A presentation *from* BARCLAYS GLOBAL INVESTORS

The tale of ‘The Statistical Hare’ versus ‘The Deterministic Tortoise’

Allan Lane

Who wins the race in probability space – the Hare or the Tortoise?

Wall Street is currently awash with more intellectual fire power than at any other time during the last 100 years. But how do the large number of mathematicians and physicists actually contribute to the ongoing success of the world's largest Financial Institutions?

By studying the tale of The Statistical Hare versus The Deterministic Tortoise, this presentation will offer an insight into what life is like on the front line in the Financial Markets.

Who wins the race in probability space – the Hare or the Tortoise?

Presentation Summary

I shall study a couple of examples arising from the Buy & Sell side of the financial markets, bringing into focus a number issues that one has to face on the implementation front.

From the Capital Markets I shall the illustrate how a very wide class of exotic derivatives can be modelled by introducing the concept of a scripting language.

This implementation takes advantage of the realization that the option's conditional expectation can be written as the solution to a set of non-linear difference equations.

The second example is from the investment side and touches upon the quite distinct modelling problems one faces trying to beat the markets, an allure that has not diminished with time.

Who wins the race in probability space – the Hare or the Tortoise?

The Rationale

Why do we engage in the very time consuming business of building financial models?

It would be very tempting to think that the use of models preceded the business proposition. Needless to say, this is generally not the case.

Least we forget this simple truism, let's start by tackling the million dollar question.

Who wins the race in probability space – the Hare or the Tortoise?

What would you do with your million?

To motivate the role played by mathematical models in the world of finance, consider what you would do if you had $1m to invest

One possibility is to put the money in your savings account and earn interest on the deposit

The more adventurous amongst would consider this rather dull, and no doubt would instead consider investing in the stock markets

Who wins the race in probability space – the Hare or the Tortoise?

S&P Composite Return Index Bull and Bear Markets 1914-2002					
Market Top	**Index High**	**% Increase**	**Market Bottom**	**Index Low**	**% Decrease**
09/01/2000	2108.8	62%	07/23/2002?	1134.0	-46%
07/17/1998	1601.1	391%	10/08/1998	1299.4	-19%
07/16/1990	403.5	81%	10/17/1990	326.1	-19%
08/25/1987	333.0	305%	10/19/1987	223.5	-33%
11/28/1980	102.9	204%	08/12/1982	82.1	-20%
01/05/1973	61.5	89%	10/03/1974	33.8	-45%
11/29/1968	48.4	59%	05/26/1970	35.5	-33%
02/09/1966	37.8	98%	10/07/1966	30.5	-19%
12/12/1961	26.0	115%	06/26/1962	19.0	-27%
07/15/1957	15.1	517%	10/22/1957	12.1	-20%
05/29/1946	3.3	214%	05/17/1947	2.4	-25%
11/09/1938	1.6	67%	04/28/1942	1.0	-34%
03/10/1937	1.9	149%	03/31/1938	0.9	-52%
02/06/1934	1.1	121%	03/14/1935	0.8	-29%
09/07/1932	0.8	115%	02/27/1933	0.5	-39%
09/07/1929	2.3	657%	06/01/1932	0.4	-84%
11/03/1919	0.4	80%	08/24/1921	0.3	-26%
11/18/1916	0.4	77%	12/19/1917	0.2	-38%

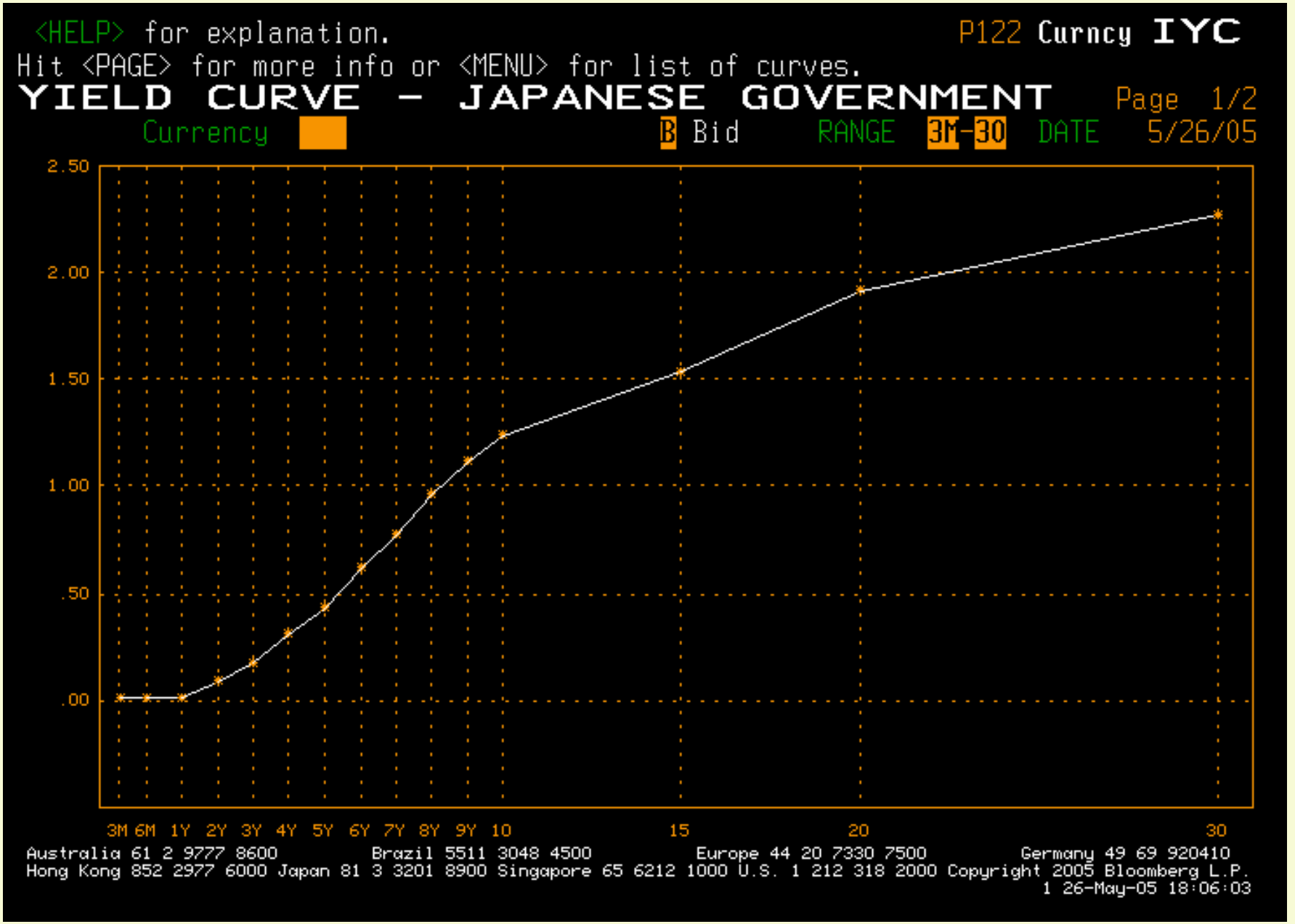
Be warned though, stock prices can go down as well as up!

