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ABSTRACT

This paper presents an empirical study of the effectiveness of various methods of hedging the risk of options positions. The data used are eight years of daily data for the CME S&P 500 index options contract. The paper examines the performance of delta-gamma and delta-kappa hedges as well as simple delta hedges. The hedging errors are also decomposed and attributed to the effect of the changes in the underlying, (including components from the portfolio's delta and gamma), the effect of changes in volatility and the effect of the changes in the interest rate. This decomposition is (loosely) based on the analysis given in Bookstaber (1991). The analysis shows that although the (kappa) risk attributable to changes in volatility is small relative to that (delta) due to changes in the underlying, it becomes very significant after delta hedging has been done. Naive kappa hedging is less effective than gamma hedging because the variation of implied volatilities depends on the option maturity.

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1. Introduction

This paper presents an empirical study of the effectiveness of various methods of hedging the risk of options positions. The data used are eight years of daily data for the CME S&P 500 index options contract. The paper examines the performance of delta-gamma and delta-kappa hedges as well as simple delta hedges. The hedging errors are decomposed and attributed to the effect of the changes in the underlying, (including components from the portfolio's delta and gamma), the effect of changes in implied volatility and the effect of changes in the interest rate. This decomposition is (loosely) based on the analysis given in Bookstaber (1991).

The framework adopted enables us easily to examine the effect of adopting different hedging procedures, including the method and frequency of rebalancing (for example to ameliorate the effect of transactions costs) and alternative approaches to forecasting future variances. Since there is no limit to the range of possibilities for accomplishing these hedge revisions, our analysis here can only indicate broad guidelines for optimal hedging.

The analysis shows that although the (kappa) risk attributable to changes in volatility is small relative to the (delta) risk of changes in the underlying, it becomes very significant after delta hedging has been done. Naive kappa hedging is less effective than gamma hedging because the variation of implied volatilities depends on the option maturity.

2. Related Literature

Starting from the seminal paper of Black and Scholes (1973) an enormous literature has developed on the theory of pricing contingent claims under all manner of assumptions and stochastic processes. Most of this work is based on the exact replication principle. Financial institutions participating in the derivatives markets have needs for risk management which are every bit as pressing as those for valuation. It is therefore surprising that, despite a reasonable amount of empirical work on testing options models, rather few published papers have focused explicitly on the effectiveness of hedging methods. We will pick out a few notable contributions.

The first modern empirical work is contained in the Black and Scholes (1972) paper which examined the pricing of over the counter stock options over a four year period. Like many later papers although delta hedging portfolios were constructed the risk aspects of their performance were decidedly secondary to their average returns. One important aspect of this early work was the focus it placed on the role of volatility in options pricing and trading. They showed that although the historical volatilities were on average unbiased, their variation is too great, and buying options at the higher historical volatilities and selling them at the lower ones is a loss making strategy.

A theoretical paper (also employing some simulation analysis) by Boyle and Emmanuel (1980) considered the magnitudes and distributions of delta hedging errors in a world which satisfies the Black and Scholes assumptions, but where hedge revisions are made at discrete time intervals. These relationships are now well understood: for small but finite time increments the expected hedge error is zero, and its distribution is a (displaced) chi-squared distribution with variance proportional to a half gamma times the stock variance rate times the time interval. Our interest in the current paper stems of course from the problem that the Black Scholes assumptions are not exactly met in reality, and the market prices of options also have varying implied volatilities.

Figlewski (1989) uses a simulation model to examine the impact of a variety market imperfections, including uncertain volatility, transactions costs, indivisibilities and rebalancing only at discrete intervals. He also confirms the importance of the volatility of the asset, including the role of sampling error even when the true underlying volatility is constant and known. Further work along similar lines has been developed in a recent paper by Kat (1993). The simulation model has been developed to represent the behaviour of the S&P index and includes a GARCH model representation for the evolution of volatility. The paper compares various methods of delta hedging under transactions costs, and concludes that "none of the strategies studied is able to deliver the desired pay-off with an acceptable degree of accuracy". However, in interpreting these conclusions two things should be borne in mind. First, positions are seldom hedged in isolation of each other: the economies of hedging a portfolio of positions can be substantial. Second, all of the hedging strategies applied are at best suboptimal. They include the variance adjustment approach of Leland (1985) and also the use of a trigger whereby revisions are only made when the delta has departed substantially from its ideal figure. Nevertheless, significantly greater hedging efficiency can be obtained by only rebalancing to stay within a control region, instead of to a central figure. This optimal control approach is described in Hodges and Neuberger (1989) and is developed further, numerically and with Monte Carlo simulation analysis, in Hodges and Clewlow (1993).

