Sharpe Style Analysis in the MSCI Sector Portfolios : A Monte Carlo Integration Approach

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Abstract

We examine a decision-theoretic Bayesian framework for the estimation of Sharpe Style portfolio weights of the MSCI sector returns. Following van Dijk and Kloek (1980) an appropriately defined prior density of style weights can incorporate non-negativity and other constraints. We use factor-mimicking portfolios as proxies to global style factors such as Value, Growth, Debt and Size. Our computational approach is based on Monte Carlo Integration (MCI) of Kloek and van Dijk (1978) for the estimation of the posterior moments and distribution of portfolio weights. MCI provides a number of advantages, such as a flexible choice of prior distributions, improved numerical accuracy of the estimated parameters, the use of inequality restrictions in prior distributions and exact inference procedures. Our empirical findings suggest that, contrary to existing evidence, style factors do explain the MSCI sector portfolio returns for the particular sample period. Further, non-negativity constraints on portfolio weights were found to be binding in all cases.

1. Introduction

The Style Analysis introduced by Sharpe (1988, 1992) is probably the most popular portfolio performance attribution methodology. It is based on the simple idea that asset returns can be attributed to the returns of investment management style factors such as *value* and *size*. In its original form, relevant style factors should form a (non-hedge) portfolio which replicates the returns of the asset under assessment, thus style factor coefficients should be positive and sum to unity.

Given time series data of asset and style factor returns, style analysis forms a constrained linear regression problem without intercept. The least squares estimation of the style portfolio weights –the regression coefficients- under linear *equality* constraints, is a typical quadratic programming problem with closed-form solution and known distribution for the estimator, thus it has become a standard practice. When linear *inequality* constraints are imposed to ensure non-negative portfolio weights, it is not possible to obtain a closed–form solution, thus Judge and Takayama (1966) proposed a modified simplex algorithm for an iterative solution of the inequality-constrained quadratic program. In univariate regression, the style coefficient estimator has a truncated normal distribution if the regression error is normally distributed. However, when there are more than two independent variables, it can be very difficult to obtain the desired sampling distributions using standard methods. One could at most assess the superiority or inferiority of the solution vs. the maximum likelihood estimator using the results of Judge and Yancey (1986).

In this paper we adopt a Bayesian perspective to formally impose the inequality parameter restrictions, in the form of a prior probability density of the model parameters. The latter is then combined with the sampling information as captured by the likelihood function to provide the joint posterior density function of the model parameters. For a normal linear model, the posterior density is a function of a multivariate *t*, thus making the analytical calculation of functions of the parameters difficult. We use Monte Carlo

Integration (MCI) as proposed by Kloek and van Dijk (1978) and van Dijk and Kloek (1980) and further studied by Geweke (1986). This methodology is sufficiently general, allowing the computation of the posterior distribution of arbitrary functions of the parameters of interest and enables exact inference procedures that is impossible to treat in a sampling-theoretic approach. We apply this methodology on monthly MSCI country and sector returns and such style factor mimicking portfolios as *value*, *growth*, *debt* and *size*, from 1988 until 1998.

The structure of the paper is as follows. In the next section we develop our Bayesian MCI methodological framework for style analysis under both equality and inequality constraints. Section three is devoted to the analysis and interpretation of our empirical results on monthly MSCI data. We conclude and provide thoughts on future research in section four.

2. Methodology

Following the seminal work of Sharpe (1988, 1992) our portfolio returns Y can be attributed to a number of style factors X such that

$$\begin{aligned} & Y = Xb + U \\ & \text{s.t.} \\ & \mathbf{1}'b = 1 \text{ and } b \geq 0 \end{aligned} \qquad (1)$$

where Y is a vector of T observations of portfolio returns, X a matrix of T observations for K style factor returns, b a vector of K style factor betas, $\mathbf{1}$ is a vector of units and $U \sim N(0, s^2 I)$. The least squares estimation of b in the above model is a constrained quadratic program. The solution under equality constraints is available in closed-form and its distributional properties known. When inequality constraints are imposed in addition, the solution requires iterative optimization, see Judge and Takayama (1966), but the distributional properties of the estimator are not known. Davis (1978) provides a solution for the latter problem which requires that one knows which constraints are binding, an implausible assumption for Sharpe style analysis. One solution to that problem is to view the style regression from a Bayesian perspective and impose the parameter restrictions in the form of information encapsulated in the prior distribution. Then, using the posterior distribution one can estimates moments and other functions of the style parameters by means of Monte Carlo Integration.