Who wins the race in probability space – the Hare or the Tortoise?

With that sobering thought, maybe investing your savings in a bank isn't such a bad idea after all.

However, suppose your home town was Tokyo rather than Warwick. Would you stick to your initial decision and place the equivalent of 100m Yen in your bank account?

Before you make that decision you might want to consider what the Government Yield Curve looks like in Japan.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YIELD CURVE - JAPANESE GOVERNMENT
Page 1/2
Currency
B Bid
RANGE 3M-30
DATE 5/26/05
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Who wins the race in probability space – the Hare or the Tortoise?

We didn't actually discuss the length of time for which we wanted to plan our investment. Was it 1 year or maybe 5 years?

Unfortunately, on a time scale of 1 year, a Japanese savings account will yield less than one tenth of 1%

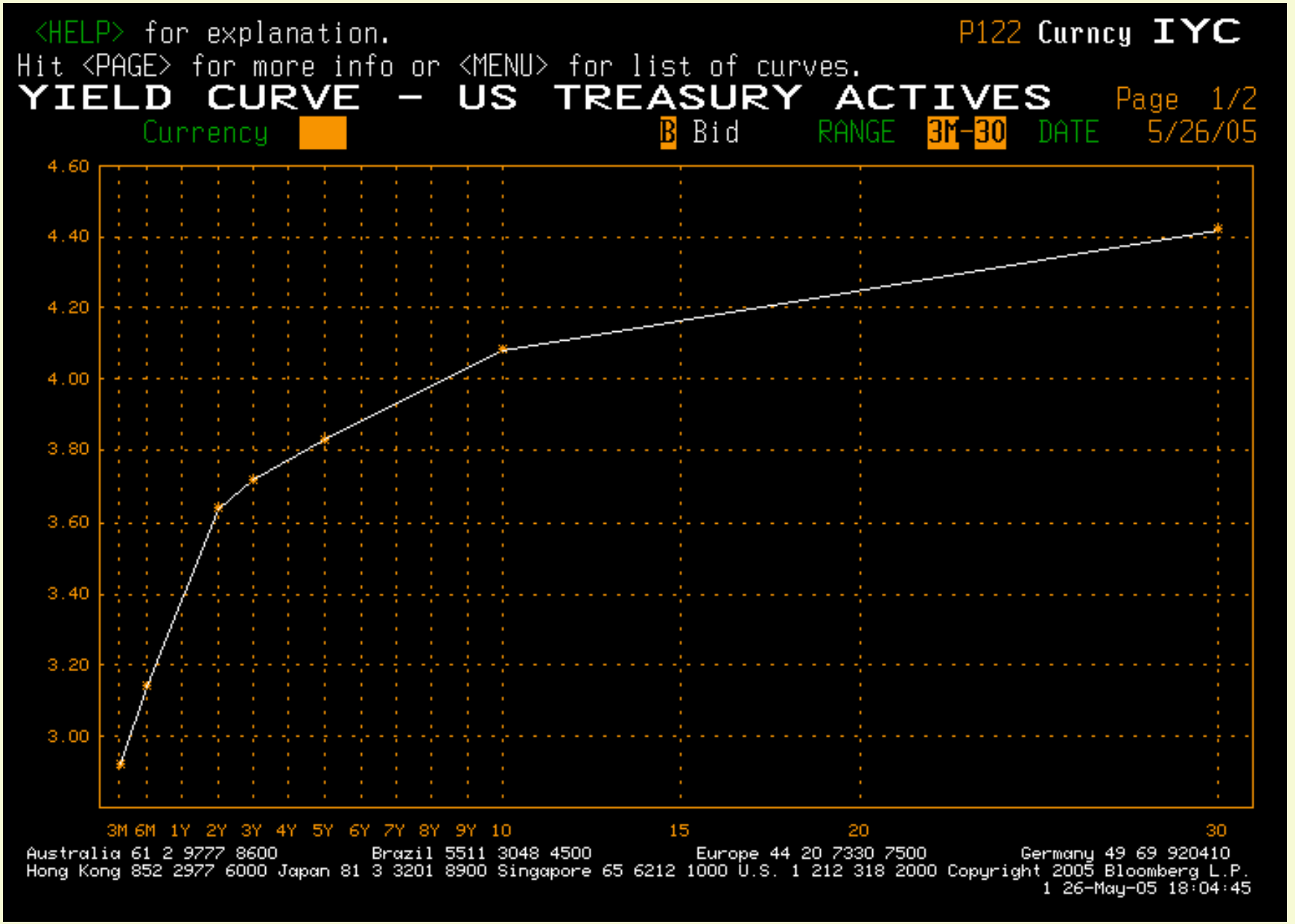
On a timescale of 5 years, your return barely reaches the grand height of less than one half of 1%

Who wins the race in probability space – the Hare or the Tortoise?

With almost no effort, we have already learnt that what often seems like a good idea at first, may not hold up under further scrutiny.

Continuing in this vein, let’s make the next logical step and focus on a country where the interest rates are a little bit higher than in Japan.

