On the Evolution of Global Style Factors in the MSCI Universe of Assets

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Abstract

We explore the effects of global style factors on the Morgan Stanley Capital International universe of stocks from 1988 to 1998. An initial Bayesian analysis by Hall, Hwang and Satchell (2001) shows very low posterior probabilities for the significance of the (constant) beta coefficients in a linear factor model. When we allow for a flexible structure, in which betas follow conditionally autoregressive discrete time random processes, introduced by Christodoulakis and Satchell (2000), this result is reversed in nearly half of the cases. Style factor betas are often found to fluctuate significantly around zero, exhibiting serial correlation, persistence and thus predictability. We report results for such style factors as Value, Growth, Debt and Size on all the individual stocks of the MSCI universe as well as on capitalization-weighted and equally-weighted aggregate sector returns.

Keywords: MSCI Universe, Random Beta, Risk Premia, Style Factors

1 Introduction[1]

Linear factor models constitute the most popular class of models, amongst both academics and practitioners, for the description of dependencies and dynamics of financial asset returns. Their origin goes back to the seminal work of Sharp(1964), Lintner (1965) and Black (1972) who jointly created the Capital Asset Pricing Model (CAPM), and Ross (1976) who introduced the Arbitrage Pricing Theory (APT).

Since then, a large number of extensions have been proposed in the literature to accommodate the observed properties of data, see Christodoulakis and Satchell (2000), and several studies perform a comparative pricing analysis, see for example Kan and Wang (2000), Hwang and Satchell (2000), Wang (2000) and references therein. The importance of these models originates from the fact that expected returns on risky assets constitute an indispensable input for converting financial decisions into optimization problems. For example optimal asset allocation in a mean-variance world (Markowitz (1952)), forms a constrained quadratic optimization problem the optimal solution of which is conditioned on the assets' expected returns and covariance matrix. Also, some recent approaches propose multicriteria methods for portfolio selection, see for example Zopounidis (1993), Zopounidis et al (1995) and Hurson and Ricci-Xella (2000), the latter link a Multicriteria Decision Aid to APT, thus conditioning the optimality of their solution on assets' expected returns.

The range and nature of factors varies in different contexts. A particular class concerns the *style factors* which, broadly speaking, can be thought

[1]We would like to thank Peter Pope, Steve Taylor and Constantine Zopounidis for helpful comments, as well as participants of the 10$^{\text{th}}$ IFORS Special Conference in Banking Management, Athens, April, 2001, and finance seminar participants at the Lancaster University Management School and the City University Business School, UK.

of as the aggregate value associated with risks pertaining to a particular investment philosophy, based on associated information. Thus, for example, a *Value* factor could be interpreted as the risk term of an investment philosophy that buys (sells) cheap (dear) stocks, where "cheap" compares the book value of the company as contained in the company's accounts versus its market value as reflected in the stock price[2]. Our use of style factors in this paper is motivated by the fact that styles such as *Size*, *Value*, *Growth* and *Leverage* (debt financing within the corporate structure of the firm) turn out to be frequently used explanatory variables in stock selection. We are agnostic as to whether these variables actually refer to risks, to fads or to behavioural phenomena.

The factor loadings known as *beta coefficients*, quantify the exposure of an asset to the corresponding factor risk and determine its risk premia. The importance of beta is also enhanced because of its contribution into the conditional second moments of asset returns. Its interpretation as a measure of price risk and its central role for financial decision making has motivated a sequence of papers addressing its dynamic properties. Early studies detect *time variation* of beta coefficients, and Blume (1975) and Collins et al (1987) present theoretical and empirical arguments documenting a *regression tendency* towards its steady state. Also, a *co-movement* between the factor beta coefficient and the factor conditional variance has been found in studies such as Schwert and Seguin (1990), Koutmos et al (1994) and Episcopos (1996). A recent stream of papers, see Ferson and Harvey (1993), Ferson and Korajczyc (1995) as well as Bekaert and Harvey (1997) and Christopherson et al (1999) presents evidence for exact conditional *systematic variation* of factor betas

[2]The term *style* is also used to identify the management style, so if the returns of a managed fund are highly correlated with US S&P500 returns and with other index returns, proponents of management style claim that the fund has a US large-capitalization equity style. We shall not follow this approach.

in that they correlate to economic macro- and micro-structure variables.

Hall, Hwang and Satchell (2001), henceforth HHS, develop a Bayesian variable selection method in linear factor models and examine the significance of (constant beta) global style factors for the MSCI universe of assets. Their methodology, conditional on the sample data, estimates a posterior probability that a particular style variable should be included. Empirical results on such global styles as *Value*, *Growth*, *Debt* and *Size*, do not find compelling evidence for global styles as useful explanatory variables.

