Panel Intensity Models with Latent Factors*

Ingmar Nolte[†]
University of Konstanz, CoFE

Valeri Voev[‡]
University of Konstanz, CoFE

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[†]Department of Economics, Box D124, University of Konstanz, 78457 Konstanz, Germany. Phone +49-7531-88-3753, Fax -4450, email: Ingmar.Nolte@uni-konstanz.de.

[‡]Department of Economics, Box D124, University of Konstanz, 78457 Konstanz, Germany. Phone +49-7531-88-2556, Fax -4450, email: Valeri.Voev@uni-konstanz.de.

Abstract

We develop a survival model for panel data based on an intensity approach. The model is extended by a time varying latent factor, which captures the influence of unobserved time effects and allows for correlation across individuals. The model extends the stochastic conditional intensity model of Bauwens & Hautsch (2005) to panel duration data.

We show how to estimate the model by a simulated maximum likelihood technique adopting the efficient importance sampling approach of Richard & Zhang (2005).

The proposed approach can be used to characterize the trading behavior of investors in multiple assets over time.

JEL classification: $G10, F31, C32$

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1 Introduction

Trading behavior of investors are influenced by a broad set of decision variables. If we were able to observe this complete information set, we could fully characterize the time varying correlation structure across individuals based on this observable information. Individual investment opportunity sets as well as unobservable macroeconomic factors are just two examples of information which is not observed by the econometrician. Time varying latent factors can be used to approximate this unobservable information and improve the characterization of the correlation structure across individuals.

In this paper we introduce latent factors to panel intensity models, which are used to investigate the stochastics of trading decisions of investors for multiple assets over time. This framework allows for a rigorous exploration of financial decision making theories such as rational expectations and behavioral finance theories.

The proposed model can be viewed on the one hand as an extension of the stochastic conditional intensity (SCI) model of Bauwens & Hautsch (2005) to panel data and on the other hand as an augmentation of the class of panel survival models by a latent factor. The intensity based specification is chosen to allow for an intuitive incorporation of time varying covariates. The latent factor is assumed to evolve on an arrival process resulting from aggregation of individual arrival processes. We use a simulated maximum likelihood (SML) technique to estimate the model. Due to the complexity of the model we develop an adjustment of the efficient importance sampling method of Richard & Zhang (2005).

The model should serve to analyze the trading behavior of retail investors in the foreign exchange market with the help of an trading activity data-set of OANDA FXTrade, which allows to trace every action of around 5000 investors in up to 30 currency pairs over the period from 1^{st} October 2003 to 15^{th} May 2004.

In this preliminary version of the paper we provide a theoretical description of the model (Section 2) and detail the SML estimation procedure (Section 3). Section 4 concludes.

2 Panel Intensity Model

Let $t \in [0, T]$ denote the physical calendar time, let $n = 1, \ldots, N$ denote the n^{th} investor and let $k = 1, \ldots, K$ denote the k^{th} currency (ccy) pair in which an investor can trade. The i^{th} action of the n^{th} investor in the k^{th} ccy pair is denoted by $i = 1, \ldots, I^{k,n}$ and the corresponding arrival time[1] is denoted by $t_i^{k,n}$. For all n and all k the sequences $\{t_i^{k,n} | 0 \leq t_{i-1}^{k,n} \leq t_i^{k,n} \leq T; i = 1, \ldots, I^{k,n}\}$ represent point processes with corresponding right-continuous counting processes $N^{k,n}(t) = N^{k,n}([0,t]) = \sum_{i=1}^{I^{k,n}} \mathbb{1}_{\{t_i^{k,n} \leq t\}}$ which count the number of actions in the time interval $[0, t]$. The corresponding left-continuous counting process is denoted by $\breve{N}^{k,n}(t) = N^{k,n}([0,t)) = \sum_{i=1}^{I^{k,n}} \mathbb{1}_{\{t_i^{k,n} < t\}}$. Let $\{\Omega, \mathfrak{F}, \mathfrak{F}_t, \mathcal{P}\}$ denote the associated joint probability space, where the filtrations of the individual processes are denoted by $\mathfrak{F}_t^{k,n} \subset \mathfrak{F}_t$. We assume that each individual point process is orderly (simple), i.e.

$$\mathrm{P}(N^{k,n}(t + \delta) - N^{k,n}(t) > 0 | \mathfrak{F}_t^{k,n}) = o(\delta), \tag{1}$$

where $o(\cdot)$ denotes the little Landau symbol, which ensures that there are no simultaneous arrivals and it implies (almost surely) that $t_{i-1}^{k,n} < t_i^{k,n}$ for $i = 1, \ldots, I^{k,n}$. The inter-event duration between two consecutive actions is denoted by $\tau_i^{k,n} = t_i^{k,n} - t_{i-1}^{k,n}$. By $u^{k,n}(t) = t - t_{\breve{N}^{k,n}(t)}^{k,n}$ we denote the corresponding backward recurrence time at t. For each investor and for each ccy pair the arrival times $\{t_i^{k,n} | i = 1, \ldots, I^{k,n}\}$ constitute a pooled process, induced by S sub-processes. The corresponding arrival times of the s^{th} sub-process is denoted by $t_i^{s,k,n}$ with $i = 1, \ldots, I^{s,k,n}$. Since the pooled process is orderly the sub-processes are orderly as well. With $N^{s,k,n}(t) = \sum_{i=1}^{I^{s,k,n}} \mathbb{1}_{\{t_i^{s,k,n} \leq t\}}$ being the corresponding counting functions we get that $N^{k,n}(t) = \sum_{s=1}^{S} N^{s,k,n}(t)$. In our application we observe $S = 2$ sub-processes which are:

- $s = 1$: The process which is related to an increase in a given ccy pair exposure, i.e. the process which characterizes whether a position is (further) opened;
- $s = 2$: The process which is related to a decrease in a given ccy pair exposure, i.e. the process which characterizes whether a position is (partly) closed.

[1] By action we understand any event that changes the investor's portfolio value. Thus it can be initiated by the investor at that particular time or be a consequence of an earlier activity of the investor, e.g. an executed limit order.

