The Delivery Options in Bond Futures Contracts: An Empirical Analysis of the LIFFE Long Gilt Futures Contract

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Abstract

Using data on the London International Financial Futures Exchange long gilt futures contract, we look at the implicit quality option contained in that contract. We look at three different 'valuations' of the quality option; an ex-ante value given by the excess of the forward price of the cheapest to deliver bond over its conversion factor multiplied by the futures price; an ex-post value of the payoffs to a strategy of buying and holding a short-futures long-forward position and delivering the cheapest bond at the expiration of the futures contract; and a Monte Carlo simulated value. We show that the delivery experience for the LIFFE contract is consistent with the delivery incentives created by the calculation of the LIFFE price factors.

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Introduction

Under the terms of a futures contract the short side frequently has options as to what (quality option), when (timing option), where (location option), and how much (quantity option) is delivered. Examples of such delivery options in traded futures contracts include the Treasury Bond Futures Contract traded on the Chicago Board of Trade (CBOT) which permits the short to deliver any one of a pre-defined set of long term Government bonds at any time during the delivery month; wheat futures contacts, traded at the CBOT, which permit the short to deliver any one of eleven different types of wheat at either Chicago or Toledo; and certain commodity futures contracts which provide flexibility regarding the amount delivered.

If these delivery options have any value to the holder of the short position then, in market equilibrium, the futures price should be bid down, by the value of the implicit option, to compensate the long for the delivery risk. Recent research, eg, Hegde [1990], Hemler [1988], and Gay and Manaster [1986] on quality options in bond futures contracts using US data, has investigated the affect on the futures price of these options.

This paper looks at the delivery option implicit in bond futures contracts. We look particularly at the implicit quality option contained in the Long Gilt Futures contract traded on the London International Financial Futures Exchange (LIFFE). The standard delivery asset for this contract is a 20-year maturity government gilt with a 9% coupon. A number of bonds with varying characteristics, however, are eligible for delivery with LIFFE publishing price factors to relate each of the bonds to the delivery asset. The delivery option exists because these factors do not perfectly reflect the relative values of each deliverable bond.

In this paper we present two empirical estimates of the value of the delivery option imbedded in the LIFFE contract. The first is an ex-ante value given by the excess of the forward price of the cheapest to deliver bond over its conversion factor multiplied by the futures price. The second is an ex-post value equal to the payoffs to a strategy of buying and holding a short-futures long-forward position at the initial time and delivering the cheapest bond at the expiration of the futures contract. Both of these measures have been used in the analysis of delivery options in exchange traded US bond futures contracts, eg Hegde[1990], allowing us to compare our results.

One of the objectives of this paper is to develop an approach to determine the value of the delivery option within an option theoretic framework. In order to achieve this we develop a Monte Carlo simulation model to simulate changes in the term structure of interest rates over the life of the option. End of period bond prices are then inferred from this term structure allowing us to derive values for the delivery option.

The paper is organised as follows. In section I we look at the institutional characteristics of the LIFFE long gilt futures contract. In section II we describe, in detail, the quality and timing options implicit in this futures contract, we also describe other delivery options implicit in other bond futures contracts. In section III we describe the price factors used by LIFFE to relate each of the deliverable bonds to the benchmark asset. We look at how there might exist incentives to deliver one bond over another, caused by these factors, if the particular term structure of interest rates, assumed by LIFFE (ex-ante), is inconsistent with the observed term structure at delivery (ex-post). In section IV we develop and describe pricing models for the value of the quality option. An empirical assessment of the delivery experience and of the value of the quality option during the period January 1987 to December 1988 is performed in section V. Our conclusions and ideas for further research are contained in section VI.

I. The LIFFE Long Gilt Futures Contract

Nominally, the LIFFE long gilt futures contract is an agreement between the buyer and the seller to exchange a British Government gilt-edged security with £50,000 face value, a 9% coupon, and 20 years to maturity for cash at a specified date. The exchange designated delivery months are March, June, September, and December. In reality the 9% coupon bond need not exist and the short position may choose to make delivery on any business day in the delivery month. Additionally, since December 1989 onwards, any gilt with a redemption date between 1st January 2003 and 31st December 2009 is eligible for delivery if they are not trading cum-dividend. Contracts which matured before September 1988 had a 12% nominal coupon. LIFFE changed the nominal coupon at this time to reflect the prevailing level of interest rates. Deliverable bonds are not eligible for delivery during the three week special ex-dividend period. At delivery, cash is exchanged for one of the gilts in the deliverable set; the amount of cash receivable by the short depending on the prevailing futures price and which of the eligible gilts is delivered. The invoice price is calculated by multiplying the prevailing futures price by a price factor and adding the accrued interest. The function of the price factor is to bring the various deliverable stocks with different coupons and maturities onto a common basis for delivery. Price factors and the cheapest gilt to deliver gilt are discussed below.

The long gilt futures contract stipulates a three-day delivery sequence. On the first day, notice day, the short gives notice to the clearing house of his intention to make delivery and of which gilt he wishes to deliver. This has to be done by 11.00am when the

Exchange Determined Settlement Price (EDSP) is calculated. On the following (second) business day the clearing house assigns the short's delivery notice to the oldest long position. The actual delivery of the nominated gilt and payment takes place on the third business day.

II. The Delivery Options in Bond Futures Contracts

The quality option, in the LIFFE long gilt futures contract, derives its value from the delivery pricing conventions adopted by LIFFE who allow the short futures position to satisfy the contract by delivering one of a variety of gilts. The underlying instrument for a Long Gilt futures contract is £50,000 par value of a hypothetical 20 year 9% coupon bond. Prices and yields of Long Gilt futures are quoted in terms of this hypothetical bond. This gilt does not actually exist and so to make delivery equitable to both buyer and seller, and to tie cash to delivery prices, LIFFE publish conversion (price) factors to determine the invoice price of each of the acceptable deliverable bonds against the bond futures contract. This pricing method is discussed below. The delivery pricing mechanism prices each eligible bond to yield 9% to maturity as of the first day of the delivery month. This price factor is calculated by the exchange before trading for the relevant contract month has commenced and applies for the life of the contract.

An important point in LIFFE's setting of price factors is that the method discounts bond cash flows at 9% regardless of the level and shape of the market yield curve at delivery. If market rates are above or below 9%, instead of exactly 9%, the price factors established by LIFFE will be slightly 'inaccurate' and will result in an incentive, for the short, to deliver one bond over another. Given this pricing convention for multiple deliverable bonds it can be argued that the short's right to choose the optimal delivery bond acquires value.

The LIFFE long gilt futures contract ensures that the short has some flexibility regarding the timing of actual delivery, as delivery may take place on any delivery day during the delivery month. Empirical results by Hemler [1990] suggest that players in the Chicago T-Bond futures contract tend to postpone delivery towards the end of the delivery month. However, in determining the conversion factor for a deliverable bond, its term to maturity is counted from the first day of the delivery month, so the term to maturity used in the price factor can be up to 30 calendar days longer than the actual maturity of the delivered bonds. The decision as to when, during the delivery month, to deliver against a futures contract will critically depend upon the relationship between the running yield on the gilt and the financing cost of the position. If an investor earns more on the gilt position in accrued interest than he pays to finance the position, it makes sense to maintain the position as long as possible, and so delivery will be at the end of the month. The timing option therefore has some value.

Boyle [1989] analyses the interaction of the timing option with the quality option. Using standard arbitrage arguments he deduces that the optimal strategy for the short, when there is just one deliverable asset, is to deliver at the first permitted opportunity. One of the assumptions in his analysis is that the deliverable asset pays no coupons and so the intuition is that the short, who has to deliver during the delivery month, receives no gain by waiting and so the timing option has no value. If there is more than one deliverable asset there is an interaction between the timing option and the quality option; one asset could be the cheapest to deliver without the timing option whilst another asset could be the cheapest to deliver when delivery is allowed during the delivery month. The timing option when there is more than one deliverable asset, therefore, has value. We should expect there to be an even greater interaction between the timing and quality option when the assumption of zero-coupon bonds is relaxed.

We now mention two extra types of delivery option which are present in the T-Bond futures contract, but not in the LIFFE futures contract. They represent important institutional differences between the two contracts and may aid our comparison of empirical results later in the paper. The first is an end-of-the-month option and represents the option to deliver during the last seven business days of the month (when the futures have ceased trading) at a price based on the last futures settlement price. The second is termed the Wild card option and is the option to decide in a 6 hour period after the futures price has been settled whether or not to deliver at that established price.

Most of the evidence regarding the presence of the quality option in bond futures contracts is centred on experience in the US markets and especially the US Treasury Bond Futures contracts. This evidence suggests that it can affect the futures price, in some contracts substantially. As we noted above there exist differences in the institutional characteristics of the LIFFE long gilt futures contract and the Chicago T-Bond futures contract which create some extra, quite important, delivery options for the latter contract. In the next part of this section we present a summary of results from previous US studies.