This paper is motivated by the non-significance result of HHS and the existing evidence on time-varying beta coefficients. In particular we apply a recent methodology by Christodoulakis & Satchell (2000), who propose a fully general framework in which assets and factors are jointly determined, betas follow conditionally autoregressive processes and the conditional covariance matrix of the system is guaranteed positive definite at each point in time. We apply this methodology on the HHS data set and find that the global style factor non-significance result is reversed in nearly half of the cases of the MSCI universe of assets. In particular, style beta coefficients are often found to significantly fluctuate around zero, exhibiting autocorrelation and persistence.

In section two we present our modelling approach and its basic properties, in section three we present the MSCI data set and discuss the style factors, their nature and construction. Section four contains our empirical results for the individual assets as well as sector aggregate data and in section five we present our conclusions.

2 AutoRegressive Conditional Beta Model

We follow the methodology and notation of Christodoulakis and Satchell (2000), henceforth CS (2000), who propose an unconstrained modelling of the covariance matrix originating from a multifactor model of asset returns. The model adopts a Cholesky decomposition of a covariance matrix proposed by Pourahmadi (1999a,b). This guarantees the positive definiteness of the covariance matrix at each time and allows for a meaningful statistical interpretation of its parameters in terms of time-varying conditional variances and conditional factor betas.

Let $\mathbf{y}_t$ be an $N \times 1$ vector of asset excess returns generated by the following process

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\mu}_{y,t} + \sum_{j=1}^{k} \boldsymbol{\beta}_{j,t}\ e_{j,t} + \boldsymbol{\varepsilon}_t \\ x_{j,t} &= \mu_{x_j,t} + e_{j,t} \quad \text{for} \quad j = 1, ..., k \end{aligned} \tag{1}$$

and

$$\begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \mathbf{e}_t \end{pmatrix} \mid I_{t-1} \sim D\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_{\varepsilon,t} & 0 \\ 0 & \Sigma_{e,t} \end{pmatrix} \right)$$

where $x_{j,t}, \boldsymbol{\beta}_{j,t}$ is the j-th factor and $N \times 1$ vector of conditional betas respectively, and $\boldsymbol{\mu}_{i,t}$, for $i = y, x_j, j = 1...k$, are $N \times 1$ vectors of conditional means. I_{t-1} denotes the information set available at time t. The covariance matrices $\Sigma_{\varepsilon,t}$, $\Sigma_{e,t}$ are diagonal with dimensions $N \times N$ and $K \times K$ respectively, but conditionally time-varying. In partitioned matrix notation the system writes

$$\begin{bmatrix} \underset{(N\times 1)}{\boldsymbol{\varepsilon}_t} \\ \underset{(K\times 1)}{\mathbf{e}_t} \end{bmatrix} = \begin{bmatrix} \underset{(N\times N)}{I_N} & \underset{(N\times K)}{-B_t} \\ \underset{(K\times N)}{0} & \underset{(K\times K)}{I_K} \end{bmatrix} \begin{bmatrix} \underset{(N\times 1)}{\mathbf{y}_t - \boldsymbol{\mu}_{y,t}} \\ \underset{(K\times 1)}{\mathbf{x}_t - \boldsymbol{\mu}_{x,t}} \end{bmatrix} \tag{2}$$

where $\mathbf{x}_t$ is the $K \times 1$ vector of common factors and B_t the $N \times K$ matrix of factor beta coefficients and I_N, I_K are identity matrices. Provided that B_t is conditionally known, the conditional covariance structure corresponding to equation (2) is

$$\begin{bmatrix} \Sigma_{\varepsilon,t} & 0 \\ 0 & \Sigma_{e,t} \end{bmatrix} = \begin{bmatrix} I_N & -B_t \\ 0 & I_K \end{bmatrix} \begin{bmatrix} \Omega_{yy,t} & \Omega_{yx,t} \\ \Omega_{xy,t} & \Omega_{xx,t} \end{bmatrix} \begin{bmatrix} I_N & -B_t \\ 0 & I_K \end{bmatrix}' \qquad (3)$$

where Ω_{yy} and Ω_{xx} are the asset excess return and factor covariance matrix respectively and $\Omega_{yx} = \Omega_{xy}^{'}$ include the covariances between the N assets and K factors. This is the Cholesky decomposition of Ω, the joint covariance matrix of N assets and K factors, in terms of the diagonal matrix Σ with positive elements as the conditional variances of asset idiosyncratic and factor shocks and the matrix M, the off-diagonal block of which corresponds to minus the factor beta coefficients $(-B_t)$, see Pourahmadi (1999a,b). The matrices Σ_ε, Σ_e and B_t are allowed to be conditionally time-varying and known. Solving with respect to $\boldsymbol{\Omega}$, its north-west block represents the asset return covariance matrix