2.1 A Bayesian Decision-Theoretic Approach

Implementing the Bayesian-Monte Carlo Integration approach, we first impose the equality constraint by restating model (1) in deviation form from the k-th style return

$$\begin{aligned} & Y^{*} = X^{*} b^{*} + U^{*} \\ & \text{s.t.} \\ & \mathbf{1'}\, b^{*} \leq 1 \text{ and } b \geq 0 \end{aligned} \tag{2}$$

where t-th elements of the new variables is $y_t^{*} = y_t - x_{k,t}$ and $x_{i,t}^{*} = x_{i,t} - x_{k,t}$, where i = 1,…,K-1 is the i-th column of X. Now b^{*} is a vector of K-1 elements and the K-th beta can be obtained from $1 - \mathbf{1'}\, b^{*}$. In our standard Bayesian framework b^{*} is formally treated as a random variable in population and all elements of X^{*} are independent of each other and of U, b^{*} and s^{2}. Then, by Bayes law the posterior density of b^{*} and s^{2} is given by

$$\text{Posterior } \left(b^{*}, s^{2} \middle| Y^{*}, X^{*}\right) = \text{Likelihood } \left(b^{*}, s^{2} \middle| Y^{*}, X^{*}\right) \times \text{ Prior } \left(b^{*}, s^{2}\right)$$

which is the product of the likelihood function and the prior density. Following van Dijk and Kloek (1980) our prior is composed of an improper uninformative component regarding s^{2} and an informative one regarding b^{*}, which for style analysis it captures our prior knowledge $\mathbf{1'}\, b^{*} \leq 1$ and $b^{*} \geq 0$. By independence

$$\text{Prior } \left(b^{*}, s^{2}\right) = s^{-1}\, q\left(b^{*}\right) \tag{3}$$

where

$$q\left(b^{*}\right) = \begin{cases} 1 & \text{if } \mathbf{1'}\, b^{*} \leq 1 \text{ and } b^{*} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Under multivariate normality for U, it can be shown that the likelihood function is proportional to

$$L\left(b^{*}, s \left| Y^{*} X^{*}\right.\right) \propto s^{-T} \exp\left\{-\frac{1}{2s^{2}}\left[v\hat{s}^{2} + (b^{*} - b)' X'^{*} X^{*} (b^{*} - b)\right]\right\}$$

where $v\hat{s}^{2} = (Y^{*} - X^{*}b)'(Y^{*} - X^{*}b)$, $b = (X'^{*} X^{*})^{-1} X'^{*} Y^{*}$ is the OLS estimator and $v = T - K + 1$. Combining the likelihood and the prior density yields a joint posterior density function which is proportional to

$$\text{Posterior } \left(b^{*}, s \left| Y^{*}, X^{*}\right.\right) \propto s^{-(T+1)} \exp\left\{-\frac{1}{2s^{2}}\left[v\hat{s}^{2} + (b^{*} - b)' X'^{*} X^{*} (b^{*} - b)\right]\right\} \times q\left(b^{*}\right)$$

Standard analysis[1] to integrate s out yields the marginal posterior probability density function of vector b^{*}, which is recognized as a multivariate t density with mean zero, variance $\frac{l}{(l-2)\hat{s}^{2}} X'^{*} X^{*}$ and l degrees of freedom

$$\text{Posterior } \left(b^{*} \left| Y^{*}, X^{*}\right.\right) = c\left[l + \frac{(b^{*} - b)' X'^{*} X^{*} (b^{*} - b)}{\hat{s}^{2}}\right]^{-\frac{1}{2}(l+K-1)} \times q\left(b^{*}\right)$$

where (4)

$$c = \frac{l^{\frac{l}{2}} G\left[\frac{1}{2}\left(l + K - 1\right)\right]}{p^{\frac{K-1}{2}} G\left[\frac{l}{2}\right] \det\left(\hat{s}^{2} (X'^{*} X^{*})^{-1}\right)^{\frac{1}{2}}}$$

and $G\left(\,.\,\right)$ is the gamma function.