Hemler [1990] estimates the value of the quality option in the US T-Bond futures market during the period 1977-1986. Hemler's analysis uses three estimation techniques to value the quality option; historical payoffs from exercising the quality option at delivery; option values prior to delivery using a T-Bond futures pricing model that explicitly incorporates the quality option; and option values using an n-asset (n=2,3) exchange option pricing formula. The author finds that; the payoff which is obtained by switching from the bond which is cheapest to deliver 3 months prior to delivery to the one which is cheapest to deliver at the time of delivery is, on average, less than 0.3 percentage points of par; option values obtained from the bond futures pricing model average less than 0.2 percentage points of par with 3 months to maturity; and option values obtained using the n-asset exchange asset option pricing formula averaging 0.7 and 1.2 percentage points of par for n=2,3 respectively[1].

Boyle [1989] develops a procedure for computing the impact of the quality option when there are several deliverable assets, and carries out numerical simulations to illustrate this impact. The author's conclusions are that the impact of the quality option increases as the number of deliverable assets increases, but as the correlation coefficient between the assets increases there is a reduction on the impact of the quality option. A high correlation coefficient can still mean that the quality option has a nontrivial effect although the number of deliverable assets will only have a marginal affect on the exchange option value.

Hegde [1990] working with weekly data on 37 delivery quarters from October 1977 to December 1986 attempts to determine the value of the implicit quality option using three empirical approaches. Firstly, he measures the excess of the forward price of the cheapest to deliver bond over its conversion factor times futures price. His results indicate that three months prior to delivery the average value is 0.4 percentage points of par. Secondly, he measures the payoff to the short of the current cheapest bond at the expiration of the futures contract rather than the cheapest bond at the initiation of a buy-and-hold strategy. This approach values the quality option at 0.33 percentage points of par. Finally, Hegde sums the profits accumulated by continuously rolling over the long forward position from the current cheapest to deliver bond to the next cheapest to deliver bond over the life of the futures contract. The mean payoff to following this strategy is found to be 2 percentage points of par. The author attributes the differences amongst the three sets of estimates to be largely due to non-synchronous spot-futures data.

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1 Hemlers values for the n-asset exchange option are mistated. He assumes that all bonds are expected to appreciate at the riskless rate of interest rather than the riskless rate of interest less the coupon yield on the bond. I wish to thank Robert Whaley for pointing this out to me.

III. LIFFE Price Factors

In respect of each delivery month LIFFE publish a list of deliverable bonds which constitute valid stock which the short can deliver to satisfy the long gilt futures contract, along with a list of price factors. The price factor (PF) for each deliverable gilt is given by P(9)/100 which is calculated as follows[2]:

$$\frac{P(9)}{100} = \frac{1}{1.045^{x/182.5}}\left[c^* + \frac{c}{0.09}\left(1 - \frac{1}{1.045^n}\right) + \frac{100}{1.045^n}\right] - \frac{c}{2}\frac{y-x}{182.5} \qquad (1)$$

c = coupon rate in percentage terms per year, payable at half yearly intervals.

c* = coupon rate in percentage terms payable at the next payment date.

n = number of half years from the next payment date to the relevant redemption date.

x = number of days from and including the first day of the delivery month up to but excluding the next payment date.

y = number of days after the previous payment date up to and including the next payment date.

The price factor for each gilt is, therefore, the cash flows of the gilt discounted at a rate of 9% and divided by 100 (the present value of the 9% notional bond discounted at 9%). This method assumes that there exists a flat term structure of interest rates of 9%. At market yields other than 9%, 'true' price factors will differ from that calculated by LIFFE. These 'true' price factors are an ex-post measure of the price factor. The LIFFE factors are ex-ante measures; differences between them provide orders of magnitude for incentives to deliver one gilt in the deliverable set over another. Using 9% as the discount

2 The above formula applies only to deliverable stock which would be cum-dividend (the buyer gets the dividend) if it were to be delivered on the earliest day of the delivery month. Adjustments, to the price factor, have to be made when the deliverable stock would be delivered ex-dividend if it were to be delivered on the earliest day of the delivery month. Price factors for deliverable stock with an optional redemption date also have to be adjusted. See LIFFE [1988].

rate when rates are above 9% will cause distant cash flows to be overvalued. Low coupon gilts which have much of their cashflow in the principal repayment will, therefore, get higher factors than they should, as will longer dated bonds. Conversely, if rates are below 9%, using 9% as the discount rate will result in the undervaluation of distant cashflows. Gilts with earlier repayment dates and those with high coupons will, therefore, get higher factors than they should. We examine the level of incentives to deliver one gilt over another caused by the level of discounting in the LIFFE price factors later in this section.

Approximately 9 different gilts are currently eligible for delivery of the LIFFE Long Gilt Futures contract, but during any one delivery period there is just one which will create the most profit or the minimum loss for the seller who makes delivery. This bond is referred to as the "cheapest to deliver". Outside of the delivery period the "cheapest to deliver" is calculated as the gilt which is expected to be the most profitable, or provide the minimum loss, for the short to deliver. Any market participant who uses the long gilt futures contract, either for hedging or speculation, should be concerned with the cheapest to deliver gilt as at all times there will be a close relationship between the CTD and the futures contract because it is assumed that the CTD will be delivered when the future expires. As a result the CTD will be most commonly used as the cash gilt in basis trading and as a consequence will be highly liquid. The cheapest to deliver gilt does not necessarily remain constant over time with changes being caused by changes in market prices and financing costs.

To examine the gilt delivery incentives caused by the level of discounting in the LIFFE price factors we look at a more appropriate (ex-post) method to determine the price factor[3]. This method involves assuming all bonds have the same yield and looking at the ratio of the cash flows of the gilt, valued at current (long term) interest rates, relative to a

3 A similiar method has also been used by Arak, Goodman, and Ross[1986] to examine biases in conversion factors published by the Chicago Board of Trade for its U.S. Treasury bond futures contract.

9% coupon bond valued at the same interest rates. To calculate the new price factors we therefore calculate PF, as defined in equation (1), with 0.09 replaced by the current long term interest rate, r, and with 1.045 replaced by (1+r/2). The term 100 is replaced by the following:

$$\sum_{i=1}^{40} \frac{4.5}{(1+r/2)^i} + \frac{100}{(1+r/2)^{40}} \qquad (2)$$

We look at three different long term interest rate scenarios 7%, 9%, and 11%, and apply them to coupon bonds of 7%, 9%, and 11% with maturities of 10, 15, and 20 years. Table 1 shows the differences in factors calculated by the two methods. The 'true' factor is the price factor calculated by the new method. If the LIFFE price factor is larger than the 'true' factor, leading to a positive figure in the "Diff." column of Table 1, the gilt will be more desirable for delivery, assuming that all deliverable gilts have the same yield.

Table 1 shows that when interest rates are above 9% the LIFFE price factor calculation favours delivery of a low coupon bond with a long maturity (the largest positive difference is for a 7% coupon 20 year bond). When interest rates are below 9% the factor favours delivery of a high coupon bond with a short maturity (the largest positive difference is for a 11% coupon 10 year bond). As we would expect, when interest rates are at 9% the LIFFE and 'true' factors are the same for all gilts.

Another issue which may also influence delivery caused by the LIFFE price factors is the shape of the yield curve. An upward sloping yield curve, during the range of deliverability, will make low coupon gilts attractive to deliver, whilst a downward sloping yield curve will make high coupon gilts attractive to deliver.

To investigate any delivery incentive that the shape of the yield curve may cause we look at the following definition of the 'true' price factor.

$$\frac{\sum_{i=0}^{n-1} \frac{\text{coupon}/2}{(1+f_0/2)(1+f_1/2)\ldots(1+f_i/2)} + \frac{100}{(1+f_0/2)(1+f_1/2)\ldots(1+f_{n-1}/2)}}{\sum_{j=0}^{39} \frac{4.5}{(1+f_0/2)(1+f_1/2)\ldots(1+f_j/2)} + \frac{100}{(1+f_0/2)(1+f_1/2)\ldots(1+f_{n-1}/2)}} \qquad (3)$$

where f_0 = current rate for a six month security;

f_i = forward interest rate for a six month security beginning i six-month periods from now.

The numerator and denominator of the 'true' price factor are both discounted by the current and relevant forward rates. Table 2 illustrates the effect of the yield curve on delivery incentives.

We use the same coupon bonds with the same maturities as in the flat term structure analysis and look at four interest rate environments; a high interest rate environment with an upward sloping yield curve and one with a downward sloping yield curve, and a low interest rate environment with the same sloping environments.

The maturity effects are as expected from our previous discussion. For a low coupon bond in a high interest upwardly sloping yield curve environment long maturities are desirable. For a high coupon bond in a low interest rate downward sloping environment short maturities are preferred.