$$\Omega_{yy,t} = \Sigma_{\varepsilon,t} + B_t \Sigma_{e,t} B_t^{'} \qquad (4)$$

which is decomposed into its idiosyncratic variance $\Sigma_{\varepsilon,t}$ plus a time-varying combination of the factor conditional variances $B_t \Sigma_{e,t} B_t^{'}$. The latter will appear as a common component but with different time-varying combinations, in all asset variances and covariances.

We now design the elements of (4) as functions of the available information set I_{t-1}. It is worth noting that (4) is guaranteed positive definite provided that the elements of $\Sigma_{\varepsilon,t}$, $\Sigma_{e,t}$ are non negative, which leaves the modelling of B_t free of restrictions. A natural candidate for modelling conditional variances could be a member of the ARCH family of processes, originated by Engle (1982) and further developed by numerous other researchers, see Bera and Higgins (1993) for an excellent survey. The choice of the form of the ARCH-type process should accommodate the empirical properties of the data, such as time dependences, non normality and asymmetries.

Following CS (2000) we allow factor beta coefficients to evolve as Auto Regressive Conditional Beta processes of order p (ARCBeta (p)), of the form

$$\begin{aligned} \beta_{ij,t} &= E\left(\frac{\varepsilon^*_{i,t} e_{j,t}}{\sigma^2_{e_j,t}} \mid I_{t-1}\right) = \alpha_{ij,0} + \alpha_{ij,1}\xi_{ij,t-1} + \cdots + \alpha_{ij,p}\xi_{ij,t-p} \\ \text{for } t &= 0, \pm 1, ... \end{aligned} \quad (5)$$

where

$$\begin{aligned} \xi_{ij,t} &= \frac{\varepsilon^*_{i,t} e_{j,t}}{\sigma^2_{e_j,t}} \\ \varepsilon^*_{i,t} &= y_{i,t} - E\left(y_{i,t} \mid I_{t-1}\right) = \sum_{j=1}^{k} \beta_{ij,t}\ e_{j,t} + \varepsilon_{i,t} \end{aligned}$$

for asset $i = 1, ..., N$ and factor $j = 1, ..., k$. By independence, the conditional beta of asset i on factor j is effectively shocked by squared innovations on factor j only so that $\xi_{ij,t} = \beta_{ij,t}\left(\frac{e_{j,t}}{\sigma_{e_{j,t}}}\right)^2$. Thus, if the j-th factor innovation $\frac{e_{j,t}}{\sigma_{e_{j,t}}} = v_{j,t} \overset{iid}{\sim} D(0,1)$ we have that $E\left(\xi_{ij,t} | I_{t-1}\right) = \beta_{ij,t}$, making equation (5) be an unrestricted ARCH-like process in the spirit of Braun et al (1995). Under stability of the process, a high order lag structure for ARCBeta(p) can be parsimoniously represented as an ARCBeta (k,l) process. CS (2000) prove results on the stationarity of the process as well as conditions for the existence of its steady-state first and second moments.

Further, the i-th diagonal element of (4) is given by

$$\sigma^2_{i,t} = \sum_{j=1}^{k} \beta^2_{ij,t}\sigma^2_{e_j,t} + \sigma^2_{\varepsilon_i,t}$$

Because of the product of random processes $\beta^2_{ij,t}\sigma^2_{e_j,t}$, stationarity of the individual processes is not sufficient to guarantee stationarity of $\sigma^2_{i,t}$. CS (2000) prove easily checkable sufficient conditions for the existence of a stationary solution for products of such random processes of any order, and provide closed-form expressions for their steady-state. They also prove closed form expressions for the steady state covariance between the factor beta and factor variance, as well as between betas on the same factor.

3 MSCI Universe and Style Factors

Our data set is identical to the one used by HHS (2001) thus making our results directly comparable. Also, the discussion of the data set draws heavily from that article. This is the Morgan Stanley Capital International (MSCI) universe, comprised of 1523 stocks. We shall examine the period of October 1988 to September 1998 with monthly frequency observations which results in time series of 120 data points. During this period there are 1154 stocks with full data, thus our data matrix of equity returns is 120×1154.