The likelihood function of the complete model without a latent factor (assuming independence across investors and currency pairs) is given by

$$\mathcal{L}(W;\theta) = \prod_{n=1}^{N}\prod_{k=1}^{K}\left(\prod_{i=1}^{I^{k,n}} f^{k,n}(\tau_i^{k,n} \mid \mathfrak{F}^{-}_{t_i^{k,n}})\right)^{d_n^k}, \tag{2}$$

where $f^{k,n}(\tau_i^{k,n} \mid \mathfrak{F}^{-}_{t_i^{k,n}})$ is the conditional duration density function. With $\mathfrak{F}^{-}_{t_i^{k,n}}$ we denote the filtration, which consists of all information up to but excluding time $t_i^{k,n}$. W denotes the generic symbol for all relevant data and θ is the generic symbol for all relevant parameters used in the estimation. By d_n^k we denote the dummy which takes on the value of one if the n^{th} investor is active in currency pair k at least once, and zero otherwise.
We can write the conditional probability of the duration $\tau_i^{k,n}$ between two arbitrary consecutive actions as the conditional probability that no process has generated an arrival over the period $[t_{i-1}^{k,n}, t_i^{k,n})$ times the instantaneous probability for arrival in the next instant $t_i^{k,n}$, which is formally given by

$$\mathrm{P}\left(\tau_i^{k,n}\middle|\mathfrak{F}^{-}_{t_i^{k,n}}\right) = \prod_{s=1}^{S}\bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n}\middle|\mathfrak{F}^{-}_{t_i^{k,n}}\right)\left(\theta^{s,k,n}\left(t_i^{k,n}\middle|\mathfrak{F}^{-}_{t_i^{k,n}}\right)\right)^{d_i^{s,k,n}}, \tag{3}$$

where $d_i^{s,k,n}$ is a dummy, which takes on the value of one whenever the corresponding duration ends with an arrival of type s, and zero otherwise. $\bar{F}^{s,k,n}$ is given by

$$\bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n}\middle|\mathfrak{F}^{-}_{t_i^{k,n}}\right) = \mathrm{P}\left(t^{s,k,n}_{N^{s,k,n}(t_{i-1}^{k,n})+1} \notin [t_{i-1}^{k,n}, t_i^{k,n}), t^{s,k,n}_{N^{s,k,n}(t_{i-1}^{k,n})+1} = t_i^{k,n}\middle|\mathfrak{F}^{-}_{t_i^{k,n}}\right) \tag{4}$$

and denotes the "survivor" function of the s-type process and

$$\theta^{s,k,n}\left(t_i^{k,n}\middle|\mathfrak{F}^{-}_{t_i^{k,n}}\right) = \lim_{h\to 0}\frac{\mathrm{P}\left(t_i^{k,n} \le t^{s,k,n}_{N^{s,k,n}(t_{i-1}^{k,n})+1} < t_i^{k,n}+h\middle|t^{s,k,n}_{N^{s,k,n}(t_{i-1}^{k,n})+1} \notin [t_{i-1}^{k,n}, t_i^{k,n}), \mathfrak{F}^{-}_{t_i^{k,n}}\right)}{h} \tag{5}$$

denotes the corresponding intensity of type s. It follows that

$$\begin{aligned}\bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n}\middle|\mathfrak{F}^{-}_{t_i^{k,n}}\right) &= \exp\left(-\int_{t_{i-1}^{k,n}}^{t_i^{k,n}} \theta^{s,k,n}(u \mid \mathfrak{F}^{-}_u)du\right)\\ &= \exp\left(-\Theta^{s,k,n}(t_{i-1}^{k,n}, t_i^{k,n} \mid \mathfrak{F}^{-}_{t_i^{k,n}})\right),\end{aligned}$$

where $\Theta^{s,k,n}(t_{i-1}^{k,n}, t_i^{k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}})$ denotes the s-type integrated intensity between $t_{i-1}^{k,n}$ and $t_i^{k,n}$. Therefore, the likelihood function of the model without a latent factor in equation (2) can be rewritten as

$$\mathcal{L}(W;\theta) = \prod_{n=1}^{N}\prod_{k=1}^{K}\prod_{i=1}^{I^{k,n}}\prod_{s=1}^{S} \bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}\right)\left(\theta^{s,k,n}\left(t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}\right)\right)^{d_i^{s,k,n}}. \tag{6}$$

Since we believe that investors' behavior is influenced by unobservable factors, like an unobservable time effect, we introduce a latent factor denoted by λ_i. To model a dynamic behavior of the latent factor, we need to introduce a time scale over which the latent factor evolves. Therefore, we define the ordered pooled point process as the sequence of arrival times $t_i, i = 1, \ldots, I$ for all actions of all investors in all currency pairs, where simultaneous arrivals at the same time are treated as one arrival only, i.e.

$$\begin{aligned}\{t_i | t_{i-1} < t_i\} = \quad & \bigcup_n \left\{ \bigcup_k \{t_i^{k,n} | \, t_{i-1}^{k,n} < t_i^{k,n}\} \setminus \bigcap_k \{t_i^{k,n} | \, t_{i-1}^{k,n} < t_i^{k,n}\} \right\} \setminus \\ & \bigcap_n \left\{ \bigcup_k \{t_i^{k,n} | \, t_{i-1}^{k,n} < t_i^{k,n}\} \setminus \bigcap_k \{t_i^{k,n} | \, t_{i-1}^{k,n} < t_i^{k,n}\} \right\}.\end{aligned}$$

The corresponding counting processes are denoted by $N(t) = \sum_{i=1}^{I} \mathbb{1}_{\{t_i \leq t\}}$ and $\breve{N}(t) = \sum_{i=1}^{I} \mathbb{1}_{\{t_i < t\}}$. Thus, at $t \in \{t_i\}$ we have $N(t) = \breve{N}(t) + 1$, whereas for $t \notin \{t_i\}$ it holds that $N(t) = \breve{N}(t)$. We introduce this pooled process since the dynamics of the latent factor will be defined on it. In particular we assume that the duration $\tau^{k,n}_{N^{k,n}(t)}$ depends on the latent factor, i.e. we assume that $\tau^{k,n}_{N^{k,n}(t)} = \tau^{k,n}_{N^{k,n}(t)}(\lambda_{\breve{N}(t)+1})$ at $t \in \bigcup_n \bigcup_k \{t_i^{k,n}\}$ is a function of the latent factor. Note, that this definition ensures that at every time t where an action occurs there is a corresponding value of the latent factor. Since the latent factor is unobservable and stochastic the likelihood is given by