IV. Pricing Models for the Quality Option

During the rest of the paper we will make the following assumptions and adopt the following notation.

We assume, along with most other academic papers on the quality option, that all contacts are delivered on some predetermined day. We use two delivery day assumptions although we report results in this draft only for the first. One is that delivery occurs on the first eligible day for delivery, ie, the first business day of the delivery month. The second is that delivery occurs on the last possible day for delivery. The assumption of a predetermined delivery day is a shortcoming of this draft, and one which will be addressed in future drafts. Let T denote the delivery day of the given delivery month for the forward and futures contracts. Let t denote time such that t ú T. We will assume that there are i = 1 to n gilt contracts which are eligible for delivery. For each t we define the following:

$S_i(t)$ = Spot price at t for ith issue bond.

$F(t,T)$ = Futures price at t for bond futures contract requiring the short to deliver at T.

$f(i,t,T)$ = Forward price of bond i at time t.

$C_i(t')$ = Coupon payment made on an ith issue bond at time t'.

$I_i(t)$ = Accrued interest at time t for an ith issue bond.

$r(t,T)$ = Risk-free rate of return over the period [t,T].

$PF(i)$ = Price factor for invoicing an ith issue bond delivered at time T.

$V_i(t,T,n)$ = Forward value of an (n,i)-quality option, which is the shorts right to deliver jth issue bonds rather than the ith issue bonds at the time of delivery.(i,j ε {1,2,....,n}).

$F(.)$, $S_i(.)$, $C_i(.)$, $I_i(.)$ will be expressed in percentage points of par.

We assume the following:

A(1) Capital markets are perfect in that there are no taxes, transaction costs or short selling restrictions. Investors have equal access to all relevant information and their activities have no effect on security prices which are taken as given.

A(2) All futures contracts expire on the same business day.

A(3) Corresponding futures and forwards are priced indistinguishably close.

Firstly, in the next part of this section we present two methods to estimate quality option values: an ex-ante value as given by the excess of the forward price of the cheapest to deliver bond over its conversion factor multiplied by the futures price; and an ex-post value equal to the payoffs of exercising this option at delivery. We then briefly describe pricing models based on an exchange option pricing formula which a number of authors have recently postulated. Finally, we present a Monte Carlo simulation model which generates changes in the term structure of interest rates, allowing us to simulate the prices of the gilts in the deliverable basket and derive a value for the delivery option.

A. Ex-Ante Value of Delivery Option based on a Bond Futures Pricing Formula

At delivery, the delivery proceeds to the short realised by delivering bond i are given by the following;

$$F(T)PF(i) - S_i(T) \tag{4}$$

The optimal delivery bond, ie, the cheapest to deliver, before delivery, at time t is that bond which maximises the anticipated delivery proceeds:

$$\max_i [F(t,T)PF(i) - f(i,t,T)] \quad (5)$$

Let bond i be the cheapest to deliver bond. The short who buys the cheapest bond and sells futures contracts in the proportion to the conversion factor of the cheapest bond is in effect buying forward the delivery option at an exercise price of F(t,T)PF(i). Market efficiency implies that the forward value of the delivery option is given by the difference between the forward price of the cheapest bond and the exercise price of the futures contract, ie:

$$V(t,T) = [f(i,t,T) - F(t,T)PF(i)] \quad (6)$$

Forward prices on the cheapest to deliver bonds are not usually available. We can, however, estimate the forward price of a bond i by a standard no-arbitrage argument[4].

$$f(i,t,T) = [S_i(t)+I_i(t)](1+r(t,T))^{(T-t)} - C_i(t')(1+r(t',T))^{(T-t')} - I_i(T) \quad (7)$$

Rearranging equation (6) we get:

$$F(t,T) = \frac{f(i,t,T) - V_i(t,T)}{PF(i)} \quad (8)$$

It can be clearly seen from equation (8) that the presence of the quality option lowers the futures equilibrium price; the lower price compensating the long for the uncertainty of the bond to be delivered by the short.

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4 See for example Duffie [1989].

B. Realised Payoff from Exercising Quality Option at Delivery

Our second empirical estimate of the value of the quality option is an ex-post measure and is generated by looking at the realised payoffs obtained from exercising the option at delivery. Suppose that at time t gilt i is the cheapest to deliver. At delivery gilt j is the cheapest to deliver. The payoff from exercising this option at delivery, ie, from switching over from bond i at t to bond j at T, is therefore equal to

$$\left.\begin{array}{ll} 0 & i=j \\ [F(T)PF(j)-S_j(T)] - [F(T)PF(i)-S_i(T)] & i\neq j \end{array}\right\} \qquad (9)$$

C. Pricing Formulas based on the option to exchange one asset for another.

We now describe delivery option valuation formulas based on an exchange option pricing formula (ie an option whereby the holder can exchange any asset j = 1,...,n for an asset i at some specified time.). Both Boyle [1989] and Hemler [1990] derive pricing models in this framework. To see that the quality option implicit in a bond futures contract qualifies as an exchange option consider the decision faced by the short at the time of delivery.

If the short delivers ith issue bonds at time T, the invoice amount that he receives is PF(i) F(t,T) + I_i(T), and the amount of bonds that he gives up are worth S_i(T)+I_i(T). His net profit is therefore the difference PF(i) F(t,T) - S_i(T). If the short used a (n,i)-quality option he would realise a profit of

$$\max_j [PF(j)F(t,T) - S_j(T)] \qquad (10)$$

rather than PF(i) F(t,T) - $S_i(T)$. If we assume that PF(j)=1 for j=1,...,n then we can rewrite (10) as

$$\max_j\{F(t,T) - S_j(T)\} \quad = F(t,T) - \min_j\{S_j(T)\}$$
$$= F(t,T) - S_i(T) + \max_j\{S_i(T) - S_j(T)\} \qquad (11)$$

The last term in equation (11) represents the extra amount received by exercising the (n,i)-quality option. By dropping the PF(j) assumption an analogue for equation (11) can be obtained.

In order to price an n-asset exchange option Hemler [1990] allows each asset to follow geometric Brownian motion. As the author himself points out, however, this assumption is made for tractability rather than for theoretical reasons; it is inconsistent with the bond price equalling par at maturity, and with the bond price variance declining as the bond approaches maturity. This restriction may not be as restrictive as it would first seem; the bond issues of his empirical work have at least 15 years to maturity/first call and as such the constraint that bond prices equal par at maturity may not be significant - an adapted Black-Scholes formula is, after all, used by many market practitioners to price short term options on long term bonds.

Boyle's approach to valuing the quality option where there are several deliverable assets generalises a result, in the one asset case, that a long forward contract has the same payoff as a portfolio consisting of a long European call option and a short European put option on the same underlying asset. He then uses results concerning order statistics of the multivariate normal distribution to obtain estimates of the value of the quality option.

D. Pricing the Quality Option using Monte Carlo Simulation.

We now develop a model for the value of the quality option using Monte Carlo simulation to simulate changes in the term structure of interest rates over the life of the option. As an illustration we use sample data of bond prices to derive a value for the option for a trading day at the beginning of the March 1987 delivery quarter for delivery at the end of March 1987.

The optimal delivery gilt will depend upon the level and shape of the term structure at delivery. The idea behind our simulation method is to simulate the evolution of an initial term structure at time t, through time, until the expiry of the option at time T.[5] We then calculate the prices of the gilts in the deliverable basket based on the simulated term structure at T, and calculate the payoff to the option.

We assume that we have a full set of pure discount bonds with maturity dates every three months between time t and the maturity of the longest dated gilt in our delivery basket. Let P(t,s) denote the price at time t of a pure discount bond which matures at time s, and let r(t,s) denote the spot rate of the s-maturity bond. Given an initial set of spot rates r(t,s) we can deduce prices of the pure discount bonds in the following way:

$$P(t,s) = e^{-(s-t)r(t,s)} \qquad (12)$$

We describe the evolution of the pure discount bond prices by the following differential equation:

$$\frac{dP(t,s)}{P(t,s)} = r_t dt + \sigma(t,s)dBt \qquad (13)$$

where the volatility of the process is given by:

$$\sigma(t,s) = \frac{\rho}{\alpha}(1-e^{-\alpha(s-t)}) \qquad (14)$$

5 By assuming a fixed delivery day we are effectively treating the option as a European option.

α and ρ are the volatility parameters describing the level of mean reversion and the annualised volatility of the short rate respectively. r_t is the level of the short rate obtained from the price of the shortest bond in the term structure, and dB_t is a standard Brownian Motion. s denotes the maturity of the bond. This is a 1 factor model and so all the discount bonds have the same dB term.

On each simulation run we evolve the term structure from the initiation of the option position until the maturity of the option. The price of each gilt is then calculated as the present value of the future coupon payments discounted by the factors implied by the simulation. The value of the option, at time T, is then calculated based on these prices.

