# The Asset Allocation Decision in a Loss Aversion World


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June 6, 2001


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The second author would like to thank Inquire for financial support.

**Abstract**

The purpose of this paper is to derive explicit formulae for the asset allocation decision for the loss aversion utility function proposed by Kahneman and Tuversky. We show that these utility functions exhibit constant absolute risk aversion. We also give analytic results which interpret the assumptions of risk-aversion with respect to gains but risk-affection with respect to losses in terms of changes of the optimal investment of equity when the probability that equity outperforms cash goes up. For the Knight, Satchell and Tran (1995) family of distributions, it is straightforward to derive closed form expressions for the optimal portfolio weights in all cases. Using UK and US data, we confirmed that the values of the parameters in the loss aversion function suggested by many previous studies are compatible with the observed proportions held in equity in both the UK and the US. The distributional assumptions are not innocuous. However, whilst modelling upside and downside returns by gamma distributions leads to plausible results, modelling upside and downside by truncated normals does not.

**Keywords:** Loss Aversion, KST family, Downside Risk

# 1 Introduction

Modern finance theory starts from a set of normatively appealing axioms about individual behavior. That is, people are assumed to be risk-averse expected utility maximizers and make rational choices based on rational expectations. However, the rational paradigm has been criticized by many behavioral economists and psychologists such as Kahnemann and Tversky (1979) and De Bondt and Thaler (1985).

In particular, dissatisfaction with power utility has been a re-occuring theme in modern financial economics. From the equity premiumm puzzle to the inability to explain the presence of gambling and holding insurance simultaneously, power utility's faults are numerous and well-documented; see Mehra and Prescott (1985) and Campbell and Viceira (1999) for example. New alternatives built around power utility have been put forward; loss aversion (Kahnemann and Tversky, 1979, 1992) and disappointment aversion (Gul, 1991 and Ang, Bekeart and Liu, 2000) may work better to name but some of the alternatives.

The purpose of this paper is to concentrate on loss aversion (LA) utility, first put forward by Kahnman and Tversky (1979, 1992) and used, among others, by Barberis, Huang and Santos (1999), Berkelaar and Kouwenberg (2000a, 2000b), and solve the asset allocation problem for an investor with LA utility in a one period world. In doing so, we revisit the two piece utility functions of Fishburn and Kochenberger (1979) who put forward this structure of utility as an example of conventional expected utility theory.

For a very broad family of distributions, the KST family, see Knight, Satchell and Tran (1995), it is possible to derive closed form expresssions for the optimal proportion of wealth held in equity. Although one can compute the optimal proportion fairly easily using numerical methods, the benefits of explicit formulae are self-evident. Furthermore, inspection of the results gives us a better understanding of the factors driving equity investment and the delicate nature of the assumptions governing upward and downward risk tolerance.

In section two, we present details of both the LA utility being considered and the KST distribution. Results are presented in section three, an application to UK equity follows in section 4 whilst section 5 concludes.

# 2 The Optiml Portfolio with Loss Aversion

The version of LA utility we use follows other authors with some minor modifications. Let $W$ be final wealth, $W_o$ initial wealth, $B$ some appropriate benchmark. Defines gains $X = W - B$, then LA utility is defined as

$$\begin{aligned} u(X) &= \frac{X^{v_1}}{v_1},\ if\ X > 0, \\ &= -\lambda\frac{(-X)^{v_2}}{v_2},\ if\ X \leq 0, \end{aligned} \tag{1}$$

where the parameters $v_1$, $v_2$ and $\lambda$ are assumed positive.

It is possible to distinguish several cases. The first case is that $0 < v_1 < 1$, $0 < v_2 < 1$. In this case $u(.)$ is 'risk averse' with respect to gains since $u''(X) = (v_1 - 1)X^{v_1 - 2}$ which is negative. However, $u(.)$ is risk loving with respect to losses since $u''(X) = -\lambda(v_2 - 1)(-X)^{v_2 - 2}$ which is positive. Another possibility studied in the literature is to let $v_i = 1$, $i = 1$ or 2, so that $u(.)$ could be risk neutral with respect to gain or loss. For example, Kahnemann and Tversky (1979) use $v_1 = v_2 = 0.88$ and $\lambda = 2.25$, Benartzi and Thaler (1995) use $v_1 = v_2 = 1$ and $\lambda = 2.25$, Ang, Bekeart, and Liu (2000) use $v_1 = v_2 = 0.88$ and a range of $\lambda$ values, although some of these applications are for multi-period optimisations rather than the one-period problem we consider here.

However, other cases could be of interest. If $v_1 > 1$ and $v_2 > 1$, then the investor is risk loving with respect to gains whilst risk averse to losses. Which of these alternatives is the more plausible is not obvious. Several risk control strategies, prevalent in the market, capture some aspect of these alternative cases. For example, a stop-loss strategy controls downside risk and is presumably consistent with $v_2 > 1$. A take-profit strategy controls upside risk and might be consistent with $v_1 < 1$. Therefore, it is not clear on a prior grounds or observed behaviour whether $0 < v_2 < 1$, $v_2 > 1$, or $v_1 > 1$, and so we include all possibilities. These appear to be no theoretical results in the literature that support any of the different assumptions about $v_1$ and $v_2$. However, Fishburn and Kochenberger (1979) present some empirical evidence $\lambda > 1$ and that $0 < v_1 < 1$ and $0 < v_2 < 1$. They also refer to other papers that present empirical support of these assumptions, namely investors are risk-averse for gains and risk-loving for losses. In choosing $B$ it is customary to let $B$ equal $W_0$ or $W_0(1 + r_f)$ where $r_f$ is the riskless rate of return and we shall adapt the latter approach.

Asset allocation problems consider optimal portfolios taken over a small number of asset classes. Suppose that there are two assets; a riskfree asset and a risky asset whose returns are denoted by $r_f$ and $r$, respectively. Denoting $\theta$ to be the proportion held in equity, then final wealth is

$$\begin{aligned} W &= W_0(1-\theta)(1+r_f) + W_0\theta(1+r) \\ &= B + W_0\theta y, \end{aligned} \tag{2}$$

where $B = W_0(1+r_f)$ and $y = r - r_f$ is the excess return of the risky asset, and thus $X = W_0\theta y$. In what follows, we implicitly assume that $\theta \geq 0$, i.e., short sales are not allowed.

In the following theorem, we show that under some condition, any utility function of $W - B$ becomes CARA class of utility functions.

**Theorem 1** *Suppose $u$ is a utility function of the form $u(W - B)$ where $B = W_0(1 + r_f)$ and $u$ does not depend upon $W_0$ in any other way. Then it follows that $u$ is CARA.*

**Proof.** For an investor whose utility function is $u(W - B)$, the problem is to solve

$$\max_{\theta} \; E(u(\theta W_0(r - r_f))).$$

The first order conditions are, putting $y = r - r_f$ and $h = \theta W_0$

$$E(u'(\theta W_0 y)W_0 y) = 0$$

or

$$E(u'(hy)y) = 0.$$

The solution $\widehat{h}$ depends only upon the form of $u$ and the distribution of $y$ but not $W_0$, since by assumption $u$ does not depend on $W_0$. So we find that $\widehat{\theta}W_0 = \widehat{h}$ and result is proved. ■

**Remark 1** *Although this may appear restrictive, it is quite easy to reparametrise this problem so that the utility function exhibits non-constant absolute risk aversion. See Pedersen and Satchell (2001) for details of this approach. In the context of this problem, let*

$$\begin{aligned} u(W - B) &= u_1(W - B) \text{ if } W \geq B \\ &= -\lambda u_2(B - W) \text{ if } W < B, \end{aligned}$$

*where $\lambda$ is some positive constant. If we let $\lambda = \lambda(W_0)$ and if $\widehat{\theta}$ is the optimal solution to A, then $\widehat{\theta}$ is, typically, decreasing in $\lambda$. The dynamics of $\widehat{\theta}$ or $\widehat{\theta}W_0$ with respect to $W_0$ can be deduced from the relationships $\widehat{\theta} = \widehat{\theta}(\lambda)$ and $\lambda = \lambda(W_0)$.*

The optimal portfolio can be obtained with an appropriate choice of $\theta$. Let $u^+ = E(y^{v_1}|y > 0)$, $u^- = E((-y)^{v_2}|y < 0)$, $p = prob(y > 0)$. Since $X$ in equation (1) is equivalent to $W_0\theta y$, expected utility $U_{LA}$ is given by

$$U_{LA} = \frac{1}{v_1}(W_0\theta)^{v_1}u^+p - \frac{\lambda}{v_2}(W_0\theta)^{v_2}u^-(1-p).$$

Thus the first derivative with respect to $\theta$ is

$$U'_{LA} = W_0^{v_1} p \theta^{v_1 - 1} u^+ - \lambda W_0^{v_2} (1-p) \theta^{v_2 - 1} u^-. \tag{3}$$

Inspection of (3) shows that if $v_1 = v_2$, the optimal solution for $\theta$ is 0 if $W_0^{v_1} u^+ p < \lambda W_0^{v_2} u^- (1-p)$, or $\infty$ if $W_0^{v_1} u^+ p > \lambda W_0^{v_2} u^- (1-p)$. If the inequality is an equality, $\theta$ is indeterminate since $U'_{LA}$ is zero.

