Testing for Overreaction in Short Sterling Options

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Abstract

This paper examines the hypothesis that Short Sterling options are rationally priced, against the alternative that the options market overreacts (or underreacts) to new volatility information in pricing longer-dated options. Two forms of exploratory analysis have been used; firstly, following Stein (1989), the joint behaviour of contemporaneous implied volatilities and secondly, the comparison of implied volatilties with realised volatilities. We are unable, using either of these forms of analysis, to reject the hypothesis that Short Sterling options are rationally priced. The methods of analysis used however are simple, approximate and weak. Suggestions for developing improved and alternative methods of analysis are made. We believe that use of such methods would lead to sharper, and quite possibly different, conclusions.

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I Introduction

This paper is concerned with the rationality of participants in options markets. The assumption of rationality has been of fundamental importance in option pricing at least since the derivation of the Black and Scholes (1973) formula for the pricing of a European call option on a non-dividend-paying asset.

The term 'rationality' is used here in a technical sense with a meaning rather stronger than its everyday one. Due to Muth (1961), the rational expectations hypothesis states that agents in an economy form their expectations on the basis of the 'true' structural model of the economy.

Thus critical assumptions in the derivation of the Black–Scholes formula are that all participants agree (it is assumed, correctly):

1. that the price of the underlying asset follows a geometric Brownian motion, leading to log-normal increments, and
2. that the volatility of this process is constant, and
3. on the value of the volatility.

In practice, at least, the second and third of these assumptions are questionable. Options markets are spoken of as vehicles for trading volatility, so that it is clear that participants do not necessarily agree on the value of volatility. And, although volatility is, strictly, unobservable, it is empirically clear that volatility of many asset prices does indeed vary through time.

If all participants in options markets are truly rational, they will be aware of the nature of this temporal variation in volatility and the resulting model (whether formal or not) of underlying asset-price behaviour that they have will be used in determining option prices. This will require them to know:

a. the current value of the volatility,
b. the current values of all other variables affecting future volatility,
c. the true model for the evolution of these other variables,
d. hence the true model for the evolution of the volatility.

These are very demanding requirements on participants' knowledge. A plausible form of irrationality and the one that is investigated in this paper, however, is that participants use models for c. and d. which are incorrect. In particular, in predicting future values of volatility they may overreact (or underreact) to new information, i.e. unexpected changes in the current values of volatility and other variables which affect it. More precisely, they may judge the effects of these changes on volatility to be more (or less) persistent than they really will be.

The plausibility of option-market overreaction as a phenomenon is suggested by two factors. Firstly, psychologists such as Tversky and Kahneman (1974) have found from experimental and survey data that people have a systematic tendency to overemphasise recent data at the expense of other information when making projections. Secondly, while progress has been made in empirical understanding (e.g. Schwert, 1989; Dimson and Marsh, 1990; models (e.g. Hull and White, 1987; Nelson, 1990) and forecasting methodology (e.g. Harvey, 1989; West and Harrison, 1989), volatility forecasting in practice remains usually rather informal.

Longer-dated options will clearly be particularly susceptible to overreaction and this has motivated us to investigate whether it occurs in the prices of LIFFE Short Sterling options, which are traded up to one year into the future.

In this paper we present the results of an initial investigation of this question. We have carried out two forms of exploratory analysis of Short Sterling option prices. We are unable on the basis of this analysis to reject the hypothesis that the options are rationally priced. The methods of analysis used so far, however, are simple, approximate and rather weak, and a number of areas of potential improvement and other approaches can be identified. Therefore there remains in our view the distinct possibility that improved methods of analysis would lead to a different conclusion.

In Section II we provide a brief discussion of overreaction in financial markets. We compare the phenomenon of overreaction in option markets with overreaction in other financial markets and we review the pioneering paper of Stein (1989) on option-market overreaction.

In Section III we describe Short Sterling options, how they are valued, and the data used in our study.

Section IV contains a discussion of the principles underlying the two types of exploratory analysis we have undertaken.

These relate to the comparison of implied volatilities with different expiry times, following the first test in Stein (1989), and of implied volatilities with measures of realised volatility. The results from these two types of analysis are shown and discussed in Sections V and VI respectively.

A summary, conclusions, and suggestions for further work are given in Section VII.

II Overreaction in financial markets

II.1 *Overreaction in financial markets as expectations formation*

A fundamental assumption in the traditional body of asset-pricing theory is that markets are efficient, i.e. broadly speaking, that participants' expectations about asset returns are correctly based on all information available to them. If it is assumed that that information includes the true model of asset returns, including parameter values, then this is equivalent to the rational expectations hypothesis. Pricing of an asset, unless it can be performed by arbitrage against other asset prices, requires the market to form expectations of future asset returns from information currently available; rational expectations, while a theoretically useful concept and with some empirical support, is but one of many conceivable expectations-forming mechanisms. Further relevant discussion of rational expectations (and alternatives), efficiency, their interrelationships and empirical testing is given in Pesaran (1987) and Ross (1987).

The expectations-forming mechanism that is the focus of this paper is overreaction (underreaction); that is, expectations of future asset returns change too much (little) in response to relevant new information. The characteristics of prices in markets in which overreaction (underreaction) occurs are that asset returns are inflated (deflated) in variance and have some excess negative (positive) autocorrelation relative to the behaviour under rational expectations.

Shiller (1981, 1986), Marsh and Merton (1986), De Bondt and Thaler (1985, 1987) and others have investigated the phenomenon of apparent overreaction or a 'rebound' effect in stock markets, whereby stocks that had once been losers later tended to perform abnormally well. The negative autocorrelations in stock returns consistent with overreaction, however, could arise in other ways; see Lo and MacKinlay (1990) for a demonstration that apparent overreaction effects in stock prices may in fact be due to thin trading and lead/lag effects between prices of large and small stocks. Stock-market overreaction, if it exists, is to new information on the present value of the stock, which implicitly reflects market expectations of future stock returns. Overreactions may occur similarly in the behaviour of currency exchange rates (where the phenomenon of 'overshooting' is well known; see e.g. Frankel and Froot (1990) and Hodrick (1990) for recent relevant research); even interest rates for a period of three months, say, will reflect not only the current instantaneous rate of return, but also expectations of the instantaneous rates of return during the three-month period.

In contrast, market expectations of values of certain economic variables at different times in the future are directly reflected in a term structure which can be imputed from a set of prices of currently traded assets. The two obvious examples are of interest rates and of volatility of a traded asset. Expectations of future interest rates are reflected in the prices of forward rate agreements, of interest rate futures, of swaps and of bonds. For example, if participants' expectations of future interest rates are too sensitive to new information which might affect the rates; bond prices may be prone to fluctuation over and above that suggested by rational-expectations-based models such as those of Cox *et al.* (1985).

Expectations of future volatility of a traded asset will be reflected in an options market for the asset, if one exists. If overreaction exists, it may conceivably have two components, temporal overreaction and current-value overreaction. The dominant component is likely to be temporal overreaction, that is, to information arriving currently on future values of volatility. The other possibility raised by the unobservability of volatility is that of current-value overreaction, that is, the market is not necessarily able to form an error-free view of instantaneous volatility currently and its view may overreact.

II.2 *Previous work on option market overreaction*

So far as we are aware, Stein (1989) is the only published work specifically on option-market overreaction. Stein investigated the existence of overreaction in the prices of Standard and Poor's 100 index options for the period December 1983 to September 1987. For each date, he had implied volatilities on the 'near' option with one month or less to expiry and on the 'distant' options with between one and two months to expiry. He conducted two types of test for overreaction.

The first type of test required the assumption of a particular stochastic volatility process. He identified and fitted a mean-reverting AR(1) process for the near implied volatility and estimated the mean-reversion and mean volatility parameters of this process. He ascribed this behaviour to the instantaneous volatility, on the claimed basis that near implied volatility behaves almost exactly like instantaneous volatility: in this way he overcame the unobservability of volatility. He derived an expression and an upper-bound approximation to this for the elasticity of distant implied volatility with respect to nearby implied volatility under rationality. To derive this he used the approximation that implied volatility of a rationally priced option will exactly equal the mean expected volatility over the time to expiry. Strictly it is more nearly true that it equals the root-mean-squared expected volatility over the time to expiry. Other factors (mentioned by Stein in his footnote 3) relating to volatility risk and the relationship of the option price with volatility also have effects.

He then found that the empirical values of this elasticity tended to exceed the upper bound. He repeated the analysis after linearly detrending the time series of near implied volatilities, with similar results. He concluded that there was evidence of overreaction.