$$\mathcal{L}(W;\theta) = \int_{\mathbb{R}^I} \prod_{n=1}^{N}\prod_{k=1}^{K}\prod_{i=1}^{I^{k,n}} f^{k,n}(\tau_i^{k,n}, \lambda_{\breve{N}(t_i^{k,n})+1} \mid \mathfrak{F}^-_{t_i^{k,n}}) d\Lambda, \tag{7}$$

where $\Lambda = (\lambda_1, \ldots, \lambda_I)'$ and the integral is taken over $\mathbb{R}^I$, and where $f^{k,n}(\tau_i^{k,n}, \lambda_{\breve{N}(t_i^{k,n})+1} \mid \mathfrak{F}^-_{t_i^{k,n}})$ is the joint conditional density of the duration $\tau_i^{k,n}$ and its corresponding latent factor $\lambda_{\breve{N}(t_i^{k,n})+1}$. The likelihood can then be factorized as the product of the density conditional on the latent factor times the conditional density of the latent factor as

$$\mathcal{L}(W;\theta) = \int_{\mathbb{R}^I} \prod_{n=1}^{N}\prod_{k=1}^{K}\prod_{i=1}^{I^{k,n}}\prod_{s=1}^{S} \bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right)$$
$$\left(\theta^{s,k,n}\left(t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right)\right)^{d_i^{s,k,n}} \rho(\lambda_{\breve{N}(t_i^{k,n})+1}|\mathfrak{F}^-_{t_i^{k,n}})d\Lambda, \quad (8)$$

where $\rho(\lambda_{\breve{N}(t_i^{k,n})+1}|\mathfrak{F}^-_{t_i^{k,n}})$ is the conditional density of the latent factor and the exact specification of the intensities and the corresponding integrated intensities is presented below. The model in (8) is formulated in terms of $t_i^{k,n}$, which is the pooled (orderly) point process of the n^{th} investor in the k^{th} currency pair. This is unfavorable for the simulated maximum likelihood (SML) estimation, which is based on the efficient important sampling (EIS) algorithm of Richard & Zhang (2005). Therefore we reformulate the model in terms of t_i, the overall pooled process, on which the latent factor is defined.

Since the pooled process may not be orderly there may be several pairs (k, n) associated with the arrival time t_i. We denote the set of such pairs by $\mathcal{C}_i = \{(k, n)|t_i = t^{k,n}_{N^{k,n}(t_i)}\}$. The likelihood in (8) can then be rewritten as

$$\mathcal{L}(W;\theta) = \int_{\mathbb{R}^I} \prod_{i=1}^{I}\prod_{\mathcal{C}_i}\prod_{s=1}^{S} \bar{F}^{s,k,n}\left(t^{k,n}_{N^{k,n}(t_i)-1}, t^{k,n}_{N^{k,n}(t_i)}\middle|\mathfrak{F}^-_{t_i}, \lambda_i\right)$$
$$\left(\theta^{s,k,n}\left(t^{k,n}_{N^{k,n}(t_i)}\middle|\mathfrak{F}^-_{t_i}, \lambda_i\right)\right)^{d^{s,k,n}_{N^{k,n}(t_i)}} \rho(\lambda_i|\mathfrak{F}^-_{t_i})d\Lambda. \quad (9)$$

As suggested by the model presentation above there are several ways to model the likelihood function. One can either specify the likelihood function (7) for the durations of the pooled process $t_i^{k,n}$ directly or one can specify the likelihood function (8) based on the intensities of the s sub-processes $t_i^{s,k,n}$ which generate the pooled process $t_i^{k,n}$. Although in different ways, both approaches ultimately allow to make inference about the durations $\tau_i^{k,n}$ of the pooled process.

An attractive feature of the intensity based modelling is that it accounts for changes in the values of time varying covariates during a duration spell in a very intuitive way since it is set up in a continuous time. The duration based approach, which is a discrete time model can also account for time varying covariates (e.g. Lunde & Timmermann (2005)), but then the likelihood function has to be additionally adjusted (effectively this again amounts to adjusting the intensity to reflect the changes in the values of the covariates). Furthermore, the intensity based approach allows to characterize the dynamic behavior among the s sub-processes, which is a source of additional information, whereas the duration approach considers the pooled process solely. A strategy to model the duration based likelihood (7) is to adopt the stochastic conditional duration (SCD) approach of Bauwens & Veredas (2004), whereas likelihood (8) can be modelled by augmenting the stochastic conditional intensity (SCI) model of Bauwens & Hautsch (2005). We rely on the latter strategy and parameterize $\theta^{s,k,n}(t|\mathfrak{F}_t^-, \lambda_{\breve{N}(t)+1})$ generally in the following way:

$$\theta^{s,k,n}(t|\mathfrak{F}_t^-, \lambda_{\breve{N}(t)+1}) = \left(b^{s,k,n}(t) S^{s,k,n}(t) \Psi^{s,k,n}(t|\mathfrak{F}_t^-)(\lambda_{\breve{N}(t)+1})^{\delta^{s,k,n}}\right) D^{s,k,n}(t). \quad (10)$$

Thereby $b^{s,k,n}(t)$ denotes a (possibly investor, currency pair or state dependent) baseline hazard rate, $S^{s,k,n}(t)$ a deterministic seasonality function, $\Psi^{s,k,n}(t|\mathfrak{F}_t^-)$ the intensity component capturing the dynamic information processing and $\delta^{s,k,n}$ is a parameter which controls for different influences of the latent component on the corresponding intensities. In our application we need to take into account that after an action which sets the exposure in a given ccy pair to zero, i.e. closes the position completely, there is no possibility for a subsequent close. Hence, the intensity $\theta^{2,k,n}(t \mid \mathfrak{F}_t^-, \lambda_{\breve{N}(t)+1})$ is zero in this case. We model this through the variable

$$D^{s,k,n}(t) = \begin{cases} 1, & \text{if} \quad s = 1 \\ 1 - d_{cc}^{k,n}(t), & \text{if} \quad s = 2, \end{cases} \quad (11)$$

where $d_{cc}^{k,n}(t)$ denotes the dummy which takes on the value one, if the previous arrival time is associated with a complete close of the position in the given currency pair k for investor n, and zero otherwise. In the following we will parameterize the different components in a parsimonious way:

Baseline Hazard

We assume that there are different baseline hazard rates for the different states, but that they are identical across currency pairs and investors, i.e. we assume that

$$b^{s,k,n}(t) = b^s(t) \quad \text{for } k = 1, \dots, K \text{ and } n = 1, \dots, N.$$