The MSCI universe we use is drawn from twenty one countries and nine sectors, it is therefore useful to think of it in terms of country-sector grids of a 21×9 matrix. Because of significant differences in the relative frequency of stocks within each grid, where some exhibit very small frequency, some countries are pooled into greater geographical groups: Canada, France, Germany, Japan, UK, US, the Other Europe group (Belgium, Denmark, Finland, Ireland, Italy, Netherlands, Norway, Spain, Sweden, and Switzerland), the Australasia group (Australia, New Zealand) and the Asia group (Hong Kong, Malaysia, Singapore). Also the nine sectors are regrouped to six: Basic Industries, Capital Goods, Consumer Goods, Energy, Financial, and the Other group (Resources, Transport, Utilities and Other Sectors). Thus our new country-sector grid matrix is 9×6 dimensional.

An inspection of the data uncovers substantial differences in both the value and the number of equities among countries and sectors. This arises naturally for a number of reasons. It is therefore useful to consider value-weighted returns versus equally weighted returns. A natural weighting scheme would be to consider, at each point in time, the value of the i-th stock relative to the value of the group of stocks within its country-sector grid. In particular

$$w_{i,t}^{k,l} = \frac{S_{i,t}^{k,l}}{\sum_{i=1}^{N^{k,l}} S_{i,t}^{k,l}}$$

where k, l denotes the k-th country - l-th sector grid, $N^{k,l}$ the number of stock in the k, l country-sector grid, $S_{i,t}^{k,l}$ the US dollar market value of equity i in the k, l country-sector grid and $\sum_{i=1}^{N^{k,l}} w_{i,t}^{k,l} = 1$ for all k, l.

3.1 Style Factor Mimicking Portfolios

The true style factors are typically latent variables and thus we need to approximate them. The most sophisticated approach, used by market specialists, is to construct portfolios of assets that mimic the style factors themselves or their equilibrium risk premiums. These are called *factor mimicking portfolios* (FMPs) and in our context their returns are designed to be highly correlated with the (unobservable) factor values.

Examples of FMPs are portfolios constructed from eigenvectors in principal component analysis. FMPs are a useful and increasingly common tool in building linear factor models and often take the form of a hedge portfolio. By construction pricing theory applies to it, so it can replace an observable or prespecified factor on which pricing theory may not apply. The theory of factor mimicking portfolios is discussed in Huberman et al (1987), Lehman and Modest (1988) and Connor and Linton (2000).

In constructing FMPs, for each factor f the entire MSCI universe is ranked according to an attribute of f. As in HHS (2001), we use style attributes for *Value* (VL_t), *Growth* (GR_t), *Debt* (DE_t) and *Size* (SZ_t) defined as

$$\begin{aligned} VL_t &= \frac{DP_t + EP_t + BP_t + CP_t}{5} \\ GR_t &= \frac{RE_t + EG_t}{3} \\ DE_t &= \frac{\textit{Total Debt per Share}}{\textit{Book Value per Share}} \\ SZ_t &= \ln\left(\textit{Share Price}_t \times \textit{Share Number}_t\right) \end{aligned}$$

where

$$\begin{aligned} DP_t &= \frac{\textit{Divident}_t}{\textit{Share Price}_t} & EP_t &= \frac{\textit{Earnings per Share}_t}{\textit{Share Price}_t} \\ CP_t &= \frac{\textit{Cash Flow per Share}_t}{\textit{Share Price}_t} & RE_t &= \frac{\textit{Earnings per Share}_t}{\textit{Book Value per Share}_{t-12}} \\ BP_t &= \frac{\textit{Book Value per Share}_t}{\textit{Share Price}_t} & EG_t &= \ln \frac{\textit{Book Value per Share}_t}{\textit{Book Value per Share}_{t-24}} \end{aligned}$$

For each attribute f_a, an equally weighted hedge portfolio is then constructed which is long the top n-tile and short the bottom n-tile of the MSCI universe ranked by f_a. The resulting hedge portfolio is the factor mimicking portfolio of factor f. A better diversification is produced for small n, thus our data set is constructed for $n = 3$.

4 Empirical Results

We aim to examine the evolution of style factor betas over time using the ARCBeta framework. As shown in section 2, the randomness of the factor may also be the source of randomness for the beta coefficient in that they are both shocked by the square factor innovation process. It is therefore important to pay attention to the adequate modelling of the first and second moments of the random style factors. This will then provide a minimal information set to condition the evolution of beta coefficients.

For the period under examination, the MSCI universe consists of 1154 stock returns, which we wish to expose to four global style factors. This requires to perform 1154×4 Maximum Likelihood estimation procedures using numerical optimization. The size of the problem prevents the application of a time series model selection procedure to identify the appropriate form for the conditional mean, beta and variance process for each individual asset. Although such a procedure would be appropriate for the exhaustive modelling of each asset returns, in the context of the present paper it is unnecessary. Our aim is to distinguish between time-varying and constant factor betas in each MSCI sector and compare with the HHS (2001) non-significance result.