Presumably from the vantage point of Tokyo, the rates in other countries offer a higher return than those offered domestically.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YIELD CURVE - US TREASURY ACTIVES
Page 1/2
Currency
B Bid
RANGE 3M-30
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4.60
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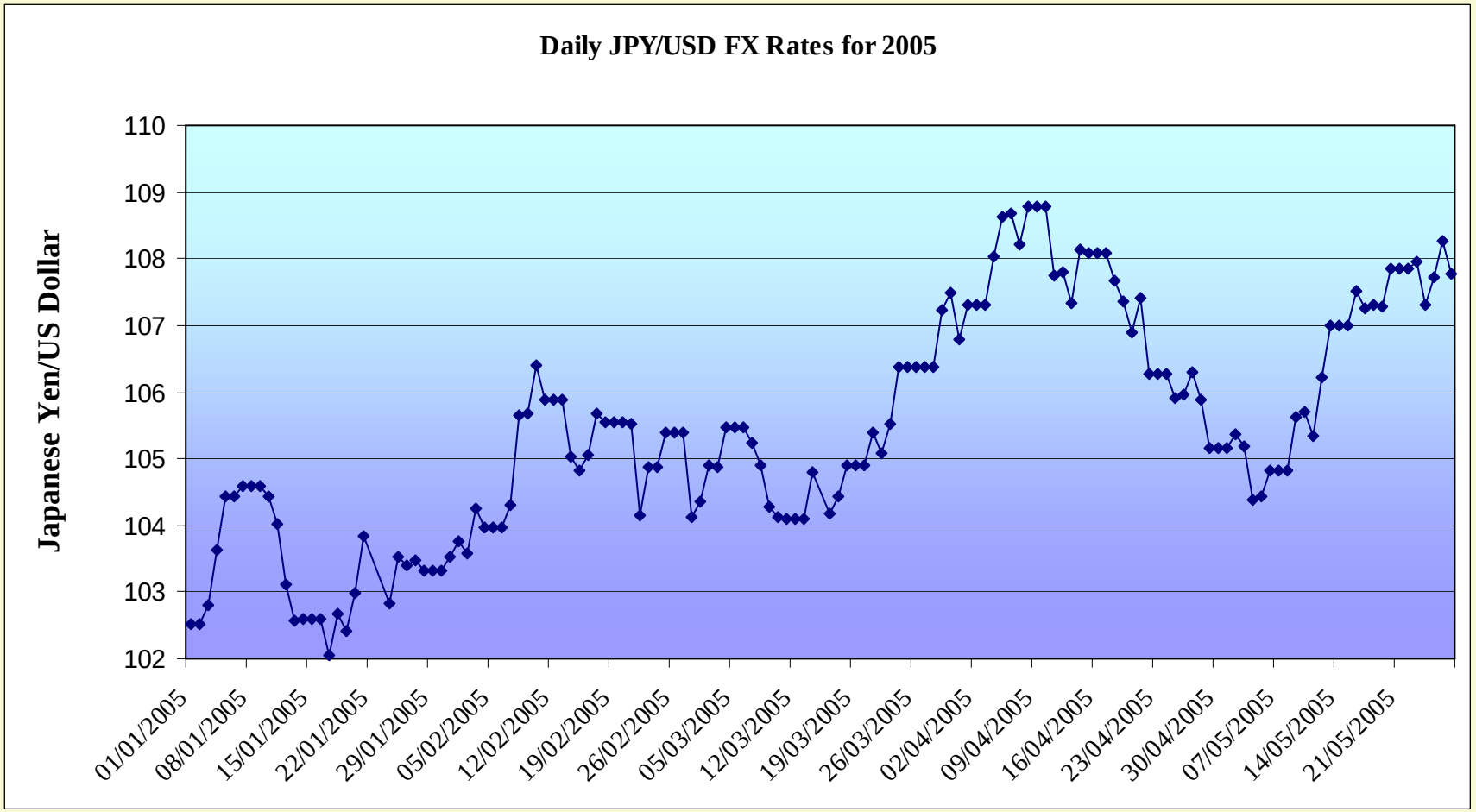
This is very encouraging, the rates offered by US banks appear to offer almost 28 times as much interest on a 1 year horizon than their Japanese counterpart.

So why not exchange your 100m Yen for a US Dollar amount, which is then put it on deposit for 1 year before exchanging the principal + interest back into Yen?

Who wins the race in probability space – the Hare or the Tortoise?



So far this year, the Japanese Yen has depreciated against the US Dollar by over 5%

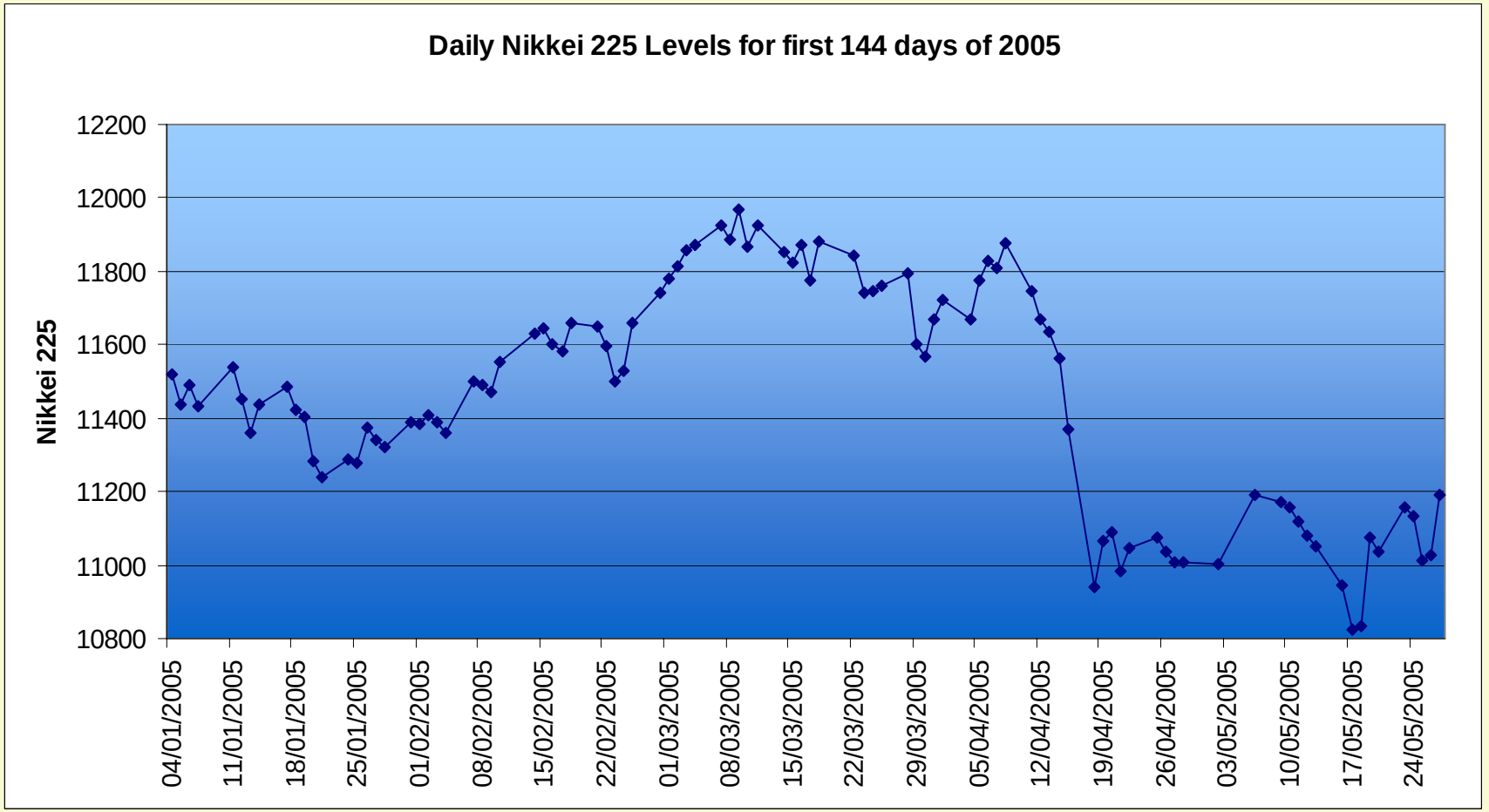
Who wins the race in probability space – the Hare or the Tortoise?