In our application we use a multivariate Weibull specification of the following form:

$$b^s(t) = \exp(\omega_s) \prod_{r=1}^{S} u^{r,k,n}(t)^{\alpha_r^s - 1} \quad \text{for } s = 1, \dots, S$$

Diurnal Seasonality

We assume that

$$S^{s,k,n}(t) = S(t) \qquad \text{for } k = 1, \ldots, K \text{ and } n = 1, \ldots, N.$$

Let $it \in [0, 24)$ denote the calendar time t projected onto time of day, where 24 corresponds to 24 hours per day. We assume that the diurnal seasonality can be approximated well enough by an exponential linear spline function with hourly knots, which is constructed in the following way. Let $w = (w_0, \ldots, w_{23})'$ denote the coefficient vector with a corresponding permutated coefficient vector $\tilde{w} = (w_1, \ldots, w_{23}, w_0)'$. Furthermore, let $\varsigma = (0, \ldots, 23)'$ and $\iota = (1, \ldots, 1)'$ denote the counting and unit vector of dimension 24×1, respectively. The vector of indicator functions is denoted by $\Upsilon = (\mathbb{1}_{\{it\in[0,1)\}}, \ldots, \mathbb{1}_{\{it\in[22,23)\}}, \mathbb{1}_{\{it\in[23,0)\}})'$. $S(t)$ is then given by

$$S(t) = \exp\left((\text{Diag}[\tilde{w}(it \cdot \iota - \varsigma)'] + \text{Diag}[w(\iota - (it \cdot \iota - \varsigma))'])' \, \Upsilon\right),$$

where w_0 is set to zero for identification purposes.

Dynamics and Explanatory Variables

The dynamic structure and the influence of the explanatory variables is modelled with $\Psi^{s,k,n}(t|\mathfrak{F}_t^-)$ in the same fashion as suggested by Russell (1999). Let $z_j^{s,k,n}$ denote the vector of all (time-varying) possibly investor, currency pair and state dependent covariates, where at least one covariate is updated at time $\tilde{t}_j^{s,k,n}$ with $j = 1, \ldots J^{s,k,n}$. $\breve{M}^{s,k,n}(t) = \sum_{j=1}^{J^{s,k,n}} \mathbb{1}_{\{\tilde{t}_j^{s,k,n} < t\}}$ is the corresponding left continuous counting function of the update times $\tilde{t}_j^{s,k,n}$. Furthermore, let $\{\check{t}_h^{s,k,n}\}$ denote the pooled process of the pooled action process $\{t_i\}$ and the covariate process $\{\tilde{t}_j^{s,k,n}\}$, with $H^{s,k,n}(t) = \sum_{h=1}^{H^{s,k,n}} \mathbb{1}_{\{\check{t}_h^{s,k,n} \leq t\}}$ denoting the corresponding right continuous counting function. We assume that

$$\Psi^{s,k,n}(t|\mathfrak{F}_t^-) = \exp\left(\tilde{\Psi}^{s,k,n}_{\breve{N}^{k,n}(t)+1} + \left(z^{s,k,n}_{\breve{M}^{s,k,n}(t)}\right)' \gamma^{s,k,n}\right).$$

Note, that $\tilde{\Psi}_{\cdot}^{s,k,n}$ is indexed by $\breve{N}^{k,n}(t) + 1$, which ensures that $\tilde{\Psi}_{\cdot}^{s,k,n}$ is updated with the value of $\tilde{\Psi}_i^{s,k,n}$ directly after but excluding $t_{i-1}^{k,n}$ and stays constant until and including $t_i^{k,n}$. The coefficient vector is denoted by $\gamma^{s,k,n}$. The vector $\tilde{\Psi}_i^{k,n} = (\tilde{\Psi}_i^{1,k,n}, \ldots, \tilde{\Psi}_i^{S,k,n})'$ is parametrized multivariately as

$$\tilde{\Psi}_i^{k,n} = \sum_{s=1}^{S} \left(A^{s,k,n} \varepsilon_{i-1}^{k,n} + B^{k,n} \tilde{\Psi}_{i-1}^{k,n}\right) d_{i-1}^{s,k,n},$$

where $A^{s,k,n} = \{\alpha_j^{s,k,n}\}$ is an $S \times 1$ parameter vector and $B^{k,n} = \{\beta_{ij}^{k,n}\}$ is an $S \times S$ parameter matrix. The innovation term $\varepsilon_i^{k,n}$ is given by

$$\varepsilon_i^{k,n} = \sum_{s=1}^{S} d_i^{s,k,n} \varepsilon_i^{s,k,n},$$

where

$$\varepsilon_i^{s,k,n} = 1 - \Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}}, \lambda_{\breve{N}(t_i^{s,k,n})+1}\right) \tag{12}$$

or

$$\varepsilon_i^{s,k,n} = -0.5772 - \ln \Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}}, \lambda_{\breve{N}(t_i^{s,k,n})+1}\right), \tag{13}$$

where the integrated hazard rate is computed as

$$\Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n}, \mid \mathfrak{F}^-_{t_i^{s,k,n}} \lambda_{\breve{N}(t_i^{s,k,n})+1}\right) = \sum_{h=H^{s,k,n}(t_{i-1}^{s,k,n})}^{H^{s,k,n}(t_i^{s,k,n})-1} \int_{\check{t}_h^{s,k,n}}^{\check{t}_{h+1}^{s,k,n}} \theta^{s,k,n}\left(u \middle| \mathfrak{F}^-_u, \lambda_{\breve{N}(u)+1}\right) du. \tag{14}$$

Note, that the hazard rate is integrated between $t_{i-1}^{s,k,n}$ and $t_i^{s,k,n}$ piecewise, where the pieces are determined either by an arrival time t_i, which includes the arrival times $t_i^{k,n}$ or by an arrival time of $\tilde{t}_j^{s,k,n}$. The innovation term in equation (12) is defined in that way, since $\Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}}, \lambda_{\breve{N}(t_i^{s,k,n})+1}\right) \sim$ i.i.d. Exp(1) and hence its mean value is 1. Equation (13) uses that $\ln \Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}}, \lambda_{\breve{N}(t_i^{s,k,n})+1}\right)$ follows an i.i.d. standard extreme value type I distribution with mean -0.5772.