2.2 Estimation by Monte Carlo Integration

We shall follow the methodology proposed by Kloek and van Dijk (1978) and further studied by van Dijk and Kloek (1980). For any function $g\left(\,.\,\right)$, the point estimator of $g\left(b^{*}\right)$ is given by

[1] See Judge et al (1985)

$$E\left(g\left(b^{*}\right)\middle|Y^{*}X^{*}\right)=\frac{\int g\left(b^{*}\right)\text{Posterior}\left(b^{*}\middle|Y^{*}X^{*}\right)db^{*}}{\int \text{Posterior}\left(b^{*}\middle|Y^{*}X^{*}\right)db^{*}}$$

The numerical implementation of the above estimator using Monte Carlo procedures requires the specification of a density function $I\left(b^{*}\right)$ from which random draws of b^{*} will be drawn; this is called *importance function* and is a proxy to the posterior density with convenient Monte Carlo properties. We can then have

$$E\left(g\left(b^{*}\right)\middle|Y^{*}X^{*}\right)=\int\left(\frac{g\left(b^{*}\right)\text{Posterior}\left(b^{*}\middle|Y^{*}X^{*}\right)}{I\left(b^{*}\right)}\right)I\left(b^{*}\right)db^{*}$$

where the expectation is now taken over $I\left(b^{*}\right)$. Let $b_1^{*}, b_2^{*},..., b_N^{*}$ be a set of N random draws from $I\left(b^{*}\right)$, then we can prove that

$$\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\frac{g\left(b_i^{*}\right)\text{Posterior}\left(b_i^{*}\middle|Y^{*}X^{*}\right)}{I\left(b_i^{*}\right)}=E\left(g\left(b^{*}\right)\middle|Y^{*}X^{*}\right) \qquad (5)$$

apart from a normalizing constant which can be calculated separately. Since $I\left(b^{*}\right)$ is supposed to be a proxy to the posterior distribution, the standard Bayesian analysis of the normal linear model in section 2.1 suggests that we could choose the multivariate t density. In this case our MCI estimator will be reduced to

$$\frac{1}{N}\sum_{i=1}^{N}g\left(b_i^{*}\right)q\left(b_i^{*}\right) \qquad (6)$$

In our Monte Carlo procedure we generate multivariate t-distributed vectors b_i^{*} as follows. We first derive the Cholesky decomposition of the OLS estimator covariance matrix such that

$$AA' = \hat{s}^{2}(X'^{*}\, X^{*})^{-1}$$

and then generate a K-1 vector $\mathbf{z}_i$ of independent standard normal random variables. Then the ***i***-th replication of b_i^{*} will be

$$b_i^{*} = b + A\,\mathbf{z}_i$$

drawn from a (K-1)-variate normal density. This can be converted to a t-distributed draw, by generating a l vector $\boldsymbol{w}_i$ of independent standard normal variables and writing

$$b_i^* = b + A\boldsymbol{z}_i \left(\frac{l}{w_i' w_i} \right)^{\frac{1}{2}} \tag{7}$$

which is t-distributed with l degrees of freedom. Thus our parameter estimates can now be obtained using (5) and $g(b_i^*) = b_i^*$. Similarly we can obtain estimates of higher moments of b^* or any other functions of interest.

The Bayesian MCI approach offers exact inference which is discussed in van Dijk and Kloek (1980), Geweke (1986) and Kim et al (2000). In a different context, Lobosco and DiBartolomeo (1997) pointed out the problem of the lack of a precision measure for the style regression coefficients and proposed an approximate method based on Taylor expansions. However, the latter approach is valid only in the special case in which none of the true style coefficients are zero or one, thus excluding empirically relevant cases. Kim et al (2000) also apply the results of Andrews (1999) and develop a comparable Bayesian method to obtain statistically valid distributions and confidence intervals regardless of the true values of style weights.

