppose that there exists an equilibrium in which market E is dominant. Then,  $S_{Ik}^* = 0$  (the book in market I is empty) and  $Q_k^* = Q_{Ek}^*$ ,  $\forall k$  (i.e. the consolidated depth is equal to the cumulative depth in market E at each price level). This means that the following condition is satisfied:

$$(1 - \gamma) + \gamma (\delta_I \overline{F}(Q_{Ek-1}^*) + \delta_E \overline{F}(Q_{Ek}^*)) \le \frac{\widehat{c}_I}{(p_k - v_0)} \,\forall k \in [1, m], \forall m$$
 (22)

Clearly, the L.H.S of inequality (22) is larger than or equal to  $(1 - \gamma) > 0$ . The RHS of inequality (22) decreases with k and goes to zero as k becomes large. Thus, we conclude that, when  $\gamma < 1$ , there always exists  $k_0(\gamma)$  such that inequality (22) is violated for  $k \ge k_0(\gamma)$ . This implies that there is no case in which E dominates when  $\gamma < 1$ .

#### **Proof of Proposition 2**

We first derive the conditions under which market I dominates. This happens **iff**  $S_{Ik}^* = 0$ ,  $\forall k$  (the book in market I is empty). In this case,  $Q_k^* = Q_{Ek}^*$ ,  $\forall k$  (i.e. the consolidated depth is equal to the cumulative depth in market E at each price level). Thus, when  $\gamma = 1$ , market E dominates **iff** the following conditions are satisfied:

$$(\delta_I \overline{F}(Q_{Ek-1}^*) + \delta_E \overline{F}(Q_{Ek}^*)) \le \frac{\widehat{c}_I}{(p_k - v_0)} \,\forall k \ge 1, \tag{23}$$

and

$$\overline{F}(Q_{Ek}^*) = \frac{\widehat{c}_E}{(p_k - v_0)} \,\forall k \ge 1, \tag{24}$$

Using equation (24) and the fact that  $Q_{E0}^* = 0$ , we rewrite Condition (23) as:

$$\frac{\delta_I \widehat{c}_E}{(p_{k-1} - v_0)} + \frac{\delta_E \widehat{c}_E}{(p_k - v_0)} \le \frac{\widehat{c}_I}{(p_k - v_0)} \text{ for } k > 1, \tag{25}$$

and

$$\delta_I + \frac{\delta_E \widehat{c}_E}{(p_1 - v_0)} \le \frac{\widehat{c}_I}{(p_1 - v_0)} \quad \text{for } k = 1$$
 (26)

As  $\delta_E = 1 - \delta_I$ , we find that equation (25) is satisfied for all k > 1 if and only if:

$$\widehat{c}_E \le \frac{\widehat{c}_I}{1 + \delta_I},$$

and equation (26) is satisfied iff:

$$\widehat{c}_E \le \frac{\widehat{c}_I - \delta_I \Delta}{1 - \delta_I}$$

Thus, market E dominates in equilibrium **iff**  $\hat{c}_E \leq Min\{\frac{\hat{c}_I}{1+\delta_I}, \frac{\hat{c}_I-\delta_I\Delta}{1-\delta_I}\}$  and  $\gamma=1$ . When  $\gamma=1$ , market I and E are symmetric. Thus, by symmetry, we deduce that when  $\gamma=1$ , market I dominates **iff**  $\hat{c}_I \leq Min\{\frac{\hat{c}_E}{1+\delta_E}, \frac{\hat{c}_E-\delta_E\Delta}{1-\delta_E}\}$  or, equivalently, market I dominates **iff**  $\hat{c}_E \geq Max\{(2-\delta_I)\hat{c}_I, \delta_I\hat{c}_I + (1-\delta_I)\Delta\}$ . We deduce from these observations that the two markets coexist **iff**  $\hat{c}_E \in (\hat{c}_E^*, \hat{c}_E^{**})$  and  $\gamma=1$ .

#### **Proof of Proposition 3**

**Benchmark:** When exchange I operates alone, the consolidated depth up to price  $p_k$ ,  $Q_k^*(0)$ , solves:

$$\overline{F}(Q_k^*(0)) = \frac{\widehat{c}_I}{(p_k - v_0)}. (27)$$

We have  $Q_k^*(0) > 0$ ,  $\forall k \in [1, m]$  since  $\hat{c}_I < p_k - v_0$ . Thus,  $Q_{Ik}^*(0) = Q_k^*(0) > 0$ ,  $\forall k \in [1, m]$ .

Case 1: Market E enters and dominates. In this case,  $Q_{Ik}^*(\gamma) = 0$ ,  $\forall k$  and therefore

 $Q_{Ik}^*(\gamma) < Q_{Ik}^*(0).$ 

Case 2: The two markets coexist. We establish that  $Q_{Ik}^*(\gamma) \leq Q_{Ik}^*(0)$  recursively. If  $S_{I1}^*(\gamma) = 0$  then  $Q_{I1}^*(\gamma) < Q_{I1}^*(0)$  and the result is true for k = 1. If  $S_{I1}^*(\gamma) > 0$  then  $Q_{I1}^*(\gamma)$  satisfies:

$$(1 - \gamma)\overline{F}(Q_{I1}^*(\gamma)) + \gamma(\delta_I \overline{F}(Q_{I1}^*(\gamma)) + \delta_E \overline{F}(Q_{I1}^*(\gamma) + S_{E1}^*)) = \frac{\widehat{c}_I}{(p_1 - v_0)}, \qquad (28)$$

Now observe that:

$$Q_{I1}^*(\gamma) \le Q_{I1}^*(\gamma) + S_{E1}^*.$$

As  $\overline{F}(.)$  is decreasing and  $\delta_E = 1 - \delta_I$ , we deduce from this inequality and equation (28), that:

$$\overline{F}(Q_{I1}^*(\gamma)) \ge \frac{\widehat{c}_I}{(p_1 - v_0)}.$$

This inequality implies (using equation (27)) that  $Q_{II}^*(\gamma) \leq Q_1^*(0)$ .

Now suppose that the property,  $Q_{Il}^*(\gamma) \leq Q_l^*(0)$ , is true up to  $l = k - 1 \leq m$ . Then at price  $p_k$ , there are two possibilities. If  $S_{Ik}^*(\gamma) = 0$  then

$$Q_{Ik}^*(\gamma) = Q_{Ik-1}^*(\gamma),$$

and, as  $Q_{Ik-1}^*(\gamma) \leq Q_{Ik-1}^*(0) < Q_{Ik}^*(0)$ , we deduce that  $Q_{Ik}^*(\gamma) < Q_{Ik}^*(0)$ . If instead,  $S_{Ik}^*(\gamma) > 0$ , then the cumulative depth up to price  $p_k$  in market I satisfies:

$$(1 - \gamma)\overline{F}(Q_{Ik}^*(\gamma)) + \gamma(\delta_I \overline{F}(Q_{k-1}^*(\gamma) + S_{Ik}^*(\gamma)) + \delta_E \overline{F}(Q_k^*(\gamma))) = \frac{\widehat{c}_I}{(p_k - v_0)}.$$
 (29)

Now observe that:

$$Q_{Ik}^*(\gamma) \le Q_{k-1}^*(\gamma) + S_{Ik}^*(\gamma) \le Q_k^*(\gamma).$$

As  $\overline{F}(.)$  is decreasing and  $\delta_E = 1 - \delta_I$ , we deduce from equation (29) that:

$$\overline{F}(Q_{Ik}^*(\gamma)) \ge \frac{\widehat{c}_I}{(p_k - v_0)},$$

which implies (using equation (27)) that  $Q_{Ik}^*(\gamma) \leq Q_k^*(0)$ . This shows that if  $Q_{Ik-1}^*(\gamma) \leq Q_{k-1}^*(0)$  then  $Q_{Ik}^*(\gamma) \leq Q_k^*(0)$ . As the property is true for k=1, the result holds for all k.