Thus we perform our empirical analysis assuming that each asset y_i and style factor x_j returns are generated from the following process

$$y_{i,t} = \mu_{y_i} + \beta_{ij,t}\, e_{j,t} + \varepsilon_{i,t} + \theta_{y_i}\varepsilon_{i,t-1} \tag{6}$$

$$x_{j,t} = \mu_{x_j} + e_{j,t} + \theta_{x_j} e_{j,t-1} \tag{7}$$

where

$$\begin{array}{ll} \varepsilon_{i,t} = z_{i,t}\sigma_{\varepsilon_i} & e_{j,t} = v_{j,t}\sigma_{e_j,t} \\ z_{i,t} \overset{iid}{\sim} N(0,1) & v_{j,t} \overset{iid}{\sim} N(0,1) \end{array}$$

and

$$\beta_{ij,t} = b_{ij,0} + b_{ij,1}\beta_{i,j,t-1}v_{j,t-1}^2 \tag{8}$$

$i = 1, ..., 1154$, $j =$ *Value, Growth, Debt, Size* and $z_{i,t}$, $v_{j,t}$ are uncorrelated for all i,j. The j style factor conditional variance, $\sigma_{e_j,t}^2$, is allowed to be generated by a process from the ARCH family.

The intuition behind this model is that dynamic betas, $\beta_{ij,t}$, are driven by past squared scaled factor innovations $v_{j,t-k}^2$. Suppose that *Size* experiences an extreme innovation. Then investment managers value stocks more as *Size* than previously and the stocks' responsiveness to the *Size* factor changes.

Whilst this story is a little loose, it does suggest a linkage between shocks to factors and changes to exposures. Assuming that factor volatility, $\sigma_{e_j,t}$, can be generated by an ARCH-like process which is also driven by past squared innovations, $\beta_{ij,t}$ is expected to co-move with $\sigma^2_{e_j,t}$. Thus, clusters of high factor volatility can be associated with clusters of high responses to the factor shocks, which reflects time-varying risk premia, see section 5.3 of CS (2000).

Prior to performing Maximum Likelihood estimation for the full data set, we model the factors as univariate processes. After appropriate model selection, we find that *Value* is well represented by an MA(1)-GARCH(1,1) process, *Growth* follows an MA(1)-EGARCH(1,1) process, while *Debt* and *Size* are found to have constant means and ARCH(1) conditional variances. We then use the factor maximum likelihood parameter estimates as starting values, to facilitate convergence in the joint asset-factor estimation.

Under gaussian innovation processes, the conditional log likelihood function can be written as

$$\begin{aligned} \ln L_{ij} &= -\frac{T(N+K)}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\left|\begin{matrix}\sigma^2_{\varepsilon_i} & 0\\ 0 & \sigma^2_{e_j,t}\end{matrix}\right| \\ &\quad -\frac{1}{2}\sum_{t=1}^{T}\begin{pmatrix}\varepsilon_{i,t}\\ e_{j,t}\end{pmatrix}'\begin{pmatrix}\sigma^2_{\varepsilon_i} & 0\\ 0 & \sigma^2_{e_j,t}\end{pmatrix}^{-1}\begin{pmatrix}\varepsilon_{i,t}\\ e_{j,t}\end{pmatrix}\end{aligned}$$

where $T = 120$, $N = K = 1$. We wish to find the unknown parameters of the asset and factor conditional means, variance and ARCBeta process that maximize the log likelihood function. Equations (6), (7) and (8) and the structure of GARCH-type processes suggest that the log likelihood function is highly non linear and a closed form solution is not available. Thus, we use the BFGS algorithm to perform numerical optimization of the likelihood function. Other algorithms such as the Newton-Raphson can be as appropriate although substantially less fast.

It might be thought that the assumption of gaussian innovations may well be unrealistic given the evidence on excess kurtosis in stock return data. however this is present in data of higher frequency than monthly. Moreover, gaussian innovations do not imply that returns are unconditionally gaussian. Indeed, they follow a complex distribution based on sums and products of normals which are capable of exhibiting some excess kurtosis, if indeed it is present.

Further, we rely on the arguments of Bollerslev and Wooldridge (1992) who prove that the maximum likelihood estimator based on a (falsely assumed) gaussian distribution, will still be consistent and asymptotically normal, provided that the first two condition al moments are correctly specified.