Note that for most of the papers we have described, Monte Carlo simulation is the preferred technique over processing market data. An interesting exception to this is provided in the work of Dolbear (1992) in which market data are used to compare the effectiveness of alternative methods of hedging foreign exchange lookback options. The study concludes that hedging with appropriately revised straddle positions is both more efficient and more robust than the alternative of delta hedging.

A number of models of option valuation under stochastic volatility have been developed in recent years. Clewlow and Xu (1992) provides a useful survey of these, and properties of the historical and implied volatilities of the CME dataset used in this study are examined in Clewlow and Xu (1993). These and other papers provide evidence on the presence of volatility smiles and skews in empirical options data, and on the consistency (or lack of it with) particular pricing models. However, there seems to be no empirical work on whether the use of a stochastic volatility model provides significantly better risk control. We shall not attempt to fill that gap in this paper either. On the other hand, both Kat's (1993) simulation study and the current empirical paper do examine the effect of alternative volatility inputs.

3. Methodology

It is convenient to describe our approach as consisting of a number of stages. First, we establish rules that will be used for marking positions to market. Where options which are held do not trade we need a method of estimating likely market values for the purpose of measuring return. We also need to estimate how values would have differed under differing values of the main driving variables (such as futures prices, volatilities and interest rates) in order to decompose performance into components due to different sources. These rules are maintained independently of the methods used to create risks to be hedged or portfolios to hedge them.

Second, we identify a rule for picking an option or portfolio of options which will represent the target to be hedged using other instruments. For example, our first simulation is for the problem of hedging a short call written at the money and of the longest expiry date traded.

Third, we choose a rule for hedging the risk exposure of the target option or portfolio of options. We routinely compare the risk characteristics of the following four strategies: no hedging, delta hedging, delta-gamma hedging, and delta-kappa hedging. In addition we necessarily have considerable scope for variations in what options positions are regarded as eligible for hedging purposes, the frequency and nature of rebalancing, and the method of calculating ideal hedge ratios (for example whether these are based on current implied volatilities or on forecasts of future volatility).

3.1 Marking to Market and Return Decomposition

In our analysis, we revalue the portfolio every day using market closing prices. Interest is credited to the cash account at the current short term interest rate, and the day's profit or loss on futures positions are immediately posted to cash. If an option in the portfolio has no closing price given for it in the database on a particular day, then it is instead valued using the implied volatility of an option with the same expiry and with the nearest possible exercise price.

The implied volatilities are also used to provide a decomposition attributing the return on the portfolio to various components. Table 1 provides an example showing how this decomposition is accomplished. For the purposes of illustration we show the return on an unhedged portfolio consisting of a European style option on a future. The principle of this decomposition is the same as that described in Bookstaber (1991), but the details differ. First we account for the passage of time and create a value yardstick adding in the interest and revaluing all options at the previous day's inputs but closer to expiry. Next we account for what return is attributable to the changes in the futures prices, by revaluing at the new prices for these. This return attributable to the change in the underlying is further decomposed into components due to the delta and gamma plus a small residual. Two further components are calculated attributing returns to changes in implied volatilities and in the interest rates. Finally, there is a small additional residual due to interactions among the various components. It is worth pointing out that we have calculated the implied volatility component using the latest implied volatilities from each of the options in the portfolio. This makes it something of a catch all for many kinds of liquidity effects and non-synchronous data problems. These effects could be isolated to some extent if we used appropriate averages of implieds across various strikes, but we have not thought it worthwhile to do that here.