#### **Proof of Proposition 4**

When the two markets coexist, there exists  $k \leq m$  such that  $S_{Ek}^* > 0$ . Let  $k_0$  be

the smallest integer such that this inequality holds true. For  $k < k_0$ , the equilibrium cumulative depth in the incumbent market solves (see equation (7)):

$$\overline{F}(Q_{Ik}^*(\gamma)) = \frac{\widehat{c}_I}{(p_k - v_0)}.$$

Thus,  $Q_{Ik}^*(\gamma) = Q_k^*(0)$  (see see equation (27)) for  $k < k_0$ . Moreover as  $S_{Ek}^* = 0$  for  $k < k_0$ , we have  $Q_k^*(\gamma) = Q_{Ik}^*(\gamma)$  for  $k < k_0$ . Hence  $Q_k^*(\gamma) = Q_k^*(0)$  for  $k < k_0$ . Thus the property stated in the proposition is true up to price  $p_{k_0-1}$ .

Now we show recursively that  $Q_k^*(\gamma) > Q_k^*(0)$  for  $k \geq k_0$ . Consider the case in which  $k = k_0$ . Equilibrium condition (7) imposes:

$$(1 - \gamma)\overline{F}(Q_{Ik_0}^*) + \gamma(\delta_I \overline{F}(Q_{k_0-1}^* + S_{Ik_0}^*) + \delta_E \overline{F}(Q_{k_0}^*)) \le \frac{\widehat{c}_I}{(p_k - v_0)}.$$
 (30)

As  $S_{Ek_0}^* > 0$  and  $Q_{k_0-1}^* = Q_{Ik_0-1}^*$ , we have

$$Q_{Ik_0}^* = Q_{k_0-1}^* + S_{Ik_0}^* < Q_{k_0}^*.$$

But then, as  $\overline{F}(.)$  decreases and  $\delta_E = 1 - \delta_I$ , Condition (30) imposes:

$$\overline{F}(Q_{k_0}^*(\gamma)) < \frac{\widehat{c}_I}{(p_k - v_0)},$$

which rewrites (using equation (27) in the proof of Proposition 3)

$$\overline{F}(Q_{k_0}^*(\gamma)) < \overline{F}(Q_{k_0}^*(0)).$$

This implies  $Q_{k_0}^*(\gamma) > Q_{k_0}^*(0)$  as  $\overline{F}(.)$  decreases. Now suppose that the property is true up to price  $p_{k-1}$  with  $k_0 \leq k-1 \leq m-1$ . That is:  $Q_l^*(\gamma) > Q_l^*(0)$  for  $l \in [k_0, k-1]$ . Then we prove that it holds at price  $p_k$ . At this price, equilibrium condition (7) imposes:

$$(1 - \gamma)\overline{F}(Q_{Ik}^*(\gamma)) + \gamma(\delta_I \overline{F}(Q_{k-1}^*(\gamma) + S_{Ik}^*(\gamma)) + \delta_E \overline{F}(Q_k^*(\gamma))) \le \frac{\widehat{c}_I}{(p_k - v_0)}$$
(31)

Now observe that:

$$Q_{Ik-1}^*(\gamma) \le Q_{Ik-1}^*(0) < Q_{k-1}^*(\gamma).$$

The first inequality follows from Proposition 3 and the second from the recursive hypothesis  $(Q_l^*(\gamma) > Q_l^*(0))$  for  $l \in [k_0, k-1]$ . We deduce:

$$Q_{Ik-1}^*(\gamma) + S_{Ik}^*(\gamma) < Q_{k-1}^*(\gamma) + S_{Ik}^*(\gamma) \le Q_k^*(\gamma).$$

Now, Condition (31) implies that:

$$\overline{F}(Q_k^*(\gamma)) < \frac{\widehat{c}_I}{(p_k - v_0)},$$

which rewrites (using equation (27) in the proof of Proposition 3):

$$\overline{F}(Q_k^*(\gamma)) < \overline{F}(Q_k^*(0)).$$

This implies  $Q_k^*(\gamma) > Q_k^*(0)$ . This shows that if  $Q_{k-1}^*(\gamma) > Q_{k-1}^*(0)$  then  $Q_k^*(\gamma) > Q_k^*(0)$ . As the property is true for  $k = k_0$ , the result holds for all  $k > k_0$ .

#### **Proof of Proposition 5**

Available upon request.

■

#### **Proof of Corollary 1**

When market E dominates, the consolidated depth up to price  $p_k$  after entry solves:

$$\overline{F}(Q_k^*(\gamma)) = \frac{\widehat{c}_E}{(p_k - v_0)}.$$
(32)

Combining equations (27) and (32), we get:

$$\overline{F}(Q_k^*(0)) - \overline{F}(Q_k^*(\gamma)) = \frac{\widehat{c}_I}{(p_k - v_0)} - \frac{\widehat{c}_E}{(p_k - v_0)} \ge 0,$$

where the last inequality follows from the fact that  $\widehat{c}_E$  is smaller than  $\widehat{c}_I$  when market E dominates. Note that the inequality is strict if  $\widehat{c}_E < \widehat{c}_I$ . As  $\overline{F}(.)$  decreases, we conclude that:  $Q_k^*(\gamma) \geq Q_k^*(0)$ ,  $\forall k \in [1, m]$ , with a strict inequality if  $\widehat{c}_E < \widehat{c}_I$ .

#### Proof of Lemma 2

Immediate from the argument following the lemma.

■

#### **Proof of Corollary 2**

The corollary derives from Proposition 2. For instance, consider the case in which  $\delta_I$  goes to 1. In this case,  $c_E^* < 0$  since  $\Delta > \widehat{c}_I$ . Moreover,  $c_E^{**}$  goes to  $\widehat{c}_I$ . We deduce from Proposition 2 and these observations that markets E and I coexists iff  $0 < \widehat{c}_E < \widehat{c}_I$  and that market I dominates otherwise when  $\delta_I = 1$ . As  $\gamma = 1$ , the case in which  $\delta_E = 1$  delivers a symmetric conclusion.

#### Proof of Proposition 6

Let  $\widehat{c}_I^b$  ( $\widehat{c}_I^a$ ) be the order entry cost in the incumbent market before (after) the introduction of market E. Assume  $\widehat{c}_I^a \leq \widehat{c}_I^b$ . Suppose that  $\widehat{c}_I^b > \Delta$  (i.e. A1 does not hold) and let  $a^*$  be the first price on the grid such that  $a^* - v_0 > \widehat{c}_I^b$ . Let  $k^*$  be such that  $a^* = p_{k^*}$ . Thus (see Section 2.2)  $(a^* - v_0)$  is (half) the inside spread when the incumbent market operates alone.

We first establish that the inside spread after entry of market E cannot be larger than  $a^* - v_0$ . Suppose (to be contradicted) that this is not the case. Then  $Q_{Ik}^* = 0$  and  $Q_{Ek}^* = 0$ ,  $\forall k \leq k^*$ . This means, in particular, that posting a limit order at price  $a^* = p_{k^*}$  is not profitable, which means:  $(a^* - v_0) \leq \hat{c}_I^a$ . But this implies  $(a^* - v_0) \leq \hat{c}_I^b$  since  $\hat{c}_I^a \leq \hat{c}_I^b$ , a contradiction. We deduce that after introduction of the entrant market, we necessarily have  $Q_{Ik^*}^* > 0$  or  $Q_{Ek^*}^* > 0$ . Thus, the bid-ask spread in the consolidated market is smaller than or equal to its level before entry of market E.