The survivor function $\bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right)$ in equation (8) is given by

$$\bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right) = \exp\left(-\Theta^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \mid \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right)\right),$$

where the integrated intensity is obtained piecewise according to equation (14).

Latent Factor

We assume that the dynamics of the latent factor are defined on the time scale t_i. This means the latent factor potentially changes whenever there is an action of some investor in some currency pair. Since each hazard $\theta^{s,k,n}$ and each integrated hazard

$\Theta^{s,k,n}$ depends at every time t on the current (one discrete time step ahead) value of the latent factor we induce at every time t a contemporaneous correlation between all hazard rates $\theta^{s,k,n}$ through the latent factor. The amount of this possibly investor, currency pair or state dependent correlation is steered through the parameters $\delta^{s,k,n}$. The latent factor therefore can be interpreted as an unobservable time effect which affects the decisions (open, close) of all investors at every time t by influencing the intensities of the corresponding processes. There are many explanations which justify the existence of such an unobservable time effect in our model: i) (News) not modelled effects of (macroeconomic) news announcements, due to data limitations, ii) (Order Flow) buy or sell pressure from the interbank market, which we do not observe directly since we consider an internet trading platform or iii) (Herding) similar behavior of traders, due to similar interpretations of any kind of technical chart patterns, which are not modelled.

In our model we assume that the latent factor follows conditional on $\mathfrak{F}_{t_i}^-$ a lognormal distribution, i.e.

$$\ln \lambda_i | \mathfrak{F}_{t_i}^- \overset{i.i.d.}{\sim} \mathrm{N}(\mu_i, 1)$$

where the dynamics is modelled through an AR(1) process

$$\ln \lambda_i = a \ln \lambda_{i-1} + \nu_i \quad \text{for} \quad i = 1, \ldots, I,$$

with $\nu_i \overset{i.i.d.}{\sim} \mathrm{N}(0,1)$. Let l_i denote the log of latent factor at t_i, i.e

$$l_i \equiv \ln \lambda_i,$$

and let L_i denote the history of the log latent factor up to and including t_i, i.e.

$$L_i = \{l_j\}_{j=1}^{i}.$$

With this specification, the (log) latent factor depends only on its own past, so we denote its conditional distribution by $p(l_i|L_{i-1})$. From Equation (10) it follows that the influence of the log latent factor on the s type intensity is given by $\delta^{s,k,n} \ln \lambda_i$, which we can denote by $\lambda_i^{s,k,n}$. Then we have that

$$\lambda_i^{s,k,n} = a\lambda_{i-1}^{s,k,n} + \delta^{s,k,n}\nu_i \quad \text{for} \quad i = 1, \ldots, I.$$

Therefore the variance of ν_i is set to unity, so that the conditional variance of $\lambda_i^{s,k,n}$ is equal to ${\delta^{s,k,n}}^2$, which eases the interpretation of the parameter[2].

Figure 1 depicts the stylized representation of the structure of the panel intensity model with a latent factor.

[2] Note that this does not preclude that $\delta^{s,k,n}$ could be negative.

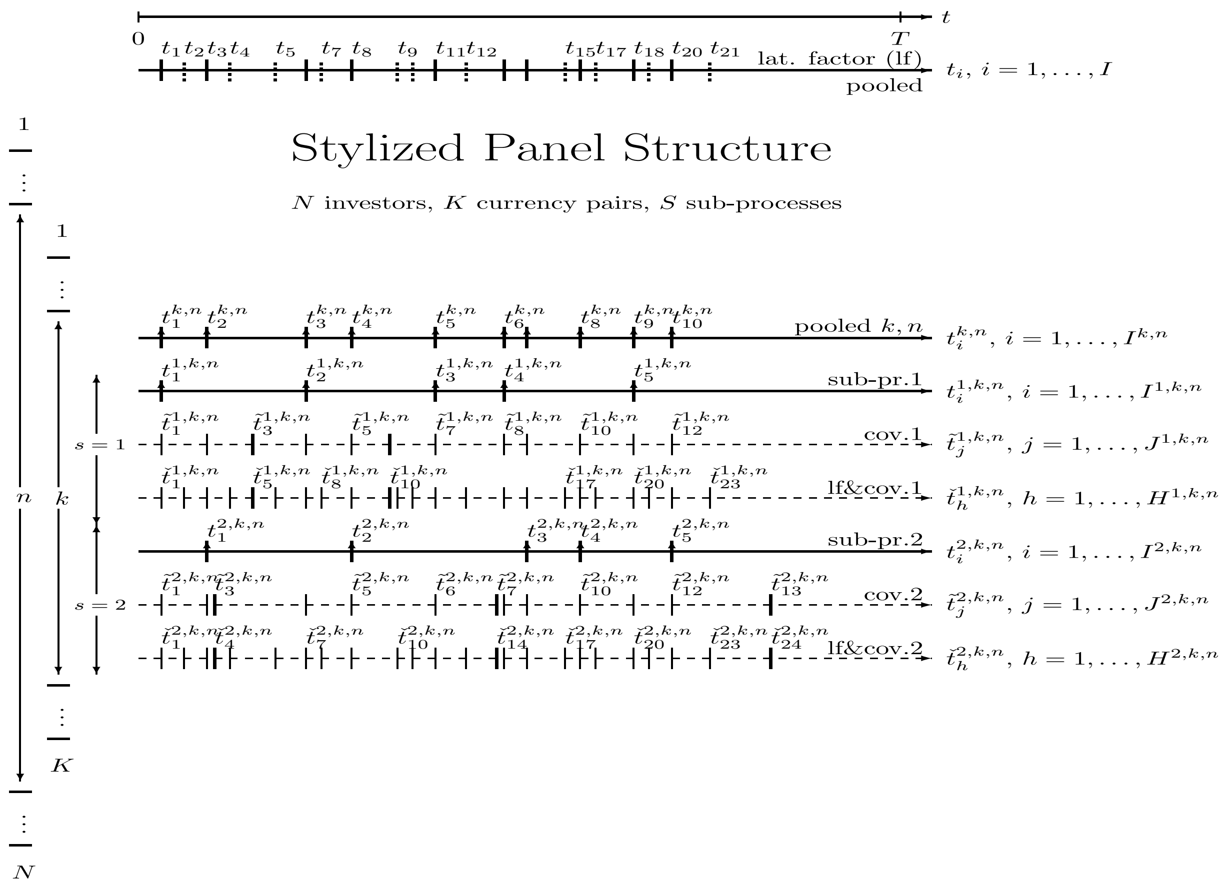

Figure 1: Stylized Model Structure. The figure represents for s=2 the time scales associated with the arrival times of the processes (sub-pr.1 and sub-pr.2), the times of the covariate processes (cov.1 an cov.2) as well as the pooled arrival processes $\check{t}_h^{s,k,n}$ and t_i.