It is worth noting that the S&P options are options on futures and that early exercise can be optimal for both call and put options. Our numerical calculations on the CME database therefore use the Barone-Adesi and Whaley (1987) model for American options, using the futures price as the underlying (and therefore treating it as if this underlying paid a dividend equal to the interest rate). We also have to be careful that options contracts with different expiry dates are with respect to different futures contracts, and that a unit change in one futures contract will not usually result in a unit change in contracts for other dates.

3.2 Choice of Positions to Hedge

The analysis described in this paper is for hedging one option at a time. We hope to explore the economies of hedging combinations of options in later work. We start with an at-the-money option and hedge for most of its life using the futures contract for delta hedging, and using the future and the shortest available option for delta-gamma and delta-kappa hedging. Our target option is liquidated when it has less than 20 (calendar) days to expiration and trades. If it has not traded and the option position is still open with less than 15 (calendar) days to expiration, we use data on other options to provide an implied volatility to enable us to estimate a fair liquidation price. In this situation the option is usually far in or far out of-the -money and its value is not sensitive to the volatility assumption.

3.3 Management of the Hedge

We have already described that we consider the results of four basic hedging strategies:

1. No hedging,
2. Delta hedging,
3. Delta-gamma hedging,
4. Delta-kappa hedging.

Our main analysis is based on rebalancing daily at recorded closing prices, and assuming we can trade at these without incurring any transactions costs. This enables us to put a "best case" interpretation on our results. As a check on the robustness of our findings, we also did some analysis using daily settlement prices. We considered using opening prices, but rejected this as there were too many days, particularly in earlier years, when these where not available.

Maintenance of the hedge can be complicated by either the target option or the option used to hedge with not trading (or both). When our target open does not trade we simply revalue it using current futures information and the implied volatility of the nearest strike option with the same expiry date (as for marking to market). The hedge can then be revised in the usual way. When the option we are using to hedge with does not trade, we have a more difficult choice of whether to leave the hedge unbalanced, to estimate a price at which we could have rebalanced it, or to do something in between. We have used nearby implied volatilities to reestimate our delta and gamma exposures, and we have used the futures contract to rebalance the delta exposure. We have also kept a tally of the number of occasions on which these conditions occur.

Option hedges are used when we are delta-gamma hedging and when we are delta-kappa hedging. We select a new option to hedge with when our hedge option is close to expiry or sufficiently in or out of the money to be contributing very little gamma or kappa. Specifically we switch to a new at-the-money option to hedge with when any of the following conditions occur:

- the hedge option doesn't trade for more than 5 days,
- the hedge option has less than 20 days to expiry,
- the hedge option's delta is outside the range 0.2 to 0.8,
- the hedge option's gamma is less than 0.01 for gamma hedging, or
- the hedge option's kappa is less than 1.0 for kappa hedging.

Again, we have kept a tally of the frequencies of these various conditions occurring.

4. Empirical Results

In this section we describe the results of our analysis. We have hedged ten non-overlapping contracts between 1985 and 1992, and can report on both the terminal replication errors and on the daily replication errors from marking to market. The data we have used is that of the daily closing prices. We have done the same analyses using settlement prices. These gave very similar results but with slightly smaller replication errors, confirming our view that the settlement prices contain some smoothing which is undesirable for our purposes. The results are summarised in Table 2A.

The table shows the contract by contract replication errors under the four basic hedging strategies, and also the size of the original option premium in each case. We also report the mean and standard deviations of these figures.

Ten contracts, of course, represent a very small sample, so more reliable evidence on the hedging behaviour is provided by the daily errors from marking to market. The bottom three lines of the table show average root mean square errors calculated from the daily hedging errors, and given for the overall error and for the separate effects of the change in the underlying and the change in the volatility. The bottom figure giving an estimate of the standard deviation of the expiry date surplus is probably biased upwards, for we detect some negative serial correlation in the hedging errors, and particularly in the volatility component.

It is evident from this table that the replication errors resulting even from the best of the methods tried are still roughly almost of the same order of magnitude as the original option premium.