The second test did not rely explicitly on a specific stochastic model of volatility. The principle of the test is similar to that used in some other tests of rational expectations (e.g. as described in Campbell and Shiller (1983), Froot (1989)) and is that the evolution of volatility expectations through time (as reflected in the term structure of implied volatility) should be random, in particular, that changes through time should be uncorrelated with the level of volatility. (Although there is no explicit dependence on a particular volatility model, some implicit restriction is introduced by the number of contemporaneous option prices from which the implied volatility term structure can be inferred.) It led to a procedure to test whether white-noise residuals resulted when the time series of differences between the near and distant implied volatilities was used to forecast future volatility changes. The negative coefficients obtained when the residuals were regressed on the near volatility reinforced the conclusions drawn from the first test.

Stein noted that, through overreaction appeared to occur, the mispricing effect on his data set was fairly small, but that this was bound to so in one- and two-month options. He posed the question as to whether investors in longer-dated options also tend to overreact and stated that if they did, the mispricings involved were likely to be much more significant.

III LIFFE Short Sterling options

III.1 *The instrument*

A long (short) position in a LIFFE Short Sterling Futures contract is used to fix a rate for lending (borrowing) £500,000 for 3 months at a certain delivery date, at the interest rate implied by the futures price at the time at which the contract was taken out. The futures price F is quoted as $F = 100 - f$, where f is called the implied futures interest rate, in ticks of 0.01 (which have a value of £12.50). At delivery, which occurs on the day after the last trading day, the contract is settled in cash against the Exchange Delivery Settlement Price which is a close proxy for the London Inter-Bank Offer Rate (LIBOR) fixing on the last trading day. Futures contracts are marked to market by daily transfers from loser to gainer of the changes in the values of futures contracts via the London Clearing House (LCH). Counterparties are obliged to put up margin to LCH to cover default risk in marking to market; but the system is designed to obviate the need for large cash transactions and interest at a rate of (LIBOR-$\frac{1}{8}$)% is earned on margin account balances. There are currently 12 futures contracts extant at any one time, with last trading days on the third Wednesday in March, June, September and December, each year up to three years hence. Normal trading hours are from 08.20 to 16.02. On 8 March 1990, after-hours trading from 16.27 to 17.57 on the APT automated trading system was introduced.

For the four futures with the nearest delivery dates, i.e. up to a year hence, Short Sterling American call and put options are traded on the future. There are typically 13 exercise prices ranging in steps of 0.25 from well out of, to well in the money; prices of options, as of futures, move in steps of 0.01 (equivalent to £12.50). An option expires on the last trading day for the future on which it is written. Options are marked to market and margined in the same way as futures and are paid for at expiry. Trading hours are from 08.20 to 16.02, the same as normal trading hours for the future; the options, however, are not traded on the APT system.

III.2 *Valuation models*

The standard procedure for valuing Short Sterling options is to use Black's (1976) model. This assumes that the implied futures rate, f, follows a geometric Brownian motion with constant annual volatility, σ_f. The system of payment and margining for the options outlined above means that, with very little error, they can be regarded as pure futures options in the terminology of Duffie (1989). Then no discounting is required in valuation (Asay, 1982) and the American nature of the options is redundant in the sense that early exercise is never rational (Carverhill, 1990). The put and call can be valued by the respective formulae,

$$P = fN(d_1) - xN(d_2),$$
$$C = P - f + x,$$

where f is implied futures rate, $N(\cdot)$ is standard normal cumulative distribution function,

$$d_1 = \frac{\ln(f/x) + \sigma_f^2\,(T-t)/2}{\sigma_f\sqrt{T-t}},$$
$$d_2 = d_1 - \sigma_f\sqrt{T-t}\,,$$
$$x = 100 - X,$$

where X is exercise price of option, t is time now, T is expiry time of option, and the units of time are years.

This approach does not take account of the dependence of futures rate volatility on the time to delivery and correlation of the futures price movements that will arise from a realistic interest-rate model and rational pricing of the futures. Schöbel (1990), for instance, has pursued this approach to interest-rate futures option pricing under the assumption that interest rates follow Vasicek's (1977) model. In general the futures rate volatility will be attenuated with the time to delivery by a factor increasing with the strength of the tendency of the interest rate to mean-revert; also the correlation between contemporaneous price movements in futures prices with different delivery dates will decline from 1, the further apart the delivery dates are. In practice the interest rate tends to mean-revert over a sufficiently long time-period that for futures with delivery dates less than a year, say, price movements of different futures tend to have similar variance and to be almost perfectly positively correlated. This perspective will assist in our interpretation of the results and we check its empirical accuracy later.

III.3 *The data*

Our data consist essentially of daily settlement prices for the futures and options from the introduction of options on 5 November 1987 to May 1990. Data from 8 March 1990 onwards, however, were discarded because the introduction of after-hours trading on the futures would introduce problems of synchroneity in the settlement prices for futures and options.

Settlement prices are the prices used for daily marking to market. A settlement price may be a real trading price if the trade occurred near enough to the end of the day; if not, it may be based on quotes if these are judged recent enough. If not, market makers will be asked to provide a settlement price; to do this, they will invariably determine an implied volatility using Black's model from other options and use this to calculate a settlement price also using Black's model and assuming volatility of different futures rates to be equal. This happens especially frequently for options away from the money or on the third- and fourth-nearest future, since trading is thinner in these options. For this reason only settlement prices of at-the-money options (i.e. with exercise prices closest to the futures prices) on the nearest and second-nearest futures are considered to be consistently representative of real trading prices, and so these are the only prices used in earnest in the study. (There is some analysis of the third-nearest contracts is Section VI, to illustrate the caution with which that data should be treated.)

For interpretation of the data, for each option settlement price used, we have calculated an implied volatility using Black's model. Settlement prices are always fixed so that the put–call parity relationship given in the previous subsection is satisfied, so this procedure leads to unique implied volatilities.

IV An exploratory analysis of overreaction in Short Sterling options

Two types of exploratory analysis have been performed on the Short Sterling options and futures settlement prices. The motivations for these are outlined in this section. Further details and the results of the analyses are contained in Sections V and VI.

The first type of analysis is of the behaviour of implied volatilities of options with different expiry times. The basis of this type of analysis is that, for a particular model of the volatility term structure, if options are rationally priced, it should be possible to establish relationships between implied volatilities, either cross-sectionally across the term structure or through time.

Stein (1989) uses this approach in his first test. His analysis relies on the assumption that the volatility term structure follows a particular one-factor form driven by the process for the instantaneous volatility, σ_t. This process has parameters which are not allowed to vary over time and is of the mean-reverting form,

$$d\sigma_t = -\alpha\,(\sigma_t - \bar{\sigma})\,dt + \beta\sigma_t dz.$$

He estimates parameters for this model and uses the model to establish a relationship under rational pricing between the near and distant implied volatilities at any point in time. He then performs a cross-sectional analysis of observed near and distant implied volatilities, to conclude that the distant implied volatility tended to overreact to changes in the near implied volatility.

Apart from the approximations in Stein's analysis that were mentioned in Section II.2, this approach has several further limitations.

Firstly, his volatility process seems to be undesirably and unnecessarily restrictive, in not allowing parameters to vary over time. Variation in *a priori* expectations of volatility certainly exists whereas Stein assumes that they are flat. It appears that it should be possible to relax this assumption at least somewhat by a more sophisticated form of analysis, taking into account both time-series and cross-sectional properties of the volatility term structure. The assumption of constant volatility of volatility seems superfluous even to Stein's analysis.

Secondly, it would be desirable to be able to estimate the degree of overreaction, if any, in the market which is not provided by Stein's analysis.

Thirdly, unlike in Stein's analysis, some explicit allowance for errors due to such factors as discreteness of option prices, non-synchroneity between options and futures prices, trading effects within arbitrage bounds, and model mis-specification should be made, and statistical procedures should be designed to minimise the effect of these.

Fourthly, Stein's test as implemented by him relies on the use of his 'linear end-point approximation' which is likely to be too crude in general. Furthermore the size and power of his test are uncontrollable.

In our view these limitations can be overcome only by an explicit model for the arrival of and reaction to volatility information, and a firmer and more generally applicable basis for the statistical procedures. We hope to pursue this approach in future work.

We face one further difficulty in using this approach to test for overreaction in Short Sterling options, namely that it is based on the assumption that options expiring at different times are written on the same underlying asset. Recall, however, that all Short Sterling options expire at the same time as the delivery date of the future on which they are written: strictly the two nearest futures we are concerned with should be treated as separate underlyings. In practice we may be willing to gloss over this if we can show empirically that the futures move with similar variance and with high correlation, as theoretically they should (as discussed in Section III.2 above). If we cannot legitimately do this, then the approach must be extended to include a more fully specified model of joint futures price behaviour, based on a theoretical model of interest-rate term structure with allowance for empirical lack of fit. Whichever route were taken the results would need to be interpreted in terms of a common measure of futures price variability.

The second type of analysis is the comparison of *ex post* measures of realised volatility up to the time of option expiry with *ex ante* predictions of volatility provided by the option-price implied volatility. Our interpretation of such comparisons is straightforward.