We classify our empirical results on individual stocks according to the six sectors presented in section 3. Table I presents the relative frequency for three cases regarding factor betas: constant beta, zero-mean ARCBeta(1) which corresponds to time-varying beta with zero steady-state and ARC-Beta(1) which corresponds to time-varying beta with non-zero steady-state. We report results for all assets in each sector and all style factors in each subtable. We will not discuss all the results but discussion of a couple of cases should aid the reader in interpreting the tables.

Consulting Table I we can see the number of times we have significant coefficients for equation (8). In particular the second and third rows of each suitable indicate the extent of time-varying exposure. Thus, Basic Industries have a time-varying exposure to *Debt* forty per cent of the times on average $\left(\frac{1}{2}\left(.57+.23\right)\right)$; Financials have a time-varying exposure to *Size*. Generally, constant non-zero betas seem more common for *Size* and *Growth* that for *Value* and *Debt*, indicating that the former have more persistent effects that the latter. *Value* has always the largest frequency in the second row, suggesting that value exposures are dynamic but regress around zero on average. The last result can be compared with the results of HHS (2001), who find no evidence of non-zero value exposures in a fixed parameter context.

Repeating the analysis, tables II to V present results on both capitalization-weighted and equally-weighted aggregate sector returns and we see similar patterns as before. *Value* has a significant slope b_1, but an insignificant intercept b_0 in virtually all cases for the cap-weighted returns. *Growth* never has a significant slope b_1, but often has a significant intercept b_0 (except for Consumer Goods and Financials). *Debt* has an occasional significant slope (Basic and Other Industries) whilst *Size* exhibits plenty of significant intercepts but only for Financials with cap-weighted returns a significant slope. Taking these results together, we see some evidence that *Value* has average zero exposure but is significantly time varying around zero, as is *Debt* to a lesser extent. *Growth* and *Size* are both significant on average in some sectors but typically not time-varying.

5 Conclusions

Our paper has described and implemented a factor-based model of stock returns, proposed by Christodoulakis and Satchell (2000), which allows for stochastic common factors and dynamic beta coefficients. The model is applied to the problem of modelling global styles in the MSCI universe of assets from 1988 to 1998; we find evidence that extends existing results in new directions. In particular, we find evidence that *Value* and *Debt* are styles that "come and go", signifying time-varying risk premia, but that on average do not appear significant; the results suggest the opposite characterization for *Growth* and *Size*. Of course our results are likely to be sensitive to the time period as well as the universe of assets chosen. Nevertheless this model has a potential as a forecasting method for factor betas, asset second conditional moments as well as providing a detailed statistical description of the data.

6 Appendix

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Table I : Frequencies of statistically Significant Parameters, Eq. (8)

Basic Industries

	value	growth	debt	size
Constant ($b_0 \neq 0, b_1 = 0$)	8/44 (.18)	17/44 (.39)	10/44 (.23)	21/44 (.47)
Zero-mean ARCBeta(1) ($b_0 = 0, b_1 \neq 0$)	15/44 (.34)	5/44 (.11)	25/44 (.57)	9/44 (.20)
ARCBeta(1) ($b_0 \neq 0, b_1 \neq 0$)	3/44 (.16)	4/44 (.09)	10/44 (.23)	6/44 (.13)

Capital Goods

	value	growth	debt	size
Constant ($b_0 \neq 0, b_1 = 0$)	47/387 (.12)	178/387 (.46)	82/387 (.21)	197/387 (.51)
Zero-mean ARCBeta(1) ($b_0 = 0, b_1 \neq 0$)	150/387 (.38)	90/387 (.23)	171/387 (.44)	99/387 (.25)
ARCBeta(1) ($b_0 \neq 0, b_1 \neq 0$)	35/387 (.09)	65/387 (.17)	44/387 (.11)	67/387 (.17)

Consumer Goods

	value	growth	debt	size
Constant ($b_0 \neq 0, b_1 = 0$)	54/277 (.19)	94/277 (.34)	49/277 (.18)	122/277 (.44)
Zero-mean ARCBeta(1) ($b_0 = 0, b_1 \neq 0$)	133/277 (.48)	64/277 (.23)	52/277 (.19)	88/277 (.32)
ARCBeta(1) ($b_0 \neq 0, b_1 \neq 0$)	50/277 (.18)	35/277 (.13)	14/277 (.05)	50/277 (.18)

Energy

	value	growth	debt	size
Constant ($b_0 \neq 0, b_1 = 0$)	4/47 (.085)	12/47 (.25)	2/47 (.04)	17/47 (.36)
Zero-mean ARCBeta(1) ($b_0 = 0, b_1 \neq 0$)	19/47 (.40)	5/47 (.10)	9/47 (.19)	12/47 (.25)
ARCBeta(1) ($b_0 \neq 0, b_1 \neq 0$)	2/47 (.042)	2/47 (.04)	0/47 (0)	8/47 (.17)