Using actual gilt and futures prices for 04 February 1987, and an initial term structure of 11% we perform simulations to derive a value for the delivery option implicit in the March 1987 futures contract. We set the volatility of the short rate to be 0.1531 and the level of mean reversion equal to 0.01449 (estimates are derived from Babbs [1990]). Each simulation was based on 1000 runs. Each simulation run consisted of generating a weekly evolving term structure from the beginning of February until the final delivery day of the March contract using equation (13). Prices of the deliverable gilts were then recalculated based on the resulting term structure at delivery and a resulting end-of-period futures price calculated. The value of the option is then calculated as the difference between the simulated gilt price of the cheapest to deliver from 4th February and the product of the implied futures price and this bonds price factor.

The estimated option values we obtained, based on this simulation, were very low - only of the order of 4 to 5 basis points. We suspect that these low values are due to the lack of sophistication in the specification of the initial term structure.

V. Empirical Analysis

We begin this section by describing the sample of British Government bonds and futures contracts used in our empirical analysis of the LIFFE long gilt futures contract. We split our empirical analysis into two parts. The first part of this section looks at the actual delivery experience of gilts into the futures contract. Our sample period for the delivery experience are the 21 quarterly delivery cycles from March 1986 to March 1991.

In the second part of this section we provide the empirical estimates of the market value and realised value of the quality option based on the models developed in section 4. The empirical values are estimated for 8 quarterly delivery cycles from March 1987 until December 1988. The Long gilt futures prices were obtained from LIFFE. These prices represent the settlement price for the futures contract and cover the period 02 January 1987 to 30 December 1988. An important consideration in choosing the period to value the option is the liquidity of the futures contract. Trading activity is concentrated mostly in the nearby contract with a significant decrease as the nearby contract enters the delivery month. We, therefore, set t at the start of the last quarter of the life of the nearby contract and follow it until the maturity of the contract.

Spot price Gilt data were obtained from Kleinwort Benson. These prices cover the same period as the futures data and represent the closing mid prices of all the deliverable gilts for each delivery month. Prices are quoted clean, ie, without accrued interest, as is the convention in the UK Government bond market. The default-free interest rate that we use is a linear interpolation of one month and three month Treasury Bill rates which we obtained from Datastream. This is an estimate of the interest rate for a maturity exactly matching the maturity of the futures contract each trading day. Days when a full set of price data (futures or spot) is not available are treated as missing.

Figure 1 shows the number of gilts in the final list of deliverable stock for each contract month, produced by LIFFE, and the number of different gilts which were delivered into the futures contract. The number of deliverable bonds ranges from a low of 7 bonds (March and June 1986) to a high of 12 (December 1987 and December 1989). Table 3 details the number of gilt edged stock of each of the delivered gilts. The first thing to notice from Figure 1 and Table 3 is that while at any point in time there is only one gilt which is the cheapest to deliver, the practical experience is that there are usually two or three gilts, and on one occasion four, which are delivered. Only 4 out of the 21 delivery months studied (Dec 87, Jun 88, Jun 89, and Sep 89) have only 1 bond being delivered into the futures contract. Table 4 shows the distribution of the number of different gilts delivered.

Table 4

Empirical Distribution of Number of Different Gilts Delivered

Number of different gilts delivered	1	2	3	4
Frequency	4	8	8	1

Why do people seem to deliver gilts which are not the cheapest to deliver? There may be a number of reasons for this seeming lack of rationality. Firstly, while there is one gilt which is cheapest to deliver on the first day of the month there may be several gilts which are 'close' to being the cheapest to deliver. If the seller does not hold the cheapest to deliver he may deliver a different bond if it proves cheaper than switching into, and then delivering, the cheapest to deliver. Secondly, the short may chose to deliver an illiquid holding which he might otherwise be unable to sell in size without adversely affecting its price.

A closer analysis of the gilts delivered shows that the delivered gilts are consistent with our earlier observations on the level of interest rates and the type of gilt delivered. Recent delivery quarters have seen the delivery of low coupon gilts (predominantly 9 1/2% 04, 9 1/2% 05, and 10% 04) which are generally cheaper to deliver than high coupon gilts, when interest rates are greater than the notional 9%. However high coupon gilts are more deliverable in downward sloping yield curve environments like those experienced in early 1988 when the notional gilt had a 12% coupon. Figure 2 illustrates the average coupon of the delivered gilts for each contract month.

Figure 3 illustrates the average time to maturity of the delivered gilts. Although we would expect the average maturity of the delivered gilts to be short in the low interest rate environments of 1987 and 1988, the experience was that it was long maturity bonds which were delivered. This is due to the fact that the high coupon bonds at that time had long maturity dates. Similarly, the low coupon gilts delivered in the period of high prevailing interest rates had short maturity dates attached to them.

Tables 5 and 6 detail the timing of delivery notices during each delivery month. Table 5 shows the number of lots to be delivered with respect to the number of trading days left until the end of the delivery month. Table 6 shows these lots as a percentage of the total number of lots delivered into that months contract. The last column of Table 6 shows the average percentage of lots delivered over the March 1986 to March 1991 period for each of the notice days during the delivery month. The highest percentage figure occurs for the last notice day, and this is mainly derived from the contracts which matured during 1987 and the first half of 1988. Overall the bulk of the delivery notices occur in the first half of the delivery months.

Analysis of the timing of delivery notices is complicated by the fact bonds in the deliverable basket cannot be delivered during the delivery month on days when they are trading special ex-dividend. For example, during the early half of September 1989 the gilt Conv. 9.50% 2004 was consistently the cheapest to deliver for the contract maturing at the end of September. This gilt was trading special XD until Tuesday 19 September. On Friday 15 September the first delivery notices (for delivery Tuesday 19) were received for the September contract of 2142 lots of Conv 9.50% 2004.

Table 7 identifies the cheapest to deliver issues identified for each of the delivery months. The "Number of Changes" column lists the total number of times the calculated cheapest to deliver gilt changes during each delivery quarter. For example, during the quarter ending December 1987 the cheapest to deliver gilt changed 6 times. The third column lists the three most identified cheapest to deliver issues, and the fourth column the three most delivered issues as identified by LIFFE. There appears to be a very close relationship between the calculated cheapest to deliver issues and those actually delivered against the futures contract, providing an approximate check on our cheapest to deliver identification procedure.

We now turn our attention to estimating empirically the market value of the quality delivery option.

In estimating V(t,T) the price factors PF(i) are worked out according to Equation (1), coupon receipt dates according to the Financial Times, and accrued interest $I_i(t)$ and $I_i(T)$ are evaluated by following the procedure described in Phillips [1984]

Estimates of the value of the quality option from equation (6) are often negative. The terms in equation (6) are based on actual market prices; such prices can be "noisy" leading to estimation error. Whenever the estimation error is large relative to the quality option value, negative estimates are likely. Since the observed futures price will reflect both the quality and timing delivery options their values will be contained in the measure V(t,T). Although V(t,T) measures the quality option payoff ignoring the timing option these values will be affected by the timing option through the observed futures price. Estimates based on equation (6) will therefore be biased upward by the extra valued added by the timing option.

Table 8 presents estimates of the ex-ante value V(n,T) for $t < n < T$. The results are presented as averages of the daily estimates for each delivery quarter.

Average values are small, the highest being 0.3719 percentage points of par for the September 1987 delivery quarter with a standard deviation of 0.7250. This average value corresponds to £186 per futures contract of £50,000 par. Over the whole of the sample period the daily average value of V(t,T) is estimated at 0.2140 percentage points of par with a standard deviation of 0.7942. Hegde [1990] looking at delivery quarters for the US T-Bond futures contract between September 1977 and December 1986 estimates V(t,T) at 0.190 percentage points of par for the whole period. Note that the minimum values of all of the contracts are negative. Also approximately 20% of the total daily estimates are negative. Under our assumptions negative values cannot occur and we think that these estimates as well as some of the larger positive figures may be due to nonsynchronous spot-futures data.[6]

Table 9 presents summary statistics of the estimates corresponding to the payoffs from exercising the quality option at delivery. The average of the buy and hold payoffs for the whole of the sample period is 0.2270 percentage points of par with a standard deviation of 0.7170. Hedge [1990] finds his sample has an average figure of 0.259 percentage points of par. Three of the delivery quarters in 1988 (the first, third, and fourth) exhibit no switching opportunities during that quarter, and so the switching profits are zero. The presence of the timing option will mean that our estimates of the payoffs from exercising the quality option at delivery are biased downward.

By comparing the values for V(t,T) and the payoffs from the buy and hold strategy we find that the overall average is higher for the switching strategy than for the ex-ante measure but that the estimates are comparable on average.

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6 Hegde [1990] estimates in his sample that 10% of the total weekly estimates of V(t,T) are negative. Of the 37 quarters he studies 6 have negative mean estimates of V(t,T).