If option prices and hence implied volatilities are determined rationally, then the implied volatilities should be approximately unbiased predictions of measured realised volatilities. If overreaction exists, then observed implied volatilities will be more variable than they should be – when implied volatility is low, it is too low, and when it is high, it is too high. Then a simple linear regression of realised volatilities against comparable implied volatilities should give a regression coefficient of 1 under rationality, but less than 1 under overreaction. Note that this interpretation relies on the assumption that any variation in *a priori* expectations of volatility can be ignored.

As an incidental but also important question, we are also able to form some assessment of whether or not implied volatility, regardless of its magnitude, tends to overpredict or underpredict realised volatility.

The over/underreaction and over/underprediction tests may be combined in a joint test that implied volatility is a rational predictor of realised volatility, against the alternative that either over/underreaction or over/underprediction or both occurs.

In principle, analysis of this type also appears to offer some prospect of checking the assumption that the market is able to judge correctly the value of instantaneous volatility, even though it is unobservable. (This would be a complicating factor since it would introduce the possibility of contemporaneous errors and lead/lag effects between the option market's view of and the true value of instantaneous volatility.)

Unfortunately this Olympian ability is certainly not enjoyed by the econometrician: this means that realised volatility up to option expiry must be measured imprecisely by the sample standard deviation of returns up to expiry, and that it would be possible to measure instantaneous volatility only very imprecisely using some form of filtering procedure. In principle, it would be possible to answer all questions about the relative behaviour of implied and realised volatility, but the imprecision of actual volatility measurement (at least when using daily returns data) poses a considerable obstacle to efficient, straightforward analysis.

V **Joint behaviour of implied volatilities**

As discussed in Section IV, to apply this kind of analysis we need strictly to have data on options of different expiry times on the same underlying instrument. We have instead two options expiring at different times but each option is written on the future being delivered at the same time. To form a judgement as to how much this may affect the validity of the analysis we briefly investigate the joint distribution of daily increments in the nearest and second-nearest futures prices.

Figure 7.1 shows a scatterplot of the daily increments in the second-nearest futures price, ΔF_2, and in the nearest futures price, ΔF_1 (using obvious notation). The similarity of the marginal distributions and the strength of the relationship are clear. In fact, the sample means of the two distributions are both very marginally less than zero (–0.008 for ΔF_1 and –0.012 for ΔF_2) due to the upward drift in interest rates during the period under study (November 1987 to March 1990). The sample standard deviations are 0.124 for ΔF_1, and 0.133 for ΔF_2. The sample correlation between ΔF_1 and ΔF_2 is 0.928, a reassuringly (even surprisingly) high figure. Due to the large sample size (631), the hypothesis of equal standard deviations can easily be rejected at any conventional significance level, but it is the size of the difference in proportional terms that is important for practical purposes and this seems acceptably small (less than 7% of the larger). (As an aside, it is rather surprising that the second-nearest futures price appears to be more variable than the nearest futures price; the opposite would be expected from rational futures pricing under a Vasicek (1977) interest-rate model, for instance.) We therefore feel justified, at least in this exploratory analysis, in directly comparing contemporaneous implied volatilities of the two options expiring at different times, ignoring the fact that strictly they are written on different underlying assets.

To give a feel for the behaviour of the daily implied volatilities, we have

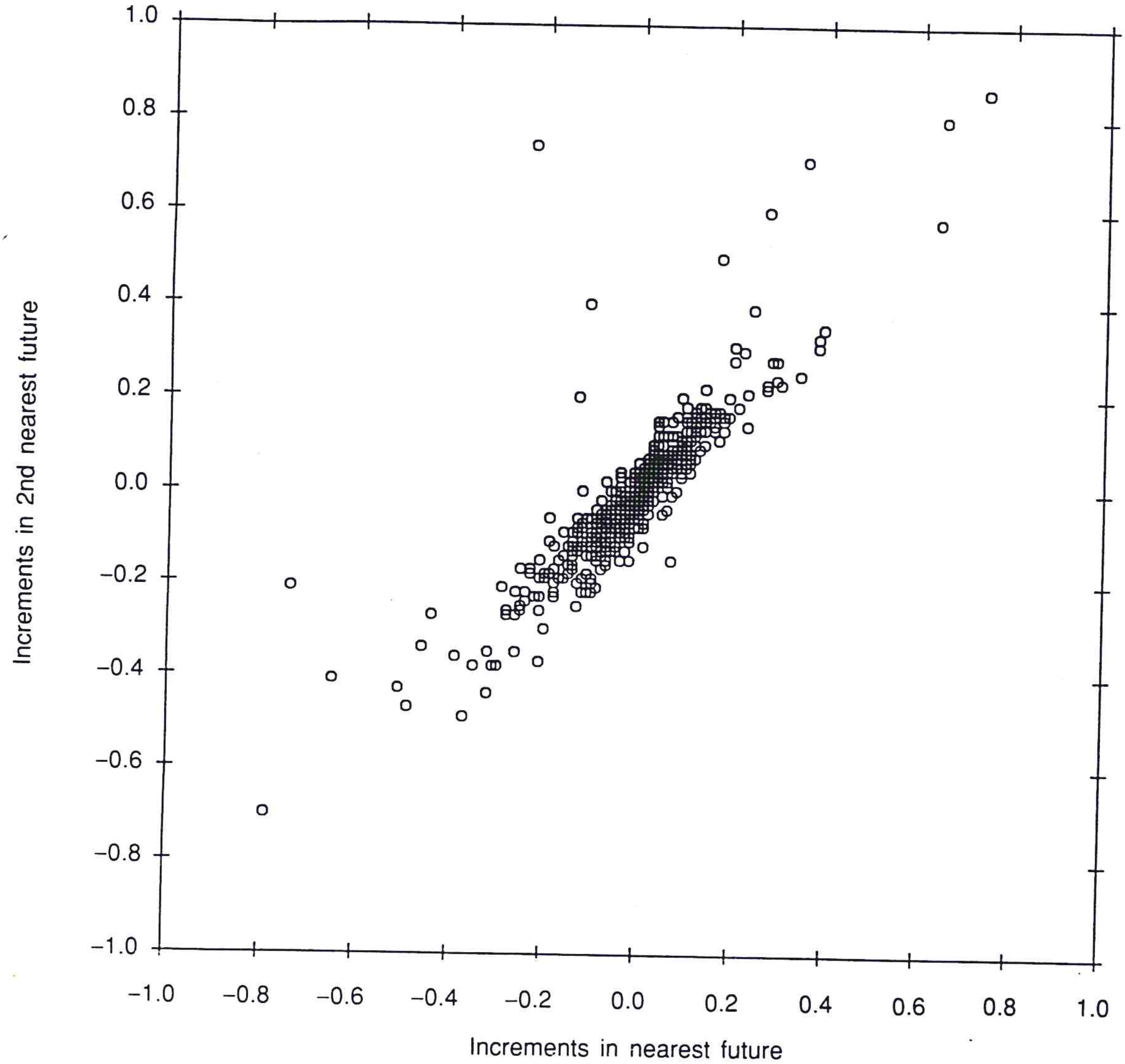

Figure 7.1 Scatterplot of increments in second nearest against nearest futures price

plotted three figures. Figure 7.2 shows that joint behaviour through time of the implied volatilities of the nearest and second-nearest options and Figure 7.3 is a scatterplot of the second-nearest implied against the nearest implied. It is evident that the nearest and second-nearest implied volatilities are quite highly correlated ($\rho = 0.752$), but that the nearest implied is quite a bit more variable than the second-nearest implied (the standard deviations are respectively 0.042 and 0.026). The fact that the mean of the second-nearest implied (0.156) exceeds that of the nearest implied (0.138) indicates a generally upward-sloping term-structure of volatility during the period under study.