The idea of using the superior returns offered by a US savings account is luckily enhanced by the 5% depreciation of the Japanese currency. This strategy has turned out to be an FX bet in disguise.

One could reasonably argue that since the Japanese Stock Market has recovered from the bear market of 2000 to 2003, it may be reasonable to re-enter that market.

Such hypotheses in the investment world are often unforgiving, so let's examine what happened next.

Who wins the race in probability space – the Hare or the Tortoise?



So far this year, the Nikkei 225 Index has depreciated by 2.8%

Who wins the race in probability space – the Hare or the Tortoise?

Simple Trading of Complex Financial Products

- The business proposition involves selling structured products to the client, complete with an up front margin which is then locked in by appropriately hedging the risks throughout the lifetime of the transaction.
- Typically found on the ‘Sell’ side
- Recent examples include Snowballs, CMS Spreads, Target Redemption Notes
- The mathematics behind this business plan is founded on the ‘No Arbitrage’ theorem, and the belief that under the Risk Neutral measure, one can replicate the market risks of an exotic derivative with a carefully constructed portfolio of vanilla products.
- Compared to a typical business plan found in other industries, this is quite an extraordinary proposition. Nonetheless this is the strategy of **The Deterministic Hare**.

Who wins the race in probability space – the Hare or the Tortoise?

Exotic Products To The Rescue

- One popular way to invest is to purchase a **Corporate Bond**.
  - typically pays a regular coupon that is determined at the outset of the deal
  - size of the coupon is governed by the prevailing market conditions and the credit rating of the lender.

- An alternative investment strategy is to invest in a **Snowball Bond**
  - here the coupon size is determined by the future path of LIBOR

$$\textbf{Coupon}(t_1) = \textbf{pre-determined in contract}$$
$$\textbf{Coupon}(t_{n+1}) = \textbf{Max}(\ \textbf{Coupon}(t_n)) + K(t_n) - \textbf{LIBOR}_{3M}, 0)$$

where

$$K(t_n) = \textbf{fixed schedule of pre-determined rates}$$

Who wins the race in probability space – the Hare or the Tortoise?

The Key Characteristics of a Snowball

- Each successive coupons depend on the size of the previous coupon
- All coupons are guaranteed to be positive
- The issuer of the Bond can cancel the agreement on any coupon date beyond an initial grace period known as the ‘No Call’ period
- The magnitude of the coupon goes down if an interest rate specified in the contract goes up

Who wins the race in probability space – the Hare or the Tortoise?

At long last, the need for a model is upon us, but how does one actually price such a bond?

- Needless to say I have chosen a particularly difficult example.
- Apologies to any of you who may have chosen Financial Engineering as your 2nd degree, but are there any mathematicians or physicists in the room?
- One needs to model the Yield Curve before deciding how many factors one needs to represent the dynamics of the interest rate market.
- Be warned though, some financial instruments are sensitive to the number of factors chosen. In many cases a simple 1-factor 'Short Rate' model will suffice. This case, I feel obliged to point out, is not one of them!

Who wins the race in probability space – the Hare or the Tortoise?

… but how does one actually price such a bond? … continued

- Principal Component Analysis is one way to identify the factors.
- From this one needs to propose a multi-dimensional model for the dynamics of the evolution of the conditional probability distribution, e.g. BGM
- Least we have forgotten, this particular example allows the issuer to ‘Call’ the bond on each coupon date (excepting the first).
- This last point, is one of those occasions when the investor’s healthy appetite can misguide him into believing a ‘free’ lunch has been left on the table. His first coupon may pay him or her 800bp, not bad for a Japanese based investor.

Who wins the race in probability space – the Hare or the Tortoise?

MTNs – Medium Term Notes

- This Snowball is typical of a type of product that is known as an MTN, they are typified by the fact they are callable and have coupons that are constructed from stochastic variables
- Software development is a very time consuming business, ideally what you build today you would like to re-use tomorrow
- Fortunately for this class of product it is possible to implement a solution that is fairly generic. The re-use is obtained by separating the 'Product' from the 'Model'.
- To see how one can do this one needs to introduce the concept of conditional expectation for the value of the MTN at time t, V(t) say.

Who wins the race in probability space – the Hare or the Tortoise?

MTN Valuation Via Backward Induction

- One uses backward induction to calculate the time evolution of the MTN's fair value

- To get an idea of how this algorithm proceeds, let us solve the much easier problem of pricing an option written on X(t), with two strikes K_1 and K_2 , at times t_1 and t_2

- Denoting the value of the option at time t_i as $V_i(X(t_i))$, and the discount factor from t_1 to t_2 as $DF(t_1,t_2)$, then

$$\mathbf{V_2(X(t_2)) = Max(X(t_2) - K_2, 0)}$$

$$\mathbf{V_1(X(t_1)) = Max(X(t_1) - K_1,\ E[DF(t_1,t_2)V_2(X(t_2) \mid X(t_1)])}$$

$$\mathbf{V_0(X(t_0)) = E[DF(t_0,t_1)\ V_1(X(t_1) \mid X(t_0))]}$$

- The equivalent set of equations for our Snowball are not too dissimilar, but in that case the expectation involves a multi-dimensional integration

Who wins the race in probability space – the Hare or the Tortoise?