Next, we establish that the bid-ask spread can be strictly smaller after entry of market E. This requires  $\hat{c}_I^a$  or  $\hat{c}_E$  to be small enough relative to  $\hat{c}_I^a$ . For instance, suppose that  $\hat{c}_E < \gamma \Delta$  while  $\hat{c}_I^a = \hat{c}_I^b$ . In this case, it is not profitable to submit a limit order at price  $p_1$  in the incumbent market since  $(p_1 - v_0) < \hat{c}_I^a$ . Hence, the execution probability of the first share offered at price  $p_1$  in the entrant market is  $\alpha \gamma$ . The expected profit on this share is then  $\alpha \gamma \Delta - c_E = \alpha(\gamma \Delta - \hat{c}_E) > 0$ . Thus, the ask price in the entrant market is  $p_1$  and the inside spread in the consolidated market is strictly smaller after entry of the entrant market.

### **Proof of Proposition 7**

We first show that  $Q_{E1}^*(\gamma)$  increases with  $\gamma$  and  $Q_{I1}^*(\gamma)$  decreases with  $\gamma$ . To this end consider two values of  $\gamma$ ,  $\gamma_0$  and  $\gamma_1$  such that  $\gamma_0 < \gamma_1$ . Assume (to be contradicted) that

$$Q_{E1}^*(\gamma_1) < Q_{E1}^*(\gamma_0).$$

Then, as  $Q_{E_1}^*(\gamma_1) \ge 0$ , we must have  $Q_{E_1}^*(\gamma_0) > 0$ . In this case, equation (8) imposes that in equilibrium:

$$\gamma_1(\delta_E \overline{F}(Q_{E1}^*(\gamma_1)) + \delta_I \overline{F}(Q_1^*(\gamma_1)) \le \frac{\widehat{c}_E}{(p_k - v_0)},$$

and

$$\gamma_0(\delta_E \overline{F}(Q_{E1}^*(\gamma_0)) + \delta_I \overline{F}(Q_1^*(\gamma_0)) = \frac{\widehat{c}_E}{(p_k - v_0)},$$

These two inequalities imply:

$$\delta_I(\gamma_0 \overline{F}(Q_1^*(\gamma_0)) - \gamma_1 \overline{F}(Q_1^*(\gamma_1)) \ge \delta_E(\gamma_1 \overline{F}(Q_{E1}^*(\gamma_1)) - \gamma_0 \overline{F}(Q_{E1}^*(\gamma_0))). \tag{33}$$

The RHS of this inequality is strictly positive since  $Q_{E1}^*(\gamma_1) < Q_{E1}^*(\gamma_0)$  and  $\gamma_0 < \gamma_1$ . Thus:

$$\delta_I(\gamma_0 \overline{F}(Q_1^*(\gamma_0)) - \gamma_1 \overline{F}(Q_1^*(\gamma_1)) > 0 \tag{34}$$

We deduce that  $Q_1^*(\gamma_0) < Q_1^*(\gamma_1)$ . But then  $Q_{I1}^*(\gamma_1) > Q_{I1}^*(\gamma_0)$  as  $Q_{E1}^*(\gamma_1) < Q_{E1}^*(\gamma_0)$ . As  $Q_{I1}^*(\gamma_0) \ge 0$ , we deduce (from equation 7) that in equilibrium the two following conditions hold true:

$$(1 - \gamma_1)\overline{F}(Q_{I1}^*(\gamma_1)) + \gamma_1(\delta_I \overline{F}(Q_{I1}^*(\gamma_1)) + \delta_E \overline{F}(Q_1^*(\gamma_1)) = \frac{\widehat{c}_I}{(p_k - v_0)},$$

and

$$(1 - \gamma_0)\overline{F}(Q_{I1}^*(\gamma_0)) + \gamma_0(\delta_I \overline{F}(Q_{I1}^*(\gamma_0)) + \delta_E \overline{F}(Q_1^*(\gamma_0)) \le \frac{\widehat{c}_I}{(p_k - v_0)}.$$

Together, these two inequalities imply:

$$(1 - \gamma_1 \delta_E) \overline{F}(Q_{II}^*(\gamma_1)) - (1 - \gamma_0 \delta_E) \overline{F}(Q_{II}^*(\gamma_0)) \ge \delta_E(\gamma_0 \overline{F}(Q_1^*(\gamma_0)) - \gamma_1 \overline{F}(Q_1^*(\gamma_1)). \tag{35}$$

We know from equation (34) that the RHS of this inequality is positive. But this yields an impossibility as the L.H.S of this inequality must be negative as  $\gamma_1 > \gamma_0$  and  $Q_{I1}^*(\gamma_1) > Q_{I1}^*(\gamma_0)$ . Thus, we deduce that in equilibrium it must be the case that

$$Q_{E1}^*(\gamma_1) \ge Q_{E1}^*(\gamma_0).$$

Now, suppose (to be contradicted) that  $Q_{I1}^*(\gamma_1) \geq Q_{I1}^*(\gamma_0)$  and  $Q_{E1}^*(\gamma_1) \geq Q_{E1}^*(\gamma_0)$ . This implies that in equilibrium inequality (35) holds true (as we can repeat the argument leading to this inequality). But this inequality can be rewritten (after some manipulations):

$$(1 - \gamma_1 \delta_E)(\overline{F}(Q_{I1}^*(\gamma_1)) - \overline{F}(Q_{I1}^*(\gamma_0)) + \gamma_1 \delta_E(\overline{F}(Q_1^*(\gamma_1)) - \overline{F}(Q_1^*(\gamma_0)) + (\gamma_1 - \gamma_0)\delta_E(\overline{F}(Q_1^*(\gamma_0)) - \overline{F}(Q_{I1}^*(\gamma_0)) > 0,$$

which cannot hold as all the terms on the L.H.S of this inequality are negative if  $Q_{I1}^*(\gamma_1) \geq Q_{I1}^*(\gamma_0)$ ,  $Q_{E1}^*(\gamma_1) \geq Q_{E1}^*(\gamma_0)$  and  $\gamma_1 > \gamma_0$ . We deduce that in equilibrium  $Q_{I1}^*(\gamma_1) < Q_{I1}^*(\gamma_0)$  and  $Q_{E1}^*(\gamma_1) \geq Q_{E1}^*(\gamma_0)$ . As  $C_{1E}(\gamma) = \frac{\rho}{1+\rho}$  where  $\rho \stackrel{def}{=} \frac{Q_{E1}^*(\gamma)}{Q_{I1}^*(\gamma)}$ , we immediately deduce that  $C_{1E}(\gamma_1) \geq C_{1E}(\gamma_0)$ , i.e.  $C_{1E}(\gamma)$  increases with  $\gamma$ .

#### **Proof of Proposition 8**

We first derive necessary and sufficient conditions for the entrant market to be the only market showing depth at price  $p_1$  (the 3rd case considered in the proposition). In this situation,  $S_{E1}^* > 0$  and  $S_{I1}^* = 0$ . This happens in equilibrium if and only if the following

conditions are satisfied (see Conditions (7) and (8)):

$$P_{I1}(0,0,0,S_{E1}^*) < R_{I1},$$

and

$$P_{E1}(0,0,S_{E1}^*) = R_{E1}.$$

That is:

$$(1-\gamma) + \gamma(\delta_I + \delta_E \overline{F}(S_{E1}^*)) < \frac{\widehat{c}_I}{\Delta},$$

and

$$\overline{F}(S_{E1}^*)) = \frac{\widehat{c}_E}{\Delta}.$$

The second condition imposes  $\hat{c}_E < \Delta$ . Moreover, substituting  $\overline{F}(S_{E1}^*)$  by  $(\frac{\hat{c}_E}{\Delta})$  in the first inequality, we find that the latter is satisfied iff  $\gamma > \frac{\Delta - \hat{c}_I + (1 - \delta_I) \hat{c}_E}{(1 - \delta_I) \Delta}$ . It is immediate that  $\frac{\Delta - \hat{c}_I + (1 - \delta_I) \hat{c}_E}{(1 - \delta_I) \Delta} < 1$  iff  $\hat{c}_E < \frac{\hat{c}_I - \delta_I \Delta}{1 - \delta_I}$ . Observe that this condition is more stringent than the condition  $\hat{c}_E < \Delta$  since  $\hat{c}_I < \Delta$ . We conclude that the equilibrium is such that  $S_{E1}^* > 0$  and  $S_{I1}^* = 0$  iff  $\frac{\Delta - \hat{c}_I + (1 - \delta_I) \hat{c}_E}{(1 - \delta_I) \Delta} < \gamma < 1$ . In this case, the best ask price in the incumbent market is strictly larger than in the entrant market. This proves the last claim of Proposition 8. The other claims in the proposition are proved using a similar reasoning. The proof is skipped for brevity.