It is also of interest that the correlation between daily increments in the nearest and second-nearest implied volatilities is quite high (0.592). A scatterplot of these increments is given in Figure 7.4. (Note that this figure excludes 17 outlying points.) It is reassuring that real changes in expecta-

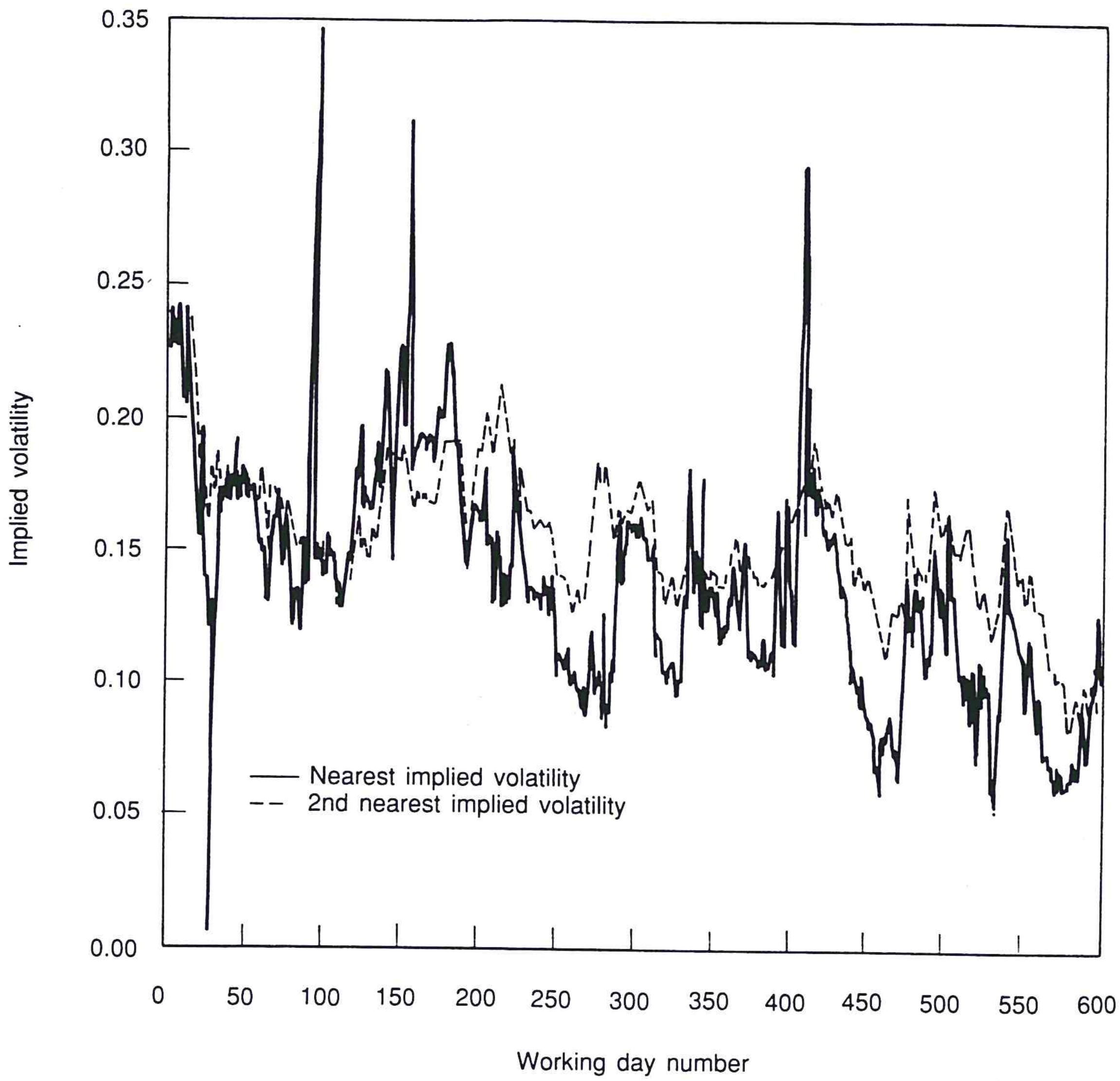

Figure 7.2 Nearest implied and second nearest implied volatility

tions of future volatility are not swamped by error sources such as option-price discreteness.

For an exploratory analysis of overreaction, we have followed Stein's (1989) first procedure. The first step is to consider the accuracy of the mean-reverting AR(1) process for volatility, σ_t, on which this procedure depends. The process is

$$d\sigma_t = -\alpha\,(\sigma_t - \overline{\sigma})\,dt + \beta\sigma_t dz.$$

Stein fits this process to the nearest implied volatility, which we denote by $\tilde{\sigma}_{(t,T_1)}$, and ascribes it to the volatility σ_t. There is a slight downward trend in volatility apparent from Figure 7.2 which we ignore for the purposes of this exploratory analysis: it is anyway swamped by short-term variation in volatility.

We examine the autocorrelogram and partial autocorrelogram for $\tilde{\sigma}_{(t,T_1)}$ at daily lags, shown in Table 7.1. There is some evidence in the table of

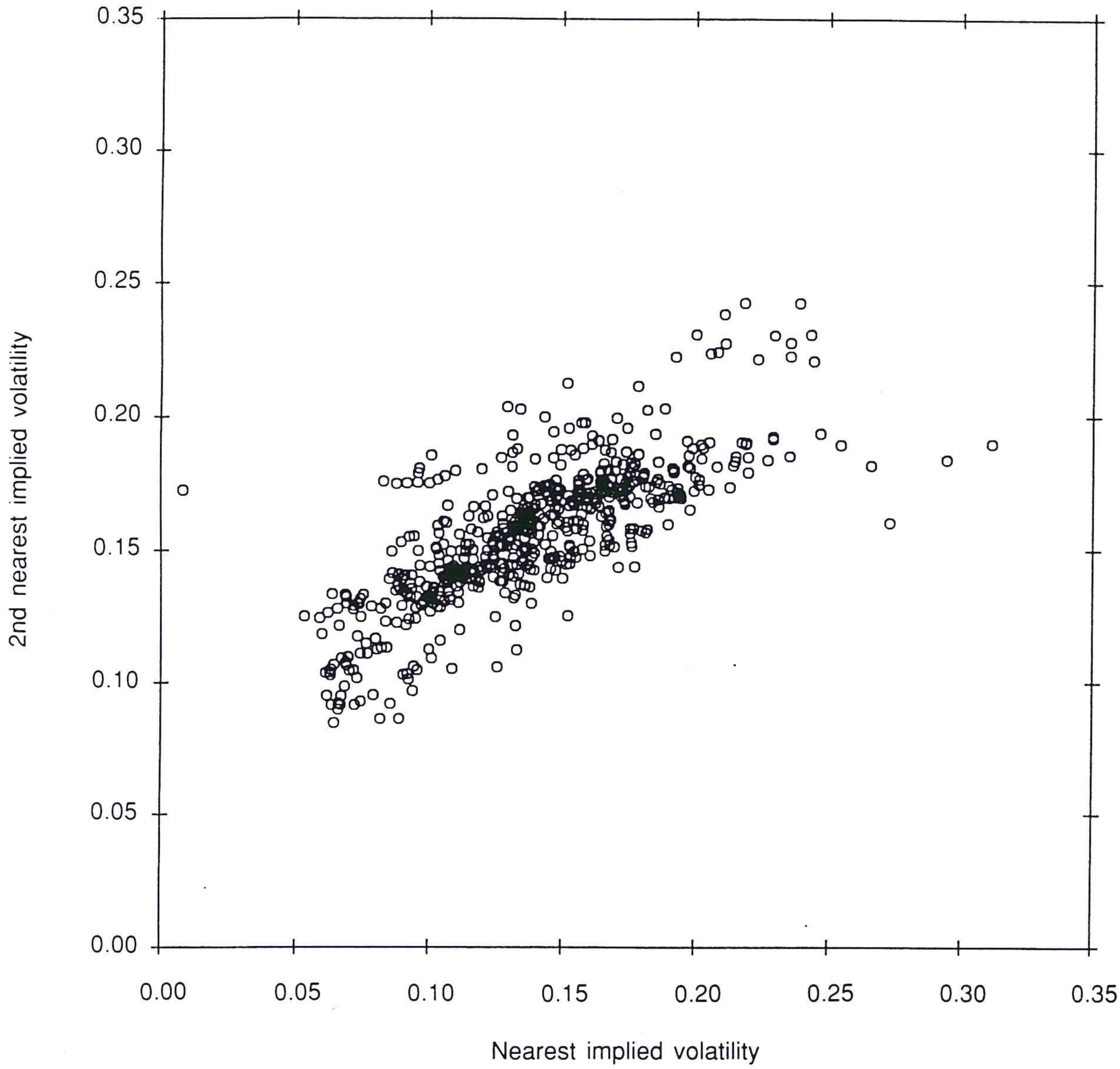

Figure 7.3 Scatterplot of second nearest against nearest implied volatility

positive partial autocorrelation up to lags of five days at least. This is reflected also in steadily climbing values of the implied daily autocorrelations, obtained by raising the autocorrelation at lag n to the power of $1/n$. This might be an accurate reflection that some more general ARMA representation for volatility is appropriate or that the volatility process is non-stationary. Alternatively, it may arise because the use of the nearest implied volatility as a proxy for the instantaneous volatility is too inaccurate; if that is the case, the positive partial autocorrelations of the near implied volatility would seem to indicate some underreaction to volatility information.