Setting $U'_{LA} = 0$ and solving for the case $v_1 \neq v_2$, we see that

$$\begin{aligned} \theta &= \left( \frac{u^+ p W_0^{v_1 - v_2}}{\lambda u^- (1-p)} \right)^{\frac{1}{v_2 - v_1}} \qquad (4) \\ &= \frac{1}{W_0} \left( \frac{u^+ p}{\lambda u^- (1-p)} \right)^{\frac{1}{v_2 - v_1}} \end{aligned}$$

and

$$\ln \theta = \frac{1}{v_2 - v_1} \ln(u^+ p) - \frac{1}{v_2 - v_1} \ln(\lambda u^- (1-p)) \tag{5}$$

when $W_0 = 1$.

In equations (4) and (5) the solution for $\theta$ is general. We have, therefore, the following proposition.

**Proposition 2** *If the investor has an LA utility function with $v_2 \neq v_1$ and benchmark $B = W_0(1 + r_f)$, then the optimal equity investment proportion $\theta$ in equation (4) can be obtained for any arbitrary probability density function. Furthermore, $\theta W_0$ is constant so we have constant absolute risk aversion.*

**Proof.** For the first part, the proof is given above. The second argument can be proved with equation (4), since the dollar amount in equity is $\theta W_0$, and,

$$\theta W_0 = \left( \frac{u^+ p}{\lambda u^- (1-p)} \right)^{\frac{1}{v_2 - v_1}},$$

which is independent of $W_0$. *QED.* ■

**Proposition 3** *If the investor has an LA utility function with $v_2 \neq v_1$, $B = W_0(1 + r_f)$, and the proportion of wealth held in equity is an increasing function of the probability that equity outperforms cash, then we have $v_2 - v_1 > 0$.*

**Proof.** Since the proportion of wealth in equity goes up with the probability of a gain, the partial derivative of $\ln \theta$ with respect to $\ln p$ should be positive, i.e., $\frac{\partial \ln \theta}{\partial \ln p} > 0$. Thus, from equation (5),

$$\frac{\partial \ln \theta}{\partial \ln p} = \frac{1}{(v_2 - v_1)(1-p)} > 0.$$

The condition that satisfies the above equation is $v_2 - v_1 > 0$. *QED.* ■

Proposition 2 is a sensible constraint since it implies that the proportion of wealth held in equity goes up if the probability of a gain goes up (ceteris paribus, other than that the probability of a loss goes down automatically). We now ask what behaviour is ruled out? For example, if $v_1 > 1$ and $v_2 < 1$ so that we are risk loving for both gains and losses, this would be ruled out by the obvious constraint. If $v_2 > 1$ so that we are risk averse for losses we can still be risk loving for gains as long as $v_1 < v_2$. Obviously risk neutrality for losses and risk loving for gains is also excluded.

**Remark 2** *In the general case, if the investor is risk averse for gains and losses, then $v_2 > 1$, $v_1 < 1$ and $v_2 - v_1$ is positive, in this case $\frac{\partial \ln \theta}{\partial \ln u^+}$ is positive and $\frac{\partial \ln \theta}{\partial \ln u^-}$ and $\frac{\partial \ln \theta}{\partial \ln \lambda}$ are negative.*

**Remark 3** *If $W_0$ is set to 1 and $0 < \theta < 1$, then any positive $v_2 - v_1$ would imply from (4) that $u^+ p < \lambda u^-(1-p)$.*

# 3 Empirical Tests

In this section, we suggest appropriate sets of $v_1$, $v_2$ and $\theta$ for the UK and US markets under the assumption that the excess returns of risky asset in (2) are normally distributed. However, in many cases, normality is not an appropriate assumption. As an alternative to the normal distribution, we use the KST distribution. Some additional explanation on our distributional assumptions and our empirical results follow.

## 3.1 KST and Normal Distributions

Since the optimal $\theta$ can be obtained with any distribution, we can calculate the optimal $\theta$ for the most widely used distribution, the normal distribution, and an alternative, the KST distribution which we define next. Let $y_t$ be the excess return at time $t$,

$$y_t = \mu + \varepsilon_{1t} z_t - \varepsilon_{2t}(1 - z_t), \tag{6}$$

where $\mu$ is the mean of $y_t$, and $z_t$ is a binary indicator variable with probability $p$, and $\varepsilon_{1t}$ and $\varepsilon_{2t}$ are independent non-negative randon variables. Then, the pdf of $y_t$ is,

$$pdf(y_t) = \left\{ \begin{array}{l} p f_1(y_t - \mu) \text{ for } y_t \geq \mu \\ (1-p) f_2(\mu - y_t) \text{ for } y_t < \mu, \end{array} \right. \tag{7}$$

where $f_1$ and $f_2$ are pdf's. In our study, $\mu$ is set to zero because the expectations $u^+$and $u^-$ are conditional on $y_t \geq 0$ or $y_t < 0$.

The first case we consider is the KST distribution. The KST distribution is designed to capture the asymmetry in upward and downward asset returns. The pdf's of the KST distribution, $f_1$ and $f_2$, are assumed in this paper to be the scale Gamma distribution;

$$f_i(x) = \{ \begin{array}{l} \frac{\xi_i^{\alpha_i} x^{\alpha_i - 1}}{\Gamma(\alpha_i)} \exp(-\xi_i x) \text{ for } x > 0 \\ 0 \text{ otherwise,} \end{array} \tag{8}$$

for $i$ =1 and 2. Thus, when $p = 1/2$, $\xi_1 = \xi_2$ and $\alpha_1 = \alpha_2$, the pdf becomes symmmetric. Although in this paper, we assume that $r_t$ is *iid*, it is straightforward to extend this model to make $\mu$ time dependent and let $z_t$ follow a Markov process. See Bond (2001) or Damant, Hwang, and Satchell (1999) for example.

With the KST distribution, we can obtain analytical results for $u^+$ and $u^-$ with the KST distribution. As explained above, the KST distribution provides us with a closed form solution for $u^+$ and $u^-$ in (4) as well as capturing the asymmetric properties of asset returns. Note that in the case of the gamma distribution,

$$\begin{aligned} u^+ &= E(y^{v_1}|y > 0) \\ &= \int_0^\infty y^{v_1} f_1(y) dy \\ &= \int_0^\infty y^{v_1} \frac{\xi_1^{\alpha_1} y^{\alpha_1 - 1}}{\Gamma(\alpha_1)} \exp(-\xi_1 y) dy \\ &= \frac{\Gamma(v_1 + \alpha_1)}{\xi_1^{v_1} \Gamma(\alpha_1)} \int_0^\infty \frac{\xi_1^{v_1 + \alpha_1} y^{v_1 + \alpha_1 - 1}}{\Gamma(v_1 + \alpha_1)} \exp(-\xi_1 y) dy \\ &= \frac{\Gamma(v_1 + \alpha_1)}{\xi_1^{v_1} \Gamma(\alpha_1)}, \end{aligned}$$

since

$$\int_0^\infty \frac{\xi_1^{v_1 + \alpha_1} y^{v_1 + \alpha_1 - 1}}{\Gamma(v_1 + \alpha_1)} \exp(-\xi_1 y) dy = 1.$$

Likewise

$$\begin{aligned} u^- &= E((-y)^{v_2}|y < 0) \\ &= \int_0^\infty y^{v_2} f_2(y) dy \\ &= \int_0^\infty y^{v_2} \frac{\xi_2^{\alpha_2} y^{\alpha_2 - 1}}{\Gamma(\alpha_2)} \exp(-\xi_2 y) dy \\ &= \frac{\Gamma(v_2 + \alpha_2)}{\xi_2^{v_2} \Gamma(\alpha_2)}, \end{aligned}$$

since

$$\int_0^\infty \frac{\xi_2^{v_2+\alpha_2} y^{v_2+\alpha_2-1}}{\Gamma(v_2+\alpha_2)} \exp(-\xi_2 y) dy = 1.$$