# Appendix B: Composition of Quartiles

This appendix presents the composition of quartiles with stock names, codes, volume, and market capitalization. We start with the 25 AEX index stocks and eliminate Unilever as it exceeds the €50 bound where Euronext tick size changes. We further remove Gucci as drops from the index on May 5 and Numico as it cancels its ADR program on July 22. The remaining 22 stocks are ranked based on out-of-sample 2003 volume to ensure an exogenous ranking.

	NT.	Euronext	2003 Volume	2004 Volume	2004 Market
	Name	Code	(bln €)	(bln €)	Cap (bln €)
$\overline{Q1}$	Royal Dutch	RDA	73.2	83.5	85.2
	ING Group	INGA	43.8	46.9	38.9
	Philips	PHIA	39.6	42.2	29.9
	ABN AMRO	AABA	32.7	32.8	29.8
	Aegon	AGN	25.7	23.8	16.2
	Fortis	FORA	19.1	19.3	23.2
Q2	ASML	ASML	18.7	27.2	6.45
	$\mathbf{A}\mathbf{hold}$	AH	17.8	14.6	9.66
	KPN	KPN	17.4	18.7	14.6
	AKZO Nobel	AKZA	8.6	8.3	8.53
	Heineken	HEIA	8.4	7.3	14.7
	Reed Elsevier	REN	7.7	7.8	8.55
Q3	$\overline{V}NU$	VNUA	7.0	7.1	5.86
	DSM	DSM	5.6	5.3	3.98
	Wolters Kluwer	WKL	5.3	4.5	4.09
	TPG	TPG	5.3	7.4	8.63
	$\operatorname{Getronics}$	GTN	1.9	5.1	1.13
Q4	IHC Caland	IHC	1.8	2.4	1.29
	Versatel	VRSA	1.4	3.9	0.89
	${f Hage meijer}$	$_{ m HGM}$	1.3	3.5	0.92
	Buhrmann	BUHR	1.4	1.7	1.00
	Van der Moolen	MOO	1.1	0.5	0.25

# References

Barclay, M., T. Hendershott and T. McCormick, 2003, "Competition among Trading Venues: Information and Trading on Electronic Communication Networks", *Journal of Finance*, 2637-2665.

Battalio, R., 1997, "Third Market Broker-Dealers: Cost Competitors or Cream-Skimmers", Journal of Finance, **52**, 241-252.

Battalio, R., Hatch, B. and R. Jennings, 2004, "Towards a National Market System for US Exchange-Listed Equity Options", *Journal of Finance*, **59**, 933-962.

Bessembinder, H., 2003, "Quote-Based Competition and Trade Execution Costs in NYSE-listed stocks", *Journal of Financial Economics*, **70**, 385-422.

Biais, B., C. Bisière and C. Spatt, 2004, "Imperfect Competition in Financial Markets", working paper, Toulouse University.

Biais, B., Martimort, D. and Rochet, J.C., 2000, "Competing Mechanism in a Common Value Environment", *Econometrica*, **68**, 799-837.

Boehmer, B. and E. Boehmer, 2004, "Trading your Neighbor's ETFs': Competition and Fragmentation", *Journal of Banking and Finance*, **27**, 1667-1703.

Board, J. and Wells, S. (2000): "Liquidity and Best Exceution in the UK: A Comparison of SETS and Tradepoint", Working paper, FMG.

Chowdhry, B. and V. Nanda, 1991, "Multimarket Trading and Market Liquidity", *Review of Financial Studies*, 4, 483-511.

DeFontnouvelle, P., R. Fishe and J. Harris, 2003, "The Behavior of Bid-Ask Spreads and Volume in Options Markets during the Competition for Listings in 1999", *Journal of Finance*, 2437-2463.

Dybvig, and Spatt, C. (1983), "Adoption Externalities as Public Good", *Journal of Public Economics*, 20, 231-247.

Fink, J., Fink, K. and Weston, J. (2004), "Competition on the Nasdaq and the Growth of Electronic Communication Networks", Working Paper, Rice University.

Glosten, L., 1994, "Is the Electronic Order Book Inevitable", *Journal of Finance*, **49**, 1127–1161.

Glosten, L., 1998, "Competition, Design of Exchanges and Welfare", mimeo, Columbia University.

Hallam, N. and Idelson, N. (2003): "Breaking the Barriers: A technological Study of the Obstacles to a Pan-European Best Execution in Equities", mimeo, Tradeserve.

Hamilton, J.(1978), "Marketplace Fragmentation, Competition and the Efficiency of the Stock Exchange", *Journal of Finance*, **34**, 171-187.

Harris, L., 1990, "Liquidity, Trading Rules, and Electronic Trading Systems", New-York University Salomon Center, Monograph Series in Finance, n°1990-4.

Hendershott, T. and Mendelson, H., (2000), "Crossing Networks and Dealer Markets: Competition and Performance", *Journal of Finance*, 55, 2071-2115.

Hendershott, T. and Jones, C. (2005a), "Trade-Throughs Prohibitions and Market Quality", Journal of Financial Markets, 8, 1-23.

Hendershott, T. and Jones, C. (2005b), "Island goes Dark: Transparency, Fragmentation and Regulation", *Review of Financial Studies*, **19**, 743-793.

Lee, R., 2002, "A Survey of the Evidence on Fragmentation, Internalization and Market Transparency", mimeo, Oxford Finance Group.

Mayhew, P. 2002, "Competition, Market Structure and Bid-Ask Spreads", *Journal of Finance*, 57, 931-958.

McCleskey, (2004), Achieving Market Integration, Elsevier Finance (Ed.).

Mendelson, H., 1987, "Consolidation, Fragmentation and Market Performance", *Journal of Financial and Quantitative Analysis*, 22, 187-207.

Pagano, M., 1989, "Trading Volume and Asset Liquidity", Quarterly Journal of Economics, 104, 255-274.

Parlour, C., and D. Seppi, 2003, "Liquidity-Based Competition for Order Flow", *Review of Financial Studies*, **16**, 301-343.

Petersen, M.A., 2005, "Estimating standard errors in finance panel data sets: comparing approaches", Technical Report, Northwestern University.

Stoll, H., 2001, "Market Fragmentation", Financial Analysts Journal, 57, 16-20.

Sandås, P., 2001, Adverse Selection and Competitive Market Making: Evidence from a Limit Order Market, *Review of Financial Studies*, **14**, 705-734.

Sabourin, D., 2004, "Competition between dealer markets and electronic limit order books", mimeo, CEREMADE.

Seppi, D., 1997, "Liquidity Provision with Limit Orders and a Strategic Specialist", *Review of Financial Studies*, **10**, 103-150.

U.S. Securities and Exchange Commission, 2000, Release N°34-42450

Viswanathan, V. and J. Wang, "Market Architecture: Limit Order Books vs. Dealership Markets", Journal of Financial Markets, 5, 127-167.