We do not investigate these possibilities further at this stage. Formally we reject the hypothesis that the near implied volatility process follows the mean-reverting AR(1) process described above. For the purposes of this exploratory analysis, however, in order to replicate Stein's analysis, we

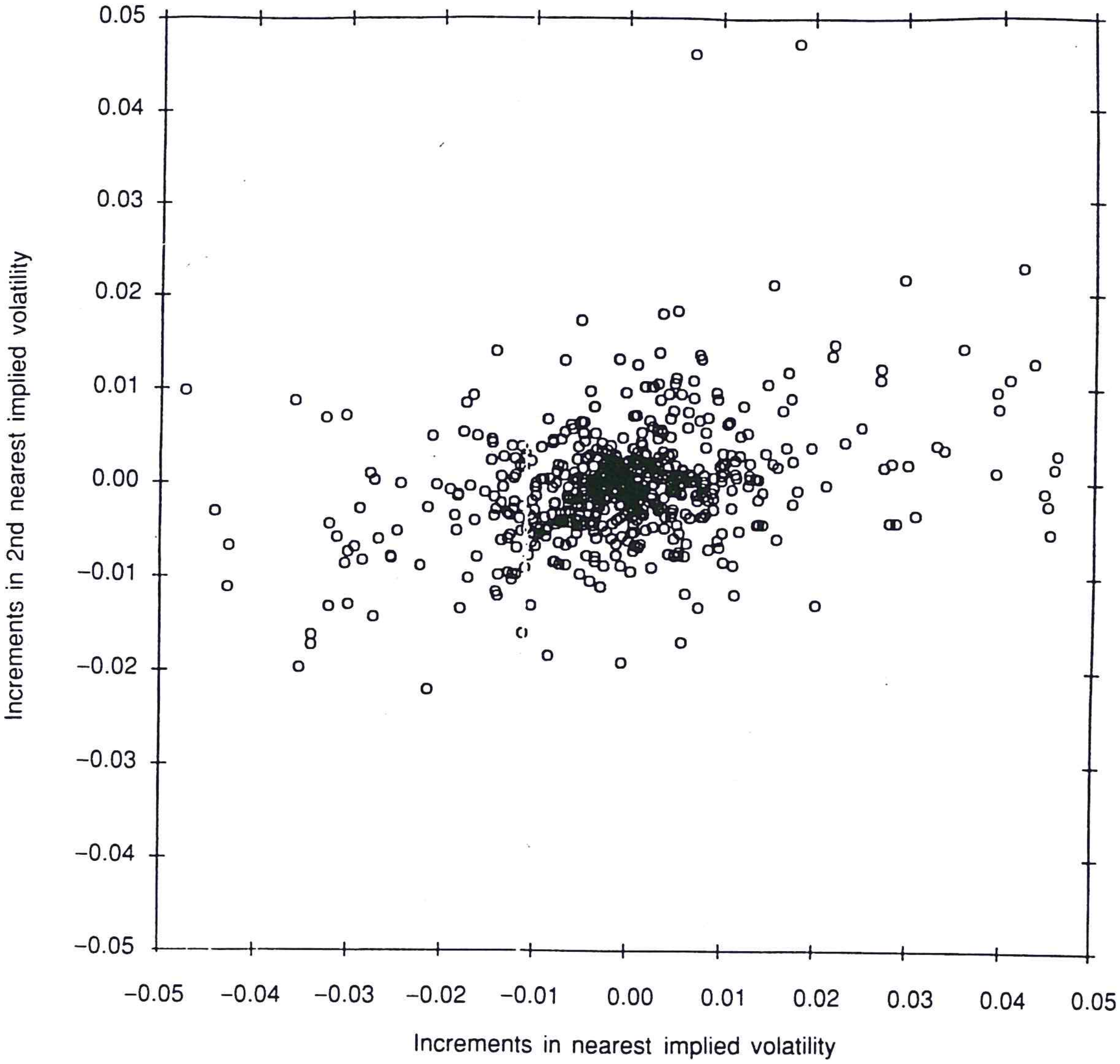

Figure 7.4 Scatterplot of increments in second nearest against nearest implied volatility

proceed under the assumption that the instantaneous volatility does indeed follow that process.

Stein shows, for this process and using the approximation that the implied volatility of an option equals the mean volatility over the time to expiry, that the following elasticity relationship holds if options are rationally priced:

$$(\tilde{\sigma}_{(t,T_2)} - \overline{\sigma}) = \gamma(\rho, T_1 - t)\,(\tilde{\sigma}_{(t,T_1)} - \overline{\sigma}),$$

where

$$\gamma(\rho, T_1 - t) = \frac{(T_1 - t)\,(\rho^{T_2 - t} - 1)}{(T_2 - t)\,(\rho^{T_1 - t} - 1)},$$

and $\tilde{\sigma}_{(t,T_2)}$ denotes the second-nearest implied volatility. Here $\rho = e^{-\alpha}$ and it is assumed that the difference in times to expiry of the two options,

Table 7.1 Correlogram and partial correlogram for nearest implied volatility, $\tilde{\sigma}_{(t,T_1)}$

Lag (days)	*Auto correlation*	*Implied daily auto correlation*	*Partial correlation*
1	0.854	0.854	0.854
2	0.748	0.865	0.067
3	0.703	0.889	0.184
4	0.672	0.905	0.086
5	0.654	0.919	0.108
6	0.619	0.923	−0.009
7	0.580	0.925	0.002
8	0.548	0.928	0.008
9	0.524	0.931	0.017
10	0.499	0.933	−0.001
15	0.412	0.943	0.020
20	0.313	0.944	0.040

Note: all autocorrelation and partial autocorrelation estimates have approximate standard errors 0.043.

$T_2 - T_1$, is constant. (Note that this notation is slightly different from that of Stein. In particular we use γ rather than his β.)

If we measure time in days, we can construct Table 7.2, of values of γ, assuming $T_2 - T_1 = 64$ (true or nearly true in all cases). We have considered three possible values of ρ, representative of the implied daily ρ's calculated for different values of the lag, n, and four values of the time expiry, $T_1 - t$.

It is evident that Stein's use of $\gamma(\rho, T_2 - T_1)$ as a linear end-point approximation to $\gamma(\rho, T_1 - t)$ is really far too crude for our data, as it is likely to be for any options expiring three months apart. Nevertheless we continue to follow his procedure and estimate a market value of γ, say $\tilde{\gamma}$, by ordinary least squares (OLS), making no allowance for variation in time to expiry. The estimated $\tilde{\gamma}$ is 0.373.

Stein compared the estimated $\tilde{\gamma}$ with values in a table similar to Table 7.2 for his data. His estimated $\tilde{\gamma}$ comfortably exceeded values of $\tilde{\gamma}$ for any plausible ρ and all $(T_1 - t)$. He was thus able to conclude that distant implied volatility reacted too much to changes in near implied volatility.

We can draw no such conclusion, since our $\tilde{\gamma}$ is consistent with plausible values of ρ, allowing for the averaging effect of different times to expiry. Thus we cannot use Stein's logic to reject the hypothesis that options are priced rationally in favour of the hypothesis that there is overreaction or