Therefore, for the KST distribution,

$$U_{LA} = p\frac{(W_0\theta)^{v_1}}{v_1}u^+ - (1-p)\lambda\frac{(W_0\theta)^{v_2}}{v_2}u^-,$$

where

$$\begin{aligned} u^+ &= \frac{\Gamma(v_1+\alpha_1)}{\xi_1^{v_1}\Gamma(\alpha_1)}, \\ u^- &= \frac{\Gamma(v_2+\alpha_2)}{\xi_2^{v_2}\Gamma(\alpha_2)}. \end{aligned}$$

The second case we consider is the frequently used normal distribution which is used for comparison purpose in this study. The normal distribution function for a variable $x$ is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \text{ for } x > 0 \tag{9}$$

where $\mu$ and $\sigma$ are mean and standard deviation of the variable $x$. Note that in the above normal distribution, we do not use different paramaters for the negative and positive returns and thus it does not capture the asymmertic property of the asset returns. In addition, analytical derivation for $u^+$ and $u^+$ is difficult and in this study, we calculate them numerically. It is worth noting that

$$u^+ = \int_0^\infty y^{v_1} \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left[-\frac{(y-\mu_1)^2}{2\sigma_1^2}\right] dy$$

and

$$u^- = \int_{-\infty}^0 (-y)^{v_2} \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left[-\frac{((-y)-\mu_2)^2}{2\sigma_2^2}\right] dy$$

## 3.2 Empirical Testing

For the two distributions above, we use UK and US financial data to capture the asset allocation decisions of typical UK and US investors. That is, we use equation (4) to obtain sensible sets of $v_1$ and $v_2$ for a given level of the investment proportion on risky assets.

As pointed out in Damant, Hwang, and Satchell (2000), if we treat such an investor as a pension scheme, it is necessary to consider at least several different asset classes which need a great deal of data and information. Instead, as discussed earlier, we shall concentrate on a stripped-down version of the problem, namely, two asset classes. This simplified version which consists of a riskfree asset and an equity (risky asset) is consistent with the setting we used above for the optimal portfolio with loss aversion. Note that the investment proportion in domestic and foreign equity in 1993 was 83% for large pension funds in the UK, while in the USA it was 46%[1].

For the riskfree and equity, we use the three month UK treasury bill and the FT All-share for the UK market and the three month US treasury bill and the S&P500 for the US market. Thus in the above loss aversion utility the benchmark is represented by the three month treasury bill interest rate.

The monthly treasury bill rate and the market index return from January 1980 to February 2001 for a total of 254 monthly observations are used. In addition, to investigate time-varying properties of the returns, we divide the entire sample period into two arbitrary sub-samples; the first sub-sample from January 1980 to December 1989 and the second sub-sample from January 1990 to February 2001.

We first report the statistical properties of the data in table 1. The mean and standard deviation of the UK returns are similar to those of the US returns. Because of last several years' high economic growth in the US, the US market outperformed the UK market in the second sub-sample period. The table also shows that the UK market returns are more negatively skewed and fat-tailed than the US market returns. But most of the non-normality in the UK market comes from the first sub-sample period, and during the second sub-sample period, the US market is more skewed and fat-tailed than the UK market.

Overall, the monthly excess returns are not normal both in the UK and US markets; Jarque-Bera statistics show that most excess returns are non-normal. As is well-known, when empirical distributions are not normal, any results obtained with the assumption of normality may be wrong. We need to model the impact of asymmetry and fat-tail in returns.

We first use the double gamma distribution as in (8) proposed by KST. The double gamma distribution can be combined with the regime switching model introduced by Hamilton (1989), if we assume that the binary indicator variable $z_t$ in equation (6) follows a two-state Markov process. In this case, the probability density function becomes complicated. See KST for further discussion on regime switching double gamma pdf. The autocorrelation coefficients reported in table 1 suggest that the excess returns are not serially

[1] These numbers were found in the following unpublished mimeos; MERCERS, European Pension Fund Managers (1993) and PDFM, Pension Fund Indicators (1991).

correlated. Thus, we do not pursue this complicated version any more here.

Table 2 reports the estimates of KST parameters with the excess returns for $\mu = 0$. All estimates are significant and similar to those reported in KST; large values of $\xi_1$ and $\xi_2$, and $\alpha_1 > 1$ and $\alpha_2 > 1$. We can test symmetry by the hypothesis $\xi_1 = \xi_2$, and $\alpha_1 = \alpha_2$. During the first sub-period, the excess returns seem to be asymmetric, but the estimates of the second sub-period show that excess returns may be symmetric. In addition, the density has maximum value at $(\alpha_i - 1)/\xi_i$ when $\alpha_i > 1$. In our case, the estimates of $\alpha_i$ are all larger than one and thus has miximum value; for example, for the second sub-period in the UK market, the conditional densities for both positive and negative excess returns have maximum value at $(\widehat{\alpha}_1 - 1)/\widehat{\xi}_1 = (1.244 - 1)/40.403 = 0.006$, and $(\widehat{\alpha}_2 - 1)/\widehat{\xi}_2 = (1.109 - 1)/33.002 = 0.003$, respectively. In addition, the likelihood ratio tests with the maximum likelihood values (not reported) suggest that there is no significant difference between the first and the second sub-periods. Thus we use the estimates obtained with the full samples.

Using the paramater values estimated in table 2, we first calibrated the values of $\theta$ for given values of $v_1$, $v_2$, and $\lambda$. The ranges of $v_1$ and $v_2$ were set from 0.2 to 20, respectively, and $\lambda$ was set to 1.5, 2.25, and 3. We used the Newton-Raphson algorithm on equation (4) with the assumption of $W_0 = 1$ to calculate $\theta$.

Table 3 shows the investment proportion, $\theta$, for the settings in the UK market. We first investigate the results for the case of $\lambda = 2.25$ in panel A, which shows that when $v_1 > v_2$, the values of $\theta$ were at least more than 2000%, which is too large to accept. The results confirms the admissible ranges suggested in Proposition 2. However, for the investors who are risk averse for gains and losses as in Remark 1, we still have too large investment proportions, i.e., 300% to 1600%. See bold numbers in panel A of table 3. The table also suggests that when $v_2 - v_1 > 0$ and $v_1$ and $v_2$ are close to each other, it is possible to explain the UK and USA investment proportions in equity (i.e., 83% and 46% for large pension funds in the UK and USA, respectively).[2] These results are roughly consistent with other multi-period studies where $v_1$ and $v_2$ are assumed to have the same value. See Kahnemann and Tversky (1979), Ang, Bekeart, and Liu (2000) and Benartzi and Thaler (1995), for example. In addition, as expected, comparing panel A with panels B and C shows that when the penalty to loss becomes larger, $\lambda = 3$, investment proportions on equity decrease, and vice versa.

We now further investigate what may be appropriate sets of $v_1$ and $v_2$ to

[2] We do not report the USA cases which are similar to those of the UK. The results in the USA market can be obtained upon request from the authors.

explain the UK and USA markets. We first set $v_1$ to a value between 0.1 to 2. Other previous studies used 0.88 (Kahnemann and Tversky, 1979, and Ang, Bekeart, and Liu, 2000) or 1 (Benartzi and Thaler, 1995). The choice of $v_1$ is closely related with the admissible range of the coefficient of the CRRA. Many previous studies, either theorectically or empirically, suggest that the admissable range of the coefficient of the CRRA should be between one and two. See Arrow(1971), Tobin and Dolde (1971), Friend and Blume (1975), Kydland and Prescott (1982), and Kehoe (1984), for example.

Then, appropriate values of $v_2$ are calculated with equation (4) for given values of $v_1$, $\theta$, and $\lambda$, and the results are reported in tables 4 and 5 for the UK and USA markets, respectively. The Newton-Raphson algorithm is used on equation (4) with the assumption of $W_0 = 1$ to calculate $v_2$. Since the investment proportions in equity in the UK and USA are different, we use different values of $\theta$ for the UK and US markets; for the UK market, we use $0.5 \leq \theta \leq 1$ and for the US market we use $0.3 \leq \theta \leq 1$.

The first panels of tables 4 and 5 shows the case of $\lambda = 2.25$ which is used in most studies. In this case, we have $v_2 - v_1 > 0$ for all given values. However, we find evidence that the representative agents on both side of the atlantic are risk averse for gains and losses. That is, $v_2 > 1$ and $v_1 < 1$ can be obtained only when $v_1$ is very close to one, since the difference between $v_1$ and $v_2$ is not large. In most cases, $0 < v_2 - v_1 < 0.3$. In particular, for the representative UK investor ($\theta = 0.83$) who is risk averse for gain ($v_1 < 1$), the difference between $v_2$ and $v_1$ is less than 0.1, and a similar pattern is found in the USA market. As explained above, these results do differ from many other multi-period studies which simply use $v_1 = v_2$. However, to be more precise, we need $v_2$ to be larger than $v_1$.