The risk of the unhedged positions is very large and consists mostly of the risk due to movements in the futures prices. Delta hedging brings about a dramatic reduction in the risk exposure, but leaves the risk component stemming from volatility movements unchanged. Delta-gamma hedging gives a significant further reduction in the hedging errors, and reduces both the risk due to movements in the futures prices and movements in the volatilities. Our delta-kappa hedging strategy is quite naive. We are hedging a long option with short ones but our calculations ignore the attenuation of the volatility of volatility as a function of the expiry date. It is therefore not surprising that the results for this strategy are rather poor. It is worth noting that the strategy provides some reduction in the risk of changes in volatility, but at the expense of an even greater exposure to the futures.

Table 2B gives the same results but weighted so that the initial sale of options had a value of $100. The values in this table can therefore be conveniently interpreted as giving the hedge errors as a percentage of the initial premium.

5. Extensions

The program provides us with a test bed that can be used to compare a wide variety of hedging strategies on both individual options and on portfolios of options. In particular, it enables calculations to be made to examine the implications of using different volatility inputs, and of adopting alternative hedging policies to take account of the transaction costs involved in rebalancing hedge positions.

Given the considerable variation in the implied volatilities in our database there is a good case to be made that rebalancing using these is likely to introduce unnecessary transactions and probably also additional hedging noise. The changes in implied volatilities exhibit greater negative serial correlation than seems likely to be attributable to conventional mean reversion. Part of the changes must be attributable to short term trading pressures, non-synchronous data and other transient effects. We have therefore examined the performances of hedges which have been calculated using a constant volatility figure. We have used a volatility of 17% for this, which is consistent with estimates taken over quite long time periods, see for example Turner and Weigel (1993). This figure is slightly above Clewlow and Xu's figures for the average historical volatility in our data set, but slightly below the average of the implied volatilities.

Table 3 shows the hedging results we have computed under this assumption. Note that while a constant volatility of 17% has been used for all the calculations of the positions to hedge with, the return calculations and their decomposition are done as before using the implied volatilities. The results show a marked improvement in the performance of the simple delta-hedging strategy. Not surprisingly, there seems to be little change in the strategies which employ options. Table 4 records the extraordinarily high turnover of the delta-gamma and delta-kappa hedges. This is slightly reduced by the constant volatility assumption.

6. Conclusions

This paper provides an empirical study of the effectiveness of various methods of hedging the risk of options positions based on eight years of daily data for the CME S&P 500 index options contract. We have examined the performance of delta-gamma hedges and delta-kappa hedges as well as simple delta hedges. The hedging errors are also decomposed and attributed to the effect of the changes in the underlying, (including components from the portfolio's delta and gamma), the effect of changes in volatility and the effect of the changes in the interest rate. This decomposition is (loosely) based on the analysis given in Bookstaber (1991). The analysis shows that although the (kappa) risk attributable to changes in volatility is small relative to that (delta) due to changes in the underlying, it becomes very significant after delta hedging has been done. Naive kappa hedging is less effective than gamma hedging if we neglect to take account of the difference in variation in implied volatilities for different expiries. Work is in progress to estimate an appropriate model for these volatility relationships. The sample period was punctuated by a number of episodes of abnormally high volatility in both the historical and implied volatilities. This and the changing relationships between expiry dates complicates the estimation. Using a constant volatility for delta-hedging calculations gave reduced hedging errors, but no real improvement for delta-gamma hedging or delta-kappa hedging.

The results obtained so far are rather preliminary and much remains to be done to extend them and to consider a range of other issues. In particular, we intend to do further work on the choice of volatility inputs, including consideration of the changing nature of volatility skews and its implications for hedging positions across a range of strikes. We also plan to compare empirically the properties of adopting alternative hedging policies to take account of the transaction costs involved in rebalancing hedge positions.

Finally, some related work is being undertaken on hedging properties of options on interest rates. We are using a database containing five years of data on the LIFFE short sterling options. For these options, the American feature has no value and we can treat them as European options. Preliminary results are qualitatively very similar to those for the equity futures options.