Financials

	value	growth	debt	size
Constant ($b_0 \neq 0, b_1 = 0$)	29/193 (.15)	54/193 (.28)	46/193 (.24)	106/193 (.55)
Zero-mean ARCBeta(1) ($b_0 = 0, b_1 \neq 0$)	101/193 (.52)	45/193 (.23)	42/193 (.22)	88/193 (.45)
ARCBeta(1) ($b_0 \neq 0, b_1 \neq 0$)	20/193 (.10)	26/193 (.13)	23/193 (.12)	64/193 (.33)

Other Sectors

	value	growth	debt	size
Constant ($b_0 \neq 0, b_1 = 0$)	21/206 (.10)	86/206 (.42)	37/206 (.18)	94/206 (.46)
Zero-mean ARCBeta(1) ($b_0 = 0, b_1 \neq 0$)	94/206 (.45)	56/206 (.27)	38/206 (.18)	53/206 (.26)
ARCBeta(1) ($b_0 \neq 0, b_1 \neq 0$)	17/206 (.08)	35/206 (.17)	13/206 (.06)	38/206 (.18)

Notes to Table I: we present the number of statistically significant parameters in each sector/factor grid for equation (8) $\beta_{ij,t} = b_{ij,0} + b_{ij,1}\beta_{i,j,t-1}v_{j,t-1}^2$. It is given as a ratio with respect to the number of stocks in the corresponding sector and as a percentage in brackets.

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Table II : Sector Aggregate Results: fmp 1 (value)

Basic. Ind / value	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	2.43	.81	2.30	.98
ma(1)	.13	1.14	.14	1.68
var	.80	5.20	.78	5.99
b_0	-.07	-.12	.05	.16
b_1	.40	.35	-.33	-.23

Cap. Goods/value	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.92	2.16	.90	1.2
ma(1)	.12	.60	-.07	-.72
var	27.58	5.50	58.24	6.33
b_0	-.32	-1.38	.08	.29
b_1	.35	2.94	-.75	1.31

Cons.Goods/value	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.3	3.8	.87	2.23
ma(1)	.07	.36	-.04	-.24
var	16.02	6.18	15.99	5.79
b_0	-.42	-1.78	-.20	-1.08
b_1	.27	2.56	.32	3.19

Energy Sect/value	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.17	3.33	.82	1.84
ma(1)	.09	.63	-.03	-.21
var	18.35	7.35	22.99	5.98
b_0	.21	.99	.20	.79
b_1	-.35	-2.51	-.29	-1.75

Financials / value	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.92	1.70	.93	2.05
ma(1)	-.07	-.76	-.14	-1.42
var	30.81	5.72	18.96	5.46
b_0	.64	2.41	.35	2.29
b_1	-.31	-3.66	-.38	-4.44

Other Sect /value	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.73	1.86	.67	1.52
ma(1)	.03	.27	-.11	-.82
var	20.47	6.12	18.11	6.09
b_0	.40	1.75	.32	1.48
b_1	-.32	-2.75	-.31	-2.65

Notes to Table II: we estimate the system of equations (6), (7) and (8) for cap-weighted and equally weighted aggregate sector returns and for all style factors. Mean and variance parameters for the style factor are not reported since they do not vary between asset categories. We thus report estimates for equations (6) and (8).

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Table III : Sector Aggregate Results: fmp 2 (growth)

Basic. Ind/growth	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.11	.18	0.18	0.42
ma(1)	-.13	-1.08	.22	1.07
var	36	6.6	25	5.65
b_0	1.5	2.32	1.48	2.83
b_1	0.1	0.90	0.08	0.7

Cap. Goods/growth	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.94	2.5	.73	1.64
ma(1)	-.21	-1.10	.04	.31
var	27	5.57	21	5.52
b_0	.95	1.82	1.39	3.0
b_1	.04	.67	0.1	1.06

Cons.Goods/growth	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.33	4.4	.89	2.43
ma(1)	-.20	-1.01	-.02	-.08
var	17	6.43	16	5.53
b_0	.04	.14	.53	1.38
b_1	.16	.86	.14	.84

Energy Sect/value	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.16	3.9	.82	2.1
ma(1)	-.21	-1.5	-.03	-.16
var	18	6.9	21	5.11
b_0	.88	2.5	1.16	2.37
b_1	-.05	-.83	-.01	-.05