- If we focus on the 2nd of these equations one notices that is it a non-linear difference equation that relates the conditional expectations.

- It may be easier if one simply takes the view that these equations provide a recipe to construct the sequence of functions $\mathbf{V_2(x)}$, $\mathbf{V_1(x)}$ and $\mathbf{V_0(x)}$

- To make good of the proposition one has to give real meaning to the terms on the right hand side
  - What is the probability space in which the expectation operator is defined?
  - What measure and numeraire has one chosen?
  - How can perform the multi-dimensional integration?

- More generally one needs to tackle the case where there are a set of 'special' dates which define the payoff features of the MTN.
  - Typical features include coupons that change through time
  - Call and Put schedules
  - Redemption payments

Who wins the race in probability space – the Hare or the Tortoise?

Financial Grammars & Scripting Languages

- The big idea is to introduce a financial grammar which in some sense captures the various payoff features of a wide class of MTNs
- One then passes this script to a calculation engine which numerically evaluates the conditional expectation of the MNT for each of the key dates defined in the contract.
- A number of investment banks have been using such an approach for the last decade. Amongst the commercial software packages that use this framework, Reech's is probably the best known.
- By studying MTNs such as Cancelable Swaps and Target Redemption Notes one very quickly gets the idea of how the idea is implemented in practice.
- What's interesting about this approach is that is it involves numerical integration, rather than the more familiar use of PDEs.

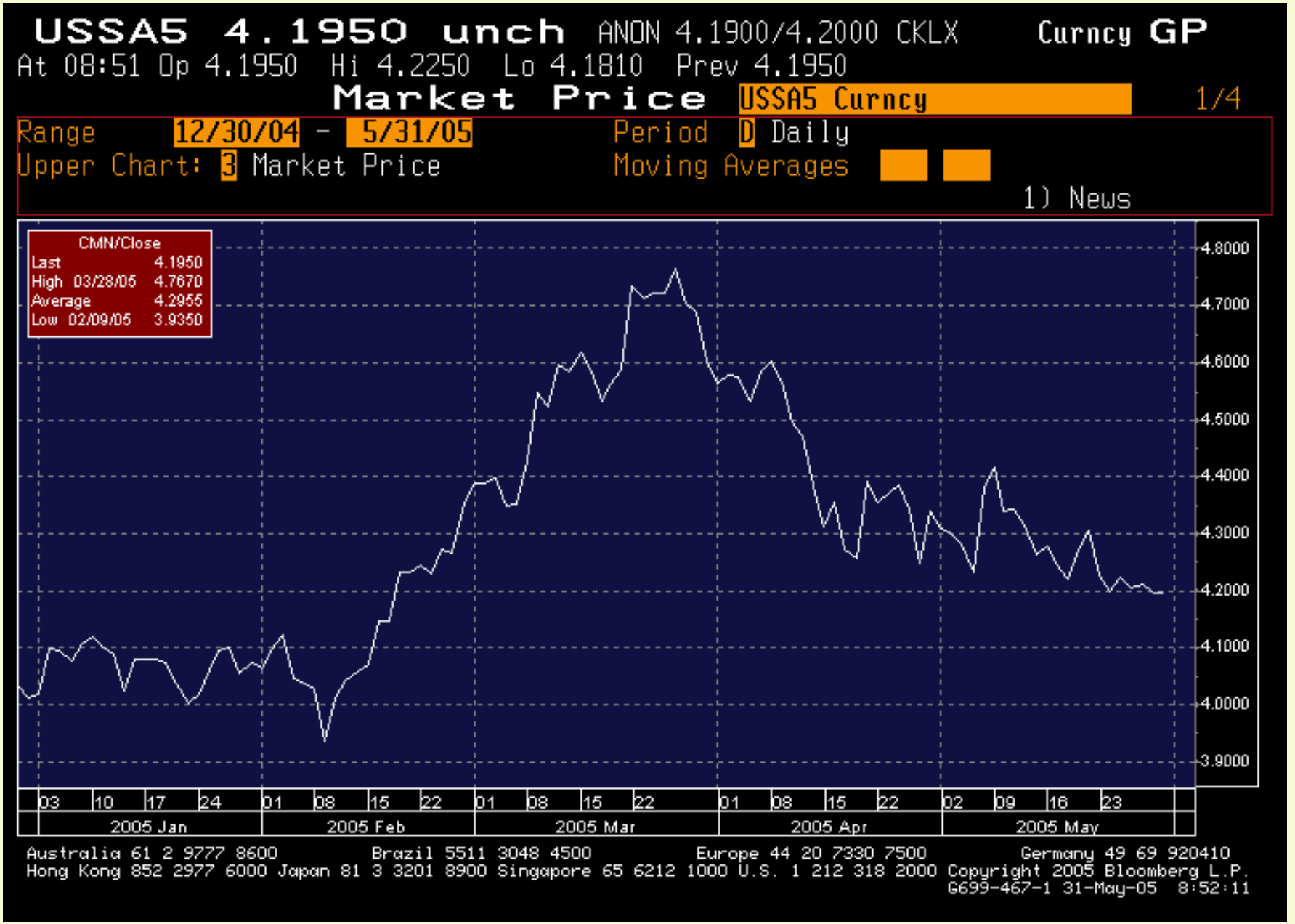
Who wins the race in probability space – the Hare or the Tortoise?

Complex Trading of Simple Financial Products

- The key idea here is to buy a linear combination of products such as Bonds and Equities, let's call this a portfolio, and then to rebalance the weights of each security at regular or irregular time intervals, to realize one's profit.
- Typically found on both the 'Buy' and 'Sell' sides. Oh, and let's not forget to include Hedge funds in that list.
- Unlike the case of the Tortoise, the business model here is to make one's profit by beating the market. Unless one matches the return offered by a simple deposit account, there is no point in using this approach to run one's business.
- Unless I am mistaken, this does seam to hint that the principal of 'No Arbitrage' needs to be suspended.
- For reason alluded to above, this business plan is also founded on around another somewhat extraordinary proposition. This is the strategy of **The Statistical Hare.**

Who wins the race in probability space – the Hare or the Tortoise?

Although the ideas presented on the next few slides could apply to any financial time series, for the sake of definiteness let's go back to the USD yield curve and focus on 2 particular points, the 5y and 10y swap rates, say.