3 Estimation of the Panel Intensity Model

We now consider the explicit form and the estimation of the parameters in the likelihood function. Let W denote the set of data matrices $W^{k,n}$ for $k = 1, \dots, K$ and $n = 1, \dots, N$ where the i^{th} row of $W^{k,n}$, $w_i^{k,n}$ consists of the following data:

$$w_i^{k,n} = (t_i^{k,n}, d_i^{1,k,n}, \dots, d_i^{S,k,n}), \qquad \text{with } i = 1, \dots, I^{k,n}.$$

With $W_i^{k,n}$ we denote the history of $w_i^{k,n}$ up to and including $t_i^{k,n}$, i.e.

$$W_i^{k,n} = \{w_j^{k,n}\}_{j=1}^{i}.$$

Furthermore, let $\breve{Z}_i^{k,n}$ for $k = 1, \dots, K$ and $n = 1, \dots, N$ denote the set which consists of the following time-varying covariate data:

$$\breve{Z}_i^{k,n} = \left\{ \{z_j^{1,k,n} | j = 1, \dots, \breve{M}^{1,k,n}(t_i^{k,n})\}, \dots, \{z_j^{S,k,n} | j = 1, \dots, \breve{M}^{S,k,n}(t_i^{k,n})\} \right\}.$$

Recall that the likelihood function of our model is given by

$$\begin{aligned}
\mathcal{L}(W;\theta) &= \int\limits_{\mathbb{R}^I} \prod_{n=1}^{N} \prod_{k=1}^{K} \prod_{i=1}^{I^{k,n}} \prod_{s=1}^{S} \bar{F}^{s,k,n} \left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1} \right) \\
&\quad \left(\theta^{s,k,n} \left(t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1} \right) \right)^{d_i^{s,k,n}} \rho(\lambda_{\breve{N}(t_i^{k,n})+1} | \mathfrak{F}^-_{t_i^{k,n}}) d\Lambda \\
&= \int\limits_{\mathbb{R}^{+I}} \prod_{n=1}^{N} \prod_{k=1}^{K} \prod_{i=1}^{I^{k,n}} \prod_{s=1}^{S} \bar{F}^{s,k,n} \left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \exp(l_{N(t_i^{k,n})}) \right) \\
&\quad \left(\theta^{s,k,n} \left(t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \exp(l_{N(t_i^{k,n})}) \right) \right)^{d_i^{s,k,n}} p(l_{N(t_i^{k,n})} | L_{N(t_i^{k,n})-1}) dL \\
&= \int\limits_{\mathbb{R}^{+I}} \prod_{i=1}^{I} \prod_{\mathcal{C}_i} \prod_{s=1}^{S} \bar{F}^{s,k,n} \left(t_{N^{k,n}(t_i)-1}^{k,n}, t_{N^{k,n}(t_i)}^{k,n} \middle| \mathfrak{F}^-_{t_i}, l_i \right) \\
&\quad \left(\theta^{s,k,n} \left(t_{N^{k,n}(t_i)}^{k,n} \middle| \mathfrak{F}^-_{t_i}, l_i \right) \right)^{d_{N^{k,n}(t_i)}^{s,k,n}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(l_i - \mu_i)^2}{2} \right) dL.
\end{aligned}$$

where $L = \ln \Lambda$ and the second equality follows from a change of the variable λ to l. Using the data sets defined above the likelihood function can be rewritten as

$$\begin{aligned}
\mathcal{L}(W;\theta) &= \int\limits_{\mathbb{R}^{+I}} \prod_{i=1}^{I} \prod_{\mathcal{C}_i} g^{k,n} \left(w_{N^{k,n}(t_i)}^{k,n} | W_{N^{k,n}(t_i)-1}^{k,n}, L_i, \breve{Z}_{N^{k,n}(t_i)}^{k,n} \right) p(l_i | L_{i-1}) dL \\
&= \int\limits_{\mathbb{R}^{+I}} \prod_{i=1}^{I} \prod_{\mathcal{C}_i} \varphi^{k,n} \left(w_{N^{k,n}(t_i)}^{k,n}, l_i | W_{N^{k,n}(t_i)-1}^{k,n}, L_{i-1}, \breve{Z}_{N^{k,n}(t_i)}^{k,n} \right) dL, \qquad (15)
\end{aligned}$$

where $g^{k,n}$ denotes the product of the survival and the intensity functions, p the density of the conditional normal distribution and $\varphi^{k,n}$ denotes the resulting corresponding joint conditional density. Since this likelihood involves the computation of an I-dimensional integral, we employ the Efficient Importance Sampling (EIS) technique of Liesenfeld & Richard (2003), which has been used for estimating stochastic conditional intensity models by Bauwens & Hautsch (2005). The EIS technique is based on simulation of the likelihood function (15) which can be rewritten as

$$\mathcal{L}(W;\theta) = \int_{\mathbb{R}^{+I}} \prod_{i=1}^{I} \prod_{\mathcal{C}_i} \frac{\varphi^{k,n}\left(w^{k,n}_{N^{k,n}(t_i)}, l_i | W^{k,n}_{N^{k,n}(t_i)-1}, L_{i-1}, \breve{Z}^{k,n}_{N^{k,n}(t_i)}\right)}{m(l_i|L_{i-1},\phi_i)}$$
$$\prod_{i=1}^{I} \prod_{\mathcal{C}_i} m(l_i|L_{i-1},\phi_i) dL,$$

where $m(l_i|L_{i-1},\phi_i)$ is a sequence of auxiliary importance samplers which are used to draw a path of the latent factor, given some additional parameters ϕ_i of the sampler. The estimation then proceeds by generating R trajectories of the latent factor and averaging over the draws