3. Style Analysis in the MSCI Sector Portfolios[2]

We apply the Bayesian MCI approach to perform Sharpe Style analysis for capitalization- and equally-weighted portfolio returns, representing the sectors of Morgan Stanley Capital International universe from 1988 until 1998. Our data set is identical to the one used by Hall et al (2002) and Christodoulakis and Satchell (2002), thus making some direct comparisons possible. Briefly, our time series consist of 120 data points for 1154 stocks, thus our data matrix of equity returns is 120×1154. The MSCI universe we use is drawn from twenty one countries and nine sectors, where the nine sectors are regrouped to six: Basic Industries, Capital Goods, Consumer Goods, Energy, Financial, and the

[2] I would like to thank Steve Satchell and Soosung Hwang for providing the data set.

Other group (Resources, Transport, Utilities and Other Sectors). An inspection of the data uncovers substantial differences in the value and the number of equities in different sectors. This arises naturally for a number of reasons. It is therefore useful to consider value-weighted returns versus equally weighted returns. A natural value weighting scheme would be to consider, at each point in time, the value of the *i*-th stock relative to the value of the group of stocks within its sector. In particular

$$w_{i,t}^k = \frac{S_{i,t}^k}{\sum_{i=1}^{N^k} S_{i,t}^k}$$

where k denotes the k-th sector, N^k is the number of stocks in the k-th sector, $S_{i,t}^k$ is the US dollar market value of equity i in the k-th sector and $\sum_{i=1}^{N^k} S_{i,t}^k = 1$ for all k.

Since style factors are typically latent, we use *style factor mimicking portfolios* (FMPs) as a proxy. That is we construct portfolios of assets that mimic the style factors themselves in that their returns are designed to be highly correlated with the (unobservable) factor values or their equilibrium risk premiums. The theory of factor mimicking portfolios is discussed in Huberman et al (1987), Lehman and Modest (1988) and Connor and Linton (2000). In constructing FMPs, for each factor X_i the entire MSCI universe is ranked according to an attribute of X_i. As in Hall et al (2002) and Christodoulakis and Satchell (2002), we use style attributes for Value, Growth, Debt and Size defined using observable company data.[3] For each attribute, X_a, an equally weighted hedge portfolio is then constructed which is long the top n-tile and short the bottom n-tile of the MSCI universe ranked by X_a. The resulting hedge portfolio is the factor mimicking portfolio of factor X. A better diversification is produced for small n, thus our data set is constructed for $n = 3$. Some data providers construct style indices based on measures which attribute Growth, say, to non-Value stocks. In contrast to the latter approach we prefer a dual sort, thus recognizing stocks that are ‘growth at the right price’, i.e. cheap (Value) Growth stocks.

We have set the number of Monte Carlo replications equal to 10^6 and have used GAUSS language as our computational platform. In performing MCI we need to specify the importance function $I\left(b_i^*\right)$. A first candidate is the multivariate t distribution as dictated by standard Bayesian analysis of the normal regression model with an uninformative volatility prior. We specify its parameters by adopting the OLS estimators b and $\hat{s}^2 (X'^* X^*)^{-1}$ and experimenting with l . We found our results to be insensitive to the choice of l , so we set $l = 4$. We also found it was not necessary to multiply $\hat{s}^2 (X'^* X^*)^{-1}$ by any constant as van Dijk and Kloek (1980) mention in page 315. Our normalization constant in equation (5) is obtained by setting g = 1 in (5) and taking the inverse.

We present our empirical results for the six capitalization- and value-weighted MSCI sector portfolios in tables I to VI. For comparison reasons we also report OLS and equality-restricted OLS estimates. We observe that unrestricted OLS produces Value and Growth portfolio weights that violate both the positivity and equality constraints in all six sector portfolios. Equality-restricted OLS still violates the positivity but to a lesser extent, primarily for the Value factor.