Table 1: Summary Statistics

This table reports summary statistics for all Dutch index stocks. We group them into volume quartiles where Q1 contains the most actively traded stocks. We report volume, number of trades, trade size, volatility, and price. For both post-entry periods, we disaggregate volume and the number of trades to report NSC and EuroSETS market shares. And, we use the disaggregation to report NSC and EuroSETS average trade size. All statistics are based on quotes and trades through the limit order book, i.e. off-market block trades are excluded. The pre-entry period runs from April 23 through May 21, 2004, post-entry 1 from August 2 through August 30, 2004, and post-entry 2 from January 3 through January 31, 2005. t-values are based on standard errors that control for commonalities across stocks, heteroskedasticity and non-zero stock-specific autocorrelation (see Section 4.1.3).

	I	Pre-Entry		Post	Post-Entry 1	1			Post	Post-Entry 2	2	
			Consoli-Change t-value NSC dated	hange t-	value N		Euro- SETS	Consoli-Change t-value	hange t-	value N	NSC E	Euro- SETS
Daily volume <sup>a</sup>	Q1	167.33	147.36	-19.98	-2.79	94.9%	5.1%	176.03	8.7	1.47	96.4%	3.6%
$(\in \text{mio})$	$Q_2$	57.12	48.85	-8.27	-2.75	99.7%	0.3%	58.07	0.95	0.11	99.9%	0.2%
	Q3	25.29	19.43	-5.87	-3.11	99.7%	0.3%	25.85	0.55	0.18	99.9%	0.1%
	Q4	9.4	9.25	-0.15	-0.06	99.8%	0.2%	9.26	-0.14	-0.07	99.9%	0.1%
	All	69.1	60.03	-9.07	-2.31	96.5%	3.5%	71.82	2.72	0.45	%9'.26	2.4%
Daily #Trades <sup>a</sup>	Q1	4.84	4.33	-0.51	-5.77	93.9%	6.1%	4.88	0.04	0.22	96.4%	3.6%
(1,000  trades)	$Q_2$	2.23	2.02	-0.21	-4.91	99.6%	0.4%	2.2	-0.02	-0.15	99.9%	0.1%
	<b>Q</b> 3	1.39	1.22	-0.16	-3.58	88.66	0.2%	1.41	0.02	0.14	99.9%	0.1%
	Q4	0.76	0.68	-0.08	-0.57	99.8%	0.2%	0.83	90.0	0.51	99.9%	0.1%
	All	2.42	2.17	-0.25	-3.71	96.5%	3.5%	2.44	0.03	0.17	98.0%	2.0%
Trade $\mathrm{Size}^a$	$Q_1$	1.66	1.71	0.05	1.50	1.73	1.45	1.61	-0.05	-0.97	1.62	1.47
(1,000  shares)	$Q_2$	2.41	2.46	0.05	0.30	2.46	1.54	2.42	0.01	0.05	2.42	1.45
	Q3	1.80	2.04	0.25	1.21	2.04	0.70	1.78	-0.02	-0.21	1.78	0.67
	Q4	3.29	4.08	0.79	1.87	4.08	0.41	2.58	-0.71	-2.56	2.58	0.14
	All	2.27	2.53	0.26	2.17	2.53	1.07	2.09	-0.17	-2.20	2.09	0.98
Annualized	$Q_1$	19.13	17.21	-1.92	-3.80			12.21	-6.92	-11.69		
$Volatility^b$	$Q_2$	22.61	21.38	-1.23	-3.59			15.39	-7.31	-3.97		
(%)	Q3	27.13	26.05	-1.07	-0.92			16.42	-10.71	-8.28		
	Q4	35.25	32.02	-3.24	-1.02			23.04	-12.21	-4.20		
	All	25.57	23.72	-1.85	-2.92			16.49	-9.10	-9.26		
Price	$Q_1$	21.22	20.44	-0.78	-1.47			22.67	1.45	1.81		
€	$Q_2$	16.17	14.10	-2.08	-3.18			15.40	-0.78	-1.20		
	Q3	19.30	18.76	-0.53	-1.02			21.06	1.77	1.37		
	Q4	11.20	10.34	-0.86	-3.09			13.04	1.84	0.69		
	All	17.13	16.04	-1.10	-4.68			18.13	1.00	1.78		
-												

a: The trade statistics are based on all trades through the limit order book, i.e. off-market block trades are not included.

 $<sup>^{</sup>b}$ : This is realized volatility based on 5-minute midquote returns. We annualize using 225 trading days per year.

Table 2: Spread and Depth Pre- and Post-Entry

market with pre-entry liquidity. We also report liquidity in NSC only to analyze to what extent the total change is due to additional liquidity in EuroSETS. For completeness, we also report spread and depth in EuroSETS only. The pre-entry period runs from April 23 through May 21, 2004, post-entry 1 from August 2 through August 30, 2004, and post-entry 2 from January 3 through January 31, 2005. For all four liquidity measures, we calculate the difference between the pre- and post-entry levels and add a "\*" to the post-entry levels if these are significantly different at the 99% level. In the test, we use standard errors that control for commonalities across stocks, heteroskedasticity and non-zero stock-specific autocorrelation This table reports (i) time-weighted quoted spread and (ii) ask-side depth and transaction-weighted effective spread for volume-ranked quartiles, where I contains the most actively traded securities. We report quoted spread, (visible) depth at the best ask using the midquote to calculate the value (Deptho), cumulative depth up until 4 ticks behind the best ask (Depth4), and effective spread. We compare post-entry liquidity in the consolidated see Section 4.1.3)

			$_{\rm los}$	t-Entry	П				$\operatorname{Post-Entry}$	ry 2	
	1	Q1	Q2	Q3_	Q4	All	Q1	Q2	Q3	Q4	All
Quoted Spread	Pre-Entry	7.91	13.80	21.43	43.48	20.67	7.91	13.80	21.43	43.48	20.68
(basispoints)	Post-Entry, Consolidated	6.65	13.55	$24.80^{*}$	$46.28^{*}$	21.66	$5.59^{*}$	$10.89^{*}$	$17.93^{*}$	$32.60^{*}$	$15.98^{*}$
	Post-Entry, NSC only	7.80	13.86	$25.13^{*}$	46.41*	22.17	6.05	11.03*	$18.19^{*}$	$32.86^{*}$	$16.26^{*}$
Depth0	Pre-Entry	1.21	1.35	0.93	1.14	1.17	1.21	1.33	0.93	1.14	1.17
(€100,000)	Post-Entry, Consolidated	$1.72^{*}$	$1.80^{*}$	1.24	$1.69^{*}$	$1.63^{*}$	$2.65^{*}$	2.87*	$1.54^{*}$	$1.93^{*}$	$2.30^{*}$
	Post-Entry, NSC only	1.44*	1.62	1.15	1.63	$1.47^{*}$	$2.30^{*}$	2.78*	$1.59^{*}$	1.92*	$2.19^{*}$
Depth4	Pre-Entry	7.71	2.76	5.84	8.23	7.42	7.71	7.64	5.84	8.23	7.41
(€100,000)	Post-Entry, Consolidated	13.37*	$11.93^{*}$	8.66*	11.08*	$11.39^*$	$17.52^{*}$	18.98*	9.47*	$11.85^{*}$	$14.80^{*}$
	Post-Entry, NSC only	$9.36^{*}$	9.50	6.84	9.77	8.92*	$15.06^{*}$	17.07*	$9.56^*$	$13.30^{*}$	$13.96^{*}$
Eff. Spread <sup>a</sup>	Pre-Entry	7.12	11.93	18.29	39.03	18.22	7.12	11.94	18.26	39.11	18.23
(basispoints)	Post-Entry, Consolidated	6.41*	12.26	22.02	42.50*	19.76	5.30*	10.44*	17.27	$31.69^{*}$	15.42*
,	Post-Entry, NSC only	7.24	$12.51^{*}$	22.27	$42.70^{*}$	$20.15^{*}$	5.80*	10.52*	17.38	$31.83^{*}$	$15.63^{*}$
-	t		1	3	0	000	Ċ	0	1	0	7
Spread	Post-Entry, EuroSETS only		27.78	55.64	112.36	48.39	6.91	.29.80	49.17	88.01	41.19
Depth0	Post-Entry, EuroSETS only		1.05	1.13	0.71	0.92	0.65	1.48	1.41	0.77	1.08
Depth4	Post-Entry, EuroSETS only	4.32	2.64	2.16	1.54	2.74	2.62	2.13	2.05	1.12	2.01
Eff. Spread $^a$	Post-Entry, EuroSETS only		18.44	32.84	91.39	32.57	5.91	21.52	34.16	41.17	24.89
					9/						