USSA5 4.1950 unch ANON 4.1900/4.2000 CKLX Curncy GP
At 08:51 Op 4.1950 Hi 4.2250 Lo 4.1810 Prev 4.1950
Market Price USSA5 Curncy 1/4
Range 12/30/04 - 5/31/05 Period D Daily
Upper Chart: 3 Market Price Moving Averages
1) News
CMN/Close
Last 4.1950
High 03/28/05 4.7670
Average 4.2955
Low 02/09/05 3.9350
4.8000
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At 9:25 Op 4.4910 Hi 4.4950 Lo 4.4540 Prev 4.4950
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Upper Chart: 3 Market Price Moving Averages
1) News
CMN/Close
Last 4.4580
High 03/22/05 5.0900
Average 4.6895
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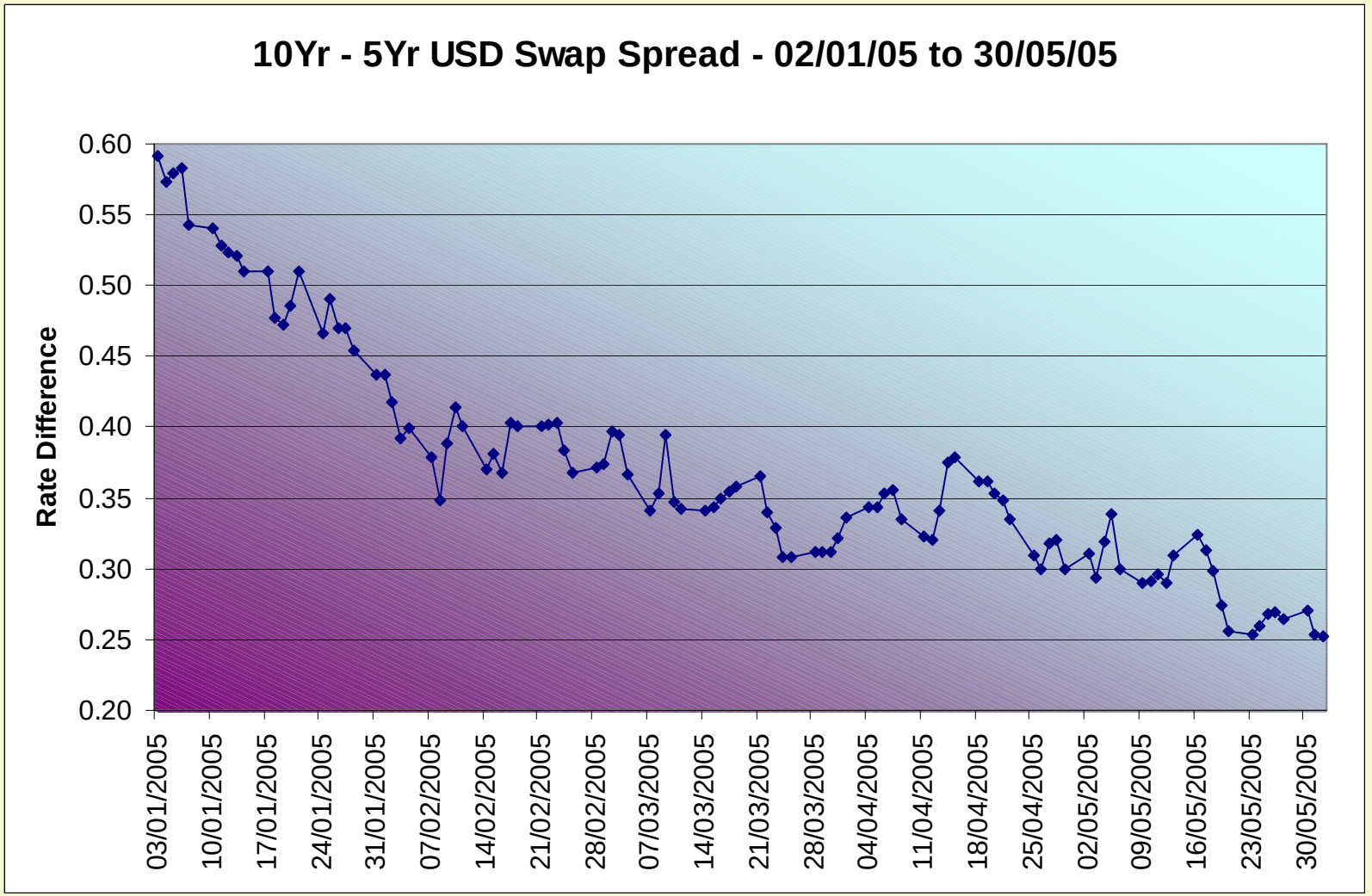
Relative Value Trading

- One of the first things you should notice is that while both graphs are very similar, they are not exactly the same.
- For the purpose of this discussion, let us consider the role played by each daily quote as if it was the closing price of a stock.
- The stigma of ‘Shorting’ a stock doesn’t really exist when one trades in the Swap Market, Paying or Receiving ‘Fixed’, who cares?
- Based on this one could consider buying one contract and selling the other. Things start to look promising when one looks at the time series of the 10y – 5y spread

Who wins the race in probability space – the Hare or the Tortoise?