-- Insert Tables I and II around here --

Inspecting our results from the Bayesian Monte Carlo Integration approach, we observe that this methodology always produces positive portfolio weights which sum to unity. Since the MCI results are based on the empirical posterior density of the b^* vector, it is flexible enough to produce estimates of more complicated functions than the mean. In particular, tables I to VI report estimates of standard errors, skewness and kurtosis coefficients as well as the Bera-Jarque normality statistic. It is evident that most of the beta coefficients are highly non-normal exhibiting positive skewness and in some cases excess kurtosis. There is only one case in which normality cannot be rejected, namely the

[3] For the explicit definition see Christodoulakis and Satchell (2002)

weight of *size* style factor for the capitalization-weighted energy sector portfolio. Also, a small number of style portfolio weights exhibit non-normality to a smaller extent compared to the majority of weights, e.g. the *value* factor weight for the capital goods sector and the *debt* factor for consumer goods and capital goods, due to platikurtosis.

-- Insert Tables III and IV around here --

The Bayesian MIC values of the *value* and *growth* factor weights for our six different portfolios range from 0.20 to 0.27 for both the capitalization- and equally-weighted portfolios. Similarly, the *size* style weight takes values from 0.08 to 0.31 whilst the *debt* factor takes values from 0.09 to 0.46. Thus, contrary to existing evidence presented in Hall et all (2002) our approach presents evidence that style factors do explain the return performance of the MSCI sector portfolios fairly uniformly for the period of 1988 to 1998.

-- Insert Tables V and VI around here --

Our exact inference procedure provides easily constructed confidence intervals for the point parameter estimates. The latter can take the form of a Bayesian Highest Posterior Density (HPD) interval (L,U) which, for a given confidence level 1-*a*, is given by the shortest interval over which the cumulative posterior probability equals 1-*a*. Following Kim et al (2000) the interval (L,U) is be given by $(0, b^*_{i,1-a})$ if posterior $(0|Y^*, X^*) >$ posterior $(b^*_{i,1-a}|Y^*, X^*)$ where $b^*_{i,1-a}$ is the value of factor weight at which the cumulative posterior probability equals $1-a$. Further, if posterior $(0|Y^*, X^*) =$ posterior $(b^*_{i,1-a}|Y^*, X^*)$ then the shortest interval (L,U) can be found numerically. We graph the empirical posterior distribution for the four style factor weights on the MSCI Energy sector[4]. An inspection uncovers clearly the effects of the non-negativity constraints which appear to be binding in all eight cases, thus truncating the posterior density of the beta coefficients. Note that the effect of the truncation is smaller for the *size* style factor which also deviated less from the normal distribution.

-- Insert Graphs around here --

4. Conclusions

We have presented a framework for Sharpe Style Analysis in the MSCI sector portfolios from 1988 to 1998. Following Kloek and van Dijk (1978) and van Dijk and Kloek (1980) we consider style portfolios from a Bayesian perspective and can formally incorporate non-negativity constraints for the beta coefficients through a appropriately specified prior density function. We can estimate any function of the parameters of interest using the Monte Carlo Integration method. The framework allows for exact inference procedures that have been further studied in an asymptotic framework by Kim et al (2000). Also, Andrews (1999) provides an asymptotically valid inference procedure for parameters on the boundary.

Contrary to existing studies, our empirical results provide evidence for a relatively uniform significance of style factors in determining the MSCI sector portfolio returns for the given sample period. We also observe that non-negativity constrains are strikingly binding in the majority of the cases, thus truncating the posterior distribution of beta coefficients. In a few cases beta coefficients can be well represented by normal densities.

Future research involves the development of a MCI methodology for betas and volatility that follow conditionally stochastic processes over time as in Christodoulakis and Satchell (2002). This approach would maintain the normality assumption in its conditional form whilst would allow for unconditional non-normality, see Geweke (1989) and Koop (1994) for similar work in the ARCH volatility framework.

[4] Because of space requirements we do not present distribution graphs for the remaining five portfolios.