a: The effective spread is twice the amount by which the trade price exceeds (for market buys) or is below (for market sells) the contemporaneous midquote. In case of market buys (sells), the trade price is defined as the volume-weighted average price of all buys (sells) reported at the same second. The reason for this aggregation is that orders that run up the book are reported as separate trades by the exchanges. For the consolidated market, we use the midquote in the consolidated market and aggregate market buys (sells) across both markets. For the "NSC only" or "EuroSETS only" market, we use the market's own midquote.

\*: Significant at a 99% significance level.

Table 3: Liquidity Change with Controls for Volume, Volatility, and Price

the best ask (Depth0), and cumulative depth up until 4 ticks behind the best ask (Depth4). The econometric specification assumes that the variable For the post-entry books, both the consolidated and the NSC-only book are considered. We analyze three liquidity measures: quoted spread, depth at This table reports time-weighted liquidity change by comparing pre- and post-entry order books with the controls volume, realized volatility, and price. of interest  $y_{it}$  (e.g. quoted spread) for stock i on day t can be expressed as the sum of a stock-specific mean  $(\mu_i)$  and an event effect  $(\delta_i)$ , potential control variables  $(X_{it})$ , and an error term  $(\varepsilon_{it})$ :

$$y_{it} = \mu_i + \delta_i \mathbf{1}_{[t]}$$
 in post-entry period]  $+ \beta' X_{it} + \varepsilon_{it}$   
 $\varepsilon_{it} = \xi_t + \eta_{it}$ 

means and event-effects based on the model estimates. Section 4.1.3 describes the methodology in more detail. The pre-entry period runs from April 23 through May 21, 2004, post-entry 1 from August, 2 through August 30, 2004, and post-entry 2 from January 3 through January 31, 2005. t-values are based on standard errors that control for commonalities across stocks, heteroskedasticity and non-zero stock-specific autocorrelation (see Section where  $I_{[A]}$  is an indicator function that is 1 if A is true, zero otherwise, and  $\xi_t$  is a (potential) common factor across all stocks. We calculate quartile

			Post-F	st-Entry $1$					Post-Entry 2			
	Spread	ead	Dep	th0	Dep	th4	$\operatorname{Spr}$	ead	Dep	th0	Dept	h4
	Consoli	_NSC	Consoli	- NSC	Consoli	_NSC	$-$ Consoli- $_{ m Ng}$	- NSC	Consoli	i- NSC	Consoli- N	NSC
Change Q1 $(\delta 1)$	-1.16	-0.13	0.56	0.30	00.9		-1.05	-0.62	1.22	0.90	8.17	6.25
Rel. Change $Q1^a$ -14.7%	-14.7%	-1.6%	46.3%	24.8%	77.8%		-13.3%	-7.8%	100.8%	74.4%	106.0%	81.1%
t-value	-4.77	-0.72	4.71	5.23	12.58		-2.84	-1.81	5.30	5.43	10.23	2.08
Change Q2 $(\delta 2)$	-0.28	-0.05	0.65	0.43	5.12		-1.88	-1.74	1.32	1.21	9.75	7.88
Rel. Change $Q2^a$	-2.0%	-0.4%	48.1%	31.9%	90.99		-13.6%	-12.6%	97.8%	89.6%	125.6%	101.5%
t-value	-1.04	-0.20	2.70	1.92	4.56		-3.97	-3.68	2.46	2.27	2.96	2.36
Change Q3 $(\delta 3)$	3.49	3.79	0.33	0.24	2.92		-1.74	-1.42	0.33	0.39	1.59	2.11
Rel. Change $Q3^a$	16.3%	17.7%	35.5%	25.8%	50.0%		-8.1%	<b>%9</b> .9-	35.5%	41.9%	27.2%	36.1%
t-value	1.18	1.29	1.91	1.38	3.05		-0.68	-0.55	1.04	1.07	1.34	1.01
Change Q4 $(\delta 4)$	3.11	3.21	0.57	0.50	2.90		-9.06	-8.73	0.51	0.50	1.54	3.43
Rel. Change $Q4^a$	7.2%	7.4%	50.0%	43.9%	35.2%		-20.8%	-20.1%	44.7%	43.9%	18.7%	41.7%
t-value	1.68	1.77	2.94	2.47	4.10		-5.56	-5.31	1.58	1.53	1.44	1.69
Daily Volume	-0.20	-0.20	0.06	0.06	0.44		-0.28	-0.28	0.11		0.89	0.83
t-value	-2.51	-2.51	2.99	3.00	3.15	3.28	-2.60	-2.65	3.00		3.67	3.26
Daily Volatility	2.18	2.26	-0.42	-0.42	-2.42	-2.36	2.33	2.36	-0.46	-0.48	-3.30	-3.03
t-value	4.93	5.06	-3.58	-3.47	-3.46	-3.35	6.70	7.01	-2.96		-3.65	-3.31
Avg Daily Price	-0.07	-0.10	0.10	0.08	0.49	0.38	-0.18	-0.20	0.02		0.20	0.05
t-value	-0.63	-0.89	2.70	2.31	2.45	1.98	-1.23	-1.36	0.45		0.97	0.21
$\mathbb{R}^2$	0.53	0.53	0.78	0.52	0.46	0.28	0.51	0.51	0.18		0.23	0.14
#Ops	920	920	920	920	920	920	920	920	920		920	920
$\sigma_{\xi}$	1.1	1.1	0.1	0.1	0.8	8.0	0.0	0.0	0.2		0.8	0.0

a: Relative to pre-entry levels.

### Table 4: Proxies for the Proportion of Smart Routers $(\gamma_i)$

This table reports estimates of the model's most important structural parameter: the proportion of liquidity demand that uses smart routers. To identify this proportion, we condition on all transactions that execute on strictly better prices in the EuroSETS system i.e. market buys when EuroSETS has strictly lower asks and markets sells when it has strictly higher bids. For these transactions, we consider the proportion that materializes on EuroSETS,  $\hat{\gamma}_{1i}$ , a first proxy for the proportion of smart routers. We realize that  $\hat{\gamma}_{1i}$  might overstate the proportion of smart routers, as non-smart routers might be willing under some market conditions to incur the search "cost" of checking EuroSETS and (also) trade there if prices are better. To identify these conditions, we estimate a PROBIT model with a vector of explanatory variables  $X_{it}$  for trade t of stock i:

$$\gamma_{it} = P_t[\text{Trade on EuroSETS}|\text{Strictly Better EuroSETS Quote}] = E[\alpha_i + \beta' X_{it} + \varepsilon_{it} > 0], \quad \varepsilon_{it} \sim \text{IIN}(0, 1)$$

where  $\alpha_i$  is a stock-specific dummy and  $X_{it}$  contains EuroSETS quoted depth, order size, the number of trades in prior 10 minutes, and volatility based on the midquote high minus low in prior 10 minutes. We use the estimates to produce a more refined estimate of gamma,  $\hat{\gamma}_{2i}$ , which is the proportion of EuroSETS trades conditional on adverse market conditions for EuroSETS. The sign of the explanatory factors in the PROBIT equation suggests that we use the following fixed values for these factors: zero for EuroSETS quoted depth and the 0.90-quantile of the unconditional (full-sample) distribution of order size, the number of trades, and volatility:

$$\widehat{\gamma}_{2i} = \widehat{\gamma}_{it}(X_{it} = X^*) = \Phi(\widehat{\alpha}_i + \widehat{\beta}'X^*), \quad X^* = [0, \ Q_{0.90}(X_2), \ Q_{0.90}(X_3), \ Q_{0.90}(X_4)]'$$

where hats indicate coefficient estimates and  $Q_{0.90}(.)$  indicates the 0.90-quantile of the unconditional (full-sample) distribution. Panel A reports both  $\hat{\gamma}_{i1}$  and  $\hat{\gamma}_{i2}$ . Panel B reports the PROBIT coefficient estimates, t-values, the probability slope which denotes the estimated change in probability for a one-unit change in the explanatory variable, the j<sup>th</sup> explanatory variable average  $(\overline{X}_j)$ , and the product of these last two variables to indicate economic significance. Post-entry 1 runs from August 2 through August 30, 2004 and post-entry 2 from January 3 through January 31, 2005.