$$\mathcal{L}_R(W;\theta) = \frac{1}{R}\sum_{r=1}^{R} \frac{\prod_{i=1}^{I}\prod_{\mathcal{C}_i} \varphi^{k,n}\left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L^{(r)}_{i-1}, \breve{Z}^{k,n}_{N^{k,n}(t_i)}\right)}{\prod_{i=1}^{I}\prod_{\mathcal{C}_i} m(l_i^{(r)}|L^{(r)}_{i-1},\phi_i)}, \quad (16)$$

where the bracketed superscript r indicates the values of the corresponding variable or set for the r-th repetition. The idea of the EIS approach is to find the values of the parameters ϕ_i for $i = 1,\ldots,I$ such that the sampling variance of $\mathcal{L}_R(W;\theta)$ is minimized. For ease of illustration denote the numerator in equation (16) by $\varphi(W, L^{(r)}|\theta) = g(W|L^{(r)},\theta)p(L^{(r)})$, where the generic parameter vector θ appears now, and the denominator by $m(L^{(r)}|\phi)$. A more elaborate presentation can be found in Richard & Zhang (2005). The sampling variance of $\mathcal{L}_R(W;\theta)$ is given by

$$\begin{aligned} \mathrm{V}\left(\mathcal{L}_R(W;\theta)\right) &= \frac{\mathcal{L}(W;\theta)}{R}\frac{1}{\mathcal{L}(W;\theta)}\mathrm{V}\left(\frac{\varphi(W,L^{(r)}|\theta)}{m(L^{(r)}|\phi)}\right) \\ &= \frac{\mathcal{L}(W;\theta)}{R}\frac{1}{\mathcal{L}(W;\theta)}\int_{\mathbb{R}^{+I}}\left(\frac{\varphi(W,L|\theta)}{m(L|\phi)} - \mathcal{L}(W;\theta)\right)^2 m(L|\phi)dL (17) \end{aligned}$$

If we are able to choose ϕ such that $m(L|\phi) = \frac{\varphi(W,L|\theta)}{\mathcal{L}(W;\theta)}$ the sampling variance would be zero. Since this case is very unrealistic the aim is to find ϕ such that $m(L|\phi)$ is very close to $\varphi(W,L|\theta)$ under the restriction that $m(L|\phi)$ is analytically integrable.

Furthermore $m(L|\phi)$ can be decomposed into

$$m(L|\phi) = \frac{k(L, \phi)}{\chi(\phi)} \tag{18}$$

where $k(L, \phi)$ and $\chi(\phi) = \int_{\mathbb{R}^{+I}} k(L, \phi) dL$ can either be interpreted as joint and marginal density or as kernel and integration constant. Defining $d(L; \varphi, \theta)$ as

$$\begin{aligned} d(L; \phi, \theta) &= \ln\left(\frac{\varphi(W, L|\theta)}{\mathcal{L}(W; \theta) m(L|\phi)}\right) & (19) \\ &= \ln(\varphi(W, L|\theta)) - \ln(\mathcal{L}(W; \theta)) - \ln(m(L, \phi)) & (20) \\ &= \ln(\varphi(W, L|\theta)) - \ln(\mathcal{L}(W; \theta)) + \ln(\chi(\phi)) - \ln(k(L, \phi)) & (21) \end{aligned}$$

and defining $h(x)$ as

$$h(x) = \exp(\sqrt{x}) + \exp(-\sqrt{x}) - 2 \tag{22}$$

allows to rewrite equation (17) as

$$\mathrm{V}\left(\mathcal{L}_R(W; \theta)\right) = \frac{\mathcal{L}(W; \theta)}{R} \int\limits_{\mathbb{R}^{+I}} h\left(d(L; \phi, \theta)^2\right) \varphi(W, L|\theta) dL. \tag{23}$$

This equation defines a nonlinear Generalized Least Squares problem in ϕ, since h is monotone and convex on $\mathbb{R}^+$. The power series representation of h is given by

$$h(x) = \sum_{i=1}^{\infty} \frac{x^i}{(2i)!}. \tag{24}$$

Using the series expansion of order one for h, which is $h(x) = x$ equation (23) simplifies to

$$\mathrm{V}\left(\mathcal{L}_R(W; \theta)\right) = \frac{\mathcal{L}(W; \theta)}{R} \int\limits_{\mathbb{R}^{+I}} d(L; \phi, \theta)^2 \varphi(W, L|\theta) dL, \tag{25}$$

and the minimization problem becomes

$$\begin{aligned} \hat{\phi}(\theta) &= \underset{\phi}{\operatorname{argmin}} \int\limits_{\mathbb{R}^{+I}} d(L; \phi, \theta)^2 \varphi(W, L|\theta) dL \\ &= \underset{\phi}{\operatorname{argmin}} \int\limits_{\mathbb{R}^{+I}} d(L; \phi, \theta)^2 g(W|L, \theta) p(L) dL & (26) \end{aligned}$$

The integral in equation (26) is computed by its Monte Carlo proxy given by

$$\frac{1}{R} \sum_{r=1}^{R} d(L^{(r)}; \phi, \theta)^2 g(W|L^{(r)}, \theta)$$

where $L^{(r)}$ denote trajectories of length I sampled from the initial sampler p and $\hat{\phi}(\theta)$ is determined based on this approximation. Since the $L^{(r)}$ generate a high variance of g Richard & Zhang (2005) propose to drop the weight function g from the equation and compute $\hat{\phi}(\theta)$ on the basis of the unweighted problem. Therefore the minimization problem is given by

$$\hat{\phi}(\theta) = \underset{\phi}{\operatorname{argmin}} \sum_{r=1}^{R} d(L^{(r)}; \phi, \theta)^2. \tag{27}$$

Writing $d(L^{(r)}; \phi, \theta)$ explicitly yields

$$d(L^{(r)}; \phi, \theta) = \ln \left(\frac{\prod_{i=1}^{I} \prod_{\mathcal{C}_i} \varphi^{k,n} \left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L^{(r)}_{i-1}, \breve{Z}^{k,n}_{N^{k,n}(t_i)} \right)}{\prod_{i=1}^{I} \prod_{\mathcal{C}_i} m(l_i^{(r)} | L^{(r)}_{i-1}, \phi_i)} \right) - \ln \left(\mathcal{L}(W; \theta) \right) \tag{28}$$