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Table I. Basic Industries, 1988-1998

	OLS		OLS-Restricted		Bayesian Monte Carlo Integration				
	Beta	Std Error	Beta	Std Error	Beta	Std Error	Skew	Kurtosis	B-J Stat
b_v^c	-0.1458	0.0209	0.0494	0.2009	0.2574	0.1883	0.7477	2.9115	166.4
b_v^e	0.0305	0.0195	0.3359	0.1919	0.2786	0.2011	0.6977	2.7880	179.8
b_g^c	-1.0434	0.0426	-0.3779	0.2816	0.2244	0.1758	0.9509	3.4066	280.4
b_g^e	-1.2444	0.0397	-0.2033	0.2690	0.2150	0.1760	0.9883	3.4611	371.9
b_s^c	0.5222	0.0278	0.7256	0.2819	0.1310	0.1083	1.3381	5.0076	830.1
b_s^e	0.2955	0.0259	0.6137	0.2692	0.1054	0.0940	1.6101	6.1976	1859
b_d^c	0.5493	0.0008	0.6029	0.0865	0.3873	0.2228	0.1936	2.1110	69.73
b_d^e	0.1700	0.0078	0.2537	0.0826	0.4010	0.2318	0.1563	2.0657	87.64

Note: (c) capitalization-weighted, (e) equally-weighted, (v) value, (g) growth, (s) size, (d) debt, 10^6 replications

Table II. MSCI Capital Goods, 1988-1998

	OLS		OLS-Restricted		Bayesian Monte Carlo Integration				
	Beta	Std Error	Beta	Std Error	Beta	Std Error	Skew	Kurtosis	B-J Stat
b_v^c	-0.5714	0.0179	-0.2725	0.1770	0.2669	0.2034	0.6148	2.2711	19.83
b_v^e	-0.2260	0.0175	0.0917	0.1742	0.2360	0.1883	0.9584	3.4135	175.7
b_g^c	-0.6752	0.0364	0.3441	0.2481	0.2530	0.2030	0.9369	3.2782	34.83
b_g^e	-1.0714	0.0356	0.0118	0.2442	0.2365	0.1953	1.1146	3.8451	259.7
b_s^c	0.0624	0.0238	0.3739	0.2483	0.1012	0.1003	1.6857	6.2235	211.2
b_s^e	0.2000	0.0232	0.5311	0.2445	0.0926	0.0967	2.1739	9.0893	2558
b_d^c	0.4725	0.0072	0.5545	0.0762	0.3790	0.2413	0.3297	2.0854	12.34
b_d^e	0.2782	0.0070	0.3653	0.0750	0.4349	0.2454	0.0495	1.9605	49.83

Note: (c) capitalization-weighted, (e) equally-weighted, (v) value, (g) growth, (s) size, (d) debt, 10^6 replications

Table III. MSCI Consumer Goods, 1988-1998

	OLS		OLS-Restricted		Bayesian Monte Carlo Integration				
	Beta	Std Error	Beta	Std Error	Beta	Std Error	Skew	Kurtosis	B-J Stat
b_v^c	-0.6730	0.0150	-0.4514	0.1467	0.2251	0.1977	1.2277	4.2956	74.82
b_v^e	-0.3765	0.0153	-0.1382	0.1510	0.2030	0.1745	1.1098	3.7228	233.6
b_g^c	-0.0174	0.0305	0.7383	0.2057	0.2721	0.2101	0.7939	4.2956	24.68
b_g^e	-0.4242	0.0313	0.3884	0.2116	0.2460	0.1974	0.9097	3.1606	143.0
b_s^c	0.1481	0.0199	0.3790	0.2059	0.1024	0.1035	1.7552	6.4459	234.9
b_s^e	0.2492	0.0204	0.4975	0.2118	0.0860	0.0856	1.9238	8.0499	1728
b_d^c	0.2732	0.0060	0.3340	0.0632	0.4005	0.2477	0.1656	1.9433	11.90
b_d^e	0.1869	0.0061	0.2522	0.0650	0.4650	0.2402	-0.045	2.0654	37.81

Note: (c) capitalization-weighted, (e) equally-weighted, (v) value, (g) growth, (s) size, (d) debt, 10^6 replications