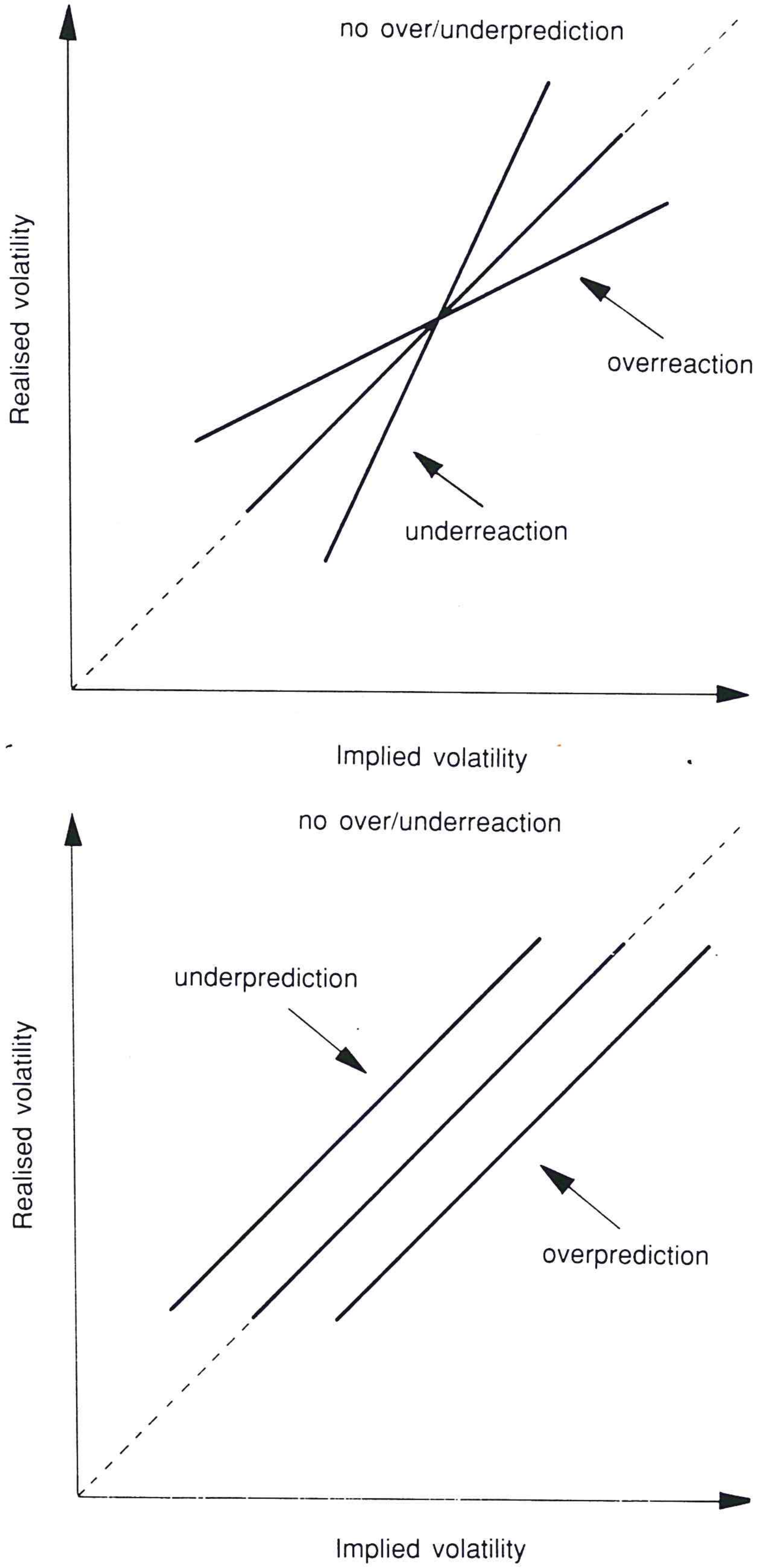

Figure 7.5 Diagram of over/underreaction and over/underprediction

VI Joint behaviour of implied volatility and realised volatility

If the implied volatility is a rational forecast of the realised volatility over the time to expiry a scatterplot of realised volatilities against comparable implied volatilities will result in a distribution of points about a straight line with a gradient of 1 and passing through the origin. If the implied volatility is not a rational forecast of the realised volatility then the distribution of points will deviate from this line. In particular if there is overreaction then the implied volatility will be more variable than it should be. Thus, if the implied volatility is low then it will tend to be too low (under the implicit assumption that the *a priori* expected volatility does not change through time) and on average will be exceeded by the measured realised volatility. Conversely if the implied volatility is high then on average it will exceed the measured realised volatility. Therefore the slope of a fitted regression line will tend to be less than 1. Conversely, underreaction will result in a slope greater than 1.

The lines will have positive and negative intercepts respectively if the implied volatility is still, on average, an unbiased forecast of the realised volatility. If the market tends to underpredict the volatility consistently this will shift the regression line to the left and overprediction will shift the regression line to the right. Figure 7.5 illustrates these points.

We have at least partial data on ten futures contracts and their associated options (expiry dates of December 1987 to March 1990). For each whole number of months to expiry (up to 9) we have computed the implied volatility and the realised volatility (as measured by the sample standard deviation of daily futures returns up to delivery) for each contract for which they were both available. Thus for 1 month to expiry we were able to compute and plot implied–realised volatility pairs for all ten contracts, but down to as few as five pairs for 7, 8 and 9 months to expiry. All data points on the same plot are independent since they relate to different contracts. There is some dependence between the plots with different times to expiry, but the simple form of analysis we have adopted here neither relies on nor makes use of this dependence. As an example, the plot for 1 month to expiry is shown in Figure 7.6.

The first test we performed was a *t*-test on the gradient of the regression of the measured realised against the implied volatility. This allowed us to confirm or reject the hypothesis that the gradient was significantly different from 1 and so confirm or reject the hypothesis of overreaction.

The second test was a paired *t*-test that the means of the realised and implied volatilities were equal. This allowed us the confirm or reject the hypothesis that the means of the distributions were equal and so confirm or reject the hypothesis of over/underprediction.

The third test was a test of the hypothesis that implied volatility was an unbiased predictor of realised volatility, at all levels of implied volatility,

Table 7.2 Values of $\gamma(\rho, T_1 - t)$ for selected values of ρ and T_{l-t}

T_{l-t}	$\rho = 0.85$	$\rho = 0.90$	$\rho = 0.95$
10	0.168	0.207	0.329
30	0.322	0.333	0.403
50	0.438	0.441	0.474
64	0.500	0.501	0.519

underreaction. The drawback to this test procedure as it stands is that the type I error, and thus also the power, are extremely low and uncontrollable. The test is very unlikely to reject in any circumstances and this cannot be changed.

We conclude that Stein's procedure is crude and weak for our purposes and we can identify several desirable points of improvement in the method and its application to our data set:

1. A more careful analysis of the path of instantaneous volatility is required.
2. The approximation that implied volatility exactly equals the mean expected volatility up to expiry should be replaced by something more realistic, at least, by the approximation that it equals the root-mean-squared expected volatility.
3. The method should be able to deal with a wider range of volatility processes.
4. The path of instantaneous volatility should not be inferred to be exactly equal to that of the nearest implied volatility.
5. The trade-off between the generality of the volatility process that can be accommodated and the apparent power of the conclusions that can be drawn should be explored.
6. It should be possible to incorporate the exact elasticity relationship between the near and far implied volatilities into the analysis, rather than relying on observing exceeding of the upper bound; for instance, Stein's procedure might be extended to inference of the market view of ρ, by non-linear regression, rather than of the market view of γ.
7. It would be desirable to allow some measure of overreaction to be estimated.
8. Some allowance must be made for errors and lack of fit.

In our view these limitations can be overcome only by an explicit model for the arrival of, and reaction to, volatility information; and a firmer and more generally applicable basis for the statistical procedures. We hope to pursue this approach in future work.

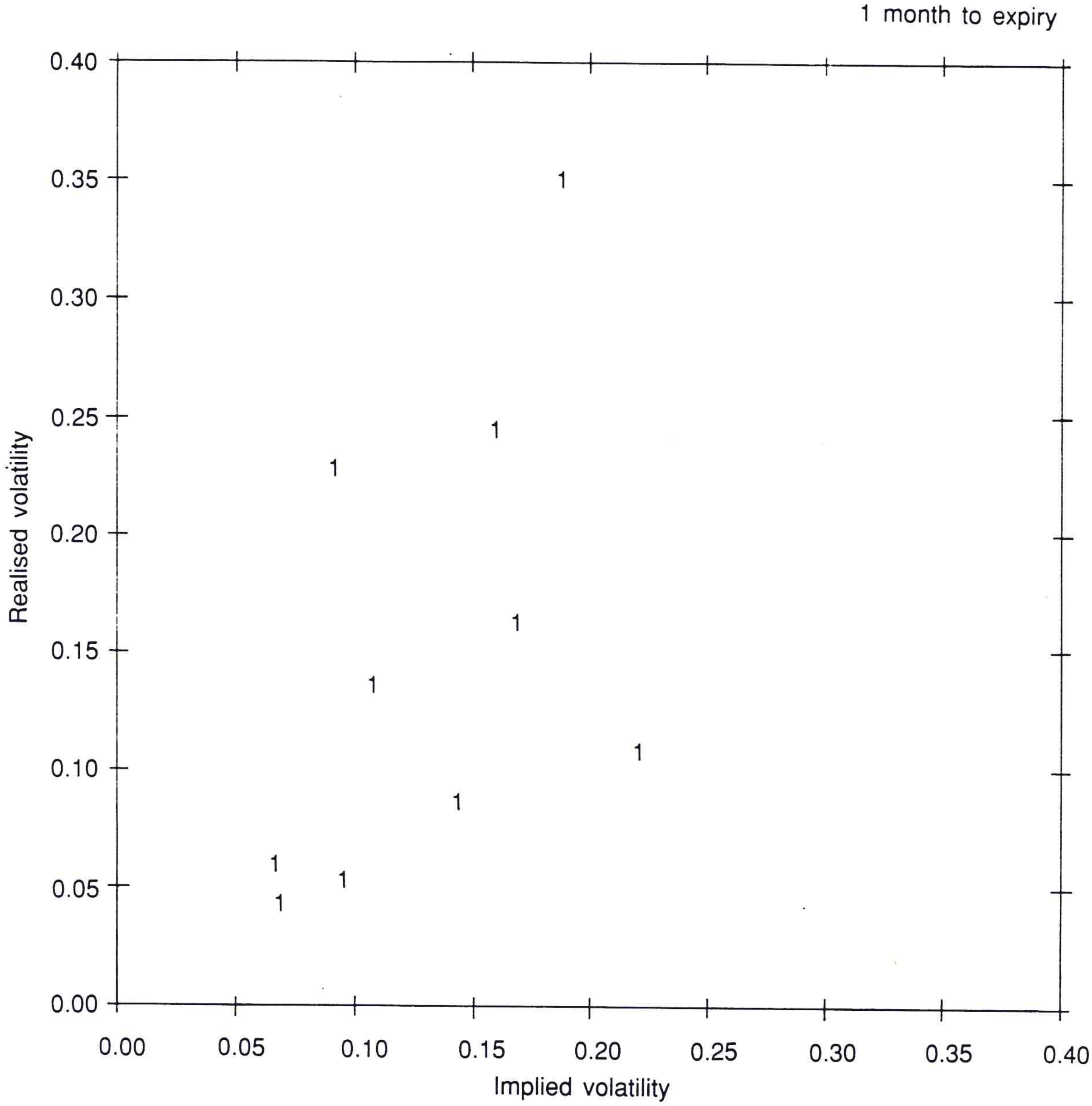

Figure 7.6 Scatterplot of realised volatility against implied volatility

against the alternative that either the intercept of the regression is non-zero or the slope not equal to 1. This was performed by comparing the relevant mean square ratios with the $F_{2,n-2}$ distribution where n is the number of points.