—continued on next page—

Panel A: Overall, Quartile-, and Stock-Specific Estimate<sup>a</sup> of  $\widehat{\gamma}_s$ 

				artile-, $ar$	nd Stock-Specific Esti			
	P	ost-Entry	1			Post-Entry	y 2	
	Proportion Eur EuroSETS Trades Adv Mai	erse	$\sigma_{\widehat{\gamma}_{2i}}$	#Obs	$rac{ ext{Proportion Er}}{ ext{EuroSETS}} rac{ ext{Ac}}{ ext{Ac}}$	$\widehat{\gamma}_{2i}$ roportion uroSETS ades for dverse arket onditions	$\sigma_{\widehat{\gamma}_{2i}}$	#Obs
$\overline{Q1}$	0.54	0.37	0.01	$22,\!239$	0.54	0.37	0.01	$12,\!688$
Q2	0.22	0.15	0.02	677	0.11	0.05	0.02	475
Q1 Q2 Q3 Q4 All	0.10	0.05	0.05	423	0.04	0.02	0.01	298
Q4	0.23	0.20	0.08	72	0.01	0.01	0.02	72
All	0.27	0.19	0.05	$23,\!411$	0.18	0.11	0.02	$13,\!533$
RDA	0.43	0.26	0.01	$5,\!248$	0.61	0.43	0.01	5,583
INGA	0.50	0.33	0.01	$3,\!641$	0.53	0.36	0.01	1,972
PHIA	0.58	0.44	0.01	$4,\!524$	0.53	0.37	0.01	1,763
AABA		0.37	0.01	$3,\!136$	0.46	0.29	0.01	1,525
AGN	0.67	0.52	0.01	$3,\!174$	0.62	0.45	0.01	891
FORA	0.48	0.31	0.01	$2,\!516$	0.47	0.30	0.01	954
ASML		0.11	0.02	$^{'}169$	0.21	0.12	0.03	56
AH	0.31	0.20	0.02	93	0.09	0.03	0.02	107
KPN	0.56	0.41	0.02	61	0.09	0.04	0.02	107
AKZA	0.16	0.08	0.02	190	0.17	0.08	0.03	54
HEIA	0.05	0.02	0.01	119	0.06	0.02	0.02	105
$\operatorname{REN}$	0.09	0.05	0.03	45	0.04	0.01	0.02	46
VNUA		0.03	0.01	174	0.06	0.02	0.01	117
DSM	0.02	0.00	0.01	105	0.00	0.00	0.00	49
WKL	0.10	0.05	0.03	41	0.02	0.01	0.02	46
TPG	0.05	0.02	0.02	99	0.14	0.05	0.02	66
GTN	0.25	0.13	0.10	4	0.00	0.00	0.00	20
IHC	0.03	0.01	0.02	38	0.00	0.00	0.00	14
VRSA					0.00	0.00	0.00	11
$_{\rm HGM}$	0.00	0.00	0.01	2	0.00	0.00	0.00	25
BUHF		0.17	0.03	26	0.00	0.00	0.00	3
MOO	0.67	0.61	0.15	6	0.05	0.03	0.05	19

 $<sup>\</sup>frac{1}{a}$ : The quartile-specific and overall estimate of  $\hat{\gamma}$  is based on equally weighted averages over all stocks in the quartile and sample, respectively.

Panel B: Control Variable Coefficients

	Coef	t-value	P-Slope	$\overline{X}$	$P$ -Slope* $\overline{X}$
Quoted Depth EuroSETS (€100,000)	0.211	26.27	0.084	0.984	0.082
Order Size <sup>a</sup> ( $\leq 100,000$ )	-0.267	-23.80	-0.107	0.518	-0.055
#Trades in Prior 10 Minutes (1,000 trades)	-0.356	-2.08	-0.142	0.070	-0.010
Volatility <sup><math>b</math></sup> (basispoints)	-0.003	-7.86	-0.001	26.424	-0.030

<sup>\*\*</sup>All transactions at the same second are aggregated into one order, as exchanges report orders running up the book as separate transactions.

\*\*b\*: Volatility is defined as the maximum minus the minimum midquote in the previous ten minutes scaled by the average midquote.

Table 5: Can the Proportion of Smart Routers Explain EuroSETS Share of Liquidity Cross-Sectionally?

This table reports cross-sectional regressions that relate two measures of EuroSETS share of liquidity to our two measures of the proportion of smart routers:  $\hat{\gamma}_{1i}$  and  $\hat{\gamma}_{2i}$ . Panel A reports results for the first measure: the ratio of time-weighted NSC spread and EuroSETS spread. Panel B reports the results for the second measure: the ratio of EuroSETS depth at the best ask in the consolidated market (EuroSETS depth at this ask could, therefore, be zero) and total depth at the best ask in the consolidated market. In a second set of regressions, we add two control variables: average daily volume and annualized volatility. t-statistics are in brackets.

		${ m R}^2$			0.88		0.89		0.89		0.90			0.63		0.61		0.68		0.67		p
		annualized volatility							0.003	(1.29)	0.002	(0.97)						-0.002	(-1.51)	-0.002	(-1.65)	efore, cou
2)		(00	(%)						0.000	(0.49)	0.000	(0.49)						0.000	(0.29)	0.000	(0.40)	es and, ther
Post-Entry 2 $(N=22)$	$\widehat{\gamma}_{2i}$ Proportion	EuroSETS volume Trades for $(\in 100.0)$	Adverse Market	Successions			$1.452^{*}$	(13.06)			1.424*	(7.28)	epth			0.352*	(5.55)			0.269*	(2.57)	SETS price
Post-En		Proportion Eur FuroSETS		TS Spread	1.018*	(12.01)			1.012*	(6.75)			Panel B: Dependent variable is the Ratio of EuroSETS Depth and Consolidated Depth	0.254*	(5.86)			$0.203^{*}$	(2.71)			did not have any transaction conditional on strictly better EuroSETS prices and, therefore, could
		CI FE		Panel A: Dependent variable is the Ratio of NSC Spread and EuroSETS Spread									Pepth and Co									al on strictļ
		${f R}^2$		pread	09.0		0.47		0.77		0.72		ELS	0.14		0.10		0.34		0.32		dition
		annualized volatility		of NSC S					-0.004	(-1.74)	-0.004	(-1.57)	of EuroS					-0.004 0.34	(-1.79)	-0.004	(-1.76)	saction cor
Post-Entry 1 $(N=21)^a$			(%)	is the Ratic					0.001*	(2.17)	0.001*	(2.68)	is the Ratic					0.000	(0.59)	0.000	(0.81)	ve any trans
Post-Entry	$\widehat{\gamma}_{2i}$ Proportion	EuroSETS volume Trades for (€100,000)	Adverse Market	ent variable			$0.586^*$	(4.09)			0.417*	(3.45)	ent variable			0.155	(1.42)			0.088	(0.81)	
		$\frac{\gamma_{1i}}{\text{Proportion}} \stackrel{\text{Eu}}{ ext{Eu}}$	Trades Ma	A: Depende	0.529*	(5.53)			$0.393^{*}$	(4.23)			B: Depend	0.150	(1.77)			0.093	(1.02)			a: For VRSA we
		Pr.	Ţ	$\overline{Panel}$	(1)		(3)		(3)		(4)		Panel	(1)		(3)		(3)		(4)		a: Fo

not identify  $\hat{\gamma_i}$ .
\*: Significant at a 95% significance level.