As explained in table 3, for a given value of $v_1$, the values of $v_2$ increase as $\lambda$ increases; see panels B and C in tables 4 and 5. Interestingly, when $\lambda = 1.5$, and for some ranges of $v_1$, the values of $v_2$ are less than those of $v_1$. See panel B of tables 4 and 5. Since we expect $v_2 > v_1$, these results are unacceptable and suggest that the arbitrary value $\lambda = 1.5$ is too small. Thus, our results indirectly indicate that $\lambda = 2.25$ which is widely used in empirical finance is in the admissible range. In fact, all three parameters in the LA utility function, $v_1$, $v_2$ and $\lambda$, are closely connected to each other and a value of one parameter may not be justified independently of other parameters. Unfortunately, our study cannot reveal a unique set of $v_1$ and $v_2$, but it does tell us what are appropriate sets of $v_1$ and $v_2$. If we assume $v_1 = 0.88$ as in Kahnemann and Tversky (1979) and others, then $v_2$ should be around one.

Finally we use normal distribution to find out the relationship between $\theta$, $v_1$, $v_2$ and $\lambda$. The same method described above is used here for the UK market except the normal distribution.[3] Using the estimates of mean and standard deviation in table 1, we numerically calculated the values of $\theta$ for given values of $v_1$, $v_2$, and $\lambda$ and reported the results in table 6. We could not obtain the values of $\theta$ when $v_2 - v_1 \leq 0$ and $v_1 > 1$ because of convergence errors. Table 6 suggests that the values of $\theta$ for the admissable ranges of $v_2 - v_1 > 0$ are extremely large and unrealistic; i.e., at least 2600% when $\lambda = 2.25$! As explained in table 3, when $\lambda = 1.5$ or $\lambda = 3$, the equity investment proportion increases or decreases.

We next calculate appropriate values of $v_2$ with equation (4) for given values of $v_1$, $\theta$, and $\lambda$ under the normality assumption. The results in table 7 also confirm that $v_2$ and $v_2$ are close. However, when tables 4 and 7 are compared, the values of $v_2$ obtained with normality are always less than those with the KST distribution. As a result of this, when $\lambda = 1.5$, we have $v_1 > v_2$ which contradicts proposition 2. In addition, when $\lambda = 2.25$ and $v_1 > 1$, we also have $v_1 > v_2$. Only when $\lambda$ is large enough, i.e., large panalty for losses, proposition 2 is satistied. This is because normality does not capture extreme events adequately and thus we need large value of $\lambda$ to compensate. For any distribution such as the KST distribution which can explain fat-tails of returns, we do not need large $\lambda$. Therefore, the results in tables 6 and 7 indicate that we should be more careful when we use normality for the LA utility function.

# 4 Conclusions

In this study we used the LA utility function to explain the asset allocation. We first developed a few conditions that the LA utility function should satisfy. We showed that under fairly general conditions, the LA utility function becomes CARA class of utility functions. This is an interesting result in the sense that bahavoural finance such as prospect theory can be explained with the expected utility theory. In addition, we also proved that the curvature for the losses ($v_2$) should be larger than that for the gains ($v_1$). This result is important because so far most studies such as Kahnemann and Tversky (1979), Benartzi and Thaler (1995), and Ang, Bekeart, and Liu (2000) used the same value for $v_1$ and $v_2$.

These results are supported by the empirical tests. In this study, we used a fairly general asymmetric distribution, Knight, Satchell and Tran (1995) distribution for the UK and US markets. We found that $v_1$ and $v_2$ are close to each other and $v_2 - v_1$ is positive for the ranges of parameter values used by other previous studies. For comparison purpose, however, when we assume

[3] The results of the US market are available from the authors upon request.

normality for asset returns, our values of $\lambda$ need to be larger since the normal distribution does not capture the fat-tails of returns adequately. Thus under normality, $\lambda$ may be larger than 2.25 which is used by many previous studies.

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## Table 1 Properties of FT All-share and S&P500 Index Returns

**A. FT All-share Index Returns**

| | | Mean | Standard Deviation | Skewness | Excess Kurtosis | Jarque-Bera Statistic |
|---|---|---|---|---|---|---|
| Entire Sample Period (January 1980 - February 2001) | FT All-share Index Returns | 1.4778 | 4.7757 | -1.0937 * | 4.6576 * | 280.23 * |
| | 3 Month Treasury Bill Returns | 0.7706 | 0.2668 | 0.3514 * | -1.0206 * | 16.25 * |
| | Excess Returns | 0.7071 | 4.7782 | -1.1444 * | 4.6906 * | 288.30 * |
| Sub-period 1 (January 1980 - December 1989) | FT All-share Index Returns | 1.9545 | 5.4103 | -1.6334 * | 6.3792 * | 256.83 * |
| | 3 Month Treasury Bill Returns | 0.9449 | 0.1858 | 0.4593 * | -0.7005 | 6.67 * |
| | Excess Returns | 1.0096 | 5.4156 | -1.6298 * | 6.2323 * | 247.33 * |
| Sub-period 2 (January 1990 - February 2001) | FT All-share Index Returns | 1.0509 | 4.0996 | -0.1943 | 0.2596 | 1.22 |
| | 3 Month Treasury Bill Returns | 0.6146 | 0.2293 | 1.4753 * | 1.0741 * | 55.05 * |
| | Excess Returns | 0.4363 | 4.1271 | -0.2576 | 0.1899 | 1.68 |

| | Lags | Entire Sample Period (January 1980 - February 2001) | Sub-period 1 (January 1980 - December 1989) | Sub-period 2 (January 1990 - February 2001) |
|---|---|---|---|---|
| Autocorrelations of Excess Returns | 1 | -0.0267 | -0.0805 | 0.0584 |
| | 2 | -0.1133 | -0.1010 | -0.1219 |
| | 3 | -0.1016 | -0.0860 | -0.1444 |
| | 4 | 0.0240 | 0.0186 | 0.0125 |
| | 5 | -0.0032 | -0.0448 | 0.0316 |
| | 6 | -0.1109 | -0.1188 | -0.1548 |

Notes: A total number of 254 monthly returns are used to calculate the above table. * represents significance at 90% level.

**B. S&P500 Index Returns**

| | | Mean | Standard Deviation | Skewness | | Excess Kurtosis | | Jarque-Bera Statistic |
|---|---|---|---|---|---|---|---|---|
| Entire Sample Period (January 1980 - February 2001) | S&P500 Index Returns | 1.3398 | 4.4019 | -0.6739 | * | 2.9716 | * | 112.68 * |
| | 3 Month Treasury Bill Returns | 0.5620 | 0.2349 | 1.1985 | * | 1.1447 | * | 74.68 * |
| | Excess Returns | 0.7777 | 4.4230 | -0.6741 | * | 2.8098 | * | 102.79 * |
| Sub-period 1 (January 1980 - December 1989) | S&P500 Index Log-returns | 1.4721 | 4.7445 | -0.7913 | * | 4.0931 | * | 96.29 * |
| | 3 Month Treasury Bill Returns | 0.7307 | 0.2280 | 0.8750 | * | -0.1112 | | 15.38 * |
| | Excess Returns | 0.7414 | 4.7837 | -0.7458 | * | 3.7082 | * | 79.88 * |
| Sub-period 2 (January 1990 - February 2001) | S&P500 Index Log-returns | 1.2213 | 4.0851 | -0.5389 | * | 1.1294 | * | 13.61 * |
| | 3 Month Treasury Bill Returns | 0.4110 | 0.0992 | 0.2801 | | 0.2538 | | 2.11 |
| | Excess Returns | 0.8103 | 4.0910 | -0.5582 | * | 1.1411 | * | 14.23 * |

| | Lags | Entire Sample Period (January 1980 - February 2001) | Sub-period 1 (January 1980 - December 1989) | Sub-period 2 (January 1990 - February 2001) |
|---|---|---|---|---|
| Autocorrelations of Excess Returns | 1 | -0.0290 | 0.0563 | -0.1349 |
| | 2 | -0.0293 | -0.0791 | 0.0349 |
| | 3 | -0.0365 | -0.0672 | -0.0141 |
| | 4 | -0.0733 | -0.0311 | -0.1355 |
| | 5 | 0.1098 | 0.1571 | 0.0542 |
| | 6 | -0.0329 | 0.0361 | -0.1005 |

Notes: A total number of 254 monthly returns are used to calculate the above table. * represents significance at 90% level.