VI. Conclusions and Further Research

Our paper looks at the delivery option implicit in bond futures contracts. The option has value because the bonds in the deliverable set are not perfect substitutes for each other. The extent to which one bond is more deliverable than another depends on the term structure of interest rates at the expiry of the contract. We analyse the incentives to deliver one bond over another by looking at different (ex-post) term structures to that of the constant (ex-ante) 9% used by LIFFE in determining price factors. We find that when interest rates are below 9% the LIFFE price factor calculation favours delivery of a high coupon bond with a short maturity, and when interest rates are above 9% the calculation favours delivery of low coupon bond with longer maturities. Analysis of the actual delivered gilts for the 17 delivery quarters from March 1986 to March 1991 shows that they are consistent with our observations.

We use two estimation techniques to obtain market values of the quality option in the LIFFE long gilt futures contract for the eight delivery quarters beginning March 1987. We find that the market price of the quality option based on the excess of the forward price of the cheapest to deliver gilt over its price factor times the price of the futures contract is on average 0.214 percentage points of par for the whole sample period. We estimate that the daily average payoff from exercising the delivery option at expiry to be 0.227 percentage points of par. Both of these figures are comparable to those obtained by Hegde[1990] in his empirical analysis of the US Treasury over the period August 1977 to December 1984.

The fact that our figures for the LIFFE futures contract and those of the US Treasury Bond futures contract are similar is quite surprising. Due to institutional differences the US Treasury Bond futures contract has two implicit options which are not present in the LIFFE contract; the end of the month option and the wild card option. We would expect these differences to be reflected in a higher figure for the first empirical estimate as the futures price will reflect the value of these options.

Our paper develops an option theoretic model to value the quality option by simulating changes in the term structure of interest rates over the life of the option. Using data from a trading day in early February 1987 and an initial assumed term structure we find that the value of the option to be only 4 to 5 basis points.

A shortcoming of this draft is that we assume the delivery day to be predetermined. The next stage in this research will be to relax this assumption and to look more closely at the interaction of the timing option and the quality option. V(t,T) is a joint test of the quality and timing options as the observed futures price will reflect both of them. The empirical estimate based on the payoffs from exercising the delivery option at expiry can be adapted to analyse the switching profits at different times during the delivery month. Finally our Monte Carlo simulation model can be adapted to incorporate a decision rule as to whether or not to deliver the futures contract depending on the relationship between the running yield on the cheapest to deliver gilt and the financing cost of the position.

Another major area for future research will be to develop the Monte Carlo simulation model to start from an initial term structure which is consistent with the observed term structure. We will use work performed by Steeley [1989] on B-Splines to construct an initial term structure based on the prices of gilts. We can also use these simulated term structures to extend the analysis of the LIFFE price factors.

Finally, we hope to perform a deeper analysis of our findings to see if they are consistent with option valuation theory, ie an increasing function of the term to delivery of the futures contract, the level and volatility of long term interest rates, and the coupon and maturity structure of the deliverable bonds. An extended database of contracts will allow us to analyse the estimates of the value of the delivery option with different times to maturity.

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Table 1

LIFFE Price Factor vs Calculated Factor

Int Rate		0.07			0.09			0.11		
	COUP	LIFFE	FAIR	DIFF	LIFFE	FAIR	DIFF	LIFFE	FAIR	DIFF
10 yrs	7	0.8706	0.8239	0.0467	0.8706	0.8706	0.0000	0.8706	0.9080	-0.0374
	9	1.0000	0.9400	0.0600	1.0000	1.0000	0.0000	1.0000	1.0493	-0.0493
	11	1.1288	1.0561	0.0727	1.1288	1.1288	0.0000	1.1288	1.1906	-0.0618
15 yrs	7	0.8375	0.8239	0.0136	0.8375	0.8375	0.0000	0.8375	0.8293	-0.0082
	9	1.0000	0.9747	0.0253	1.0000	1.0000	0.0000	1.0000	1.0182	-0.0182
	11	1.1619	1.1255	0.0364	1.1619	1.1619	0.0000	1.1619	1.1906	-0.0287
20 yrs	7	0.8161	0.8239	-0.0078	0.8161	0.8161	0.0000	0.8161	0.8092	0.0069
	9	1.0000	0.9993	0.0007	1.0000	1.0000	0.0000	1.0000	0.9999	0.0001
	11	1.1833	1.1748	0.0085	1.1833	1.1833	0.0000	1.1833	1.1906	-0.0073

Table 2

Effect of Yield Curve Shape on Price Factors

10 YEARS	9.1% - 11.0%			7.1% - 9.0%		
	LIFFE	FAIR	DIFF	LIFFE	FAIR	DIFF
7%	0.8706	0.9274	-0.0568	0.8706	0.8930	-0.0224
9%	0.9997	1.0708	-0.0711	0.9997	1.0243	-0.0246
11%	1.1288	1.2142	-0.0854	1.1288	1.1557	-0.0269

15 YEARS	9.1% - 12.0%			6.1% - 9.0%		
	LIFFE	FAIR	DIFF	LIFFE	FAIR	DIFF
7%	0.8375	0.8558	-0.0183	0.8375	0.8459	-0.0082
9%	0.9997	1.0310	-0.0313	0.9997	1.0057	-0.0060
11%	1.1619	1.2031	-0.0442	1.1619	1.1657	-0.0038

20 YEARS	9.1% - 13.0%			5.1% - 9.0%		
	LIFFE	FAIR	DIFF	LIFFE	FAIR	DIFF
7%	0.8161	0.8071	0.0090	0.8161	0.8215	-0.0054
9%	0.9997	1.0000	-0.0003	0.9997	1.0000	-0.0003
11%	1.1833	1.1929	-0.0096	1.1833	1.1785	0.0048

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10 YEARS	12.0% - 9.1%			9.0% - 7.1%		
	LIFFE	FAIR	DIFF	LIFFE	FAIR	DIFF
7%	0.8706	0.8490	0.0216	0.8706	0.8016	0.0690
9%	0.9997	0.9783	0.0214	0.9997	0.9177	0.0820
11%	1.1288	1.1076	0.0212	1.1288	1.0338	0.0950

15 YEARS	12.0% - 9.1%			9.0% - 6.1%		
	LIFFE	FAIR	DIFF	LIFFE	FAIR	DIFF
7%	0.8375	0.8296	0.00079	0.8375	0.8086	0.0289
9%	0.9997	0.9959	0.0038	0.9997	0.9578	0.0419
11%	1.1619	1.1623	-0.0004	1.1619	1.1070	0.0549

20 YEARS	13.0% - 9.1%			9.0% - 5.1%		
	LIFFE	FAIR	DIFF	LIFFE	FAIR	DIFF
7%	0.8161	0.8103	0.0058	0.8161	0.8262	-0.0101
9%	0.9997	1.0000	-0.0003	0.9997	1.0000	-0.0003
11%	1.1833	1.1897	-0.0064	1.1833	14.1738	0.0095

Table 3(i)

Delivery Experience For LIFFE Long Gilt Future
March 1986 - March 1991

BOND				CONTRACT MAR86	JUN86	SEP86	DEC86	MAR87	JUN87	SEP87
TRES	5.50	%	2008-12	-	-	-	-	-	-	-
TRES	7.75	%	2012-15	X	X	X	X	X	X	-
TRES	8.00	%	2002-06	-	-	-	-	-	-	-
TRES	8.00	%	2009	X	X	X	X	X	X	X
TRES	8.50	%	2007	X	X	X	X	X	X	X
TRES	9.00	%	2008	X	X	X	X	X	X	X
CONV	9.50	%	2004	X	X	X	-	-	-	-
CONV	9.50	%	2005	X	X	X	X	X	X	X
TRES	9.75	%	2002	X	X	X	X	-	-	-
CONV	9.75	%	2006	X	X	X	X	X	X	X
EXCH	10.00	%	2002	X	-	X	X	X	X	X
CONV	10.00	%	2002	-	-	-	-	-	X	X
TRES	10.00	%	2003	X	X	X	X	X	X	X
TRES	10.00	%	2004	X	X	-	-	-	-	-
EXCH	10.50	%	2005	X	X	X	-	-	-	-
TRES	11.50	%	2001-04	4738	X	X	X	X	X	X
TRES	11.75	%	2003-07	-	447	30	199	-	647	1202
TRES	12.50	%	2003-05	3	-	2462	2762	2612	3081	3708
TRES	13.50	%	2004-08	-	2322	2355	1507	755	478	880

BOND				CONTRACT DEC87	MAR88	JUN88	SEP88	DEC88	MAR89	JUN89
TRES	5.50	%	2008-12	-	-	-	-	-	-	-
TRES	7.75	%	2012-15	-	-	-	X	X	X	X
TRES	8.00	%	2002-06	X	X	X	X	X	X	X
TRES	8.00	%	2009	-	-	-	-	-	-	-
TRES	8.50	%	2007	-	-	-	-	-	-	-
TRES	9.00	%	2008	X	X	X	-	-	-	-
CONV	9.50	%	2004	-	-	-	-	-	-	5931
CONV	9.50	%	2005	-	-	-	-	-	-	-
TRES	9.75	%	2002	X	X	X	X	X	X	X
CONV	9.75	%	2006	X	X	X	X	X	X	-
EXCH	10.00	%	2002	X	X	X	X	X	X	X
CONV	10.00	%	2002	X	X	X	X	X	X	X
TRES	10.00	%	2003	-	-	-	73	X	X	X
TRES	10.00	%	2004	-	-	-	-	291	796	X
EXCH	10.50	%	2005	-	-	-	-	-	-	-
TRES	11.50	%	2001-04	X	X	X	X	X	X	X
TRES	11.75	%	2003-07	3554	X	X	X	X	X	X
TRES	12.50	%	2003-05	-	2066	5861	10939	X	X	X
TRES	13.50	%	2004-08	-	1576	-	-	13672	11062	X

X = BOND NOT ELIGIBLE FOR DELIVERY.