We cannot reject the hypothesis that the gradient is 1 and so we reject the overreaction hypothesis for the 1 to 6 months to expiry data sets. For 7 and 9 months to expiry (and almost for 8 months), we can reject the hypothesis that the gradient is 1 at the 5% significance level. As discussed in Section III.3, however, for this length of time to expiry there is generally little trading in the contracts and the prices are computed from the Black model using the implied volatility of the nearer, more liquid, options. This would lead to the apparent overreaction we find, but does not imply inefficiency since it would not necessarily be possible to trade at those prices.

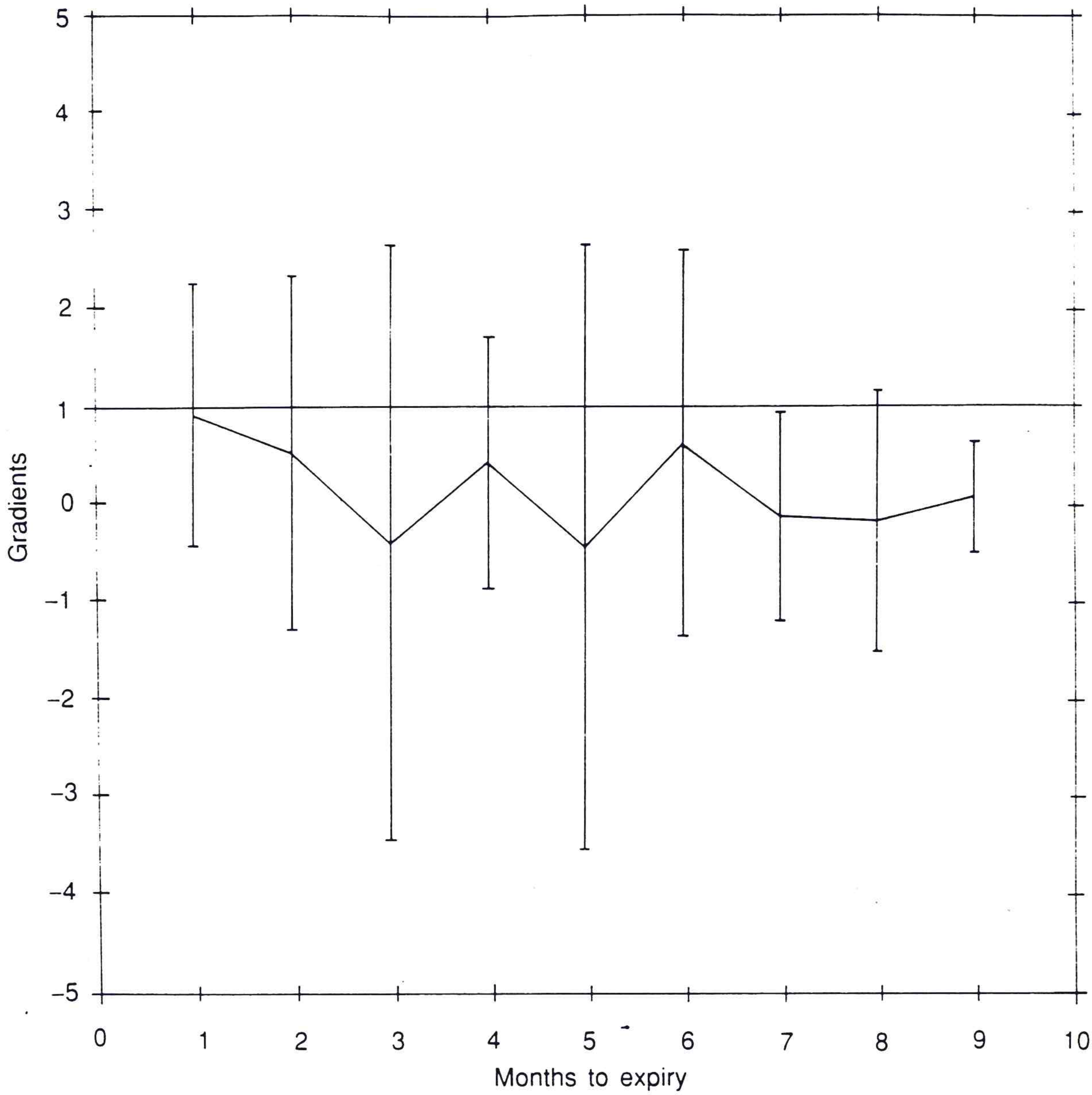

Figure 7.7 Gradients of realised against implied volatility

Figure 7.7 summarises the results with 95% confidence intervals for the slopes. There is some evidence of a slight downward trend as time to expiry increases and this may be indicative of overreaction. However, a more sophisticated test on the combined data taking into account the dependencies is required before any conclusions can be reached.

The second test does not allow us to reject the hypothesis that the means are equal at the 5% significance level for any month to expiry. Figure 7.8 summarises the results with 95% confidence intervals for the mean differences between the realised and the implied volatility. There is evidence of a downward trend from slight underprediction at short times to expiry decreasing to no over/underprediction at around 9 months to expiry. Again a more sophisticated test on the full data set is needed before any definite conclusions can be drawn.