## Table 2  Estimates of KST Parameters for the UK Returns

**A. FT All-share Index Excess Returns**

| | Entire Sample Period (January 1980 - February 2001) | Sub-period 1 (January 1980 - December 1989) | Sub-period 2 (January 1990 - February 2001) |
|---|---|---|---|
| $\xi_1$ | 42.256 (5.136) | 48.420 (8.162) | 40.403 (7.061) |
| $\alpha_1$ | 1.467 (0.150) | 1.870 (0.275) | 1.244 (0.178) |
| $\xi_2$ | 27.751 (4.484) | 23.495 (5.818) | 33.002 (7.008) |
| $\alpha_2$ | 1.066 (0.136) | 1.054 (0.206) | 1.109 (0.188) |
| Probability of Positive Excess Return (p) | 0.622 | 0.658 | 0.590 |
| Maximum Likelihood Value | 427.896 | 194.958 | 238.115 |

**B. S&P500 Index Excess Returns**

| | Entire Sample Period (January 1980 - February 2001) | Sub-period 1 (January 1980 - December 1989) | Sub-period 2 (January 1990 - February 2001) |
|---|---|---|---|
| $\xi_1$ | 40.296 (4.968) | 37.426 (6.868) | 43.439 (7.225) |
| $\alpha_1$ | 1.395 (0.144) | 1.394 (0.213) | 1.410 (0.196) |
| $\xi_2$ | 34.249 (5.384) | 29.747 (6.698) | 40.644 (8.917) |
| $\alpha_2$ | 1.173 (0.149) | 1.022 (0.180) | 1.388 (0.254) |
| Probability of Positive Excess Return (p) | 0.610 | 0.583 | 0.634 |
| Maximum Likelihood Value | 437.035 | 199.495 | 239.119 |

Notes: The estimates are obatined with maximum likelihood estimation for the pdf function in equations (3) and (4).

## Table 3 Proportion in Equity for Given Sets of $v_1$ and $v_2$ for the Entire Sample Period for the UK Market with the KST Distribution

**A. The Proportion in Equity (q) for the Entire Sample Period (January 1980-February 2001) with $l$ =2.25**

| $v_2 \backslash v_1$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 3.0 | 5.0 | 10.0 | 20.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | | 154.03 | 66.04 | 47.78 | 39.53 | 34.58 | 31.16 | 28.58 | 26.53 | 24.84 | 19.26 | 13.73 | 8.25 | 4.69 |
| 0.4 | 6.48 | | 142.43 | 59.68 | 43.04 | 35.65 | 31.27 | 28.26 | 25.99 | 24.20 | 18.59 | 13.26 | 8.03 | 4.60 |
| 0.6 | 12.79 | 5.01 | | 141.21 | 56.23 | 40.01 | 32.99 | 28.90 | 26.12 | 24.06 | 18.12 | 12.88 | 7.84 | 4.52 |
| 0.8 | 15.19 | 10.36 | 3.79 | | 147.60 | 54.69 | 38.11 | 31.15 | 27.18 | 24.53 | 17.85 | 12.57 | 7.67 | 4.44 |
| 1.0 | 15.98 | 12.59 | 8.40 | 2.82 | | 160.78 | 54.54 | 36.99 | 29.89 | 25.92 | 17.78 | 12.33 | 7.52 | 4.38 |
| 1.2 | **16.07** | **13.46** | **10.50** | **6.81** | 2.07 | | 181.06 | 55.51 | 36.48 | 29.05 | 17.93 | 12.14 | 7.39 | 4.32 |
| 1.4 | **15.83** | **13.70** | **11.43** | **8.80** | 5.52 | 1.50 | | 209.54 | 57.46 | 36.45 | 18.38 | 12.02 | 7.27 | 4.26 |
| 1.6 | **15.44** | **13.63** | **11.78** | **9.76** | 7.40 | 4.47 | 1.08 | | 248.09 | 60.33 | 19.25 | 11.94 | 7.17 | 4.21 |
| 1.8 | **14.99** | **13.41** | **11.84** | **10.20** | 8.38 | 6.24 | 3.62 | 0.77 | | 299.46 | 20.78 | 11.93 | 7.07 | 4.16 |
| 2.0 | **14.51** | **13.11** | **11.75** | **10.36** | 8.88 | 7.22 | 5.28 | 2.94 | 0.55 | | 23.53 | 11.98 | 6.99 | 4.11 |
| 3.0 | **12.26** | **11.37** | **10.54** | **9.74** | 8.96 | 8.19 | 7.39 | 6.56 | 5.67 | 4.69 | | 13.79 | 6.74 | 3.93 |
| 5.0 | **9.17** | **8.68** | **8.23** | **7.80** | 7.40 | 7.02 | 6.65 | 6.29 | 5.94 | 5.60 | 3.79 | | 7.01 | 3.70 |
| 10.0 | **5.61** | **5.41** | **5.22** | **5.04** | 4.88 | 4.72 | 4.58 | 4.44 | 4.30 | 4.17 | 3.58 | 2.52 | | 3.81 |
| 20.0 | **3.17** | **3.10** | **3.03** | **2.96** | 2.90 | 2.84 | 2.78 | 2.73 | 2.68 | 2.63 | 2.40 | 2.03 | 1.28 | |

**B. The Proportion in Equity (q) for the Entire Sample Period (January 1980-February 2001) with $l$ =1.5**

| $v_2 \backslash v_1$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 3.0 | 5.0 | 10.0 | 20.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | | 20.28 | 23.96 | 24.31 | 23.81 | 23.06 | 22.23 | 21.39 | 20.59 | 19.83 | 16.67 | 12.62 | 7.92 | 4.59 |
| 0.4 | 49.21 | | 18.76 | 21.66 | 21.90 | 21.48 | 20.85 | 20.15 | 19.46 | 18.78 | 15.90 | 12.14 | 7.70 | 4.50 |
| 0.6 | 35.23 | 38.08 | | 18.60 | 20.40 | 20.36 | 19.88 | 19.27 | 18.63 | 18.01 | 15.31 | 11.75 | 7.51 | 4.42 |
| 0.8 | 29.85 | 28.56 | 28.81 | | 19.44 | 19.84 | 19.39 | 18.77 | 18.12 | 17.49 | 14.84 | 11.42 | 7.34 | 4.35 |
| 1.0 | 26.52 | 24.76 | 23.15 | 21.43 | | 21.17 | 19.79 | 18.82 | 18.00 | 17.28 | 14.51 | 11.14 | 7.19 | 4.29 |
| 1.2 | **24.10** | **22.34** | **20.65** | **18.76** | 15.72 | | 23.84 | 20.15 | 18.56 | 17.50 | 14.31 | 10.91 | 7.05 | 4.22 |
| 1.4 | **22.19** | **20.55** | **18.97** | **17.30** | 15.21 | 11.41 | | 27.59 | 20.85 | 18.55 | 14.27 | 10.74 | 6.93 | 4.17 |
| 1.6 | **20.63** | **19.12** | **17.67** | **16.20** | 14.55 | 12.32 | 8.21 | | 32.67 | 21.89 | 14.41 | 10.60 | 6.83 | 4.12 |
| 1.8 | **19.31** | **17.92** | **16.60** | **15.30** | 13.91 | 12.27 | 9.99 | 5.86 | | 39.43 | 14.82 | 10.51 | 6.73 | 4.07 |
| 2.0 | **18.17** | **16.89** | **15.69** | **14.52** | 13.31 | 11.99 | 10.37 | 8.10 | 4.16 | | 15.68 | 10.47 | 6.65 | 4.02 |
| 3.0 | **14.17** | **13.29** | **12.48** | **11.71** | 10.98 | 10.25 | 9.52 | 8.77 | 7.95 | 7.03 | | 11.26 | 6.36 | 3.83 |
| 5.0 | **9.98** | **9.48** | **9.02** | **8.59** | 8.19 | 7.81 | 7.45 | 7.09 | 6.75 | 6.41 | 4.64 | | 6.46 | 3.60 |
| 10.0 | **5.84** | **5.64** | **5.45** | **5.27** | 5.10 | 4.95 | 4.80 | 4.65 | 4.52 | 4.39 | 3.79 | 2.74 | | 3.66 |
| 20.0 | **3.24** | **3.16** | **3.09** | **3.02** | 2.96 | 2.90 | 2.84 | 2.79 | 2.74 | 2.69 | 2.46 | 2.08 | 1.33 | |