Substituting

$$m(l_i^{(r)} | L^{(r)}_{i-1}, \phi_i) = \frac{k(L_i^{(r)}, \phi_i)}{\chi(\phi_i, L^{(r)}_{i-1})} \tag{29}$$

yields

$$\begin{aligned} d(L^{(r)}; \phi, \theta) &= \ln \left(\prod_{i=1}^{I} \prod_{\mathcal{C}_i} \varphi^{k,n} \left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L^{(r)}_{i-1}, \breve{Z}^{k,n}_{N^{k,n}(t_i)} \right) \chi \left(\phi_i, L^{(r)}_{i-1} \right) \right) \\ &\quad - \ln \left(\prod_{i=1}^{I} \prod_{\mathcal{C}_i} k(L_i^{(r)}, \phi_i) \right) - \ln \left(\mathcal{L}(W; \theta) \right) \\ &= \ln \left(\prod_{i=1}^{I} \prod_{\mathcal{C}_i} \varphi^{k,n} \left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L^{(r)}_{i-1}, \breve{Z}^{k,n}_{N^{k,n}(t_i)} \right) \chi \left(\phi_{i+1}, L^{(r)}_{i} \right) \right) \\ &\quad - \ln \left(\prod_{i=1}^{I} \prod_{\mathcal{C}_i} k(L_i^{(r)}, \phi_i) \right) - \ln \left(\mathcal{L}(W; \theta) \right) + \ln \left(\chi \left(\phi_1, L_0^{(r)} \right) \right) \end{aligned}$$

where $\chi \left(\phi_{I+1}, L_I^{(r)} \right) \equiv 1$. The thereto related minimization problem (27) can now be solved sequentially using a backward recursion from $I \to 1$ which yields $\phi = \{\phi_i | i = I, \ldots, 1\}$. The sequential problem consists then at each $i = 1, \ldots, I$ of approximating

$$\ln \left(\prod_{\mathcal{C}_i} \varphi^{k,n} \left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L^{(r)}_{i-1}, \breve{Z}^{k,n}_{N^{k,n}(t_i)} \right) \chi \left(\phi_{i+1}, L_i^{(r)} \right) \right)$$

by

$$\ln\left(k\left(L_i^{(r)},\phi_i\right)\right).$$

Thus $\hat{\phi}_i(\theta)$ is obtained through

$$\hat{\phi}_i(\theta) = \underset{\phi_i}{\operatorname{argmin}} \sum_{r=1}^{R}\Bigg(\ln\left(\prod_{\mathcal{C}_i}\varphi^{k,n}\left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)}|W^{k,n}_{N^{k,n}(t_i)-1}, L_{i-1}^{(r)}, \breve{Z}^{k,n}_{N^{k,n}(t_i)}\right)\chi\left(\phi_{i+1}, L_i^{(r)}\right)\right)$$
$$-\phi_{0,i} - \ln\left(k\left(L_i^{(r)},\phi_i\right)\right)\Bigg)^2 \tag{30}$$

The additional coefficients $\phi_{0,i}$ are scalars which capture corresponding components of $\ln\left(\mathcal{L}(W;\theta)\right)$, which are still unobservable. As Liesenfeld & Richard (2003) note, a sensible choice for the class of kernels for the auxiliary samplers m is a parametric extension to the direct samplers p given by

$$k\left(L_i,\phi_i\right) = p(l_i|L_{i-1})\zeta\left(l_i,\phi_i\right),$$

where ζ is itself a Gaussian density kernel given by

$$\zeta\left(l_i,\phi_i\right) = \exp\left(\phi_{1,i}l_i + \phi_{2,i}l_i^2\right).$$

Since a product of normal kernels is a normal kernel as well, we obtain for $k\left(L_i,\phi_i\right)$

$$\begin{aligned} k(L_i,\phi_i) &\propto \exp\left((\phi_{1,i}+\mu_i)l_i + \left(\phi_{2,i}-\frac{1}{2}\right)l_i^2 - \frac{1}{2}\mu_i^2\right) \\ &= \exp\left(-\frac{1}{2\pi_i^2}(l_i-\kappa_i)^2\right)\exp\left(\frac{\kappa_i^2}{2\pi_i^2}-\frac{1}{2}\mu_i^2\right), \end{aligned}$$

where

$$\pi_i^2 = (1-2\phi_{2,i})^{-1}, \qquad \text{and} \tag{31}$$
$$\kappa_i = (\phi_{1,i}+\mu_i)\pi_i^2. \tag{32}$$

It follows that

$$\chi(\phi_i, L_{i-1},) = \exp\left(\frac{\kappa_i^2}{2\pi_i^2} - \frac{\mu_i^2}{2}\right). \tag{33}$$

Under this choice of kernels class, $p(l_i|L_{i-1})$ cancels out in the minimization problem (30), which can then be rewritten as

$$\hat{\phi}_i(\theta) = \underset{\phi_i}{\operatorname{argmin}} \sum_{r=1}^{R}\Bigg(\ln\left(\prod_{\mathcal{C}_i} g^{k,n}\left(w^{k,n}_{N^{k,n}(t_i)}|W^{k,n}_{N^{k,n}(t_i)-1}, L_i^{(r)}, \breve{Z}^{k,n}_{N^{k,n}(t_i)}\right)\chi\left(\phi_{i+1}, L_i^{(r)}\right)\right)$$
$$-\phi_{0,i} - \ln\left(\zeta\left(l_i^{(r)},\phi_i\right)\right)\Bigg)^2. \tag{34}$$

The implementation of the sequential ML-EIS approach involves then the following steps:

STEP 1. Draw R trajectories $\{l_i^{(r)}\}_{i=1}^{I}$ from $\{\mathrm{N}(\mu_i, 1)\}_{i=1}^{I}$.
STEP 2. For each i with $i : I \to 1$ solve the R-dimensional OLS problem in (34).
STEP 3. Calculate the sequences $\{\pi_i^2\}_{i=1}^{I}$ and $\{\kappa_i\}_{i=1}^{I}$ from equations (31) and (32) and draw R trajectories of $\{l_i^{(r)}\}_{i=1}^{I}$ from $\{\mathrm{N}(\kappa_i, \pi_i^2)\}_{i=1}^{I}$ to compute the likelihood function given in (16).

4 Conclusion

In this preliminary version of the paper we provide a theoretical treatment of a panel intensity model augmented by a latent factor. The latent factor allows for a refined characterization of the time-varying correlation structure across individuals. The choice of the intensity specification enables us to capture the impact of time-varying covariates. We show how to adjust the efficient importance sampling algorithm of Richard & Zhang (2005) in order to estimate the model.

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