\- = ELIGIBLE FOR DELIVERY BUT NOT DELIVERED.

Table 3(ii)
Delivery Experience For LIFFE Long Gilt Future
March 1986 - March 1991

	CONTRACT						
BOND	SEP89	DEC89	MAR90	JUN90	SEP90	DEC90	MAR91
TRES 5.50 % 2008-12	-	X	X	X	X	X	X
TRES 7.75 % 2012-15	X	X	X	X	X	X	X
TRES 8.00 % 2002-06	X	X	X	X	X	X	X
TRES 8.00 % 2009	-	-	-	-	-	-	-
TRES 8.50 % 2007	-	-	-	-	-	-	-
TRES 9.00 % 2008	-	-	-	-	-	-	-
CONV 9.50 % 2004	3890	-	199	3854	36	5	27
CONV 9.50 % 2005	-	-	-	510	872	2709	2090
TRES 9.75 % 2002	X	X	X	X	X	X	X
CONV 9.75 % 2006	-	-	X	X	X	X	X
EXCH 10.00 % 2002	X	X	X	X	X	X	X
CONV 10.00 % 2002	X	X	X	X	X	X	X
TRES 10.00 % 2003	X	4867	8620	4706	-	-	-
TRES 10.00 % 2004	X	-	54	2761	4147	X	X
EXCH 10.50 % 2005	-	-	-	-	X	X	X
TRES 11.50 % 2001-04	X	X	X	X	X	X	X
TRES 11.75 % 2003-07	X	-	-	-	-	-	-
TRES 12.50 % 2003-05	X	1088	-	-	-	219	122
TRES 13.50 % 2004-08	X	-	-	-	-	-	-

X = BOND NOT ELIGIBLE FOR DELIVERY.

\- = ELIGIBLE FOR DELIVERY BUT NOT DELIVERED.

Table 5
Timing of Delivery Notices
March 1986 - March 1991

^C

NUMBER OF LOTS DELIVERED

CONTRACT	MAR86	JUN86	SEP86	DEC86	MAR87	JUN87	SEP87	DEC87	MAR88	JUN88	SEP88
23					378	200					
22		404	2225		65	37	0	0			
21		791	822	0	5	171	0	0	100		2688
20	2683	38	225	935	193	1	80	0	0		885
19	775	10	91	60	500	0	0	0	4		92
18	135		260	23	181	17	0	0	0		876
17	122	167	80	0	187	0	0	0	0		951
16	58		4	0	0	0	0	0	0		501
15	4		20	0	0	0	0	0	0		212
14	10	447	11	0	281	0	0	0	0		0
13	86			0	60	0	0	0	0		0
12	63			0	60	0	0	0	1019	0	90
11	5	63	4	0	253	0	0	0	943	0	0
10	0		12	0	643	0	0	0	0	0	0
9	0			0	20	0	2	0	0	240	309
8	0			752	0	0	0	0	0	0	178
7	585		39	297	0	0	3706	0	0	500	44
6	169	849	455	0	111	0	0	220	0	437	2
5	10			1935	0	0	5	0	0	200	0
4	0		4	23	0	0	40	0	0	20	0
3	0			0	0	60	195	0	250	739	16
2	0			0	0	0	0	708	368	0	11
1	36		595	443	430	3720	1772	3334	958	3725	0

CONTRACT	DEC88	MAR89	JUN89	SEP89	DEC89	MAR90	JUN90	SEP90	DEC90	MAR91
23			2000			0				
22		332	524			408				
21		25	103	0		3602	3240	1393		
20	1007	9059	603	0	386	1090	536	514		
19	724	1018	1231	0	45	212	184	878	72	
18	339	301	794	0	563	920	3313	291	608	
17	4978	391	184	0	1370	730	1340	360	131	
16	1712	299	2	0	679	30	1156	125	34	21
15	200	15	120	0	193	1043	460	297	684	0
14	1107	123	153	0	889	246	79	469	269	1
13	499	161	76	0	27	248	649	90	66	100
12	107	87	0	0	285	25	154	40	412	1997
11	776	0	46	0	8	100	100	263	69	1
10	267	3	50	2142	69	61	108	2	63	5
9	661	30	0	1550	17	0	135	120	45	0
8	121	0	0	158	827	0	101	27	132	48
7	259	0	0	0	124	148	44	0	114	29
6	625	11	0	0	20	0	0	50	5	0
5	441	1	20	1	1	10	1	130	8	0
4	7	0	0	0	142	0	0	0	0	0
3	130	2	5	0	211	0	41	0	20	0
2	0	0	20	0	0	0	50	1	7	37
1	3	0	0	39	100	0	140	5	194	0

Table 6
Timing of Delivery Notices
March 1986 - March 1991
(Percentage of Total Delivered Lots)

PERCENTAGE OF DELIVERED LOTS

CONTRACT	MAR86	JUN86	SEP86	DEC86	MAR87	JUN87	SEP87	DEC87	MAR88	JUN88	SEP88
23	0.00	0.00	0.00	0.00	11.23	4.76	0.00	0.00	0.00	0.00	11.17
22	0.00	14.59	45.90	0.00	1.93	0.88	0.00	0.00	0.00	0.00	26.58
21	0.00	28.57	16.96	0.00	0.15	4.07	0.00	0.00	2.75	0.00	24.41
20	56.59	1.37	4.64	20.93	5.73	0.02	1.38	0.00	0.00	0.00	8.04
19	16.35	0.36	1.88	1.34	14.85	0.00	0.00	0.00	0.11	0.00	0.84
18	2.85	0.00	5.36	0.51	5.38	0.40	0.00	0.00	0.00	0.00	7.95
17	2.57	6.03	1.65	0.00	5.55	0.00	0.00	0.00	0.00	0.00	8.64
16	1.22	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.55
15	0.08	0.00	0.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.93
14	0.21	16.14	0.23	0.00	8.35	0.00	0.00	0.00	0.00	0.00	0.00
13	1.81	0.00	0.00	0.00	1.78	0.00	0.00	0.00	0.00	0.00	0.00
12	1.33	0.00	0.00	0.00	1.78	0.00	0.00	0.00	27.98	0.00	0.82
11	0.11	2.28	0.08	0.00	7.51	0.00	0.00	0.00	25.89	0.00	0.00
10	0.00	0.00	0.25	0.00	19.10	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.59	0.00	0.03	0.00	0.00	4.09	2.81
8	0.00	0.00	0.00	16.83	0.00	0.00	0.00	0.00	0.00	0.00	1.62
7	12.34	0.00	0.80	6.65	0.00	0.00	63.90	0.00	0.00	8.53	0.40
6	3.56	30.66	9.39	0.00	3.30	0.00	0.00	5.16	0.00	7.46	0.02
5	0.21	0.00	0.00	43.31	0.00	0.00	0.09	0.00	0.00	3.41	0.00
4	0.00	0.00	0.08	0.51	0.00	0.00	0.69	0.00	0.00	0.34	0.00
3	0.00	0.00	0.00	0.00	0.00	1.43	3.36	0.00	6.86	12.61	0.15
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.61	10.10	0.00	0.10
1	0.76	0.00	12.28	9.91	12.77	88.45	30.55	78.23	26.30	63.56	0.00