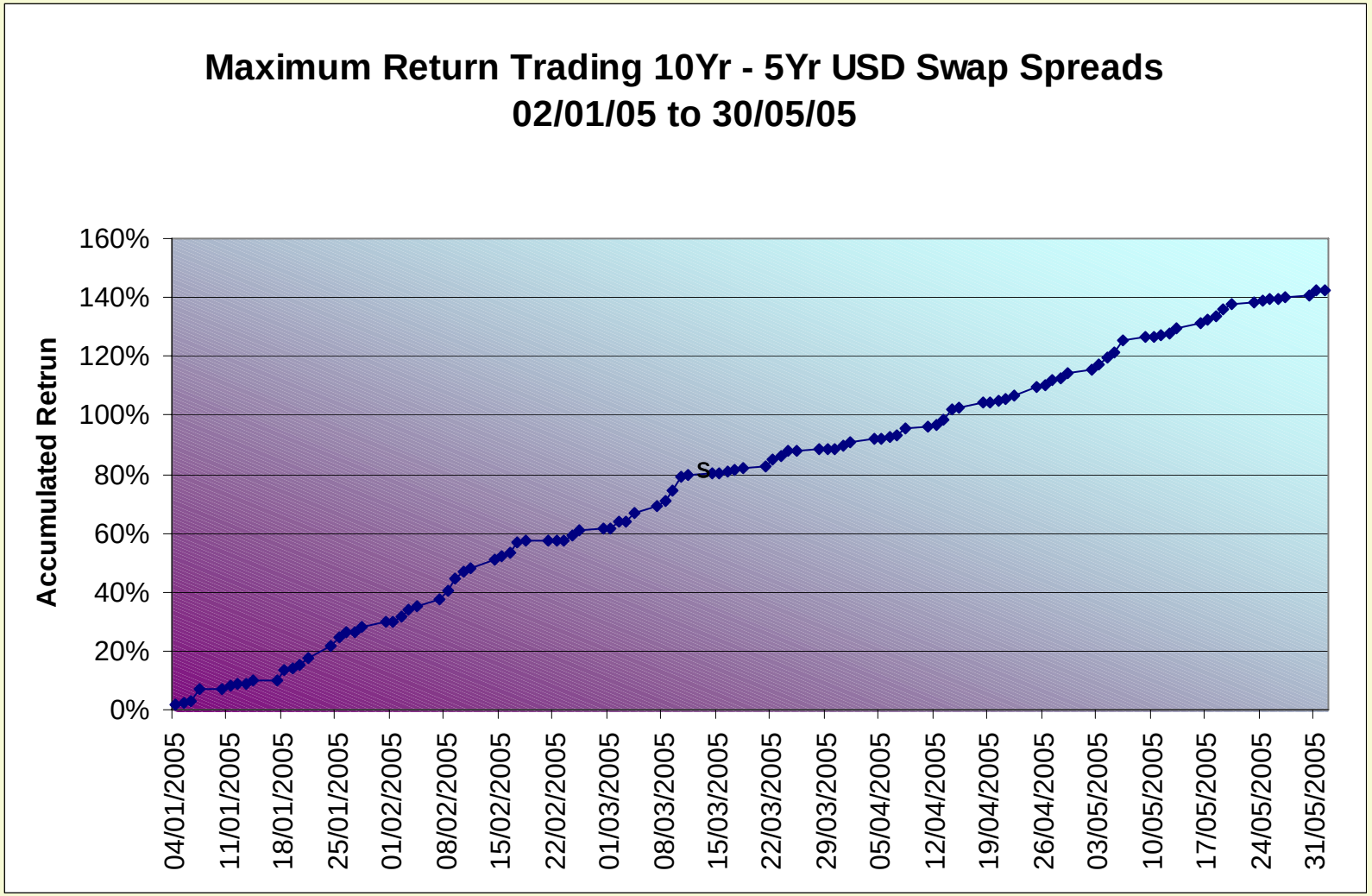


Who wins the race in probability space – the Hare or the Tortoise?

Relative Value Trading Cont.

- At this stage, the name of the game is to look for statistical outliers, operating with the belief that these anomalies are temporary in nature.
- At first glance it looks like that this time series is less random than the original pair of separate time series. From a mathematical point of view it is interesting to ponder what this statement might actually mean?
- The fluctuations present in the data can easily be wished away if one looks at the data over longer rather than shorter timescales.
- If one could only forecast the direction of the change in spread (regardless of size), then from an investment perspective that would be ideal.
- Choosing to operate on a timescale of daily forecasting, the maximum return is obtained by correctly guessing the market direction on a daily basis.

Who wins the race in probability space – the Hare or the Tortoise?



Who wins the race in probability space – the Hare or the Tortoise?

Information Ratios & Theoretical Upper Bounds

- This accumulated performance provides a theoretical upper bound for what can be achieved when trading that particular investment strategy.
- In fact it will set an upper limit to the 'Information Ratio', which is defined as the excess return of the investment divided by the volatility of that return.
- For simplicity we have ignored transaction costs, bid/offer spreads, and slippage. At best these will reduce the information ratio, at worst they will kill the strategy dead.

Who wins the race in probability space – the Hare or the Tortoise?

What are the modelling issues when implementing this business plan?

- At the risk of stating the obvious, the challenges facing the modeller on this side of the fence are quite different to that of pricing an MTN.
- One of the first things that notices is that there isn't the need to restrict one's analysis to the Risk Neutral Measure. In practice, one uses what might be best described as the Statistical Measure.
- As a result, the practitioner has a much wider set of models and tools available. With such a wide open field, the list of available choices is daunting :-
  - Wavelets?
  - Particle Filters?
  - Bayesian Techniques?

Who wins the race in probability space – the Hare or the Tortoise?

What are the modelling issues … continued

- Back Testing
  - The real dilemma lies with the realization that the proof is in the pudding. One cannot talk of the best model until one has tried them all. As a direct consequence, one spends most of one's time performing back testing with no guarantee it will work in the future.

- Combinatorial Issues
  - What particular relative value trade should one analyse? It would be a good guess that all of the trades made up of two parts have been well studied by the rest of the market
  - But exactly how many trades made up of four part swaps are there?
  - For each combinatorial choice one needs to study the historical time series, construct a forecasting model for a variety of in-sample window sizes, etc etc.

Who wins the race in probability space – the Hare or the Tortoise?