Financials/growth	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.97	1.8	.96	2.28
ma(1)	-.02	.20	.18	-.24
var	33	5.9	20	5.24
b_0	.44	.66	.60	1.32
b_1	.19	1.4	.09	.80

Other Sect/growth	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.78	2.1	.70	1.9
ma(1)	-.10	-.76	-.01	-.14
var	20	6.11	17	5.3
b_0	.99	2.34	1.16	2.8
b_1	.16	1.8	.11	1.2

Notes to Table III: we estimate the system of equations (6), (7) and (8) for cap-weighted and equally weighted aggregate sector returns and for all style factors. Mean and variance parameters for the style factor are not reported since they do not vary between asset categories. We thus report estimates for equations (6) and (8).

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Table IV : Sector Aggregate Results: fmp 3 (debt)

Basic. Ind / debt	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.089	.05	.17	.29
ma(1)	-.13	-.86	.14	1.1
var	37	6.19	27	6.5
b_0	-.49	-1.36	-.27	-1.1
b_1	.32	2.43	.39	1.94

Cap. Goods/debt	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.98	2.3	.75	1.55
ma(1)	-.15	-.83	.07	.47
var	28	5.81	23	6.3
b_0	.20	.74	.12	.64
b_1	-.16	-1.4	-.21	-1.31

Cons.Goods/debt	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.34	4.4	.87	2.4
ma(1)	-.19	-.93	-.03	-.20
var	17	6.5	16	5.8
b_0	.03	.52	-.001	-.29
b_1	-.24	-1.4	-.97	-2.29

Energy Sect/debt	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.05	3.2	.77	1.8
ma(1)	-.17	-1.1	-.006	-.057
var	18	7.1	23.1	5.98
b_0	-.10	-.65	-.12	-.55
b_1	.36	.71	.11	.17

Financials / debt	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	.86	1.07	.90	2.11
ma(1)	-.06	-.54	.006	.18
var	33	6.10	20	5.56
b_0	-.025	-.55	-.008	-.31
b_1	-.75	-1.86	-.87	-1.20

Other Sect/growth	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	parameter	t-statistic
c	.71	1.92	.67	1.62
ma(1)	-.13	-.86	.008	.18
var	21	6.17	19	6.21
b_0	-.002	-.43	-.001	-.23
b_1	-.99	-2.8	-.94	-1.08

Notes to Table IV: we estimate the system of equations (6), (7) and (8) for cap-weighted and equally weighted aggregate sector returns and for all style factors. Mean and variance parameters for the style factor are not reported since they do not vary between asset categories. We thus report estimates for equations (6) and (8).

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Table V : Sector Aggregate Results: fmp 4 (size)

Basic. Ind / size	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	parameter	t-statistic
c	0.27	.50	0.21	0.59
ma(1)	.003	.03	.18	1.71
var	29	6.67	25	6.51
b_0	-0.6	-5.45	-0.28	-2.85
b_1	.04	.63	.08	.90

Cap. Goods/size	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.12	2.88	.79	1.75
ma(1)	-.09	-.72	.12	1.21
var	21	5.85	20	6.34
b_0	-0.5	-4.68	-.39	-4.31
b_1	.02	0.5	.07	.88

Cons.Goods/size	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.38	4.71	.89	2.52
ma(1)	-.17	-1.24	-.007	-.23
var	14	6.10	15	5.61
b_0	-.31	-4.48	-.25	-3.60
b_1	.08	1.6	.12	1.5

Energy Sect/size	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.19	3.16	.82	1.86
ma(1)	-.15	-.91	-.006	-.15
var	18.6	7.35	23	6.0
b_0	-.042	-.48	-.02	-.75
b_1	.07	.07	.85	2.49

Financials / size	Cap.	Weighted	Equally	Weighted
parameter	estimate	t-statistic	estimate	t-statistic
c	1.02	2.36	.87	2.15
ma(1)	-.093	-1.43	.001	.06
var	25	4.58	18.82	5.40
b_0	-.56	-5.3	-.15	-1.68
b_1	.10	3.16	.43	1.33

Other Sect /size	Cap.	Weighted	Equally	Weighted
	estimate	t-statistic	estimate	t-statistic
c	.86	2.47	.65	1.63
ma(1)	-.08	-.76	.003	.17
var	18	6.92	18	6.14
b_0	.37	.99	05	.55
b_1	.035	.76	.77	2.84

Notes to Table V: we estimate the system of equations (6), (7) and (8) for cap-weighted and equally weighted aggregate sector returns and for all style factors. Mean and variance parameters for the style factor are not reported since they do not vary between asset categories. We thus report estimates for equations (6) and (8).

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