Table IV. MSCI Energy, 1988-1998

	OLS		OLS-Restricted		Bayesian Monte Carlo Integration				
	Beta	Std Error	Beta	Std Error	Beta	Std Error	Skew	Kurtosis	B-J Stat
b_v^c	-0.6238	0.0167	-0.2088	0.1733	0.2067	0.1637	1.0315	3.7633	692.30
b_v^e	-0.4408	0.0180	-0.0331	0.1844	0.2179	0.1693	0.9241	3.3913	504.06
b_g^c	-1.0485	0.0340	0.3666	0.2429	0.2144	0.1697	0.9453	3.3659	530.61
b_g^e	-1.3722	0.0367	0.0179	0.2585	0.2248	0.1776	0.9340	3.3608	511.26
b_s^c	0.2886	0.0222	0.7212	0.2431	0.3102	0.1339	-0.017	2.8026	5.7456
b_s^e	0.4928	0.0240	0.9177	0.2588	0.2577	0.1286	0.3057	3.1525	56.076
b_d^c	0.0072	0.0067	0.1210	0.0746	0.2687	0.1724	0.4473	2.4532	157.31
b_d^e	-0.0141	0.0072	0.0976	0.0794	0.2995	0.1880	0.4072	2.4255	140.30

Note: (c) capitalization-weighted, (e) equally-weighted, (v) value, (g) growth, (s) size, (d) debt, 10^6 replications

Table V. MSCI Financials, 1988-1998

	OLS		OLS-Restricted		Bayesian Monte Carlo Integration				
	Beta	Std Error	Beta	Std Error	Beta	Std Error	Skew	Kurtosis	B-J Stat
b_v^c	-0.6947	0.0198	-0.5659	0.1888	0.2137	0.1707	0.9687	3.4139	292.07
b_v^e	-0.4466	0.0175	-0.1557	0.1728	0.2083	0.1747	1.0972	3.7558	756.55
b_g^c	-0.2324	0.0403	0.2067	0.2646	0.2687	0.1952	0.7178	2.8463	155.14
b_g^e	-0.6233	0.0356	0.3686	0.2423	0.2477	0.1943	0.8932	3.1602	451.87
b_s^c	0.7166	0.0263	0.8508	0.2649	0.1851	0.1228	0.8937	4.3954	382.62
b_s^e	0.3305	0.0232	0.6337	0.2425	0.1058	0.0894	1.6719	7.1986	4046
b_d^c	0.4731	0.0079	0.5084	0.0813	0.3325	0.2084	0.3581	2.2777	77.002
b_d^e	0.0736	0.0070	0.1533	0.0744	0.4382	0.2362	-0.040	1.9649	151.40

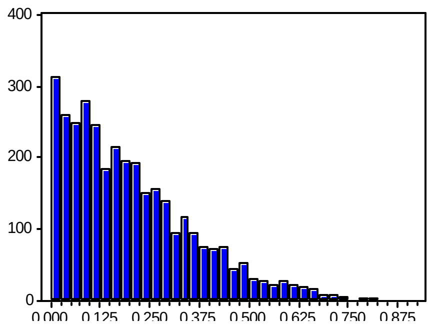
Note: (c) capitalization-weighted, (e) equally-weighted, (v) value, (g) growth, (s) size, (d) debt, 10^6 replications

Table VI. MSCI Other Sectors, 1988-1998

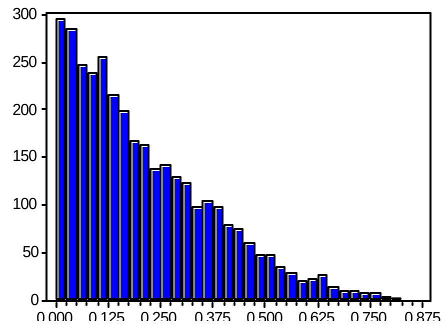
	OLS		OLS-Restricted		Bayesian Monte Carlo Integration				
	Beta	Std Error	Beta	Std Error	Beta	Std Error	Skew	Kurtosis	B-J Stat
b_v^c	-0.5608	0.0165	-0.2758	0.1634	0.2088	0.1709	1.0630	3.6624	294.62
b_v^e	-0.3842	0.0165	0.0130	0.1703	0.2316	0.1797	0.9523	3.3994	625.03
b_g^c	-0.7548	0.0335	0.2172	0.2290	0.2389	0.1895	0.8549	2.9717	173.73
b_g^e	-1.1676	0.0336	0.1867	0.2387	0.2301	0.1846	0.9415	3.2998	599.99
b_s^c	0.3311	0.0219	0.6282	0.2293	0.1039	0.0907	1.5919	6.4776	1320.8
b_s^e	0.2687	0.0219	0.6826	0.2390	0.1231	0.0905	1.2536	5.3875	1978.1
b_d^c	0.3521	0.0066	0.4303	0.0703	0.4485	0.2332	-0.049	2.0008	59.915
b_d^e	0.0088	0.0066	0.1177	0.0733	0.4151	0.2247	0.0342	2.0392	153.13