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Table 1

Return Decomposition

Value Under	Cash	Option	Gives Return Due to	
$F_0\ \sigma_0\ r_0\ t_0$	6.7500	-6.7500		
$F_0\ \sigma_0\ r_0\ t_1$	6.7517	-6.7381	Time:	0.0136
			Future: $\Delta = 0.6179$	
			$\Gamma = -0.0167$	
			Res = -0.0001	
$F_1\ \sigma_0\ r_0\ t_1$	6.7517	-6.1370	Total Future:	0.6011
$F_0\ \sigma_1\ r_0\ t_1$	6.7517	-6.8565	Volatility	-0.1184
$F_0\ \sigma_0\ r_1\ t_1$	6.7517	-6.7333	Int Rate	0.0048
			Residual:	0.0006
$F_1\ \sigma_1\ r_1\ t_1$	6.7517	-6.2500	Total	0.5017

t	τ	F	σ	r	C	Δ	Γ
0	261	174.3	12.81%	9.00	6.75	0.4753	0.0198
1	260	173.0	13.02	9.10	6.25	0.4501	0.0196

Option has strike = 175.

Table 2A

Summary of Hedging Results

Contract		Expiry Hedging Errors Under			
Date	Premium	No Hedge	Delta	D-Gamma	D-Kappa
8509	6.75	-5.5	0.8	-2.4	-2.7
8606	4.90	-45.5	-1.4	0.8	0.7
8703	13.00	-10.1	-2.1	0.8	2.7
8712	15.50	16.5	-10.5	-0.9	10.2
8809	22.50	16.2	11.2	3.1	-10.5
8906	20.00	-29.4	6.8	0.5	-14.8
9003	5.00	5.5	-0.1	0.9	6.9
9012	22.50	24.4	1.1	1.0	-5.7
9112	25.00	-16.2	5.4	-5.0	-11.3
9209	18.00	6.7	7.4	-1.1	-13.9
Mean		-3.75	1.86	-0.21	-3.84
SD		20.98	5.83	2.13	8.38
SDs from daily errors:					
underlying:		23.43	3.33	1.46	7.16
volatility:		7.82	7.82	5.52	4.76
Total		22.75	6.50	4.27	8.27

Table 2B

Summary of Hedging Results: $100 Initial Premium

Contract Date	Expiry Hedging Errors Under			
	No Hedge	Delta	D-Gamma	D-Kappa
8509	-81.8	11.2	-35.4	-39.9
8606	-929.5	-28.3	16.9	15.1
8703	-77.8	-16.0	6.4	20.8
8712	104.8	-66.9	-5.6	64.6
8809	75.8	52.6	14.7	-49.2
8906	-141.3	32.5	2.5	-70.9
9003	104.9	-1.7	17.0	132.6
9012	104.9	4.6	4.5	-24.4
9112	-64.9	21.8	-20.0	-45.3
9209	31.8	35.0	-5.1	-65.8
Mean	-87.3	4.5	-0.4	-6.2
SD	294.0	33.2	16.0	61.7
SD from daily errors:				
underlying	207.0	28.1	14.7	87.9
volatility	58.5	58.5	45.1	34.3
Total	202.3	52.4	36.1	89.5

Table 3

Hedging Results at Constant Volatility

Contract Date	Expiry Hedging Errors Under Delta	D-Gamma	D-Kappa
8509	0.3	-3.4	-5.5
8606	-2.0	0.3	1.2
8703	-1.7	1.2	3.4
8715	-6.5	-0.3	4.7
8809	8.2	-0.5	-8.2
8906	6.6	0.0	-14.2
9003	-2.7	-0.8	6.6
9012	1.5	0.5	-11.4
9112	6.8	-8.9	-7.2
9209	7.6	-0.1	-12.1
Mean	1.80	-1.20	-4.27
SD	4.91	2.81	7.25
SDs from daily:			
underlying	4.09	4.09	8.68
volatility	7.82	6.97	6.28
Total	4.65	4.21	9.00

Table 4

Average Turnover

	Futures Turnover (#'s of contracts)		
Volatility	Delta	D-Gamma	D-Kappa
Implied	3.1	6.5	17.0
Constant	3.2	3.6	18.2

	Options Turnover (#'s of contracts)	
	D-Gamma	D-Kappa
Implied	25.1	38.3
Constant	20.5	39.6