PERCENTAGE OF DELIVERED LOTS

CONTRACT	DEC88	MAR89	JUN89	SEP89	DEC89	MAR90	JUN90	SEP90	DEC90	MAR91	AVG
23	0.00	0.00	33.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.90
22	0.00	2.80	8.83	0.00	0.00	4.60	0.00	0.00	0.00	0.00	5.05
21	0.00	0.21	1.74	0.00	0.00	40.60	27.39	27.56	0.00	0.00	8.30
20	7.21	76.40	10.17	0.00	6.48	12.28	4.53	10.17	0.00	0.00	10.76
19	5.19	8.58	20.76	0.00	0.76	2.39	1.56	17.37	2.45	0.00	4.51
18	2.43	2.54	13.39	0.00	9.45	10.37	28.00	5.76	20.73	0.00	5.48
17	35.65	3.30	3.10	0.00	23.00	8.23	11.33	7.12	4.47	0.00	5.74
16	12.26	2.52	0.03	0.00	11.40	0.34	9.77	2.47	1.16	0.94	2.23
15	1.43	0.13	2.02	0.00	3.24	11.75	3.89	5.88	23.32	0.00	2.58
14	7.93	1.04	2.58	0.00	14.93	2.77	0.67	9.28	9.17	0.04	3.49
13	3.57	1.36	1.28	0.00	0.45	2.79	5.49	1.78	2.25	4.47	1.29
12	0.77	0.73	0.00	0.00	4.79	0.28	1.30	0.79	14.05	89.19	6.85
11	5.56	0.00	0.78	0.00	0.13	1.13	0.85	5.20	2.35	0.04	2.47
10	1.91	0.03	0.84	55.06	1.16	0.69	0.91	0.04	2.15	0.22	3.92
9	4.73	0.25	0.00	39.85	0.29	0.00	1.14	2.37	1.53	0.00	2.75
8	0.87	0.00	0.00	4.06	13.89	0.00	0.85	0.53	4.50	2.14	2.16
7	1.85	0.00	0.00	0.00	2.08	1.67	0.37	0.00	3.89	1.30	4.94
6	4.48	0.09	0.00	0.00	0.34	0.00	0.00	0.99	0.17	0.00	3.12
5	3.16	0.01	0.34	0.03	0.02	0.11	0.01	2.57	0.27	0.00	2.55
4	0.05	0.00	0.00	0.00	2.38	0.00	0.00	0.00	0.00	0.00	0.19
3	0.93	0.02	0.08	0.00	3.54	0.00	0.35	0.00	0.68	0.00	1.43
2	0.00	0.00	0.34	0.00	0.00	0.00	0.42	0.02	0.24	1.65	1.40
1	0.02	0.00	0.00	1.00	1.68	0.00	1.18	0.10	6.61	0.00	15.88

Table 7

Cheapest to Deliver Gilts

Contract	Number of Changes	Calculated[1]			Actual[2]		
Mar 87	(8)	TRES	12.50	03-05	TRES	12.50	03-05
		TRES	13.50	04-08	TRES	13.50	04-08
		CONV	10.00	02			
Jun 87	(8)	TRES	13.5	04-08	TRES	12.50	03-05
		TRES	11.75	03-07	TRES	11.75	03-07
		TRES	12.50	03-05	TRES	13.50	04-08
Sep 87	(11)	TRES	12.50	03-05	TRES	12.50	03-05
		TRES	11.75	03-07	TRES	11.75	03-07
		TRES	13.50	04-08	TRES	13.50	04-08
Dec 87	(6)	TRES	11.75	03-07	TRES	11.75	03-07
		CONV	9.50	05			
		TRES	12.50	03-05			
Mar 88	(0)	TRES	12.50	03-05	TRES	12.50	03-05
					TRES	13.50	04-08
Jun 88	(1)	TRES	12.50	03-05	TRES	12.50	03-05
		CONV	9.50	05			
Sep 88	(0)	TRES	12.50	03-05	TRES	12.50	03-05
					TRES	10.00	03
Dec 88	(0)	TRES	13.50	04-08	TRES	13.50	04-08

1. The three most regulary identified CTD gilts during the period that we follow the contract.

2. The three gilts most delivered into the contract.

Table 8

Ex-Ante Estimate of Market Price V(t,T)

	Number Observations	AVG	MAX	MIN	STD
Mar 87	41	0.1029	1.7982	-1.5825	0.7102
Jun 87	40	0.1107	1.5102	-1.7869	0.8712
Sep 87	41	0.3719	2.2760	-1.1308	0.7250
Dec 87	43	0.1348	2.3690	-2.1010	1.0397
Total 87	165	0.1737	2.3590	-2.1010	0.8562
Mar 88	34	0.3079	1.6521	-1.1020	0.6397
Jun 88	31	0.1976	2.4076	-1.6847	1.0756
Sep 88	41	0.2932	1.4955	-1.0020	0.5612
Dec 88	41	0.2315	1.0116	-0.9645	0.5473
Total 88	147	0.2592	2.4076	-1.6847	0.7155
Total	**312**	**0.2140**	**2.4076**	**-2.1010**	**0.7942**

Table 9

Ex-Post Payoffs from Exercising Option at Delivery

	Number Observations	AVG	MAX	MIN	STD
Mar 87	41	0.0257	0.2499	0.000	0.0630
Jun 87	40	0.1019	0.1607	0.000	0.0692
Sep 87	41	0.0234	0.1231	0.000	0.0417
Dec 87	43	0.4160	2.7478	0.000	1.1843
Total 87	165	0.2313	2.7478	0.000	0.6800
Mar 88	34	0	0	0	0
Jun 88	31	0.7532	2.9185	0.000	1.2770
Sep 88	41	0	0	0	0
Dec 88	43	0	0	0	0
Total 88	147	0.2203	2.9185	0.000	0.7709
Total	**312**	**0.2270**	**2.9185**	**0.0000**	**0.7170**