Note: (c) capitalization-weighted, (e) equally-weighted, (v) value, (g) growth, (s) size, (d) debt, 10^6 replications

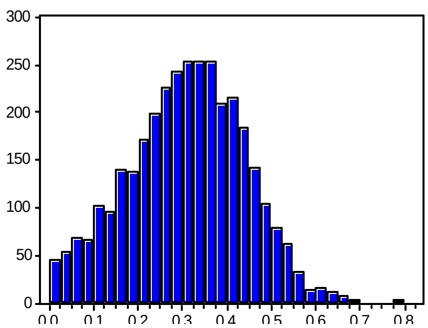
Posterior Distribution of Value Style Factor Weight
(MSCI Energy, Cap-Weighted Returns)



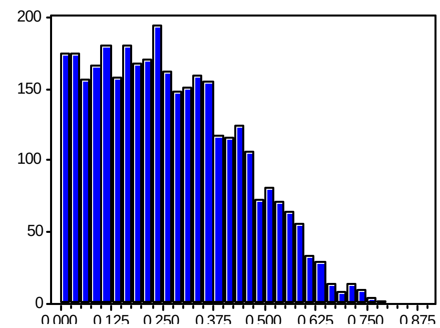
Posterior Distribution of Growth Style Factor Weight
(MSCI Energy, Cap-Weighted Returns)



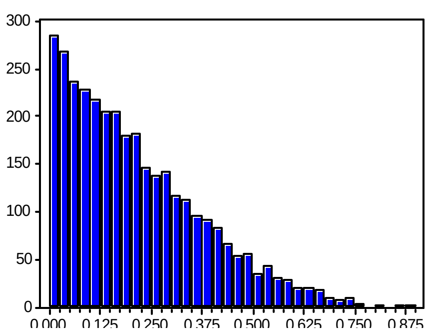
Posterior Distribution of Size Style Factor Weight
(MSCI Energy, Cap-Weighted Returns)



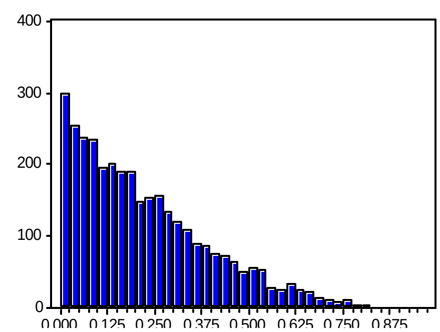
Posterior Distribution of Debt Style Factor Weight
(MSCI Energy, Cap-Weighted Returns)



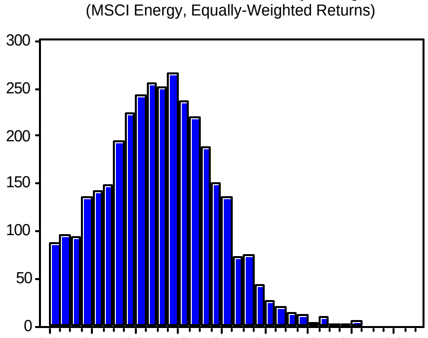
Posterior Distribution of Value Style Weight
(MSCI Energy, Equally-Weighted Returns)



Posterior Distribution of Growth Style Weight
(MSCI Energy, Equally-Weighted Returns)



Posterior Distribution of Size Style Weight
(MSCI Energy, Equally-Weighted Returns)



Posterior Distribution of Debt Style Weight
(MSCI Energy, Equally-Weighted Returns)