**C. The Proportion in Equity (q) for the Entire Sample Period (January 1980-February 2001) with $l$ =3**

| $v_2 \backslash v_1$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 3.0 | 5.0 | 10.0 | 20.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | | 649.07 | 135.56 | 77.17 | 56.64 | 46.11 | 39.60 | 35.10 | 31.76 | 29.15 | 21.35 | 14.58 | 8.50 | 4.75 |
| 0.4 | 1.54 | | 600.22 | 122.52 | 69.52 | 51.08 | 41.69 | 35.91 | 31.92 | 28.97 | 20.76 | 14.12 | 8.28 | 4.67 |
| 0.6 | 6.23 | 1.19 | | 595.07 | 115.42 | 64.63 | 47.27 | 38.54 | 33.20 | 29.54 | 20.43 | 13.75 | 8.08 | 4.58 |
| 0.8 | 9.40 | 5.05 | 0.90 | | 621.96 | 112.26 | 61.55 | 44.63 | 36.24 | 31.17 | 20.34 | 13.46 | 7.91 | 4.51 |
| 1.0 | 11.15 | 7.80 | 4.09 | 0.67 | | 677.52 | 111.96 | 59.75 | 42.82 | 34.56 | 20.53 | 13.25 | 7.76 | 4.45 |
| 1.2 | **12.05** | **9.39** | **6.50** | **3.32** | 0.49 | | 762.98 | 113.96 | 58.93 | 41.63 | 21.04 | 13.10 | 7.63 | 4.38 |
| 1.4 | **12.46** | **10.28** | **7.98** | **5.45** | 2.69 | 0.36 | | 882.99 | 117.96 | 58.88 | 22.00 | 13.01 | 7.52 | 4.33 |
| 1.6 | **12.57** | **10.73** | **8.84** | **6.81** | 4.58 | 2.18 | 0.26 | | 1045.43 | 123.84 | 23.64 | 13.00 | 7.42 | 4.27 |
| 1.8 | **12.52** | **10.92** | **9.32** | **7.65** | 5.85 | 3.86 | 1.77 | 0.18 | | 1261.91 | 26.41 | 13.05 | 7.33 | 4.23 |
| 2.0 | **12.36** | **10.95** | **9.56** | **8.15** | 6.66 | 5.04 | 3.27 | 1.43 | 0.13 | | 31.37 | 13.19 | 7.25 | 4.18 |
| 3.0 | **11.06** | **10.18** | **9.35** | **8.55** | 7.76 | 6.98 | 6.18 | 5.34 | 4.46 | 3.52 | | 15.92 | 7.02 | 3.99 |
| 5.0 | **8.64** | **8.16** | **7.71** | **7.29** | 6.89 | 6.51 | 6.14 | 5.78 | 5.43 | 5.08 | 3.28 | | 7.42 | 3.77 |
| 10.0 | **5.44** | **5.25** | **5.06** | **4.89** | 4.73 | 4.57 | 4.43 | 4.29 | 4.15 | 4.02 | 3.43 | 2.38 | | 3.92 |
| 20.0 | **3.13** | **3.05** | **2.98** | **2.92** | 2.86 | 2.80 | 2.74 | 2.69 | 2.63 | 2.58 | 2.36 | 1.99 | 1.24 | |

Notes: The proportions in the table are calculated with the estimates of the KST distribution for the entire sample period; see tables 1 and 2. That is, $x_1$ =42.256, $a_1$ =1.467, $x_2$ =27.752, $a_2$ =1.066, and $p$ =0.622. Bold Values are the cases that investors are risk averse for gains and losses and $v_2$-$v_1$ is positive as in Ramark 1.

**Table 4 The Values of $v_2$ for Given Sets of $v_1$ and Investment Proportion in Equity for the Entire Sample Period (January 1980-February 2001) in the UK Market with the KST Distribution**

**A. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =2.25**

| $v_1 \backslash q$ | 0.50 | 0.60 | 0.70 | **0.75** | **0.80** | **0.85** | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.172 | 0.175 | 0.178 | **0.179** | **0.181** | **0.182** | 0.183 | 0.185 | 0.186 |
| 0.3 | 0.377 | 0.381 | 0.384 | **0.386** | **0.388** | **0.389** | 0.391 | 0.392 | 0.394 |
| 0.5 | 0.586 | 0.590 | 0.594 | **0.596** | **0.598** | **0.600** | 0.601 | 0.603 | 0.605 |
| 0.7 | 0.797 | 0.802 | 0.807 | **0.809** | **0.811** | **0.813** | 0.815 | 0.817 | 0.819 |
| 0.9 | 1.010 | 1.016 | 1.022 | **1.025** | **1.027** | **1.030** | 1.032 | 1.035 | 1.037 |
| 1.1 | 1.226 | 1.233 | 1.240 | **1.243** | **1.246** | **1.249** | 1.252 | 1.255 | 1.258 |
| 1.3 | 1.443 | 1.451 | 1.459 | **1.463** | **1.467** | **1.471** | 1.474 | 1.478 | 1.481 |
| 1.5 | 1.662 | 1.672 | 1.681 | **1.686** | **1.690** | **1.694** | 1.699 | 1.703 | 1.707 |
| 1.7 | 1.883 | 1.894 | 1.905 | **1.910** | **1.916** | **1.921** | 1.926 | 1.931 | 1.936 |
| 1.9 | 2.105 | 2.119 | 2.131 | **2.137** | **2.143** | **2.149** | 2.155 | 2.161 | 2.167 |
| 2.0 | 2.217 | 2.231 | 2.245 | **2.251** | **2.258** | **2.264** | 2.271 | 2.277 | 2.283 |

**B. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =1.5**

| $v_1 \backslash q$ | 0.50 | 0.60 | 0.70 | **0.75** | **0.80** | **0.85** | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.078 | 0.077 | 0.076 | **0.076** | **0.075** | **0.075** | 0.075 | 0.074 | 0.074 |
| 0.3 | 0.278 | 0.277 | 0.276 | **0.276** | **0.275** | **0.275** | 0.275 | 0.274 | 0.274 |
| 0.5 | 0.482 | 0.481 | 0.480 | **0.480** | **0.479** | **0.479** | 0.478 | 0.478 | 0.478 |
| 0.7 | 0.688 | 0.688 | 0.687 | **0.687** | **0.686** | **0.686** | 0.686 | 0.686 | 0.685 |
| 0.9 | 0.897 | 0.897 | 0.897 | **0.897** | **0.897** | **0.897** | 0.897 | 0.896 | 0.896 |
| 1.1 | 1.108 | 1.109 | 1.109 | **1.109** | **1.110** | **1.110** | 1.110 | 1.110 | 1.110 |
| 1.3 | 1.322 | 1.323 | 1.324 | **1.325** | **1.325** | **1.326** | 1.326 | 1.327 | 1.327 |
| 1.5 | 1.537 | 1.539 | 1.541 | **1.542** | **1.543** | **1.544** | 1.545 | 1.546 | 1.547 |
| 1.7 | 1.754 | 1.757 | 1.760 | **1.762** | **1.763** | **1.765** | 1.766 | 1.768 | 1.769 |
| 1.9 | 1.973 | 1.977 | 1.982 | **1.984** | **1.986** | **1.988** | 1.990 | 1.992 | 1.994 |
| 2.0 | 2.083 | 2.088 | 2.093 | **2.095** | **2.098** | **2.100** | 2.102 | 2.105 | 2.107 |

**C. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =3**

| $v_1 \backslash q$ | 0.50 | 0.60 | 0.70 | **0.75** | **0.80** | **0.85** | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.240 | 0.247 | 0.252 | **0.255** | **0.258** | **0.261** | 0.263 | 0.266 | 0.268 |
| 0.3 | 0.449 | 0.457 | 0.463 | **0.466** | **0.469** | **0.472** | 0.475 | 0.478 | 0.481 |
| 0.5 | 0.661 | 0.669 | 0.677 | **0.681** | **0.684** | **0.688** | 0.691 | 0.694 | 0.698 |
| 0.7 | 0.876 | 0.885 | 0.894 | **0.898** | **0.902** | **0.906** | 0.910 | 0.913 | 0.917 |
| 0.9 | 1.092 | 1.103 | 1.113 | **1.117** | **1.122** | **1.127** | 1.131 | 1.135 | 1.140 |
| 1.1 | 1.310 | 1.322 | 1.334 | **1.339** | **1.345** | **1.350** | 1.355 | 1.360 | 1.365 |
| 1.3 | 1.531 | 1.544 | 1.557 | **1.563** | **1.569** | **1.576** | 1.581 | 1.587 | 1.593 |
| 1.5 | 1.752 | 1.768 | 1.783 | **1.790** | **1.797** | **1.804** | 1.810 | 1.817 | 1.824 |
| 1.7 | 1.976 | 1.993 | 2.010 | **2.018** | **2.026** | **2.034** | 2.041 | 2.049 | 2.057 |
| 1.9 | 2.201 | 2.221 | 2.239 | **2.248** | **2.257** | **2.266** | 2.275 | 2.284 | 2.292 |
| 2.0 | 2.314 | 2.335 | 2.355 | **2.364** | **2.374** | **2.383** | 2.393 | 2.402 | 2.411 |

Notes: The values of $v_2$ are calculated with the estimates of the KST distribution for the entire sample period; see tables 1 and 2. That is, $x_1$ =42.256, $a_1$ =1.467, $x_2$ =27.752, $a_2$ =1.066, and $p$ =0.622.