Figure 1

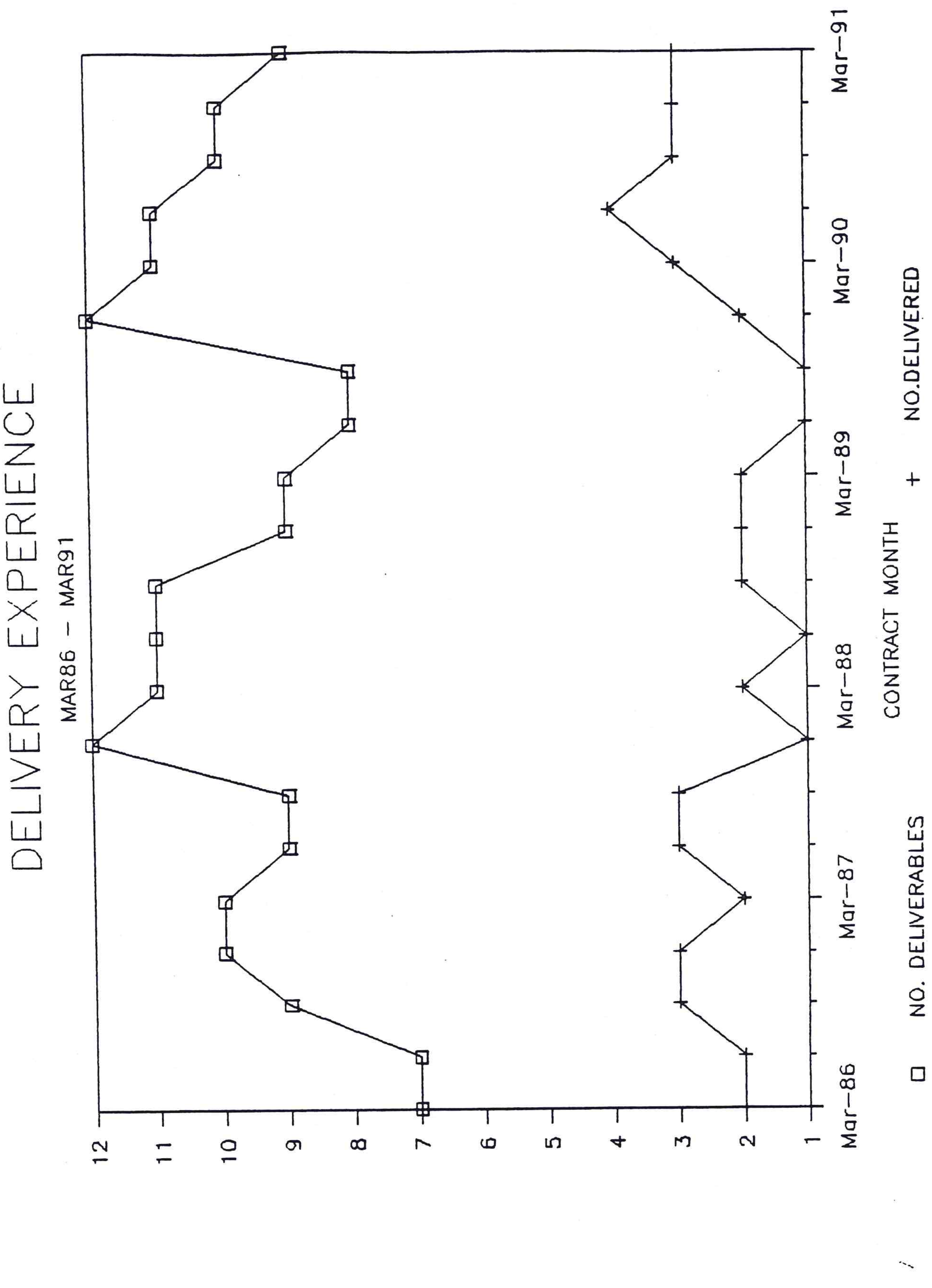
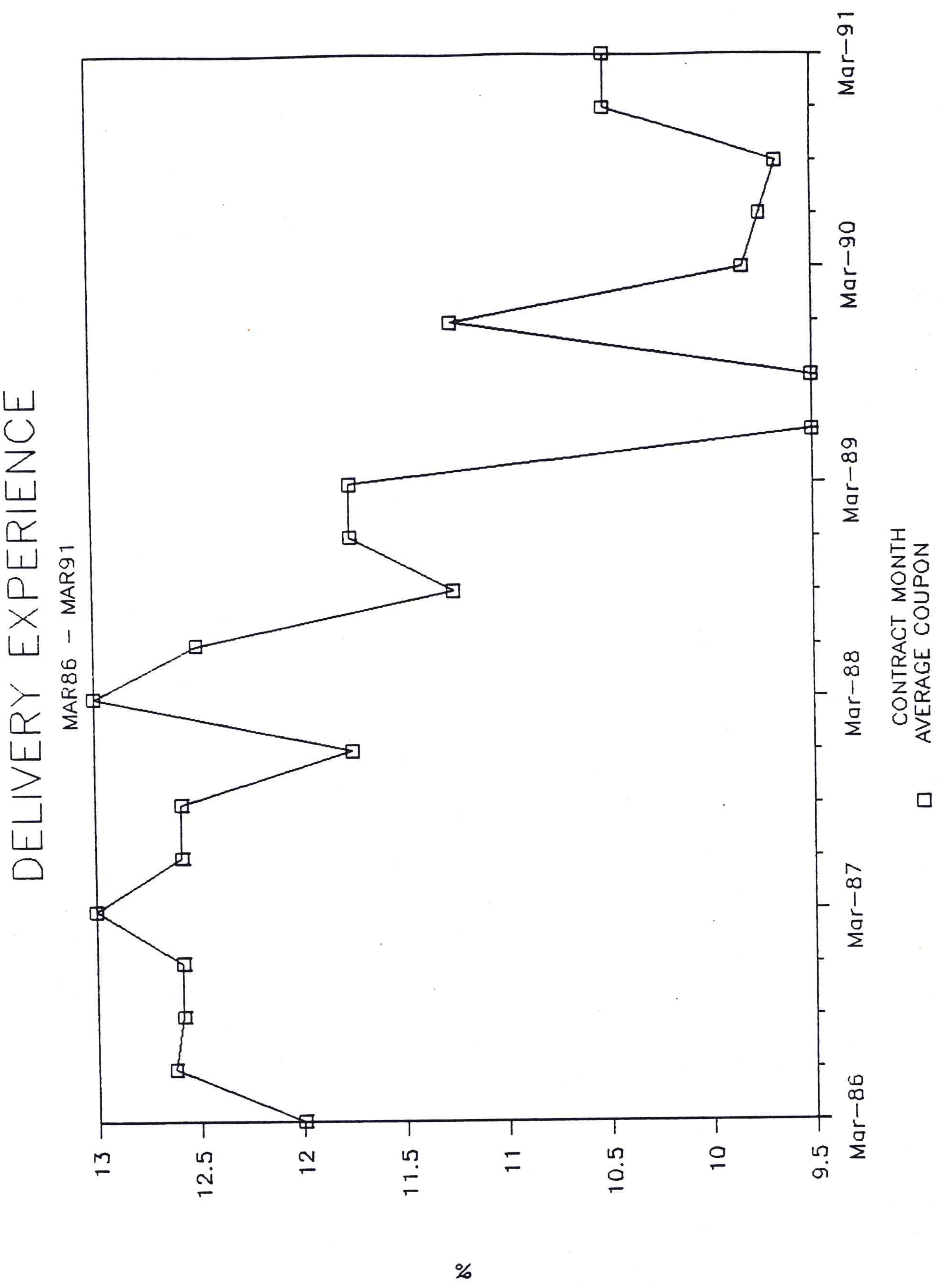
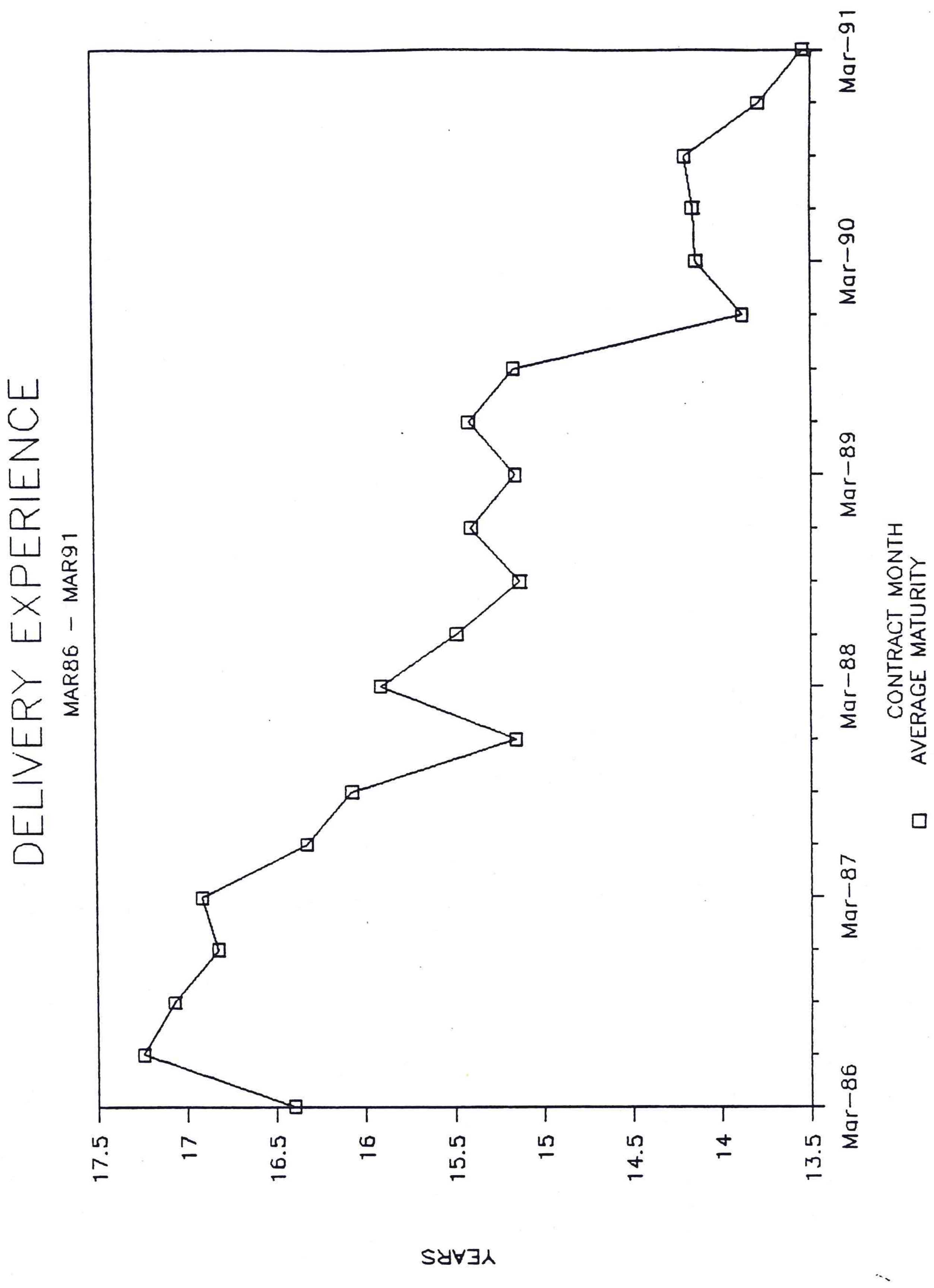

DELIVERY EXPERIENCE
MAR86 - MAR91
12
11
10
9
8
7
6
5
4
3
2
1
Mar-86
Mar-87
Mar-88
Mar-89
Mar-90
Mar-91
CONTRACT MONTH
□ NO. DELIVERABLES
+ NO.DELIVERED

Figure 2



DELIVERY EXPERIENCE
MAR86 - MAR91
%
13
12.5
12
11.5
11
10.5
10
9.5
Mar-86
Mar-87
Mar-88
Mar-89
Mar-90
Mar-91
CONTRACT MONTH
AVERAGE COUPON

Figure 3



DELIVERY EXPERIENCE
MAR86 - MAR91
YEARS
17.5
17
16.5
16
15.5
15
14.5
14
13.5
Mar-86
Mar-87
Mar-88
Mar-89
Mar-90
Mar-91
CONTRACT MONTH
AVERAGE MATURITY