Table 6: Estimates of the Tie-Breaking Rule Parameter  $(\delta_I)$ 

This table reports estimates of the tie-breaking rule parameter  $\delta_I$ , which represents the proportion of traders who start executing orders in the incumbent market when both markets are available for trade at the best quote. To identify  $\delta_I$ , we condition on market buys (sells) at times that ask (bid) prices are equal across markets and further require that the size of the order is smaller than EuroSETS depth. The proportion of these orders (buy or sell) that execute on NSC identifies  $\delta_I$ , as this proportion equals  $\kappa = (1 - \hat{\gamma}) + \hat{\gamma} \delta_I$ . In the calculations, we use  $\hat{\gamma}_{1i}$  as our proxy for  $\hat{\gamma}$  (see Table 4 for details on  $\hat{\gamma}_{1i}$ ). Post-Entry 1 runs from August, 2 through August 30, 2004 and Post-Entry 2 from January 3 through January 31, 2005.

		Pos	t-Entry 1	_				Pos	t-Entry 2	2	
_	Q1	Q2	Q3	Q4	All	-	Q1	Q2	Q3	Q4	All
$\widehat{\kappa} = (1 - \widehat{\gamma}_1) + \widehat{\gamma}_1 \delta_I$	0.975	0.997	0.999	0.998	0.992		0.981	0.982	0.985	0.997	0.986
#Obs	80,911	$15,\!383$	$4,\!552$	1,860	102,706		$129,\!957$	7,989	$2,\!539$	653	$141,\!138$
$\widehat{\delta_I}$	0.954	0.988	0.991	0.989	0.981		0.964	0.834	0.630	0.710	0.784
$\sigma(\widehat{\delta_I})^a$	0.001	0.003	0.007	0.009	0.003		0.001	0.035	0.148	0.384	0.103

<sup>&</sup>lt;sup>a</sup>: We calculate an approximate standard error based on linearization, recognizing that both  $\hat{\kappa}$  and  $\hat{\gamma}_1$  are subject to estimation error.

Table 7: Lower Bound Opportunity Cost of Non-Smart Routers

This table reports estimates of the monthly opportunity cost for NSC-only traders based on generalized trade-throughs. We screen all market orders executed on NSC and, ex-post, reallocate (part of) the order to EuroSETS to optimize execution. Panel A reports the total savings (i.e. opportunity cost) if we disregard execution fee in the decision to reallocate. It reports both gross savings and savings net of execution fee. Panel B is similar to Panel A, but reports the total savings if we reallocate optimally net of execution fee. Panel C reports the difference between Panel A and B. Post-entry 1 runs from August 2 through August 30, 2004 and post-entry 2 from January 3 through January 31, 2005.

		Po	$\operatorname{st-Entr}$	y 1			Pos	${ m st-Entr}$	y 2	
	$\overline{Q1}$	Q2	Q3 (	Q4	All	$\overline{\mathrm{Q}1}$	Q2 (	Q3 (	Q4	All
Panel A: Order Allocation Disr	regardin	g Fees								
# Sub-Optimal Orders	16,374	1,172	769	185	$18,\!500$	10,691	657	514	212	$12,\!074$
Fraction of Total	0.07	0.01	0.01	0.01	0.04	0.04	0.01	0.01	0.01	0.02
Monthly Savings <sup>a</sup> ( $\leq 1,000$ )	313	39	24	8	385	172	17	11	9	208
Monthly Net Savings <sup>a</sup> ( $\in 1,000$ )	271	37	23	8	338	143	16	10	9	177
Panel B: Order Allocation Net	of Fees									
# Sub-Optimal Orders	15,938	1,140	751	179	18,008	$10,\!262$	632	477	202	$11,\!573$
Fraction of Total	0.07	0.01	0.01	0.01	0.04	0.04	0.00	0.01	0.00	0.02
Monthly Savings <sup>a</sup> ( $\in 1,000$ )	313	39	24	8	385	171	17	11	9	208
Monthly Net Savings <sup>a</sup> ( $\in 1,000$ )	271	37	23	8	339	143	16	10	9	177
Panel C: Difference between Pa	$nel\ A\ a$	and Pa	nel B $I$	Estima	tes					
Diff. Monthly Savings (%)	-0.03	-0.03	-0.02	-0.05	-0.03	-0.06	-0.06	-0.16	-0.06	-0.06
Diff. Monthly Net Savings (%)	0.04	0.03	0.02	0.02	0.04	0.07	0.04	0.19	0.14	0.08

<sup>&</sup>lt;sup>a</sup>: For the TAQ data, we miss three days in post-entry 1 and one day in post-entry 2. We use appropriate scaling factors to estimate the amount for the full 21 trading days.

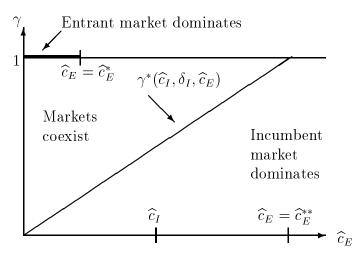


Figure 1: Potential Equilibria—One Market Dominates or Markets Coexist This figure illustrates for what values of  $(\hat{c}_E, \gamma)$  both markets coexist. The alternative is that one market captures all order flow and, by our definition, dominates.  $\hat{c}_E$  is the passive order submission fee and  $\gamma$  is the proportion of smart routers.

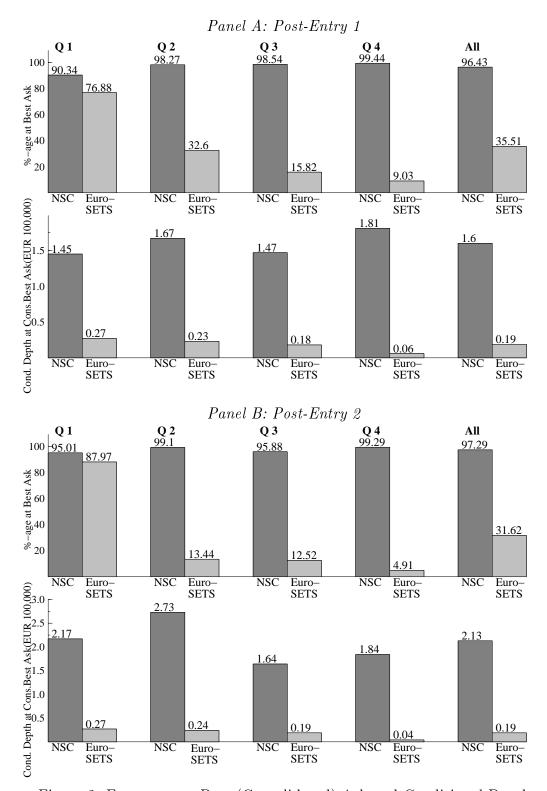


Figure 2: Frequency at Best (Consolidated) Ask and Conditional Depth This figure illustrates how often a market is at the best ask in the consolidated market and, if that is the case, how much depth (in €100,000) it offers at that quote. These statistics are based on five-minute order book snapshots and are, therefore, time-weighted averages.