**Table 5 The Values of $v_2$ for Given Sets of $v_1$ and Investment Proportion in Equity for the Entire Sample Period (January 1980-February 2001) in the US Market with the KST Distribution**

**A. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =2.25**

| $v_1 \backslash q$ | 0.30 | **0.40** | **0.45** | **0.50** | 0.55 | 0.60 | 0.65 | 0.70 | 0.80 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.172 | **0.177** | **0.179** | **0.181** | 0.183 | 0.184 | 0.186 | 0.188 | 0.191 | 0.194 | 0.196 |
| 0.3 | 0.373 | **0.378** | **0.380** | **0.383** | 0.384 | 0.386 | 0.388 | 0.390 | 0.393 | 0.396 | 0.399 |
| 0.5 | 0.576 | **0.581** | **0.583** | **0.585** | 0.587 | 0.589 | 0.591 | 0.593 | 0.597 | 0.600 | 0.603 |
| 0.7 | 0.778 | **0.784** | **0.786** | **0.789** | 0.791 | 0.793 | 0.795 | 0.797 | 0.801 | 0.805 | 0.809 |
| 0.9 | 0.982 | **0.988** | **0.991** | **0.993** | 0.996 | 0.998 | 1.000 | 1.003 | 1.007 | 1.011 | 1.015 |
| 1.1 | 1.186 | **1.193** | **1.196** | **1.199** | 1.201 | 1.204 | 1.206 | 1.209 | 1.213 | 1.218 | 1.222 |
| 1.3 | 1.391 | **1.398** | **1.401** | **1.404** | 1.407 | 1.410 | 1.413 | 1.416 | 1.421 | 1.426 | 1.430 |
| 1.5 | 1.596 | **1.604** | **1.608** | **1.611** | 1.614 | 1.617 | 1.620 | 1.623 | 1.629 | 1.634 | 1.640 |
| 1.7 | 1.802 | **1.811** | **1.814** | **1.818** | 1.822 | 1.825 | 1.828 | 1.832 | 1.838 | 1.844 | 1.850 |
| 1.9 | 2.009 | **2.018** | **2.022** | **2.026** | 2.030 | 2.033 | 2.037 | 2.040 | 2.047 | 2.054 | 2.060 |
| 2.0 | 2.112 | **2.121** | **2.126** | **2.130** | 2.134 | 2.138 | 2.141 | 2.145 | 2.152 | 2.159 | 2.166 |

**B. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =1.5**

| $v_1 \backslash q$ | 0.30 | **0.40** | **0.45** | **0.50** | 0.55 | 0.60 | 0.65 | 0.70 | 0.80 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.090 | **0.089** | **0.089** | **0.088** | 0.088 | 0.088 | 0.088 | 0.088 | 0.087 | 0.087 | 0.086 |
| 0.3 | 0.287 | **0.286** | **0.286** | **0.286** | 0.285 | 0.285 | 0.285 | 0.284 | 0.284 | 0.283 | 0.283 |
| 0.5 | 0.486 | **0.485** | **0.485** | **0.484** | 0.484 | 0.483 | 0.483 | 0.483 | 0.482 | 0.481 | 0.481 |
| 0.7 | 0.686 | **0.685** | **0.684** | **0.684** | 0.684 | 0.683 | 0.683 | 0.682 | 0.682 | 0.681 | 0.680 |
| 0.9 | 0.887 | **0.886** | **0.885** | **0.885** | 0.884 | 0.884 | 0.884 | 0.883 | 0.883 | 0.882 | 0.881 |
| 1.1 | 1.088 | **1.087** | **1.087** | **1.087** | 1.086 | 1.086 | 1.085 | 1.085 | 1.085 | 1.084 | 1.083 |
| 1.3 | 1.290 | **1.290** | **1.289** | **1.289** | 1.289 | 1.289 | 1.288 | 1.288 | 1.287 | 1.287 | 1.286 |
| 1.5 | 1.493 | **1.493** | **1.493** | **1.492** | 1.492 | 1.492 | 1.492 | 1.492 | 1.491 | 1.491 | 1.491 |
| 1.7 | 1.697 | **1.697** | **1.697** | **1.696** | 1.696 | 1.696 | 1.696 | 1.696 | 1.696 | 1.696 | 1.696 |
| 1.9 | 1.901 | **1.901** | **1.901** | **1.901** | 1.901 | 1.901 | 1.901 | 1.901 | 1.901 | 1.901 | 1.901 |
| 2.0 | 2.003 | **2.003** | **2.004** | **2.004** | 2.004 | 2.004 | 2.004 | 2.004 | 2.004 | 2.004 | 2.005 |

**C. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =3**

| $v_1 \backslash q$ | 0.30 | **0.40** | **0.45** | **0.50** | 0.55 | 0.60 | 0.65 | 0.70 | 0.80 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.232 | **0.240** | **0.244** | **0.248** | 0.251 | 0.254 | 0.257 | 0.260 | 0.266 | 0.271 | 0.277 |
| 0.3 | 0.436 | **0.445** | **0.449** | **0.453** | 0.456 | 0.460 | 0.463 | 0.466 | 0.472 | 0.478 | 0.484 |
| 0.5 | 0.640 | **0.650** | **0.654** | **0.658** | 0.662 | 0.666 | 0.669 | 0.673 | 0.680 | 0.686 | 0.692 |
| 0.7 | 0.845 | **0.855** | **0.860** | **0.864** | 0.869 | 0.873 | 0.877 | 0.881 | 0.888 | 0.895 | 0.902 |
| 0.9 | 1.051 | **1.062** | **1.067** | **1.072** | 1.076 | 1.081 | 1.085 | 1.089 | 1.097 | 1.104 | 1.112 |
| 1.1 | 1.257 | **1.269** | **1.274** | **1.279** | 1.284 | 1.289 | 1.294 | 1.298 | 1.307 | 1.315 | 1.323 |
| 1.3 | 1.463 | **1.476** | **1.482** | **1.487** | 1.493 | 1.498 | 1.503 | 1.508 | 1.517 | 1.526 | 1.535 |
| 1.5 | 1.670 | **1.684** | **1.690** | **1.696** | 1.702 | 1.708 | 1.713 | 1.718 | 1.728 | 1.738 | 1.748 |
| 1.7 | 1.878 | **1.892** | **1.899** | **1.906** | 1.912 | 1.918 | 1.923 | 1.929 | 1.940 | 1.951 | 1.961 |
| 1.9 | 2.086 | **2.101** | **2.108** | **2.115** | 2.122 | 2.128 | 2.135 | 2.141 | 2.153 | 2.164 | 2.175 |
| 2.0 | 2.190 | **2.206** | **2.213** | **2.220** | 2.227 | 2.234 | 2.240 | 2.247 | 2.259 | 2.271 | 2.283 |

Notes: The values of $v_2$ are calculated with the estimates of the KST distribution for the entire sample period; see tables 1 and 2. That is, $x_1$ =42.256, $a_1$ =1.467, $x_2$ =27.752, $a_2$ =1.066, and $p$ =0.622.