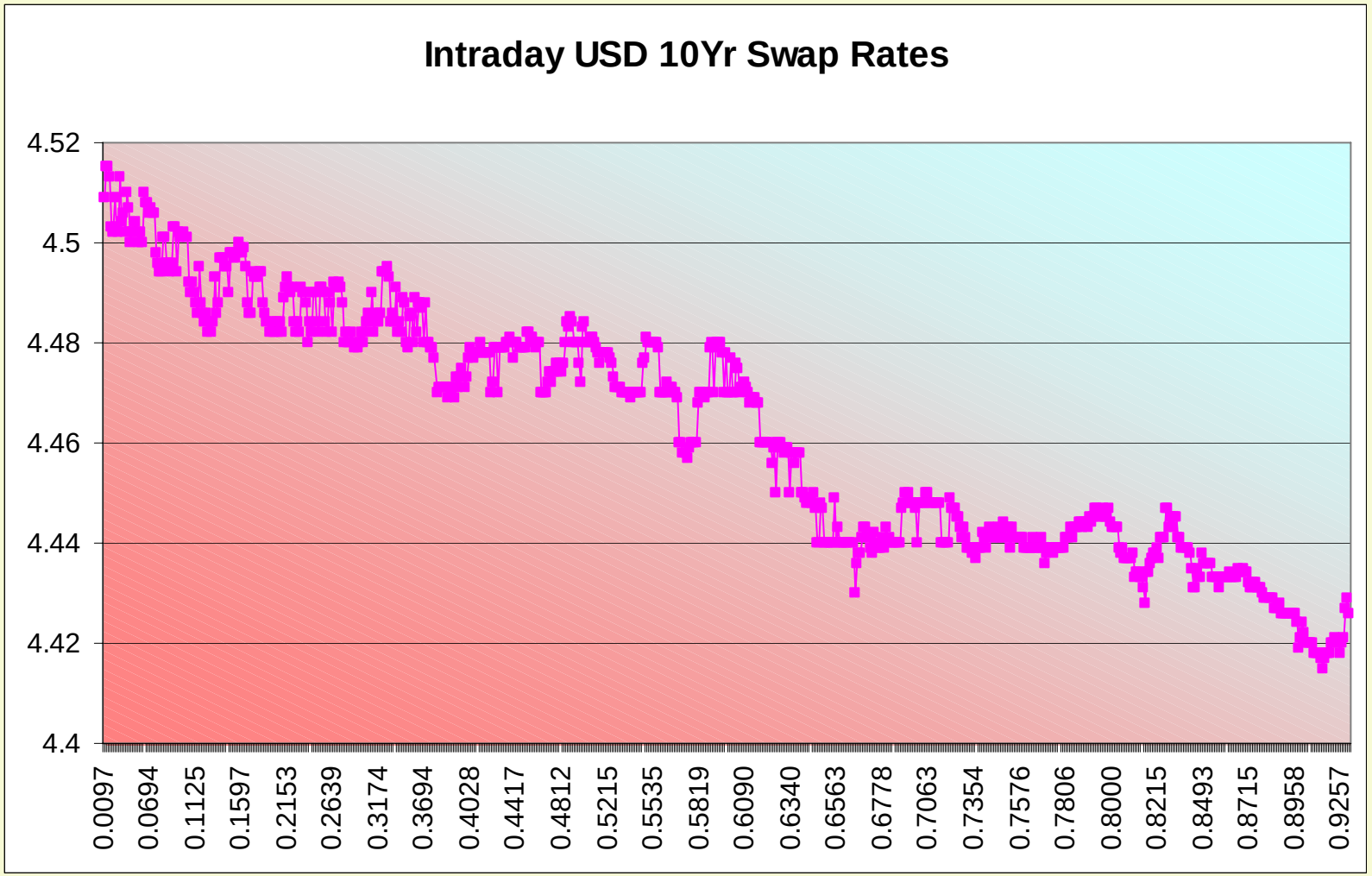
High Frequency Trading

- If we can ‘Theoretically’ increase our return by reducing the time horizon on which to operate, the logical conclusion is to take this to the limit, whereby one trades the markets on a ‘Intraday’ basis.

- To a large extent, altering the timescale in which we previously viewed the data didn’t really change the characteristics of the raw data (daily and beyond).

- Once one operates at the level of ‘Tick Data’, such an observation is less likely to remain valid. It is not uncommon to detect a much higher degree of skew and kurtosis (this is particularly true for foreign exchange rates).

Who wins the race in probability space – the Hare or the Tortoise?



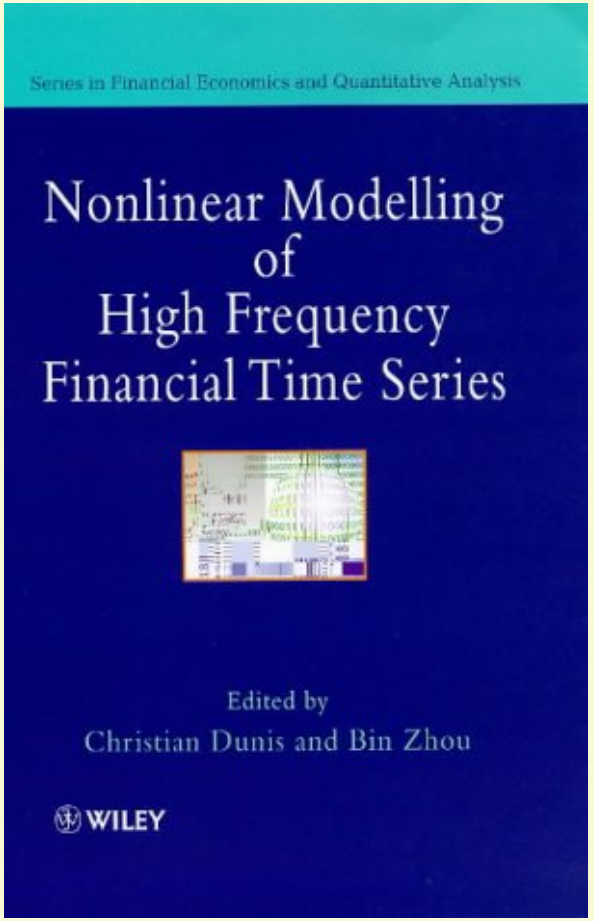
Who wins the race in probability space – the Hare or the Tortoise?

High Frequency Trading Cont.

- Those of you with a keen eye will have noticed I didn't display the spread between the two swap rates on an intraday basis.
- This is rather embarrassing. Intraday data is generally not equally spaced in time, and for this reason the time series are not both defined at the same points in time.
- This is simply reminding us that the two contracts each trade on their own account, and any attempt to make the data points lie on the same points of the time axis would be spurious information that wasn't in the original data..
- **When presenting, or indeed publishing on the topic of investment strategies it is very rare that a winning strategy is actually written down. This talk, will unfortunately stick with that tradition**.
- And so the story goes on …

Who wins the race in probability space – the Hare or the Tortoise?

Not surprisingly, the area of High Frequency economics has become an area of research in its own right.



Who wins the race in probability space – the Hare or the Tortoise?

The Conclusion

Our choices are wide and varied. We can invest our $1m in a hedge fund and follow the statistical hare who is trying to beat the market, and as a consequence, temporarily beat the no-arbitrage principle.

Alternatively we can trust our investment in the Tortoise's belief that we can lock in the up-front P&L when trading the exotics market.

The hare's strategy, offers a potentially higher return but not without risk. In many ways this combination of choices summaries the dilemma faced by all investors. There are no winners in probability space, only choices regarding one's own risk/reward preferences.