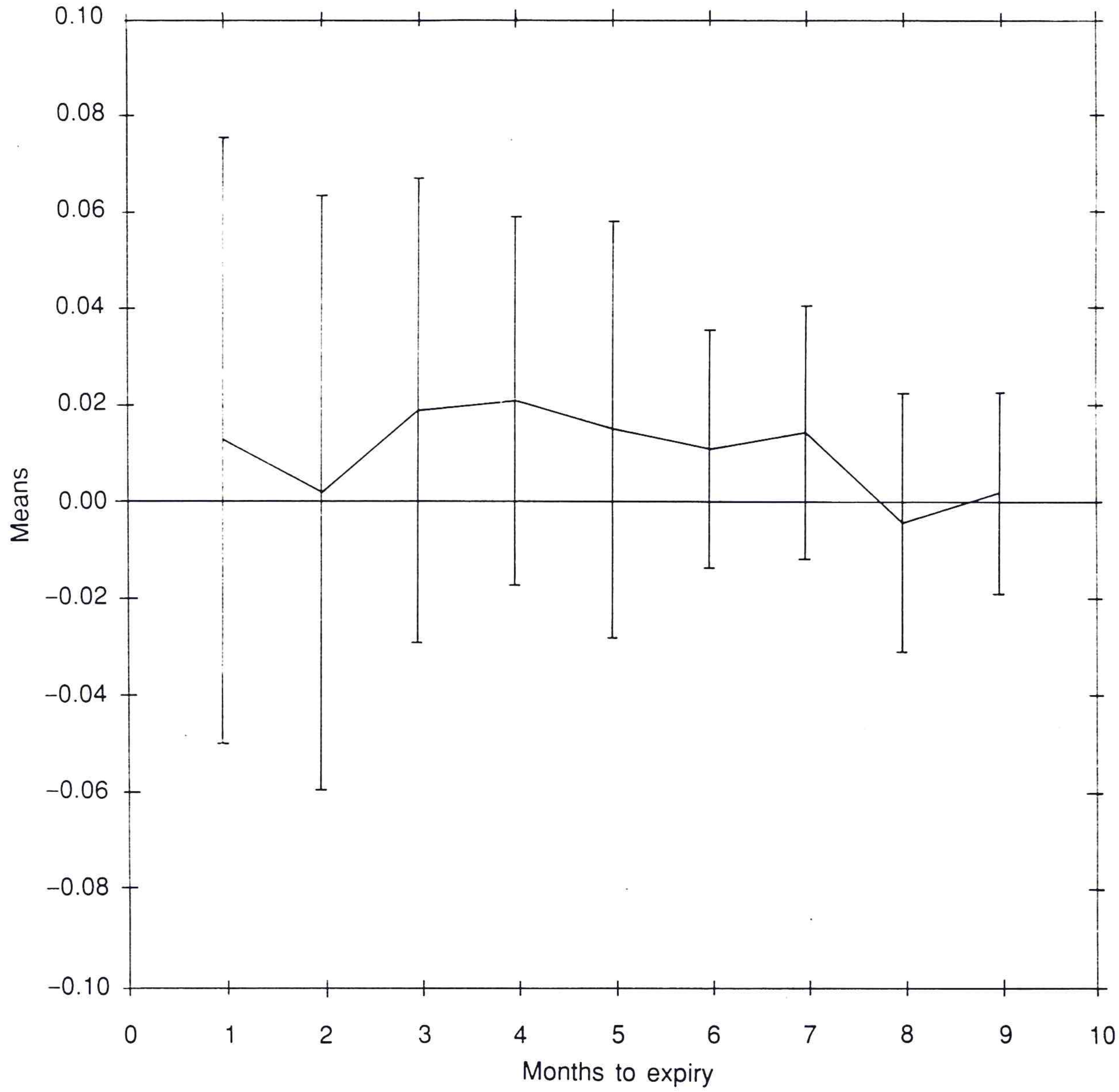

Figure 7.8 Means of differences between realised and implied volatility

The joint tests do not lead to rejection of rationality at the 5% significance level, except for 7 and 9 months to expiry. As explained above, however, prices at those lengths of time to expiry are not considered to be realistic.

The results are given in tabular form in Table 7.3. The *p*-value shown for each test indicates the significance level at which the hypothesis would be just rejected.

Some of the assumptions underlying these tests are clearly questionable (e.g. that the variance of the prediction error does not depend on the level of the implied volatility) and more sophisticated and detailed analysis of the plots is certainly possible. In practical terms, however, there seems little to be gained from such analysis since the small numbers of points means that any analysis will be very imprecise and almost certainly inconclusive. What is needed, if the comparison of implied and realised volatilities

Table 7.3 Analysis of implied and realised volatilities

a. Over/underreaction

		Estimate of slope			
No of months to expiry	*No of points*	*Lower 0.025 confidence limit*	*Point estimate*	*Upper 0.975 confidence limit*	*p-value for slope = 1*
1	10	–0.43	0.91	2.24	0.88
2	9	–1.29	0.51	2.32	0.55
3	9	–3.45	–0.41	2.63	0.31
4	9	–0.87	0.42	1.71	0.32
5	8	–3.54	–0.45	2.64	0.30
6	7	–1.36	0.61	2.58	0.63
7	5	–1.20	–0.13	0.94	0.04
8	5	–1.52	–0.18	1.16	0.07
9	5	–0.50	0.07	0.64	0.01

b. Over/underprediction

		Estimate of mean of realised – implied volatility			
No of months to expiry	*No of points*	*Lower 0.025 confidence limit*	*Point estimate*	*Upper 0.975 confidence limit*	*p-value for mean = 1*
1	10	–0.050	0.013	0.076	0.65
2	9	–0.060	0.002	0.063	0.95
3	9	–0.029	0.019	0.067	0.39
4	9	–0.017	0.021	0.059	0.25
5	8	–0.028	0.015	0.058	0.45
6	7	–0.014	0.011	0.035	0.34
7	5	–0.012	0.014	0.040	0.21
8	5	–0.032	–0.005	0.022	0.65
9	5	–0.019	0.002	0.022	0.85

Table 7.3 (Cont.)

c. Joint rationality test

No of months to expiry	*No of points*	*p-value for intercept = 0 slope = 1*
1	10	0.90
2	9	0.82
3	9	0.41
4	9	0.32
5	8	0.11
6	7	0.58
7	5	0.05
8	5	0.13
9	5	0.03

is to be pursued as a paradigm for analysis, is the development of some more sophisticated procedures which take full and correct account of the paths of implied and realised volatilities up to expiry.

VII Summary and conclusions

We have presented here the results of an initial study of the rationality of pricing of LIFFE Short Sterling options against the particular alternative that the market's view of future volatility overreacts (or underreacts) to new information. This question has important implications for the efficiency of the options market.

On the basis of the exploratory analysis we have performed to date, we are unable to reject the hypothesis that the options are rationally priced. The two methods of analysis used, however, are simple, approximate and rather weak and a number of areas of potential improvement and other approaches can be identified. Therefore we do not regard our conclusion from this analysis as a strong one and there remains the distinct possibility in our view that improved methods of analysis would lead to a different conclusion.

The first type of analysis is based on the joint behaviour of implied volatilities of options expiring at different times. In this exploratory analysis, we have followed the first procedure of Stein (1989). This is based on the theoretical elasticity relationship under rationality of a distant implied volatility to a near implied volatility, based on a fitted mean-reverting AR(1) model of volatility. Stein, for his data set, was able to demonstrate that empirical values of this elasticity exceeded upper bounds for the elasticity under rationality and was thus able to conclude the existence of overreaction. We are unable to demonstrate the upper bound is exceeded in our data set, so are unable to reject rationality in the same way as Stein. This is not altogether surprising since the elasticity is more sensitive to the time to expiry for Short Sterling options (which expire three months apart) than for those options studied by Stein (which expire one month apart): this leads the test to have very low type I error and very low power. There are other approximations and simplifications in Stein's method and there is no explicit model of option-price behaviour to support its modification and easy general application. The basis of this should be an explicit model of the arrival of, and reaction to, new volatility information and we hope in future work to develop such a model. There is another reason to be wary in applying even a more general variant of Stein's approach, namely that our options relate to different futures contracts, not the same underlying asset. Futures price movements tend to have similar dispersion and to be highly correlated, however, so this appears not be a reason for major concern.

The second type of analysis is the comparison of *ex post* measures of realised volatility over the time to option expiry with *ex ante* predictions of volatility provided by the option price implied volatility. If options are rationally priced, then the latter should be approximately unbiased predictions of the former. If overreaction exists, then observed implied volatilities will be more variable than they should be – when implied volatility is low, it is too low, and when it is high, it is too high. Then a simple linear regression of realised volatilities against comparable implied volatilities should give a regression coefficient of 1 under rationality, but less than 1 under overreaction. We have analysed on this basis the (measured) realised and implied volatility at whole months to option expiry, separately for each number of months to expiry. We are unable using this procedure to reject the hypothesis of rationality for any number of months to expiry. The confidence intervals for the regression coefficients are very wide, however, because of the small numbers of data points (10 maximum). There is some scope in principle for getting more out of the data, by using the complete time-paths of implied and realised volatilities for each contract. Difficulties with this more subtle approach are posed by the unobservability of actual volatility (and so the need to estimate it) and the need to take account of the structure of errors consequent on this and the dependence of data at different times to expiry.

There are three further possible approaches to testing for overreaction that we have not investigated.

The first approach is to test that the implied volatility term structure evolves randomly in time. This is the basis of Stein's (1989) second test.

He is able to use his linear end-point approximation (which is too coarse for our data) and our data series is rather short to follow his procedure exactly, so some subtle adaptation seems to be required. Although the procedure is based on a non-parametric description of the implied volatility term structure, some implicit restriction is placed on the forms of the term structure that can be inferred by the number of option expiry times.

The second approach is to examine the behaviour through time of at the money delta hedge returns. If options are rationally priced, then broadly speaking there should be no autocorrelation in these returns. If, however, overreaction (underreaction) exists, then they will be negatively (positively) autocorrelated. Care would be needed, since negative serial correlation can be caused also by such features as thin trading (Scholes and Williams, 1977), bid–ask spreads (Roll, 1984) and price discreteness (Gottlieb and Kalay, 1985). An attraction of this approach is that no relationship would need to be assumed between options of different expiry times, which is especially problematical in our case since we are concerned with different futures.

The third, as yet untried, approach is the most direct and concrete. If option prices consistently overreact (or underreact), then an intelligent investor who is *a priori* aware of the market's behaviour or is prepared to observe and learn it should be able to make money by a suitable trading strategy (neglecting transactions costs for the moment). This assumes, of course, that trading would be possible at recorded settlement prices and that his trading would have no effect on the market. There is some similarity here with the approach just described since a suitable trading strategy would almost certainly rely on the formation and trading of at-the-money delta hedges.

There are thus at least five possible approaches to testing overreaction, of which we have applied simple variants of two to Short Sterling options. By refining and applying these approaches, there is scope for reaching sharper and more confident conclusions on the presence or absence of overreaction in Short Sterling options. This seems a very pertinent empirical question for all longer-dated options. At theoretical and methodological levels, there is a need to develop a better understanding of the properties of, relative merits of, and links between the five approaches described, and whether anything might be gained from a combined approach.

Notes

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