## Table 6 Proportion in Equity for Given Sets of $v_1$ and $v_2$ for the Entire Sample Period for the UK Market with the Normal Distribution

**A. The Proportion in Equity (q) for the Entire Sample Period (January 1980-February 2001) with $l$ =2.25**

| $v_2 \backslash v_1$ | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|
| 0.1 | | | | | |
| 0.3 | 34.72 | | | | |
| 0.5 | 40.71 | 37.20 | | | |
| 0.7 | 40.57 | 38.72 | 38.86 | | |
| 0.9 | 39.05 | 37.37 | 36.78 | 39.46 | |
| 1.1 | **37.22** | **35.59** | **34.64** | **34.82** | **39.09** |
| 1.3 | **35.43** | **33.84** | **32.75** | **32.26** | **32.85** |
| 1.5 | **33.77** | **32.25** | **31.11** | **30.36** | **30.15** |
| 1.7 | **32.27** | **30.82** | **29.69** | **28.85** | **28.31** |
| 1.9 | **30.93** | **29.56** | **28.45** | **27.58** | **26.94** |
| 2.0 | **30.32** | **28.98** | **27.89** | **27.02** | **26.36** |

**B. The Proportion in Equity (q) for the Entire Sample Period (January 1980-February 2001) with $l$ =1.5**

| $v_2 \backslash v_1$ | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|
| 0.1 | | | | | |
| 0.3 | 263.67 | | | | |
| 0.5 | 112.18 | 282.48 | | | |
| 0.7 | 79.74 | 106.69 | 295.08 | | |
| 0.9 | 64.82 | 73.45 | 101.35 | 299.60 | |
| 1.1 | **55.83** | **59.08** | **68.10** | **95.98** | **296.73** |
| 1.3 | **49.67** | **50.76** | **54.37** | **63.41** | **90.66** |
| 1.5 | **45.11** | **45.21** | **46.67** | **50.41** | **59.26** |
| 1.7 | **41.58** | **41.17** | **41.62** | **43.27** | **47.03** |
| 1.9 | **38.75** | **38.08** | **38.01** | **38.67** | **40.41** |
| 2.0 | **37.53** | **36.78** | **36.55** | **36.91** | **38.10** |

**C. The Proportion in Equity (q) for the Entire Sample Period (January 1980-February 2001) with $l$ =3**

| $v_2 \backslash v_1$ | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|
| 0.1 | | | | | |
| 0.3 | 8.24 | | | | |
| 0.5 | 19.83 | 8.83 | | | |
| 0.7 | 25.12 | 18.86 | 9.22 | | |
| 0.9 | 27.25 | 23.14 | 17.92 | 9.37 | |
| 1.1 | **27.92** | **24.84** | **21.45** | **16.97** | **9.28** |
| 1.3 | **27.88** | **25.38** | **22.86** | **19.97** | **16.03** |
| 1.5 | **27.50** | **25.37** | **23.33** | **21.19** | **18.66** |
| 1.7 | **26.96** | **25.10** | **23.36** | **21.64** | **19.77** |
| 1.9 | **26.36** | **24.69** | **23.17** | **21.70** | **20.21** |
| 2.0 | **26.06** | **24.47** | **23.03** | **21.66** | **20.29** |

Notes: The proportions in the table are calculated with the estimates of the normal distribution for the entire sample period; see table 1 for the parameter values. Bold Values are the cases that investors are risk averse for gains and losses and losses and $v_2$-$v_1$ is positive as in Proposition 2.

## Table 7 The Values of $v_2$ for Given Sets of $v_1$ and Investment Proportion in Equity for the Entire Sample Period (January 1980-February 2001) in the UK Market with the Normal Distribution

**A. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =2.25**

| $v_1$\q | 0.50 | 0.60 | 0.70 | **0.75** | **0.80** | **0.85** | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.121 | 0.122 | 0.123 | **0.123** | **0.123** | **0.124** | 0.124 | 0.124 | 0.125 |
| 0.3 | 0.311 | 0.311 | 0.312 | **0.312** | **0.312** | **0.312** | 0.312 | 0.312 | 0.313 |
| 0.5 | 0.502 | 0.502 | 0.502 | **0.502** | **0.502** | **0.502** | 0.502 | 0.502 | 0.502 |
| 0.7 | 0.694 | 0.694 | 0.694 | **0.694** | **0.693** | **0.693** | 0.693 | 0.693 | 0.693 |
| 0.9 | 0.888 | 0.888 | 0.887 | **0.887** | **0.887** | **0.887** | 0.887 | 0.886 | 0.886 |
| 1.1 | 1.084 | 1.084 | 1.083 | **1.083** | **1.082** | **1.082** | 1.082 | 1.081 | 1.081 |
| 1.3 | 1.282 | 1.281 | 1.280 | **1.280** | **1.279** | **1.279** | 1.279 | 1.278 | 1.278 |
| 1.5 | 1.480 | 1.479 | 1.478 | **1.478** | **1.477** | **1.477** | 1.477 | 1.476 | 1.476 |
| 1.7 | 1.679 | 1.678 | 1.677 | **1.677** | **1.676** | **1.676** | 1.676 | 1.675 | 1.675 |
| 1.9 | 1.879 | 1.878 | 1.877 | **1.877** | **1.876** | **1.876** | 1.875 | 1.875 | 1.874 |
| 2.0 | 1.979 | 1.978 | 1.977 | **1.977** | **1.976** | **1.976** | 1.975 | 1.975 | 1.975 |

**B. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =1.5**

| $v_1$\q | 0.50 | 0.60 | 0.70 | **0.75** | **0.80** | **0.85** | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.038 | 0.036 | 0.033 | **0.032** | **0.031** | **0.030** | 0.029 | 0.029 | 0.028 |
| 0.3 | 0.224 | 0.221 | 0.218 | **0.217** | **0.216** | **0.215** | 0.213 | 0.212 | 0.211 |
| 0.5 | 0.412 | 0.408 | 0.405 | **0.403** | **0.401** | **0.400** | 0.399 | 0.397 | 0.396 |
| 0.7 | 0.601 | 0.597 | 0.593 | **0.591** | **0.589** | **0.587** | 0.586 | 0.584 | 0.582 |
| 0.9 | 0.792 | 0.787 | 0.783 | **0.781** | **0.779** | **0.777** | 0.775 | 0.773 | 0.771 |
| 1.1 | 0.985 | 0.980 | 0.975 | **0.973** | **0.971** | **0.969** | 0.966 | 0.964 | 0.962 |
| 1.3 | 1.180 | 1.175 | 1.169 | **1.167** | **1.164** | **1.162** | 1.160 | 1.158 | 1.155 |
| 1.5 | 1.376 | 1.370 | 1.365 | **1.362** | **1.360** | **1.357** | 1.355 | 1.352 | 1.350 |
| 1.7 | 1.574 | 1.568 | 1.562 | **1.559** | **1.556** | **1.554** | 1.551 | 1.549 | 1.546 |
| 1.9 | 1.772 | 1.765 | 1.759 | **1.757** | **1.754** | **1.751** | 1.748 | 1.746 | 1.743 |
| 2.0 | 1.871 | 1.865 | 1.859 | **1.856** | **1.853** | **1.850** | 1.847 | 1.845 | 1.842 |

**C. The Estimated Values of $v_2$ for Given Sets of $v_1$ and q with $l$ =3**

| $v_1$\q | 0.50 | 0.60 | 0.70 | **0.75** | **0.80** | **0.85** | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.181 | 0.184 | 0.187 | **0.189** | **0.190** | **0.191** | 0.192 | 0.194 | 0.195 |
| 0.3 | 0.373 | 0.376 | 0.379 | **0.380** | **0.381** | **0.383** | 0.384 | 0.385 | 0.386 |
| 0.5 | 0.566 | 0.569 | 0.572 | **0.573** | **0.574** | **0.575** | 0.577 | 0.578 | 0.579 |
| 0.7 | 0.761 | 0.764 | 0.766 | **0.768** | **0.769** | **0.770** | 0.771 | 0.772 | 0.773 |
| 0.9 | 0.958 | 0.960 | 0.963 | **0.964** | **0.965** | **0.966** | 0.967 | 0.968 | 0.969 |
| 1.1 | 1.155 | 1.158 | 1.160 | **1.162** | **1.163** | **1.164** | 1.165 | 1.166 | 1.167 |
| 1.3 | 1.354 | 1.357 | 1.359 | **1.361** | **1.362** | **1.363** | 1.364 | 1.365 | 1.366 |
| 1.5 | 1.554 | 1.557 | 1.559 | **1.561** | **1.562** | **1.563** | 1.564 | 1.565 | 1.566 |
| 1.7 | 1.755 | 1.758 | 1.760 | **1.761** | **1.763** | **1.764** | 1.765 | 1.766 | 1.767 |
| 1.9 | 1.956 | 1.959 | 1.961 | **1.963** | **1.964** | **1.965** | 1.966 | 1.967 | 1.969 |
| 2.0 | 2.056 | 2.059 | 2.062 | **2.063** | **2.065** | **2.066** | 2.067 | 2.068 | 2.069 |

Notes: The proportions in the table are calculated with the estimates of the normal distribution for the entire sample period; see table 1 for the parameter values.